Universitat Autònoma de Barcelona

Modeling and forecasting firm's asset and equity volatility
Essays on Volatility in Credit Risk Modeling

Francisco González Pla

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Autor: Francisco González Pla

Supervisor: Dr. Lidija Lovreta

Tutor: Dr. Maria Antònia Tarrazón Rodón

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Introduction

Volatility of firm’s assets is one of the fundamental variables in credit risk modeling and refers to a degree of fluctuation of firm's asset returns. While equity volatility and its time-series properties have been extensively studied in the finance literature, little is known about the time-series behavior of volatility of firm's assets. The main reason for such a gap in the literature lies in the unobservability of the underlying value of the firm’s assets. For that reason, many practical applications applied in credit risk modeling are based on the use of stock return volatility as a proxy variable of asset volatility. However, recent studies point out in the direction that equity volatility has different time-series properties when compared to asset volatility. Thus, the main goal of this research is to study the time-series properties of firm’s asset volatility, the eventual differences it presents with respect to equity volatility, and to provide an in-depth understanding of its most relevant features such as asymmetry and long-range persistence in the context of volatility modeling and forecasting.

This research is structured as follows: First, we examine the persistence properties (long memory) of firm’s asset volatility and its relationship with equity volatility. Second, we determine which model or models, considering the discrete-time family of GARCH (Generalized Auto-Regressive Conditional Heteroskedasticicty) models are the best to estimate and forecast conditional asset volatility. We analyze in detail the implications of asymmetry and long-range persistence on modeling and forecasting firm's asset volatility. Third, we provide a practical application by estimating a CDS implied firm’s asset volatility and by analyzing its time-series properties. Throughout this research we use as a baseline sample a sample of 52 non-financial iTraxx Europe companies during the 2004-2016 period, and estimate underlying firm's asset values using different, commonly applied procedures.

In the first Chapter we study the persistence properties of firm's asset and equity volatility. We estimate the degree of persistence on a firm-specific basis using the FIGARCH model and find strong evidence of long-memory in the conditional variance of both firm's asset and equity returns. The estimated degree of persistence of firm's asset and equity volatility is lower than 0.5 for the vast majority of companies considered. We find the persistence of equity volatility to be slightly higher than the persistence of firm's asset volatility. However, this difference is not statistically significant. Our findings show that the persistence of both firm's asset and equity volatility is positively related to leverage and negatively related to relative idiosyncratic volatility. A DFA analysis of absolute returns confirms the long-memory behavior of both volatility series.

In the second Chapter we analyze the relevance of asymmetry and long-memory in modeling and forecasting firm-level volatility. The degree of asymmetric effect seems to be more pronounced for equity than for firm's asset volatility, and is decreasing with financial leverage. The degree of long-memory is in general slightly higher for equity than for firm's asset volatility, and this difference is decreasing with leverage. However, once the asymmetry is allowed in the model in addition to long-memory, firm's asset volatility turns out to be more persistent than equity volatility for higher leverage groups. A horse race among different GARCH-type model specifications (GARCH, EGARCH, IGARCH, FIGARCH, HYGARCH, FIEGARCH, and FIAPARCH) in forecasting firm's asset and equity volatility, shows that more sophisticated (FIEGARCH and FIAPARCH) models outperform other specifications in out-of-sample firm-level volatility forecasting. In turn, the simplest GARCH and IGARCH models show the worst performance.

In the third Chapter we analyze time-series properties of the unobserved volatility of asset returns implied in market observable CDS spreads (i.e. CDS implied firm's asset volatility). We fit different ARFIMA and ARIMA models to the obtained time-series to forecast future firm's asset volatility for the purpose of CDS pricing. We observe that implied firm's asset volatility is also highly persistent with the degree of fractional integration lying within the non-stationary region, although, on average the process eventually mean-reverts in the very long-run. The in-sample-fit and out-of-sample forecasting performance shows that ARFIMA models on average outperform ARIMA models. Out of all ARFIMA specifications, we find that the non-stationary ARFIMA(0,d,0) model provide best in-sample fit to the data and perform best in out-of-sample forecasting of firm's asset volatility and credit spreads.

Chapter 1

Persistence in firm's asset and equity volatility[1]

1. Introduction

Equity volatility and its properties have been extensively studied in the finance literature. A general conclusion is that the equity volatility process is highly persistent and characterized by a slow decay of volatility shocks (Bollerslev and Mikkelsen, 1996). In contrast, only a few studies focus on the underlying firm's assets volatility, which represents a key parameter in credit risk assessment and management. The main reason for such a gap in the literature is that equity volatility can be easily estimated from observable stock prices, whereas estimation of the firm's asset volatility is hindered by the latent nature of the underlying firm's asset values. Recent studies by Choi and Richardson (2016) and Lovreta and Silaghi (2017), however, point to possible significant differences between the properties of equity and firm's asset volatility. Choi and Richardson (2016), for example, argue that equity volatility shows greater persistence when compared to the volatility of the corresponding firm's assets and they attribute this difference to financial leverage. Their conclusions are primarily based on the autocorrelogram characteristics of firm's asset and equity volatility as well as on the parameters of the EGARCH model proposed by Nelson (1991), which, by definition, considers only short-run effects. Formal analysis of the persistence properties of firm's asset volatility in the context of a long-memory model however, has not been done in the previous literature. In this paper, we fill this gap.

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The analysis of the long-memory properties of the levered and unlevered volatility has important theoretical and empirical implications for asset pricing and trading, risk management, and portfolio management. On the one hand, understanding the dynamics of the firm's asset value process is a critical issue in credit derivative and bond pricing. The price of a corporate bond, Credit Default Swap (CDS) or any other single credit sensitive instrument depends crucially on the firm-specific probability of the default, which in turn is a function of the firm-specific asset volatility. If firm's asset volatility has long-memory properties, many of the existing pricing models would need to be reconsidered to incorporate this feature. Furthermore, given that firm's asset volatility could only be considered at the firm-specific level, any potential analysis of its long-memory properties at the market level, as well as a comparison with its corresponding equity volatility needs to be based on the bottom-up approach, that is, by aggregating firm-level results. On the other hand, total volatility of equity returns is a key input in option pricing. Therefore, at the firm-level it becomes important to consider not only the long-memory behavior of the systematic volatility (reflected in the readily available well diversified equity indexes) but also the effect of idiosyncratic features on the total equity volatility persistence. Finally, an analysis of the persistence properties at the firm-level allows understanding of the cross-sectional behavior of equity and firm's asset volatility. These two measures of the firm-level risk are particularly important in portfolio allocation problems and risk management.

The concept of long-memory was originally introduced by Hurst (1951) in his study on the seasonal variation in river flow and later formalized in the work of Mandelbrot and Van Ness (1968), Mandelbrot (1971), Granger (1980), Granger and Joyeux (1980) and Hosking (1981), among others. It refers to a long-run dependence between observations that are far from each other. Accordingly, the degree of persistence, or the degree of long-memory, refers to a degree of decay of dependence between observations as the time distance between them increases. A large volume of literature on long-memory in financial time-series has emerged following the work of Greene and Fielitz (1977), who studied the persistence of returns on common stock. Although long-memory in raw financial returns is questioned in the literature (e.g. Lo, 1991; Lobato and Savin, 1998; Gil-Alana, 2006), it has been well documented in the volatility of financial returns (e.g. stock returns and stock index returns, commodity returns, exchange rates). There is an overwhelming amount of empirical evidence that market volatilities exhibit slow decay features, converting persistence into one of the stylized features of volatility (Ding et al. 1993; Baillie et al., 1996; Baillie, 1996; Engle and Patton, 2001).

Long-run temporal dependencies in equity volatility (volatility of stock returns and stock index returns) have been extensively studied in the context of the FIGARCH (Fractionally Integrated GARCH) model proposed by Baillie et al. (1996). Some of the contributions include, Dionisio et al. (2007), Kang and Yoon (2007), Lux and Kaizoji (2007), Kasman et al. (2009), Kang et al. (2010) and Bentes (2014).[2] When financial time-series is characterized by a long-memory, Bollerslev and Mikkelsen (1996) and Baillie et al. (1996) show that FIGARCH models are more appropriate than GARCH (or EGARCH) and IGARCH models. The FIGARCH model explicitly considers the degree of long memory, which is measured with the fractional differencing parameter *d.* The model therefore provides a precise measure of the persistence in the observed data, and is able to distinguish between process with an exponential decay of shocks and a process with a permanent effect of shocks, allowing for an intermediate range of dependence.

[2] The FIGARCH model has been considered in volatility of commodity returns (e.g. Cochran et al. 2012; Bentes, 2015), commodity futures returns (e.g. Jin and Frechette, 2004; Baillie et al. 2007), exchange rates (e.g. Baillie et al. 1996; Vilasuso, 2002; Beine et al. 2002).

In this paper, we build on the recent evidence on the persistence of firm's asset and equity volatility by providing precise parametric estimates of the degree of persistence of both volatilities at the firm-specific level. We consider 52 European non-financial companies that belong to the iTraxx Europe index during the 2004-2016 period. We contribute to the existing literature in several ways.

First, we contribute to the relatively scarce literature on firm's asset volatility by providing strong evidence of long-memory in the daily firm's asset volatility. We estimate the degree of persistence using the FIGARCH model, which although applied to conditional volatility of equity returns, has not been considered in the case of the firm's asset returns. We find that the estimated fractional differencing parameter is statistically significant for all firms in the sample, with a cross-sectional average of 0.35. This finding has implications for pricing credit-sensitive instruments due to their analogy with stock options (Merton, 1974).[3] Bollerslev and Mikkelsen (1996) illustrate the practical importance of long-run volatility characteristics on the pricing of long-maturity stock option contracts. Given that bonds or Credit Default Swaps (CDSs) embed features similar to a short position in long-maturity put options, with firm's asset value as the underlying state variable, the presence of long-memory in firm's asset volatility should be an important aspect to consider when pricing these instruments.

Second, we provide further evidence on the persistence of equity volatility. Although volatility of stock index returns has been extensively studied in the literature in the context of a long-memory model (Dionisio et al, 2007; Kang and Yoon, 2007; Kang et al, 2010; Bentes, 2014), there have been only a few applications to firm-specific stock returns. Lux and Kaizoji (2007), for example, estimate the FIGARCH model for a sample of Japanese stocks. They report the cross-sectional mean of the

[3] Debt and equity are treated as contingent claims on the underlying firm asset value.

fractional differencing parameter of 0.34 for 100 companies with the largest trading volume, and 0.37 for a random sample of 100 companies. Uctum et al. (2017) analyse the conditional volatility of four French stocks using intraday returns, and report a degree of long-memory between 0.22 and 0.59. Our results are in line with these findings. For our sample of European companies, the cross-sectional average of the estimated fractional differencing parameter amounts to 0.37.

Third, we compare the persistence of equity and firm’s asset volatility for our matched sample. Our findings show that equity volatility is on average slightly more persistent than a firm's asset volatility. Although this result is initially in line with that obtained by Choi and Richardson (2016) using a different methodology, we do not find this difference in persistence to be statistically significant. Moreover, we find no statistically significant evidence of higher persistence of equity volatility even for the more levered subsample of companies.

Fourth, the firm-specific orientation that we adopt in this paper allows us to relate the estimated degree of persistence of volatility in both a firm’s asset and equity returns to firm-specific characteristics. More specifically, we consider financial leverage, size and firm’s relative idiosyncratic volatility. We find that a firm's asset and equity volatility are both positively related to leverage and negatively related to the firm's relative idiosyncratic volatility. Although long-range dependence measures have been previously related to firm-specific variables in Cajueiro and Tabak (2005), our work differs from theirs in two major aspects. Namely, Cajueiro and Tabak (2005) relate firm-specific variables to long-range dependence in raw stock returns measured by the Hurst exponent, whereas we consider persistence of conditional volatility measured by the fractional differencing parameter.

Finally, to complement our analysis, in addition to estimating the conditional variance using the FIGARCH model, we use absolute daily returns as a direct proxy for volatility and estimate the degree of long-memory using a non-parametric DFA approach. The results of the DFA analysis are in line with our previous findings and show strong evidence of long-memory in the daily absolute firm's asset and equity returns. The scaling exponent, used as a measure of the degree of persistence, lies in the region between 0.5 and 1, and is slightly higher for absolute equity returns than for absolute firm's asset returns (0.85 vs. 0.81). In this case, the difference in mean is statistically significant.

The rest of the paper is organized as follows. Section 2 provides a summary of the FIGARCH model framework. Section 3 summarizes the estimation methods of the firm's asset value. Section 4 provides a description of our data set. Section 5 provides our empirical results on the persistence of firm’s asset and equity volatility. Section 6 conducts robustness analysis using a DFA methodology. Finally, our conclusions are presented in Section 7.

2. Model framework

The FIGARCH model, introduced by Baillie et al. (1996), is a conditional volatility model that allows a slow hyperbolic rate of decay for the lagged squared innovations in the conditional variance. In other words, the FIGARCH model allows for long-memory in volatility. This model property contrasts with the GARCH model (Bollerslev, 1986) that only considers short-term memory, or the IGARCH model (Engle and Bollerslev, 1986) that only assumes infinite memory. In the GARCH model, the effect of the lagged squared innovations in the conditional variance decays at an exponential rate. As such, the GARCH model may not be appropriate for describing processes that evidence long-range dependence. Specifically, Baillie et al. (1996) show that when a short-memory model is applied to a process that exhibits long-memory, the estimated parameters of the short-memory model quite often tend to point to an integrated process, that is, to infinite persistence. This may spuriously suggest that an IGARCH model, in which a volatility shock has a permanent effect, might be appropriate. The infinite memory assumption, however, may be quite unrealistic when applied to financial data. By contrast, the FIGARCH model is more flexible and imposes a more realistic slow hyperbolic decay in the conditional volatility, in such a way that the effect of a volatility shock is very persistent, but eventually mean reverting.

To fully define any GARCH-type model a specification of a conditional mean equation, conditional variance equation and a conditional error distribution is required. In this paper, we model the conditional mean equation of each return series as a k-order autoregressive process, $AR(k)$, given by:

$$r_t = \varphi_0 + \varphi_1 r_{t-1} + \cdots + \varphi_k r_{t-k} + \varepsilon_t, \quad (1)$$

For $k = 0$, the conditional mean equation is reduced to a model with only a constant (i.e. $r_t = \varphi_0 + \varepsilon_t$). The error term, ε_t, could be expressed as $\varepsilon_t = \sigma_t z_t$, where z_t is an i.i.d. sequence.

The FIGARCH model for the conditional variance is given by:

$$(1-L)^d \theta(L)\varepsilon_t^2 = w + [1-\beta(L)]v_t \quad (2)$$

where, $v_t = \varepsilon_t^2 - \sigma_t^2$ are innovations in the conditional variance (with an expected value of zero and serially uncorrelated), L denotes the lag operator, $(1-L)^d$ is a fractional difference operator, $0 \leq d \leq 1$ is the fractional differencing parameter and all the roots of $\theta(L)$ and $[1-\beta(L)]$ lie outside the unit circle. The process is strictly stationary and ergodic for $0 \leq d \leq 1$. Therefore, the FIGARCH model formally considers long-memory in volatility through the fractional differencing (or long-memory) parameter, d, subsuming at the same time the GARCH and IGARCH models as a special case. That is, the FIGARCH behaves as a GARCH process when $d = 0$ and as an IGARCH process when $d = 1$, whereas for $0 < d < 1$ is a long memory process.

The model assumed for z_t distribution is the generalized error distribution (GED). The choice of the GED distribution is motivated by the results of (Gao et al, 2012) who show that this distributional assumption can adequately account for fat tails. The density of a GED random variable with mean 0 and variance of 1 is given by:

$$f(z; \upsilon) = \frac{\upsilon}{\lambda_\upsilon 2^{(1+1/\upsilon)}\Gamma(1/\upsilon)} exp\left[-\left(\frac{1}{2}\right)|z/\lambda_\upsilon|^\upsilon\right] \quad (3)$$

where, υ is a tail thickness parameter that takes the value $0 < \upsilon \leq \infty$, $\Gamma(\cdot)$ is the gamma function, and $\lambda_\upsilon \equiv \left[2^{(-2/\upsilon)}\,\Gamma(1/\upsilon)/\Gamma(3/\upsilon)\right]^{1/2}$.

The FIGARCH model is estimated by maximizing the log-likelihood function. The GED log-likelihood function (of a normalized random variable) is given by:

$$LL_{GED} = \sum_{t=1}^{T}\left[ln(\upsilon/\lambda_\upsilon) - 0.5\left|\frac{z_t}{\lambda_\upsilon}\right|^\upsilon - (1+\upsilon^{-1})ln(2) - ln\Gamma(1/\upsilon) - 0.5ln(\sigma_t^2)\right] \quad (4)$$

3. Estimation of the firm's asset values

In order to be able to study time-series properties of the firm's asset volatility we must first estimate the unobserved underlying firm's asset value process and the firm's asset returns. The literature offers several approaches for estimating the latent firm's asset value: the structural model framework, the naïve approach of Charitou et al. (2013), the Bharath and Shumway (2008) KMV procedure, and the Choi and Richardson (2016) procedure based on market prices of equity, bonds and loans. To provide robustness to our results, in this paper we consider three basic procedures. A detailed description follows:

In our baseline approach, we consider the possibility of using structural models of default to infer the underlying firm asset value (Forte, 2011; Lovreta and Silaghi, 2017). Structural models originated from the seminal model of Merton (1974), which applied the option pricing theory developed by Black and Scholes (1973) to value corporate securities: debt and equity. Under the structural model framework, debt and equity are treated as contingent claims on the underlying firm's asset value, V, which is assumed to evolve according to the continuous diffusion process of the following general form:

$$dV = (\mu - \delta)Vdt + \sigma Vdz, \tag{5}$$

where μ is the expected rate of return on asset value, δ is the fraction of the asset value paid out to investors, σ is the asset return volatility, and z is a standard Brownian motion. In the structural setting, default is explicitly linked to the underlying firm's asset value and will occur when V reaches a lower threshold boundary called the default barrier (V_B). Provided that equity for traded firms is observable, for a given equity pricing equation of the structural model at hand, $E = f(V)$, we can back-out the underlying firm's asset value.[4]

In our baseline case, the structural model and the model estimation methodology is the one suggested by Forte (2011). Specifically, in Forte (2011) the default barrier (V_B) is defined as the fraction β of the nominal value of the total debt P. The exact value of β is calibrated to market observable Credit Default Swap (CDS) spreads, whereas P is approximated with the sum of the book value of short-term liabilities (STL) and long-term liabilities (LTL). The debt structure is assumed to consist of 10 coupon bonds, with maturities ranging from 1 to 10 years ($\tau = 1, \ldots, 10$). The principal of the 1-year maturity bond is equal to the book value of the STL, whereas the principal of the bonds with maturities from 2 to 10 years is equal to 1/9 of LTL each. The coupon of each individual bond is determined as a part of the total interest expenses, proportional to the ratio of the bond's principal to the nominal value of the total debt.

[4] Specific equity pricing equation will depend on the structural model at hand.

The risk-free rate for each bond is proxied with the swap rate of the corresponding maturity. The value of τ-maturity individual bond at any point in time t is then determined using the expressions provided by Leland and Toft (1996):

$$d(V_t,\tau) = \frac{c(\tau)}{r} + e^{-r\tau}\left[p(\tau) - \frac{c(\tau)}{r}\right][1 - F(\tau)] + \left[(1-\alpha)\beta p(\tau) - \frac{c(\tau)}{r}\right]G(\tau), \quad (6)$$

where, $p(\tau)$ is the principal of the τ-year maturity bond, $c(\tau)$ is the coupon of the τ-year maturity bond, r is the risk-free rate, $\alpha \in [0,1]$ represents bankruptcy costs, and specific expressions for $F(\tau)$ and $G(\tau)$ are given in the Appendix A.

In the Forte (2011) model, the total value of debt $D(V_t)$ equals the sum of the values of the 10 individual bonds, and the equity pricing equation is given by:

$$E(V_t) = V_t - D(V_t|\alpha = 0), \quad (7)$$

where $D(V_t|\alpha = 0)$ is the value of the total debt when bankruptcy costs equal zero. Finally, for a series of observable equity prices, the firm's asset values at each point in time are determined using an iterative algorithm. We refer here interested readers to the original paper for details. The market value of the firm's assets using the structural model framework will be denoted as, V_{SM} and firm's asset returns calculated as $r(V_{SM})_t = ln(V_{SM,t}/V_{SM,t-1})$.

The nominal value of the total debt, P, used in deriving the V_{SM} is determined on the basis of accounting data. Unfortunately, accounting data is not available on a daily basis. Therefore, we are in principle left with two options, either to carry out an interpolation between available observations (Forte, 2011), or to keep P constant and equal to the last available value reported in a firm's balance sheet. Our baseline approach considers the linear interpolation method with the objective to capture the evolution of the accounting variables over time. However, it seems reasonable to show that our results are not affected by this procedure. As a robustness check, we consider the second option, and use the book value of total liabilities reported in the last available annual firm's balance sheet. The market value of the firm's assets using the structural model framework and no interpolation of accounting data (STL, LTL, interest expenses and cash dividends) will be denoted as, V_{SMNI} and corresponding firm's asset returns as $r(V_{SMNI})_t$.[5] In this case, for the purpose of the time-series analysis of firm's asset returns, it is necessary to discard return observations corresponding to January 1st. If these observations were included, it would imply that all the adjustment of the book level of debt (which occurs throughout the year) occurs in one single day. This quite unrealistic assumption would then imply the existence of artificial one-day jumps spaced at regular yearly intervals, which would subsequently overestimate the volatility of firm's asset returns and underestimate its persistence. Accordingly, these 12 return observations are treated as outliers.

In order to show that our results are general and do not depend on the specific assumptions of the structural model at hand, we also use two additional naïve estimations of the firm's asset value as a robustness check. First, we consider the naïve procedure considered in Bharath and Shumway (2008), in which the firm's asset value is simply the sum of the market value of equity (E) and the default barrier (V_B), which is set as in KMV to the value of $STL + 0.5 \times LTL$. The market value of firm's assets resulting from this approach will be denoted as, V_{KMV} and firm's asset returns calculated as $r(V_{KMV})_t = ln(V_{KMV,t}/V_{KMV,t-1})$. Second, we consider the approach by Charitou et al. (2013), in which the firm's asset value is treated as observable and is proxied with the sum of the market value of equity (E) and the face value of debt (P). The market value of firm's assets using this approach will be denoted as, V_{Proxy}. In this case, however, in order to resemble the naïve approach considered for robustness purposes in Choi and Richardson (2016), firm's asset returns, $r(V_{Proxy})_t$, are then calculated as

[5] It should be noted that although the nominal value of debt is kept constant, the market value of debt, which is derived by inverting the equity pricing equation, is changing on a daily basis.

follows: $r_{t+1}^{asset} = \left(E_t/V_{Proxy,t}\right) r_{t+1}^{equity} + \left(P_t/V_{Proxy,t}\right) r_t^f$, that is, debt is assumed to be risk-free.[6]

4. Data Set

The estimation of firm's asset values requires calibration of the default barrier to market observable CDS spreads. Therefore, our sample is limited not only to companies with traded equity but also to companies with highly liquid CDS spreads. Our initial sample was chosen from constituents of the iTraxx Europe index which comprises the most liquid 125 CDS referencing European investment-grade companies, starting from the year 2004, in which the index was introduced. We exclude companies in the banking and financial sector due to their different capital structure, private companies, and companies for which we lack data on either market capitalization or CDS spreads for the overall sample period. Additionally, we exclude all companies involved in corporate operations that resulted in significant jumps in the market capitalization time-series but not in the time-series of adjusted prices. Our final sample is comprised of 52 non-financial companies that we track over the 2004-2016 period.

A complete list of the companies considered, its market capitalization and sector classification is provided in Table 1. The average company in the sample has a market capitalization of €25.4 billion, a leverage of 0.52 and a historical equity volatility of 30%. The leverage ranges from 0.27 to 0.86 and historical equity volatility from 21% to 48%. Leverage is calculated as the ratio of the book value of total liabilities to the sum of the market value of equity and the book value of total liabilities. Historical equity volatility is calculated as the annualized standard deviation of the continuously compounded returns on equity.

[6] We do not use the baseline approach of Choi and Richardson (2016) due to data limitations on the market prices of bonds and loans.

Table 1. *List of companies*

No.	Company	MC in m €	Sector	Subsector
1	AB Volvo	13,628.76	Industrial Engineering	Commercial Vehicles & Trucks
2	BMW AG	33,359.36	Automobiles & Parts	Automobiles
3	Michelin SCA	10,639.76	Automobiles & Parts	Tires
4	Continental AG	17,302.49	Automobiles & Parts	Tires
5	Daimler AG	51,141.31	Automobiles & Parts	Automobiles
6	Peugeot SA	8,266.57	Automobiles & Parts	Automobiles
7	Renault SA	17,540.90	Automobiles & Parts	Automobiles
8	Valeo SA	4,243.23	Automobiles & Parts	Auto Parts
9	Deutsche Lufthansa AG	6,176.73	Travel & Leisure	Airlines
10	Kingfisher PLC	8,147.19	General Retailers	Home Improvement Retailers
11	Koninklijke Philips NV	23,991.42	Health Care Equipment & Services	Medical Equipment
12	LVMH SE	50,950.98	Personal Goods	Clothing & Accessories
13	Marks & Spencer Group PLC	9,150.33	General Retailers	Broadline Retailers
14	Kering SA	14,755.60	General Retailers	Apparel Retailers
15	Sodexo SA	8,543.53	Travel & Leisure	Restaurants & Bars
16	BAT PLC	59,806.73	Tobacco	Tobacco
17	Carrefour SA	23,186.11	Food & Drug Retailers	Food Retailers & Wholesalers
18	Casino Guichard SA	6,979.49	Food & Drug Retailers	Food Retailers & Wholesalers
19	Diageo PLC	43,243.94	Beverages	Distillers & Vintners
20	Danone SA	29,692.63	Food Producers	Food Products
21	Henkel & Co KGaA AG	11,725.60	Household Goods & Home Construction	Nondurable Household Products
22	Imperial Tobacco Group PLC	25,595.60	Tobacco	Tobacco
23	J Sainsbury PLC	7,775.60	Food & Drug Retailers	Food Retailers & Wholesalers
24	Tesco PLC	33,715.80	Food & Drug Retailers	Food Retailers & Wholesalers
25	Unilever NV	42,970.03	Food Producers	Food Products
26	BP PLC	125,691.29	Oil & Gas Producers	Integrated Oil & Gas
27	E.ON SE	44,600.90	Gas, Water & Multiutilities	Multiutilities
28	EDP Energias de Portugal SA	9,757.37	Electricity	Alternative Electricity
29	Iberdrola SA	30,735.58	Electricity	Conventional Electricity
30	Repsol SA	23,708.21	Oil & Gas Producers	Integrated Oil & Gas
31	RWE AG	24,318.96	Gas, Water & Multiutilities	Multiutilities
32	Akzo Nobel NV	11,723.31	Chemicals	Specialty Chemicals
33	Anglo American PLC	33,491.23	Mining	General Mining
34	BAE Systems PLC	16,096.54	Aerospace & Defense	Defense
35	Bayer AG	49,976.78	Pharmaceuticals & Biotechnology	Pharmaceuticals
36	Saint Gobain SA	19,022.46	Construction & Materials	Building Materials & Fixtures
37	Investor AB	8,367.33	Financial Services	Specialty Finance
38	Linde AG	17,498.17	Chemicals	Commodity Chemicals
39	Rolls-Royce Holdings PLC	14,296.92	Aerospace & Defense	Aerospace
40	Siemens AG	69,649.35	General Industrials	Diversified Industrials
41	Stora Enso OYJ	5,040.96	Forestry & Paper	Paper
42	UPM Kymmene OYJ	7,002.66	Forestry & Paper	Paper
43	BT Group PLC	28,121.99	Fixed Line Telecommunications	Fixed Line Telecommunications
44	Deutsche Telekom AG	53,768.38	Mobile Telecommunications	Mobile Telecommunications
45	Orange SA	42,617.39	Fixed Line Telecommunications	Fixed Line Telecommunications
46	Hellenic Telecom. Org. SA	5,530.32	Fixed Line Telecommunications	Fixed Line Telecommunications
47	Koninklijke KPN NV	15,613.93	Fixed Line Telecommunications	Fixed Line Telecommunications
48	Pearson PLC	9,484.90	Media	Publishing
49	STMicroelectronics NV	7,999.00	Technology Hardware & Equipment	Semiconductors
50	Telefonica SA	65,947.77	Fixed Line Telecommunications	Fixed Line Telecommunications
51	Wolters Kluwer NV	5,740.91	Media	Publishing
52	WPP PLC	14,099.36	Media	Media Agencies

This table reports the list of the 52 non-financial companies analyzed in the paper. MC refers to market capitalization. Sector and Subsector classification is based on the ICB (Industry Classification Benchmark) codes downloaded from Datastream.

The data needed for the estimation of firm's asset values: daily data on market capitalization, CDS spreads, and 1-10 year swap rates, as well as yearly data on current liabilities, total liabilities, interest expenses and cash dividends, is downloaded from Datastream. In our baseline case, daily data on accounting items is obtained using linear interpolation. In the case of CDS spreads, we consider only the most liquid 5-year Euro-denominated CDS contracts on senior unsecured debt.

The cross-sectional average of the main descriptive statistics for daily equity and firm's asset returns are shown in Panel A of Table 2. In total, for each firm in the sample the equity and the firm's asset time-series consists of 3,391 observations of daily returns.[7] The mean equity and firm’s asset returns are both positive, but statistically not significantly different from zero. Equity returns exhibit a larger variance compared to the variance of the estimated firm’s asset returns. In other words, equity volatility is higher than the firm’s asset volatility and this finding is consistent with the literature (Choi and Richardson, 2016). Interestingly, equity returns are positively skewed while the firm’s asset returns on average show a negative-skewed behaviour. Both return series exhibit high kurtosis. The returns of the estimated firm’s asset values have similar magnitudes in terms of mean, standard deviation, skewness and kurtosis for the four estimation procedures V_{SM}, V_{SMNI},V_{KMV} and V_{Proxy}.

We perform several diagnostic statistical tests on the time-series of equity and firm’s asset returns. The main cross-sectional results are reported in Panel B of Table 2. First, we check for the presence of serial correlation in returns and squared returns using the Ljung-Box test with 10 lags. The Ljung-Box test rejects the null hypothesis of no serial correlation at the 5% level for 78.85% of the companies in the case of equity returns and for 63.46% of the companies in the case of firm’s asset returns. For those

[7] For $r(V_{SMNI})$ the total number of observations is 3,379.

companies for which return series show evidence of serial correlation, the mean equation in the FIGARCH model is modelled as an AR(k) process, where $k>0$. In the case of squared returns, the Ljung-Box test rejects the null hypothesis of no serial correlation at the 5% level for practically all of the companies considered. On this matter, we additionally perform the Engle's ARCH test for heteroscedasticity. The results confirm strong evidence of conditional heteroscedasticity in all return series. The Jarque-Bera test for normality rejects the null hypothesis of Gaussianity at the 1% significance level in all of the equity and firm's asset returns. Finally, Panel C of Table 2 presents the results of the unit-root tests. To account for possible structural breaks in the data we perform a two-break unit root test of Clemente et al. (1998), with both the additive outlier (AO) and the innovational outlier (IO) approach. The double mean shift test rejects the null hypothesis of unit-roots in all of the cases considered. We also perform an AO and IO one-break tests of Perron and Vogelsang (1992) which also rejects the unit root null hypothesis for both firm's asset and equity returns.[8] Finally, we perform the standard Augmented Dickey-Fuller test (ADF) which confirms the stationarity of all return series. Overall, the preliminary analysis shows that data employed in this study presents common features of financial returns data.

[8] The Clemente et al. (1998) and Perron and Vogelsang (1992) tests have been performed using Stata routines developed by Baum (2004). For the Perron and Vogelsang (1992) one-break test, the 5% trimming was used. For the Clemente et al. (1998) two-break tests, the calculation of the unit-root test statistics with 5% trimming was not feasible for our sample of 5×52 daily time-series with 3,391 observations each. To make the calculations feasible, we have employed a 25% trimming, which essentially implies that the possible break-dates are confined to the interval April 2007 – October 2013, including therefore the probable break events related to the global financial crisis and the European sovereign debt crisis.

Table 2. *Descriptive statistics of equity and firm's asset returns*

	***r*(E)**	***r*(V_{SM})**	***r*(V_{SMN})**	***r*(V_{KMV})**	***r*(V_{Proxy})**
Panel A					
Mean	0.00015	0.00016	0.00010	0.00015	0.00014
Variance	0.01890	0.00860	0.00856	0.00860	0.00860
Skewness	0.08304	-0.03682	-0.06909	-0.04363	-0.06960
Kurtosis	15.43038	11.77791	11.00821	11.69407	11.36487
Panel B					
Q(10)	25.6132	22.9192	22.9623	22.7408	22.4136
	(78.85%)	(63.46%)	(67.31%)	(65.38%)	(63.46%)
$Q^2(10)$	788.2130	524.7031	521.3927	520.5706	513.1666
	(98.08%)	(96.15%)	(96.15%)	(96.15%)	(98.08%)
ARCH	92.2538	77.5149	76.1948	77.0270	73.8417
	(100%)	(100%)	(100%)	(100%)	(100%)
J-B	131,614.80	25,039.54	17,961.27	24,040.52	21,520.91
	(100%)	(100%)	(100%)	(100%)	(100%)
Panel C					
$CMR_{(AO)}$	-26.7459	-27.7460	-28.3229	-27.8874	-27.9368
	(100%)	(100%)	(100%)	(100%)	(100%)
$CMR_{(IO)}$	-35.1810	-33.5940	-33.7785	-34.2121	-33.7312
	(100%)	(100%)	(100%)	(100%)	(100%)
$PV_{(AO)}$	-28.8787	-28.5874	-28.7958	-28.5148	-28.3900
	(100%)	(100%)	(100%)	(100%)	(100%)
$PV_{(IO)}$	-34.6168	-35.6646	-34.0869	-35.6102	-35.7983
	(100%)	(100%)	(100%)	(100%)	(100%)
ADF	-58.0923	-58.1000	-58.0274	-58.0179	-58.1711
	(100%)	(100%)	(100%)	(100%)	(100%)

Panel A of Table 2 reports the cross-sectional average of the main descriptive statistics (mean, variance, skewness and kurtosis) of equity $r(E)$, and firm's asset returns $r(V_{SM})$, $r(V_{SMNI})$, $r(V_{KMV})$, and $r\left(V_{Proxy}\right)$, for the set of 52 non-financial companies. Panel B of Table 2 reports the results of the four key diagnostic tests: Q(10) and $Q^2(10)$ refer to the Ljung-Box statistics for tenth-order serial correlation in returns and squared returns, respectively; ARCH is the Engle's ARCH test; J-B refers to the Jarque-Bera normality test. Panel C of Table 2 reports the results of the unit-root tests. CMR is the Clemente et al. (1998) double-break unit root test statistics for the additive outlier (AO) and innovational outlier (IO), respectively (the 5% critical value is -5.49); PV is the Perron and Vogelsang (1992) single-break unit root test statistics for AO (the 5% critical value is -3.56) and IO (the 5% critical value is -4.27) model; ADF is the Augmented Dickey-Fuller unit root test. For each test, the cross-sectional average of the test statistics as well as the percentage of companies for which the null hypothesis is rejected at least at the 5% significance level (in parenthesis), are reported.

5. Empirical results

In this section, we report the main empirical results from estimating the FIGARCH(1,d,1) model for equity and firm's asset returns. As mentioned in the Section 2, the conditional mean equation is specified as an $AR(k)$ model. The exact specification is defined on a case by case basis using the following algorithm. In the initial step, raw returns (after subtracting the mean) are tested for the presence of autocorrelation. If there is no evidence of autocorrelation the mean equation is specified as a constant only model (i.e. $k = 0$). If, on the contrary, the null hypothesis for the Ljung-Box Q(10) statistics is rejected at 5% level, one autoregressive term is added to the mean equation (i.e. $k = 1$). The fit of the lag-augmented model is subsequently tested on the basis of the Ljung-Box statistics for tenth-order serial correlation in residuals. This procedure is repeated until the adequate fit of the model, based on the 5% criteria, is achieved.

The FIGARCH(1,d,1) model is estimated using the G@RCH 6.1 developed by Laurent and Peters (2002), which is an OxMetrics module used for estimation of univariate or multivariate ARCH-type models. The FIGARCH model has been estimated using a Maximum Likelihood Estimation (MLE) approach. In particular, OxMetrics uses two main methods to estimate models. By default parameters are estimated with a quasi-Newton method of Broden, Fletcher, Goldfarb and Shanno (BFGS). However, if any constraint is imposed, parameters are estimated with a constraint optimization method that implements a sequential quadratic technique to maximize a non-linear function subject to non-linear constraints (MaxSQF algorithm). The computation method used to calculate the covariance matrix of the estimates, and therefore, to obtain the standard deviations of the estimated parameters is based on second-order derivatives. This method computes the hessian matrix, which contains the second-order partial derivatives of the Likelihood function evaluated at the estimated MLE value. OxMetrics uses numerical methods to compute the hessian matrix approximating derivatives of the log-likelihood function. More details about the implementation of these estimation methods can be found in OxMetrics documentation (Laurent and Peters, 2002).

In practical applications the conditional distribution of the error term is generally modelled with normal distribution, student's t-distribution and the generalized error distribution (GED). Normal distribution is not usually employed when modelling equity returns, as it is unable to capture fat-tailed behaviour. Student's t-distribution is fat-tailed and generally performs better fit than normal distribution in asset returns. Nelson (1991) suggested the use of GED as error distribution, which allows a range of kurtosis through the value of shape parameter, that determines the thickness of the tail generating fat-tailed and thin-tailed error term distributions. In this research we assume that the conditional distribution of the error term follows a GED distribution density. This choice is motivated by Ferenstein et al. (2004), Gao et al. (2012), and Gabriel (2012), who provide evidence that GED distribution in GARCH-type modelling of asset returns outperform other return distributions. It is important to mention that as a robustness check, all of the FIGARCH(1,d,1) estimates were obtained with both GED and Student-t distributions (which are available upon request), concluding that results do not differ depending on the error distribution selected.

The estimation results of the FIGARCH(1,d,1) model are depicted in Table 3. To save space, we report only the cross-sectional mean of the estimated coefficients and the percentage of companies for which the coefficients are statistically significant.[9] The fractional differencing parameter, as a measure of persistence of the conditional

[9] Detailed results are available upon request.

volatility, is significant at least at 5% level in virtually all of the cases, providing strong evidence of long-range dependence in firm-specific equity and firm's asset volatility. The cross-sectional mean of the estimated persistence for equity volatility is 0.37, which is in line with the common findings in the literature on the volatility of stock returns (Lux and Kaizoji, 2007) and stock index returns (Dionisio et al, 2007; Kang and Yoon, 2007; Kang et al, 2010). On the other side, the estimated persistence for firm's asset volatility is slightly lower, with the cross-sectional mean of 0.35 in our base case (V_{SM}). In addition, we find no substantial difference in the estimate of the persistence parameter among different procedures used to assess the underlying firm's asset value. This result provides robustness to our novel evidence on the long-range dependence in firm's asset volatility. Finally, we reject the null hypothesis that $d = 0$ (GARCH model) or alternatively, that $d = 1$ (IGARCH model), in all of the cases considered. This result supports the use of the flexible FIGARCH model, which allows for intermediate ranges of persistence.

The results of the Ljung-Box test on standardized residuals and squared standardized residuals, reported in Table 3, indicate an adequate fit of the model. We find no additional autocorrelation in the standardized residuals. In the case of squared standardized residuals, we fail to reject the null hypothesis of conditional homoscedasticity for 98.08% of the companies in the case of equity, and 96.15% of the companies in the case of firm's assets. As well, it is important to note that the GED parameter υ is statistically significant and considerably lower than 2 in all of the cases which indicates that the distribution of z has thicker tails than the normal distribution.[10] Overall, these results provide support that the specified AR(k)-FIGARCH(1,d,1) model fit the data reasonably well.

[10] The maximum is 1.44 in the case of equity and 1.48 in the case of baseline firm's asset data.

Table 3. *Estimated coefficients for the FIGARCH(1,d,1) model with GED errors*

	$\boldsymbol{E}$	$\boldsymbol{V_{SM}}$	$\boldsymbol{V_{SMNI}}$	$\boldsymbol{V_{KMV}}$	$\boldsymbol{V_{Proxy}}$
w	$1.5690 \cdot 10^{-5}$	$4.5348 \cdot 10^{-6}$	$4.3760 \cdot 10^{-6}$	$4.4467 \cdot 10^{-6}$	$4.4515 \cdot 10^{-6}$
	(96.15%)	(90.38%)	(90.38%)	(92.31%)	(94.23%)
d	0.3745	0.3493	0.3534	0.3495	0.3574
	(100%)	(100%)	(100%)	(100%)	(98.08%)
α	0.2643	0.3085	0.3157	0.3081	0.3005
	(80.77%)	(82.70%)	(86.54%)	(80.77%)	(82.70%)
β	0.5437	0.5683	0.5792	0.5682	0.5717
	(98.08%)	(94.23%)	(94.23%)	(94.23%)	(98.08%)
GED	1.2304	1.2639	1.2679	1.2654	1.2637
	(100%)	(100%)	(100%)	(100%)	(100%)
Q(10)	9.9097	10.6097	10.7374	10.4691	10.5634
	(100%)	(100%)	(100%)	(100%)	(100%)
$\mathbf{Q^2(10)}$	6.4845	6.8839	6.9538	6.8178	6.8143
	(98.08%)	(96.15%)	(96.15%)	(96.15%)	(96.15%)

This table reports the cross-sectional average of the estimated FIGARCH(1,d,1) coefficients. The figures in parenthesis correspond to the percentage of companies for which the estimated coefficients of the model are statistically significant at least at the 5% level. The Q(10) and $Q^2(10)$ refer to the Ljung-Box statistics for tenth-order serial correlation in standardized residuals and squared standardized residuals, respectively.

Table 4 reports main cross-sectional descriptive statistics for our parameter of interest. The standard deviation of the persistence parameter for equity volatility is higher than the standard deviation of the persistence parameter for firm's asset volatility, which suggests a larger variability in the estimated equity volatility persistence. The equity volatility persistence ranges between 0.17 and 0.72, whereas firm's asset volatility persistence ranges between 0.15 and 0.54 (in our baseline case). In general, for the absolute majority of the companies considered, the parameter *d* is lower than 0.5.

Table 4. *Descriptive statistics and distribution of companies by the value of d*

	E	V_{SM}	V_{SMNI}	V_{KMV}	V_{Proxy}
Mean	0.3745	0.3493	0.3534	0.3495	0.3574
Median	0.3684	0.3474	0.3569	0.3469	0.3602
Minimum	0.1706	0.1524	0.1498	0.1607	0.1498
Maximum	0.7235	0.5367	0.5322	0.5341	0.5466
Std. dev.	0.0898	0.0833	0.0831	0.0828	0.0829
Number of firms with $d \geq 0.5$	3	4	4	4	4
Number of firms with $d < 0.5$	49	48	48	48	48

This table reports the main descriptive statistics of the estimated fractional differencing parameter within the FIGARCH(1,d,1) model for equity and firm's asset conditional variance.

5.1. Equity vs. firm's asset volatility

As previously described, the mean (and median) value of the fractional differencing parameter is higher for equity volatility. On a firm specific basis, equity volatility is more persistent than firm's asset volatility for as many as 38 companies, that is, for 73% of the sample. If this difference is significant it would imply that shocks to equity volatility die at a slower rate than shocks to firm's asset volatility. We therefore find it suitable to analyse whether this apparent difference is statistically significant or not. For that, we use the Welch t-test for equality of means. The results, depicted in Table 5, demonstrate that we fail to reject the null hypothesis of equality of means. In other words, according to the results, there are no significant differences between the persistence of firm's asset and equity volatility, and this result holds true irrespective of how a firm's asset values are computed.

We further check the statistical significance of the difference in volatility persistence after controlling for financial leverage. According to Choi and Richardson (2016), we should expect a higher persistence for the more levered firms. We divide our sample of 52 companies into two groups according to financial leverage, setting as the cut-off the mean level of 0.52. The high leverage group has a mean leverage of 0.631

and the low leverage group a mean of 0.417. For both leverage groups the estimated fractional differencing parameter (d) for asset volatility is on average lower than the one for equity volatility. However, as before, the test for equality of means fails to find significant differences in the level of persistence (see Table 5).

Table 5. *Welch t-test for equality of means*

	mean *d*		**Welch t-test**	
	firm's assets	**equity**	**t-stat**	***p-val***
$d(V_{SM})$	0.3493	0.3745	-1.4831	0.1411
$d(V_{SMNI})$	0.3534	0.3745	-1.2464	0.2155
$d(V_{KMV})$	0.3495	0.3745	-1.4765	0.1429
$d(V_{Proxy})$	0.3574	0.3745	-1.0087	0.3155
leverage > 0.52				
$d(V_{SM})$	0.3702	0.3929	-0.9751	0.3343
$d(V_{SMNI})$	0.3768	0.3929	-0.7105	0.4807
$d(V_{KMV})$	0.3679	0.3929	-1.0639	0.2926
$d(V_{Proxy})$	0.3812	0.3929	-0.4992	0.6199
leverage < 0.52				
$d(V_{SM})$	0.3285	0.3561	-1.5642	0.2535
$d(V_{SMNI})$	0.3300	0.3561	-1.0796	0.2859
$d(V_{KMV})$	0.3311	0.3561	-1.0533	0.2978
$d(V_{Proxy})$	0.3337	0.3561	-0.9612	0.3417

This table reports the results of the Welch t-test for the equality of means.

Our empirical findings differ from Choi and Richardson (2016) who argue that equity volatility is significantly more persistent than asset volatility for levered firms. However, it is important to mention that Choi and Richardson (2016) do not estimate the degree of persistence within a formal long-memory framework as we do. That is, their approach doesn't include the estimation of the long-memory parameter but is based on the autocorrelogram characteristics of firm's asset and equity volatility as well as on the parameters of the EGARCH(1,1) model, which by definition considers only short-run effects. At the same time, and consistent with Choi and Richardson (2016), we are able to observe that the persistence increases with leverage for both firm's asset and equity volatility. The persistence of equity volatility increases from 0.36 to 0.39 when moving from the low to the high leverage group. The persistence of firm's asset volatility, in turn, increases from 0.33 to 0.37 for our base case (V_{SM}). The two naïve approaches show similar behaviour of values for the *d* parameter when moving from the low to the high leverage group. We investigate this issue in more detail below.

5.2. Volatility persistence and firm-specific characteristics

In this section, we relate the persistence level to several firm-specific characteristics shown in the literature to be related to persistence in volatility. To be precise, we estimate a parsimonious cross-sectional regression model in which we relate the parameter d to a small set of explanatory variables: financial leverage (LEV), firm size ($SIZE$) and relative idiosyncratic volatility ($IVOL$).

$$d_i = \gamma_0 + \gamma_1 LEV_i + \gamma_2 SIZE_i + \gamma_3 IVOL_i, \tag{8}$$

The first explanatory variable that we consider is financial leverage, calculated as before, as the ratio of the book value of total liabilities to the proxy for the market value of the firm. Choi and Richardson (2016) report higher persistence for both firm's asset and equity volatility for highly levered firms. Similarly, although they were considering the relationship between firm-specific variables and long-range dependence in raw stock returns, Cajueiro and Tabak (2005), find positive relationship between financial leverage and the long-range dependence parameter measured by the Hurst exponent. Therefore, we expect a positive relationship between persistence and leverage.

The second firm-specific variable that we consider is firm size, measured by the log of market capitalization. Brooks et al. (2001) find that small firms exhibit higher persistence in equity volatility. Along the same lines, Cajueiro and Tabak (2005) argue that market capitalization should be negatively related to long-range dependence in stock returns given that firms with higher market capitalization have higher trading activity, which ultimately fosters market efficiency. We therefore expect persistence in volatility to be negatively related to firm size.

Finally, we introduce a measure of relative idiosyncratic volatility. Xu and Malkiel (2003) report that idiosyncratic volatilities are much less persistent than the total market volatility. We therefore would expect the persistence of firm's asset and equity volatility to be negatively related to the ratio of idiosyncratic to market volatility. Moreover, Fu (2009) argues that idiosyncratic volatility reflects firm-specific information which is highly volatile in nature, and, therefore, there is no theoretical reason to assume high persistence in idiosyncratic volatility. In this paper we use the log-transformed relative idiosyncratic volatility of Ferreira and Laux (2007) and Kapadia and Pu (2012). The relative idiosyncratic volatility is calculated as the $ln[(1 - R^2)/R^2]$, where $1\text{-}R^2$ represents the ratio of the idiosyncratic variance to total variance calculated from the market model. To estimate the market model we estimate a regression of excess firm-specific stock returns on the excess return on STOXX Europe 50.

The results of the cross-sectional regression described in Equation 8 are reported in Table 6. These results show the equity volatility persistence parameter is positively related to financial leverage which is statistically significant at 5% level and negatively related to relative idiosyncratic volatility, which, in turn, is statistically significant at 1% level. Firm size is not found to be statistically significant. The overall adjusted R^2 in equity regressions amounts to 17.15%. The persistence of firm asset volatility is also positively related to leverage and negatively related to relative idiosyncratic volatility. The leverage variable is statistically significant at 10% in our base case $d(V_{SM})$, not statistically significant for $d(V_{KMV})$ and statistically significant at 5% level for $d(V_{SMNI})$ and $d(V_{Proxy})$. In turn, the relative idiosyncratic volatility variable is statistically significant at 1% level in all of the cases. Interestingly, the adjusted R^2 is considerably higher in the case of firm's asset volatility persistence and amounts to 29.30% for $d(V_{SM})$, 33.53% for $d(V_{SMNI})$, 31.16% for $d(V_{KMV})$, and 32.64% for $d(V_{Proxy})$. This difference in the explanatory power seems to be mainly related to the relative idiosyncratic volatility which explains more of the cross-sectional variation in the degree of a firm's asset volatility persistence than in the degree of equity volatility persistence.

Table 6. *Cross-sectional regression*

Variable	**Coeff.**	**Equity *d*(E)**	**Firm's Asset *d***			
			$d(V_{SM})$	**$d(V_{SMNI})$**	**$d(V_{KMV})$**	**$d(V_{Proxy})$**
const.	γ_0	0.3697 ***	0.2715 *	0.2817 **	0.2870 **	0.3174 **
		(0.0829)	*(0.1363)*	*(0.1286)*	*(0.1348)*	*(0.1368)*
LEV	γ_1	0.1203 **	0.1231 *	0.1554 **	0.1027	0.1641 **
		(0.0558)	*(0.0666)*	*(0.0638)*	*(0.0668)*	*(0.0708)*
SIZE	γ_2	-0.0020	0.0058	0.0036	0.0055	-0.0002
		(0.0093)	*(0.0126)*	*(0.0116)*	*(0.0125)*	*(0.0124)*
IVOL	γ_3	-0.0763 ***	-0.0855 ***	-0.0885 ***	-0.0896 ***	-0.0873 ***
		(0.0219)	*(0.0223)*	*(0.0224)*	*(0.0197)*	*(0.0213)*
Adj R^2		0.1715	0.2930	0.3353	0.3116	0.3264

This table reports coefficients and standard errors (in parentheses) from a cross-sectional regression of the persistence in volatility, measured by the parameter *d* from the FIGARCH(1,d,1) model, on leverage (LEV), log of market capitalization (SIZE) and relative idiosyncratic volatility (IVOL). Standard errors are calculated as White Standard Errors. *, **, and *** denote significance at the 10%, 5% and 1% levels, respectively.

6. Detrended Fluctuation Analysis (DFA)

In this section, we complement our findings on the long-memory property in volatility of firm's asset and equity returns by considering a non-parametric measure of persistence and a direct proxy for volatility – the absolute value of the daily log returns. More specifically, as a measure of the degree of long-range dependence we consider the scaling exponent calculated using the Detrended Fluctuation Analysis (DFA) method developed by Peng et al. (1994) and widely used to study long-range dependence in financial returns and volatility (Grau-Carles, 2001; Cajueiro and Tabak, 2005; Oh et al, 2008; among others).[11]

The DFA method comprises several steps that could be briefly described as follows: First, the original time-series $y(t)$ of length N is integrated by computing the cumulative sum of the demeaned series, $x(i) = \sum_{t=1}^{i}[y(t) - \bar{y}]$, i $= 1,\ldots N$. In the second step, $x(i)$ is divided into N/n boxes (i.e. non-overlapping subsamples) of equal length, n. Third, for each box a local trend is determined by fitting a least squares line to the data and the integrated time series $x(i)$ is subsequently detrended by subtracting the local trend $x_n(i)$. Fourth, the DFA fluctuation function is defined as the square root of the average variance of the detrended time series:

$$F(n) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}[x(i) - x_n(i)]^2}, \tag{9}$$

This procedure is repeated for different box sizes, that is, for different values of n. The fluctuation function $F(n)$ behaves as a power-low of n, $F(n) \sim n^{\alpha}$, and the power-law assumption is tested by running a regression of the $log(F(n))$ on the $log(n)$. The parameter of interest is the scaling exponent α (or the Hurst exponent) which is therefore determined by the slope of the line. If $0 < \alpha < 0.5$, the time series is anti-persistent, if $0.5 < \alpha < 1$, it is persistent, and in the case of $\alpha = 0.5$, it becomes a random walk. A detailed description of the DFA algorithm is given in Peng et al. (1995).

The results of the DFA analysis presented in Table 7, Table 8 and Table 9, undoubtedly indicate that the absolute returns have a positive long-range dependence with a long-memory behavior.[12] First, from the main descriptive statistics presented in

[11] We thank the anonymous referee for suggesting this line of research.

[12] In line with Oh et al. (2008), we have also tested the effect of autocorrelation in raw returns and calculated the scaling exponent by considering the absolute errors from the conditional mean equation

Table 7, we can observe that the scaling exponent ranges between 0.73 and 0.9 for our base case firm's asset volatility and between 0.75 and 0.97 for equity volatility. In all of the cases, scaling exponent is significantly higher than 0.5 and lies within the region $0.5 < \alpha \leq 1$, which provides strong evidence on the long-memory characteristics of the examined volatility series. In line with our previous analysis, the standard deviation of the persistence parameter for equity volatility is higher than the standard deviation of the persistence parameter for firm's asset volatility. As before, we find that equity volatility persistence (with a mean of 0.85) is on average slightly higher than asset volatility persistence (with a mean of 0.81). In this case, we find that the difference in mean is statistically significant (see Table 8). However, and in line with our previous analysis, although the persistence increases with leverage the difference in mean cannot be attributed to leverage (the difference is statistically significant for both high and low leveraged firms). Finally, Table 9 replicates the analysis on volatility persistence and firm-specific characteristics. The relative idiosyncratic volatility, as before, seems to be the crucial explanatory variable of the degree of persistence for both firm's asset and equity volatility. In all of the cases, relative idiosyncratic volatility is statistically significant at 1% level and is negatively related to the degree of persistence. Leverage, as before, is positively related to firm's asset volatility persistence. Interestingly, when we consider the scaling exponents of the absolute return series, we find that firm size is statistically significant and positively related to the degree of persistence, although a negative sign was expected.[13] This could be due to the fact that daily volatility is proxied using one-day observation data, because all considered firms are big in size, highly liquid and investment-grade so that the size variable is not adequately reflecting

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instead of the absolute raw returns. The results obtained with $AR(k)$ filtered returns are virtually identical to those reported in the paper.

[13] As a robustness check, we have also examined the relationship between the firm-specific variables and scaling exponents for return series. In line with Cajueiro and Tabak (2005), we find that firm size in this case is negatively related to the degree of long-memory in return series.

the trading activity effect, or simply because for such a firm category an opposite effect holds.

Table 7. *Scaling exponent from the DFA analysis*

	E	V_{SM}	V_{SMNI}	V_{KMV}	V_{Proxy}
Mean	0.8455	0.8088	0.8106	0.8093	0.8115
Median	0.8463	0.8107	0.8091	0.8108	0.8132
Minimum	0.7451	0.7264	0.7312	0.7065	0.7232
Maximum	0.9728	0.8995	0.8999	0.9002	0.8842
Std. dev.	0.0421	0.0368	0.0386	0.0376	0.0372
No of firms with $0.5 < \alpha < 1$	52	52	52	52	52

This table reports the main descriptive statistics of the DFA scaling exponent for equity and firm's asset absolute returns.

Table 8. *DFA analysis - Welch t-test for equality of means*

	mean α		**Welch t-test**	
	firm's assets	**equity**	**t-stat**	***p-val***
$\alpha(V_{SM})$	0.8088	0.8455	22.3389	0.0000
$\alpha(V_{SMNI})$	0.8106	0.8455	19.4303	0.0000
$\alpha(V_{KMV})$	0.8093	0.8455	21.3711	0.0000
$\alpha(V_{Proxy})$	0.8115	0.8455	18.9841	0.0000
leverage > 0.52				
$\alpha(V_{SM})$	0.8165	0.8496	9.7225	0.0031
$\alpha(V_{SMNI})$	0.8188	0.8496	7.9823	0.0069
$\alpha(V_{KMV})$	0.8157	0.8496	9.5604	0.0033
$\alpha(V_{Proxy})$	0.8210	0.8496	6.9553	0.0113
leverage < 0.52				
$\alpha(V_{SM})$	0.8012	0.8413	12.7684	0.0008
$\alpha(V_{SMNI})$	0.8023	0.8413	11.6909	0.0013
$\alpha(V_{KMV})$	0.8028	0.8413	11.8325	0.0012
$\alpha(V_{Proxy})$	0.8020	0.8413	12.7076	0.0008

This table reports the results of the Welch t-test for the equality of means.

Table 9. *DFA analysis - Cross-sectional regression*

Variable	Coeff.	Equity α(E)	Firm's Asset α			
			$\alpha(V_{SM})$	$\alpha(V_{SMNI})$	$\alpha(V_{KMV})$	$\alpha(V_{Proxy})$
const.	γ_0	0.7219 ***	0.6455 ***	0.6527 ***	0.6532 ***	0.6437 ***
		(0.0654)	*(0.0671)*	*(0.0668)*	*(0.0669)*	*(0.0677)*
LEV	γ_1	0.0540	0.0648 *	0.0730 **	0.0559 *	0.0863 **
		(0.0373)	*(0.0325)*	*(0.0344)*	*(0.0326)*	*(0.0348)*
SIZE	γ_2	0.0115 *	0.0147 **	0.0140 **	0.0145 **	0.0140 **
		(0.0061)	*(0.0055)*	*(0.0055)*	*(0.0056)*	*(0.0055)*
IVOL	γ_3	-0.0353 ***	-0.0287 ***	-0.0345 ***	-0.0314 ***	-0.0290 ***
		(0.0059)	*(0.0086)*	*(0.0085)*	*(0.0093)*	*(0.0092)*
Adj R^2		0.2669	0.3362	0.3852	0.3439	0.3581

This table reports coefficients and standard errors (in parentheses) from a cross-sectional regression of the persistence in volatility, measured by the scaling exponent from the DFA analysis, on leverage (LEV), log of market capitalization (SIZE) and relative idiosyncratic volatility (IVOL). Standard errors are calculated as White Standard Errors. *, **, and *** denote significance at the 10%, 5% and 1% levels, respectively.

7. Conclusions

The main objective of this paper is to understand and analyse the persistence properties of firm's asset volatility and the eventual difference it presents with respect to the equity volatility of the same underlying firm. Using the FIGARCH model, we find strong evidence of long memory in the conditional variance of both firm's asset and equity returns. The firm's asset volatility is less persistent than equity volatility for the vast majority of the companies considered. However, we do not find statistically significant differences in the level of firm's asset and equity volatility persistence. In a parsimonious econometric model we show that both equity and firm's asset volatility are related to firm-specific variables: positively to leverage and negatively to relative idiosyncratic volatility. A complementary DFA analysis based on an alternative definition of volatility confirms strong evidence on the long-memory characteristics of both volatility series.

These results have several important implications in finance. In particular, the presence of long-memory in firm's asset volatility should be an important aspect to consider in pricing credit-sensitive instruments (e.g. bonds, CDS), and therefore, in credit risk assessment and management. As these instruments are fundamentally analogous to long-maturity equity options, the pricing effect of long-memory characteristics of firm's asset volatility could be even more pronounced in comparison to the effect long-memory has on pricing equity options. In the same way the averge option pricing errors substantially decline when moving from short- to long-memory models (Bollerslev and Mikkelsen, 1996; Bollerslev and Mikkelsen, 1999), pricing errors of corporate credit instruments could be potentially reduced if long-memory feauters are accounted for in a volatility model specification. This is, of course, an empirical question and represents an important area for future research. From the equity volatility perspective, although the implications of long-memory in equity volatility have been previously studied at the aggregate market level, this paper brings a new aspect to consider when pricing individual options or defining the optimal efficient portfolio. Namely, the relative proportion of idiosyncratic risk seems to be an important determinant of the degree of long-memory in the total equity volatility. The results of the paper further suggest that there are no substantiall differences between the level and the determinants of the degree of long-memory in firm's asset and equity volatility. This implies that the existing knowledge on the equity volatility dynamics could be directly used to improve the modelling of the unlevered volatility process.

References

Baillie, R. T., Bollerslev, T., Mikkelsen, H. -O., 1996. Fractionally integrated generalized autoregressive conditional heteroskedasticity, *Journal of Econometrics*, 74(1): 3–30

Baillie, R. T., 1996. Long memory process and fractional integration in econometrics, *Journal of Econometrics*, 73(1): 5–59

Baillie, R. T., Han, Y.-W., Myers, R. J., and Song, J. 2007. Long memory models for daily and high frequency commodity futures returns. *Journal of Futures Markets*, 27(7): 643–668

Baum, C.F. 2004. CLEMAO_IO: Stata module to perform unit root tests with one or two structural breaks, Statistical Software Components S444302, Boston College Department of Economics

Beine, M., Bénassy-Quéré, A., and Lecourt, C. 2002. Central bank intervention and foreign exchange rates: new evidence from FIGARCH estimations. *Journal of International Money and Finance*, 21(1): 115-144

Bentes, S. R. 2014. Measuring persistence in stock market volatility using the FIGARCH approach, *Physica A*, 408: 190–197

Bentes, S. R. 2015. Forecasting volatility in gold returns under the GARCH, IGARCH and FIGARCH frameworks: New evidence, *Physica A*, 438: 355–364

Bharath, S. T. and Shumway, T. 2008. Forecasting Default with the Merton Distance to Default Model, *Review of Financial Studies*, 21(3): 1339–1369

Black, F. and Scholes, M. 1973. The Pricing of Options and Corporate Liabilities, *The Journal of Political Economy*, 81(3): 637–54

Bollerslev, T. 1986. Generalized Autoregressive Conditional Heteroskedasticity, *Journal of Econometrics*, 31(3): 307–327

Bollerslev, T., and Mikkelsen, H.O. 1996. Modeling and pricing long-memory in stock market volatility. *Journal of Econometrics*. 73(1): 151-184

Bollerslev, T., and Mikkelsen, H.O. 1999. Long-term equity anticipation securities and stock market volatility dynamics. *Journal of Econometrics*. 92(1): 75-99

Brooks, C., Burk, S., and Persand, G. 2001. Benchmarks and the accuracy of GARCH model estimation, *International Journal of Forecasting*. 17(1): 45–56

Cajueiro, D. O., and Tabak B. M. 2005. Possible causes of long-range dependence in the Brazilian stock market, *Physica A*, 345(3-4): 635-645

Charitou, A., Dionysiou, D., Lambertides, N., and Trigeorgis, L. 2013. Alternative bankruptcy prediction models using option-pricing theory, *Journal of Banking and Finance*, 37(7): 2329–2341

Choi, J. and Richardson, M. 2016. The volatility of a firm’s assets and the leverage effect, *Journal of Financial Economics*, 121(2): 254–277

Clemente, J., Montañes, A., and Reyes, M. 1998. Testing for a unit root in variables with a double change in the mean. *Economic Letters*, 59: 175–182

Cochran, S. J., Mansur, I., and Odusami, B. 2012. Volatility persistence in metal returns: A FIGARCH approach, *Journal of Economics and Business*, 64(4): 287–305

Dionisio, A., Menezes, R., and Mendes, D. A. 2007. On the integrated behaviour of non-stationary volatility in stock markets, *Physica A*, 382(1): 58-65

Ding, Z., Granger, C.W.J., and Engle, R.F. 1993. A long memory property of stock market returns and a new model. *Journal of Empirical Finance*, 1(1): 83-106

Engle, R. F. and Bollerslev, T. 1986. Modelling the persistence of conditional variances, *Econometric Reviews*, 5(1): 1–50

Engle, R. F. and Patton, A. J. 2001. What good is a volatility model?, *Quantitative Finance*, 1(2): 237–245

Ferreira, M. A. and Laux, P. A. 2007. Corporate Governance, Idiosyncratic Risk, and Information Flow, *The Journal of Finance*, 62(2): 951–989

Ferenstein, E., and Gasowski, M. 2004. Modelling stock returns with AR-GARCH processes. *SORT*, 28(1), 55–68

Forte, S. 2011. Calibrating structural models: a new methodology based on stock and credit default swap data, *Quantitative Finance*, 11(12): 1745–1759

Fu, F. 2009. Idiosyncratic risk and the cross-section of expected stock returns. *Journal of Financial Economics*, 91(1): 24–37

Gao, Y., Zhang, C., and Zhang, L. 2012. Comparison of GARCH Models based on Different Distributions, *Journal of Computers*, 7(8): 1967–1973

Gil-Alana, L.A. 2006. Fractional integration in daily stock market indexes. *Review of Financial Economics*, 15(1): 28–48

Granger, C.W.J. 1980. Long Memory Relationships and the Aggregation of Dynamic Models. *Journal of Econometrics*, 14(2): 227-238

Granger, C.W.J. and Joyeux, R. 1980. An Introduction to Long-Memory Time Series Models and Fractional Differencing. *Journal of Time Series Analysis*, 1(1): 15-29

Grau-Carles, P. 2001. Long-range power-law correlations in stock returns. *Physica A*, 299: 521-527

Greene, M. T. and Fielitz, B. D. 1977. Long-term dependence in common stock returns. *Journal of Financial Economics*, 4(3): 339-349.

Hosking, J.R.M. 1981. Fractional Differencing, *Biometrika*, 68: 165-176

Hurst, H. E. 1951. Long term storage capacity of reservoirs, *Transactions of the American Society of Civil Engineering*, 116(1): 770-799

Jin, H.J., & Frechette, D. (2004). Fractional integration in agricultural futures price volatilities. *American Journal of Agricultural Economics*, 86(2): 432–443

Kang, S. H. and Yoon, S.-M. 2007. Long memory properties in return and volatility: Evidence from the Korean stock market, *Physica A*, 385(2): 591–600

Kang, S. H., Cheong, C., and Yoon, S.-M. 2010. Long memory volatility in Chinese stock markets, *Physica A*, 389(7): 1425–1433

Kapadia, N. and Pu, X. 2012. Limited arbitrage between equity and credit markets, *Journal of Financial Economics*, 105(3): 542–564

Kasman, A., Kasman, S., and Torun, E. 2009. Dual long memory property in returns and volatility: Evidence from the CEE countries' stock markets, *Emerging Markets Review*, 10(2): 122–39

Laurent, S., and Peters, J.-P. 2002. G@RCH 2.2: An Ox Package for Estimating and Forecasting Various ARCH Models, *Journal of Economic Surveys*, 16: 447–485

Leland, H. E. and Toft, K. B. 1996. Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads. *The Journal of Finance*, 51(3): 987-1019

Lobato, I.N., and Savin, N.E. 1998. Real and spurious long-memory properties of stock-market data. *Journal of Business and Economic Statistics*, 16(3): 261–268

Lo, A.W, 1991. Long-term memory in stock market prices. *Econometrica*, 59(5): 1279–1313

Lovreta, L. and Silaghi, F. 2017. The surface of implied firm's asset volatility, *Journal of Banking & Finance*, *forecoming*

Lux, T. and Kaizoji, T. 2007. Forecasting volatility and volume in the Tokyo Stock Market: Long memory, fractality and regime switching, *Journal of Economic Dynamics & Control*, 31(6): 1808–1843

Mandelbrot B.B., and Van Ness, J.W., 1968, Fractional Brownian Motion, Fractional Noises and Applications, *SIAM Review*, 10(4): 422–437

Mandelbrot B.B., 1971, When can a price be arbitraged efficiently? A limit to the validity of the random walk and martingale models, *Review of Economics and Statistics*, 53(3): 225–236

Merton, R. C. 1974. On the Pricing of Corporate Debt: The Risk Structure of Interest Rates, *The Journal of Finance,* 29(2): 449

Nelson, D. B. 1991. Conditional Heteroskedasticity in Asset Returns: A New Approach, *Econometrica*, 59(2): 347–70

Oh, G., Kim, S., and Eom, C. 2008. Long-term memory and volatility clustering in high-frequency price changes. *Physica A*, 387: 1247–1254.

Perron, P., and Vogelsang, T. 1992. Nonstationarity and level shifts with an application to purchasing power parity. *Journal of Business and Economic Statistics*, 10: 301–320

Peng, C.-K., Buldyrev S.V., Havlin, S., Simons, M., Stanley, H.E., and Goldberger, A.L. 1994. Mosaic organization of DNA nucleotides. *Physical Review E,* 49: 1685–1689

Peng, C.-K., Havlin, S., Stanley, H.E., Goldberger, A.L. 1995. Quantification of scaling exponents and crossover phenomena in nonstationary heartbeat time series. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 5: 82–87

Gabriel, A.S. 2012. Evaluating the forecasting performance of GARCH models. Evidence from Romania. *Procedia - Social and Behavioral Sciences*, 62: 1006-1010

Uctum, R., Renou-Maissant, P., Prat, G. and Lecarpentier-Moyal, S. 2017. Persistence of announcement effects on the intraday volatility of stock returns: Evidence from individual data. *Review of Financial Economics*, 35: 43-56

Vilasuso, J. 2002. Forecasting Exchange Rate Volatility, *Economics Letters*, 76(1): 59–64

Xu, Y. and Malkiel, B. G. 2003. Investigating the Behavior of Idiosyncratic Volatility, *The Journal of Business*, 76(4): 613–645

Appendix A

$$F(\tau) = N[h_1(\tau)] + \left(\frac{V}{V_B}\right)^{-2a} N[h_2(\tau)]$$

$$G(\tau) = \left(\frac{V}{V_B}\right)^{-a+z} N[q_1(\tau)] + \left(\frac{V}{V_B}\right)^{-a-z} N[q_2(\tau)]$$

with

$$q_1(\tau) = \frac{-b - z\sigma^2\tau}{\sigma\sqrt{\tau}}; \qquad q_2(\tau) = \frac{-b + z\sigma^2\tau}{\sigma\sqrt{\tau}};$$

$$h_1(\tau) = \frac{-b - a\sigma^2\tau}{\sigma\sqrt{\tau}}; \qquad h_2(\tau) = \frac{-b + a\sigma^2\tau}{\sigma\sqrt{\tau}};$$

$$a = \frac{r - \delta - \sigma^2/2}{\sigma^2}; \qquad b = \ln\left(\frac{V}{V_B}\right); \qquad z = \frac{\left[(a\sigma^2)^2 + 2r\sigma^2\right]^{1/2}}{\sigma^2}.$$

Appendix B

comp	AO model					IO model				
	r (E)	r (V_{SM})	r (V_{SMN})	r (V_{KMV})	r (V_{Proxy})	r (E)	r (V_{SM})	r (V_{SMN})	r (V_{KMV})	r (V_{Proxy})
c1	-31.0840	-28.9612	-28.7185	-28.9771	-28.9581	-28.6678	-29.0266	-28.5754	-29.0449	-29.0231
c2	-27.4000	-27.5972	-33.4667	-27.6257	-27.7786	-27.8339	-28.1121	-27.8309	-28.1372	-28.3346
c3	-25.3890	-25.3851	-25.2997	-25.4018	-25.4418	-28.6797	-28.8581	-28.7787	-28.8682	-28.8674
c4	-27.5610	-27.7026	-27.7266	-27.8216	-24.3184	-57.2822	-27.9671	-27.9517	-27.9760	-27.9709
c5	-25.2760	-25.3575	-25.2806	-25.3457	-26.8371	-27.4969	-27.2208	-27.1841	-27.2058	-27.2147
c6	-41.9940	-27.3582	-28.0111	-27.2382	-27.3292	-41.9849	-27.1902	-28.0258	-27.0645	-27.1155
c7	-25.7850	-26.6863	-26.8732	-26.6928	-26.6725	-34.6318	-54.9146	-54.9481	-54.8920	-54.9845
c8	-30.2090	-24.7372	-24.7252	-24.7216	-24.8597	-30.7527	-30.9261	-30.9223	-30.9012	-31.0880
c9	-27.1170	-26.8597	-27.0078	-26.8464	-26.8040	-27.5539	-27.3212	-27.4789	-27.3042	-27.2541
c10	-25.8300	-28.3228	-28.3713	-28.3053	-28.3193	-27.8175	-28.4179	-28.4844	-28.4043	-28.4381
c11	-27.0650	-29.7018	-29.6832	-29.6984	-29.7434	-43.2797	-59.4905	-59.2843	-59.4608	-59.4667
c12	-31.0550	-36.6282	-31.0850	-31.1096	-31.1413	-36.8909	-36.7220	-36.6865	-36.7416	-36.7330
c13	-25.1710	-24.8953	-27.3946	-24.9159	-24.9701	-57.6596	-57.3980	-27.5727	-27.6812	-27.6577
c14	-25.5600	-41.5985	-41.5342	-41.6560	-41.6218	-27.4887	-27.8346	-27.7548	-27.8386	-27.8851
c15	-28.0300	-27.9754	-28.0365	-27.9636	-27.9795	-28.5835	-28.2346	-28.3055	-28.2208	-28.2451
c16	-25.3940	-25.5003	-25.2835	-25.5095	-25.5394	-38.7121	-37.8166	-37.6221	-37.8350	-37.7690
c17	-43.6260	-29.4224	-29.4415	-29.4159	-29.5168	-43.6132	-43.3413	-43.2829	-43.3273	-43.4353
c18	-25.6810	-24.4115	-25.1181	-24.4306	-24.5294	-26.6106	-42.7836	-43.1686	-42.6950	-42.9504
c19	-25.9630	-25.9051	-25.9149	-25.9471	-25.9980	-37.9706	-37.4587	-37.3163	-37.3835	-37.4060
c20	-29.5520	-29.1124	-29.2533	-29.1719	-29.1874	-38.3257	-37.6057	-37.8018	-37.7002	-37.8170
c21	-27.7200	-27.4088	-27.3072	-27.4143	-27.3024	-27.9396	-27.5964	-27.5139	-27.5160	-27.5958
c22	-26.0240	-37.5218	-36.8288	-37.3308	-37.4071	-38.3778	-38.2005	-37.2199	-37.9544	-38.0046
c23	-25.8670	-25.9023	-26.0223	-25.9085	-25.9430	-36.3902	-26.0615	-26.2273	-26.0675	-26.0773
c24	-29.7640	-29.6719	-27.0047	-26.8238	-29.7118	-30.0212	-29.6930	-29.7555	-29.7279	-29.7095
c25	-26.7950	-26.7142	-26.6938	-26.7001	-26.7199	-30.1508	-30.0256	-29.8819	-30.0154	-30.0389
c26	-43.3920	-24.9930	-25.0153	-25.0252	-25.0866	-36.2521	-36.3180	-36.2433	-36.2210	-36.1036
c27	-24.8410	-25.0936	-25.5645	-25.0304	-25.0058	-25.3876	-26.2655	-26.4041	-26.2033	-26.2165
c28	-25.8900	-25.1334	-25.2297	-25.2278	-25.3955	-26.4236	-25.4435	-25.5286	-25.5507	-25.8742
c29	-37.1880	-37.2804	-45.7659	-37.2344	-37.2513	-29.0287	-38.4978	-37.9096	-38.4189	-38.3786
c30	-24.3160	-26.8678	-27.2317	-26.9390	-26.8635	-35.5444	-56.1959	-27.6793	-56.2041	-56.2170
c31	-27.0340	-26.6643	-26.8697	-26.6506	-26.8969	-56.7540	-42.3727	-42.4848	-42.3866	-56.1185
c32	-25.9730	-30.0453	-29.9716	-30.0628	-30.0552	-26.2403	-25.6717	-25.4759	-25.6554	-25.6766
c33	-25.9400	-36.3690	-30.9712	-36.3334	-31.0041	-31.7279	-36.6711	-36.2555	-36.6295	-28.6071
c34	-26.2610	-26.0519	-26.1443	-26.0117	-26.1195	-28.6920	-43.6984	-43.5094	-43.6139	-43.6217
c35	-25.6190	-25.1894	-25.2588	-25.1661	-25.1350	-36.1165	-35.5269	-35.5918	-61.3775	-61.5134
c36	-28.3580	-28.3577	-28.3282	-28.3606	-28.4619	-28.8131	-28.5859	-28.5795	-28.5966	-28.6636
c37	-28.5030	-28.4358	-28.3395	-28.4953	-28.5074	-29.0622	-28.9403	-28.8416	-29.0174	-29.0302
c38	-24.7810	-26.7278	-24.5422	-34.8855	-26.7105	-35.6951	-35.0118	-35.5602	-35.0065	-34.9743
c39	-28.1920	-28.2573	-26.3426	-28.3514	-28.3856	-28.5600	-28.4479	-28.4258	-28.5506	-28.5036
c40	-27.9110	-27.3634	-27.3086	-27.3742	-27.3552	-28.1311	-27.6343	-27.5726	-27.6379	-27.6180
c41	-41.1790	-26.0181	-26.0651	-25.9948	-26.2068	-57.1885	-57.6687	-57.7035	-57.5924	-57.7695
c42	-27.3450	-24.7562	-34.7797	-24.7188	-24.9328	-35.1044	-57.3005	-57.0816	-57.2842	-57.2796
c43	-26.4370	-28.3346	-28.2733	-28.3276	-28.4363	-26.8052	-26.6695	-26.6741	-28.6976	-28.7904
c44	-27.7320	-25.7631	-25.8245	-25.7487	-27.3708	-26.9336	-26.7041	-26.7728	-26.6758	-28.4608
c45	-28.2820	-26.1835	-26.0757	-26.1409	-28.2889	-28.8649	-26.8939	-26.7795	-26.8261	-27.1129
c46	-31.5470	-33.2323	-33.1089	-33.0349	-33.0784	-31.7751	-33.1437	-33.1158	-32.9362	-33.0019
c47	-31.8060	-36.4342	-36.5623	-36.4582	-36.5547	-66.2596	-36.9990	-37.0815	-37.0394	-37.0078
c48	-28.1770	-31.2329	-31.1704	-25.8402	-25.8983	-31.3386	-31.3947	-31.3211	-31.3954	-31.4092
c49	-41.7270	-41.7304	-41.7009	-41.7193	-41.7404	-23.7375	-56.9906	-35.1204	-56.9469	-56.9957
c50	-27.9760	-25.2927	-25.3825	-25.2584	-25.3276	-28.7057	-28.0664	-28.1502	-28.0535	-28.1709
c51	-27.3730	-26.9342	-26.8442	-26.9352	-27.0805	-59.0661	-58.4842	-58.4442	-58.4144	-58.4614
c52	-27.0000	-26.4656	-26.6282	-28.4718	-28.4997	-27.1478	-28.7452	-28.6376	-28.8368	-28.8525

Test statistics for the Perron and Vogelsang (1992) unit root test with single mean shift (AO and IO models).

Appendix C

comp	AO model					IO model				
	r **(E)**	***r*** **(V_{SM})**	***r*** **(V_{SMN})**	***r*** **(V_{KMV})**	***r*** **(V_{Proxy})**	***r*** **(E)**	***r*** **(V_{SM})**	***r*** **(V_{SMN})**	***r*** **(V_{KMV})**	***r*** **(V_{Proxy})**
c1	-25.5390	-28.8660	-28.7380	-28.8650	-28.8500	-28.8550	-29.1715	-29.0577	-29.1981	-29.1589
c2	-27.4970	-27.6170	-27.3640	-27.6460	-27.6670	-28.0800	-27.8370	-27.6727	-27.8982	-27.9823
c3	-24.8640	-25.6610	-25.5840	-25.6780	-25.7270	-28.9460	-29.1842	-29.1152	-28.8408	-29.2005
c4	-24.4970	-28.0270	-40.9480	-28.1020	-28.0310	-58.0770	-28.0521	-28.1167	-28.1860	-28.0115
c5	-27.5640	-27.0150	-26.9920	-26.9960	-27.0070	-28.7090	-27.8019	-27.7709	-27.7849	-27.7589
c6	-26.7840	-24.4760	-28.2030	-27.6460	-27.7690	-42.2110	-27.8307	-28.2801	-27.6399	-27.8332
c7	-24.0660	-27.1880	-27.3120	-27.1960	-27.1620	-35.4460	-27.6173	-27.7262	-27.6309	-27.6165
c8	-30.8440	-31.0290	-30.9510	-31.0080	-31.1010	-31.4770	-31.3321	-31.2614	-31.3176	-31.3631
c9	-24.4530	-24.3270	-24.5190	-24.2990	-24.3430	-27.7870	-27.5027	-27.6635	-27.4790	-27.4784
c10	-25.7450	-28.2930	-28.3490	-28.2720	-28.3040	-28.2120	-28.5352	-28.6045	-28.5268	-28.5823
c11	-30.3420	-30.0300	-30.0120	-30.0370	-30.0500	-43.8570	-59.9927	-59.7777	-59.9740	-59.9317
c12	-28.1510	-31.3350	-31.3060	-31.3540	-31.3810	-37.4600	-37.1203	-37.0831	-37.1690	-37.1498
c13	-25.8540	-25.6860	-28.1740	-25.6580	-25.7210	-36.7459	-28.8545	-28.6572	-28.8589	-28.8333
c14	-27.4870	-41.8420	-41.7690	-41.8950	-28.0390	-28.4388	-27.8973	-27.7926	-27.8976	-28.0146
c15	-25.9940	-25.6760	-25.6850	-25.6680	-25.7410	-29.0609	-28.2903	-28.3626	-28.2820	-28.2619
c16	-25.6420	-25.5790	-25.3820	-25.5890	-25.6400	-39.4784	-38.4288	-38.2269	-38.4484	-38.3618
c17	-24.7670	-24.0370	-24.1190	-24.0470	-24.1950	-44.0063	-43.7668	-43.7238	-43.7530	-43.8470
c18	-25.9810	-24.6590	-25.3580	-24.6550	-24.8520	-43.7570	-43.5163	-43.9311	-43.4043	-43.7121
c19	-26.1040	-25.9980	-25.1700	-26.0560	-26.1360	-38.2117	-37.6490	-37.0953	-37.5878	-37.6326
c20	-29.2490	-28.8730	-28.9460	-28.9160	-28.9690	-38.4888	-37.6517	-37.9729	-37.7604	-37.8819
c21	-25.6370	-27.6360	-27.2190	-27.6200	-27.5960	-28.3838	-27.8925	-27.7349	-27.8594	-27.8591
c22	-25.7980	-32.3970	-31.8400	-32.3100	-32.3650	-33.9427	-38.0994	-37.5499	-33.3708	-37.9328
c23	-25.8610	-25.8890	-25.9400	-25.8900	-25.9290	-37.3140	-26.7368	-26.8281	-26.7092	-26.7478
c24	-26.4610	-26.6820	-26.8370	-26.6410	-26.6540	-29.6225	-29.7174	-29.7849	-29.7526	-30.0475
c25	-24.4070	-24.3270	-24.3530	-24.3080	-24.3100	-30.5231	-30.3619	-30.2241	-30.3463	-30.3709
c26	-24.1050	-24.7660	-24.7850	-24.8000	-24.8540	-36.5682	-36.6741	-36.5804	-36.5797	-36.4129
c27	-24.7640	-24.9910	-25.6060	-24.9310	-24.8840	-25.7556	-26.9665	-27.1522	-26.8971	-27.0141
c28	-25.7890	-25.3170	-25.4030	-25.3990	-25.6620	-26.0743	-26.0069	-25.3200	-26.1027	-26.4757
c29	-25.4710	-27.6880	-25.1530	-27.7250	-27.8260	-29.7258	-39.3845	-38.5140	-39.2866	-39.2653
c30	-24.4500	-24.3590	-24.7030	-24.4290	-23.7920	-36.3380	-42.1151	-56.9898	-42.1809	-42.1126
c31	-26.9240	-26.0380	-26.4960	-26.0490	-26.2600	-56.9828	-43.0573	-28.3484	-43.0428	-42.8018
c32	-26.2390	-26.1910	-26.0490	-26.1570	-26.1660	-25.9868	-25.7685	-25.5681	-25.7508	-25.7720
c33	-31.3790	-26.3470	-36.1750	-36.5090	-36.6050	-32.0356	-28.7557	-28.7398	-28.7943	-28.8124
c34	-25.8340	-26.0600	-26.6400	-26.0220	-26.1410	-28.6976	-36.4772	-36.3235	-43.8813	-36.3760
c35	-25.4030	-25.4780	-25.0900	-25.4520	-25.4200	-36.4081	-35.7194	-35.7986	-61.6821	-35.6360
c36	-28.6580	-28.5440	-28.4910	-28.5540	-28.6620	-29.6411	-28.8014	-28.7769	-28.8182	-28.8974
c37	-26.2860	-26.2870	-26.3780	-26.3810	-26.3420	-29.5668	-29.3298	-29.2195	-29.3887	-29.4193
c38	-24.7490	-23.9060	-24.5460	-23.9090	-23.7900	-36.5111	-35.5207	-36.2273	-35.5000	-35.5112
c39	-28.0740	-28.2330	-35.4700	-26.3080	-28.3850	-28.6629	-28.5263	-28.5149	-28.7429	-28.6802
c40	-24.9530	-29.3290	-29.2690	-29.3430	-29.3250	-28.8153	-27.9668	-27.9795	-28.0437	-27.9317
c41	-26.5710	-26.1310	-26.1570	-26.1160	-26.3420	-57.4867	-57.9179	-57.9400	-57.8626	-58.0200
c42	-27.2890	-24.9550	-24.8220	-24.9210	-29.9270	-35.3914	-57.3373	-57.1163	-57.3218	-57.2645
c43	-27.0430	-28.5680	-28.5710	-28.5790	-28.7110	-27.4827	-26.9069	-26.9675	-28.9818	-29.0808
c44	-25.6720	-25.6610	-25.7590	-27.1480	-34.0870	-28.9057	-26.5087	-26.5715	-26.4821	-28.4190
c45	-28.3120	-25.9920	-25.8820	-25.9560	-26.2570	-28.9129	-26.7792	-26.6701	-26.7115	-27.0272
c46	-31.4390	-33.3160	-33.2220	-33.1080	-33.0460	-31.4570	-33.3269	-33.2375	-33.1168	-33.0661
c47	-33.7030	-35.2740	-35.4310	-35.2660	-35.5830	-60.2619	-36.7141	-36.8659	-36.7267	-36.9042
c48	-30.8230	-31.0670	-31.3100	-28.0200	-28.0750	-43.6123	-31.5836	-31.5187	-31.6116	-31.5429
c49	-29.7110	-41.6900	-41.6830	-41.6930	-41.6850	-35.4187	-23.7408	-35.0975	-23.7381	-23.7358
c50	-25.5550	-25.4930	-25.5700	-25.4430	-25.5450	-28.9003	-26.5964	-26.6318	-28.5108	-28.6474
c51	-24.6500	-26.8580	-26.8230	-26.8610	-27.1060	-59.2877	-58.5163	-58.6669	-58.4448	-58.4959
c52	-27.3540	-31.1080	-26.2380	-28.7160	-28.6980	-27.4330	-29.0503	-29.1319	-29.1519	-29.1278

Test statistics for the Clemente et al. (1998) unit root test with double mean shift (AO and IO models).

Appendix D

comp	w	s.e.	*d*	s.e.	α	s.e.	β	s.e.	GED	s.e.
1	0.180	*(0.071)*	0.3855	*(0.062)*	0.1971	*(0.085)*	0.5156	*(0.104)*	1.2765	*(0.040)*
2	0.084	*(0.037)*	0.4195	*(0.068)*	0.2813	*(0.060)*	0.6272	*(0.082)*	1.3466	*(0.045)*
3	0.148	*(0.061)*	0.3668	*(0.058)*	0.1954	*(0.073)*	0.5150	*(0.097)*	1.3962	*(0.046)*
4	0.208	*(0.095)*	0.3973	*(0.075)*	0.1950	*(0.130)*	0.4452	*(0.166)*	1.2161	*(0.041)*
5	0.125	*(0.045)*	0.4471	*(0.077)*	0.1443	*(0.054)*	0.5527	*(0.091)*	1.2898	*(0.043)*
6	0.164	*(0.078)*	0.4208	*(0.069)*	0.3083	*(0.089)*	0.6059	*(0.113)*	1.2113	*(0.037)*
7	0.103	*(0.047)*	0.4633	*(0.086)*	0.2590	*(0.047)*	0.6761	*(0.084)*	1.4293	*(0.046)*
8	0.115	*(0.049)*	0.3940	*(0.061)*	0.3746	*(0.067)*	0.6686	*(0.075)*	1.2764	*(0.041)*
9	0.230	*(0.085)*	0.3149	*(0.053)*	0.3161	*(0.085)*	0.5466	*(0.092)*	1.2187	*(0.039)*
10	0.123	*(0.052)*	0.3627	*(0.062)*	0.3579	*(0.076)*	0.6177	*(0.091)*	1.2537	*(0.041)*
11	0.205	*(0.086)*	0.3014	*(0.052)*	0.2188	*(0.087)*	0.4834	*(0.110)*	1.2737	*(0.041)*
12	0.199	*(0.070)*	0.3080	*(0.043)*	0.0767	*(0.101)*	0.3528	*(0.120)*	1.3107	*(0.042)*
13	0.298	*(0.121)*	0.3138	*(0.100)*	0.2993	*(0.103)*	0.4807	*(0.128)*	1.0378	*(0.029)*
14	0.067	*(0.026)*	0.4532	*(0.071)*	0.3033	*(0.055)*	0.6703	*(0.065)*	1.1422	*(0.036)*
15	0.047	*(0.019)*	0.4183	*(0.072)*	0.4812	*(0.069)*	0.7287	*(0.064)*	1.2163	*(0.037)*
16	0.117	*(0.051)*	0.2938	*(0.060)*	0.2932	*(0.088)*	0.5099	*(0.107)*	1.4362	*(0.049)*
17	0.071	*(0.030)*	0.4052	*(0.066)*	0.3773	*(0.060)*	0.6886	*(0.068)*	1.2123	*(0.039)*
18	0.122	*(0.049)*	0.4106	*(0.084)*	0.2688	*(0.085)*	0.5626	*(0.115)*	1.0506	*(0.034)*
19	0.053	*(0.022)*	0.3729	*(0.067)*	0.3874	*(0.059)*	0.6635	*(0.065)*	1.3560	*(0.046)*
20	0.095	*(0.039)*	0.3406	*(0.065)*	0.3398	*(0.083)*	0.5603	*(0.101)*	1.2695	*(0.041)*
21	0.155	*(0.089)*	0.2653	*(0.053)*	0.3717	*(0.165)*	0.5564	*(0.175)*	1.2147	*(0.039)*
22	0.501	*(0.000)*	0.1706	*(0.022)*	0.0052	*(0.000)*	0.0146	*(0.000)*	1.2343	*(0.038)*
23	0.121	*(0.044)*	0.3853	*(0.070)*	0.3295	*(0.079)*	0.6063	*(0.085)*	1.0300	*(0.029)*
24	0.070	*(0.027)*	0.4642	*(0.091)*	0.3666	*(0.069)*	0.7047	*(0.078)*	1.2097	*(0.036)*
25	0.080	*(0.033)*	0.3313	*(0.059)*	0.3380	*(0.082)*	0.5808	*(0.095)*	1.2459	*(0.040)*
26	0.093	*(0.036)*	0.3829	*(0.061)*	0.2668	*(0.088)*	0.5381	*(0.107)*	1.3520	*(0.045)*
27	0.158	*(0.054)*	0.3667	*(0.061)*	0.1851	*(0.092)*	0.4451	*(0.109)*	1.1650	*(0.036)*
28	0.190	*(0.068)*	0.3911	*(0.079)*	0.1337	*(0.109)*	0.4321	*(0.144)*	1.0435	*(0.031)*
29	0.067	*(0.023)*	0.5425	*(0.078)*	0.2403	*(0.078)*	0.5793	*(0.092)*	1.1319	*(0.035)*
30	0.082	*(0.031)*	0.4701	*(0.070)*	0.1986	*(0.061)*	0.5901	*(0.083)*	1.2164	*(0.040)*
31	0.166	*(0.073)*	0.3430	*(0.059)*	0.3112	*(0.137)*	0.4995	*(0.149)*	1.1972	*(0.039)*
32	0.225	*(0.085)*	0.3019	*(0.051)*	0.1790	*(0.128)*	0.3782	*(0.139)*	1.1110	*(0.033)*
33	0.091	*(0.040)*	0.7235	*(0.167)*	0.1082	*(0.082)*	0.8076	*(0.092)*	1.4111	*(0.048)*
34	0.219	*(0.091)*	0.2835	*(0.057)*	0.3104	*(0.104)*	0.4876	*(0.121)*	1.2957	*(0.041)*
35	0.186	*(0.074)*	0.3204	*(0.059)*	0.1940	*(0.084)*	0.4520	*(0.108)*	1.2948	*(0.044)*
36	0.049	*(0.021)*	0.5504	*(0.081)*	0.2620	*(0.053)*	0.7201	*(0.055)*	1.2829	*(0.042)*
37	0.114	*(0.039)*	0.4266	*(0.066)*	0.1115	*(0.061)*	0.4796	*(0.090)*	1.4148	*(0.049)*
38	0.115	*(0.058)*	0.3303	*(0.077)*	0.3763	*(0.099)*	0.6042	*(0.120)*	1.1146	*(0.033)*
39	0.437	*(0.163)*	0.2858	*(0.047)*	0.0110	*(0.172)*	0.2269	*(0.191)*	1.1492	*(0.034)*
40	0.116	*(0.048)*	0.3660	*(0.064)*	0.2118	*(0.077)*	0.5215	*(0.104)*	1.2053	*(0.040)*
41	0.094	*(0.043)*	0.4159	*(0.068)*	0.3767	*(0.058)*	0.6979	*(0.069)*	1.2549	*(0.044)*
42	0.257	*(0.112)*	0.3150	*(0.048)*	0.1950	*(0.133)*	0.4475	*(0.151)*	1.0714	*(0.035)*
43	0.099	*(0.039)*	0.3919	*(0.080)*	0.3813	*(0.058)*	0.6823	*(0.066)*	1.1570	*(0.034)*
44	0.129	*(0.049)*	0.3653	*(0.065)*	0.1282	*(0.097)*	0.4301	*(0.125)*	1.0960	*(0.033)*
45	0.212	*(0.086)*	0.2641	*(0.047)*	0.3317	*(0.101)*	0.5435	*(0.111)*	1.2131	*(0.038)*
46	0.244	*(0.093)*	0.3700	*(0.054)*	0.3016	*(0.088)*	0.5496	*(0.097)*	1.1661	*(0.041)*
47	0.571	*(0.000)*	0.1938	*(0.038)*	0.0133	*(0.000)*	0.0437	*(0.000)*	0.9806	*(0.029)*
48	0.122	*(0.162)*	0.2767	*(0.090)*	0.5744	*(0.306)*	0.7152	*(0.322)*	1.2089	*(0.037)*
49	0.181	*(0.070)*	0.3535	*(0.053)*	0.3932	*(0.068)*	0.6710	*(0.068)*	1.3297	*(0.044)*
50	0.072	*(0.022)*	0.4899	*(0.082)*	0.2124	*(0.059)*	0.6027	*(0.076)*	1.2717	*(0.041)*
51	0.070	*(0.028)*	0.3834	*(0.066)*	0.3556	*(0.068)*	0.6395	*(0.077)*	1.2693	*(0.042)*
52	0.118	*(0.050)*	0.3654	*(0.063)*	0.2916	*(0.090)*	0.5511	*(0.111)*	1.4232	*(0.049)*

This table reports the firm level estimates of the FIGARCH(1,d,1) coefficients for equity returns. Standard errors for the estimated coefficients are given in parenthesis.

Appendix E

comp	w	s.e.	*d*	s.e.	α	s.e.	β	s.e.	GED	s.e.
1	0.034	*(0.014)*	0.3530	*(0.054)*	0.2578	*(0.076)*	0.5604	*(0.090)*	1.4108	*(0.045)*
2	0.010	*(0.004)*	0.3478	*(0.059)*	0.3714	*(0.065)*	0.6452	*(0.077)*	1.3745	*(0.046)*
3	0.029	*(0.012)*	0.3854	*(0.059)*	0.2252	*(0.067)*	0.5584	*(0.087)*	1.3840	*(0.045)*
4	0.139	*(0.189)*	0.2630	*(0.100)*	0.0962	*(0.684)*	0.2280	*(0.765)*	1.2581	*(0.043)*
5	0.004	*(0.002)*	0.5357	*(0.077)*	0.1936	*(0.044)*	0.6830	*(0.064)*	1.3208	*(0.043)*
6	0.006	*(0.004)*	0.3397	*(0.064)*	0.3162	*(0.201)*	0.5257	*(0.230)*	1.2491	*(0.036)*
7	0.008	*(0.004)*	0.4516	*(0.097)*	0.3117	*(0.048)*	0.7218	*(0.076)*	1.4329	*(0.046)*
8	0.008	*(0.003)*	0.5107	*(0.091)*	0.3777	*(0.059)*	0.7814	*(0.055)*	1.2875	*(0.040)*
9	0.010	*(0.004)*	0.3500	*(0.059)*	0.3502	*(0.073)*	0.6141	*(0.081)*	1.2447	*(0.038)*
10	0.073	*(0.029)*	0.2995	*(0.054)*	0.4430	*(0.074)*	0.6479	*(0.078)*	1.3232	*(0.042)*
11	0.093	*(0.037)*	0.2667	*(0.047)*	0.2526	*(0.088)*	0.4872	*(0.107)*	1.2933	*(0.042)*
12	0.109	*(0.038)*	0.2969	*(0.041)*	0.0890	*(0.102)*	0.3572	*(0.121)*	1.3208	*(0.043)*
13	0.168	*(0.068)*	0.2195	*(0.053)*	0.3349	*(0.132)*	0.4454	*(0.143)*	1.0859	*(0.030)*
14	0.014	*(0.005)*	0.4450	*(0.061)*	0.3306	*(0.049)*	0.7038	*(0.052)*	1.1621	*(0.036)*
15	0.012	*(0.005)*	0.4201	*(0.075)*	0.5049	*(0.067)*	0.7496	*(0.058)*	1.2203	*(0.037)*
16	0.059	*(0.024)*	0.3120	*(0.059)*	0.2914	*(0.073)*	0.5378	*(0.091)*	1.4823	*(0.051)*
17	0.019	*(0.008)*	0.3114	*(0.057)*	0.4368	*(0.068)*	0.6642	*(0.074)*	1.2265	*(0.039)*
18	0.019	*(0.009)*	0.2483	*(0.056)*	0.2589	*(0.152)*	0.4151	*(0.172)*	1.1125	*(0.035)*
19	0.032	*(0.014)*	0.3596	*(0.066)*	0.3992	*(0.057)*	0.6665	*(0.067)*	1.4220	*(0.048)*
20	0.053	*(0.022)*	0.2869	*(0.054)*	0.3552	*(0.095)*	0.5306	*(0.108)*	1.2737	*(0.041)*
21	0.024	*(0.011)*	0.3337	*(0.058)*	0.4536	*(0.086)*	0.6930	*(0.087)*	1.2262	*(0.039)*
22	0.043	*(0.035)*	0.2654	*(0.081)*	0.5782	*(0.140)*	0.6951	*(0.175)*	1.2790	*(0.040)*
23	0.035	*(0.012)*	0.3559	*(0.061)*	0.3238	*(0.068)*	0.6114	*(0.072)*	1.1412	*(0.032)*
24	0.035	*(0.014)*	0.3841	*(0.075)*	0.3446	*(0.070)*	0.6299	*(0.089)*	1.3111	*(0.039)*
25	0.021	*(0.008)*	0.3681	*(0.059)*	0.3393	*(0.070)*	0.6208	*(0.079)*	1.2433	*(0.039)*
26	0.027	*(0.012)*	0.3724	*(0.064)*	0.3343	*(0.081)*	0.6073	*(0.100)*	1.4196	*(0.048)*
27	0.014	*(0.005)*	0.3632	*(0.062)*	0.2259	*(0.096)*	0.4742	*(0.111)*	1.1768	*(0.036)*
28	0.018	*(0.008)*	0.3470	*(0.078)*	0.0949	*(0.171)*	0.3352	*(0.214)*	1.1059	*(0.031)*
29	0.014	*(0.005)*	0.5367	*(0.082)*	0.2808	*(0.088)*	0.5896	*(0.102)*	1.1530	*(0.035)*
30	0.025	*(0.010)*	0.3843	*(0.064)*	0.2277	*(0.072)*	0.5365	*(0.093)*	1.2434	*(0.041)*
31	0.008	*(0.006)*	0.3063	*(0.075)*	0.4472	*(0.210)*	0.5800	*(0.226)*	1.2166	*(0.039)*
32	0.071	*(0.029)*	0.2973	*(0.053)*	0.2321	*(0.136)*	0.4241	*(0.148)*	1.1073	*(0.033)*
33	0.047	*(0.018)*	0.5073	*(0.079)*	0.2188	*(0.043)*	0.6952	*(0.064)*	1.4409	*(0.048)*
34	0.054	*(0.021)*	0.3005	*(0.053)*	0.2949	*(0.086)*	0.5181	*(0.106)*	1.3605	*(0.043)*
35	0.035	*(0.014)*	0.3769	*(0.061)*	0.2442	*(0.066)*	0.5611	*(0.085)*	1.2999	*(0.043)*
36	0.010	*(0.004)*	0.4979	*(0.073)*	0.3017	*(0.050)*	0.7194	*(0.051)*	1.3252	*(0.043)*
37	0.040	*(0.015)*	0.4428	*(0.064)*	0.1576	*(0.056)*	0.5391	*(0.081)*	1.4354	*(0.049)*
38	0.032	*(0.015)*	0.3056	*(0.065)*	0.4069	*(0.101)*	0.6105	*(0.113)*	1.1438	*(0.034)*
39	0.131	*(0.057)*	0.2381	*(0.045)*	0.2483	*(0.155)*	0.4204	*(0.170)*	1.2229	*(0.037)*
40	0.033	*(0.014)*	0.3384	*(0.057)*	0.2471	*(0.078)*	0.5354	*(0.100)*	1.2197	*(0.039)*
41	0.017	*(0.007)*	0.3893	*(0.067)*	0.4212	*(0.060)*	0.7154	*(0.063)*	1.2781	*(0.043)*
42	0.109	*(0.051)*	0.2456	*(0.043)*	0.1647	*(0.196)*	0.3550	*(0.214)*	1.1073	*(0.034)*
43	0.031	*(0.012)*	0.3811	*(0.074)*	0.3788	*(0.051)*	0.6861	*(0.061)*	1.2241	*(0.037)*
44	0.016	*(0.006)*	0.4052	*(0.073)*	0.1724	*(0.078)*	0.5147	*(0.106)*	1.0998	*(0.033)*
45	0.026	*(0.011)*	0.2820	*(0.052)*	0.3733	*(0.092)*	0.5949	*(0.101)*	1.1997	*(0.037)*
46	0.077	*(0.034)*	0.2891	*(0.053)*	0.3227	*(0.134)*	0.4931	*(0.145)*	1.1935	*(0.042)*
47	0.173	*(0.000)*	0.1524	*(0.032)*	0.0000	*(0.000)*	0.0386	*(0.000)*	1.0122	*(0.029)*
48	0.057	*(0.031)*	0.2684	*(0.052)*	0.5772	*(0.125)*	0.7291	*(0.121)*	1.2862	*(0.039)*
49	0.170	*(0.072)*	0.2618	*(0.050)*	0.4423	*(0.091)*	0.6353	*(0.093)*	1.3398	*(0.045)*
50	0.015	*(0.005)*	0.4568	*(0.085)*	0.2489	*(0.062)*	0.6016	*(0.081)*	1.2861	*(0.041)*
51	0.021	*(0.008)*	0.3654	*(0.060)*	0.3757	*(0.069)*	0.6444	*(0.074)*	1.2779	*(0.042)*
52	0.030	*(0.013)*	0.3441	*(0.061)*	0.3685	*(0.070)*	0.6240	*(0.085)*	1.4638	*(0.050)*

This table reports the firm level estimates of the FIGARCH(1,d,1) coefficients for firm's asset returns (the base case estimation of the firm's asset values). Standard errors for the estimated coefficients are given in parenthesis.

Chapter 2

Modeling and Forecasting Firm-specific Volatility: the Role of Asymmetry and Long-memory

1. Introduction

Estimation and forecasting of the time-varying volatility has been widely analyzed in the literature. A general conclusion is that volatility possesses important stylized features such as volatility clustering, asymmetric effect of positive and negative shocks, mean-reversion and long-memory, among others. In order to account for these features many different GARCH-type models have been developed, essentially by modifying the conditional variance equation. As a result, a crucial question to answer became which of these models provides the best fit to data and provides the best forecast of volatility over some future time horizon. A number of papers have previously addressed this question for equities and equity indices, commodities and currencies (Vilasuso, 2002; Andersen et al. 2003; Pong et al. 2004; Martens and Zein, 2004; Hansen and Lunde, 2005; among others). However, the previous literature is silent about the performance of different models for the firm's asset volatility process. In their seminal paper, Choi and Richardson (2016) directly apply an EGARCH model to firm-level returns but do not analyze which volatility model is best suited to equity and asset returns. In this paper, we fill this gap and perform a horse race among different GARCH-type model specifications (GARCH, EGARCH, IGARCH, FIGARCH, HYGARCH, FIEGARCH, and FIAPARCH) in terms of modeling and forecasting firm's asset and equity volatility.

We especially focus on two important volatility characteristics: the asymmetry and the long-range persistence. Asymmetry in this context denotes a different response of conditional volatility to positive and negative shocks of the same magnitude, whereas long-memory denotes a slow hyperbolic rate of decay of the lagged squared innovations in the conditional variance. In general, previous literature on performance of the GARCH-type models reveals that asymmetric GARCH models provide better out-of-sample forecasts when compared to symmetric GARCH model (Awartani and Corradi, 2005), and that long-memory properties have important implications for volatility forecasting (Poon and Granger, 2003; Dionisio et al. 2007; Kang and Yoon, 2007). However, literature on performance of the GARCH-type models has largely focused on stock indexes rather than individual firms and to the best of our knowledge no previous study has analyzed forecasting performance of different models for the firm's asset volatility. In this paper we contribute to the current literature by analyzing to what extent the two important properties of volatility, the asymmetry and long-memory, improve modeling and forecasting accuracy of the firm's asset and equity volatility. We also study eventual differences in asymmetry and long-memory between the two firm-level volatilities and its relation to financial leverage.

We employ a variety of GARCH-type models for a sample of 52 non-financial iTraxx Europe companies that we track during the 2004-2016 period. Even though the underlying data set is the same, different conditional volatility models are expected to produce different volatility forecasts due to their different underlying assumptions. We apply out-of-sample forecasting at monthly horizon using iterated forecasts based on daily data and a rolling window scheme.[14] Our main findings could be summarized as follows.

[14] It has been shown in the literature that iterated forecasts dominate direct forecasts (Marcellino et al. 2006; Ghysels et al. 2019).

First, all estimated models are in general able to properly capture the dynamics in the conditional variance of equity and firm's asset returns. In the case of asymmetric GARCH models, the asymmetry coefficient is negative and statistically significant for most of the considered return series suggesting that negative shocks affect more conditional volatility than positive shocks of the same magnitude. However, the importance of asymmetry seems to be more pronounced for equity than for firm's asset volatility: the magnitude of the sign coefficient is higher in absolute terms and is statistically significant for a higher percentage of companies when compared to firm's asset volatility. In the case of long-memory GARCH models, fractional integrating parameter is statistically significant for all the series considered, suggesting the presence of long-memory in firm-level volatility.

Second, we analyze the impact of financial leverage on asymmetry and long-range persistence. We find that asymmetry is more pronounced in equity than in firm's asset volatility. This difference in the level of asymmetry is statistically significant and increases with leverage being higher for high leverage firms and lower for low leverage firms. As regards long-range persistence we confirm the evidence of the long-run dependence in firm level equity and asset volatility (González-Pla and Lovreta, 2019). In general, the persistence is slightly higher for equity than for firm's asset volatility, however, the difference in equity - firm's asset volatility persistence is decreasing with leverage. Once the asymmetry is allowed in the model in addition to long-memory, the difference in equity - firm's asset volatility persistence notably decreases. Actually, in the FIEGARCH and FIAPARCH models firm's asset volatility turns out to be more persistent than equity volatility for higher leverage groups.

Third, we evaluate the relative predictive power of monthly forecasts from different GARCH models using several methods. We apply statistical tests for equal and superior predictive accuracy, and additionally test the economic importance of the volatility forecasts by estimating CDS spread forecasts, as a measure of the credit risk of a company. The results that we obtain are unambiguous. We find that more sophisticated models that simultaneously capture asymmetric and long-memory effects provide more accurate out-of-sample one month ahead volatility forecasts for both firm-specific equity and firm's asset volatility. Out of all considered models FIEGARCH model performs the best, followed by the FIAPARCH model. Models that capture only asymmetry (EGARCH) or only long-memory in volatility (FIGARCH and HYGARCH) show higher forecasting accuracy when compared to the symmetric and short-memory GARCH model. Our results also show relatively higher relevance of asymmetry vs. long-memory in equity and firm's asset volatility forecasting: EGARCH seems to perform better than FIGARCH or HYGARCH.

In summary, our out-of-sample forecasting results reveal superiority of long-memory over short-memory models, superiority of asymmetric over symmetric models, and suggest a higher relative importance of asymmetry over long-memory. The best performance could be clearly attributed to more sophisticated models that simultaneously capture asymmetry and long-memory. On the opposite pole are clearly GARCH and IGARCH models that are outperformed by alternative models in most of the cases. In terms of model ranking we do not find any substantial difference between equity and firm's asset volatility. The main implication of these findings is that long-memory features should be accounted for in models for pricing financial instruments that require an estimate of the future volatility as an input. For forecasting purposes it is important to account for the two features at the same time.

The rests of the paper is organized as follows. Section 2 provides the literature review on volatility estimation and forecasting using different GARCH-type models.

Section 3 provides a summary of the seven GARCH-type volatility models used in this paper. Section 4 provides a summary of the methods used to estimate the underlying firm's asset value process. Section 5 explains the data set used. Section 6 presents the in-sample fit of the seven models. Section 7 explores the effect of leverage on asymmetry and long-memory properties. Section 8 presents the results of the out-of-sample forecasting performance. Section 9 concludes.

2. Literature Review

Financial volatility modelling has been extensively studied in the literature. It has become a prime importance topic of research because of its broad applications in portfolio management, asset allocation, asset pricing and risk management, among others. The main features that a good volatility model has to exhibit are the flexibility to appropriately fit the data and the capability to forecast volatility. Engle and Patton (2001) pointed out some stylized facts that seem to be important in modelling and forecasting volatility of financial returns, like asymmetry, mean reversion, long-range persistence and the influence of exogenous variables. In terms of forecasting, the superiority in forecasting volatility of GARCH models over classical models (equally and exponentially moving average models) has been widely evidenced in the literature. For example, Brailsford and Faff (1996) evaluated the forecasting accuracy of volatility of the Australian stock market monthly returns with classical and GARCH volatility models. They observed that GARCH models are clearly better in forecasting terms than classical models, and that the choice of the preferred model is sensitive to the error measure employed. Literature on financial volatility and its characteristics is extensive and here we review only those papers that are closely related to our work. A detailed literature review on forecasting volatility in financial markets is provided in Poon and Granger (2003).

As noted in the introduction, we are primarily interested in two important volatility characteristics in terms of both estimation and forecasting of firm-level volatility: the long-memory (i.e. long-range persistence) and asymmetric response of volatility to negative and positive shocks of the same magnitude. It is important to note that persistence is more general term than long-memory and can be generally defined as slowly decaying auto-correlations in the volatility of financial time series. Ding and Meade (2010), for example, defined different scenarios based on persistence and observed that as persistence increased, forecasting accuracy improved for the three models they used: stochastic volatility, GARCH and EWMA (i.e. exponentially weighted moving average). Long-memory or long-range persistence refers to a slow hyperbolical decay of the autocorrelation function at long lags. This volatility feature, however, cannot be properly modelled with short-memory GARCH or EGARCH models.

The finance literature has well established that volatility of stock market returns exhibit long-memory (Ding et al. 1993; Granger and Ding, 1996; Baillie et al. 1996; Bollerslev and Mikkelsen, 1996; Lobato and Savin, 1998; Breidt et al. 1998, among others). The studies on stock market volatility, however, do not agree about the order of fractional integration (i.e. the degree of long-memory). Christensen and Nielsen (2006), for example, find that the fractional order of stock market volatility is in the stationary region, whereas Bandi et al. (2006) find the order of integration to be in the non-stationary region. Lux and Kaizoji (2007) estimated with FIGARCH approximately 1,200 stocks traded in the Tokyo Stock Exchange from 1975 to 2001. They observed that the fractional differencing (i.e. long-memory) parameter has a mean value of 0.340 for 100 companies with largest trading volume, and 0.367 for a random sample of 100 companies. Kang and Yoon (2007) have examined stock index returns for the Korean stock market within the ARFIMA-FIGARCH framework. For the best model specification, these authors report the long-memory parameter of 0.471 and 0.231 for KOSPI and KOSDAQ index return volatility. They have also compared the performance of FIGARCH to the GARCH, IGARCH models, reporting superiority of the FIGARCH estimates. Dionisio et al. (2007) have examined several international stock market indexes. They found that the long-memory parameter is always statistically significant, ranging between 0.3 and 0.6, and report the preference of the FIGARCH model over the GARCH and IGARCH models, as well. Lopes and Prass (2014) observed when estimating the Brazilian stock market index in an empirical application, that FIEGARCH is able to capture same information as ARCH, GARCH or EGARCH models, in addition to long memory in volatility. However, these authors find that the performance of fractionally integrated and non-integrated models is very similar if the time series is very persistent.

Previous literature has shown that asymmetry is another important feature of stock market volatility (Pagan and Schwert, 1990; Engle and Ng, 1993; Bekaert and Wu, 2000; Awartani and Corradi, 2005, among others). For example, Awartani and Corradi (2005) examine the role of asymmetries in the GRACH-type modes in predicting volatility of S&P-500 stock index returns. They find that asymmetric GARCH models provide better out-of-sample forecasts when compared to symmetric GARCH model. They find that predictive improvement of asymmetric models is lower for longer-horizons and higher for one-step ahead forecasts. Hansen and Lunde (2005) compared 330 GARCH-type models and showed that GARCH(1,1) outperformed all other tested models when forecasting one-day ahead exchange rate volatility. However, when they modelled the volatility of the IBM equity returns, they found that the

GARCH(1,1) model is inferior to other, more sophisticated models that take into account an asymmetric response in volatility.

As regards the firm's asset volatility and its properties the evidence is rather scarce. In their seminal paper, Choi and Richardson (2016) observed noticeable differences between the time-series properties of equity and asset volatility at the individual firm level. First, they performed an estimation for both equity and asset returns using an EGARCH model proposed by Nelson (1991). In that context, they observed that equity volatility is significantly more persistent and asymmetric than asset volatility for levered firms, and the differences between equity volatility and asset volatility generally increase with leverage. Second, they observed that the asymmetry between equity returns and volatility can be explained in combinations of three components: financial leverage, volatility feedback and operating leverage. Third, they analysed the nature structure of equity volatility and its relation to asset volatility and financial leverage, where financial leverage and asset volatility should have different dynamic properties over time, and long/short time properties may depend differently on asset volatility and financial leverage. In order to go in depth in this behaviour, they used an autocorrelogram of volatility and leverage, where implied equity volatility is taken from one-month implied volatility from equity options and firm asset volatility was estimated using and EGARCH(1,1) model. In this respect, they concluded that asset volatility is more tied to the short term equity volatility, and leverage to the long term equity volatility. Their conclusions nevertheless are based on the short-memory model, which, as indicated before cannot capture eventual long-memory properties. In contrast, González-Pla and Lovreta (2019) show that firm's asset volatility possesses long-memory features and that fractionally integrated models should be used to fit firm-level returns. Actually, Choi and Richardson (2016) indicate in their paper that they didn't address the question which volatility model best fits equity and asset returns. This research question we analyse in depth in the following sections.

3. Model Framework

In this section, we present the Generalized Auto-Regressive Conditional Heteroscedasticity (GARCH) models used to estimate and forecast firm's asset and equity volatility. These models are based on Bollerslev (1986) who generalized the ARCH model formulated initially by Engle (1982). The GARCH-type models are defined by two equations: a conditional mean and a conditional variance equation. The conditional mean equation defines the behaviour of the returns in order to isolate unexpected returns or market shocks, which are further assumed to follow some conditional distribution (i.e. a Gaussian, Student's t or Generalized Error Distribution). The GARCH-type models that we examine in this paper are GARCH, EGARCH, IGARCH, FIGARCH, HYGARCH, FIEGARCH, and FIAPARCH. For tractability purposes in this paper we assume the same conditional mean equation (first order autoregressive process) for all the models and time series, and the same conditional error distribution (Generalized Error Distribution). Detailed description of the conditional variance equation for each of the models considered is provided below.

3.1 Short-memory models

3.1.1 GARCH

The conditional variance equation of the GARCH model, proposed by Bollerslev (1986), is defined as a function of p past values of the conditional variance and q past values of the squared shocks. The GARCH(p,q) model is defined as follows:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \equiv \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2 \qquad (1)$$

where $\varepsilon_t = \sigma_t z_t$, $z_t \sim i.i.d.(0,1)$ and, $L(\cdot)$ denotes the lag operator. Defining $v_t \equiv \varepsilon_t^2 - \sigma_t^2$, the equation (1) can be rewritten as follows:

$$[1 - \alpha(L) - \beta(L)]\varepsilon_t^2 = \omega + [1 - \beta(L)]v_t \quad (2)$$

If the roots of $1 - \alpha(L) - \beta(L) = 0$ lie outside the unit circle, then the parameters accomplish the following restriction $\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1$, and the GARCH(p,q) process for ε_t^2 is covariance stationary (also called weakly stationary).[15] The unconditional volatility of the process can be easily obtained from equation (1), assuming that $E(\varepsilon_t^2) \equiv \sigma_t^2$ and $\sigma^2 = \sigma_t^2 = \sigma_{t-1}^2$, and is given by:

$$\sigma^2 = E(\varepsilon_t^2) = \frac{\omega}{1 - \sum_{i=1}^{q} \alpha_i - \sum_{j=1}^{p} \beta_j} \quad (3)$$

Due to its simplicity and forecasting performance, GARCH model has been commonly used to estimate and forecast volatility for different asset classes. However, it is important to point out that GARCH model has a symmetric behaviour, as it assumes that the response of conditional variance to negative market shocks is the same as to positive shocks of the same magnitude. In that sense, the symmetric GARCH model doesn't consider the asymmetric effect (i.e. the leverage effect) commonly observed in equity returns, where volatility increases more following a negative shock than following a positive one (of the same magnitude). The leverage effect is considered a stylized fact, particularly noticeable in equity markets, where there is usually a strong negative correlation between the equity returns and the change in volatility. Namely, this means that volatility is usually higher in response to "bad news", and lower in response to "good news".[16]

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[15] A real-valued stochastic process y_t is considered covariance stationary if the mean $E[y_t]$ and the covariance between y_t and y_{t+k} do not depend on t and are finite.

[16] The opposite effect commonly occurs in commodity markets, as price increases in commodity markets are "bad news".

3.1.2 EGARCH

The Exponential GARCH (EGARCH) is originally introduced by Nelson (1991). The main feature of this model is that it allows for asymmetric response of conditional volatility to positive and negative shocks, a feature that is commonly observed in financial markets. The conditional variance equation in the EGARCH model is formulated in terms of the log of the variance. Such a transformation ensures that conditional variance is always positive, without imposing any constraints on the coefficients (as in the case of the symmetric GARCH model).[17] Following Bollerslev and Mikkelsen (1996) and Bollerslev and Mikkelsen (1999) the EGARCH model could be formulated as follows:

$$\ln(\sigma_t^2) = \omega + [1 - \beta(L)]^{-1}[1 + \alpha(L)]g(z_{t-1}) \tag{4}$$

where, $z_t \equiv \varepsilon_t \sigma_t^{-1}$. The function $g(z_t)$ captures the asymmetric behaviour between financial returns and volatility by taking into account not only the size, but also the sign of lagged residuals. Nelson (1991) suggested $g(z_t)$ function as a linear combination of z_t and $|z_t|$:

$$g(z_t) = \gamma_1 z_t + \gamma_2[|z_t| - E|z_t|] \tag{5}$$

where, $E|z_t|$ depends on the assumption made on the error distribution. For the GED distribution $E|z_t|$ is given by:

$$E|z_t| = \sqrt{\frac{\Gamma\left(\frac{1}{v}\right)2^{-\frac{2}{v}}}{\Gamma\left(\frac{3}{v}\right)}} 2^{\left(\frac{1}{v}\right)} \frac{\Gamma\left(\frac{2}{v}\right)}{\Gamma\left(\frac{1}{v}\right)} \tag{6}$$

The constant γ_1 in equation (5) represents the sign effect, while γ_2 captures the magnitude effect similarly as in the GARCH model.[18] Therefore, an asymmetric behaviour is observed when the estimated γ_1 coefficient of the model is statistically

[17] Despite the fact that log function may be negative, the variance will always be positive.
[18] The conditional variance in the GARCH model is a function only of the magnitude of the lagged residuals.

significant. If coefficient $\gamma_1 < 0$ than conditional variances increase proportionally more following a negative than following a positive shock of the same magnitude. This effect is sometimes referred to as the "leverage effect". Note that when $E|z_t| = 0$, the function $g(z_t)$ is proportional to $\gamma_1 + \gamma_2$ in case of positive returns ($z_t > 0$), whereas for negative returns ($z_t < 0$) is proportional to $-\gamma_1 + \gamma_2$. As the conditional variance is positive regardless of the value of estimated parameters, it is not necessary to impose additional constraints. The only sufficient condition for the covariance stationarity of the model is $\sum_{j=1}^{p} \beta_j < 1$.

To align with the most popular formulation of the EGARCH model used in practical applications and to ensure comparability, a specific formulation of the EGARCH model used in this paper is given by:[19]

$$\ln(\sigma_t^2) = \omega(1-\beta) + \gamma_1 z_{t-1} + \gamma_2[|z_{t-1}| - E|z_{t-1}|] + \beta \ln(\sigma_{t-1}^2) \qquad (7)$$

3.2 Integrated models

3.2.1 IGARCH

The Integrated GARCH (IGARCH) was introduced by Engle and Bollerslev (1986) in order to capture the apparent persistence of the estimated conditional variance processes. Namely, in most empirical applications of the GARCH model $\alpha + \beta$ is found to be close to unity. The IGARCH can be deducted from the autoregressive polynomial in equation 2 which has one unit root:[20]

$$\phi(L)(1-L)\varepsilon_t^2 = \omega + [1-\beta(L)]v_t, \qquad (8)$$

where $\phi(L) = [1-\alpha(L)-\beta(L)](1-L)^{-1}$.

The model equation of an IGARCH(1,1) which satisfies that $\alpha + \beta = 1$ for $\beta \in (0,1)$ reduces to:

[19] In OxMetrics this reduces to the estimation of EGARCH(p,0) model, that is, with ARCH order set to 0.
[20] IGARCH(p,q) model can be generalized satisfying the unit root condition: $\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j = 1$.

$$\sigma_t^2 = \omega + (1-\beta)\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \quad (9)$$

In the IGARCH model shocks to conditional variance affect future forecasts at all horizons. By definition, unconditional variance (σ^2) does not exist and converges to an infinite limit $\sigma^2 = E(\sigma_t^2) \to \infty$ as $t \to \infty$, but conditional variance (σ_t^2) exists and is well defined (Bollerslev et al. 1994).

3.3 Fractionally integrated models

It has been evidenced in the literature that stock market volatility could be characterized as a long memory and mean reverting process (Ding et al. 1993; Granger and Ding, 1996; Lobato and Savin, 1998, among others). This behavior can be understood through the order of integration of a time series. Integrated processes of a time series refers to the minimum number of differences needed to obtain a covariance stationary process, usually denoted as $I(d)$ where d is the order of integration. A time-series Y_t is fractional of order d, if $\Delta^d Y_t$ is fractional of order zero and a covariance stationary process. Thus, a non-stationary process may be made covariance stationary by differencing the original time series. For instance, Y_t is an integrated process of order one $I(1)$ if the first difference of the time series is covariance stationary; this is commonly denoted in terms of the difference operator Δ; $I(1)$: $\Delta Y_t = Y_t - Y_{t-1}$. On the contrary, Y_t is an integrated process of order zero $I(0)$ if Y_t does not to be differenced to result in a covariance stationary process. As a result, an $I(0)$ time series shows short memory as its autocorrelation function decays at an exponential rate implying that distant observations in time are independent; an $I(d)$ time series, with $0 < d < 1$, dies out at a slow hyperbolic rate which implies that far distant observations in time show a weak correlation; and, finally an $I(1)$ series declines at a linear rate and doesn't not exhibit a mean reversion. A time-series is said to follow an ergodic and stationary process with long memory if d is in the interval $(0,0.5)$; if d is higher than 0.5, the time-series is said to follow a long-memory non-stationary process. The GARCH and EGARCH models described above are $I(0)$ time series, as shocks to conditional volatility converge with a fast exponential decay rate.[21] Generally, a time-series of stock market volatility is shown to be neither $I(0)$ nor $I(1)$, but fractionally integrated of order d, denoted as $I(d)$. For this reason, we consider several fractionally integrated GARCH-type models which allow for long memory in conditional volatility.

3.3.1 FIGARCH

The Fractionally Integrated GARCH (FIGARCH) model, proposed by Baillie et al. (1996), allows persistence in the conditional volatility by introducing a fractional differencing parameter d. This contrast with GARCH or EGARCH models that only consider short-term memory or IGARCH model that assumes infinite memory. In the FIGARCH processes lagged squared innovations in the conditional variance decay at a slow hyperbolic rate and are modelled through the fractional differencing parameter (d). In short, FIGARCH introduces a new parameter to model the long memory behaviour, while the short-run dynamics are modelled with the usual GARCH parameters. FIGARCH model is obtained by replacing the first difference operator in equation (8) by the fractional differencing operator:

$$\phi(L)(1-L)^d \varepsilon_t^2 = \omega + [1-\beta(L)]v_t \tag{10}$$

where $0 < d < 1$ and all the roots of $\phi(L)$ and $[1-\beta(L)]$ lie outside the unit circle. The FIGARCH process is strictly stationary and ergodic for $0 \le d \le 1$. The FIGARCH behaves as a GARCH process when $d = 0$ and as an IGARCH process when $d = 1$, whereas for $0 < d < 1$ is a long memory process. Furthermore, only if $d < 0.5$, the FIGARCH model is covariance stationary. Rearranging the terms of the equation (10) we obtain the conditional variance for the FIGARCH model:

[21] GARCH(1,1) model shows a decay rate of $(\alpha + \beta)$ and EGARCH(1,1) model a decay rate of (β).

$$\sigma_t^2 = \omega[1-\beta(L)]^{-1} + \{1-[1-\beta(L)]^{-1}\phi(L)(1-L)^d\}\varepsilon_t^2 \qquad (11)$$

The fractional differencing operator $(1-L)^d$ is a sum of infinite terms defined by its Maclaurin series expansion, and it has been truncated in this paper at 1,000 lags, according to the usual procedure adopted in the literature (Bollerslev and Mikkelsen, 1996).

3.3.2 HYGARCH

Davidson (2004) observed that the FIGARCH model does not really perform as an intermediate $I(d)$ process because of discontinuities in the amplitude (variations in the conditional volatility) or in the length of the memory as d approaches to 0 or 1. As a result, Davidson (2004) proposes a Hyperbolic GARCH (HYGARCH) model, which removes the unitary amplitude restriction inherent in the FIGARCH and introduces a new parameter κ to model this phenomenon. In the HYGARCH model the fractional differencing operator $(1-L)^d$ is replaced by $[(1-\kappa)+\kappa(1-L)^d]$. Thus, the HYGARCH is a more flexible approach than the FIGARCH that behaves as FIGARCH or GARCH models when $\kappa = 1$ and $\kappa = 0$, respectively.[22] The specification of the conditional variance in the HYGARCH model is as follows:

$$\sigma_t^2 = \omega[1-\beta(L)]^{-1} + \{1-[1-\beta(L)]^{-1}\phi(L)[1+\kappa((1-L)^d-1)]\}\varepsilon_t^2 \qquad (12)$$

3.3.3 FIEGARCH

The Fractionally Integrated EGARCH (FIEGARCH) model, proposed by Bollerslev and Mikkelsen (1996), introduces a fractional integrating parameter into an asymmetric EGARCH model. The FIEGARCH model extends the asymmetric EGARCH model by introducing a fractional differencing parameter d to model the long-memory behaviour. Bollerslev and Mikkelsen (1996) point out that evidence suggests the presence of long-run dependence in stock market volatility in such a way

[22] It also nests the IGARCH model for $k = d = 1$.

that behaves as a mean-reverting fractionally integrated process and shocks to conditional variance fade at hyperbolic rate. The procedure to obtain a fractionally integrated EGARCH model is very similar to the factorization previously explained in the case of the FIGARCH model. Namely, the polynomial in equation (4) is factorized $[1-\beta(L)]=\phi(L)(1-L)^d$ to allow the fractionally integrated behaviour, where all the roots of $\phi(z)=0$ lie outside the unit circle.[23] Under these conditions, the resultant conditional variance equation is given by:

$$log(\sigma_t^2)=\omega+\phi(L)^{-1}(1-L)^{-d}[1+\alpha(L)]g(z_{t-1}) \quad (13)$$

Consequently, the FIEGARCH model is reduced to a short-memory EGARCH model proposed by Nelson (1991), when $d=0$. Similarly to the EGARCH model, different formulations of the FIEGARCH model exist in the literature. For consistency, we estimate the FIEGARCH model in its simplest form, as in Ruiz and Veiga (2008).

3.3.4 FIAPARCH

The Fractionally Integrated Asymmetric Power ARCH (FIAPARCH) model was introduced by Tse (1998). He proposed a Fractionally Integrated extension of the Asymmetric Power ARCH model (APARCH) by Ding et al. (1993). The APARCH model is given as follows:

$$\sigma_t^\delta=\omega+\alpha(L)(|\varepsilon_t|-\gamma\varepsilon_t)^\delta+\beta(L)\sigma_t^\delta \quad (14)$$

The APARCH model introduces a Box-Cox transformation of the conditional standard deviation as a function of the parameter δ, while the parameter γ reflects the asymmetric effect of shocks. In this case, given the negative sign of the asymmetric parameter γ, a positive value for this coefficient implies that lagged negative shocks have a higher impact on conditional volatility than lagged positive shocks, and vice versa. Probably, the most interesting feature of this model, is that includes seven

[23] Lopes and Prass (2014) performed a deep analysis of the FIEGARCH processes and its main properties such as stationarity, ergodicity or invertibility conditions.

ARCH-type models as special cases (see Appendix A of Ding et al. 1993). For example, the APARCH model reduces to an ARCH model proposed by Engle (1982) when $\delta = 2, \gamma = 0$, and $\beta = 0$ or to a GARCH model proposed by Bollerslev (1986) when $\delta = 2, \gamma = 0$ and $\beta \neq 0$. Rearranging the terms of the equation (14), we obtain the following expression:

$$(1 - \alpha(L) - \beta(L))(|\varepsilon_t| - \gamma\varepsilon_t)^\delta = \omega + [1 - \beta(L)]((|\varepsilon_t| - \gamma\varepsilon_t)^\delta - \sigma_t^\delta) \quad (15)$$

Then, the polynomial is factorized to be fractionally integrated, $(1 - \alpha(L) - \beta(L)) = \phi(L)(1 - L)^d$, where all the roots of $\phi(z) = 0$ are outside the unit circle and $0 \leq d \leq 1$. Details about existence and sufficient conditions of the model can be found in Tse (1998). The resultant conditional variance equation of the FIAPARCH model considering the long memory behaviour with the fractional integrating parameter d is given by:

$$\sigma_t^\delta = \omega + \{1 - [1 - \beta(L)]^{-1}\phi(L)(1 - L)^d\}(|\varepsilon_t| - \gamma\varepsilon_t)^\delta \quad (16)$$

The FIAPARCH model could be also considered as a generalization of the FIGARCH model. Namely, for $\delta = 2$ and $\gamma = 0$ the FIAPARCH model reduces to a FIGARCH model.

4. Firm's asset value estimation

There are several approaches to estimate the latent firm's asset value. In this paper, we consider as a baseline case a method suggested by Forte (2011) who applies a structural model of default to infer the underlying firm's asset value. This model represents the debt structure as the sum of an arbitrary number of coupon bonds, each with its own principal, coupon, and maturity. Therefore, the model is flexible enough to accommodate any possible debt profile. The market value of firm's assets using the structural model framework will be denoted as, V_{SM}.

The second possibility that we consider is the approach of Charitou et al. (2013), in which the firm's asset value, V, is treated as observable and is proxied with the sum of the market value of equity and the face value of debt (P). That is, $V = S + P$. The face value of debt (i.e. the face value of total liabilities) is treated in Charitou et al. (2013) as the "original default boundary". In other words, the default boundary is set to: $STL + 1 \times LTL$, where STL stands for short-term liabilities and LTL for long-term liabilities, respectively. The market value of firm's assets using this approach will be denoted as, V_{Proxy}.

The third approach that we consider is the procedure of Bharath and Shumway (2008), in which the firm's asset value is simply the sum of the market value of equity and the default threshold which is set as in KMV to the value $STL + 0.5 \times LTL$. The market value of firm's assets resulting from this approach will be denoted as, V_{KMV}.

For each approach, the equity and firm's asset daily returns are calculated as follows:

$$r_{i,t} = ln\left(\frac{V_{i,t}}{V_{i,t-1}}\right) \tag{17}$$

where $r_{i,t}$ is the return at time t for the firm i and $V_{i,t}$ is the firm's valuation at time t for the firm i.

It is important to mention that we do not use the approach of Choi and Richardson (2016) due to data limitations. Although we could obtain the information on the market value of bonds, this would result in a significant reduction in the number of companies considered, and even more important, we would not be able to capture the full market value of debt given that only a small portion of company's debt is publically traded. In other words, we would still miss the significant portion of the firm's debt which is represented by bank loans.

5. Data and descriptive statistics

In this paper we consider a sample of 52 non-financial companies that belong to the iTraxx Europe index during the 2004-2016 period. The iTraxx Europe index comprises the most liquid 125 CDS referencing European investment-grade companies. The data frequency is daily which results in a sample size per company of 3,391 observations. Daily data on market capitalization, CDS spreads, and 1-10 year swap rates, as well as yearly data on current liabilities, total liabilities, interest expenses and cash dividends are downloaded from Datastream. We consider only 5-year Euro-denominated CDS contracts on senior unsecured debt. We exclude companies in the banking and financial sector due their different capital structure, private companies, and companies for which we lack data on either market capitalization or CDS spreads for the overall sample period. Additionally, we exclude all companies involved in corporate operations that resulted in significant jumps in the market capitalization time-series but not in the time-series of adjusted prices. The complete list of companies considered is provided in Appendix A (see Table A.1).

Table 1. *Main characteristics of the companies in the sample*

	Mean	Median	Minimum	Maximum	Std. dev.
MC in m €	25,431.38	17,400.33	4,243.23	125,691.30	22,519.31
Leverage	0.52	0.52	0.28	0.85	0.13
Equity volatility	0.30	0.29	0.21	0.48	0.06

This table reports the main descriptive statistics on a cross-sectional basis for the set of 52 non-financial companies. MC refers to market capitalization. Equity volatility refers to the unconditional historical equity volatility, calculated as the annualized standard deviation of the continuously compounded returns on equity. Leverage is defined as the ratio of the book value of total liabilities to the proxy for the market value of the firm (i.e., the sum of the market value of equity and the book value of total liabilities).

The main descriptive statistics of 52 companies under analysis are shown in Table 1. The companies have an average market capitalization of €25.43 billion, a leverage of 0.52 and a historical equity volatility of 30%. Leverage is defined as the ratio of the book value of total liabilities to the proxy for the market value of the firm, historical equity volatility is the annualized standard deviation of the continuously compounded returns on equity.

Table 2. *Descriptive statistics of equity and firm's asset returns*

	***r*(E)**	***r*(V_{SM})**	***r*(V_{KMV})**	***r*(V_{Proxy})**
Panel A				
Mean	0.00015	0.00016	0.00015	0.00014
Standard deviation	0.01890	0.00860	0.00860	0.00860
Skewness	0.08304	-0.03682	-0.04363	-0.06960
Kurtosis	15.43038	11.77791	11.69407	11.36487
Panel B				
Q(10)	25.6132	22.9192	22.7408	22.4136
	(78.85%)	(63.46%)	(65.38%)	(63.46%)
$Q^2(10)$	788.2130	524.7031	520.5706	513.1666
	(98.08%)	(96.15%)	(96.15%)	(98.08%)
ARCH	92.2538	77.5149	77.0270	73.8417
	(100%)	(100%)	(100%)	(100%)
J-B	131,614.80	25,039.54	24,040.52	21,520.91
	(100%)	(100%)	(100%)	(100%)
ADF	-58.0923	-58.1000	-58.0179	-58.1711
	(100%)	(100%)	(100%)	(100%)

Panel A of Table 2 reports the cross-sectional average of the main descriptive statistics (mean, standard deviation, skewness and kurtosis) of equity $r(E)$, and firm's asset returns $r(V_{SM})$, $r(V_{KMV})$, and $r(V_{Proxy})$, for the set of 52 non-financial companies. Panel B of Table 2 reports the results of the five key diagnostic tests: Q(10) and $Q^2(10)$ refer to the Ljung-Box statistics for tenth-order serial correlation in returns and squared returns, respectively; ARCH is the Engle's ARCH test; J-B refers to the Jarque-Bera normality test; ADF is the Augmented Dickey-Fuller unit root test. For each test, the cross-sectional average of the test statistics as well as the percentage of companies for which the null hypothesis is rejected at least at the 5% significance level (in parenthesis), are reported.

The cross-sectional average of the main descriptive statistics for daily equity and firm's asset returns are depicted in Panel A of Table 2. Equity and firm's asset returns have on average similar mean, however, equity returns evidence a larger unconditional volatility (standard deviation) compared to firm's asset returns, which is consistent with the literature (Choi and Richardson, 2016). Equity returns are on average positively skewed (0.083) while firm's asset returns show a negative-skewed behaviour (from -0.0368 to -0.0696). Both return series show a leptokurtic behaviour (excess of kurtosis), and kurtosis is generally higher for equity returns than for its corresponding firm's asset returns, indicating that equity returns are more fat-tailed. The main descriptive statistics of firm's asset returns obtained through three different estimation methods (V_{SM}, V_{KMV} and V_{Proxy}) show similar magnitudes in terms of the mean, standard deviation, skewness and kurtosis.

We perform several diagnostic statistical tests on the time-series of equity and firm's asset returns. The main cross-sectional results are reported in Panel B of Table 2. First, we check for the presence of serial correlation in returns and squared returns using the Ljung-Box test with 10 lags. The Ljung-Box test rejects the null hypothesis of no serial correlation at the 5% level for 78.85% of the companies in the case of equity returns and for 63.46% of the companies in the case of V_{SM} firm's asset returns. Similar results are obtained for V_{KMV} and V_{Proxy} firm's asset returns with rejection rates of 65.38% and 63.46%, respectively. In the case of squared returns, the Ljung-Box test rejects the null hypothesis of no serial correlation at the 5% level for practically all of the companies considered. On this matter, we additionally perform the Engle's ARCH test for heteroscedasticity. The null hypothesis of no conditional heteroscedasticity (no ARCH effects) is rejected for all considered time-series, which confirms strong evidence of conditional heteroscedasticity in our dataset. The Jarque-Bera test for normality rejects the null hypothesis of Gaussianity at the 1% significance level in all of the equity and firm's asset returns. Finally, the Augmented Dickey-Fuller test (ADF) rejects the unit root null hypothesis for both firm's asset and equity returns, indicating stationarity of return series for all companies in the sample. Overall, the preliminary analysis shows that data employed in this study presents common features of financial returns data, and supports the use of GARCH methodology. Detailed descriptive statistics at the firm level are provided in the Appendix B, Table B.1. - Table B.4.

6. Model Estimation

The sample consists of daily equity and firm's asset returns from January 1st, 2004 to December 31st, 2016, which results in a sample size of 3,391 observations per company. The initial empirical analysis shows that for the absolute majority of the companies the null hypothesis of no autocorrelation in returns is rejected, and we further model the conditional mean equation as a first-order autoregressive process. For tractability purposes, we use a common framework for all the time-series of equity and firm asset returns, and impose the $AR(1)$ model to all the companies in the sample. Specifically, for all the GARCH-type models that we consider the conditional mean equation is specified as follows: $y_t = \varphi_0 + \varphi_1 y_{t-1} + \varepsilon_t$. The conditional distribution of the error term is estimated with a Generalized Error Distribution (GED). Nelson (1991) proposed the use of GED as error distribution to be able to adjust the deviation of the tail, especially important in stock returns, which are usually fat-tailed. Gao et al. (2012) observed that GED outperforms Normal and Student-t distributions when modelling financial series that evidence fat-tails and leptokurtic behaviour. To estimate and forecast equity and firm's asset volatility we use G@RCH 6.1 developed by Laurent and Peters (2002), which is an OxMetrics module used for estimation of univariate or multivariate ARCH-type models.[24] For estimation purposes returns are expressed in percentage terms (i.e. $100 \times r_{i,t}$). To facilitate comparison between the models, all the GARCH models are estimated with one lag of the variance term and one lag of the innovation term. Next, we summarize the estimation results for the

[24] Although it is possible to estimate some of the models using different software packages, for consistency, in all of the estimations we use OxMetrics. This is to avoid eventual comparison problems as different software packages may use different parameterization of the same model, different optimization algorithms, etc.

conditional variance equation for each model that we consider. Detailed estimation results at the firm level are provided in the Appendix C, Table C.1. - Table C.28.

The summary of the estimation results for the GARCH model are presented in Table 3. All the α and β parameters in equity and firm's asset conditional volatility are statistically significant at least at the 5% level. Considering that GARCH is a short memory model with an exponential decay rate of $(\alpha + \beta)$, the sum of α and β parameters is close to 1 which indicates that the volatility of the returns is very persistent, but mean-reverting. In that context of estimation and taking $(\alpha + \beta)$ as a measure of estimated persistence, we observe that equity volatility is on average more persistent (0.9856) than firm's asset volatility (0.9844 for V_{SM}, 0.9839 for V_{KMV}, and 0.9852 for V_{Proxy}). However, the difference in mean is not statistically significant.[25] Interestingly, this slightly higher equity persistence is mainly due to the higher α coefficient for equity, whereas the coefficient β is on average lower for equity. In the cross-sectional analysis, we observe that the effect of past squared returns on the current conditional volatility is higher for equity (0.0623) than for firm's asset volatility (from 0.0525 to 0.0535), whereas the effect of past values of volatility on the current conditional volatility is higher in firm's asset (from 0.9307 to 0.9319) than in equity (0.9233) volatility modelling. Moreover, the difference in mean is statistically significant in the case of α coefficient, and not statistically significant in the case of the β coefficient.

The GARCH estimation results show a very persistent behaviour of equity and firm's asset volatility as the sum of estimated parameters $(\alpha + \beta) \rightarrow 1$, suggesting that shocks to conditional volatility have a long-lasting effect. To assess this phenomenon,

[25] We fail to reject the null hypothesis of equality of means. For our base case, V_{SM}, the Welch t-test statistics for equality of means is equal to 0.1830 (p-value 0.6699). For V_{KMV} and V_{Proxy} the Welch t-test statistics equals 0.3868 (p-value 0.5356) and 0.0230 (p-value 0.8798), respectively.

we have performed a Wald test at the firm level defining the null hypothesis as $H_0: (\alpha + \beta) = 1$. In the case of equity we fail to reject the null hypothesis for 15 companies (28.85% of the sample), whereas in the case of firm's assets we fail to reject the null hypothesis for 10 companies (19.23% of the sample), for all specifications of the firm's asset value, V_{SM}, V_{KMV} and V_{Proxy}. Therefore, the results of the Wald test undoubtedly show that for absolute majority of the companies there is a mean reversion in volatility.

Table 3. *Estimated coefficients for the GARCH model with GED errors*

	E	V_{SM}	V_{KMV}	V_{Proxy}
ω	0.0469	0.0119	0.0122	0.0118
	(100.00%)	(96.15%)	(96.15%)	(96.15%)
α	0.0623	0.0525	0.0533	0.0535
	(100.00%)	(100.00%)	(100.00%)	(100.00%)
β	0.9233	0.9319	0.9307	0.9317
	(100.00%)	(100.00%)	(100.00%)	(100.00%)
GED	1.2300	1.2630	1.2641	1.2618
	(100.00%)	(100.00%)	(100.00%)	(100.00%)
Q(10)	10.2048	10.7137	10.6966	10.5977
	(98.08%)	(96.15%)	(96.15%)	(96.15%)
Q^2(10)	6.7009	7.3019	7.1897	7.0139
	(98.08%)	(96.15%)	(96.15%)	(96.15%)
LL	-6,376.1144	-3,731.2526	-3,714.0342	-3,730.4620
Akaike	3.7630	2.2030	2.1929	2.2026
Shibata	3.7630	2.2030	2.1929	2.2026
Schwarz	3.7702	2.2103	2.2001	2.2098
Hannan-Quinn	3.7656	2.2056	2.1955	2.2052

This table reports the cross-sectional average of the estimated GARCH(1,1) coefficients. The figures in parenthesis correspond to the percentage of companies for which the estimated coefficients of the model are statistically significant at least at the 5% level. The Q(10) and Q^2(10) refer to the Ljung-Box statistics for tenth-order serial correlation in standardized residuals and squared standardized residuals, respectively.

The EGARCH estimation results are depicted in Table 4. We can observe that the β parameter is statistically significant in all of the cases at the 5% level for both equity and firm's asset conditional volatility. In the EGARCH model the value of β quantifies the persistence of shocks to conditional volatility. In our case the estimated values are very close to 1, however, the Wald test rejects the null hypothesis $H_0: \beta = 1$ in all of the cases considered. In the case of equity, the mean β coefficient is 0.9850. In the case of firm's assets, it is equal to 0.9828 (for V_{SM}) and 0.9824 (for V_{KMV} and V_{Proxy}). This would suggest, in line with GARCH estimates that shocks to firms' asset conditional volatility show slightly lower persistence than shocks to equity volatility. However, as before, the difference in mean is not statistically significant.

Table 4. *Estimated coefficients for the EGARCH model with GED errors*

	E	V_{SM}	V_{KMV}	V_{Proxy}
ω	0.9691	-0.6027	-0.6125	-0.6013
	(100.00%)	(75.00%)	(73.08%)	(73.08%)
β	0.9850	0.9828	0.9824	0.9824
	(100.00%)	(100.00%)	(100.00%)	(100.00%)
γ_1	-0.0548	-0.0369	-0.0378	-0.0383
	(100.00%)	(86.54%)	(88.46%)	(86.54%)
γ_2	0.1140	0.1151	0.1151	0.1175
	(100.00%)	(100.00%)	(100.00%)	(100.00%)
GED	1.2459	1.2691	1.2705	1.2682
	(100.00%)	(100.00%)	(100.00%)	(100.00%)
Q(10)	10.7924	10.9143	10.9174	10.8387
	(94.23%)	(94.23%)	(94.23%)	(94.23%)
Q^2(10)	11.1359	10.4388	10.4680	9.9055
	(86.54%)	(86.54%)	(86.54%)	(90.38%)
LL	-6,358.6125	-3,721.3829	-3,703.9319	-3,719.9348
Akaike	3.7532	2.1978	2.1875	2.1970
Shibata	3.7532	2.1978	2.1875	2.1969
Schwarz	3.7623	2.2068	2.1966	2.2060
Hannan-Quinn	3.7565	2.2010	2.1907	2.2002

This table reports the cross-sectional average of the estimated EGARCH(1,1) coefficients. The figures in parenthesis correspond to the percentage of companies for which the estimated coefficients of the model are statistically significant at least at the 5% level. The Q(10) and Q^2(10) refer to the Ljung-Box statistics for tenth-order serial correlation in standardized residuals and squared standardized residuals, respectively.

The sign effect in the EGARCH model is measured with the parameter γ_1, which is negative for both firm's asset and equity, suggesting that negative shocks affect more volatility than positive shocks of equal magnitude. However, the importance of asymmetry seems to be more pronounced for equity than for firm's asset volatility. In the case of equity, the asymmetry coefficient is negative and statistically significant at the 5% level for all the companies. In turn, in the case of V_{SM} firm's assets the γ_1 coefficient is significant for 86.54% of the companies. Similar results are obtained for

V_{KMV} and V_{Proxy} firm's assets with 88.46% and 86.54% significant cases, respectively. On average the γ_1 coefficient for equity is -0.0548, whereas for V_{SM} firm's assets is noticeably lower and equals -0.0369 (-0.0378 for V_{KMV} and -0.0383 for V_{Proxy}). The difference in mean for the asymmetry coefficient is statistically significant. At the firm level, practically for all the companies (51 for V_{SM}, V_{KMV} and 50 for V_{Proxy}) estimated γ_1 coefficient is more negative for equity than for all the other firm's asset estimations.

Table 5 reports the estimation results for the IGARCH model, which assumes that the sum of α and β parameters of the GARCH model equals 1 (i.e. $\alpha + \beta = 1$). In a similar way to what happens in GARCH estimation, all of the autoregressive parameters (α) estimated in conditional volatility (equity and firm's asset) are statistically significant at 5%.[26]

Table 5. *Estimated coefficients for the IGARCH model with GED errors*

	E	V_{SM}	V_{KMV}	V_{Proxy}
ω	0.0233	0.0039	0.0039	0.0040
	(100.00%)	(96.15%)	(98.08%)	(98.08%)
1-β	0.0671	0.0536	0.0538	0.0546
	(100.00%)	(100.00%)	(100.00%)	(100.00%)
β	0.9329	0.9464	0.9462	0.9454
GED	1.2080	1.2423	1.2437	1.2419
	(100.00%)	(100.00%)	(100.00%)	(100.00%)
Q(10)	10.2404	10.7276	10.7470	10.5548
	(98.08%)	(98.08%)	(98.08%)	(98.08%)
Q^2(10)	6.5867	7.5449	7.5029	7.2451
	(96.15%)	(94.23%)	(94.23%)	(92.31%)
LL	-6,380.1827	-3,735.8557	-3,718.6663	-3,734.8143
Akaike	3.7648	2.2052	2.1950	2.2046
Shibata	3.7648	2.2052	2.1950	2.2045
Schwarz	3.7702	2.2106	2.2004	2.2100
Hannan-Quinn	3.7667	2.2071	2.1970	2.2065

This table reports the cross-sectional average of the estimated IGARCH(1,1) coefficients. The figures in parenthesis correspond to the percentage of companies for which the estimated coefficients of the model are statistically significant at least at the 5% level. The Q(10) and Q^2(10) refer to the Ljung-Box statistics for tenth-order serial correlation in standardized residuals and squared standardized residuals, respectively.

[26] Note that $\alpha = 1 - \beta$.

To explore long memory properties of equity and firm's asset volatility we estimate several fractionally integrated models. Table 6 depicts the estimation results of the FIGARCH model. The cross-sectional mean of the estimated long-memory coefficient for equity volatility is 0.3765, which is in line with the common findings in the literature on the volatility of stock returns (Lux and Kaizoji, 2007) and stock index returns (Dionisio et al. 2007; Kang and Yoon, 2007). On the other side, the estimated long-memory coefficient for firm's asset volatility is slightly lower, with the cross-sectional mean of 0.3501 in our base case (V_{SM}). This result is also in line with the higher persistence observed in equity vs. firm's asset volatility estimation with GARCH and EGARCH models. In addition, we find no substantial difference in the estimate of the long-memory parameter among the three procedures used to assess the underlying firm's asset value. Next, using a Wald test we test the null hypothesis that $d = 0$ (GARCH model) or alternatively, that $d = 1$ (IGARCH model). In all of the cases we reject both null hypotheses. This result supports the use of the flexible FIGARCH model, which allows for intermediate ranges of persistence.

The standard deviation of the persistence parameter for equity volatility is higher than the standard deviation of the persistence parameter for firm's asset volatility, which suggests a larger variability in the estimated equity volatility persistence. The equity volatility persistence ranges between 0.1996 and 0.7236, whereas firm's asset volatility persistence ranges between 0.1524 and 0.5508 (in our baseline V_{SM} case). In general, for the absolute majority of the companies considered, the parameter d is lower than 0.5 (94.23% in the case of equity, and 92.31% in the case of firms assets). A comprehensive analysis of persistence of firm's asset and equity volatility with FIGARCH model is available in González-Pla and Lovreta (2019).[27]

Table 6. *Estimated coefficients for the FIGARCH model with GED errors*

	E	V_{SM}	V_{KMV}	V_{Proxy}
ω	0.1555	0.0471	0.0462	0.0474
	(100.00%)	(100.00%)	(100.00%)	(100.00%)
α	0.2653	0.2977	0.2965	0.2857
	(84.62%)	(86.54%)	(86.54%)	(88.46%)
β	0.5464	0.5572	0.5563	0.5567
	(98.08%)	(96.15%)	(96.15%)	(98.08%)
d	0.3765	0.3501	0.3502	0.3579
	(100.00%)	(100.00%)	(100.00%)	(100.00%)
GED	1.2315	1.2643	1.2657	1.2640
	(100.00%)	(100.00%)	(100.00%)	(100.00%)
Q(10)	10.0653	10.5071	10.5053	10.3880
	(98.08%)	(96.15%)	(96.15%)	(96.15%)
Q^2(10)	5.4868	6.0710	5.9912	6.0021
	(100.00%)	(96.15%)	(96.15%)	(96.15%)
LL	-6,372.1837	-3,728.8586	-3,711.6717	-3,728.0199
Akaike	3.7612	2.2022	2.1921	2.2017
Shibata	3.7612	2.2022	2.1921	2.2017
Schwarz	3.7703	2.2113	2.2011	2.2108
Hannan-Quinn	3.7645	2.2054	2.1953	2.2050

This table reports the cross-sectional average of the estimated FIGARCH(1,1) coefficients. The figures in parenthesis correspond to the percentage of companies for which the estimated coefficients of the model are statistically significant at least at the 5% level. The Q(10) and Q^2(10) refer to the Ljung-Box statistics for tenth-order serial correlation in standardized residuals and squared standardized residuals, respectively.

Estimation results for the HYGARCH model are provided in Table 7. The estimated persistence for equity volatility measured by the fractional differencing parameter (d) is 0.4176, which is higher than the estimated persistence in firms' asset volatility estimations: 0.3774 for V_{SM}, 0.3785 for V_{KMV}, and 0.3847 for V_{Proxy}. One of the advantages of HYGARCH is its flexibility in two dimensions capturing both the amplitude and memory. This implies that the HYGARCH model is able to behave as

[27] It should be noted here that although the data set is the same as in González-Pla and Lovreta (2019) the reported results for the FIGARCH model in this paper slightly differ. The reason for the small difference is that we consider here a common framework for all the estimations. Specifically, the conditional mean equation in this paper is always modelled as a first-order autoregressive process, whereas in González-Pla and Lovreta (2019) it is modelled as a k-order autoregressive process with the exact specification defined on a case by case basis.

FIGARCH when $\kappa = 1$ (i.e. $log(\kappa) = 0$) or GARCH when $\kappa = 0$. As observed, the estimation of κ approximates 1 for both equity and firm's asset volatility. In the case of equity as well as our base case firm's assets, we cannot reject the FIGARCH in favour of the HYGARCH in 86.54% of the cases. For the remaining, 13.46% of the companies κ is significantly lower than 1, or equivalently, $log(\kappa) < 0$.

Table 7. *Estimated coefficients for the HYGARCH model with GED errors*

	E	V_{SM}	V_{KMV}	V_{Proxy}
ω	0.1491	0.0502	0.0495	0.0453
	(63.46%)	(53.85%)	(51.92%)	(53.85%)
α	0.2719	0.2650	0.2630	0.2821
	(84.62%)	(84.62%)	(84.62%)	(90.38%)
β	0.5849	0.5471	0.5462	0.5757
	(96.15%)	(92.31%)	(92.31%)	(96.15%)
d	0.4176	0.3774	0.3785	0.3847
	(96.15%)	(90.38%)	(90.38%)	(92.31%)
log(κ)	-0.0103	0.0122	0.0137	0.0108
κ	0.9935	1.0232	1.0251	1.0203
	(13.46%)	(13.46%)	(17.31%)	(11.54%)
GED	1.2338	1.2658	1.2672	1.2650
	(100.00%)	(100.00%)	(100.00%)	(100.00%)
Q(10)	10.0805	10.4654	10.4556	10.3630
	(98.08%)	(96.15%)	(96.15%)	(96.15%)
Q^2(10)	5.3905	5.8152	5.7412	5.6953
	(100.00%)	(100.00%)	(100.00%)	(100.00%)
LL	-6,371.5817	-3,728.1426	-3,710.9496	-3,727.2737
Akaike	3.7615	2.2024	2.1922	2.2019
Shibata	3.7615	2.2024	2.1922	2.2019
Schwarz	3.7723	2.2132	2.2031	2.2127
Hannan-Quinn	3.7654	2.2063	2.1961	2.2057

This table reports the cross-sectional average of the estimated HYGARCH(1,1) coefficients. The figures in parenthesis correspond to the percentage of companies for which the estimated coefficients of the model are statistically significant at least at the 5% level. The Q(10) and $Q^2(10)$ refer to the Ljung-Box statistics for tenth-order serial correlation in standardized residuals and squared standardized residuals, respectively.

In context of fractionally integrated models, FIEGARCH is an interesting alternative as generalizes EGARCH model by incorporating a fractionally integrated behavior, an additional feature to its inherent ability to capture asymmetry. Estimation results for the FIEGARCH model are shown in Table 8. At the cross sectional level, the sign effect (γ_1) and the amplitude effect (γ_2) have an influence on volatility comparable to that observed in the EGARCH estimations. Similarly to the EGARCH model, asymmetry coefficient is negative and statistically significant at the 5% level for all the companies in the case of equity, and for 86.54% of the V_{SM} firm's asset volatility estimations (84.62% and 86.54% for V_{KMV} and V_{Proxy} firm's assets). On average the γ_1 coefficient is more negative for equity (-0.0729) than for firm's assets (-0.0489 for V_{SM}, -0.0497 for V_{KMV} and -0.0479 for V_{Proxy}). The difference in mean for the asymmetry coefficient is statistically significant. The fractional differencing parameter (d) is statistically significant in all of the equity and asset estimations, and its mean value for equity is 0.5866, and ranges between 0.5796 and 0.5870 for firms' asset estimations (0.5861 in our base case using the structural model).

Table 8. *Estimated coefficients for the FIEGARCH model with GED errors*

	E	V_{SM}	V_{KMV}	V_{Proxy}
ω	0.8764	-0.6748	-0.6941	-0.6847
	(94.23%)	(63.46%)	(63.46%)	(61.54%)
β	0.4753	0.4164	0.4167	0.4513
	(71.15%)	(61.54%)	(59.62%)	(63.46%)
d	0.5866	0.5861	0.5870	0.5796
	(100.00%)	(100.00%)	(100.00%)	(100.00%)
γ_1	-0.0729	-0.0489	-0.0497	-0.0479
	(100.00%)	(86.54%)	(84.62%)	(86.54%)
γ_2	0.1478	0.1613	0.1604	0.1552
	(100.00%)	(100.00%)	(100.00%)	(100.00%)
GED	1.2500	1.2740	1.2752	1.2729
	(100.00%)	(100.00%)	(100.00%)	(100.00%)
Q(10)	10.7612	10.9008	10.9065	10.8305
	(94.23%)	(94.23%)	(94.23%)	(94.23%)
Q^2(10)	8.5383	8.2958	8.3071	8.1309
	(92.31%)	(94.23%)	(94.23%)	(94.23%)
LL	-6,352.8421	-3,716.3930	-3,699.0042	-3,715.2074
Akaike	3.7504	2.1955	2.1852	2.1948
Shibata	3.7504	2.1954	2.1852	2.1947
Schwarz	3.7613	2.2063	2.1960	2.2056
Hannan-Quinn	3.7543	2.1993	2.1891	2.1986

This table reports the cross-sectional average of the estimated FIEGARCH(1,1) coefficients. The figures in parenthesis correspond to the percentage of companies for which the estimated coefficients of the model are statistically significant at least at the 5% level. The Q(10) and Q^2(10) refer to the Ljung-Box statistics for tenth-order serial correlation in standardized residuals and squared standardized residuals, respectively.

Table 9 depicts the estimations with the FIAPARCH model. The Box-Cox transformation parameter (δ) is about 1.4 and is statistically significant in all of the cases. The fractional differencing parameter (d) is 0.3905 for equity and from 0.3978 to 0.4080 in firms' asset estimates (0.4022 in our base case). At firm level, the asymmetry coefficient (γ) is positive, which means that negative shocks have greater impact on volatility than positive shocks. The asymmetry coefficient is higher for equity (0.5283 on average), than for the V_{SM} firm's assets (0.3414 on average).

Table 9. Estimated coefficients for the FIAPARCH model with GED errors

	E	V_{SM}	V_{KMV}	V_{Proxy}
ω	0.1641	0.0643	0.0664	0.0660
	(98.08%)	(90.38%)	(90.38%)	(90.38%)
α	0.2823	0.2923	0.2905	0.2867
	(96.15%)	(94.23%)	(92.31%)	(98.08%)
β	0.5931	0.6062	0.6019	0.6111
	(98.08%)	(98.08%)	(96.15%)	(98.08%)
d	0.3905	0.4022	0.3978	0.4080
	(100.00%)	(98.08%)	(96.15%)	(96.15%)
γ	0.5283	0.3414	0.3638	0.3625
	(98.08%)	(88.46%)	(84.62%)	(86.54%)
δ	1.3914	1.4257	1.4330	1.4197
	(100.00%)	(100.00%)	(100.00%)	(100.00%)
GED	1.2580	1.2790	1.2803	1.2784
	(100.00%)	(100.00%)	(100.00%)	(100.00%)
Q(10)	10.3794	10.5959	10.6053	10.5601
	(96.15%)	(96.15%)	(96.15%)	(96.15%)
Q^2(10)	6.7497	6.9709	6.9348	6.9953
	(98.08%)	(96.15%)	(96.15%)	96.15%
LL	-6,350.9694	-3,716.1823	-3,698.8482	-3,715.1377
Akaike	3.7499	2.1959	2.1857	2.1953
Shibata	3.7499	2.1959	2.1857	2.1953
Schwarz	3.7626	2.2086	2.1983	2.2080
Hannan-Quinn	3.7544	2.2004	2.1902	2.1998

This table reports the cross-sectional average of the estimated FIAPARCH(1,1) coefficients. The figures in parenthesis correspond to the percentage of companies for which the estimated coefficients of the model are statistically significant at least at the 5% level. The Q(10) and Q^2(10) refer to the Ljung-Box statistics for tenth-order serial correlation in standardized residuals and squared standardized residuals, respectively.

Finally, it is important to note that the estimated GED parameter is statistically significant and lower than 2 in all estimations suggesting a fat-tailed distribution of return series (see Table 3 to Table 9). The results of the Ljung–Box diagnostic tests for tenth-order serial correlation in standardized residuals and standardized squared residuals indicate that estimated models are in general able to properly capture the dynamics in the conditional variance of equity and firm's asset returns (see Table 3 to Table 9).

6.1 Model fit

To asses model fit we use several information criteria which are a function of the log-likelihood (i.e. a measure of goodness of fit of the estimated model), and a penalty component that changes depending on the criteria. The analytical formulation of considered information criteria is as follows:

$$Akaike = -2\frac{log\,L}{n} + 2\frac{q}{n} \tag{18}$$

$$Schwarz = -2\frac{log\,L}{n} + q\frac{log\,n}{n} \tag{19}$$

$$Shibata = -2\frac{log\,L}{n} + log\frac{n+2q}{n} \tag{20}$$

$$Hannan - Quinn = -2\frac{log\,L}{n} + 2\frac{q\,log\,(log\,n)}{n}, \tag{21}$$

where $\log L$ is the log-likelihood function, q is the number of estimated parameters and n is the number of observations.

Table 10 reports the cross sectional results of the information criteria and Table 11 the number of firms for which each of the considered models is selected as the best model. In consequence, according to the log-likelihood statistics and information criteria described above, models that simultaneously account for asymmetry and long-range persistence (i.e. FIEGARCH and FIAPARCH models) provide the best fit in the cross-section. In equity volatility estimations, FIAPARCH model provides the best fit according to the Akaike and Schibata information criteria, and FIEGARCH model according to the Schwarz and Hannan-Quinn criteria. At the firm level, the two most complete models seem to be the best choice in 94.23% (Akaike and Schibata), 61.54% (Schwarz) and 86.54% (Hannan-Quinn) of the cases. These results are not significantly different in firm's asset volatility estimations. That is, in the cross-section all four information criteria (i.e. Akaike, Schibata, Schwarz and Hannan-Quinn criteria) suggest the FIEGARCH model, followed in the second place by the FIAPARCH model. The two models seem to be the best choice in 82.69% (Akaike), 84.62% (Schibata), 40.38% (Schwarz) and 71.15% (Hannan-Quinn) of the cases in our base case firm asset estimations. The model selection using the Information Criteria or goodness of fit is consistent along the different firms' asset estimation approaches considered in this paper. Finally, it should be noted that according to the Schwarz information criteria the EGARCH model seems to provide the best goodness of fit for a significant number of companies (28.85% in the case of equity and 34.62% in the case of firm's asset volatility).

When analyzing the information criteria obtained in the model fit, it is important to contextualize them with a certain degree of prudence, and according to the characteristics of the model. Javed and Mantalos (2013) analyzed several information criteria (Akaike, Schwarz and Hannan-Quinn) in selecting GARCH-type models compared to their forecasting results, and observed that Schwarz and Hannan-Quinn performed better with low order GARCH effects. Mitchell and Mckenzie (2003) point out that a Hannan-Quinn criterion behaves particularly well choosing the ARCH model when the modelled process is linear. However, in presence of leverage or power effects (which are usually observed in stock returns) none of the criteria provide accurate results. Brooks and Burk (2003) observed a poor performance in information criteria when selecting a GARCH model, probably due to the characteristics of financial data (e.g. fat tails, structural breaks, etc). Therefore, the results about model selection according to the information criteria obtained should be interpreted with caution.

Table 10. *Cross-sectional values of information criteria by estimated model*

	E	V_{SM}	V_{KMV}	V_{Proxy}
Log-Likelihood				
GARCH	-6,376.1144	-3,731.2526	-3,714.0342	-3,730.4620
EGARCH	-6,358.6125	-3,721.3829	-3,703.9319	-3,719.9348
IGARCH	-6,380.1827	-3,735.8557	-3,718.6663	-3,734.8143
FIGARCH	-6,372.1837	-3,728.8586	-3,711.6717	-3,728.0199
HYGARCH	-6,371.5817	-3,728.1426	-3,710.9496	-3,727.2737
FIEGARCH	-6,352.8421	-3,716.3930	-3,699.0042	-3,715.2074
FIAPARCH	**-6,350.9694**	**-3,716.1823**	**-3,698.8482**	**-3,715.1377**
Akaike				
GARCH	3.7630	2.2030	2.1929	2.2026
EGARCH	3.7532	2.1978	2.1875	2.1970
IGARCH	3.7648	2.2052	2.1950	2.2046
FIGARCH	3.7612	2.2022	2.1921	2.2017
HYGARCH	3.7615	2.2024	2.1922	2.2019
FIEGARCH	3.7504	**2.1955**	**2.1852**	**2.1948**
FIAPARCH	**3.7499**	2.1959	2.1857	2.1953
Shibata				
GARCH	3.7630	2.2030	2.1929	2.2026
EGARCH	3.7532	2.1978	2.1875	2.1969
IGARCH	3.7648	2.2052	2.1950	2.2045
FIGARCH	3.7612	2.2022	2.1921	2.2017
HYGARCH	3.7615	2.2024	2.1922	2.2019
FIEGARCH	3.7504	**2.1954**	**2.1852**	**2.1947**
FIAPARCH	**3.7499**	2.1959	2.1857	2.1953
Schwarz				
GARCH	3.7702	2.2103	2.2001	2.2098
EGARCH	3.7623	2.2068	2.1966	2.2060
IGARCH	3.7702	2.2106	2.2004	2.2100
FIGARCH	3.7703	2.2113	2.2011	2.2108
HYGARCH	3.7723	2.2132	2.2031	2.2127
FIEGARCH	**3.7613**	**2.2063**	**2.1960**	**2.2056**
FIAPARCH	3.7626	2.2086	2.1983	2.2080
Hannan-Quinn				
GARCH	3.7656	2.2056	2.1955	2.2052
EGARCH	3.7565	2.2010	2.1907	2.2002
IGARCH	3.7667	2.2071	2.1970	2.2065
FIGARCH	3.7645	2.2054	2.1953	2.2050
HYGARCH	3.7654	2.2063	2.1961	2.2057
FIEGARCH	**3.7543**	**2.1993**	**2.1891**	**2.1986**
FIAPARCH	3.7544	2.2004	2.1902	2.1998

This table reports the cross-sectional average of the information criteria (Akaike, Shibata, Schwartz, Hannan-Quinn) and log-likelihood obtained in the estimation of volatility models. The best selected model according to the criteria is shown in bold.

Table 11. *Information Criteria at firm level*

	E		V_{SM}		V_{KMV}		V_{Proxy}	
	Num.	***%***	***Num.***	***%***	***Num.***	***%***	***Num.***	***%***
Log-Likelihood								
GARCH	0	0.00%	0	0.00%	0	0.00%	0	0.00%
EGARCH	1	1.92%	3	5.77%	3	5.77%	3	5.77%
IGARCH	0	0.00%	0	0.00%	0	0.00%	0	0.00%
FIGARCH	0	0.00%	0	0.00%	0	0.00%	0	0.00%
HYGARCH	0	0.00%	1	1.92%	1	1.92%	1	1.92%
FIEGARCH	15	28.85%	21	40.38%	19	36.54%	20	38.46%
FIAPARCH	**36**	**69.23%**	**27**	**51.92%**	**29**	**55.77%**	**28**	**53.85%**
Akaike								
GARCH	0	0.00%	2	3.85%	2	3.85%	2	3.85%
EGARCH	3	5.77%	6	11.54%	6	11.54%	7	13.46%
IGARCH	0	0.00%	0	0.00%	0	0.00%	0	0.00%
FIGARCH	0	0.00%	1	1.92%	0	0.00%	0	0.00%
HYGARCH	0	0.00%	0	0.00%	0	0.00%	0	0.00%
FIEGARCH	20	38.46%	**24**	**46.15%**	**25**	**48.08%**	**23**	**44.23%**
FIAPARCH	**29**	**55.77%**	19	36.54%	19	36.54%	20	38.46%
Shibata								
GARCH	0	0.00%	2	3.85%	2	3.85%	2	3.85%
EGARCH	3	5.77%	5	9.62%	6	11.54%	7	13.46%
IGARCH	0	0.00%	0	0.00%	0	0.00%	0	0.00%
FIGARCH	0	0.00%	1	1.92%	0	0.00%	0	0.00%
HYGARCH	0	0.00%	0	0.00%	0	0.00%	0	0.00%
FIEGARCH	20	38.46%	**24**	**46.15%**	**25**	**48.08%**	**23**	**44.23%**
FIAPARCH	**29**	**55.77%**	20	38.46%	19	36.54%	20	38.46%
Schwarz								
GARCH	1	1.92%	5	9.62%	6	11.54%	5	9.62%
EGARCH	15	28.85%	18	34.62%	18	34.62%	**20**	**38.46%**
IGARCH	3	5.77%	7	13.46%	6	11.54%	8	15.38%
FIGARCH	1	1.92%	1	1.92%	0	0.00%	2	3.85%
HYGARCH	0	0.00%	0	0.00%	0	0.00%	0	0.00%
FIEGARCH	**22**	**42.31%**	**19**	**36.54%**	**20**	**38.46%**	14	26.92%
FIAPARCH	10	19.23%	2	3.85%	2	3.85%	3	5.77%
Hannan-Quinn								
GARCH	0	0.00%	2	3.85%	2	3.85%	2	3.85%
EGARCH	7	13.46%	10	19.23%	9	17.31%	10	19.23%
IGARCH	0	0.00%	2	3.85%	2	3.85%	2	3.85%
FIGARCH	0	0.00%	1	1.92%	1	1.92%	1	1.92%
HYGARCH	0	0.00%	0	0.00%	0	0.00%	0	0.00%
FIEGARCH	22	42.31%	**24**	**46.15%**	**26**	**50.00%**	**24**	**46.15%**
FIAPARCH	**23**	**44.23%**	13	25.00%	12	23.08%	13	25.00%

This table reports the number and percentage of firms with the best performance for each GARCH-type model according to the log-likelihood and information criteria (Akaike, Shibata, Schwartz, Hannan-Quinn) considered. The model with the highest proportion of firms is highlighted in bold.

Brooks and Burk (2003) argue that the traditional information criteria previously discussed cannot be used in the context of conditionally heteroscedastic models. The traditional Akaike and Schwartz information criteria have been extensively used in autoregressive models to determine the optimal number of lags. However, these criteria are not appropriate for assessing (heteroscedastic) variance models since model selection leads to inevitable biases. Therefore, Brooks and Burk (2003) propose a modified version of Akaike (HAIC) and Schwarz (HSIC) information criteria which are defined as follows:

$$HAIC = \sum_{t=1}^{T} log(\hat{\sigma}_t^2) + 2q \quad (22)$$

$$HSIC = \sum_{t=1}^{T} log(\hat{\sigma}_t^2) + qlog(T) \quad (23)$$

Where $\hat{\sigma}_t^2$ is the estimated conditional variance in the time *t*, q refers to the number of estimated parameters and T is the number of observations. Table 12 and Table 13 report the cross-sectional and firm level analysis of the modified information criteria by estimated model.

The obtained results of HAIC and HSIC show some differences to traditional information criteria. In equity volatility estimations EGARCH is the best choice in 50.00% (HAIC) and 53.85% (HSIC) of the cases, followed by FIAPARCH that is selected by 48.08% (HAIC) and 42.31% (HSIC) of firms. FIEGARCH, which is one of the best models according to traditional information criteria, seems to perform well overall as its cross-sectional average is very close to the values of EGARCH and FIAPARCH although it is not selected by any firm as the best model according to HAIC or HSIC criterion. On the other hand, when assessing firm's asset volatility estimations with modified information criteria we observe that FIEGARCH is the best model, selected from 59.62% to 61.54% of firms with HAIC criterion and from 61.54% to 63.46% of firms with HSIC criterion (depending on the firm's asset value estimation). The FIEGARCH is followed by the FIAPARCH which is selected from 30.77% to 32.69% of firms with HAIC criterion and from 19.23% to 25.00% with HSIC criterion. According to cross-sectional mean criteria FIEGARCH is the best overall model in firm's asset volatility estimation. In this way, according these criteria the majority of firm's asset estimations choose fractionally integrated models as the best choice as opposed to equity volatility estimations where less than 48.08% of firms select a fractionally integrated model.

Table 12. *Cross-sectional values of information criteria by estimated model*

	E	V_{SM}	V_{KMV}	V_{Proxy}
HAIC				
GARCH	-12.095,4108	-14.404,9037	-14.419,9703	-14.404,8196
EGARCH	**-12.133,5106**	-14.412,8141	-14.428,2872	-14.413,4921
IGARCH	-12.030,4301	-14.342,1337	-14.357,1351	-14.344,5967
FIGARCH	-12.093,0756	-14.403,7018	-14.418,8425	-14.405,7395
HYGARCH	-12.097,9076	-14.405,4842	-14.420,8394	-14.405,7710
FIEGARCH	-12.131,5106	**-14.431,5179**	**-14.448,9052**	**-14.434,1284**
FIAPARCH	-12.132,6125	-14.427,0547	-14.442,3967	-14.428,7122
HSIC				
GARCH	-12.089,2895	-14.398,7824	-14.413,8490	-14.398,6982
EGARCH	**-12.125,8589**	-14.405,1624	-14.420,6355	-14.405,8404
IGARCH	-12.025,8391	-14.337,5427	-14.352,5441	-14.340,0057
FIGARCH	-12.085,4240	-14.396,0502	-14.411,1909	-14.398,0879
HYGARCH	-12.088,7256	-14.396,3023	-14.411,6574	-14.396,5890
FIEGARCH	-12.122,3286	**-14.422,3359**	**-14.439,7233**	**-14.424,9464**
FIAPARCH	-12.121,9002	-14.416,3424	-14.431,6844	-14.417,9999

This table repots the cross-sectional average of the modified heteroscedastic information criteria HAIC and HSIC obtained in the estimation volatility model. The best selected model according to the criteria is shown in bold.

Table 13. *Modified information criteria at firm level*

	E		V_{SM}		V_{KMV}		V_{Proxy}	
	Num.	***%***	***Num.***	***%***	***Num.***	***%***	***Num.***	***%***
HAIC								
GARCH	1	1.92%	1	1.92%	1	1.92%	1	1.92%
EGARCH	**26**	**50.00%**	2	3.85%	2	3.85%	3	5.77%
IGARCH	0	0.00%	0	0.00%	0	0.00%	0	0.00%
FIGARCH	0	0.00%	0	0.00%	0	0.00%	0	0.00%
HYGARCH	0	0.00%	1	1.92%	1	1.92%	1	1.92%
FIEGARCH	0	0.00%	**31**	**59.62%**	**32**	**61.54%**	**31**	**59.62%**
FIAPARCH	25	48.08%	17	32.69%	16	30.77%	16	30.77%
HSIC								
GARCH	2	3,85%	5	9.62%	4	7.69%	3	5.77%
EGARCH	**28**	**53.85%**	4	7.69%	4	7.69%	3	5.77%
IGARCH	0	0.00%	0	0.00%	0	0.00%	0	0.00%
FIGARCH	0	0.00%	1	1.92%	0	0.00%	1	1.92%
HYGARCH	0	0.00%	0	0.00%	0	0.00%	0	0.00%
FIEGARCH	0	0.00%	**32**	**61.54%**	**33**	**63.46%**	**32**	**61.54%**
FIAPARCH	22	42.31%	10	19.23%	11	21.15%	13	25.00%

This table reports the number and percentage of firms with the best performance for each GARCH-type model according to the modified heteroscedastic information criteria HAIC and HSIC obtained in the estimation volatility model. The best selected with the highest proportion of firms according to the criteria is shown in bold.

7. The effect of leverage on asymmetry and long-memory

In this section we analyze the impact of financial leverage on asymmetry and long-range persistence. We particularly focus on the eventual differences between equity and firm asset volatility modeling. To analyze the effect of leverage on the two model features we divide the overall sample into low, medium and high leverage groups. Leverage is defined as before, as the ratio of the book value of total liabilities to the proxy for the market value of the firm (i.e. the sum of the market capitalization and the nominal value of debt). We rank firms in ascending order according to their leverage and divide further firms into quartiles. The low leverage group consists of firms within the first quartile (Q1), whereas the high leverage group consists of firms within the last quartile (Q4). The medium leverage group includes second (Q2) and third (Q3) quartile. Such a division implies a mean leverage of 0.3576, 0.5187 and 0.7007 for the low, medium and high leverage group, respectively.

7.1 The effect of leverage on asymmetry

The effect of asymmetry could be analyzed in the context of the short-memory EGARCH model, and long-memory FIEGARCH and FIAPARCH models. The results are reported in Table 14. In the case of the short-memory EGARCH model the asymmetry is analyzed on the basis of the coefficient γ_1. It can be observed that asymmetry is more pronounced in the case of equity than in the case of firm asset volatility. Although, the asymmetry coefficient in the case of firm asset volatility is statistically significant at 5% level for most of the firms (i.e. 86.54% in our base case), it is almost always (i.e. 98.08% in our base V_{SM} case) lower than the corresponding coefficient for equity volatility of the same firm. Moreover, the difference in asymmetry increases with leverage being higher for high leverage firms and lower for low leverage firms. The difference in asymmetry increases despite the fact that asymmetry in low leverage firms is more pronounced than asymmetry in high leverage firms (for both, equity and firm asset volatility). This tendency is also confirmed in the case of FIEGARCH and FIAPARCH models. Namely, once the EGARCH model is extended to allow for the long-memory parameter *d*, the difference in asymmetry between equity and firm asset volatility becomes even more pronounced. That is, in the case of the FIEGARCH model, the average γ_1 coefficient for the overall sample amounts to -0.0729 for equity and -0.0477 for firm's asset returns in our base case. The difference in mean is statistically significant. At the firm level, in this case the asymmetry in equity volatility is higher than asymmetry in firm's asset volatility in all of the cases considered. As before, once we account for the three leverage groups, we can observe that the difference in the level of asymmetry increases with leverage.

Table 14. *The effect of leverage on asymmetry*

	EGARCH (γ_1)			**FIEGARCH (γ_1)**			**FIAPARCH (γ)**		
lev	**E**	**V_{SM}**	**diff**	**E**	**V_{SM}**	**diff**	**E**	**V_{SM}**	**diff**
Low	-0.0542	-0.0422	-0.0120	-0.0712	-0.0531	-0.0182	0.5552	0.4363	0.1189
Medium	-0.0584	-0.0408	-0.0177	-0.0796	-0.0531	-0.0265	0.5479	0.3576	0.1903
High	-0.0483	-0.0241	-0.0242	-0.0613	-0.0318	-0.0295	0.4622	0.2139	0.2483
all	**-0.0548**	**-0.0369**	**-0.0179**	**-0.0729**	**-0.0477**	**-0.0252**	**0.5283**	**0.3414**	**0.1870**
lev	**E**	**V_{KMV}**	**diff**	**E**	**V_{KMV}**	**diff**	**E**	**V_{KMV}**	**diff**
Low	-0.0542	-0.0422	-0.0120	-0.0712	-0.0530	-0.0182	0.5552	0.4895	0.0657
Medium	-0.0584	-0.0417	-0.0167	-0.0796	-0.0541	-0.0255	0.5479	0.3702	0.1778
High	-0.0483	-0.0256	-0.0227	-0.0613	-0.0333	-0.0280	0.4622	0.2256	0.2366
all	**-0.0548**	**-0.0378**	**-0.0170**	**-0.0729**	**-0.0486**	**-0.0243**	**0.5283**	**0.3638**	**0.1645**
lev	**E**	**V_{Proxy}**	**diff**	**E**	**V_{Proxy}**	**diff**	**E**	**V_{Proxy}**	**diff**
Low	-0.0542	-0.0431	-0.0111	-0.0712	-0.0532	-0.0180	0.5552	0.4907	0.0645
Medium	-0.0584	-0.0430	-0.0155	-0.0796	-0.0537	-0.0260	0.5479	0.3771	0.1709
High	-0.0483	-0.0240	-0.0243	-0.0613	-0.0312	-0.0302	0.4622	0.2051	0.2570
all	**-0.0548**	**-0.0383**	**-0.0166**	**-0.0729**	**-0.0479**	**-0.0250**	**0.5283**	**0.3625**	**0.1658**

This table reports the cross-sectional average of the estimated asymmetry coefficient in EGARCH, FIEGARCH and FIAPRACH model for low (Q1), medium (Q2 and Q3) and high (Q4) leverage group. Column *diff* refers to the average difference between equity and firm's asset asymmetry coefficient.

7.2 The effect of leverage on long-range persistence

The effect of long-range persistence could be analyzed in the context of long-memory models. In our case, we consider FIGARCH, HYGARCH, FIEGARCH and FIAPARCH models. We should also reiterate here that persistence and long-memory are not identical concepts. Persistence is a more general term and refers to a slow decay of the autocorrelation function, whereas long-memory implies persistence over long horizons. The comparison of the persistence parameter over different models and different leverage groups is provided in Table 15. In the case of the FIGARCH model we can see that persistence in both equity and firm assets increases with leverage, however, the difference in persistence is decreasing with leverage. At the firm level, in most of the cases equity is more persistent than firm asset volatility. In our base case for 71.15% of firms equity is more persistent than firm asset volatility (69.23% for V_{KMV} and 67.31% for V_{Proxy}). For HYGARCH model, on average, equity is more persistent than firm asset volatility (0.4176 vs 0.3774 for our base case). For the highest leverage group, however, we find that asset volatility is more persistent than equity volatility. Once the asymmetry is allowed in a model, the difference in equity - asset volatility is on average lower. In the case of the FIEGARCH model, at the firm level, the number of companies for which we find that equity is higher than asset volatility reduces to 48.08% in our base case (50% for V_{KMV} and 48.08% for V_{Proxy}). Moreover, for higher leverage groups firm asset volatility turns out to be more persistent than equity. Findings are very similar for the FIAPARCH model. At the firm level, the number of companies for which we find that equity is higher than asset volatility reduces to 46.15% in our base case (44.20% for V_{KMV} and 28.85% for V_{Proxy}). As before, it is precisely that for higher leverage groups firm asset volatility turns out to be more persistent than equity.

Table 15. *The effect of leverage on long-range persistence*

	FIGARCH (*d*)			HYGARCH (*d*)			FIEGARCH (*d*)			FIAPARCH (*d*)		
lev	**E**	**V_{SM}**	**diff**	**E**	**V_{SM}**	**diff**	**E**	**V_{SM}**	**diff**	**E**	**V_{SM}**	**diff**
Low	0.3649	0.3290	0.0359	0.4556	0.3628	0.0928	0.5821	0.5546	0.0275	0.3790	0.3677	0.0113
Medium	0.3715	0.3491	0.0224	0.4066	0.3685	0.0381	0.5873	0.5890	-0.0017	0.4034	0.4133	-0.0099
High	0.3980	0.3732	0.0248	0.4016	0.4100	-0.0084	0.5899	0.6118	-0.0219	0.3763	0.4144	-0.0381
all	**0.3765**	**0.3501**	**0.0264**	0.4176	0.3774	**0.0402**	0.5866	0.5861	**0.0005**	**0.3905**	**0.4022**	**-0.0116**
lev	**E**	**V_{KMV}**	**diff**	**E**	**V_{KMV}**	**diff**	**E**	**V_{KMV}**	**diff**	**E**	**V_{KMV}**	**diff**
Low	0.3649	0.3341	0.0308	0.4556	0.3660	0.0897	0.5821	0.5626	0.0195	0.3790	0.3586	0.0204
Medium	0.3715	0.3476	0.0239	0.4066	0.3670	0.0396	0.5873	0.5883	-0.0010	0.4034	0.4115	-0.0081
High	0.3980	0.3715	0.0265	0.4016	0.4142	-0.0126	0.5899	0.6087	-0.0188	0.3763	0.4097	-0.0335
all	**0.3765**	**0.3502**	**0.0263**	0.4176	0.3785	**0.0391**	0.5866	0.5870	**-0.0003**	**0.3905**	**0.3978**	**-0.0073**
lev	**E**	**V_{Proxy}**	**diff**	**E**	**V_{Proxy}**	**diff**	**E**	**V_{Proxy}**	**diff**	**E**	**V_{Proxy}**	**diff**
Low	0.3649	0.3336	0.0313	0.4556	0.3637	0.0920	0.5821	0.5568	0.0253	0.3790	0.3579	0.0211
Medium	0.3715	0.3525	0.0190	0.4066	0.3796	0.0269	0.5873	0.5819	0.0054	0.4034	0.4191	-0.0157
High	0.3980	0.3931	0.0049	0.4016	0.4158	-0.0142	0.5899	0.5977	-0.0078	0.3763	0.4357	-0.0594
all	**0.3765**	**0.3579**	**0.0185**	**0.4176**	**0.3847**	**0.0329**	0.5866	0.5796	**0.0071**	**0.3905**	**0.4080**	**-0.0174**

This table reports the cross-sectional average of the estimated long-memory coefficient in FIGARCH, HYGARCH, FIEGARCH and FIAPRACH model for low (Q1), medium (Q2 and Q3) and high (Q4) leverage group. Column *diff* refers to the average difference between equity and firm's asset long-memory coefficient.

8. Out-of-sample forecasting

In order to explore in more detail the properties of firm’s asset and equity volatility, we evaluate which volatility model provides the best out-of-sample monthly forecasts. Monthly frequency forecast horizon is chosen for several reasons. First, the analysis of firm-level equity and asset volatility is typically conducted using monthly estimates (Choi and Richardson, 2016). Second, the firm-level intraday data, necessary for ex-post daily volatility measurement (Andersen and Bollerslev, 1998) is not generally available. Consequently, in contrast to equity indices, firm-level equity and asset volatility is typically estimated from daily data. Another possibility used in the literature (Poon and Granger, 2003; Zivot, 2008) is to consider squared returns as a proxy measure of the daily realized volatility, and compare it with a one-day ahead forecasts from a conditional volatility model. Andersen and Bollerslev (1998), however argue that daily squared returns are a poor estimator of the daily realized volatility and GARCH-type models could spuriously provide better results when comparing forecasts to squared returns. Given that we work with daily data on estimated firm's asset values, and in order to avoid problems associated with estimating daily volatility from daily data we therefore opt for monthly frequency which will by definition reduce the standard error of the volatility estimate. Moreover, Poon and Granger (2003) show that the benefit of using intraday returns to estimate realized volatility is less important for long prediction horizons. Third, one of the central issues of this paper is to analyze the importance of long-memory features in firm-level volatility forecasting. The effect of long-memory features on the performance of the one-day-ahead volatility forecasts could be questioned by the very nature of the model.

We apply forecasting at monthly horizons based on daily data. That is, we use iterated forecasts of the daily variance 21 days forward, and aggregate these forecasts to the monthly horizon. Such an approach is motivated by the results of Ghysels et al. (2019) who show that in the case of GARCH models, iterated forecasts (based on daily data) dominate the direct forecasts (based on monthly data). Specifically, they report the highest accuracy of the monthly horizon iterated GARCH for as much as 83% of the assets they consider. Along the same lines, Marcellino et al. (2006) show that iterated forecasts typically outperform direct forecasts and that the relative performance of iterated forecasts improves as the forecast horizon increases.

To perform the out-of-sample forecast analysis we divide the overall sample into estimation period and evaluation period, and subsequently apply the rolling widow scheme. An initial subsample covers a period of ten years, from January 1st, 2004 to December 31st, 2013 and includes 2,610 daily observations. We estimate all selected conditional volatility models over this initial period and generate forecasts 1 to 21 days ahead applying the conditional variance equation. In the subsequent step, the estimation period is rolled forward by adding 21 return observations and dropping the first 21 return observations, so that the size of the estimation window remains fixed (2,610 daily observations). All the considered GARCH models are then re-estimated to produce a new set of volatility forecasts. This process is repeated until the end of the out-of-sample period is reached (December 31st, 2016). Sampling at monthly frequency (i.e. using a time step of one month) allows us to finally obtain 37 non-overlapping out-of-sample monthly observations for each company ($c = 1, \dots, 52$), return series ($Equity$, V_{SM}, V_{KMV}, V_{Proxy}) and the GARCH-type model ($k = 1, \dots, 7$).

One of the important features of the procedure that we apply is that we generate a non-overlapping sample of forecasted and realized volatility. If the estimation were rolled over 1 observation instead of 21 observations we would have a substantial overlap between two adjacent forecasts and realizations which would artificially produce autocorrelation in the forecasts and forecast errors (Poon and Granger, 2003). Finally, a time step of one month makes our analysis feasible, given the significant computational time needed especially for the long-memory models.

As previously stated, we use daily data to estimate GARCH-type models. For each window we construct 1 to 21-step ahead forecasts of the daily variance, and aggregate iterated daily forecasts to obtain the monthly forecast. That is, we use the sum of the iterated forecasts:

$$\sigma_F = \sqrt{\frac{252}{N}\sum_{i=1}^{N}\sigma_{\tau,i}^2} \qquad (24)$$

where N is the length of the forecasting period (21 days), $\sigma_{\tau,i}^2$ refers to the i-days ahead variance forecast in the period τ for $\tau = 1, \dots 37$. For comparison purposes, monthly volatility forecasts are annualized assuming 252 trading days per year.

To evaluate the accuracy of forecasted volatility and determine which model provides better forecasts, it is necessary to compare the generated forecasts with subsequently realized volatility. We calculate the subsequently realized monthly volatility as the annualized standard deviation of the continuously compounded returns over the same, 21 days period. The proxy used for the subsequently realized volatility.

$$\sigma_R = \sqrt{\frac{252}{N}\sum_{i=1}^{N}r_{\tau,i}^2} \qquad (25)$$

where N is the length of the forecasting period (21 days), and $r_{\tau,i}^2$ is the squared return in the period τ for $\tau = 1, \dots 37$.

In this way, for each GARCH-type model, k, we obtain a sequence of non-overlapping one-month ahead volatility forecasts ($\sigma_{k,F,1}, \dots, \sigma_{k,F,n}$) that are compared to subsequently realized volatility ($\sigma_{R,1}, \dots, \sigma_{R,n}$). To evaluate performance of different models we proceed in several steps. First, as a preliminary analysis, for every firm in the sample (and time series) we rank models using different loss functions as criteria.

These functions are usually a measure of the error between the predicted volatility and subsequently realized volatility. The lower the value of the loss function the more precise is the forecast. Second, we use formal tests to determine the significance of relative performance of considered models: the Diebold and Mariano (1995) test for equal predictive ability and the Hansen (2005) test for superior predictive ability. Third, we evaluate the performance of different forecasts by comparing the implied credit spreads (ICS), calculated using structural credit risk model and forecasted volatility, to a market observable subsequently realized CDS spreads.

There are several loss functions to assess which model provides the most accurate prediction performance of volatility. However, as Lopez (2001) there is no the best loss function to use to assess the predictive accuracy of the model. For that reason, we consider several, commonly applied loss functions. A detailed bibliography about forecasting performance can be found in Poon and Granger (2003), and Poon (2005). In particular, we use the Mean Squared Error (MSE), Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE), and Theil's Inequality Coefficient (TIC), which are defined as follows:

$$MSE = \frac{1}{n}\sum_{\tau=1}^{n}\left(\sigma_{F,\tau} - \sigma_{R,\tau}\right)^2 \qquad (26)$$

$$MAE = \frac{1}{n}\sum_{\tau=1}^{n}\left|\sigma_{F,\tau} - \sigma_{R,\tau}\right| \qquad (27)$$

$$MAPE = \frac{1}{n}\sum_{\tau=1}^{n}\frac{\left|\sigma_{F,\tau} - \sigma_{R,\tau}\right|}{\sigma_{R,\tau}} \qquad (28)$$

$$TIC = \frac{\sqrt{\frac{1}{n}\sum_{\tau=1}^{n}\left(\sigma_{F,\tau} - \sigma_{R,\tau}\right)^2}}{\sqrt{\frac{1}{n}\sum_{\tau=1}^{n}\left(\sigma_{R,\tau}\right)^2} + \sqrt{\frac{1}{n}\sum_{\tau=1}^{n}\left(\sigma_{F,\tau}\right)^2}} \qquad (29)$$

where n is the number of forecasts and $\sigma_{F,\tau}$, $\sigma_{R,\tau}$ are the forecasted and realized volatility respectively in the evaluation period τ $(\tau = 1, \ldots, n)$. When analyzing the statistics it is important to consider that MSE is a mean of the squared error whereas

MAPE provides a mean of a relative measure of the error, so MSE is more likely to be affected by outliers. MSE, MAE and MAPE are mean error measures of the forecasts, whereas TIC is a coefficient bounded to the interval $[0,1]$, where a value of 0 corresponds to a perfect forecast whereas a value of 1 indicates an imperfect prediction.

The main results of the out-of-sample forecasting are presented in Table 16, and Table 17. In Table 16, the cross-sectional summary of the results are presented, while Table 17, presents firm level results. The selected loss functions point out in the direction that the asymmetric fractionally integrated models provide better forecasts than the short-memory and integrated models. This holds for both firm's asset and equity volatility. The firm level results go in the same direction.

Table 16. *Forecasting results*

	E	V_{SM}	V_{KMV}	V_{Proxy}
MSE				
GARCH	0.0113	0.0024	0.0025	0.0026
EGARCH	0.0094	0.0022	0.0023	0.0024
IGARCH	0.0130	0.0028	0.0029	0.0030
FIGARCH	0.0107	0.0023	0.0024	0.0025
HYGARCH	0.0104	0.0023	0.0024	0.0025
FIEGARCH	**0.0089**	**0.0020**	**0.0021**	**0.0022**
FIAPARCH	0.0093	0.0022	0.0022	0.0023
MAE				
GARCH	0.0744	0.0337	0.0337	0.0344
EGARCH	0.0679	0.0318	0.0319	0.0324
IGARCH	0.0816	0.0366	0.0367	0.0374
FIGARCH	0.0723	0.0326	0.0327	0.0333
HYGARCH	0.0715	0.0325	0.0326	0.0332
FIEGARCH	**0.0661**	**0.0308**	**0.0309**	**0.0315**
FIAPARCH	0.0663	0.0314	0.0315	0.0320
MAPE				
GARCH	0.3024	0.2815	0.2796	0.2831
EGARCH	0.2718	0.2597	0.2584	0.2609
IGARCH	0.3403	0.3128	0.3120	0.3145
FIGARCH	0.2910	0.2671	0.2662	0.2690
HYGARCH	0.2867	0.2670	0.2660	0.2692
FIEGARCH	**0.2585**	**0.2481**	**0.2471**	**0.2498**
FIAPARCH	0.2632	0.2563	0.2547	0.2580

Table 16 (cont.). *Forecasting results*

TIC				
GARCH	0.1709	0.1620	0.1627	0.1646
EGARCH	0.1604	0.1570	0.1577	0.1591
IGARCH	0.1804	0.1698	0.1704	0.1720
FIGARCH	0.1670	0.1589	0.1596	0.1610
HYGARCH	0.1660	0.1582	0.1590	0.1608
FIEGARCH	**0.1578**	**0.1528**	**0.1536**	**0.1547**
FIAPARCH	0.1595	0.1556	0.1564	0.1575

This table reports the cross-sectional average of several error measures between the forecasted and realized volatility (MSE, MAE, MAPE, and TIC). The model with the lowest forecast errors is highlighted in bold.

In the evaluation of the forecasts of equity volatility at the cross-sectional level, we observe that the FIEGARCH model is the best model according to all of the criteria considered, followed by the FIAPARCH model. In contrast, GARCH and IGARCH models show the poorest results. The use of asymmetric and fractionally integrated models does improve equity volatility forecasts. However, the performance showed by EGARCH is better than performance of FIGARCH or HYGARCH, suggesting a relatively higher relevance of asymmetry vs. long-memory property in equity volatility modeling. This pattern is also observed at the firm level: for the majority of firms FIEGARCH or FIAPARCH model have the lowest value of the loss function (from 65.38% to 76.92% of firms depending on the error measure criteria). It is noticeable that in a significant number of firms EGARCH is the best model (from 17.31% to 21.15% of firms, depending on the error measure criteria) outreaching the individual importance of only fractionally integrated models.

Table 17. *Forecasting results at firm level*

	E		***V_{SM}***		***V_{KMV}***		***V_{Proxy}***	
	Num	***%***	***Num***	***%***	***Num***	***%***	***Num***	***%***
MSE								
GARCH	1	1.92%	1	1.92%	1	1.92%	0	0.00%
EGARCH	9	17.31%	9	17.31%	7	13.46%	8	15.38%
IGARCH	0	0.00%	0	0.00%	0	0.00%	0	0.00%
FIGARCH	0	0.00%	3	5.77%	3	5.77%	4	7.69%
HYGARCH	2	3.85%	3	5.77%	4	7.69%	2	3.85%
FIEGARCH	**25**	**48.08%**	**26**	**50.00%**	**27**	**51.92%**	**26**	**50.00%**
FIAPARCH	15	28.85%	10	19.23%	10	19.23%	12	23.08%
MAE								
GARCH	0	0.00%	3	5.77%	3	5.77%	3	5.77%
EGARCH	11	21.15%	10	19.23%	10	19.23%	10	19.23%
IGARCH	0	0.00%	0	0.00%	0	0.00%	0	0.00%
FIGARCH	4	7.69%	3	5.77%	3	5.77%	4	7.69%
HYGARCH	3	5.77%	4	7.69%	3	5.77%	2	3.85%
FIEGARCH	**19**	**36.54%**	**23**	**44.23%**	**25**	**48.08%**	**24**	**46.15%**
FIAPARCH	15	28.85%	9	17.31%	8	15.38%	9	17.31%
MAPE								
GARCH	0	0.00%	2	3.85%	2	3.85%	2	3.85%
EGARCH	9	17.31%	12	23.08%	11	21.15%	13	25.00%
IGARCH	0	0.00%	0	0.00%	0	0.00%	0	0.00%
FIGARCH	1	1.92%	3	5.77%	3	5.77%	3	5.77%
HYGARCH	4	7.69%	3	5.77%	3	5.77%	2	3.85%
FIEGARCH	**23**	**44.23%**	**22**	**42.31%**	**23**	**44.23%**	**23**	**44.23%**
FIAPARCH	15	28.85%	10	19.23%	10	19.23%	9	17.31%
TIC								
GARCH	0	0.00%	2	3.85%	2	3.85%	2	3.85%
EGARCH	11	21.15%	8	15.38%	9	17.31%	9	17.31%
IGARCH	1	1.92%	2	3.85%	2	3.85%	2	3.85%
FIGARCH	2	3.85%	2	3.85%	2	3.85%	3	5.77%
HYGARCH	1	1.92%	3	5.77%	5	9.62%	3	5.77%
FIEGARCH	**23**	**44.23%**	**22**	**42.31%**	**19**	**36.54%**	**21**	**40.38%**
FIAPARCH	14	26.92%	13	25.00%	13	25.00%	12	23.08%

This table reports the number and percentage of firm's with best forecasting results compared along the GARCH-type models estimated and according to the selected loss function to evaluate (MSE, MAE, MAPE and TIC). The model with the highest proportion of firms is highlighted in bold.

Next we evaluate the forecasting error of firm's asset volatility. Firstly, we can observe that forecasting performance of different models is consistent among the three methodologies employed to estimate firm's asset returns. Secondly, at first glance, no substantial difference in model performance with respect to equity volatility could be observed. By way of example, focusing on V_{SM} as our baseline case, according to all of the criteria considered asymmetric and long-memory models outperform simpler models. At firm-level FIEGARCH and FIAPARCH models yield the lowest loss function value for more than 60% of the companies. As in the case of equity volatility, the importance of the EGARCH model vs. only long-memory models (FIGARCH and HYGARCH) is noticeable.

As a robustness check we use variances instead of volatilities (i.e. standard deviations), and consider the Mean Squared Error (MSE2), Mean Absolute Error (MSE2) and following Patton (2011, the maximum likelihood based QLIKE function:

$$MSE2 = \frac{1}{n}\sum_{t=1}^{n}\left(\sigma_{F,\tau}^{2} - \sigma_{R,\tau}^{2}\right)^{2} \quad (30)$$

$$MAE2 = \frac{1}{n}\sum_{\tau=1}^{n}\left|\sigma_{F,\tau}^{2} - \sigma_{R,\tau}^{2}\right| \quad (31)$$

$$QLIKE = \frac{1}{N}\sum_{t=1}^{N}\left[log\left(\sigma_{F,\tau}^{2}\right) + \frac{\sigma_{R,\tau}^{2}}{\sigma_{F,\tau}^{2}}\right] \quad (32)$$

These additional results are provided in the Appendix D (see Table D.1 and Table D.2). The results obtained with variances instead of volatilities are virtually the same as those previously reported.

In conclusion, preliminary analysis on the basis of the loss function comparison points out in the direction that FIEGARCH and FIAPARCH models provide the best forecasts for both equity and firm asset volatility. To assess if the observed difference in forecasts produced from different models is also statistically significant we further apply tests for equal and superior predictive ability.

8.1 The equal predictive ability test

We first consider a widely used test for equal predictive ability of Diebold and Mariano (1995) which is designed to compare predictive accuracy of two competing forecasts. The predictive accuracy of each model is measured on the basis of a particular loss function. Therefore, for each model we define a sequence of losses as: $L_\tau \equiv L(\sigma_{R,\tau}, \sigma_{F,\tau}), \tau = 1, \dots, n$. In empirical applications, either the mean squared error (MSE) or the mean absolute error (MAE), are commonly used as a benchmark loss function to evaluate the performance of volatility models. In our main analysis we use the MSE as a benchmark loss function, $L(\sigma_{R,\tau}, \sigma_{F,\tau}) = (e_\tau)^2$, where e_τ represents a set of forecast errors defined as the difference between realized and forecasted volatility $(\sigma_{R,\tau} - \sigma_{F,\tau})$.

The Diebold and Mariano (1995) test is based on the loss differential between two competing forecasts. Accordingly, for each pair of competing forecasts we define the loss differential as:

$$d_\tau \equiv L_{1,\tau} - L_{2,\tau} \tag{33}$$

The $L_{1,\tau}$ is the sequence of losses for the benchmark model, and $L_{2,\tau}$ is the sequence of losses for the alternative model. If forecasts from the two competing models have equal predictive accuracy the loss differential will have zero expectation. Therefore, the null hypothesis to test is specified as follows: $H_0: E(d_\tau) = 0$. If we reject null hypothesis the two models differ in their predictive accuracy. The DM test statistics is given by:

$$T^{DM} = \frac{\bar{d}}{\left[\widehat{var}(\bar{d})\right]^{1/2}} \tag{34}$$

where $\bar{d} = n^{-1} \sum_{\tau=1}^{n} d_\tau$ and $\widehat{var}(\bar{d})$ is an estimate of the asymptotic variance of $\bar{d}$. Specifically, $\widehat{var}(\bar{d}) \approx n^{-1}\left[\gamma_0 + 2\sum_{k=1}^{h-1} \gamma_k\right]$, where γ_k is the kth autocovariance of d_τ estimated by $\hat{\gamma}_k = n^{-1} \sum_{\tau=k+1}^{n} (d_\tau - \bar{d})(d_{\tau-k} - \bar{d})$.

The results of the Diebold and Mariano (1995) test are reported in Table 18. The table reports the percentage of companies for which the null hypothesis is rejected at the 5% level and the resulted test statistics is negative (i.e. the alternative model outperforms the benchmark model).[28] We report a modification of the Diebold and Mariano (1995) test suggested by Harvey, Leybourne, and Newbold (1997) that leads to better small-sample properties.

Table 18. *Diebold and Mariano (1995) test*

Equity	GARCH	EGARCH	IGARCH	FIGARCH	HYGARCH	FIEGARCH
EGARCH	84.62%					
IGARCH	0.00%	0.00%				
FIGARCH	73.08%	23.08%	94.23%			
HYGARCH	86.54%	26.92%	100.00%	55.77%		
FIEGARCH	94.23%	71.15%	98.08%	88.46%	86.54%	
FIAPARCH	94.23%	57.69%	98.08%	73.08%	75.00%	32.69%

V_{SM}	GARCH	EGARCH	IGARCH	FIGARCH	HYGARCH	FIEGARCH
EGARCH	80.77%					
IGARCH	0.00%	5.77%				
FIGARCH	73.08%	26.92%	92.31%			
HYGARCH	78.85%	23.08%	92.31%	40.38%		
FIEGARCH	88.46%	59.62%	96.15%	78.85%	76.92%	
FIAPARCH	78.85%	51.92%	88.46%	61.54%	63.46%	19.23%

V_{KMV}	GARCH	EGARCH	IGARCH	FIGARCH	HYGARCH	FIEGARCH
EGARCH	78.85%					
IGARCH	0.00%	7.69%				
FIGARCH	71.15%	25.00%	94.23%			
HYGARCH	76.92%	23.08%	94.23%	44.23%		
FIEGARCH	88.46%	59.62%	94.23%	78.85%	78.85%	
FIAPARCH	78.85%	53.85%	86.54%	59.62%	59.62%	23.08%

V_{Proxy}	GARCH	EGARCH	IGARCH	FIGARCH	HYGARCH	FIEGARCH
EGARCH	80.77%					
IGARCH	1.92%	1.92%				
FIGARCH	71.15%	26.92%	98.08%			
HYGARCH	78.85%	23.08%	96.15%	42.31%		
FIEGARCH	88.46%	61.54%	98.08%	78.85%	78.85%	
FIAPARCH	80.77%	51.92%	96.15%	59.62%	65.38%	23.08%

[28] The conclusions are not materially different from those reported in the paper if the 10% significance level is used instead.

This table reports the results for the Diebold and Mariano (1995) test with Harvey et al. (2017) modification. The table reports the percentage of companies for which the row model outperforms the column model under a MSE loss function at the 5% significance level.

The Diebold and Mariano (1995) test show the superioriority of models that allow for long-memory over models that allow only for short-memory in the return volatility process. In the case of equity, FIGARCH and HYGARCH outperform GARCH in 73.08% and 86.54% of the cases, respectively. When asymmetric long-memory models, are compared with the asymmetric short-memory EGARCH model, we can observe that FIEGARCH and FIAPARCH outperform EGARCH in 71.15% and 57.69% of the cases, respectively. In the case of firm assets (V_{SM}), FIGARCH and HYGARCH outperform GARCH in 73.08% and 78.85% of the cases, respectively, whereas FIEGARCH and FIAPARCH outperform EGARCH in 59.62% and 51.92% of the cases, respectively. The results are very similar for V_{KMV} and V_{Proxy}.

The results of the Diebold and Mariano (1995) test also reveal the superiority of asymmetric over symmetric models. Namely, for equity, EGARCH outperforms GARCH in 84.62% of the cases, whereas FIEGARCH and FIAPARCH outperform FIGARCH in 88.46% and 73.08% of the cases, respectively. For our base case firm asset volatility EGARCH outperforms GARCH in 80.77% of the cases, whereas FIEGARCH and FIAPARCH outperform FIGARCH in 78.85% and 61.54%, respectively. The V_{KMV} and V_{Proxy} volatility confirm this result.

Another interesting result that we can derive from Table 18 is that models that account only for long-memory in forecasting volatility (i.e. FIGARCH and HYGARCH) outperform the EGARCH model in only 25% of the cases on average. This finding holds for both equity and firm's asset volatility suggesting a higher relative importance of asymmetry over long-memory. Such a conclusion could also be reached by comparing FIEGARCH to EGARCH and FIGARCH, respectively. Namely, when

EGARCH is extended to allow for long-memory, the performance of the model significantly improves in 71.15% (equity volatility) and 59.62% (firm's asset volatility) of the cases. In contrast, when FIGARCH is extended to account for asymmetry, the performance of the model improves even more, in 88.46% (equity volatility) and 78.85% (firm's asset volatility) of the cases.

Finally, we can also observe a very poor performance of the IGARCH model with regards to all other models. In the case of equity, the IGARCH model is outperformed by all other models in almost 100% the cases. For firm's asset volatility, the IGARCH model is outperformed by other models in more than 90% of the cases, on average (except for the FIAPARCH model which outperforms IGARCH in 88.46% of the cases). We shouldn't be surprised with the poor performance of the IGARCH model. As noted by Baillie et al. (1996), when a short-memory model is applied to a process that exhibits long-memory, the estimated parameters of the short-memory model quite often tend to point to infinite persistence, spuriously suggesting IGARCH model. The infinite memory assumption, however, may be quite unrealistic when applied to financial data.

It could be argued, that Diebold and Mariano (1995) test might not be suitable for comparing nested models (Clark and McCracken, 2001). Although Giacomini and White (2006) justify the validity of the Diebold and Mariano (1995) critical values when rolling window approach is used to estimate parameters of the forecasting models, for robustness purposes we have also considered the Clark and West (2007) test for equal predictive ability in the case of nested models. Such a strategy has been pursued by Awartani and Corradi (2005). Even if we use two different testing approaches for nested (Clark and West, 2007) and non-nested (Diebold and Mariano, 1995) models our previously reported conclusions remain the same. For robustness purposes we have also considered a Giacomini and White (2006) test that can handle both nested and non-nested models under the same testing framework. Moreover, the Giacomini and White (2006) test is suitable for our forecasting methodology in which we use a rolling window of fixed size. The results of the Giacomini and White (2006) shown in Table 19 are not materially different from those previously reported and are thus completely consistent with previous analysis.

Table 19. *Giacomini and White (2006) test*

Equity	GARCH	EGARCH	IGARCH	FIGARCH	HYGARCH	FIEGARCH
EGARCH	82.69%					
IGARCH	0.00%	0.00%				
FIGARCH	71.15%	21.15%	94.23%			
HYGARCH	84.62%	23.08%	100.00%	51.92%		
FIEGARCH	94.23%	71.15%	98.08%	86.54%	86.54%	
FIAPARCH	94.23%	57.69%	98.08%	73.08%	73.08%	26.92%

V_{SM}	GARCH	EGARCH	IGARCH	FIGARCH	HYGARCH	FIEGARCH
EGARCH	80.77%					
IGARCH	0.00%	3.85%				
FIGARCH	69.23%	26.92%	90.38%			
HYGARCH	78.85%	19.23%	92.31%	40.38%		
FIEGARCH	86.54%	59.62%	94.23%	76.92%	75.00%	
FIAPARCH	75.00%	48.08%	86.54%	59.62%	61.54%	15.38%

V_{KMV}	GARCH	EGARCH	IGARCH	FIGARCH	HYGARCH	FIEGARCH
EGARCH	78.85%					
IGARCH	0.00%	5.77%				
FIGARCH	69.23%	25.00%	92.31%			
HYGARCH	73.08%	21.15%	92.31%	44.23%		
FIEGARCH	86.54%	59.62%	94.23%	76.92%	76.92%	
FIAPARCH	76.92%	48.08%	86.54%	57.69%	57.69%	21.15%

V_{Proxy}	GARCH	EGARCH	IGARCH	FIGARCH	HYGARCH	FIEGARCH
EGARCH	80.77%					
IGARCH	1.92%	1.92%				
FIGARCH	69.23%	26.92%	96.15%			
HYGARCH	78.85%	23.08%	96.15%	40.38%		
FIEGARCH	88.46%	59.62%	98.08%	76.92%	76.92%	
FIAPARCH	80.77%	50.00%	92.31%	55.77%	59.62%	23.08%

The table reports the percentage of companies for which the row model outperforms the column model at the 5% significance level using the Giacomini and White (2006) test, under a MSE loss function.

8.2 The superior predictive ability (SPA) test

To establish the ranking of volatility models in terms of their forecasting power we apply the Hansen (2005) test for superior predictive ability (SPA). The SPA test is also used in Hansen and Lunde (2005), Lux et al. (2016), among others. The Hansen (2005) approach evaluates forecasts of volatility models on the basis of their expected loss, without necessity of making an assumption that any of the volatility models is correctly specified. In comparison to Diebold and Mariano (1995) approach, the SPA test, allows taking into consideration multiple forecasting models at once.

Formally, the Hansen (2005) test compares m alternative forecasts ($k = 1, \dots, m$) with a benchmark forecast ($k = 0$), in terms of its predictive ability, defined by expected loss. For each model, k, we use a sequence of obtained volatility forecasts ($\sigma_{k,F,1}, \dots, \sigma_{k,F,n}$) that are compared to subsequently realized volatility ($\sigma_{R,1}, \dots, \sigma_{R,n}$) using a loss function L. In our main analysis we use a mean squared error (MSE) as a benchmark loss function, $L(\sigma_{R,\tau}, \sigma_{F,\tau}) = (\sigma_{R,\tau} - \sigma_{F,\tau})^2$. Accordingly, for each model we define a sequence of losses as: $L_{k,\tau} \equiv L(\sigma_{R,\tau}, \sigma_{F,\tau}), \tau = 1, \dots, n$. Each set of competing model forecasts ($k = 1, \dots, m$) is compared to those of the benchmark model ($k = 0$). The performance of competing models relative to the benchmark, for τ ($\tau = 1, \dots n$) is defined as:

$$d_{k,\tau} \equiv L_{0,\tau} - L_{k,\tau} \tag{35}$$

The null hypothesis is that the benchmark model is not inferior to any alternative forecast. Formally, the null hypothesis is specified as follows:

$$H_0\colon \max_{k=1,\dots,m} E(d_{k,\tau}) \leq 0 \tag{36}$$

If we reject the null hypothesis there is at least one alternative forecasting model that outperforms the benchmark. The SPA test statistics is defined as follows:

$$T^{SPA} = \max_{k=1,\dots,m} \frac{n^{1/2}\bar{d}_k}{\widehat{\omega}_k} \tag{37}$$

Where, $\bar{d}_k$ is the average relative performance of model k, $\bar{d}_k \equiv n^{-1}\sum_{\tau=1}^{n} d_{k,\tau}$, and $\widehat{\omega}_k^2$ is a consistent estimator of $\omega_k^2 \equiv \lim_{n\to\infty} \text{var}\left(n^{1/2}\bar{d}_k\right)$. The p-values are obtained using stationary bootstrap procedure. Following Hansen and Lunde (2005), the number of bootstrap resamples is set to 10,000.

Low p-values indicate rejection of the null hypothesis, i.e. the benchmark model is outperformed by at least one of the other competing models. In contrast, the higher the p-value the better the forecasting performance of the benchmark model relative to all other models. We sequentially set each of the models as the benchmark model and test it against the remaining six models. The main results from the model comparisons are provided in Table 20. We report the mean p-value of the SPA test for a set of 52 companies, the number of companies for which p-value (for the benchmark model) is higher than 5%, and the number of companies for which the model has the highest p-value among all the models when set as a benchmark. Several conclusions could be made.

First, when GARCH model is set as a benchmark, p-values are very low for both equity and firm assets and are similar between the firms. More specifically, p-value is lower than the 5% cut-off for as much as 47 companies (90.38%) in the case of equity (i.e. we reject the null hypothesis), and 45 (86.54%) in the case of V_{SM} (86.54% for V_{KMV}, 90.38% for V_{Proxy}). That is, the test indicates that for almost all the companies there is at least one alternative forecasting model with superior predictive ability relative to the GARCH. Similar results are obtained for IGARCH forecasts. In the case of equity IGARCH model is significantly outperformed by other models in all of the cases. In the case of firm assets the null hypothesis cannot be rejected at 5% only for 2 companies in the case of V_{SM} and V_{KMV} and for one company in the case of V_{Proxy}. Importantly, neither GARCH nor IGARCH result in a highest p-value for any of the companies. This holds for both equity and firm assets.

Table 20. *SPA test*

Benchmark	mean *p-val*	N° comp *p-val* > 0.05		N° comp max *p-val*	
Equity					
GARCH	0.0191	5	9.62%	0	0.00%
EGARCH	0.1166	19	36.54%	3	5.77%
IGARCH	0.0005	0	0.00%	0	0.00%
FIGARCH	0.1223	14	26.92%	3	5.77%
HYGARCH	0.0805	13	25.00%	0	0.00%
FIEGARCH	0.6555	44	84.62%	30	57.69%
FIAPARCH	0.3979	28	53.85%	16	30.77%
V_{SM}					
GARCH	0.0311	7	13.46%	0	0.00%
EGARCH	0.3007	29	55.77%	12	23.08%
IGARCH	0.0033	2	3.85%	0	0.00%
FIGARCH	0.1559	19	36.54%	5	9.62%
HYGARCH	0.1538	20	38.46%	4	7.69%
FIEGARCH	0.5908	41	78.85%	23	44.23%
FIAPARCH	0.3121	28	53.85%	8	15.38%
V_{KMV}					
GARCH	0.0322	7	13.46%	0	0.00%
EGARCH	0.2992	29	55.77%	12	23.08%
IGARCH	0.0037	2	3.85%	0	0.00%
FIGARCH	0.1436	18	34.62%	4	7.69%
HYGARCH	0.1364	19	36.54%	3	5.77%
FIEGARCH	0.6031	42	80.77%	24	46.15%
FIAPARCH	0.3127	30	57.69%	9	17.31%
V_{Proxy}					
GARCH	0.0246	5	9.62%	0	0.00%
EGARCH	0.3076	30	57.69%	12	23.08%
IGARCH	0.0027	1	1.92%	0	0.00%
FIGARCH	0.1501	18	34.62%	3	5.77%
HYGARCH	0.1142	17	32.69%	1	1.92%
FIEGARCH	0.6132	42	80.77%	26	50.00%
FIAPARCH	0.3175	27	51.92%	10	19.23%

This table reports results for the Superior Predictive Ability (SPA) test under a MSE loss function where: *mean (p-value)* refers to the mean p-value of the SPA test for a set of 52 companies; *N° comp (p-val > 0.05)* reports the number and percentage of companies for which p-values (for the benchmark column model) are greater than 5% level; *N° comp (max p-val)* reports the number and percentage of companies for which the model (in the first column) has the highest p-value among all the models when set as a benchmark.

In contrast, when FIEGARCH model is set as a benchmark, for most of the companies in the sample there is no statistical evidence that any of the remaining forecasts are better than the FIEGARCH forecasts. Specifically, in the case of equity we fail to reject the null hypothesis in 84.62% of the cases. In the case of firm asset volatility, we fail to reject the null hypothesis in 78.85% (V_{SM}) and 80.77% (V_{KMV} and V_{Proxy}) of the cases. The alternative, FIAPARCH model, which also accounts for asymmetry and long-range persistence, cannot be outperformed by other models in more than 50% of the cases. Although the performance of the FIAPARCH model is also quite high, it is still lower when compared to the FIEGARCH model. This is consistent with previously reported results of the Diebold and Mariano (1995) test.

For a number of companies, we also fail to reject the null hypothesis in the case of only asymmetric (EGARCH) or only long-memory models (FIGARCH and HYGARCH). These models, on average, perform better than GARCH and IGARCH and worse than FIEGARCH. In the case of equity, p-values are higher than the 5% cut-off for 36.54%, 26.92%, and 25.00% of the companies for the EGARCH, FIGARCH and HYGARCH, respectively. In the case of firm assets, these models seem to perform even slightly better. For the V_{SM}, we are able to reject the null hypothesis for 55.77%, 36.54%, 38.46% of the companies, for the EGARCH, FIGARCH and HYGARCH, respectively. Similar results could also be observed for V_{KMV} and V_{Proxy}.

In line with previous findings, if we treat the model with the highest p-value as the best model for a company, we can clearly observe the underperformance of GARCH and IGARCH models and outperformance of the asymmetry and long-memory models (FIEGARCH and FIAPARCH). This difference in the forecasting power is particularly pronounced in the case of equity. While the symmetric GARCH and IGARCH were never selected as the best model, the asymmetry and long-memory models were selected as the best models in 88.46% of the cases (FIEGARCH in 57.69% and FIAPARCH in 30.77%), that is, for the total of 46 companies. For the remaining 6 companies, EGARCH (in 5.77% of the cases) and FIGARCH (in 5.77% of the cases), were selected as being the best. This leads to a conclusion that for forecasting purposes, in the case of equity it is important to account for the two features at the same time. In the case of firm assets, the ranking of the models seems to be slightly different. The best model is still the FIEGARCH model (44.23% for V_{SM}, 46.15% for V_{KMV}, and 50.00% for V_{Proxy}), while together FIEGARCH and FIAPARCH models were selected for 59.62% (V_{SM}), 63.46% (V_{KMV}), and 69.23% (V_{Proxy}) of the companies. For the firm assets, EGARCH was selected as the best model in 23.08% of the cases (for V_{SM}, V_{KMV}, and V_{Proxy}) whereas only long-memory models (FIGARCH and HYGARCH) were selected for a smaller proportion of companies, 17.31% for V_{SM}, 13.46% for V_{KMV}, and 7.69% for V_{Proxy}.

We perform several robustness checks. First, we use a mean absolute error (MAE) as a loss function, $L(\sigma_{R,\tau}, \sigma_{F,\tau}) = |\sigma_{R,\tau} - \sigma_{F,\tau}|$. The choice of the loss function, doesn't affect the forecast evaluation results. Second, we measure the out-of-sample performance of volatility models in terms of variances, rather than standard deviations, by MSE, MAE and QLIKE loss functions. Third, we also perform the robustness check by setting the cut-off level to 10%. In all the cases, the results remain virtually the same. These additional results are provided in the Appendix E, Table E.1 to Table E.4.

8.3 The credit spread forecasts

In this section we analyze economic implications of forecasted conditional volatilities from different models. We have shown that asymmetry and long-memory significantly enhance the performance of volatility forecasts. To illustrate the economic implications and ensure the robustness of our empirical findings, we utilize the firm's asset volatility forecasts to determine the theoretical credit spread forecasts. The theoretical credit spread is determined as the premium from issuing at par value a hypothetical 5-year maturity coupon bond. A detailed procedure on credit spread estimation from a structural model is given in Forte (2011).

We compare the forecasted credit spreads (CDS_F) with the ex-post, market observable CDS spreads over the following month (CDS_R) using absolute basis as a standard measure of pricing discrepancy. Absolute basis is defined as: $abasis = |CDS_R - CDS_F|$. In other words, we are using a mean absolute error as criteria to differentiate among models. The results, presented in Table 21, are completely in line with our previous findings. The lowest pricing discrepancy could be observed for the FIEGARCH model, followed by the FIAPARCH model. This holds independently on the way in which firm asset values are estimated. The two more sophisticated models outperform simpler models for 57.69% (V_{SM}), 65.38% (V_{KMV}), and 67.31% (V_{Proxy}) of the companies.

We can also observe that pricing errors are lower for the Forte (2011) default barrier, than for the KMV or nominal value of debt. This is completely in line with the previous literature (Forte and Lovreta, 2012). It is important to note that for the purpose of comparison among GARCH-type models the default barrier is assumed to be constant. If default barrier is adjusted to the forecasted volatility level (and this is possible only in our base V_{SM} case) the mean absolute basis would significantly improve. It could be also argued, that while the level of theoretical credit spreads is substantially influenced by the level of default barrier (Forte and Lovreta, 2012) its pattern over time will not be materially affected. For that reason, to evaluate economic implications it becomes even more important to compare models on the basis of their ability to predict the direction of change in the CDS spread level. We calculate mean correct prediction (MCP) as a proportion of observations for which the change in the CDS level over the following month (i.e. increase or decrease) is correctly predicted. The MCP results show the highest percentage of correct sign predictions precisely for the FIEGARCH model (again followed by the FIAPARCH model). As expected, in this case we do not observe substantial differences in the percentage of correct sign predictions among V_{SM}, V_{KMV} and V_{Proxy} firm asset values.

Table 21. *CDS forecasts*

	V_{SM}			V_{KMV}			V_{Proxy}		
		Num	***%***		***Num***	***%***		***Num***	***%***
abasis									
GARCH	68.37	6	11.54%	78.13	3	5.77%	145.05	6	11.54%
EGARCH	67.57	11	21.15%	77.37	9	17.31%	138.72	5	9.62%
IGARCH	79.33	1	1.92%	77.93	3	5.77%	161.78	0	0.00%
FIGARCH	65.48	2	3.85%	76.87	1	1.92%	137.34	5	9.62%
HYGARCH	64.04	2	3.85%	76.91	2	3.85%	137.05	1	1.92%
FIEGARCH	**61.74**	**17**	**32.69%**	**74.47**	**20**	**38.46%**	**130.28**	**18**	**34.62%**
FIAPARCH	62.75	13	25.00%	76.15	14	26.92%	133.68	17	32.69%
MCP									
GARCH	59.2%	6	11.54%	58.3%	8	15.38%	59.6%	7	13.46%
EGARCH	60.7%	5	9.62%	60.1%	6	11.54%	61.9%	8	15.38%
IGARCH	58.0%	1	1.92%	57.5%	1	1.92%	58.4%	0	0.00%
FIGARCH	59.6%	9	17.31%	58.8%	6	11.54%	59.3%	7	13.46%
HYGARCH	59.7%	1	1.92%	58.8%	2	3.85%	59.9%	3	5.77%
FIEGARCH	**62.8%**	18	**34.62%**	**61.6%**	18	**34.62%**	**62.0%**	15	**28.85%**
FIAPARCH	61.7%	12	23.08%	61.2%	11	21.15%	61.9%	12	23.08%

This table reports the cross-sectional average of abasis and MCP error measures between the forecasted and realized CDS spreads, as well as the number and percentage of firm's with best forecasting results according to the selected criteria. The model with the highest proportion of firms is highlighted in bold.

9. Conclusions

In this paper we analyze the relevance of asymmetry and long memory in modeling and forecasting equity and firm's asset volatility. The degree of asymmetry seems to be more pronounced for equity than for firm's asset volatility, whereas the degree of long-memory is in general slightly higher for equity than for firm's asset volatility. However, we find that this difference changes with financial leverage of the company. The difference in asymmetry is increasing with leverage. The difference in long-memory is decreasing with leverage, and once the asymmetry is allowed in the model in addition to long-memory, firm's asset volatility turns out to be more persistent than equity volatility for higher leverage groups.

We provide a comparative ranking of different volatility models in terms of their ability to produce accurate forecasts of firm-level volatility. The results of the paper undoubtedly show that asymmetric models perform better than symmetric models, that long-range dependence models perform better than short-range dependence models and that more sophisticated models that simultaneously account for asymmetry and long-memory work the best: they produce the best in-sample fit and the best out-of-sample forecasts. The main implication of these findings is that the two features should be accounted for in models for pricing financial instruments that require an estimate of the future volatility as an input.

Finally, we answer the question of which volatility model is best suited to equity and asset returns. To compare different models we use a number of tests (both, statistical and economical) and loss functions. Different tests always lead to the same conclusion, the FIEGARCH model (followed by the FIAPARCH model) outperforms other models, and GARCH and IGARCH underperform and should not be used for volatility forecasting.

Further possible extensions of the conducted research might be directed to comparison of the out-of sample forecasts over different forecasting horizons. The main reason for such a comparison is that over the short-term forecasting horizons the difference between the different models should not be significant. However, as the forecasting horizon increases we should observe important differences.

References

Andersen, T. G. and Bollerslev, T. 1998. Answering the skeptics: yes, standard volatility models do provide accurate forecasts, *International Economic Review*, vol. 39, no. 4, 885–905

Andersen, T.G., Bollerslev, T., Diebold, F.X., and Labys, P. 2003. Modeling and forecasting realized voatility, *Econometrica*, vol. 71, no. 2, 579-625

Awartani, B.M.A., and Corradi, V. 2005. Predicting the volatility of the S&P-500 stock index via GARCH models: the role of asymmetries, *International Journal of Forecasting,* vol. 21, 167– 183

Baillie, R.T., Bollerslev, T. and Mikkelsen, H.O. 1996. Fractionally integrated generalized autoregressive conditional heteroskedasticity, Journal of Econometrics, vol. 74, 3–30

Bandi, F. M., Perron, B. 2006. Long memory and the relation between implied and realized volatility, *Journal of Financial Econometrics*, vol. 4, no. 4, 636–670

Black, F., 1976. Studies of stock price volatility changes. In: Proceedings of the 1976 Meetings of the Business and Economic Statistics.American Statistical Society, pp. 177–181

Bekaert, G., and Wu, G. 2000. Asymmetric volatility and risk in equity markets. *Review of Financial Studies*, vol. 13, 1 – 42.

Bollerslev, T. 1986. Generalized Autoregressive Conditional Heteroskedasticity, *Journal of Econometrics*, vol. 31, 307–27

Bollerslev, T., Engle, R. F., and Nelson, D. B. 1994. ARCH models, pp. 2959-3038, in Engle, R. F. and McFadden, D. L. (eds.), *Handbook of econometrics*, Elsevier Science

Bollerslev, T. and Mikkelsen, H. O. 1996. Modeling and pricing long memory in stock market volatility, *Journal of Econometrics*, vol. 73, 151–84

Bollerslev, T. and Mikkelsen, H. O. 1999. Long-term equity anticipation securities and stock market volatility dynamics, *Journal of Econometrics*, vol. 92, 75–99.

Brailsford, T. J. and Faff, R. W. 1996. An evaluation of volatility forecasting techniques, *Journal of Banking & Finance*, vol. 20, no. 3, 419–38

Breidt, F.J, Crato, N., and De Lima, P. 1998. The detection and estimation of long memory in stochastic volatility, *Journal of Econometrics*, 83, 325–348

Brooks, C. and Burke, S. P. 2003. Information criteria for GARCH model selection, *The European Journal of Finance*, vol. 9, no. 6, 557–80

Bharath, S.T. and Shumway, T. 2008. Forecasting default with the merton distance to default model, *Review of Financial Studies*, vol. 21(3), 1339–1369

Charitou, A., Dionysiou, D., Lambertides, N. and Trigeorgis, L. 2013. Alternative bankruptcy prediction models using option-pricing theory, *Journal of Banking and Finance*, vol. 37 (7), 2329–2341

Choi, J. and Richardson, M. 2016. The volatility of a firm's assets and the leverage effect, *Journal of Financial Economics*, vol. 121, no. 2, 254–77

Christensen, B. J. and Nielsen, M. Ø. 2006. Asymptotic normality of narrow-band least squares in the stationary fractional cointegration model and volatility forecasting, *Journal of Econometrics*, vol. 133, 343–71

Clark, T.E., and McCracken. 2001. Tests of equal forecast accuracy and encompassing for nested models, *Journal of Econometrics*, vol. 105, issue 1, 85–110.

Clark, T.E., and West, K.D. 2007. Approximately normal tests for equal predictive accuracy in nested models, *Journal of Econometrics*, 138, 291-311

Davidson, J. 2004. Moment and memory properties of linear conditional heteroscedasticity models, and a new model, *Journal of Business & Economic Statistics*, vol. 22, no. 1, 16–29

Diebold, F.X., and Mariano, R.S. 1995. Comparing predictive accuracy, *Journal of Business and Economic Statistics*, 13, 253-263

Ding, Z., Granger, C. W. J., and Engle, R. F. 1993. A long memory property of stock market returns and a new model, *Journal of Empirical Finance*, vol. 1, 83–106

Ding, J. and Meade, N. 2010. Forecasting accuracy of stochastic volatility, GARCH and EWMA models under different volatility scenarios, *Applied Financial Economics*, vol. 20, no. 10, 771–83

Dionisio, A., Menezes, R. and Mendes, D.A. 2007. On the integrated behaviour of non-stationary volatility in stock markets, *Physica A*, vol. 382 (1), 58–65

Engle, R.F. 1982. Autoregressive conditional heteroskedasticity with estimates of the variance of U.K. inflation. *Econometrica*, vol. 50, 987-1008

Engle, R. F. and Bollerslev, T. 1986. Modelling the persistence of conditional variances, *Econometric Reviews*, vol. 5, no. 1, 1–50

Engle, R. F. and Ng, V.K. 1993. Measuring and testing the impact of news on volatility, *Journal of Finance*, vol. 48, 1749–1778

Engle, R. F. and Patton, A. J. 2001. What good is a volatility model?, *Quantitative Finance*, vol. 1, no. 2, 237–45

Forte, S. 2011. Calibrating structural models: a new methodology based on stock and credit default swap data, *Quantitative Finance*, vol. 11, no. 12, 1745–59

Forte, S., and Lovreta, L., 2012. Endogenizing exogenous default barrier models: the MM algorithm. *Journal of Banking and Finance*, vol. 36, 1639-1652

Gao, Y., Zhang, C., and Zhang, L. 2012. Comparison of GARCH models based on different d istributions, *Journal of Computers*, vol. 7, no. 8, 1967–73

Granger, C. W. J., and Ding, Z., 1996. Varieties of long memory models, *Journal of Econometrics*, vol. 73, 61–77

Giacomini, R, and White, H. 2006. Tests of conditional predictive ability, *Econometrica*, vol. 74, No. 6, 1545-1578

González-Pla, F. and Lovreta, L. 2019. Persistence in firm's asset and equity volatility, *Physica A: Statistical Mechanics and its Applications*, vol. 535, 122265

Ghysels, E., Plazzi, A., Valkanov, R., Rubia, A., and Dossani, A., 2019, Direct versus iterated multiperiod volatility forecasts, *Annual Review of Financial Economics*, vol. 11, 173-195

Hansen, P. R. 2005. A test for superior predictive ability. *Journal of Business & Economic Statistics*, vol. 23, issue 4, 365-380

Hansen, P. R. and Lunde, A. 2005. A forecast comparison of volatility models: Does anything beat a GARCH(1,1)?, *Journal of Applied Econometrics*, vol. 20, no. 7, 873–89

Harvey, D., Leybourne, S., and Newbold, P. 1997. Testing the equality of prediction mean squared error, *International Journal of Forecasting*, vol. 13, 281–291

Javed, F. and Mantalos, P. 2013. GARCH-Type Models and Performance of Information Criteria, *Communications in Statistics - Simulation and Computation*, vol. 42, no. 8, 1917–33

Kang, S. H. and Yoon, S.-M. 2007. Long memory properties in return and volatility: Evidence from the Korean stock market, *Physica A*, vol. 385, 591–600

Laurent, S. and Peters, J.P. 2002. G@RCH 2.2: An ox package for estimating and forecasting various ARCH models, *Journal of Economic Surveys*, vol. 16, 447–485

Lobato, I. N. and Savin, N. E. 1998. Real and spurious long-memory properties of stock-market data, *Journal of Business & Economic Statistics*, vol. 16, issue 3, 261-268

Lopes, S. and Prass, T. 2014. Theoretical results on fractionally integrated exponential generalized autoregressive conditional heteroskedastic processes, *Physica A*, 278–307

Lopez, J.A. 2001. Evaluating the predictive accuracy of volatility models, *Journal of Forecasting*, vol(20), 87-109

Lux, T. and Kaizoji, T. 2007. Forecasting volatility and volume in the Tokyo Stock Market: Long memory, fractality and regime switching, *Journal of Economic Dynamics & Control*, vol. 31, 1808–43

Lux, T, Segnon, M. and Gupta, R. 2016. Forecasting crude oil price volatility and value-at-risk: Evidence from historical and recent data, *Energy Economics*, vol. 56, 117-133

Martens, M., and Zein, J. 2004. Predicting financial volatility: High-frequency time-series forecasts vis-a-vis implied volatility, *Journal of Futures Markets*, vol. 24, 1005–1028

Marcellino, M, Stock, J.H, Watson, M.W. 2006. A comparison of direct and iterated multistep AR methods for forecasting macroeconomic time series, *Journal of Econometrics*, 135, 499–526

Mitchell, H. and Mckenzie, M. D. 2003. GARCH model selection criteria, *Quantitative Finance*, vol. 3, no. 4, 262–84

Nelson, D. B. 1991. Conditional Heteroskedasticity in Asset Returns: A New Approach, *Econometrica*, vol. 59, no. 2, 347–70

Pagan, A.R., and Schwert, G.W. 1990. Alternative models for conditional stock volatility. *Journal of Econometrics*, 45, 267-290

Patton, A. 2011. Volatility Forecast Comparison Using Imperfect Volatility Proxies. *Journal of Econometrics*, 160:246–256.

Pong, S., Shackleton, M.B., Taylor, S.J., and Xu, X. 2004. Forecasting currency volatility: A comparison of implied volatilities and AR(FI)MA models, *Journal of Banking and Finance*, vol. 28, 2541-2563

Poon, S.-H. 2005. *A Practical Guide to Forecasting Financial Market Volatility*, Wiley

Poon, S.-H. and Granger, C. W. J. 2003. Forecasting volatility in financial markets: A review, *Journal of Economic Literature*, vol. XLI, no. 2, 478–539

Ruiz, E., and Veiga, H., 2008. Modelling long-memory volatilities with leverage effect: A-LMSV versus FIEGARCH, *Computational Statistics & Data Analysis.* 52, 2846–2862.

Tse, Y. K. 1998. The conditional heteroscedasticity of the Yen-Dollar exchange rate, *Journal of Applied Econometrics*, vol. 13, no. 1, 49–55

Vilasuso, J. 2002. Forecasting exchange rate volatility, *Economics Letters*, vol. 76(1), 59–64

Zivot, E. 2008. Practical issues in the analysis of univariate GARCH models, pp. 113–55, in Mikosch, T., KreiB, J., Davis, R., and Andersen, T. (eds.), *Handbook of Financial Time Series*, Berlin, Heidelberg, Springer

Appendix A: Table A.1. List of companies

No.	Company	MC in m €	Sector	Subsector
1	AB Volvo	13,628.76	Industrial Engineering	Commercial Vehicles & Trucks
2	BMW AG	33,359.36	Automobiles & Parts	Automobiles
3	Michelin SCA	10,639.76	Automobiles & Parts	Tires
4	Continental AG	17,302.49	Automobiles & Parts	Tires
5	Daimler AG	51,141.31	Automobiles & Parts	Automobiles
6	Peugeot SA	8,266.57	Automobiles & Parts	Automobiles
7	Renault SA	17,540.90	Automobiles & Parts	Automobiles
8	Valeo SA	4,243.23	Automobiles & Parts	Auto Parts
9	Deutsche Lufthansa AG	6,176.73	Travel & Leisure	Airlines
10	Kingfisher PLC	8,147.19	General Retailers	Home Improvement Retailers
11	Koninklijke Philips NV	23,991.42	Health Care Equipment & Services	Medical Equipment
12	LVMH SE	50,950.98	Personal Goods	Clothing & Accessories
13	Marks & Spencer Group PLC	9,150.33	General Retailers	Broadline Retailers
14	Kering SA	14,755.60	General Retailers	Apparel Retailers
15	Sodexo SA	8,543.53	Travel & Leisure	Restaurants & Bars
16	BAT PLC	59,806.73	Tobacco	Tobacco
17	Carrefour SA	23,186.11	Food & Drug Retailers	Food Retailers & Wholesalers
18	Casino Guichard SA	6,979.49	Food & Drug Retailers	Food Retailers & Wholesalers
19	Diageo PLC	43,243.94	Beverages	Distillers & Vintners
20	Danone SA	29,692.63	Food Producers	Food Products
21	Henkel & Co KGaA AG	11,725.60	Household Goods & Home Construction	Nondurable Household Products
22	Imperial Tobacco Group PLC	25,595.60	Tobacco	Tobacco
23	J Sainsbury PLC	7,775.60	Food & Drug Retailers	Food Retailers & Wholesalers
24	Tesco PLC	33,715.80	Food & Drug Retailers	Food Retailers & Wholesalers
25	Unilever NV	42,970.03	Food Producers	Food Products
26	BP PLC	125,691.29	Oil & Gas Producers	Integrated Oil & Gas
27	E.ON SE	44,600.90	Gas, Water & Multiutilities	Multiutilities
28	EDP Energias de Portugal SA	9,757.37	Electricity	Alternative Electricity
29	Iberdrola SA	30,735.58	Electricity	Conventional Electricity
30	Repsol SA	23,708.21	Oil & Gas Producers	Integrated Oil & Gas
31	RWE AG	24,318.96	Gas, Water & Multiutilities	Multiutilities
32	Akzo Nobel NV	11,723.31	Chemicals	Specialty Chemicals
33	Anglo American PLC	33,491.23	Mining	General Mining
34	BAE Systems PLC	16,096.54	Aerospace & Defense	Defense
35	Bayer AG	49,976.78	Pharmaceuticals & Biotechnology	Pharmaceuticals
36	Saint Gobain SA	19,022.46	Construction & Materials	Building Materials & Fixtures
37	Investor AB	8,367.33	Financial Services	Specialty Finance
38	Linde AG	17,498.17	Chemicals	Commodity Chemicals
39	Rolls-Royce Holdings PLC	14,296.92	Aerospace & Defense	Aerospace
40	Siemens AG	69,649.35	General Industrials	Diversified Industrials
41	Stora Enso OYJ	5,040.96	Forestry & Paper	Paper
42	UPM Kymmene OYJ	7,002.66	Forestry & Paper	Paper
43	BT Group PLC	28,121.99	Fixed Line Telecommunications	Fixed Line Telecommunications
44	Deutsche Telekom AG	53,768.38	Mobile Telecommunications	Mobile Telecommunications
45	Orange SA	42,617.39	Fixed Line Telecommunications	Fixed Line Telecommunications
46	Hellenic Telecom. Org. SA	5,530.32	Fixed Line Telecommunications	Fixed Line Telecommunications
47	Koninklijke KPN NV	15,613.93	Fixed Line Telecommunications	Fixed Line Telecommunications
48	Pearson PLC	9,484.90	Media	Publishing
49	STMicroelectronics NV	7,999.00	Technology Hardware & Equipment	Semiconductors
50	Telefonica SA	65,947.77	Fixed Line Telecommunications	Fixed Line Telecommunications
51	Wolters Kluwer NV	5,740.91	Media	Publishing
52	WPP PLC	14,099.36	Media	Media Agencies

Appendix B: Table B.1. Descriptive statistics of equity returns

Company	Mean	St. Dev.	Skew	Kurt	Q(10)	$Q^2(10)$	ARCH	JB	ADF
1	2.67E-04	0.0231	-0.0999	7.9285	21.7568 **	1,141.30 ***	140.0618 ***	3,437.59 ***	-56.5347 ***
2	2.49E-04	0.0192	0.0356	8.1139	21.7525 **	955.03 ***	134.6904 ***	3,695.82 ***	-55.4699 ***
3	3.81E-04	0.0211	-0.0357	7.1421	25.7646 ***	1,003.53 ***	63.7277 ***	2,424.82 ***	-56.9293 ***
4	6.49E-04	0.0262	-0.2864	19.8161	13.7850	320.98 ***	80.0692 ***	40,000.77 ***	-56.2564 ***
5	2.05E-04	0.0207	0.2014	10.2145	37.2552 ***	1,448.24 ***	117.9724 ***	7,377.09 ***	-55.1424 ***
6	7.22E-05	0.0272	0.9121	17.0218	23.8412 ***	200.89 ***	8.9281 ***	28,249.71 ***	-54.8904 ***
7	1.39E-04	0.0252	-0.2197	7.8284	28.8246 ***	1,812.68 ***	204.1410 ***	3,321.26 ***	-53.9043 ***
8	4.75E-04	0.0232	0.0488	6.3437	30.5485 ***	1,095.46 ***	102.6709 ***	1,581.07 ***	-53.8902 ***
9	3.92E-05	0.0190	-0.1239	7.0539	15.3126	169.79 ***	24.4356 ***	2,330.71 ***	-56.8125 ***
10	-1.58E-07	0.0199	-0.1585	7.1193	19.8099 **	1,284.41 ***	223.2909 ***	2,411.75 ***	-58.3415 ***
11	-3.61E-05	0.0182	-0.1326	7.1875	10.1763	687.85 ***	23.5719 ***	2,487.55 ***	-58.5731 ***
12	3.48E-04	0.0175	0.1120	8.5080	22.2611 **	694.30 ***	23.9564 ***	4,293.65 ***	-59.8993 ***
13	-9.86E-05	0.0208	-2.2619	39.8967	17.9156 *	22.81 **	9.5736 ***	195,241.48 ***	-55.6755 ***
14	3.11E-04	0.0193	0.2825	9.8791	18.6801 **	1,701.84 ***	178.7026 ***	6,731.27 ***	-56.4021 ***
15	4.38E-04	0.0153	-0.1886	8.6893	36.6429 ***	579.89 ***	74.3105 ***	4,593.43 ***	-44.4088 ***
16	4.40E-04	0.0141	0.0197	10.4272	36.7546 ***	2,169.94 ***	382.2024 ***	7,794.31 ***	-37.6918 ***
17	-1.73E-04	0.0181	-0.1665	7.0357	14.9490	752.32 ***	91.9448 ***	2,316.92 ***	-58.3634 ***
18	-1.04E-04	0.0168	-0.1698	9.4027	23.7566 ***	908.75 ***	140.0565 ***	5,808.49 ***	-57.4228 ***
19	1.88E-04	0.0131	0.1146	7.7586	40.1896 ***	1,376.10 ***	172.1253 ***	3,206.87 ***	-37.5658 ***
20	2.37E-04	0.0144	0.3734	11.7857	44.6968 ***	389.30 ***	35.6244 ***	10,984.78 ***	-38.0366 ***
21	4.84E-04	0.0146	0.0875	6.5351	33.4059 ***	288.10 ***	30.7069 ***	1,770.08 ***	-43.6620 ***
22	3.69E-04	0.0155	1.6677	36.8778	27.6291 ***	48.42 ***	24.4711 ***	163,732.72 ***	-61.4261 ***
23	-8.85E-05	0.0184	-0.9003	20.3363	18.3845 **	111.10 ***	8.2754 ***	42,922.67 ***	-59.3233 ***
24	-8.90E-05	0.0172	-0.2479	12.3608	26.7433 ***	238.76 ***	40.3564 ***	12,415.28 ***	-58.1712 ***
25	2.41E-04	0.0133	-0.0732	7.4975	50.2421 ***	1,486.68 ***	256.0541 ***	2,860.95 ***	-45.4301 ***
26	-5.95E-05	0.0171	-0.1441	9.1492	24.3201 ***	2,458.37 ***	236.5360 ***	5,354.29 ***	-58.7491 ***
27	-2.91E-04	0.0184	-0.3468	11.2583	28.5130 ***	1,559.36 ***	190.1788 ***	9,704.00 ***	-56.1239 ***
28	1.54E-04	0.0167	-0.0122	21.6745	18.7325 **	169.32 ***	15.9992 ***	49,273.47 ***	-57.3643 ***
29	3.04E-04	0.0179	0.9053	22.7100	34.2484 ***	449.61 ***	81.8100 ***	55,352.83 ***	-43.7141 ***
30	1.83E-05	0.0190	-0.2632	9.0419	29.9110 ***	1,111.81 ***	67.3489 ***	5,196.89 ***	-55.3603 ***
31	-2.62E-04	0.0192	-0.0585	9.7482	23.3419 ***	781.14 ***	125.7870 ***	6,436.09 ***	-56.1681 ***
32	1.58E-04	0.0180	0.2223	11.8739	19.7737 **	249.53 ***	14.8357 ***	11,154.10 ***	-57.0330 ***
33	-8.35E-05	0.0305	-0.0845	9.5995	20.1308 **	1,927.92 ***	90.4418 ***	6,157.84 ***	-57.7958 ***
34	3.25E-04	0.0171	-0.2160	7.0152	30.1984 ***	1,009.44 ***	153.4073 ***	2,304.23 ***	-62.7461 ***
35	4.61E-04	0.0170	-0.1225	6.5859	31.1704 ***	1,077.31 ***	136.9416 ***	1,825.28 ***	-61.0928 ***
36	1.77E-04	0.0228	0.5436	16.1843	40.4260 ***	589.84 ***	40.9887 ***	24,727.11 ***	-59.0023 ***
37	4.53E-04	0.0176	-0.0009	7.3777	21.6927 **	1,179.87 ***	85.3833 ***	2,707.81 ***	-58.8038 ***
38	5.14E-04	0.0162	0.5552	18.4341	24.2463 ***	200.41 ***	19.9214 ***	33,831.46 ***	-60.5036 ***
39	3.64E-04	0.0203	-0.2599	12.4862	29.6176 ***	186.07 ***	19.9056 ***	12,752.73 ***	-58.4528 ***
40	1.63E-04	0.0179	-0.3318	15.1151	15.0158	1,351.64 ***	55.9007 ***	20,800.48 ***	-57.7575 ***
41	-4.56E-05	0.0220	0.0914	6.4046	10.0465	864.42 ***	69.1670 ***	1,642.45 ***	-56.8011 ***
42	1.34E-04	0.0207	-0.0698	7.0351	15.7693	427.27 ***	34.2239 ***	2,303.26 ***	-56.7906 ***
43	1.82E-04	0.0193	-0.3994	19.0305	30.3704 ***	174.26 ***	14.3927 ***	36,398.76 ***	-44.6101 ***
44	6.54E-05	0.0150	-0.0416	11.7451	31.5405 ***	824.78 ***	226.9040 ***	10,806.60 ***	-58.3653 ***
45	-1.03E-04	0.0161	0.0610	6.6070	28.6642 ***	176.75 ***	43.6189 ***	1,840.38 ***	-58.2615 ***
46	-5.54E-05	0.0269	-0.1071	8.4613	41.1009 ***	501.25 ***	145.4225 ***	4,220.64 ***	-55.8631 ***
47	-7.02E-05	0.0201	6.7542	207.6993	13.9297	1.62	1.0638	5,946,159.09 ***	-59.4973 ***
48	3.16E-05	0.0162	-0.1279	13.0011	17.9007 *	77.34 ***	22.5536 ***	14,141.60 ***	-59.2890 ***
49	-2.00E-04	0.0228	-0.1174	5.9280	28.6792 ***	173.95 ***	19.0097 ***	1,219.14 ***	-56.2777 ***
50	-7.70E-05	0.0155	-0.4810	12.0240	26.1673 ***	405.30 ***	61.0037 ***	11,636.61 ***	-57.9940 ***
51	3.20E-04	0.0146	-0.1894	6.1798	7.1731	520.59 ***	54.4908 ***	1,448.90 ***	-58.8560 ***
52	3.19E-04	0.0176	-0.2314	7.2501	28.1285 ***	1,649.39 ***	178.0404 ***	2,582.49 ***	-59.8793 ***

Appendix B: Table B.2. Descriptive statistics of firm asset returns (V_{SM})

Company	Mean	St. Dev.	Skew	Kurt	Q(10)	$Q^2(10)$	ARCH	JB	ADF
1	2.31E-04	0.0091	-0.1009	5.4486	20.7476 **	340.83 ***	34.9443 ***	852.87 ***	-57.7573 ***
2	3.12E-04	0.0049	-0.0959	5.0848	18.4420 **	293.70 ***	58.7884 ***	619.31 ***	-55.3974 ***
3	2.14E-04	0.0095	-0.0474	5.7311	23.5349 ***	533.45 ***	31.3014 ***	1,055.12 ***	-56.9773 ***
4	5.11E-04	0.0102	-0.1289	6.8254	14.6032	177.62 ***	98.2716 ***	2,076.99 ***	-56.9611 ***
5	1.21E-04	0.0058	-0.1560	6.0806	33.2649 ***	770.81 ***	42.6170 ***	1,354.61 ***	-54.8927 ***
6	-6.39E-05	0.0036	-0.4784	18.0463	18.2499 *	233.60 ***	94.2973 ***	32,116.37 ***	-56.3422 ***
7	1.58E-04	0.0057	-0.2069	5.7639	22.0824 **	442.43 ***	37.0026 ***	1,103.51 ***	-54.6518 ***
8	3.32E-04	0.0081	0.1458	6.5441	30.7043 ***	402.81 ***	88.5482 ***	1,786.75 ***	-55.0327 ***
9	1.79E-04	0.0048	-0.1488	8.4998	10.6000	163.35 ***	27.7669 ***	4,286.19 ***	-57.0672 ***
10	2.19E-06	0.0117	-0.3409	7.4579	15.6771	316.89 ***	198.5092 ***	2,873.53 ***	-57.6312 ***
11	-2.84E-06	0.0104	-0.1035	5.5988	9.9385	326.15 ***	9.7694 ***	960.31 ***	-59.1101 ***
12	3.41E-04	0.0118	0.0600	6.3940	20.5344 **	441.88 ***	16.4215 ***	1,629.60 ***	-59.8445 ***
13	-5.30E-05	0.0125	-2.1710	36.8125	14.3365	14.79	8.6165 ***	164,200.25 ***	-55.9696 ***
14	1.43E-04	0.0089	0.1802	7.0389	7.0543	251.27 ***	28.9061 ***	2,323.25 ***	-57.8933 ***
15	3.26E-04	0.0074	-0.2120	8.4641	29.7282 ***	371.82 ***	52.1617 ***	4,243.82 ***	-43.8844 ***
16	3.72E-04	0.0104	-0.0360	7.9656	27.3968 ***	1,881.15 ***	334.6426 ***	3,484.58 ***	-37.0628 ***
17	-3.26E-05	0.0064	-0.1976	6.7118	17.8865 *	517.58 ***	98.4841 ***	1,968.76 ***	-58.4055 ***
18	1.71E-04	0.0043	-0.0203	9.3124	10.8449	575.99 ***	89.1504 ***	5,630.23 ***	-56.9025 ***
19	1.80E-04	0.0097	0.0598	6.6623	31.9993 ***	1,069.58 ***	133.7823 ***	1,897.10 ***	-37.1897 ***
20	3.10E-04	0.0091	0.2748	9.0950	32.1381 ***	383.20 ***	49.5387 ***	5,291.62 ***	-37.3958 ***
21	3.34E-04	0.0077	0.0308	6.2616	26.5078 ***	112.30 ***	15.1676 ***	1,503.58 ***	-60.6896 ***
22	3.84E-04	0.0096	1.6925	33.0407	22.5345 **	26.10 ***	14.0175 ***	129,126.68 ***	-60.3304 ***
23	1.15E-05	0.0100	-1.3322	24.0065	23.5463 ***	107.48 ***	6.2569 **	63,351.41 **	-59.2921 ***
24	1.26E-04	0.0101	-0.2070	9.8299	35.1732 ***	645.95 ***	96.7965 ***	6,615.15 ***	-37.2686 ***
25	1.95E-04	0.0078	-0.0857	7.2028	40.9730 ***	1,382.38 ***	231.7154 ***	2,499.81 ***	-44.9189 ***
26	6.33E-05	0.0096	-0.1322	7.7563	21.7755 **	2,217.13 ***	200.4697 ***	3,206.26 ***	-59.3102 ***
27	-9.21E-05	0.0060	-0.1732	15.6639	60.6463 ***	1,977.89 ***	263.3975 ***	22,676.68 ***	-42.6683 ***
28	2.44E-04	0.0046	0.9066	23.5244	11.8638	111.11 ***	21.6396 ***	59,983.79 ***	-57.3637 ***
29	3.76E-04	0.0075	2.7454	58.7545	16.6066 *	76.57 ***	15.4649 ***	443,473.19 ***	-58.0506 ***
30	9.87E-05	0.0081	-0.4491	9.7999	17.1259 *	1,055.77 ***	45.2829 ***	6,647.12 ***	-55.7758 ***
31	-5.62E-05	0.0043	-0.0131	13.9617	39.9630 ***	1,122.74 ***	260.0163 ***	16,977.67 ***	-54.8555 ***
32	1.00E-04	0.0097	0.1199	11.9238	17.0386 *	101.62 ***	5.7947 **	11,259.68 **	-57.0588 ***
33	1.73E-05	0.0175	-0.1277	8.1650	20.6106 **	1,198.24 ***	57.1769 ***	3,778.53 ***	-58.1500 ***
34	2.19E-04	0.0090	-0.3346	7.5083	21.0522 **	652.98 ***	107.7564 ***	2,935.03 ***	-61.4143 ***
35	3.46E-04	0.0094	-0.1694	5.7941	26.3956 ***	695.18 ***	82.3402 ***	1,119.31 ***	-61.0972 ***
36	1.32E-04	0.0086	0.1530	7.5110	27.8347 ***	426.01 ***	50.9130 ***	2,888.44 ***	-59.2591 ***
37	3.73E-04	0.0121	0.0107	7.3889	23.2101 ***	1,327.98 ***	97.9087 ***	2,721.70 ***	-58.7868 ***
38	4.13E-04	0.0077	0.0654	14.7817	14.0078	108.53 ***	12.2360 ***	19,614.92 ***	-59.5293 ***
39	3.90E-04	0.0105	-0.2974	8.4141	25.4423 ***	60.74 ***	8.5533 ***	4,191.66 ***	-58.6051 ***
40	1.82E-04	0.0088	-0.3762	12.2581	9.5896	574.03 ***	44.5423 ***	12,190.52 ***	-57.8876 ***
41	-5.00E-05	0.0081	-0.0351	6.1268	5.7124	624.86 ***	52.0968 ***	1,382.07 ***	-57.3420 ***
42	4.68E-05	0.0094	-0.2016	8.0070	12.2993	98.02 ***	7.7960 ***	3,565.16 ***	-57.0168 ***
43	1.20E-04	0.0096	-0.1775	26.6939	23.9645 ***	81.70 ***	10.6933 ***	79,339.09 ***	-58.4099 ***
44	1.15E-04	0.0058	-0.0086	11.5180	35.1564 ***	910.52 ***	235.4506 ***	10,251.75 ***	-57.4915 ***
45	-3.22E-05	0.0064	0.0413	8.0381	33.2394 ***	300.69 ***	87.9558 ***	3,587.34 ***	-58.1401 ***
46	-4.48E-05	0.0100	0.0458	8.0584	51.8096 ***	281.95 ***	147.1819 ***	3,616.45 ***	-33.0634 ***
47	-5.29E-05	0.0079	1.3461	32.7388	16.7282 *	8.51	5.1267 **	125,981.79 ***	-59.3136 ***
48	7.44E-05	0.0106	-0.2921	9.6932	21.3141 **	66.03 ***	17.8643 ***	6,378.01 ***	-58.8660 ***
49	-1.67E-04	0.0152	-0.1462	5.3701	24.3403 ***	67.31 ***	9.0284 ***	805.80 ***	-56.5864 ***
50	1.27E-04	0.0067	-0.2430	9.9573	29.8554 ***	809.11 ***	141.4675 ***	6,872.51 ***	-57.9843 ***
51	2.52E-04	0.0075	-0.2447	6.1663	5.2759	292.68 ***	36.3215 ***	1,450.38 ***	-58.0543 ***
52	3.24E-04	0.0081	-0.3016	6.9980	20.4408 **	253.56 ***	108.8279 ***	2,309.83 ***	-59.0562 ***

Appendix B: Table B.3. Descriptive statistics of firm asset returns (V_{KMV})

Company	Mean	St. Dev.	Skew	Kurt	Q(10)	$Q^2(10)$	ARCH	JB	ADF
1	2.20E-04	0.0092	-0.0976	5.4857	22.3033 **	390.08 ***	36.2367 ***	878.39 ***	-57.3714 ***
2	3.23E-04	0.0049	-0.0922	5.1331	18.5744 **	300.04 ***	60.6941 ***	647.71 ***	-55.4171 ***
3	2.09E-04	0.0093	-0.0442	5.6812	23.5974 ***	513.49 ***	29.6886 ***	1,016.85 ***	-56.9591 ***
4	5.14E-04	0.0101	-0.1272	6.9206	15.2777	181.91 ***	103.8106 ***	2,180.97 ***	-57.0253 ***
5	1.23E-04	0.0058	-0.1549	6.1023	33.5329 ***	772.34 ***	43.5005 ***	1,373.38 ***	-54.8610 ***
6	-5.31E-05	0.0035	-0.3837	17.2694	18.5307 **	224.06 ***	83.9292 ***	28,852.38 ***	-56.2381 ***
7	1.63E-04	0.0056	-0.2079	5.7484	22.3415 **	442.66 ***	37.5812 ***	1,091.69 ***	-54.6309 ***
8	3.42E-04	0.0080	0.1354	6.5166	30.9775 ***	407.66 ***	90.3162 ***	1,757.65 ***	-55.0049 ***
9	1.63E-04	0.0047	-0.1558	8.4397	10.5306	158.37 ***	27.1987 ***	4,194.52 ***	-57.0362 ***
10	-5.42E-05	0.0117	-0.3435	7.6253	15.6840	354.02 ***	203.9494 ***	3,089.41 ***	-57.5134 ***
11	-6.42E-06	0.0103	-0.1055	5.6818	9.8507	348.83 ***	10.4730 ***	1,022.44 ***	-59.0648 ***
12	3.30E-04	0.0120	0.0693	6.6859	20.7310 **	477.22 ***	17.4793 ***	1,922.32 ***	-59.8654 ***
13	-2.71E-05	0.0127	-2.1922	37.7348	15.5058	16.58 *	10.0900 ***	173,185.34 ***	-55.8144 ***
14	1.21E-04	0.0089	0.1892	7.0893	7.1443	279.88 ***	32.0452 ***	2,382.96 ***	-57.8137 ***
15	3.01E-04	0.0073	-0.2088	8.4620	29.8297 ***	388.59 ***	54.1561 ***	4,239.90 ***	-43.8822 ***
16	3.54E-04	0.0104	-0.0363	8.1093	27.6396 ***	1,863.81 ***	341.5934 ***	3,689.15 ***	-37.0587 ***
17	-3.72E-05	0.0063	-0.1941	6.6929	17.7717 *	517.25 ***	97.8947 ***	1,948.11 ***	-58.3809 ***
18	1.67E-04	0.0042	-0.0044	9.3396	10.7427	525.86 ***	83.4567 ***	5,678.62 ***	-56.8718 ***
19	1.58E-04	0.0098	0.0635	6.7577	30.1653 ***	1,099.70 ***	138.3587 ***	1,997.39 ***	-37.1078 ***
20	2.90E-04	0.0092	0.2895	9.3930	33.4894 ***	389.91 ***	48.0007 ***	5,822.09 ***	-37.4842 ***
21	3.68E-04	0.0076	0.0154	6.3343	26.6506 ***	107.87 ***	14.2356 ***	1,570.92 ***	-60.8014 ***
22	3.61E-04	0.0096	1.5210	29.8772	20.7427 **	29.70 ***	14.4337 ***	103,373.98 ***	-59.9658 ***
23	1.17E-05	0.0100	-1.3040	23.1670	21.0964 **	104.02 ***	6.7442 ***	58,425.50 ***	-58.9980 ***
24	1.21E-04	0.0100	-0.2338	9.7766	34.0671 ***	627.44 ***	98.2449 ***	6,519.41 ***	-37.0804 ***
25	1.63E-04	0.0077	-0.0728	7.1791	40.4247 ***	1,384.87 ***	232.5857 ***	2,470.65 ***	-44.9043 ***
26	6.67E-05	0.0097	-0.1576	7.7215	21.1626 **	2,097.56 ***	192.7845 ***	3,163.78 ***	-59.0156 ***
27	-9.82E-05	0.0059	-0.1863	15.3460	57.4347 ***	1,906.70 ***	257.9067 ***	21,555.92 ***	-42.5631 ***
28	2.55E-04	0.0046	0.8262	22.7060	12.9592	128.44 ***	24.4344 ***	55,253.24 ***	-57.4568 ***
29	3.65E-04	0.0074	2.6280	55.9673	16.6868 *	79.53 ***	16.2018 ***	400,302.26 ***	-57.9751 ***
30	8.49E-05	0.0081	-0.4416	9.8853	17.5269 *	1,073.88 ***	46.0623 ***	6,808.53 ***	-55.7870 ***
31	-8.55E-05	0.0043	-0.0212	13.6464	39.4335 ***	1,121.21 ***	257.7906 ***	16,015.16 ***	-54.8713 ***
32	1.01E-04	0.0095	0.1244	11.6863	16.6844 *	96.26 ***	5.7742 **	10,669.44 **	-57.0827 ***
33	1.91E-05	0.0174	-0.1370	8.0876	20.9281 **	1,198.55 ***	55.5771 ***	3,667.72 ***	-58.0449 ***
34	2.13E-04	0.0090	-0.3650	7.7984	19.5550 **	594.77 ***	107.0135 ***	3,328.44 ***	-61.1306 ***
35	3.36E-04	0.0093	-0.1664	5.7639	25.9324 ***	678.23 ***	80.3729 ***	1,094.97 ***	-61.0453 ***
36	1.31E-04	0.0085	0.1570	7.6493	28.1996 ***	426.75 ***	51.0007 ***	3,068.05 ***	-59.2375 ***
37	3.56E-04	0.0121	0.0095	7.3595	23.4869 ***	1,317.77 ***	91.7631 ***	2,685.39 ***	-58.5263 ***
38	3.95E-04	0.0076	0.0865	15.3695	13.8976	93.51 ***	10.9136 ***	21,622.34 ***	-59.5309 ***
39	3.77E-04	0.0105	-0.3062	8.4925	25.7690 ***	69.00 ***	9.3876 ***	4,315.44 ***	-58.5395 ***
40	1.79E-04	0.0087	-0.3647	12.0401	9.7648	589.36 ***	43.6330 ***	11,621.89 ***	-57.8573 ***
41	-6.84E-05	0.0081	-0.0249	6.0736	5.9057	634.54 ***	52.3141 ***	1,335.15 ***	-57.2612 ***
42	4.96E-05	0.0096	-0.1841	7.7964	12.2135	112.37 ***	9.0709 ***	3,269.67 ***	-56.9972 ***
43	7.62E-05	0.0096	-0.2776	27.7778	23.1202 **	88.92 ***	12.1077 ***	86,788.17 ***	-58.1332 ***
44	7.10E-05	0.0057	-0.0105	11.3800	35.5880 ***	887.90 ***	231.9444 ***	9,922.09 ***	-57.4636 ***
45	-6.68E-05	0.0062	0.0422	7.8552	31.7357 ***	279.29 ***	82.4921 ***	3,331.64 ***	-58.0908 ***
46	-3.15E-05	0.0101	0.0226	7.5210	49.4593 ***	247.06 ***	136.2098 ***	2,888.18 ***	-32.8693 ***
47	-7.15E-05	0.0078	1.4181	34.2983	17.2225 *	8.39	5.0859 **	139,543.81 ***	-59.3730 ***
48	5.83E-05	0.0107	-0.2860	10.0785	21.8021 **	68.03 ***	18.4187 ***	7,125.73 ***	-58.7699 ***
49	-1.75E-04	0.0151	-0.1455	5.3795	24.2532 ***	66.05 ***	9.1988 ***	811.97 ***	-56.5493 ***
50	1.17E-04	0.0066	-0.2503	9.7310	27.6544 ***	726.45 ***	133.7246 ***	6,436.80 ***	-57.9011 ***
51	2.36E-04	0.0074	-0.2375	6.1440	5.3607	294.01 ***	35.9923 ***	1,428.54 ***	-57.9970 ***
52	3.26E-04	0.0081	-0.3452	7.3339	21.5852 **	279.01 ***	111.5304 ***	2,721.20 ***	-58.7879 ***

Appendix B: Table B.4. Descriptive statistics of firm asset returns (V_{Proxy})

Company	Mean	St. Dev.	Skew	Kurt	Q(10)	$Q^2(10)$	ARCH	JB	ADF
1	2.04E-04	0.0090	-0.1232	5.3925	21.3879 **	367.76 ***	34.1085 ***	817.35 ***	-57.5248 ***
2	3.06E-04	0.0048	-0.1335	5.4963	15.8633	376.29 ***	67.4178 ***	890.52 ***	-56.0615 ***
3	1.91E-04	0.0094	-0.0561	5.6700	22.7169 **	510.95 ***	28.7926 ***	1,009.02 ***	-57.1321 ***
4	5.08E-04	0.0101	-0.1614	6.4365	14.2717	190.35 ***	98.4152 ***	1,683.34 ***	-57.0667 ***
5	1.04E-04	0.0057	-0.2119	6.0125	29.3300 ***	745.22 ***	32.5262 ***	1,307.62 ***	-55.2173 ***
6	-7.50E-05	0.0035	-0.6618	19.5024	17.5720 *	255.59 ***	112.8063 ***	38,725.47 ***	-56.4250 ***
7	1.44E-04	0.0056	-0.2203	5.8128	20.4432 **	426.04 ***	34.7576 ***	1,145.28 ***	-54.7559 ***
8	3.26E-04	0.0080	0.1188	6.8439	30.8864 ***	456.76 ***	99.2874 ***	2,095.67 ***	-55.1530 ***
9	1.54E-04	0.0047	-0.2081	8.1381	11.3425	163.61 ***	24.3562 ***	3,754.65 ***	-57.3093 ***
10	-5.56E-05	0.0117	-0.3539	7.6751	15.7059	343.44 ***	206.1349 ***	3,158.86 ***	-57.5774 ***
11	-1.44E-05	0.0104	-0.1084	5.5146	9.6635	300.74 ***	9.0169 ***	900.08 ***	-59.1157 ***
12	3.30E-04	0.0119	0.0593	6.5999	20.3703 **	457.39 ***	18.3770 ***	1,833.04 ***	-59.8128 ***
13	-4.41E-05	0.0127	-2.0437	34.3865	14.7682	20.03 **	12.7215 ***	141,548.63 ***	-55.9954 ***
14	1.20E-04	0.0090	0.1682	7.1453	7.2816	271.44 ***	30.1140 ***	2,443.85 ***	-57.9369 ***
15	3.05E-04	0.0074	-0.2126	8.2666	29.1362 ***	360.03 ***	49.8130 ***	3,944.51 ***	-43.8493 ***
16	3.43E-04	0.0105	-0.0465	7.9072	27.5193 ***	1,806.25 ***	318.8494 ***	3,403.62 ***	-37.0362 ***
17	-4.55E-05	0.0064	-0.1979	6.5022	18.0854 *	476.13 ***	88.6619 ***	1,755.10 ***	-58.6161 ***
18	1.57E-04	0.0043	-0.0329	8.8387	12.6293	589.43 ***	81.6980 ***	4,817.23 ***	-57.0295 ***
19	1.62E-04	0.0100	0.0591	6.7291	30.6135 ***	1,094.72 ***	133.7934 ***	1,966.77 ***	-37.1200 ***
20	2.84E-04	0.0092	0.2893	9.3998	35.3534 ***	357.59 ***	45.1444 ***	5,834.27 ***	-37.5761 ***
21	3.70E-04	0.0077	0.0052	6.3003	26.9627 ***	119.25 ***	14.8450 ***	1,538.96 ***	-60.8107 ***
22	3.56E-04	0.0096	1.4785	29.1867	21.7198 **	29.97 ***	14.1795 ***	98,125.65 ***	-60.1850 ***
23	4.98E-06	0.0099	-1.2744	22.3878	21.8764 **	111.56 ***	7.0829 ***	54,027.74 ***	-59.1567 ***
24	1.18E-04	0.0100	-0.2497	9.8308	34.5870 ***	607.40 ***	94.9117 ***	6,627.93 ***	-37.1278 ***
25	1.66E-04	0.0078	-0.0727	7.0130	42.9699 ***	1,299.52 ***	212.0329 ***	2,278.37 ***	-45.0492 ***
26	5.75E-05	0.0095	-0.1842	7.5631	19.7349 **	2,017.97 ***	178.9664 ***	2,961.09 ***	-59.1289 ***
27	-1.20E-04	0.0058	-0.2071	15.1613	50.6806 ***	1,941.17 ***	237.3822 ***	20,921.02 ***	-56.4562 ***
28	2.44E-04	0.0047	0.8089	23.7736	12.4231	128.37 ***	18.1018 ***	61,343.44 ***	-57.1830 ***
29	3.60E-04	0.0074	2.5555	54.1350	15.4379	78.34 ***	14.9036 ***	373,138.14 ***	-58.0909 ***
30	7.97E-05	0.0082	-0.4277	9.3572	16.2825 *	965.58 ***	41.2740 ***	5,813.54 ***	-55.8095 ***
31	-1.02E-04	0.0042	-0.0719	13.9116	34.7414 ***	1,178.66 ***	231.8597 ***	16,825.63 ***	-55.2151 ***
32	8.59E-05	0.0097	0.1000	11.2146	16.4059 *	109.61 ***	6.8118 ***	9,539.93 ***	-57.2257 ***
33	1.05E-05	0.0169	-0.1389	8.2765	21.6586 **	1,164.09 ***	56.4054 ***	3,944.69 ***	-58.1298 ***
34	2.14E-04	0.0090	-0.3773	7.9616	20.9049 **	575.99 ***	108.3400 ***	3,558.68 ***	-61.2318 ***
35	3.32E-04	0.0095	-0.1777	5.6573	25.9496 ***	660.17 ***	78.0830 ***	1,015.56 ***	-61.2305 ***
36	1.17E-04	0.0086	0.1043	7.2984	27.4110 ***	424.53 ***	53.2725 ***	2,616.70 ***	-59.1693 ***
37	3.45E-04	0.0123	0.0042	7.3206	23.2333 ***	1,298.68 ***	88.1498 ***	2,637.63 ***	-58.6124 ***
38	3.87E-04	0.0078	-0.0263	14.6284	14.5907	104.84 ***	12.9757 ***	19,105.83 ***	-59.4629 ***
39	3.75E-04	0.0104	-0.3184	8.4267	25.0370 ***	66.92 ***	9.7349 ***	4,218.22 ***	-58.5993 ***
40	1.67E-04	0.0087	-0.3857	11.6409	9.1523	566.16 ***	43.4084 ***	10,633.57 ***	-57.9643 ***
41	-9.22E-05	0.0081	-0.0560	6.0156	6.5734	615.88 ***	45.6441 ***	1,286.63 ***	-57.4506 ***
42	3.83E-05	0.0097	-0.1908	8.0615	13.3886	99.35 ***	7.2894 ***	3,640.26 ***	-57.0091 ***
43	8.49E-05	0.0097	-0.2383	28.8493	24.9122 ***	88.19 ***	12.4627 ***	94,441.18 ***	-58.2901 ***
44	6.82E-05	0.0059	0.0019	10.4245	26.1721 ***	845.68 ***	197.1559 ***	7,788.44 ***	-58.1035 ***
45	-6.32E-05	0.0065	0.0608	7.6549	37.2655 ***	292.67 ***	74.9162 ***	3,063.64 ***	-36.6329 ***
46	-5.98E-05	0.0103	0.0563	8.2841	49.0189 ***	284.09 ***	147.2025 ***	3,946.96 ***	-32.9187 ***
47	-7.80E-05	0.0080	0.9405	24.0215	19.2760 **	13.60	7.1832 ***	62,937.21 ***	-59.3800 ***
48	5.42E-05	0.0108	-0.3003	10.0907	22.4366 **	67.70 ***	18.2718 ***	7,154.74 ***	-58.8860 ***
49	-1.81E-04	0.0151	-0.1473	5.3908	24.0349 ***	65.28 ***	8.6986 ***	819.86 ***	-56.5879 ***
50	1.14E-04	0.0069	-0.2364	9.5122	27.5962 ***	731.99 ***	119.9441 ***	6,023.61 ***	-58.1613 ***
51	2.33E-04	0.0076	-0.2266	5.9240	5.7776	290.06 ***	36.1431 ***	1,237.06 ***	-58.0478 ***
52	3.18E-04	0.0082	-0.3505	7.4287	22.3571 **	271.62 ***	115.3453 ***	2,840.65 ***	-58.9914 ***

Appendix C: Table C.1. Estimated coefficients for the GARCH model - Equity

comp	***LogL***	**ω**	**s.e.**	**α**	**s.e.**	**β**	**s.e.**	**GED**	**s.e.**
1	-7,106.55	0.065	*(0.019)*	0.069	*(0.010)*	0.919	*(0.012)*	1.270	*(0.031)*
2	-6,516.35	0.017	*(0.007)*	0.045	*(0.006)*	0.950	*(0.007)*	1.350	*(0.042)*
3	-6,853.99	0.037	*(0.012)*	0.058	*(0.008)*	0.933	*(0.009)*	1.390	*(0.039)*
4	-7,181.29	0.106	*(0.024)*	0.125	*(0.014)*	0.862	*(0.014)*	1.222	*(0.037)*
5	-6,713.51	0.059	*(0.015)*	0.078	*(0.011)*	0.908	*(0.012)*	1.288	*(0.037)*
6	-7,603.66	0.030	*(0.011)*	0.058	*(0.008)*	0.941	*(0.008)*	1.197	*(0.022)*
7	-7,376.03	0.028	*(0.011)*	0.049	*(0.006)*	0.947	*(0.007)*	1.436	*(0.038)*
8	-7,160.51	0.022	*(0.009)*	0.043	*(0.007)*	0.953	*(0.007)*	1.261	*(0.034)*
9	-6,723.27	0.032	*(0.012)*	0.037	*(0.007)*	0.954	*(0.008)*	1.227	*(0.029)*
10	-6,579.68	0.029	*(0.010)*	0.045	*(0.007)*	0.947	*(0.008)*	1.258	*(0.034)*
11	-6,466.05	0.040	*(0.012)*	0.045	*(0.007)*	0.942	*(0.010)*	1.266	*(0.035)*
12	-6,256.51	0.027	*(0.007)*	0.050	*(0.007)*	0.941	*(0.008)*	1.312	*(0.036)*
13	-6,600.47	0.192	*(0.036)*	0.112	*(0.016)*	0.842	*(0.021)*	1.049	*(0.015)*
14	-6,310.56	0.025	*(0.008)*	0.053	*(0.008)*	0.940	*(0.009)*	1.143	*(0.034)*
15	-5,770.76	0.016	*(0.005)*	0.045	*(0.007)*	0.948	*(0.007)*	1.214	*(0.028)*
16	-5,549.71	0.038	*(0.010)*	0.056	*(0.008)*	0.922	*(0.012)*	1.447	*(0.044)*
17	-6,348.13	0.020	*(0.007)*	0.046	*(0.007)*	0.948	*(0.008)*	1.210	*(0.033)*
18	-5,999.11	0.064	*(0.016)*	0.083	*(0.013)*	0.896	*(0.015)*	1.056	*(0.028)*
19	-5,371.94	0.025	*(0.008)*	0.054	*(0.008)*	0.931	*(0.011)*	1.354	*(0.045)*
20	-5,626.91	0.059	*(0.013)*	0.078	*(0.010)*	0.892	*(0.014)*	1.272	*(0.033)*
21	-5,782.19	0.025	*(0.008)*	0.038	*(0.007)*	0.950	*(0.009)*	1.214	*(0.034)*
22	-5,796.73	0.050	*(0.013)*	0.054	*(0.008)*	0.922	*(0.012)*	1.205	*(0.025)*
23	-6,190.71	0.025	*(0.007)*	0.037	*(0.006)*	0.954	*(0.007)*	1.026	*(0.018)*
24	-6,093.83	0.036	*(0.009)*	0.062	*(0.010)*	0.926	*(0.011)*	1.211	*(0.020)*
25	-5,325.37	0.017	*(0.005)*	0.043	*(0.007)*	0.946	*(0.008)*	1.241	*(0.035)*
26	-6,031.17	0.033	*(0.008)*	0.067	*(0.009)*	0.920	*(0.010)*	1.352	*(0.040)*
27	-6,224.17	0.071	*(0.015)*	0.084	*(0.011)*	0.894	*(0.012)*	1.159	*(0.026)*
28	-5,952.89	0.078	*(0.019)*	0.090	*(0.014)*	0.886	*(0.016)*	1.044	*(0.018)*
29	-5,737.84	0.054	*(0.012)*	0.157	*(0.016)*	0.835	*(0.015)*	1.129	*(0.029)*
30	-6,356.90	0.035	*(0.010)*	0.076	*(0.009)*	0.916	*(0.010)*	1.212	*(0.034)*
31	-6,399.60	0.051	*(0.013)*	0.068	*(0.009)*	0.917	*(0.011)*	1.189	*(0.034)*
32	-6,255.09	0.093	*(0.021)*	0.082	*(0.013)*	0.888	*(0.016)*	1.104	*(0.026)*
33	-7,838.70	0.042	*(0.015)*	0.056	*(0.008)*	0.939	*(0.008)*	1.422	*(0.037)*
34	-6,302.30	0.125	*(0.029)*	0.081	*(0.013)*	0.874	*(0.020)*	1.303	*(0.033)*
35	-6,273.10	0.082	*(0.020)*	0.071	*(0.011)*	0.899	*(0.015)*	1.295	*(0.040)*
36	-6,825.66	0.032	*(0.010)*	0.078	*(0.010)*	0.917	*(0.009)*	1.278	*(0.037)*
37	-6,279.71	0.066	*(0.016)*	0.087	*(0.011)*	0.890	*(0.013)*	1.416	*(0.048)*
38	-5,868.66	0.040	*(0.010)*	0.050	*(0.009)*	0.933	*(0.011)*	1.128	*(0.021)*
39	-6,724.76	0.049	*(0.013)*	0.041	*(0.007)*	0.946	*(0.009)*	1.150	*(0.022)*
40	-6,128.58	0.025	*(0.007)*	0.050	*(0.007)*	0.941	*(0.008)*	1.211	*(0.033)*
41	-7,014.00	0.015	*(0.007)*	0.036	*(0.006)*	0.961	*(0.006)*	1.263	*(0.038)*
42	-6,779.43	0.014	*(0.007)*	0.030	*(0.005)*	0.967	*(0.005)*	1.079	*(0.028)*
43	-6,428.87	0.056	*(0.014)*	0.056	*(0.008)*	0.926	*(0.010)*	1.154	*(0.027)*
44	-5,676.81	0.061	*(0.014)*	0.081	*(0.011)*	0.891	*(0.014)*	1.095	*(0.027)*
45	-6,158.61	0.019	*(0.007)*	0.027	*(0.005)*	0.965	*(0.006)*	1.215	*(0.032)*
46	-7,628.21	0.046	*(0.015)*	0.062	*(0.008)*	0.934	*(0.007)*	1.151	*(0.035)*
47	-6,194.84	0.113	*(0.021)*	0.081	*(0.010)*	0.882	*(0.013)*	0.988	*(0.020)*
48	-6,050.35	0.015	*(0.005)*	0.025	*(0.005)*	0.969	*(0.006)*	1.212	*(0.027)*
49	-7,320.82	0.015	*(0.007)*	0.026	*(0.004)*	0.972	*(0.005)*	1.322	*(0.037)*
50	-5,779.39	0.051	*(0.011)*	0.096	*(0.013)*	0.884	*(0.014)*	1.271	*(0.029)*
51	-5,723.93	0.029	*(0.008)*	0.054	*(0.009)*	0.932	*(0.011)*	1.274	*(0.039)*
52	-6,299.74	0.046	*(0.012)*	0.067	*(0.009)*	0.917	*(0.011)*	1.423	*(0.044)*

Appendix C: Table C.2. Estimated coefficients for the GARCH model - V_{SM}

comp	***LogL***	**ω**	**s.e.**	**α**	**s.e.**	**β**	**s.e.**	**GED**	**s.e.**
1	-4,179.62	0.006	*(0.002)*	0.040	*(0.006)*	0.952	*(0.008)*	1.399	*(0.037)*
2	-2,158.69	0.002	*(0.001)*	0.035	*(0.006)*	0.958	*(0.007)*	1.379	*(0.044)*
3	-4,269.54	0.005	*(0.002)*	0.052	*(0.007)*	0.943	*(0.008)*	1.374	*(0.037)*
4	-4,616.74	0.054	*(0.012)*	0.107	*(0.014)*	0.845	*(0.020)*	1.268	*(0.038)*
5	-2,497.42	0.001	*(0.000)*	0.060	*(0.008)*	0.939	*(0.007)*	1.311	*(0.037)*
6	-735.98	0.001	*(0.000)*	0.043	*(0.006)*	0.953	*(0.007)*	1.240	*(0.021)*
7	-2,592.39	0.003	*(0.001)*	0.041	*(0.006)*	0.950	*(0.008)*	1.448	*(0.037)*
8	-3,659.60	0.001	*(0.001)*	0.036	*(0.006)*	0.963	*(0.006)*	1.278	*(0.029)*
9	-1,992.20	0.001	*(0.001)*	0.032	*(0.006)*	0.962	*(0.007)*	1.253	*(0.028)*
10	-5,079.88	0.016	*(0.005)*	0.037	*(0.006)*	0.951	*(0.009)*	1.323	*(0.033)*
11	-4,709.29	0.016	*(0.005)*	0.038	*(0.007)*	0.946	*(0.010)*	1.285	*(0.037)*
12	-5,047.17	0.012	*(0.004)*	0.044	*(0.007)*	0.947	*(0.008)*	1.319	*(0.036)*
13	-5,021.74	0.079	*(0.017)*	0.093	*(0.015)*	0.853	*(0.023)*	1.095	*(0.016)*
14	-3,942.57	0.001	*(0.001)*	0.032	*(0.005)*	0.967	*(0.005)*	1.154	*(0.033)*
15	-3,381.97	0.004	*(0.001)*	0.041	*(0.006)*	0.951	*(0.007)*	1.222	*(0.028)*
16	-4,603.30	0.018	*(0.005)*	0.053	*(0.008)*	0.929	*(0.011)*	1.488	*(0.049)*
17	-2,998.08	0.005	*(0.002)*	0.037	*(0.007)*	0.949	*(0.009)*	1.226	*(0.034)*
18	-1,564.05	0.007	*(0.002)*	0.070	*(0.012)*	0.890	*(0.019)*	1.117	*(0.028)*
19	-4,397.87	0.013	*(0.004)*	0.048	*(0.007)*	0.937	*(0.010)*	1.424	*(0.047)*
20	-4,148.49	0.033	*(0.007)*	0.077	*(0.010)*	0.882	*(0.016)*	1.273	*(0.034)*
21	-3,640.28	0.003	*(0.001)*	0.030	*(0.005)*	0.966	*(0.006)*	1.229	*(0.033)*
22	-4,229.83	0.007	*(0.002)*	0.028	*(0.005)*	0.962	*(0.007)*	1.251	*(0.023)*
23	-4,181.98	0.004	*(0.001)*	0.024	*(0.004)*	0.971	*(0.005)*	1.132	*(0.018)*
24	-4,366.11	0.014	*(0.004)*	0.052	*(0.009)*	0.933	*(0.012)*	1.315	*(0.023)*
25	-3,514.82	0.003	*(0.001)*	0.037	*(0.006)*	0.957	*(0.006)*	1.239	*(0.035)*
26	-4,215.26	0.008	*(0.002)*	0.055	*(0.007)*	0.936	*(0.008)*	1.418	*(0.044)*
27	-2,319.58	0.005	*(0.001)*	0.078	*(0.010)*	0.905	*(0.010)*	1.176	*(0.029)*
28	-1,655.69	0.007	*(0.002)*	0.095	*(0.014)*	0.872	*(0.019)*	1.106	*(0.018)*
29	-2,692.40	0.011	*(0.002)*	0.154	*(0.015)*	0.829	*(0.015)*	1.151	*(0.028)*
30	-3,640.31	0.011	*(0.003)*	0.067	*(0.009)*	0.915	*(0.011)*	1.245	*(0.034)*
31	-1,423.78	0.003	*(0.001)*	0.058	*(0.008)*	0.922	*(0.011)*	1.208	*(0.034)*
32	-4,247.58	0.021	*(0.005)*	0.063	*(0.010)*	0.914	*(0.013)*	1.104	*(0.024)*
33	-6,165.98	0.013	*(0.005)*	0.046	*(0.007)*	0.950	*(0.007)*	1.445	*(0.036)*
34	-4,144.82	0.015	*(0.004)*	0.054	*(0.008)*	0.926	*(0.011)*	1.360	*(0.032)*
35	-4,270.69	0.010	*(0.003)*	0.059	*(0.009)*	0.931	*(0.010)*	1.293	*(0.039)*
36	-3,875.14	0.004	*(0.001)*	0.062	*(0.008)*	0.934	*(0.008)*	1.314	*(0.037)*
37	-4,968.18	0.017	*(0.005)*	0.073	*(0.009)*	0.915	*(0.010)*	1.438	*(0.049)*
38	-3,482.24	0.009	*(0.003)*	0.044	*(0.008)*	0.939	*(0.010)*	1.151	*(0.021)*
39	-4,711.27	0.013	*(0.004)*	0.031	*(0.006)*	0.957	*(0.008)*	1.226	*(0.026)*
40	-3,871.80	0.005	*(0.002)*	0.041	*(0.006)*	0.952	*(0.007)*	1.219	*(0.033)*
41	-3,748.31	0.003	*(0.001)*	0.031	*(0.005)*	0.965	*(0.006)*	1.286	*(0.039)*
42	-4,260.08	0.004	*(0.002)*	0.023	*(0.004)*	0.972	*(0.005)*	1.108	*(0.028)*
43	-4,197.25	0.013	*(0.004)*	0.047	*(0.007)*	0.938	*(0.010)*	1.218	*(0.029)*
44	-2,447.97	0.007	*(0.002)*	0.064	*(0.009)*	0.915	*(0.011)*	1.107	*(0.029)*
45	-2,945.77	0.003	*(0.001)*	0.025	*(0.005)*	0.968	*(0.006)*	1.211	*(0.031)*
46	-4,471.97	0.018	*(0.005)*	0.057	*(0.008)*	0.925	*(0.009)*	1.184	*(0.037)*
47	-3,534.66	0.061	*(0.013)*	0.090	*(0.016)*	0.805	*(0.033)*	1.012	*(0.019)*
48	-4,729.70	0.005	*(0.002)*	0.020	*(0.004)*	0.975	*(0.005)*	1.296	*(0.029)*
49	-6,079.49	0.014	*(0.006)*	0.020	*(0.004)*	0.974	*(0.006)*	1.335	*(0.037)*
50	-3,025.14	0.011	*(0.003)*	0.087	*(0.012)*	0.888	*(0.014)*	1.289	*(0.030)*
51	-3,513.67	0.006	*(0.002)*	0.045	*(0.007)*	0.945	*(0.009)*	1.280	*(0.039)*
52	-3,856.79	0.009	*(0.003)*	0.052	*(0.008)*	0.935	*(0.010)*	1.457	*(0.040)*

Appendix C: Table C.3. Estimated coefficients for the GARCH model - V_{KMV}

comp	*LogL*	ω	s.e.	α	s.e.	β	s.e.	GED	s.e.
1	-4,218.12	0.006	*(0.002)*	0.041	*(0.006)*	0.951	*(0.008)*	1.402	*(0.037)*
2	-2,173.65	0.002	*(0.001)*	0.036	*(0.006)*	0.958	*(0.007)*	1.376	*(0.043)*
3	-4,225.27	0.005	*(0.002)*	0.052	*(0.007)*	0.943	*(0.008)*	1.374	*(0.037)*
4	-4,562.80	0.050	*(0.011)*	0.106	*(0.014)*	0.847	*(0.020)*	1.265	*(0.038)*
5	-2,488.66	0.001	*(0.000)*	0.060	*(0.008)*	0.939	*(0.007)*	1.314	*(0.037)*
6	-721.03	0.001	*(0.000)*	0.043	*(0.006)*	0.952	*(0.007)*	1.242	*(0.021)*
7	-2,564.16	0.003	*(0.001)*	0.041	*(0.006)*	0.950	*(0.008)*	1.449	*(0.037)*
8	-3,608.66	0.001	*(0.001)*	0.036	*(0.006)*	0.963	*(0.006)*	1.279	*(0.029)*
9	-1,934.86	0.001	*(0.000)*	0.031	*(0.006)*	0.963	*(0.007)*	1.254	*(0.028)*
10	-5,070.36	0.014	*(0.005)*	0.037	*(0.006)*	0.952	*(0.008)*	1.325	*(0.033)*
11	-4,659.66	0.015	*(0.005)*	0.038	*(0.007)*	0.947	*(0.010)*	1.285	*(0.037)*
12	-5,077.79	0.012	*(0.004)*	0.045	*(0.007)*	0.946	*(0.008)*	1.316	*(0.036)*
13	-5,063.50	0.076	*(0.016)*	0.092	*(0.014)*	0.858	*(0.022)*	1.092	*(0.016)*
14	-3,930.31	0.001	*(0.001)*	0.034	*(0.005)*	0.966	*(0.005)*	1.153	*(0.033)*
15	-3,327.89	0.004	*(0.001)*	0.041	*(0.006)*	0.951	*(0.007)*	1.222	*(0.028)*
16	-4,587.85	0.018	*(0.005)*	0.053	*(0.008)*	0.929	*(0.011)*	1.491	*(0.049)*
17	-2,931.62	0.005	*(0.002)*	0.037	*(0.007)*	0.949	*(0.009)*	1.227	*(0.034)*
18	-1,463.32	0.007	*(0.002)*	0.069	*(0.012)*	0.889	*(0.019)*	1.120	*(0.028)*
19	-4,436.95	0.013	*(0.004)*	0.049	*(0.007)*	0.937	*(0.010)*	1.423	*(0.047)*
20	-4,172.83	0.033	*(0.007)*	0.078	*(0.010)*	0.881	*(0.016)*	1.271	*(0.034)*
21	-3,596.80	0.003	*(0.001)*	0.028	*(0.005)*	0.967	*(0.006)*	1.234	*(0.034)*
22	-4,239.25	0.008	*(0.002)*	0.030	*(0.005)*	0.959	*(0.007)*	1.258	*(0.023)*
23	-4,177.99	0.004	*(0.001)*	0.025	*(0.004)*	0.969	*(0.005)*	1.142	*(0.018)*
24	-4,356.72	0.013	*(0.004)*	0.052	*(0.009)*	0.934	*(0.012)*	1.318	*(0.023)*
25	-3,467.14	0.003	*(0.001)*	0.038	*(0.006)*	0.957	*(0.006)*	1.236	*(0.035)*
26	-4,243.48	0.008	*(0.003)*	0.056	*(0.007)*	0.934	*(0.009)*	1.426	*(0.044)*
27	-2,308.71	0.006	*(0.001)*	0.078	*(0.010)*	0.903	*(0.011)*	1.178	*(0.029)*
28	-1,656.75	0.008	*(0.002)*	0.097	*(0.015)*	0.865	*(0.020)*	1.111	*(0.018)*
29	-2,671.05	0.011	*(0.002)*	0.155	*(0.015)*	0.828	*(0.015)*	1.151	*(0.028)*
30	-3,608.55	0.010	*(0.003)*	0.068	*(0.009)*	0.916	*(0.010)*	1.240	*(0.034)*
31	-1,440.28	0.004	*(0.001)*	0.061	*(0.009)*	0.917	*(0.012)*	1.206	*(0.034)*
32	-4,200.32	0.024	*(0.006)*	0.068	*(0.011)*	0.905	*(0.014)*	1.106	*(0.024)*
33	-6,142.20	0.012	*(0.005)*	0.046	*(0.007)*	0.950	*(0.007)*	1.448	*(0.036)*
34	-4,147.47	0.016	*(0.004)*	0.054	*(0.008)*	0.925	*(0.012)*	1.364	*(0.032)*
35	-4,214.97	0.008	*(0.003)*	0.058	*(0.008)*	0.933	*(0.009)*	1.292	*(0.039)*
36	-3,828.08	0.004	*(0.001)*	0.062	*(0.008)*	0.933	*(0.008)*	1.313	*(0.037)*
37	-4,963.84	0.017	*(0.005)*	0.072	*(0.009)*	0.916	*(0.010)*	1.434	*(0.049)*
38	-3,434.30	0.009	*(0.003)*	0.044	*(0.008)*	0.940	*(0.010)*	1.150	*(0.021)*
39	-4,705.15	0.012	*(0.004)*	0.031	*(0.006)*	0.958	*(0.007)*	1.241	*(0.026)*
40	-3,852.44	0.005	*(0.002)*	0.041	*(0.006)*	0.952	*(0.007)*	1.219	*(0.033)*
41	-3,730.10	0.002	*(0.001)*	0.031	*(0.005)*	0.965	*(0.006)*	1.290	*(0.039)*
42	-4,333.65	0.004	*(0.002)*	0.023	*(0.004)*	0.972	*(0.005)*	1.107	*(0.028)*
43	-4,189.88	0.011	*(0.003)*	0.046	*(0.007)*	0.941	*(0.009)*	1.223	*(0.029)*
44	-2,387.58	0.007	*(0.002)*	0.063	*(0.009)*	0.915	*(0.011)*	1.107	*(0.029)*
45	-2,849.15	0.003	*(0.001)*	0.025	*(0.004)*	0.968	*(0.006)*	1.211	*(0.031)*
46	-4,578.85	0.058	*(0.013)*	0.087	*(0.014)*	0.858	*(0.022)*	1.178	*(0.034)*
47	-3,477.91	0.051	*(0.011)*	0.087	*(0.015)*	0.824	*(0.029)*	1.008	*(0.019)*
48	-4,731.47	0.005	*(0.002)*	0.021	*(0.004)*	0.975	*(0.005)*	1.301	*(0.029)*
49	-6,064.75	0.013	*(0.006)*	0.020	*(0.004)*	0.974	*(0.006)*	1.336	*(0.037)*
50	-2,975.28	0.011	*(0.003)*	0.087	*(0.012)*	0.886	*(0.015)*	1.287	*(0.030)*
51	-3,467.29	0.006	*(0.002)*	0.045	*(0.007)*	0.945	*(0.009)*	1.278	*(0.039)*
52	-3,869.09	0.009	*(0.003)*	0.052	*(0.007)*	0.936	*(0.010)*	1.459	*(0.040)*

Appendix C: Table C.4. Estimated coefficients for the GARCH model - V_{Proxy}

comp	***LogL***	**ω**	**s.e.**	**α**	**s.e.**	**β**	**s.e.**	**GED**	**s.e.**
1	-4,138.59	0.006	*(0.002)*	0.042	*(0.007)*	0.951	*(0.008)*	1.405	*(0.037)*
2	-2,080.43	0.002	*(0.001)*	0.039	*(0.006)*	0.954	*(0.007)*	1.364	*(0.043)*
3	-4,248.62	0.005	*(0.002)*	0.053	*(0.007)*	0.942	*(0.008)*	1.381	*(0.038)*
4	-4,589.52	0.049	*(0.011)*	0.107	*(0.014)*	0.849	*(0.020)*	1.263	*(0.038)*
5	-2,379.83	0.001	*(0.000)*	0.065	*(0.008)*	0.934	*(0.007)*	1.307	*(0.037)*
6	-551.32	0.000	*(0.000)*	0.050	*(0.007)*	0.951	*(0.006)*	1.204	*(0.021)*
7	-2,534.80	0.003	*(0.001)*	0.044	*(0.007)*	0.946	*(0.009)*	1.445	*(0.036)*
8	-3,578.07	0.001	*(0.001)*	0.038	*(0.006)*	0.962	*(0.006)*	1.276	*(0.029)*
9	-1,911.08	0.001	*(0.001)*	0.035	*(0.006)*	0.959	*(0.007)*	1.237	*(0.029)*
10	-5,070.37	0.014	*(0.005)*	0.037	*(0.006)*	0.952	*(0.008)*	1.319	*(0.033)*
11	-4,725.89	0.018	*(0.006)*	0.039	*(0.007)*	0.944	*(0.011)*	1.283	*(0.037)*
12	-5,057.64	0.012	*(0.004)*	0.045	*(0.007)*	0.946	*(0.008)*	1.323	*(0.036)*
13	-5,078.88	0.075	*(0.016)*	0.089	*(0.014)*	0.862	*(0.022)*	1.094	*(0.016)*
14	-3,963.31	0.001	*(0.001)*	0.035	*(0.005)*	0.964	*(0.005)*	1.151	*(0.033)*
15	-3,389.40	0.005	*(0.001)*	0.043	*(0.007)*	0.949	*(0.007)*	1.224	*(0.028)*
16	-4,631.63	0.018	*(0.005)*	0.053	*(0.008)*	0.928	*(0.011)*	1.493	*(0.050)*
17	-2,976.42	0.005	*(0.002)*	0.039	*(0.007)*	0.948	*(0.010)*	1.235	*(0.034)*
18	-1,556.45	0.007	*(0.002)*	0.071	*(0.012)*	0.892	*(0.019)*	1.108	*(0.028)*
19	-4,501.39	0.013	*(0.004)*	0.049	*(0.007)*	0.937	*(0.010)*	1.411	*(0.047)*
20	-4,197.33	0.032	*(0.007)*	0.076	*(0.010)*	0.885	*(0.016)*	1.277	*(0.034)*
21	-3,631.71	0.003	*(0.001)*	0.030	*(0.005)*	0.966	*(0.006)*	1.227	*(0.034)*
22	-4,265.55	0.009	*(0.002)*	0.032	*(0.006)*	0.957	*(0.007)*	1.252	*(0.023)*
23	-4,164.69	0.004	*(0.001)*	0.025	*(0.004)*	0.970	*(0.005)*	1.140	*(0.018)*
24	-4,352.53	0.014	*(0.004)*	0.054	*(0.010)*	0.930	*(0.012)*	1.313	*(0.023)*
25	-3,528.28	0.004	*(0.001)*	0.039	*(0.006)*	0.955	*(0.007)*	1.242	*(0.035)*
26	-4,174.96	0.008	*(0.002)*	0.058	*(0.008)*	0.933	*(0.009)*	1.440	*(0.045)*
27	-2,212.69	0.004	*(0.001)*	0.073	*(0.009)*	0.911	*(0.010)*	1.202	*(0.029)*
28	-1,627.63	0.005	*(0.001)*	0.080	*(0.011)*	0.901	*(0.014)*	1.080	*(0.017)*
29	-2,722.35	0.011	*(0.002)*	0.147	*(0.014)*	0.838	*(0.015)*	1.137	*(0.028)*
30	-3,702.05	0.011	*(0.003)*	0.067	*(0.009)*	0.916	*(0.011)*	1.244	*(0.035)*
31	-1,257.78	0.002	*(0.001)*	0.052	*(0.008)*	0.934	*(0.009)*	1.213	*(0.034)*
32	-4,280.56	0.022	*(0.005)*	0.066	*(0.010)*	0.911	*(0.013)*	1.105	*(0.025)*
33	-6,049.11	0.012	*(0.005)*	0.046	*(0.007)*	0.950	*(0.007)*	1.445	*(0.036)*
34	-4,139.49	0.017	*(0.005)*	0.057	*(0.008)*	0.920	*(0.012)*	1.364	*(0.032)*
35	-4,289.85	0.008	*(0.003)*	0.060	*(0.008)*	0.932	*(0.009)*	1.288	*(0.039)*
36	-3,874.08	0.004	*(0.001)*	0.063	*(0.008)*	0.934	*(0.008)*	1.299	*(0.036)*
37	-5,023.91	0.018	*(0.005)*	0.075	*(0.009)*	0.913	*(0.011)*	1.436	*(0.049)*
38	-3,530.03	0.009	*(0.003)*	0.046	*(0.008)*	0.939	*(0.010)*	1.149	*(0.022)*
39	-4,661.16	0.012	*(0.004)*	0.032	*(0.006)*	0.956	*(0.008)*	1.241	*(0.027)*
40	-3,858.80	0.005	*(0.002)*	0.043	*(0.007)*	0.950	*(0.007)*	1.223	*(0.033)*
41	-3,751.61	0.002	*(0.001)*	0.032	*(0.006)*	0.964	*(0.006)*	1.283	*(0.039)*
42	-4,339.31	0.004	*(0.002)*	0.023	*(0.004)*	0.973	*(0.005)*	1.091	*(0.028)*
43	-4,223.91	0.014	*(0.004)*	0.050	*(0.007)*	0.934	*(0.010)*	1.226	*(0.030)*
44	-2,550.10	0.009	*(0.002)*	0.076	*(0.011)*	0.897	*(0.014)*	1.110	*(0.029)*
45	-3,025.46	0.003	*(0.001)*	0.028	*(0.005)*	0.964	*(0.006)*	1.220	*(0.031)*
46	-4,583.80	0.025	*(0.006)*	0.063	*(0.009)*	0.913	*(0.010)*	1.164	*(0.036)*
47	-3,592.17	0.058	*(0.013)*	0.083	*(0.016)*	0.820	*(0.032)*	1.020	*(0.019)*
48	-4,758.88	0.005	*(0.002)*	0.021	*(0.004)*	0.974	*(0.005)*	1.296	*(0.029)*
49	-6,047.16	0.013	*(0.006)*	0.020	*(0.004)*	0.974	*(0.006)*	1.339	*(0.037)*
50	-3,146.18	0.011	*(0.003)*	0.087	*(0.012)*	0.890	*(0.014)*	1.295	*(0.030)*
51	-3,574.19	0.006	*(0.002)*	0.047	*(0.008)*	0.942	*(0.009)*	1.286	*(0.039)*
52	-3,883.08	0.009	*(0.003)*	0.052	*(0.007)*	0.935	*(0.010)*	1.444	*(0.040)*

Appendix C: Table C.5. Estimated coefficients for the EGARCH model - Equity

comp	***LogL***	**ω**	**s.e.**	**β**	**s.e.**	**γ_1**	**s.e.**	**γ_2**	**s.e.**	**GED**	**s.e.**
1	-7,098.09	1.391	*(0.150)*	0.988	*(0.003)*	-0.036	*(0.010)*	0.123	*(0.015)*	1.263	*(0.030)*
2	-6,509.20	1.032	*(0.150)*	0.989	*(0.003)*	-0.044	*(0.010)*	0.115	*(0.014)*	1.358	*(0.043)*
3	-6,844.83	1.228	*(0.164)*	0.991	*(0.003)*	-0.034	*(0.009)*	0.105	*(0.014)*	1.388	*(0.037)*
4	-7,159.39	1.451	*(0.161)*	0.981	*(0.004)*	-0.081	*(0.012)*	0.212	*(0.019)*	1.217	*(0.034)*
5	-6,711.69	1.174	*(0.119)*	0.981	*(0.004)*	-0.051	*(0.011)*	0.145	*(0.018)*	1.276	*(0.034)*
6	-7,581.40	1.757	*(0.250)*	0.995	*(0.002)*	-0.025	*(0.008)*	0.091	*(0.011)*	1.215	*(0.023)*
7	-7,363.12	1.552	*(0.157)*	0.991	*(0.002)*	-0.047	*(0.009)*	0.098	*(0.012)*	1.453	*(0.038)*
8	-7,141.94	1.416	*(0.165)*	0.990	*(0.003)*	-0.053	*(0.009)*	0.112	*(0.015)*	1.291	*(0.034)*
9	-6,708.44	1.186	*(0.086)*	0.980	*(0.005)*	-0.044	*(0.010)*	0.095	*(0.016)*	1.235	*(0.029)*
10	-6,576.99	1.071	*(0.198)*	0.993	*(0.002)*	-0.033	*(0.009)*	0.094	*(0.013)*	1.265	*(0.034)*
11	-6,450.11	1.002	*(0.097)*	0.984	*(0.004)*	-0.058	*(0.010)*	0.091	*(0.013)*	1.277	*(0.035)*
12	-6,220.46	0.775	*(0.130)*	0.990	*(0.002)*	-0.082	*(0.010)*	0.090	*(0.013)*	1.354	*(0.036)*
13	-6,578.43	1.211	*(0.121)*	0.994	*(0.002)*	-0.025	*(0.007)*	0.042	*(0.006)*	1.056	*(0.017)*
14	-6,284.39	0.917	*(0.175)*	0.992	*(0.002)*	-0.062	*(0.010)*	0.103	*(0.014)*	1.174	*(0.035)*
15	-5,753.84	0.585	*(0.138)*	0.988	*(0.003)*	-0.053	*(0.012)*	0.116	*(0.014)*	1.234	*(0.029)*
16	-5,543.96	0.445	*(0.101)*	0.983	*(0.004)*	-0.049	*(0.011)*	0.113	*(0.015)*	1.451	*(0.045)*
17	-6,329.41	0.912	*(0.146)*	0.992	*(0.002)*	-0.062	*(0.010)*	0.082	*(0.013)*	1.232	*(0.035)*
18	-5,968.21	0.779	*(0.116)*	0.985	*(0.004)*	-0.078	*(0.011)*	0.106	*(0.015)*	1.073	*(0.029)*
19	-5,359.88	0.347	*(0.105)*	0.984	*(0.004)*	-0.047	*(0.011)*	0.110	*(0.016)*	1.369	*(0.045)*
20	-5,622.82	0.531	*(0.100)*	0.974	*(0.006)*	-0.045	*(0.012)*	0.158	*(0.016)*	1.260	*(0.032)*
21	-5,756.47	0.617	*(0.081)*	0.981	*(0.005)*	-0.068	*(0.011)*	0.086	*(0.016)*	1.245	*(0.034)*
22	-5,776.98	0.616	*(0.109)*	0.983	*(0.005)*	-0.061	*(0.012)*	0.113	*(0.015)*	1.223	*(0.025)*
23	-6,169.22	0.949	*(0.144)*	0.991	*(0.002)*	-0.039	*(0.008)*	0.080	*(0.012)*	1.046	*(0.018)*
24	-6,077.01	0.810	*(0.152)*	0.993	*(0.002)*	-0.048	*(0.010)*	0.076	*(0.013)*	1.228	*(0.020)*
25	-5,310.11	0.338	*(0.126)*	0.988	*(0.003)*	-0.041	*(0.011)*	0.103	*(0.015)*	1.257	*(0.036)*
26	-6,009.18	0.719	*(0.131)*	0.987	*(0.003)*	-0.066	*(0.010)*	0.114	*(0.014)*	1.385	*(0.040)*
27	-6,211.88	0.908	*(0.108)*	0.979	*(0.004)*	-0.056	*(0.011)*	0.137	*(0.016)*	1.176	*(0.028)*
28	-5,931.86	0.796	*(0.108)*	0.980	*(0.005)*	-0.043	*(0.013)*	0.125	*(0.016)*	1.068	*(0.019)*
29	-5,710.31	0.623	*(0.152)*	0.977	*(0.005)*	-0.076	*(0.012)*	0.227	*(0.019)*	1.133	*(0.027)*
30	-6,331.84	0.949	*(0.143)*	0.986	*(0.003)*	-0.065	*(0.010)*	0.136	*(0.015)*	1.243	*(0.036)*
31	-6,388.10	1.005	*(0.126)*	0.984	*(0.004)*	-0.050	*(0.011)*	0.135	*(0.017)*	1.204	*(0.036)*
32	-6,219.76	0.918	*(0.098)*	0.977	*(0.004)*	-0.078	*(0.012)*	0.128	*(0.018)*	1.129	*(0.025)*
33	-7,833.30	1.849	*(0.207)*	0.993	*(0.002)*	-0.043	*(0.009)*	0.113	*(0.014)*	1.445	*(0.040)*
34	-6,301.99	0.929	*(0.076)*	0.959	*(0.009)*	-0.040	*(0.013)*	0.168	*(0.020)*	1.295	*(0.032)*
35	-6,242.31	0.863	*(0.075)*	0.973	*(0.005)*	-0.099	*(0.012)*	0.108	*(0.017)*	1.343	*(0.043)*
36	-6,795.15	1.181	*(0.159)*	0.989	*(0.002)*	-0.086	*(0.012)*	0.131	*(0.015)*	1.306	*(0.035)*
37	-6,241.79	0.830	*(0.076)*	0.970	*(0.005)*	-0.116	*(0.012)*	0.132	*(0.018)*	1.463	*(0.048)*
38	-5,837.89	0.689	*(0.110)*	0.987	*(0.003)*	-0.068	*(0.011)*	0.089	*(0.014)*	1.163	*(0.023)*
39	-6,705.37	1.207	*(0.109)*	0.983	*(0.004)*	-0.053	*(0.012)*	0.116	*(0.017)*	1.170	*(0.023)*
40	-6,109.85	0.811	*(0.125)*	0.986	*(0.003)*	-0.065	*(0.011)*	0.114	*(0.015)*	1.220	*(0.033)*
41	-7,007.12	1.350	*(0.187)*	0.994	*(0.002)*	-0.038	*(0.009)*	0.083	*(0.012)*	1.272	*(0.037)*
42	-6,764.58	1.231	*(0.173)*	0.993	*(0.002)*	-0.042	*(0.009)*	0.079	*(0.012)*	1.095	*(0.028)*
43	-6,408.22	1.038	*(0.126)*	0.986	*(0.003)*	-0.057	*(0.011)*	0.111	*(0.015)*	1.185	*(0.027)*
44	-5,674.52	0.601	*(0.095)*	0.969	*(0.007)*	-0.040	*(0.013)*	0.163	*(0.019)*	1.089	*(0.027)*
45	-6,151.03	0.835	*(0.096)*	0.984	*(0.005)*	-0.035	*(0.010)*	0.093	*(0.014)*	1.223	*(0.032)*
46	-7,622.29	1.726	*(0.182)*	0.991	*(0.002)*	-0.052	*(0.010)*	0.110	*(0.010)*	1.145	*(0.033)*
47	-6,155.27	0.887	*(0.134)*	0.990	*(0.003)*	-0.051	*(0.009)*	0.088	*(0.013)*	1.021	*(0.020)*
48	-6,034.55	0.793	*(0.125)*	0.993	*(0.002)*	-0.038	*(0.008)*	0.059	*(0.011)*	1.227	*(0.027)*
49	-7,308.75	1.524	*(0.159)*	0.993	*(0.002)*	-0.029	*(0.007)*	0.077	*(0.012)*	1.336	*(0.038)*
50	-5,760.62	0.592	*(0.103)*	0.974	*(0.005)*	-0.071	*(0.012)*	0.167	*(0.020)*	1.288	*(0.028)*
51	-5,705.72	0.556	*(0.102)*	0.982	*(0.004)*	-0.065	*(0.011)*	0.114	*(0.017)*	1.291	*(0.040)*
52	-6,288.21	0.895	*(0.107)*	0.979	*(0.004)*	-0.063	*(0.012)*	0.149	*(0.017)*	1.429	*(0.043)*

Appendix C: Table C.6. Estimated coefficients for the EGARCH model - V_{SM}

comp	*LogL*	ω	s.e.	β	s.e.	γ_1	s.e.	γ_2	s.e.	GED	s.e.
1	-4,172.79	-0.384	*(0.144)*	0.990	*(0.003)*	-0.020	*(0.009)*	0.103	*(0.014)*	1.403	*(0.037)*
2	-2,161.91	-1.545	*(0.092)*	0.979	*(0.006)*	-0.026	*(0.011)*	0.119	*(0.017)*	1.377	*(0.045)*
3	-4,265.68	-0.293	*(0.166)*	0.991	*(0.003)*	-0.019	*(0.010)*	0.114	*(0.014)*	1.370	*(0.037)*
4	-4,596.81	-0.071	*(0.068)*	0.939	*(0.012)*	-0.078	*(0.015)*	0.205	*(0.024)*	1.264	*(0.036)*
5	-2,501.56	-1.327	*(0.180)*	0.988	*(0.003)*	-0.026	*(0.010)*	0.151	*(0.017)*	1.300	*(0.036)*
6	-738.97	-2.348	*(0.167)*	0.988	*(0.003)*	-0.009	*(0.010)*	0.140	*(0.015)*	1.233	*(0.022)*
7	-2,602.89	-1.288	*(0.112)*	0.984	*(0.004)*	-0.023	*(0.011)*	0.114	*(0.015)*	1.431	*(0.037)*
8	-3,653.29	-0.670	*(0.218)*	0.993	*(0.002)*	-0.024	*(0.009)*	0.113	*(0.015)*	1.292	*(0.031)*
9	-1,986.87	-1.659	*(0.141)*	0.990	*(0.003)*	-0.011	*(0.008)*	0.090	*(0.014)*	1.252	*(0.029)*
10	-5,079.56	0.188	*(0.116)*	0.989	*(0.004)*	-0.021	*(0.009)*	0.086	*(0.013)*	1.324	*(0.032)*
11	-4,693.42	-0.035	*(0.070)*	0.977	*(0.006)*	-0.056	*(0.011)*	0.083	*(0.014)*	1.297	*(0.037)*
12	-5,016.53	0.105	*(0.110)*	0.987	*(0.003)*	-0.073	*(0.010)*	0.089	*(0.013)*	1.352	*(0.036)*
13	-4,996.26	0.230	*(0.135)*	0.995	*(0.002)*	-0.014	*(0.007)*	0.040	*(0.007)*	1.103	*(0.018)*
14	-3,923.00	-0.463	*(0.181)*	0.992	*(0.003)*	-0.043	*(0.010)*	0.106	*(0.014)*	1.177	*(0.034)*
15	-3,370.20	-0.813	*(0.127)*	0.986	*(0.004)*	-0.041	*(0.012)*	0.114	*(0.014)*	1.235	*(0.029)*
16	-4,602.81	-0.114	*(0.105)*	0.984	*(0.004)*	-0.037	*(0.011)*	0.110	*(0.015)*	1.483	*(0.048)*
17	-2,983.70	-1.055	*(0.094)*	0.985	*(0.004)*	-0.047	*(0.010)*	0.084	*(0.014)*	1.243	*(0.037)*
18	-1,551.12	-1.842	*(0.081)*	0.969	*(0.007)*	-0.057	*(0.012)*	0.133	*(0.019)*	1.122	*(0.029)*
19	-4,392.12	-0.237	*(0.108)*	0.985	*(0.004)*	-0.035	*(0.011)*	0.107	*(0.016)*	1.432	*(0.047)*
20	-4,148.57	-0.341	*(0.081)*	0.963	*(0.008)*	-0.033	*(0.012)*	0.166	*(0.017)*	1.258	*(0.032)*
21	-3,627.56	-0.646	*(0.103)*	0.984	*(0.004)*	-0.053	*(0.011)*	0.099	*(0.016)*	1.232	*(0.033)*
22	-4,221.36	-0.331	*(0.149)*	0.990	*(0.003)*	-0.026	*(0.010)*	0.099	*(0.014)*	1.257	*(0.022)*
23	-4,168.16	-0.273	*(0.163)*	0.993	*(0.002)*	-0.028	*(0.008)*	0.072	*(0.011)*	1.149	*(0.019)*
24	-4,355.07	-0.303	*(0.189)*	0.995	*(0.002)*	-0.026	*(0.008)*	0.066	*(0.011)*	1.319	*(0.023)*
25	-3,505.32	-0.739	*(0.157)*	0.990	*(0.003)*	-0.030	*(0.011)*	0.107	*(0.015)*	1.250	*(0.036)*
26	-4,211.11	-0.353	*(0.149)*	0.989	*(0.003)*	-0.035	*(0.010)*	0.120	*(0.015)*	1.424	*(0.043)*
27	-2,318.57	-1.432	*(0.145)*	0.983	*(0.004)*	-0.011	*(0.010)*	0.162	*(0.016)*	1.172	*(0.029)*
28	-1,646.77	-1.777	*(0.117)*	0.981	*(0.006)*	-0.014	*(0.011)*	0.134	*(0.017)*	1.119	*(0.018)*
29	-2,684.27	-1.206	*(0.150)*	0.974	*(0.006)*	-0.044	*(0.012)*	0.260	*(0.020)*	1.140	*(0.028)*
30	-3,628.56	-0.654	*(0.116)*	0.981	*(0.004)*	-0.045	*(0.010)*	0.142	*(0.016)*	1.257	*(0.036)*
31	-1,421.96	-1.964	*(0.109)*	0.978	*(0.006)*	-0.024	*(0.011)*	0.153	*(0.018)*	1.202	*(0.034)*
32	-4,214.71	-0.268	*(0.092)*	0.976	*(0.005)*	-0.066	*(0.012)*	0.126	*(0.018)*	1.128	*(0.024)*
33	-6,164.02	0.845	*(0.216)*	0.994	*(0.002)*	-0.023	*(0.009)*	0.104	*(0.014)*	1.458	*(0.037)*
34	-4,147.98	-0.355	*(0.101)*	0.982	*(0.005)*	-0.022	*(0.010)*	0.114	*(0.014)*	1.351	*(0.030)*
35	-4,244.46	-0.300	*(0.096)*	0.980	*(0.004)*	-0.083	*(0.012)*	0.115	*(0.017)*	1.325	*(0.041)*
36	-3,848.09	-0.533	*(0.116)*	0.985	*(0.003)*	-0.076	*(0.012)*	0.116	*(0.015)*	1.342	*(0.035)*
37	-4,940.26	0.068	*(0.095)*	0.979	*(0.004)*	-0.093	*(0.011)*	0.129	*(0.016)*	1.471	*(0.049)*
38	-3,465.00	-0.700	*(0.104)*	0.983	*(0.005)*	-0.042	*(0.010)*	0.107	*(0.017)*	1.163	*(0.022)*
39	-4,700.70	-0.007	*(0.094)*	0.986	*(0.004)*	-0.029	*(0.010)*	0.078	*(0.014)*	1.240	*(0.026)*
40	-3,859.26	-0.521	*(0.128)*	0.987	*(0.003)*	-0.049	*(0.011)*	0.113	*(0.015)*	1.221	*(0.033)*
41	-3,744.55	-0.595	*(0.151)*	0.993	*(0.002)*	-0.022	*(0.009)*	0.079	*(0.012)*	1.293	*(0.039)*
42	-4,252.90	-0.247	*(0.113)*	0.988	*(0.004)*	-0.027	*(0.010)*	0.082	*(0.014)*	1.113	*(0.027)*
43	-4,189.59	-0.298	*(0.130)*	0.988	*(0.004)*	-0.030	*(0.010)*	0.100	*(0.014)*	1.226	*(0.027)*
44	-2,445.73	-1.321	*(0.102)*	0.976	*(0.005)*	-0.020	*(0.012)*	0.143	*(0.017)*	1.098	*(0.028)*
45	-2,941.52	-1.094	*(0.134)*	0.991	*(0.003)*	-0.015	*(0.009)*	0.080	*(0.012)*	1.213	*(0.031)*
46	-4,471.86	-0.126	*(0.075)*	0.954	*(0.010)*	-0.059	*(0.015)*	0.159	*(0.015)*	1.145	*(0.031)*
47	-3,505.90	-0.657	*(0.074)*	0.966	*(0.009)*	-0.048	*(0.013)*	0.118	*(0.019)*	1.030	*(0.019)*
48	-4,719.21	-0.020	*(0.146)*	0.995	*(0.002)*	-0.024	*(0.007)*	0.050	*(0.010)*	1.307	*(0.030)*
49	-6,071.22	0.785	*(0.075)*	0.977	*(0.007)*	-0.027	*(0.009)*	0.094	*(0.017)*	1.348	*(0.038)*
50	-3,007.99	-1.044	*(0.091)*	0.969	*(0.006)*	-0.058	*(0.012)*	0.170	*(0.020)*	1.305	*(0.030)*
51	-3,500.51	-0.754	*(0.113)*	0.987	*(0.003)*	-0.046	*(0.010)*	0.099	*(0.016)*	1.289	*(0.039)*
52	-3,849.68	-0.557	*(0.090)*	0.977	*(0.006)*	-0.037	*(0.012)*	0.128	*(0.017)*	1.458	*(0.038)*

Appendix C: Table C.7. Estimated coefficients for the EGARCH model - V_{KMV}

comp	*LogL*	ω	s.e.	β	s.e.	γ_1	s.e.	γ_2	s.e.	GED	s.e.
1	-4,211.57	-0.365	*(0.151)*	0.990	*(0.003)*	-0.021	*(0.009)*	0.103	*(0.014)*	1.407	*(0.037)*
2	-2,175.87	-1.537	*(0.093)*	0.979	*(0.006)*	-0.028	*(0.011)*	0.119	*(0.017)*	1.376	*(0.045)*
3	-4,221.46	-0.318	*(0.161)*	0.990	*(0.003)*	-0.019	*(0.010)*	0.115	*(0.014)*	1.369	*(0.037)*
4	-4,542.17	-0.102	*(0.069)*	0.940	*(0.012)*	-0.080	*(0.015)*	0.205	*(0.023)*	1.261	*(0.036)*
5	-2,492.11	-1.330	*(0.175)*	0.988	*(0.003)*	-0.028	*(0.010)*	0.150	*(0.017)*	1.303	*(0.037)*
6	-723.71	-2.357	*(0.154)*	0.986	*(0.004)*	-0.010	*(0.010)*	0.138	*(0.015)*	1.234	*(0.022)*
7	-2,574.50	-1.304	*(0.110)*	0.984	*(0.005)*	-0.025	*(0.011)*	0.114	*(0.015)*	1.431	*(0.037)*
8	-3,601.73	-0.698	*(0.211)*	0.993	*(0.002)*	-0.025	*(0.009)*	0.112	*(0.015)*	1.293	*(0.030)*
9	-1,929.79	-1.686	*(0.135)*	0.990	*(0.003)*	-0.012	*(0.008)*	0.090	*(0.015)*	1.252	*(0.029)*
10	-5,069.66	0.180	*(0.135)*	0.991	*(0.003)*	-0.021	*(0.009)*	0.083	*(0.012)*	1.326	*(0.032)*
11	-4,643.96	-0.064	*(0.072)*	0.978	*(0.005)*	-0.055	*(0.010)*	0.083	*(0.014)*	1.296	*(0.036)*
12	-5,046.66	0.119	*(0.115)*	0.988	*(0.003)*	-0.074	*(0.010)*	0.089	*(0.013)*	1.349	*(0.036)*
13	-5,038.03	0.254	*(0.134)*	0.995	*(0.002)*	-0.016	*(0.006)*	0.040	*(0.007)*	1.100	*(0.018)*
14	-3,910.68	-0.473	*(0.190)*	0.992	*(0.002)*	-0.043	*(0.010)*	0.108	*(0.014)*	1.176	*(0.034)*
15	-3,316.45	-0.844	*(0.127)*	0.986	*(0.004)*	-0.041	*(0.012)*	0.116	*(0.015)*	1.235	*(0.029)*
16	-4,587.15	-0.123	*(0.108)*	0.985	*(0.004)*	-0.036	*(0.011)*	0.110	*(0.014)*	1.485	*(0.048)*
17	-2,917.51	-1.095	*(0.094)*	0.985	*(0.004)*	-0.047	*(0.010)*	0.085	*(0.015)*	1.243	*(0.037)*
18	-1,450.62	-1.902	*(0.079)*	0.968	*(0.008)*	-0.057	*(0.013)*	0.135	*(0.019)*	1.124	*(0.029)*
19	-4,430.47	-0.217	*(0.112)*	0.986	*(0.004)*	-0.035	*(0.011)*	0.110	*(0.016)*	1.431	*(0.047)*
20	-4,172.95	-0.326	*(0.083)*	0.964	*(0.008)*	-0.033	*(0.012)*	0.167	*(0.017)*	1.256	*(0.032)*
21	-3,581.90	-0.674	*(0.092)*	0.983	*(0.004)*	-0.057	*(0.011)*	0.091	*(0.016)*	1.238	*(0.033)*
22	-4,230.62	-0.321	*(0.147)*	0.990	*(0.003)*	-0.025	*(0.010)*	0.099	*(0.014)*	1.265	*(0.022)*
23	-4,164.03	-0.281	*(0.163)*	0.993	*(0.002)*	-0.029	*(0.008)*	0.071	*(0.011)*	1.157	*(0.019)*
24	-4,345.68	-0.304	*(0.185)*	0.995	*(0.002)*	-0.027	*(0.008)*	0.066	*(0.011)*	1.323	*(0.023)*
25	-3,458.43	-0.764	*(0.161)*	0.990	*(0.003)*	-0.028	*(0.011)*	0.111	*(0.016)*	1.246	*(0.036)*
26	-4,239.28	-0.337	*(0.145)*	0.989	*(0.003)*	-0.036	*(0.009)*	0.120	*(0.014)*	1.431	*(0.043)*
27	-2,307.52	-1.436	*(0.137)*	0.982	*(0.004)*	-0.013	*(0.010)*	0.163	*(0.017)*	1.175	*(0.029)*
28	-1,648.08	-1.780	*(0.113)*	0.980	*(0.006)*	-0.016	*(0.011)*	0.133	*(0.017)*	1.124	*(0.019)*
29	-2,662.52	-1.216	*(0.147)*	0.973	*(0.006)*	-0.045	*(0.012)*	0.261	*(0.020)*	1.140	*(0.028)*
30	-3,596.37	-0.674	*(0.122)*	0.982	*(0.004)*	-0.046	*(0.010)*	0.143	*(0.015)*	1.254	*(0.036)*
31	-1,438.10	-1.947	*(0.101)*	0.974	*(0.006)*	-0.028	*(0.012)*	0.159	*(0.019)*	1.201	*(0.035)*
32	-4,167.68	-0.296	*(0.088)*	0.974	*(0.006)*	-0.067	*(0.012)*	0.126	*(0.018)*	1.129	*(0.024)*
33	-6,139.54	0.830	*(0.213)*	0.994	*(0.002)*	-0.024	*(0.009)*	0.103	*(0.014)*	1.462	*(0.037)*
34	-4,150.77	-0.353	*(0.097)*	0.981	*(0.005)*	-0.024	*(0.010)*	0.112	*(0.014)*	1.353	*(0.030)*
35	-4,189.53	-0.331	*(0.100)*	0.981	*(0.004)*	-0.081	*(0.012)*	0.117	*(0.017)*	1.323	*(0.041)*
36	-3,801.18	-0.562	*(0.115)*	0.985	*(0.003)*	-0.076	*(0.012)*	0.116	*(0.015)*	1.341	*(0.035)*
37	-4,935.58	0.063	*(0.095)*	0.979	*(0.004)*	-0.094	*(0.011)*	0.128	*(0.016)*	1.468	*(0.049)*
38	-3,417.34	-0.728	*(0.105)*	0.983	*(0.005)*	-0.042	*(0.010)*	0.108	*(0.017)*	1.162	*(0.022)*
39	-4,694.63	-0.015	*(0.101)*	0.988	*(0.004)*	-0.028	*(0.010)*	0.075	*(0.013)*	1.255	*(0.027)*
40	-3,839.81	-0.531	*(0.125)*	0.986	*(0.003)*	-0.049	*(0.011)*	0.114	*(0.015)*	1.221	*(0.033)*
41	-3,725.93	-0.597	*(0.149)*	0.992	*(0.002)*	-0.023	*(0.009)*	0.078	*(0.012)*	1.296	*(0.039)*
42	-4,325.39	-0.206	*(0.117)*	0.989	*(0.004)*	-0.029	*(0.010)*	0.080	*(0.014)*	1.113	*(0.027)*
43	-4,182.56	-0.302	*(0.143)*	0.990	*(0.003)*	-0.026	*(0.010)*	0.098	*(0.014)*	1.230	*(0.028)*
44	-2,386.62	-1.352	*(0.101)*	0.975	*(0.005)*	-0.019	*(0.012)*	0.145	*(0.018)*	1.097	*(0.028)*
45	-2,845.91	-1.142	*(0.129)*	0.990	*(0.003)*	-0.014	*(0.009)*	0.081	*(0.012)*	1.212	*(0.031)*
46	-4,574.30	-0.068	*(0.059)*	0.925	*(0.015)*	-0.077	*(0.017)*	0.170	*(0.021)*	1.162	*(0.031)*
47	-3,447.89	-0.694	*(0.082)*	0.974	*(0.008)*	-0.045	*(0.012)*	0.109	*(0.018)*	1.028	*(0.019)*
48	-4,720.11	-0.016	*(0.153)*	0.995	*(0.002)*	-0.025	*(0.007)*	0.050	*(0.010)*	1.313	*(0.030)*
49	-6,056.36	0.776	*(0.075)*	0.977	*(0.007)*	-0.027	*(0.009)*	0.093	*(0.017)*	1.349	*(0.038)*
50	-2,958.08	-1.071	*(0.087)*	0.967	*(0.007)*	-0.060	*(0.012)*	0.169	*(0.020)*	1.303	*(0.030)*
51	-3,453.87	-0.781	*(0.113)*	0.986	*(0.003)*	-0.046	*(0.010)*	0.099	*(0.016)*	1.288	*(0.039)*
52	-3,861.80	-0.552	*(0.096)*	0.979	*(0.005)*	-0.036	*(0.012)*	0.126	*(0.016)*	1.460	*(0.038)*

Appendix C: Table C.8. Estimated coefficients for the EGARCH model - V_{Proxy}

comp	*LogL*	ω	s.e.	β	s.e.	γ_1	s.e.	γ_2	s.e.	GED	s.e.
1	-4,131.64	-0.405	*(0.144)*	0.989	*(0.003)*	-0.022	*(0.009)*	0.107	*(0.014)*	1.410	*(0.038)*
2	-2,081.46	-1.590	*(0.096)*	0.979	*(0.006)*	-0.032	*(0.011)*	0.126	*(0.017)*	1.360	*(0.044)*
3	-4,244.41	-0.305	*(0.166)*	0.990	*(0.003)*	-0.018	*(0.010)*	0.116	*(0.014)*	1.377	*(0.038)*
4	-4,570.46	-0.086	*(0.071)*	0.942	*(0.012)*	-0.075	*(0.015)*	0.206	*(0.024)*	1.260	*(0.037)*
5	-2,387.05	-1.391	*(0.184)*	0.988	*(0.003)*	-0.027	*(0.010)*	0.156	*(0.017)*	1.292	*(0.036)*
6	-551.01	-2.464	*(0.213)*	0.990	*(0.003)*	-0.003	*(0.010)*	0.150	*(0.015)*	1.205	*(0.022)*
7	-2,544.97	-1.322	*(0.108)*	0.983	*(0.005)*	-0.023	*(0.011)*	0.118	*(0.016)*	1.428	*(0.037)*
8	-3,571.66	-0.719	*(0.230)*	0.993	*(0.002)*	-0.023	*(0.009)*	0.117	*(0.015)*	1.290	*(0.031)*
9	-1,904.57	-1.694	*(0.135)*	0.988	*(0.004)*	-0.012	*(0.009)*	0.104	*(0.016)*	1.235	*(0.029)*
10	-5,069.02	0.184	*(0.130)*	0.990	*(0.003)*	-0.022	*(0.009)*	0.084	*(0.012)*	1.320	*(0.032)*
11	-4,709.29	-0.024	*(0.068)*	0.975	*(0.006)*	-0.058	*(0.011)*	0.083	*(0.015)*	1.298	*(0.037)*
12	-5,025.87	0.108	*(0.112)*	0.988	*(0.003)*	-0.075	*(0.010)*	0.090	*(0.013)*	1.359	*(0.036)*
13	-5,053.70	0.262	*(0.133)*	0.995	*(0.002)*	-0.015	*(0.007)*	0.042	*(0.007)*	1.101	*(0.018)*
14	-3,943.41	-0.449	*(0.187)*	0.992	*(0.003)*	-0.045	*(0.010)*	0.110	*(0.014)*	1.175	*(0.034)*
15	-3,376.28	-0.808	*(0.119)*	0.985	*(0.004)*	-0.045	*(0.012)*	0.119	*(0.015)*	1.238	*(0.029)*
16	-4,630.97	-0.097	*(0.106)*	0.984	*(0.004)*	-0.038	*(0.011)*	0.111	*(0.015)*	1.487	*(0.049)*
17	-2,962.75	-1.065	*(0.090)*	0.984	*(0.004)*	-0.049	*(0.010)*	0.086	*(0.015)*	1.250	*(0.037)*
18	-1,541.32	-1.847	*(0.083)*	0.970	*(0.007)*	-0.060	*(0.013)*	0.134	*(0.019)*	1.115	*(0.030)*
19	-4,493.93	-0.176	*(0.111)*	0.985	*(0.004)*	-0.038	*(0.011)*	0.109	*(0.016)*	1.421	*(0.047)*
20	-4,196.96	-0.312	*(0.082)*	0.964	*(0.008)*	-0.036	*(0.012)*	0.166	*(0.017)*	1.260	*(0.032)*
21	-3,614.89	-0.651	*(0.097)*	0.984	*(0.004)*	-0.059	*(0.011)*	0.095	*(0.016)*	1.235	*(0.033)*
22	-4,256.51	-0.305	*(0.143)*	0.989	*(0.003)*	-0.028	*(0.011)*	0.101	*(0.014)*	1.259	*(0.023)*
23	-4,150.84	-0.282	*(0.160)*	0.993	*(0.002)*	-0.029	*(0.008)*	0.072	*(0.011)*	1.156	*(0.019)*
24	-4,341.66	-0.303	*(0.179)*	0.995	*(0.002)*	-0.028	*(0.008)*	0.068	*(0.012)*	1.316	*(0.023)*
25	-3,518.98	-0.727	*(0.155)*	0.990	*(0.003)*	-0.030	*(0.011)*	0.111	*(0.016)*	1.253	*(0.036)*
26	-4,171.19	-0.376	*(0.147)*	0.988	*(0.003)*	-0.034	*(0.009)*	0.124	*(0.015)*	1.444	*(0.044)*
27	-2,212.12	-1.507	*(0.155)*	0.985	*(0.004)*	-0.007	*(0.009)*	0.159	*(0.016)*	1.198	*(0.030)*
28	-1,609.97	-1.793	*(0.131)*	0.984	*(0.005)*	-0.014	*(0.011)*	0.128	*(0.015)*	1.096	*(0.018)*
29	-2,712.33	-1.184	*(0.153)*	0.975	*(0.006)*	-0.045	*(0.012)*	0.253	*(0.020)*	1.129	*(0.028)*
30	-3,688.31	-0.617	*(0.115)*	0.981	*(0.005)*	-0.047	*(0.010)*	0.144	*(0.016)*	1.259	*(0.037)*
31	-1,258.27	-2.061	*(0.118)*	0.979	*(0.005)*	-0.019	*(0.011)*	0.159	*(0.019)*	1.205	*(0.035)*
32	-4,247.00	-0.249	*(0.093)*	0.976	*(0.005)*	-0.067	*(0.012)*	0.128	*(0.018)*	1.130	*(0.025)*
33	-6,047.60	0.774	*(0.218)*	0.994	*(0.002)*	-0.019	*(0.009)*	0.104	*(0.014)*	1.456	*(0.037)*
34	-4,142.32	-0.357	*(0.097)*	0.980	*(0.005)*	-0.025	*(0.010)*	0.117	*(0.014)*	1.353	*(0.030)*
35	-4,264.03	-0.283	*(0.100)*	0.981	*(0.004)*	-0.083	*(0.012)*	0.117	*(0.017)*	1.319	*(0.041)*
36	-3,847.16	-0.520	*(0.112)*	0.984	*(0.003)*	-0.078	*(0.012)*	0.117	*(0.015)*	1.326	*(0.035)*
37	-4,995.39	0.100	*(0.096)*	0.979	*(0.004)*	-0.094	*(0.011)*	0.130	*(0.016)*	1.470	*(0.050)*
38	-3,512.17	-0.667	*(0.107)*	0.983	*(0.005)*	-0.042	*(0.010)*	0.109	*(0.017)*	1.160	*(0.022)*
39	-4,650.03	-0.041	*(0.098)*	0.987	*(0.004)*	-0.030	*(0.010)*	0.078	*(0.013)*	1.256	*(0.027)*
40	-3,845.99	-0.525	*(0.121)*	0.985	*(0.003)*	-0.051	*(0.011)*	0.115	*(0.015)*	1.226	*(0.033)*
41	-3,748.59	-0.582	*(0.150)*	0.992	*(0.002)*	-0.022	*(0.009)*	0.082	*(0.013)*	1.286	*(0.039)*
42	-4,330.90	-0.195	*(0.120)*	0.989	*(0.004)*	-0.029	*(0.010)*	0.083	*(0.014)*	1.098	*(0.027)*
43	-4,216.02	-0.283	*(0.134)*	0.989	*(0.004)*	-0.030	*(0.010)*	0.101	*(0.014)*	1.233	*(0.028)*
44	-2,547.84	-1.255	*(0.095)*	0.970	*(0.006)*	-0.026	*(0.013)*	0.160	*(0.019)*	1.101	*(0.029)*
45	-3,020.82	-1.038	*(0.122)*	0.989	*(0.004)*	-0.018	*(0.009)*	0.087	*(0.013)*	1.222	*(0.031)*
46	-4,581.75	-0.053	*(0.071)*	0.947	*(0.011)*	-0.068	*(0.016)*	0.165	*(0.016)*	1.125	*(0.030)*
47	-3,564.73	-0.621	*(0.068)*	0.960	*(0.011)*	-0.052	*(0.013)*	0.124	*(0.020)*	1.037	*(0.020)*
48	-4,747.30	0.001	*(0.150)*	0.995	*(0.002)*	-0.025	*(0.007)*	0.052	*(0.010)*	1.308	*(0.030)*
49	-6,038.61	0.766	*(0.075)*	0.977	*(0.007)*	-0.026	*(0.009)*	0.094	*(0.017)*	1.352	*(0.038)*
50	-3,127.46	-0.973	*(0.091)*	0.969	*(0.006)*	-0.061	*(0.012)*	0.168	*(0.021)*	1.315	*(0.030)*
51	-3,559.67	-0.717	*(0.105)*	0.985	*(0.004)*	-0.050	*(0.010)*	0.104	*(0.016)*	1.298	*(0.040)*
52	-3,875.43	-0.541	*(0.094)*	0.978	*(0.005)*	-0.039	*(0.012)*	0.127	*(0.017)*	1.446	*(0.038)*

Appendix C: Table C.9. Estimated coefficients for the IGARCH model - Equity

comp	*LogL*	ω	s.e.	1−β	s.e.	β	GED	s.e.
1	-7,109.72	0.033	*(0.009)*	0.071	*(0.010)*	0.929	1.245	*(0.030)*
2	-6,517.65	0.010	*(0.004)*	0.047	*(0.006)*	0.953	1.337	*(0.042)*
3	-6,856.32	0.018	*(0.006)*	0.061	*(0.008)*	0.939	1.371	*(0.038)*
4	-7,182.72	0.086	*(0.019)*	0.140	*(0.014)*	0.860	1.207	*(0.037)*
5	-6,716.72	0.035	*(0.009)*	0.087	*(0.011)*	0.913	1.261	*(0.034)*
6	-7,603.75	0.027	*(0.008)*	0.059	*(0.008)*	0.941	1.194	*(0.022)*
7	-7,377.15	0.017	*(0.006)*	0.051	*(0.006)*	0.949	1.425	*(0.038)*
8	-7,161.37	0.013	*(0.005)*	0.043	*(0.006)*	0.957	1.247	*(0.031)*
9	-6,725.87	0.009	*(0.004)*	0.034	*(0.006)*	0.966	1.210	*(0.030)*
10	-6,582.38	0.011	*(0.004)*	0.045	*(0.007)*	0.955	1.233	*(0.033)*
11	-6,471.34	0.012	*(0.004)*	0.045	*(0.007)*	0.955	1.237	*(0.033)*
12	-6,260.00	0.013	*(0.004)*	0.055	*(0.007)*	0.945	1.289	*(0.034)*
13	-6,608.87	0.119	*(0.022)*	0.148	*(0.018)*	0.852	1.016	*(0.015)*
14	-6,312.46	0.013	*(0.004)*	0.054	*(0.008)*	0.946	1.124	*(0.033)*
15	-5,773.04	0.009	*(0.003)*	0.050	*(0.006)*	0.950	1.199	*(0.028)*
16	-5,558.56	0.010	*(0.003)*	0.059	*(0.008)*	0.941	1.417	*(0.044)*
17	-6,349.86	0.010	*(0.004)*	0.048	*(0.007)*	0.952	1.194	*(0.033)*
18	-6,003.31	0.037	*(0.009)*	0.096	*(0.013)*	0.904	1.025	*(0.028)*
19	-5,377.09	0.008	*(0.003)*	0.056	*(0.007)*	0.944	1.328	*(0.045)*
20	-5,635.65	0.021	*(0.005)*	0.085	*(0.010)*	0.915	1.239	*(0.034)*
21	-5,786.47	0.006	*(0.003)*	0.036	*(0.006)*	0.964	1.193	*(0.033)*
22	-5,803.71	0.005	*(0.002)*	0.030	*(0.005)*	0.970	1.174	*(0.022)*
23	-6,194.70	0.014	*(0.004)*	0.046	*(0.006)*	0.954	1.006	*(0.018)*
24	-6,097.68	0.021	*(0.005)*	0.071	*(0.010)*	0.929	1.194	*(0.021)*
25	-5,329.27	0.005	*(0.002)*	0.042	*(0.006)*	0.958	1.223	*(0.036)*
26	-6,034.92	0.017	*(0.004)*	0.073	*(0.009)*	0.927	1.327	*(0.038)*
27	-6,229.66	0.038	*(0.008)*	0.095	*(0.010)*	0.905	1.131	*(0.026)*
28	-5,956.76	0.044	*(0.010)*	0.101	*(0.012)*	0.899	1.021	*(0.018)*
29	-5,738.19	0.049	*(0.009)*	0.165	*(0.015)*	0.835	1.123	*(0.029)*
30	-6,358.37	0.024	*(0.006)*	0.081	*(0.009)*	0.919	1.197	*(0.034)*
31	-6,404.00	0.024	*(0.007)*	0.071	*(0.009)*	0.929	1.164	*(0.034)*
32	-6,262.81	0.037	*(0.009)*	0.085	*(0.011)*	0.915	1.072	*(0.025)*
33	-7,839.68	0.028	*(0.009)*	0.059	*(0.008)*	0.941	1.410	*(0.038)*
34	-6,316.25	0.038	*(0.009)*	0.090	*(0.011)*	0.910	1.250	*(0.031)*
35	-6,282.10	0.026	*(0.007)*	0.077	*(0.010)*	0.923	1.258	*(0.039)*
36	-6,826.22	0.026	*(0.007)*	0.082	*(0.009)*	0.918	1.269	*(0.037)*
37	-6,286.09	0.029	*(0.008)*	0.094	*(0.011)*	0.906	1.387	*(0.048)*
38	-5,875.00	0.017	*(0.005)*	0.059	*(0.009)*	0.941	1.104	*(0.022)*
39	-6,730.79	0.017	*(0.005)*	0.045	*(0.006)*	0.955	1.126	*(0.023)*
40	-6,131.39	0.012	*(0.004)*	0.054	*(0.007)*	0.946	1.191	*(0.033)*
41	-7,014.73	0.008	*(0.004)*	0.037	*(0.006)*	0.963	1.252	*(0.037)*
42	-6,780.47	0.007	*(0.003)*	0.031	*(0.004)*	0.969	1.067	*(0.028)*
43	-6,435.03	0.024	*(0.006)*	0.065	*(0.008)*	0.935	1.129	*(0.028)*
44	-5,683.26	0.028	*(0.006)*	0.089	*(0.010)*	0.911	1.058	*(0.025)*
45	-6,162.73	0.005	*(0.002)*	0.028	*(0.004)*	0.972	1.194	*(0.031)*
46	-7,628.74	0.033	*(0.011)*	0.063	*(0.007)*	0.937	1.142	*(0.035)*
47	-6,203.55	0.045	*(0.010)*	0.090	*(0.009)*	0.910	0.964	*(0.020)*
48	-6,054.13	0.004	*(0.002)*	0.027	*(0.005)*	0.973	1.196	*(0.027)*
49	-7,321.81	0.007	*(0.003)*	0.027	*(0.004)*	0.973	1.311	*(0.037)*
50	-5,783.66	0.032	*(0.007)*	0.111	*(0.013)*	0.889	1.249	*(0.031)*
51	-5,728.70	0.011	*(0.003)*	0.056	*(0.008)*	0.944	1.248	*(0.038)*
52	-6,304.80	0.020	*(0.005)*	0.073	*(0.009)*	0.927	1.396	*(0.044)*

Appendix C: Table C.10. Estimated coefficients for the IGARCH model - V_{SM}

comp	***LogL***	**ω**	**s.e.**	**1−β**	**s.e.**	**β**	**GED**	**s.e.**
1	-4,182.52	0.002	*(0.001)*	0.039	*(0.006)*	0.961	1.376	*(0.035)*
2	-2,161.37	0.001	*(0.000)*	0.037	*(0.005)*	0.963	1.358	*(0.043)*
3	-4,270.63	0.003	*(0.001)*	0.053	*(0.007)*	0.947	1.361	*(0.037)*
4	-4,626.74	0.023	*(0.005)*	0.133	*(0.015)*	0.867	1.228	*(0.038)*
5	-2,497.48	0.001	*(0.000)*	0.061	*(0.007)*	0.939	1.308	*(0.035)*
6	-736.78	0.000	*(0.000)*	0.044	*(0.006)*	0.956	1.230	*(0.021)*
7	-2,595.88	0.001	*(0.000)*	0.043	*(0.006)*	0.957	1.432	*(0.037)*
8	-3,659.65	0.001	*(0.000)*	0.036	*(0.006)*	0.964	1.275	*(0.029)*
9	-1,994.77	0.000	*(0.000)*	0.034	*(0.005)*	0.966	1.237	*(0.029)*
10	-5,085.00	0.004	*(0.001)*	0.036	*(0.006)*	0.964	1.298	*(0.033)*
11	-4,715.95	0.003	*(0.001)*	0.036	*(0.006)*	0.964	1.255	*(0.034)*
12	-5,050.65	0.005	*(0.002)*	0.048	*(0.007)*	0.952	1.295	*(0.034)*
13	-5,034.37	0.033	*(0.007)*	0.114	*(0.015)*	0.886	1.055	*(0.016)*
14	-3,942.65	0.001	*(0.000)*	0.033	*(0.005)*	0.967	1.151	*(0.033)*
15	-3,384.88	0.002	*(0.001)*	0.045	*(0.006)*	0.955	1.204	*(0.028)*
16	-4,610.56	0.005	*(0.002)*	0.054	*(0.007)*	0.946	1.460	*(0.048)*
17	-3,003.50	0.001	*(0.000)*	0.037	*(0.006)*	0.963	1.200	*(0.033)*
18	-1,575.89	0.002	*(0.000)*	0.070	*(0.009)*	0.930	1.074	*(0.028)*
19	-4,403.48	0.004	*(0.001)*	0.050	*(0.007)*	0.950	1.401	*(0.048)*
20	-4,160.52	0.009	*(0.002)*	0.082	*(0.009)*	0.918	1.232	*(0.034)*
21	-3,641.65	0.001	*(0.001)*	0.030	*(0.005)*	0.970	1.218	*(0.033)*
22	-4,235.56	0.002	*(0.001)*	0.030	*(0.005)*	0.970	1.234	*(0.022)*
23	-4,185.71	0.002	*(0.001)*	0.028	*(0.004)*	0.972	1.116	*(0.018)*
24	-4,372.21	0.005	*(0.002)*	0.054	*(0.009)*	0.946	1.286	*(0.021)*
25	-3,516.95	0.001	*(0.001)*	0.038	*(0.006)*	0.962	1.225	*(0.035)*
26	-4,217.98	0.004	*(0.001)*	0.059	*(0.008)*	0.941	1.400	*(0.044)*
27	-2,323.85	0.003	*(0.001)*	0.087	*(0.009)*	0.913	1.151	*(0.029)*
28	-1,661.65	0.003	*(0.001)*	0.101	*(0.013)*	0.899	1.083	*(0.018)*
29	-2,693.87	0.009	*(0.002)*	0.168	*(0.015)*	0.832	1.141	*(0.028)*
30	-3,645.51	0.005	*(0.001)*	0.075	*(0.009)*	0.925	1.219	*(0.035)*
31	-1,430.18	0.001	*(0.000)*	0.057	*(0.008)*	0.943	1.181	*(0.034)*
32	-4,252.50	0.002	*(0.001)*	0.035	*(0.005)*	0.965	1.081	*(0.024)*
33	-6,167.04	0.008	*(0.003)*	0.049	*(0.007)*	0.951	1.435	*(0.037)*
34	-4,151.93	0.004	*(0.001)*	0.052	*(0.006)*	0.948	1.331	*(0.032)*
35	-4,273.17	0.005	*(0.002)*	0.062	*(0.008)*	0.938	1.276	*(0.039)*
36	-3,875.81	0.003	*(0.001)*	0.065	*(0.008)*	0.935	1.305	*(0.037)*
37	-4,971.43	0.008	*(0.003)*	0.077	*(0.009)*	0.923	1.419	*(0.049)*
38	-3,489.00	0.003	*(0.001)*	0.051	*(0.008)*	0.949	1.125	*(0.022)*
39	-4,717.79	0.003	*(0.001)*	0.031	*(0.005)*	0.969	1.204	*(0.027)*
40	-3,874.33	0.002	*(0.001)*	0.044	*(0.006)*	0.956	1.201	*(0.033)*
41	-3,749.88	0.001	*(0.001)*	0.033	*(0.005)*	0.967	1.270	*(0.037)*
42	-4,262.65	0.001	*(0.001)*	0.023	*(0.004)*	0.977	1.092	*(0.028)*
43	-4,203.02	0.004	*(0.001)*	0.046	*(0.006)*	0.954	1.196	*(0.029)*
44	-2,454.28	0.002	*(0.001)*	0.062	*(0.007)*	0.938	1.073	*(0.026)*
45	-2,950.28	0.001	*(0.000)*	0.028	*(0.004)*	0.972	1.191	*(0.031)*
46	-4,476.53	0.004	*(0.002)*	0.044	*(0.007)*	0.956	1.175	*(0.037)*
47	-3,551.36	0.002	*(0.001)*	0.029	*(0.004)*	0.971	0.983	*(0.019)*
48	-4,733.14	0.001	*(0.001)*	0.021	*(0.004)*	0.979	1.283	*(0.030)*
49	-6,082.26	0.002	*(0.001)*	0.020	*(0.004)*	0.980	1.319	*(0.037)*
50	-3,031.15	0.006	*(0.001)*	0.103	*(0.012)*	0.897	1.263	*(0.031)*
51	-3,517.32	0.002	*(0.001)*	0.045	*(0.007)*	0.955	1.259	*(0.038)*
52	-3,861.24	0.003	*(0.001)*	0.054	*(0.007)*	0.946	1.436	*(0.041)*

Appendix C: Table C.11. Estimated coefficients for the IGARCH model - V_{KMV}

comp	***LogL***	**ω**	**s.e.**	**1−β**	**s.e.**	**β**	**GED**	**s.e.**
1	-4,221.00	0.002	*(0.001)*	0.040	*(0.006)*	0.960	1.380	*(0.036)*
2	-2,176.22	0.001	*(0.000)*	0.037	*(0.005)*	0.963	1.356	*(0.043)*
3	-4,226.41	0.003	*(0.001)*	0.053	*(0.007)*	0.947	1.360	*(0.037)*
4	-4,572.47	0.022	*(0.005)*	0.133	*(0.015)*	0.867	1.227	*(0.038)*
5	-2,488.74	0.001	*(0.000)*	0.061	*(0.007)*	0.939	1.310	*(0.035)*
6	-722.25	0.000	*(0.000)*	0.045	*(0.006)*	0.955	1.230	*(0.021)*
7	-2,567.63	0.001	*(0.000)*	0.043	*(0.006)*	0.957	1.432	*(0.038)*
8	-3,608.71	0.001	*(0.000)*	0.036	*(0.005)*	0.964	1.276	*(0.029)*
9	-1,937.48	0.000	*(0.000)*	0.034	*(0.005)*	0.966	1.238	*(0.029)*
10	-5,074.87	0.004	*(0.001)*	0.037	*(0.006)*	0.963	1.302	*(0.033)*
11	-4,666.10	0.003	*(0.001)*	0.036	*(0.006)*	0.964	1.255	*(0.034)*
12	-5,081.19	0.005	*(0.002)*	0.049	*(0.007)*	0.951	1.293	*(0.034)*
13	-5,075.37	0.034	*(0.007)*	0.115	*(0.015)*	0.885	1.055	*(0.016)*
14	-3,930.39	0.001	*(0.001)*	0.034	*(0.005)*	0.966	1.150	*(0.033)*
15	-3,330.77	0.002	*(0.001)*	0.045	*(0.006)*	0.955	1.204	*(0.028)*
16	-4,594.76	0.005	*(0.002)*	0.054	*(0.007)*	0.946	1.464	*(0.049)*
17	-2,937.09	0.001	*(0.000)*	0.036	*(0.006)*	0.964	1.200	*(0.033)*
18	-1,475.37	0.002	*(0.000)*	0.069	*(0.009)*	0.931	1.077	*(0.028)*
19	-4,442.12	0.004	*(0.001)*	0.051	*(0.007)*	0.949	1.400	*(0.048)*
20	-4,184.66	0.009	*(0.002)*	0.084	*(0.010)*	0.916	1.232	*(0.034)*
21	-3,598.57	0.001	*(0.000)*	0.028	*(0.005)*	0.972	1.222	*(0.033)*
22	-4,244.94	0.002	*(0.001)*	0.031	*(0.005)*	0.969	1.241	*(0.023)*
23	-4,181.59	0.002	*(0.001)*	0.030	*(0.004)*	0.970	1.126	*(0.018)*
24	-4,362.80	0.005	*(0.002)*	0.054	*(0.009)*	0.946	1.290	*(0.022)*
25	-3,469.14	0.001	*(0.001)*	0.038	*(0.006)*	0.962	1.223	*(0.035)*
26	-4,246.59	0.004	*(0.001)*	0.060	*(0.008)*	0.940	1.407	*(0.044)*
27	-2,313.47	0.003	*(0.001)*	0.088	*(0.010)*	0.912	1.152	*(0.029)*
28	-1,663.80	0.003	*(0.001)*	0.103	*(0.013)*	0.897	1.086	*(0.018)*
29	-2,672.52	0.009	*(0.002)*	0.168	*(0.015)*	0.832	1.142	*(0.028)*
30	-3,613.00	0.005	*(0.001)*	0.075	*(0.009)*	0.925	1.217	*(0.034)*
31	-1,447.14	0.001	*(0.000)*	0.059	*(0.008)*	0.941	1.177	*(0.034)*
32	-4,206.08	0.002	*(0.001)*	0.035	*(0.005)*	0.965	1.081	*(0.024)*
33	-6,143.18	0.007	*(0.003)*	0.049	*(0.007)*	0.951	1.439	*(0.037)*
34	-4,155.11	0.004	*(0.001)*	0.051	*(0.006)*	0.949	1.333	*(0.032)*
35	-4,216.99	0.004	*(0.001)*	0.061	*(0.008)*	0.939	1.277	*(0.039)*
36	-3,828.81	0.003	*(0.001)*	0.066	*(0.008)*	0.934	1.304	*(0.037)*
37	-4,967.02	0.008	*(0.003)*	0.076	*(0.009)*	0.924	1.415	*(0.049)*
38	-3,440.79	0.003	*(0.001)*	0.051	*(0.008)*	0.949	1.125	*(0.022)*
39	-4,710.96	0.003	*(0.001)*	0.032	*(0.005)*	0.968	1.221	*(0.027)*
40	-3,855.06	0.002	*(0.001)*	0.044	*(0.006)*	0.956	1.201	*(0.033)*
41	-3,731.48	0.001	*(0.000)*	0.032	*(0.005)*	0.968	1.275	*(0.037)*
42	-4,336.08	0.001	*(0.001)*	0.024	*(0.004)*	0.976	1.092	*(0.028)*
43	-4,194.90	0.003	*(0.001)*	0.046	*(0.006)*	0.954	1.204	*(0.029)*
44	-2,394.17	0.002	*(0.001)*	0.060	*(0.007)*	0.940	1.073	*(0.026)*
45	-2,853.81	0.001	*(0.000)*	0.027	*(0.004)*	0.973	1.190	*(0.031)*
46	-4,588.20	0.005	*(0.002)*	0.047	*(0.007)*	0.953	1.175	*(0.038)*
47	-3,492.45	0.002	*(0.001)*	0.029	*(0.004)*	0.971	0.983	*(0.019)*
48	-4,734.72	0.001	*(0.001)*	0.022	*(0.004)*	0.978	1.290	*(0.030)*
49	-6,067.52	0.002	*(0.001)*	0.020	*(0.004)*	0.980	1.319	*(0.037)*
50	-2,981.91	0.006	*(0.001)*	0.103	*(0.012)*	0.897	1.260	*(0.031)*
51	-3,470.96	0.002	*(0.001)*	0.045	*(0.007)*	0.955	1.257	*(0.038)*
52	-3,873.29	0.003	*(0.001)*	0.054	*(0.007)*	0.946	1.439	*(0.041)*

Appendix C: Table C.12. Estimated coefficients for the IGARCH model - V_{Proxy}

comp	*LogL*	ω	s.e.	1−β	s.e.	β	GED	s.e.
1	-4,141.26	0.002	*(0.001)*	0.040	*(0.006)*	0.960	1.383	*(0.036)*
2	-2,082.87	0.001	*(0.000)*	0.040	*(0.006)*	0.960	1.345	*(0.042)*
3	-4,249.59	0.003	*(0.001)*	0.055	*(0.007)*	0.945	1.368	*(0.038)*
4	-4,598.25	0.023	*(0.005)*	0.134	*(0.015)*	0.866	1.225	*(0.038)*
5	-2,379.83	0.001	*(0.000)*	0.066	*(0.007)*	0.934	1.306	*(0.035)*
6	-551.35	0.000	*(0.000)*	0.049	*(0.006)*	0.951	1.206	*(0.020)*
7	-2,538.72	0.001	*(0.000)*	0.046	*(0.006)*	0.954	1.428	*(0.037)*
8	-3,578.10	0.001	*(0.000)*	0.038	*(0.006)*	0.962	1.274	*(0.029)*
9	-1,913.35	0.001	*(0.000)*	0.037	*(0.006)*	0.963	1.222	*(0.030)*
10	-5,074.90	0.004	*(0.001)*	0.037	*(0.006)*	0.963	1.295	*(0.033)*
11	-4,732.92	0.003	*(0.001)*	0.037	*(0.006)*	0.963	1.252	*(0.034)*
12	-5,061.00	0.005	*(0.002)*	0.049	*(0.007)*	0.951	1.301	*(0.034)*
13	-5,090.83	0.033	*(0.007)*	0.112	*(0.015)*	0.888	1.056	*(0.016)*
14	-3,963.36	0.001	*(0.001)*	0.035	*(0.005)*	0.965	1.149	*(0.033)*
15	-3,392.59	0.002	*(0.001)*	0.047	*(0.006)*	0.953	1.205	*(0.029)*
16	-4,638.59	0.005	*(0.002)*	0.055	*(0.007)*	0.945	1.466	*(0.049)*
17	-2,981.78	0.001	*(0.000)*	0.038	*(0.006)*	0.962	1.208	*(0.033)*
18	-1,566.65	0.002	*(0.001)*	0.073	*(0.009)*	0.927	1.065	*(0.028)*
19	-4,506.30	0.004	*(0.001)*	0.051	*(0.007)*	0.949	1.387	*(0.047)*
20	-4,208.60	0.009	*(0.002)*	0.081	*(0.009)*	0.919	1.238	*(0.035)*
21	-3,632.99	0.001	*(0.001)*	0.031	*(0.005)*	0.969	1.216	*(0.033)*
22	-4,271.35	0.002	*(0.001)*	0.032	*(0.005)*	0.968	1.234	*(0.023)*
23	-4,168.20	0.002	*(0.001)*	0.029	*(0.004)*	0.971	1.124	*(0.018)*
24	-4,358.84	0.005	*(0.002)*	0.057	*(0.009)*	0.943	1.284	*(0.022)*
25	-3,530.38	0.002	*(0.001)*	0.040	*(0.006)*	0.960	1.228	*(0.036)*
26	-4,178.03	0.004	*(0.001)*	0.062	*(0.008)*	0.938	1.422	*(0.045)*
27	-2,216.68	0.002	*(0.001)*	0.081	*(0.009)*	0.919	1.179	*(0.029)*
28	-1,630.69	0.002	*(0.001)*	0.086	*(0.011)*	0.914	1.062	*(0.018)*
29	-2,723.65	0.009	*(0.002)*	0.160	*(0.014)*	0.840	1.128	*(0.028)*
30	-3,706.76	0.005	*(0.001)*	0.075	*(0.009)*	0.925	1.219	*(0.035)*
31	-1,262.27	0.001	*(0.000)*	0.053	*(0.007)*	0.947	1.191	*(0.035)*
32	-4,285.56	0.003	*(0.001)*	0.038	*(0.005)*	0.962	1.081	*(0.024)*
33	-6,050.25	0.007	*(0.003)*	0.048	*(0.007)*	0.952	1.435	*(0.037)*
34	-4,147.46	0.004	*(0.001)*	0.055	*(0.007)*	0.945	1.332	*(0.032)*
35	-4,291.70	0.005	*(0.002)*	0.063	*(0.008)*	0.937	1.273	*(0.039)*
36	-3,874.61	0.003	*(0.001)*	0.065	*(0.008)*	0.935	1.290	*(0.036)*
37	-5,027.14	0.009	*(0.003)*	0.079	*(0.009)*	0.921	1.416	*(0.049)*
38	-3,536.15	0.003	*(0.001)*	0.053	*(0.008)*	0.947	1.125	*(0.023)*
39	-4,667.25	0.003	*(0.001)*	0.033	*(0.005)*	0.967	1.221	*(0.028)*
40	-3,861.45	0.002	*(0.001)*	0.046	*(0.007)*	0.954	1.204	*(0.033)*
41	-3,752.85	0.001	*(0.001)*	0.034	*(0.005)*	0.966	1.268	*(0.037)*
42	-4,341.16	0.001	*(0.001)*	0.024	*(0.004)*	0.976	1.077	*(0.028)*
43	-4,229.72	0.004	*(0.001)*	0.050	*(0.007)*	0.950	1.204	*(0.029)*
44	-2,556.96	0.004	*(0.001)*	0.079	*(0.009)*	0.921	1.075	*(0.026)*
45	-3,030.35	0.001	*(0.000)*	0.030	*(0.005)*	0.970	1.198	*(0.031)*
46	-4,589.06	0.004	*(0.002)*	0.046	*(0.007)*	0.954	1.156	*(0.037)*
47	-3,609.01	0.001	*(0.001)*	0.025	*(0.004)*	0.975	0.990	*(0.020)*
48	-4,762.16	0.001	*(0.001)*	0.022	*(0.004)*	0.978	1.284	*(0.030)*
49	-6,049.81	0.002	*(0.001)*	0.020	*(0.004)*	0.980	1.323	*(0.037)*
50	-3,151.86	0.006	*(0.001)*	0.103	*(0.013)*	0.897	1.271	*(0.032)*
51	-3,578.06	0.002	*(0.001)*	0.047	*(0.007)*	0.953	1.264	*(0.039)*
52	-3,887.10	0.003	*(0.001)*	0.054	*(0.007)*	0.946	1.424	*(0.041)*

Appendix C: Table C.13. Estimated coefficients for the FIGARCH model - Equity

comp	***LogL***	**ω**	**s.e.**	**α**	**s.e.**	**β**	**s.e.**	***d***	**s.e.**	**GED**	**s.e.**
1	-7,099.67	0.180	*(0.063)*	0.197	*(0.072)*	0.516	*(0.091)*	0.386	*(0.061)*	1.277	*(0.029)*
2	-6,518.65	0.084	*(0.031)*	0.281	*(0.053)*	0.627	*(0.067)*	0.419	*(0.067)*	1.347	*(0.042)*
3	-6,850.43	0.148	*(0.051)*	0.195	*(0.054)*	0.515	*(0.077)*	0.367	*(0.057)*	1.396	*(0.040)*
4	-7,174.53	0.204	*(0.074)*	0.188	*(0.093)*	0.441	*(0.114)*	0.399	*(0.062)*	1.228	*(0.037)*
5	-6,708.37	0.125	*(0.038)*	0.144	*(0.051)*	0.553	*(0.081)*	0.447	*(0.068)*	1.290	*(0.035)*
6	-7,599.35	0.164	*(0.057)*	0.308	*(0.063)*	0.606	*(0.074)*	0.421	*(0.068)*	1.211	*(0.022)*
7	-7,379.29	0.103	*(0.037)*	0.259	*(0.042)*	0.676	*(0.058)*	0.463	*(0.070)*	1.429	*(0.038)*
8	-7,157.01	0.115	*(0.042)*	0.375	*(0.059)*	0.669	*(0.064)*	0.394	*(0.068)*	1.276	*(0.034)*
9	-6,717.71	0.228	*(0.084)*	0.315	*(0.087)*	0.546	*(0.091)*	0.315	*(0.056)*	1.222	*(0.029)*
10	-6,578.05	0.123	*(0.047)*	0.358	*(0.067)*	0.618	*(0.081)*	0.363	*(0.062)*	1.254	*(0.033)*
11	-6,462.79	0.204	*(0.076)*	0.221	*(0.082)*	0.486	*(0.101)*	0.302	*(0.050)*	1.274	*(0.036)*
12	-6,251.06	0.199	*(0.062)*	0.077	*(0.088)*	0.353	*(0.106)*	0.308	*(0.042)*	1.311	*(0.034)*
13	-6,600.62	0.298	*(0.109)*	0.293	*(0.114)*	0.478	*(0.136)*	0.315	*(0.056)*	1.043	*(0.015)*
14	-6,305.39	0.067	*(0.025)*	0.303	*(0.054)*	0.670	*(0.062)*	0.453	*(0.073)*	1.142	*(0.033)*
15	-5,762.78	0.047	*(0.018)*	0.481	*(0.066)*	0.729	*(0.057)*	0.418	*(0.073)*	1.216	*(0.027)*
16	-5,549.83	0.117	*(0.047)*	0.293	*(0.092)*	0.510	*(0.105)*	0.294	*(0.051)*	1.436	*(0.046)*
17	-6,347.24	0.071	*(0.026)*	0.378	*(0.054)*	0.688	*(0.057)*	0.404	*(0.066)*	1.215	*(0.033)*
18	-5,996.91	0.122	*(0.044)*	0.269	*(0.080)*	0.563	*(0.104)*	0.411	*(0.078)*	1.051	*(0.028)*
19	-5,368.68	0.053	*(0.021)*	0.387	*(0.056)*	0.663	*(0.063)*	0.373	*(0.061)*	1.356	*(0.045)*
20	-5,623.83	0.095	*(0.036)*	0.340	*(0.082)*	0.560	*(0.094)*	0.341	*(0.053)*	1.269	*(0.034)*
21	-5,779.60	0.155	*(0.064)*	0.372	*(0.101)*	0.556	*(0.108)*	0.265	*(0.052)*	1.215	*(0.034)*
22	-5,779.88	0.086	*(0.029)*	0.685	*(0.083)*	0.770	*(0.062)*	0.247	*(0.047)*	1.234	*(0.032)*
23	-6,184.13	0.121	*(0.044)*	0.329	*(0.073)*	0.606	*(0.085)*	0.385	*(0.067)*	1.030	*(0.019)*
24	-6,090.70	0.070	*(0.024)*	0.367	*(0.067)*	0.705	*(0.062)*	0.464	*(0.081)*	1.210	*(0.021)*
25	-5,321.78	0.080	*(0.031)*	0.338	*(0.076)*	0.581	*(0.087)*	0.331	*(0.059)*	1.246	*(0.035)*
26	-6,028.51	0.093	*(0.030)*	0.267	*(0.071)*	0.538	*(0.091)*	0.383	*(0.060)*	1.352	*(0.041)*
27	-6,216.57	0.158	*(0.057)*	0.185	*(0.101)*	0.445	*(0.126)*	0.367	*(0.065)*	1.165	*(0.027)*
28	-5,950.42	0.190	*(0.056)*	0.134	*(0.096)*	0.432	*(0.124)*	0.391	*(0.070)*	1.044	*(0.019)*
29	-5,726.28	0.067	*(0.020)*	0.240	*(0.069)*	0.579	*(0.084)*	0.542	*(0.076)*	1.132	*(0.029)*
30	-6,353.63	0.082	*(0.029)*	0.199	*(0.057)*	0.590	*(0.084)*	0.470	*(0.074)*	1.216	*(0.033)*
31	-6,392.09	0.166	*(0.065)*	0.311	*(0.115)*	0.499	*(0.130)*	0.343	*(0.061)*	1.197	*(0.035)*
32	-6,244.80	0.225	*(0.091)*	0.179	*(0.148)*	0.378	*(0.160)*	0.302	*(0.049)*	1.111	*(0.025)*
33	-7,838.51	0.091	*(0.031)*	0.108	*(0.060)*	0.808	*(0.056)*	0.724	*(0.107)*	1.411	*(0.037)*
34	-6,301.38	0.219	*(0.091)*	0.310	*(0.118)*	0.488	*(0.137)*	0.284	*(0.051)*	1.296	*(0.032)*
35	-6,272.40	0.186	*(0.069)*	0.194	*(0.086)*	0.452	*(0.107)*	0.320	*(0.053)*	1.295	*(0.041)*
36	-6,819.55	0.049	*(0.020)*	0.262	*(0.052)*	0.720	*(0.055)*	0.550	*(0.081)*	1.283	*(0.037)*
37	-6,274.99	0.114	*(0.040)*	0.112	*(0.064)*	0.480	*(0.090)*	0.427	*(0.061)*	1.415	*(0.048)*
38	-5,869.92	0.115	*(0.045)*	0.376	*(0.081)*	0.604	*(0.085)*	0.330	*(0.059)*	1.115	*(0.021)*
39	-6,719.75	0.437	*(0.147)*	0.011	*(0.140)*	0.227	*(0.163)*	0.286	*(0.049)*	1.149	*(0.025)*
40	-6,124.80	0.115	*(0.043)*	0.211	*(0.071)*	0.522	*(0.093)*	0.368	*(0.060)*	1.210	*(0.033)*
41	-7,016.96	0.096	*(0.040)*	0.374	*(0.054)*	0.694	*(0.059)*	0.413	*(0.070)*	1.262	*(0.038)*
42	-6,776.52	0.250	*(0.088)*	0.200	*(0.095)*	0.452	*(0.107)*	0.314	*(0.050)*	1.086	*(0.028)*
43	-6,423.76	0.099	*(0.035)*	0.381	*(0.051)*	0.682	*(0.056)*	0.392	*(0.061)*	1.157	*(0.028)*
44	-5,670.71	0.129	*(0.042)*	0.128	*(0.088)*	0.430	*(0.111)*	0.365	*(0.060)*	1.096	*(0.027)*
45	-6,160.01	0.212	*(0.078)*	0.332	*(0.083)*	0.543	*(0.092)*	0.264	*(0.049)*	1.213	*(0.032)*
46	-7,614.96	0.244	*(0.089)*	0.302	*(0.082)*	0.550	*(0.091)*	0.370	*(0.060)*	1.166	*(0.035)*
47	-6,184.84	0.975	*(0.193)*	-0.727	*(0.140)*	-0.677	*(0.159)*	0.200	*(0.026)*	0.991	*(0.021)*
48	-6,046.01	0.077	*(0.023)*	0.697	*(0.062)*	0.820	*(0.042)*	0.294	*(0.044)*	1.209	*(0.024)*
49	-7,321.66	0.181	*(0.069)*	0.393	*(0.060)*	0.671	*(0.061)*	0.353	*(0.057)*	1.330	*(0.037)*
50	-5,775.38	0.072	*(0.022)*	0.212	*(0.062)*	0.603	*(0.077)*	0.490	*(0.075)*	1.272	*(0.031)*
51	-5,722.66	0.069	*(0.025)*	0.362	*(0.065)*	0.643	*(0.068)*	0.383	*(0.064)*	1.267	*(0.038)*
52	-6,298.20	0.118	*(0.043)*	0.292	*(0.074)*	0.551	*(0.091)*	0.365	*(0.058)*	1.423	*(0.044)*

Appendix C: Table C.14. Estimated coefficients for the FIGARCH model - V_{SM}

comp	*LogL*	ω	s.e.	α	s.e.	β	s.e.	*d*	s.e.	GED	s.e.
1	-4,176.63	0.034	*(0.012)*	0.258	*(0.064)*	0.560	*(0.079)*	0.353	*(0.055)*	1.411	*(0.037)*
2	-2,162.01	0.010	*(0.004)*	0.371	*(0.056)*	0.645	*(0.064)*	0.348	*(0.061)*	1.375	*(0.043)*
3	-4,267.75	0.029	*(0.010)*	0.225	*(0.048)*	0.558	*(0.071)*	0.385	*(0.063)*	1.384	*(0.039)*
4	-4,616.38	0.131	*(0.057)*	0.110	*(0.196)*	0.247	*(0.217)*	0.268	*(0.047)*	1.269	*(0.039)*
5	-2,494.95	0.004	*(0.001)*	0.194	*(0.044)*	0.683	*(0.061)*	0.536	*(0.079)*	1.321	*(0.036)*
6	-731.57	0.006	*(0.002)*	0.329	*(0.082)*	0.541	*(0.096)*	0.343	*(0.058)*	1.249	*(0.021)*
7	-2,597.82	0.008	*(0.003)*	0.312	*(0.043)*	0.722	*(0.049)*	0.452	*(0.070)*	1.433	*(0.037)*
8	-3,662.80	0.008	*(0.003)*	0.378	*(0.060)*	0.781	*(0.047)*	0.511	*(0.090)*	1.288	*(0.028)*
9	-1,993.16	0.009	*(0.003)*	0.353	*(0.068)*	0.619	*(0.067)*	0.354	*(0.060)*	1.237	*(0.028)*
10	-5,077.15	0.073	*(0.028)*	0.443	*(0.062)*	0.647	*(0.074)*	0.299	*(0.055)*	1.324	*(0.034)*
11	-4,707.01	0.092	*(0.035)*	0.257	*(0.086)*	0.492	*(0.104)*	0.267	*(0.048)*	1.292	*(0.037)*
12	-5,042.46	0.109	*(0.034)*	0.089	*(0.089)*	0.357	*(0.108)*	0.297	*(0.042)*	1.321	*(0.035)*
13	-5,019.48	0.165	*(0.073)*	0.334	*(0.160)*	0.448	*(0.177)*	0.222	*(0.041)*	1.095	*(0.015)*
14	-3,940.39	0.014	*(0.005)*	0.331	*(0.049)*	0.701	*(0.052)*	0.442	*(0.071)*	1.170	*(0.034)*
15	-3,375.57	0.012	*(0.004)*	0.505	*(0.065)*	0.750	*(0.052)*	0.420	*(0.072)*	1.220	*(0.028)*
16	-4,602.47	0.059	*(0.024)*	0.291	*(0.078)*	0.538	*(0.094)*	0.312	*(0.053)*	1.482	*(0.050)*
17	-2,997.39	0.019	*(0.007)*	0.437	*(0.063)*	0.664	*(0.064)*	0.311	*(0.056)*	1.225	*(0.034)*
18	-1,564.15	0.019	*(0.009)*	0.247	*(0.156)*	0.409	*(0.173)*	0.253	*(0.052)*	1.113	*(0.028)*
19	-4,396.46	0.032	*(0.013)*	0.399	*(0.054)*	0.667	*(0.061)*	0.360	*(0.059)*	1.422	*(0.047)*
20	-4,144.42	0.053	*(0.021)*	0.355	*(0.098)*	0.531	*(0.109)*	0.287	*(0.046)*	1.274	*(0.034)*
21	-3,642.14	0.024	*(0.009)*	0.454	*(0.067)*	0.693	*(0.059)*	0.334	*(0.059)*	1.226	*(0.033)*
22	-4,217.98	0.032	*(0.010)*	0.656	*(0.072)*	0.769	*(0.048)*	0.276	*(0.051)*	1.279	*(0.032)*
23	-4,177.76	0.035	*(0.013)*	0.324	*(0.060)*	0.611	*(0.076)*	0.356	*(0.059)*	1.141	*(0.020)*
24	-4,364.82	0.035	*(0.012)*	0.345	*(0.065)*	0.630	*(0.077)*	0.384	*(0.064)*	1.311	*(0.023)*
25	-3,512.48	0.021	*(0.008)*	0.339	*(0.064)*	0.621	*(0.072)*	0.368	*(0.062)*	1.243	*(0.035)*
26	-4,214.65	0.027	*(0.010)*	0.334	*(0.061)*	0.607	*(0.073)*	0.372	*(0.060)*	1.420	*(0.044)*
27	-2,309.96	0.014	*(0.005)*	0.226	*(0.102)*	0.474	*(0.120)*	0.363	*(0.061)*	1.177	*(0.028)*
28	-1,654.05	0.020	*(0.006)*	0.058	*(0.124)*	0.281	*(0.156)*	0.334	*(0.059)*	1.102	*(0.020)*
29	-2,680.84	0.014	*(0.004)*	0.283	*(0.070)*	0.598	*(0.082)*	0.551	*(0.079)*	1.151	*(0.029)*
30	-3,638.80	0.025	*(0.009)*	0.223	*(0.068)*	0.539	*(0.091)*	0.389	*(0.063)*	1.243	*(0.034)*
31	-1,415.61	0.008	*(0.004)*	0.447	*(0.124)*	0.580	*(0.128)*	0.306	*(0.057)*	1.217	*(0.035)*
32	-4,237.88	0.072	*(0.030)*	0.224	*(0.146)*	0.414	*(0.158)*	0.297	*(0.049)*	1.112	*(0.024)*
33	-6,165.75	0.047	*(0.017)*	0.219	*(0.046)*	0.695	*(0.054)*	0.507	*(0.074)*	1.441	*(0.035)*
34	-4,142.43	0.054	*(0.020)*	0.295	*(0.084)*	0.518	*(0.103)*	0.301	*(0.047)*	1.361	*(0.033)*
35	-4,267.41	0.035	*(0.013)*	0.244	*(0.059)*	0.561	*(0.079)*	0.377	*(0.062)*	1.300	*(0.040)*
36	-3,869.38	0.010	*(0.004)*	0.302	*(0.051)*	0.719	*(0.053)*	0.498	*(0.078)*	1.325	*(0.037)*
37	-4,964.63	0.040	*(0.015)*	0.158	*(0.055)*	0.539	*(0.078)*	0.443	*(0.064)*	1.435	*(0.048)*
38	-3,482.25	0.033	*(0.013)*	0.405	*(0.090)*	0.601	*(0.096)*	0.302	*(0.057)*	1.145	*(0.021)*
39	-4,710.43	0.131	*(0.052)*	0.248	*(0.136)*	0.420	*(0.148)*	0.238	*(0.046)*	1.223	*(0.026)*
40	-3,868.91	0.033	*(0.012)*	0.243	*(0.071)*	0.534	*(0.090)*	0.342	*(0.055)*	1.219	*(0.032)*
41	-3,753.02	0.017	*(0.007)*	0.418	*(0.055)*	0.712	*(0.054)*	0.387	*(0.066)*	1.277	*(0.038)*
42	-4,257.74	0.108	*(0.040)*	0.169	*(0.134)*	0.360	*(0.147)*	0.246	*(0.042)*	1.109	*(0.027)*
43	-4,193.30	0.031	*(0.011)*	0.379	*(0.045)*	0.686	*(0.055)*	0.381	*(0.061)*	1.224	*(0.031)*
44	-2,446.47	0.016	*(0.005)*	0.172	*(0.071)*	0.515	*(0.089)*	0.405	*(0.065)*	1.100	*(0.028)*
45	-2,952.05	0.026	*(0.009)*	0.373	*(0.074)*	0.595	*(0.076)*	0.282	*(0.050)*	1.200	*(0.030)*
46	-4,462.80	0.077	*(0.032)*	0.323	*(0.122)*	0.493	*(0.130)*	0.289	*(0.053)*	1.193	*(0.036)*
47	-3,528.59	0.299	*(0.064)*	-0.705	*(0.173)*	-0.660	*(0.192)*	0.152	*(0.025)*	1.016	*(0.020)*
48	-4,729.40	0.043	*(0.012)*	0.661	*(0.061)*	0.798	*(0.042)*	0.277	*(0.042)*	1.287	*(0.027)*
49	-6,079.40	0.170	*(0.070)*	0.442	*(0.079)*	0.635	*(0.082)*	0.262	*(0.052)*	1.340	*(0.038)*
50	-3,023.38	0.015	*(0.005)*	0.249	*(0.065)*	0.602	*(0.078)*	0.457	*(0.072)*	1.286	*(0.031)*
51	-3,511.92	0.021	*(0.007)*	0.384	*(0.064)*	0.649	*(0.065)*	0.366	*(0.060)*	1.275	*(0.038)*
52	-3,854.40	0.030	*(0.012)*	0.369	*(0.055)*	0.624	*(0.074)*	0.344	*(0.059)*	1.464	*(0.041)*

Appendix C: Table C.15. Estimated coefficients for the FIGARCH model - V_{KMV}

comp	*LogL*	ω	s.e.	α	s.e.	β	s.e.	*d*	s.e.	GED	s.e.
1	-4,215.30	0.035	*(0.013)*	0.247	*(0.064)*	0.556	*(0.079)*	0.356	*(0.055)*	1.414	*(0.037)*
2	-2,177.18	0.009	*(0.004)*	0.370	*(0.055)*	0.651	*(0.063)*	0.355	*(0.062)*	1.372	*(0.043)*
3	-4,223.32	0.029	*(0.010)*	0.224	*(0.048)*	0.554	*(0.072)*	0.382	*(0.063)*	1.384	*(0.039)*
4	-4,562.00	0.109	*(0.047)*	0.178	*(0.169)*	0.325	*(0.189)*	0.280	*(0.050)*	1.268	*(0.039)*
5	-2,486.24	0.004	*(0.001)*	0.195	*(0.044)*	0.682	*(0.061)*	0.534	*(0.079)*	1.323	*(0.036)*
6	-716.27	0.006	*(0.002)*	0.316	*(0.093)*	0.513	*(0.108)*	0.327	*(0.056)*	1.250	*(0.021)*
7	-2,569.55	0.007	*(0.003)*	0.313	*(0.043)*	0.722	*(0.049)*	0.451	*(0.070)*	1.433	*(0.037)*
8	-3,611.76	0.008	*(0.003)*	0.380	*(0.059)*	0.781	*(0.047)*	0.508	*(0.089)*	1.288	*(0.028)*
9	-1,936.04	0.009	*(0.003)*	0.354	*(0.069)*	0.616	*(0.068)*	0.350	*(0.059)*	1.236	*(0.027)*
10	-5,068.29	0.066	*(0.025)*	0.427	*(0.059)*	0.648	*(0.071)*	0.315	*(0.056)*	1.325	*(0.034)*
11	-4,657.31	0.088	*(0.034)*	0.256	*(0.085)*	0.493	*(0.104)*	0.269	*(0.048)*	1.291	*(0.037)*
12	-5,073.02	0.108	*(0.034)*	0.090	*(0.088)*	0.361	*(0.107)*	0.300	*(0.042)*	1.318	*(0.035)*
13	-5,061.70	0.155	*(0.066)*	0.332	*(0.144)*	0.459	*(0.162)*	0.236	*(0.043)*	1.092	*(0.015)*
14	-3,927.93	0.013	*(0.005)*	0.328	*(0.049)*	0.702	*(0.053)*	0.447	*(0.071)*	1.169	*(0.034)*
15	-3,321.35	0.012	*(0.004)*	0.505	*(0.065)*	0.749	*(0.052)*	0.420	*(0.072)*	1.220	*(0.028)*
16	-4,586.59	0.056	*(0.023)*	0.292	*(0.075)*	0.545	*(0.091)*	0.318	*(0.054)*	1.486	*(0.050)*
17	-2,930.98	0.018	*(0.007)*	0.437	*(0.063)*	0.664	*(0.064)*	0.310	*(0.056)*	1.226	*(0.034)*
18	-1,463.40	0.018	*(0.008)*	0.244	*(0.159)*	0.403	*(0.175)*	0.250	*(0.051)*	1.117	*(0.028)*
19	-4,434.65	0.033	*(0.013)*	0.398	*(0.054)*	0.663	*(0.061)*	0.359	*(0.058)*	1.422	*(0.047)*
20	-4,169.02	0.052	*(0.021)*	0.354	*(0.098)*	0.532	*(0.109)*	0.292	*(0.047)*	1.271	*(0.034)*
21	-3,598.28	0.027	*(0.009)*	0.469	*(0.070)*	0.690	*(0.061)*	0.314	*(0.056)*	1.230	*(0.033)*
22	-4,226.95	0.033	*(0.010)*	0.646	*(0.071)*	0.763	*(0.048)*	0.277	*(0.051)*	1.286	*(0.033)*
23	-4,174.08	0.034	*(0.013)*	0.326	*(0.056)*	0.620	*(0.074)*	0.360	*(0.060)*	1.151	*(0.020)*
24	-4,355.83	0.035	*(0.012)*	0.337	*(0.063)*	0.628	*(0.076)*	0.384	*(0.063)*	1.315	*(0.023)*
25	-3,464.67	0.020	*(0.007)*	0.335	*(0.065)*	0.620	*(0.072)*	0.371	*(0.062)*	1.241	*(0.035)*
26	-4,242.67	0.030	*(0.011)*	0.322	*(0.063)*	0.590	*(0.077)*	0.364	*(0.058)*	1.428	*(0.044)*
27	-2,299.80	0.014	*(0.005)*	0.225	*(0.103)*	0.473	*(0.120)*	0.363	*(0.061)*	1.178	*(0.028)*
28	-1,655.89	0.021	*(0.007)*	0.048	*(0.135)*	0.261	*(0.166)*	0.325	*(0.058)*	1.106	*(0.020)*
29	-2,659.42	0.014	*(0.004)*	0.286	*(0.071)*	0.596	*(0.082)*	0.547	*(0.079)*	1.152	*(0.029)*
30	-3,606.92	0.022	*(0.008)*	0.223	*(0.064)*	0.553	*(0.088)*	0.404	*(0.065)*	1.239	*(0.034)*
31	-1,432.97	0.009	*(0.004)*	0.436	*(0.128)*	0.570	*(0.133)*	0.308	*(0.058)*	1.213	*(0.035)*
32	-4,190.87	0.074	*(0.031)*	0.219	*(0.153)*	0.403	*(0.165)*	0.289	*(0.048)*	1.113	*(0.024)*
33	-6,142.08	0.045	*(0.016)*	0.219	*(0.046)*	0.704	*(0.054)*	0.516	*(0.075)*	1.444	*(0.035)*
34	-4,145.02	0.058	*(0.021)*	0.295	*(0.088)*	0.509	*(0.108)*	0.291	*(0.046)*	1.365	*(0.033)*
35	-4,211.49	0.032	*(0.012)*	0.245	*(0.058)*	0.568	*(0.078)*	0.383	*(0.063)*	1.300	*(0.040)*
36	-3,822.30	0.010	*(0.004)*	0.303	*(0.051)*	0.718	*(0.053)*	0.496	*(0.078)*	1.324	*(0.037)*
37	-4,960.42	0.041	*(0.014)*	0.160	*(0.055)*	0.539	*(0.078)*	0.442	*(0.064)*	1.432	*(0.048)*
38	-3,434.15	0.032	*(0.013)*	0.407	*(0.089)*	0.605	*(0.094)*	0.305	*(0.057)*	1.144	*(0.020)*
39	-4,704.17	0.116	*(0.045)*	0.268	*(0.122)*	0.449	*(0.133)*	0.250	*(0.046)*	1.239	*(0.026)*
40	-3,849.44	0.033	*(0.013)*	0.241	*(0.072)*	0.529	*(0.091)*	0.338	*(0.055)*	1.220	*(0.033)*
41	-3,734.78	0.016	*(0.006)*	0.420	*(0.054)*	0.718	*(0.053)*	0.392	*(0.066)*	1.283	*(0.039)*
42	-4,331.35	0.110	*(0.041)*	0.165	*(0.133)*	0.359	*(0.146)*	0.249	*(0.043)*	1.108	*(0.028)*
43	-4,185.72	0.028	*(0.010)*	0.365	*(0.043)*	0.694	*(0.053)*	0.398	*(0.063)*	1.230	*(0.031)*
44	-2,386.05	0.016	*(0.005)*	0.173	*(0.073)*	0.504	*(0.091)*	0.393	*(0.063)*	1.100	*(0.028)*
45	-2,855.17	0.026	*(0.009)*	0.373	*(0.076)*	0.588	*(0.078)*	0.274	*(0.049)*	1.200	*(0.031)*
46	-4,571.24	0.114	*(0.051)*	0.286	*(0.168)*	0.420	*(0.177)*	0.248	*(0.047)*	1.203	*(0.036)*
47	-3,471.09	0.278	*(0.058)*	-0.707	*(0.161)*	-0.659	*(0.180)*	0.159	*(0.025)*	1.013	*(0.020)*
48	-4,731.40	0.042	*(0.012)*	0.649	*(0.061)*	0.793	*(0.042)*	0.285	*(0.042)*	1.293	*(0.027)*
49	-6,064.56	0.173	*(0.071)*	0.444	*(0.081)*	0.634	*(0.083)*	0.258	*(0.051)*	1.341	*(0.038)*
50	-2,974.13	0.015	*(0.005)*	0.252	*(0.066)*	0.598	*(0.079)*	0.451	*(0.072)*	1.284	*(0.031)*
51	-3,465.58	0.020	*(0.007)*	0.382	*(0.064)*	0.649	*(0.065)*	0.367	*(0.061)*	1.274	*(0.038)*
52	-3,866.54	0.029	*(0.011)*	0.364	*(0.054)*	0.625	*(0.073)*	0.349	*(0.059)*	1.466	*(0.041)*

Appendix C: Table C.16. Estimated coefficients for the FIGARCH model - V_{Proxy}

comp	***LogL***	**ω**	**s.e.**	**α**	**s.e.**	**β**	**s.e.**	***d***	**s.e.**	**GED**	**s.e.**
1	-4,135.88	0.030	*(0.011)*	0.266	*(0.061)*	0.579	*(0.076)*	0.366	*(0.057)*	1.417	*(0.038)*
2	-2,083.33	0.009	*(0.003)*	0.331	*(0.055)*	0.634	*(0.065)*	0.370	*(0.063)*	1.362	*(0.043)*
3	-4,247.12	0.027	*(0.009)*	0.222	*(0.046)*	0.569	*(0.070)*	0.396	*(0.065)*	1.390	*(0.039)*
4	-4,590.18	0.180	*(0.067)*	-0.124	*(0.229)*	0.004	*(0.250)*	0.255	*(0.041)*	1.264	*(0.039)*
5	-2,376.13	0.004	*(0.001)*	0.164	*(0.043)*	0.673	*(0.062)*	0.547	*(0.079)*	1.322	*(0.036)*
6	-550.85	0.002	*(0.001)*	0.350	*(0.054)*	0.684	*(0.056)*	0.458	*(0.075)*	1.220	*(0.021)*
7	-2,540.15	0.008	*(0.003)*	0.299	*(0.042)*	0.700	*(0.052)*	0.439	*(0.069)*	1.432	*(0.037)*
8	-3,581.81	0.007	*(0.003)*	0.364	*(0.060)*	0.783	*(0.047)*	0.525	*(0.093)*	1.286	*(0.028)*
9	-1,910.12	0.009	*(0.003)*	0.327	*(0.069)*	0.599	*(0.072)*	0.359	*(0.061)*	1.222	*(0.028)*
10	-5,068.04	0.065	*(0.025)*	0.436	*(0.058)*	0.657	*(0.069)*	0.316	*(0.056)*	1.319	*(0.034)*
11	-4,723.68	0.094	*(0.036)*	0.253	*(0.086)*	0.489	*(0.105)*	0.268	*(0.048)*	1.290	*(0.037)*
12	-5,052.84	0.105	*(0.033)*	0.087	*(0.086)*	0.361	*(0.105)*	0.302	*(0.042)*	1.326	*(0.035)*
13	-5,077.14	0.157	*(0.067)*	0.342	*(0.143)*	0.467	*(0.160)*	0.234	*(0.043)*	1.093	*(0.016)*
14	-3,961.15	0.014	*(0.005)*	0.316	*(0.049)*	0.697	*(0.053)*	0.451	*(0.072)*	1.167	*(0.034)*
15	-3,382.89	0.013	*(0.005)*	0.503	*(0.066)*	0.740	*(0.055)*	0.410	*(0.071)*	1.223	*(0.028)*
16	-4,630.21	0.058	*(0.023)*	0.288	*(0.075)*	0.541	*(0.092)*	0.318	*(0.053)*	1.488	*(0.051)*
17	-2,976.01	0.018	*(0.007)*	0.430	*(0.062)*	0.664	*(0.064)*	0.316	*(0.057)*	1.234	*(0.035)*
18	-1,555.68	0.016	*(0.007)*	0.246	*(0.135)*	0.431	*(0.155)*	0.277	*(0.056)*	1.105	*(0.028)*
19	-4,498.87	0.032	*(0.012)*	0.398	*(0.053)*	0.671	*(0.060)*	0.368	*(0.060)*	1.411	*(0.047)*
20	-4,193.45	0.052	*(0.021)*	0.354	*(0.096)*	0.536	*(0.107)*	0.295	*(0.047)*	1.278	*(0.034)*
21	-3,633.80	0.024	*(0.009)*	0.439	*(0.067)*	0.685	*(0.059)*	0.337	*(0.059)*	1.224	*(0.033)*
22	-4,254.41	0.033	*(0.011)*	0.635	*(0.069)*	0.759	*(0.048)*	0.280	*(0.051)*	1.277	*(0.032)*
23	-4,161.34	0.032	*(0.012)*	0.336	*(0.054)*	0.633	*(0.071)*	0.364	*(0.062)*	1.147	*(0.020)*
24	-4,351.08	0.036	*(0.013)*	0.341	*(0.066)*	0.619	*(0.079)*	0.377	*(0.063)*	1.309	*(0.023)*
25	-3,525.72	0.021	*(0.008)*	0.332	*(0.064)*	0.616	*(0.072)*	0.368	*(0.062)*	1.247	*(0.036)*
26	-4,173.62	0.028	*(0.010)*	0.320	*(0.062)*	0.596	*(0.077)*	0.372	*(0.059)*	1.442	*(0.045)*
27	-2,203.20	0.012	*(0.004)*	0.219	*(0.093)*	0.478	*(0.111)*	0.359	*(0.058)*	1.204	*(0.029)*
28	-1,625.08	0.011	*(0.004)*	0.189	*(0.084)*	0.497	*(0.112)*	0.406	*(0.071)*	1.075	*(0.018)*
29	-2,711.58	0.014	*(0.004)*	0.260	*(0.069)*	0.581	*(0.082)*	0.534	*(0.076)*	1.137	*(0.029)*
30	-3,699.87	0.026	*(0.010)*	0.215	*(0.068)*	0.534	*(0.092)*	0.390	*(0.062)*	1.245	*(0.034)*
31	-1,249.14	0.009	*(0.004)*	0.334	*(0.144)*	0.475	*(0.155)*	0.286	*(0.050)*	1.225	*(0.036)*
32	-4,270.68	0.069	*(0.028)*	0.223	*(0.136)*	0.425	*(0.148)*	0.306	*(0.051)*	1.114	*(0.024)*
33	-6,047.73	0.049	*(0.018)*	0.226	*(0.046)*	0.671	*(0.056)*	0.476	*(0.069)*	1.444	*(0.035)*
34	-4,137.00	0.056	*(0.020)*	0.295	*(0.086)*	0.513	*(0.106)*	0.297	*(0.046)*	1.365	*(0.033)*
35	-4,286.38	0.032	*(0.012)*	0.241	*(0.056)*	0.574	*(0.076)*	0.392	*(0.064)*	1.297	*(0.040)*
36	-3,868.29	0.010	*(0.004)*	0.288	*(0.050)*	0.718	*(0.054)*	0.503	*(0.079)*	1.312	*(0.036)*
37	-5,020.25	0.041	*(0.014)*	0.151	*(0.053)*	0.546	*(0.077)*	0.455	*(0.065)*	1.433	*(0.049)*
38	-3,529.99	0.031	*(0.012)*	0.403	*(0.084)*	0.612	*(0.089)*	0.316	*(0.058)*	1.143	*(0.021)*
39	-4,659.83	0.113	*(0.045)*	0.274	*(0.124)*	0.452	*(0.135)*	0.248	*(0.046)*	1.240	*(0.027)*
40	-3,855.35	0.033	*(0.012)*	0.239	*(0.071)*	0.532	*(0.090)*	0.344	*(0.056)*	1.225	*(0.033)*
41	-3,756.58	0.016	*(0.006)*	0.405	*(0.054)*	0.718	*(0.053)*	0.402	*(0.068)*	1.276	*(0.039)*
42	-4,337.90	0.095	*(0.035)*	0.201	*(0.114)*	0.414	*(0.127)*	0.265	*(0.045)*	1.093	*(0.027)*
43	-4,219.44	0.029	*(0.010)*	0.361	*(0.044)*	0.692	*(0.054)*	0.401	*(0.064)*	1.232	*(0.032)*
44	-2,547.56	0.018	*(0.006)*	0.142	*(0.075)*	0.482	*(0.099)*	0.402	*(0.065)*	1.104	*(0.028)*
45	-3,030.76	0.027	*(0.009)*	0.351	*(0.073)*	0.585	*(0.076)*	0.288	*(0.050)*	1.210	*(0.031)*
46	-4,575.02	0.092	*(0.037)*	0.283	*(0.129)*	0.456	*(0.139)*	0.283	*(0.052)*	1.177	*(0.036)*
47	-3,586.73	0.327	*(0.066)*	-0.735	*(0.163)*	-0.693	*(0.181)*	0.148	*(0.024)*	1.023	*(0.020)*
48	-4,758.63	0.044	*(0.012)*	0.639	*(0.063)*	0.785	*(0.044)*	0.284	*(0.042)*	1.288	*(0.027)*
49	-6,046.90	0.171	*(0.069)*	0.434	*(0.080)*	0.628	*(0.083)*	0.260	*(0.051)*	1.345	*(0.038)*
50	-3,144.13	0.015	*(0.005)*	0.220	*(0.061)*	0.599	*(0.077)*	0.469	*(0.073)*	1.293	*(0.031)*
51	-3,572.86	0.021	*(0.007)*	0.383	*(0.063)*	0.655	*(0.064)*	0.373	*(0.062)*	1.281	*(0.039)*
52	-3,880.66	0.029	*(0.011)*	0.360	*(0.053)*	0.626	*(0.072)*	0.353	*(0.059)*	1.451	*(0.041)*

Appendix C: Table C.17. Estimated coefficients for the HYGARCH model - Equity

comp	*LogL*	ω	s.e.	α	s.e.	β	s.e.	*d*	s.e.	*log (κ)*	s.e.	GED	s.e.
1	-7,099.67	0.181	*(0.102)*	0.197	*(0.073)*	0.517	*(0.100)*	0.387	*(0.095)*	-0.001	*(0.062)*	1.277	*(0.030)*
2	-6,518.64	0.082	*(0.053)*	0.283	*(0.059)*	0.626	*(0.070)*	0.416	*(0.089)*	0.003	*(0.045)*	1.346	*(0.042)*
3	-6,850.43	0.142	*(0.086)*	0.196	*(0.058)*	0.511	*(0.082)*	0.361	*(0.087)*	0.006	*(0.063)*	1.396	*(0.040)*
4	-7,174.45	0.228	*(0.100)*	0.194	*(0.086)*	0.459	*(0.111)*	0.416	*(0.078)*	-0.019	*(0.045)*	1.231	*(0.037)*
5	-6,707.85	0.173	*(0.059)*	0.134	*(0.048)*	0.593	*(0.083)*	0.513	*(0.095)*	-0.037	*(0.032)*	1.297	*(0.037)*
6	-7,598.33	0.095	*(0.087)*	0.297	*(0.086)*	0.533	*(0.109)*	0.342	*(0.081)*	0.070	*(0.062)*	1.206	*(0.023)*
7	-7,379.23	0.118	*(0.056)*	0.252	*(0.047)*	0.688	*(0.061)*	0.487	*(0.092)*	-0.010	*(0.027)*	1.431	*(0.038)*
8	-7,156.42	0.062	*(0.066)*	0.398	*(0.073)*	0.639	*(0.078)*	0.321	*(0.090)*	0.065	*(0.078)*	1.270	*(0.035)*
9	-6,717.59	0.175	*(0.140)*	0.325	*(0.100)*	0.527	*(0.118)*	0.267	*(0.129)*	0.073	*(0.196)*	1.220	*(0.029)*
10	-6,577.92	0.145	*(0.068)*	0.348	*(0.070)*	0.631	*(0.082)*	0.397	*(0.110)*	-0.029	*(0.063)*	1.258	*(0.035)*
11	-6,462.78	0.212	*(0.094)*	0.221	*(0.079)*	0.493	*(0.116)*	0.313	*(0.100)*	-0.014	*(0.095)*	1.274	*(0.036)*
12	-6,250.81	0.155	*(0.091)*	0.060	*(0.108)*	0.308	*(0.141)*	0.263	*(0.082)*	0.069	*(0.125)*	1.308	*(0.035)*
13	-6,596.49	0.389	*(0.112)*	0.206	*(0.072)*	0.621	*(0.122)*	0.612	*(0.166)*	-0.128	*(0.048)*	1.054	*(0.015)*
14	-6,305.23	0.083	*(0.039)*	0.292	*(0.057)*	0.681	*(0.064)*	0.481	*(0.094)*	-0.019	*(0.033)*	1.146	*(0.034)*
15	-5,762.68	0.037	*(0.027)*	0.499	*(0.076)*	0.726	*(0.061)*	0.392	*(0.097)*	0.022	*(0.055)*	1.214	*(0.027)*
16	-5,547.40	0.166	*(0.049)*	0.265	*(0.064)*	0.605	*(0.096)*	0.478	*(0.133)*	-0.132	*(0.048)*	1.446	*(0.046)*
17	-6,347.10	0.054	*(0.041)*	0.394	*(0.066)*	0.680	*(0.063)*	0.371	*(0.097)*	0.026	*(0.060)*	1.212	*(0.033)*
18	-5,996.61	0.155	*(0.061)*	0.257	*(0.073)*	0.602	*(0.106)*	0.483	*(0.137)*	-0.046	*(0.054)*	1.056	*(0.028)*
19	-5,368.13	0.077	*(0.030)*	0.349	*(0.068)*	0.692	*(0.066)*	0.466	*(0.121)*	-0.057	*(0.044)*	1.361	*(0.045)*
20	-5,622.33	0.150	*(0.047)*	0.289	*(0.068)*	0.621	*(0.081)*	0.498	*(0.118)*	-0.103	*(0.043)*	1.277	*(0.034)*
21	-5,779.06	0.115	*(0.093)*	0.346	*(0.160)*	0.475	*(0.173)*	0.161	*(0.101)*	0.251	*(0.353)*	1.213	*(0.034)*
22	-5,779.51	0.113	*(0.045)*	0.617	*(0.101)*	0.747	*(0.054)*	0.321	*(0.105)*	-0.089	*(0.087)*	1.234	*(0.032)*
23	-6,183.94	0.142	*(0.056)*	0.318	*(0.069)*	0.623	*(0.086)*	0.425	*(0.102)*	-0.032	*(0.048)*	1.033	*(0.019)*
24	-6,090.06	0.089	*(0.029)*	0.323	*(0.087)*	0.749	*(0.069)*	0.569	*(0.145)*	-0.035	*(0.025)*	1.215	*(0.021)*
25	-5,321.55	0.097	*(0.040)*	0.326	*(0.071)*	0.600	*(0.087)*	0.379	*(0.105)*	-0.047	*(0.065)*	1.248	*(0.035)*
26	-6,028.09	0.118	*(0.040)*	0.266	*(0.061)*	0.576	*(0.090)*	0.438	*(0.095)*	-0.042	*(0.040)*	1.357	*(0.041)*
27	-6,216.09	0.209	*(0.076)*	0.184	*(0.082)*	0.485	*(0.125)*	0.428	*(0.103)*	-0.058	*(0.054)*	1.170	*(0.027)*
28	-5,949.73	0.250	*(0.076)*	0.136	*(0.075)*	0.498	*(0.128)*	0.486	*(0.122)*	-0.075	*(0.054)*	1.049	*(0.019)*
29	-5,726.18	0.075	*(0.030)*	0.237	*(0.068)*	0.582	*(0.082)*	0.551	*(0.081)*	-0.014	*(0.030)*	1.135	*(0.029)*
30	-6,353.60	0.091	*(0.047)*	0.197	*(0.056)*	0.597	*(0.091)*	0.483	*(0.098)*	-0.009	*(0.037)*	1.218	*(0.034)*
31	-6,392.02	0.183	*(0.079)*	0.311	*(0.103)*	0.518	*(0.131)*	0.371	*(0.112)*	-0.027	*(0.074)*	1.199	*(0.035)*
32	-6,244.80	0.229	*(0.110)*	0.180	*(0.148)*	0.382	*(0.175)*	0.306	*(0.088)*	-0.006	*(0.103)*	1.111	*(0.025)*
33	-7,836.62	0.041	*(0.017)*	-0.068	*(0.048)*	0.944	*(0.017)*	1.038	*(0.059)*	-0.004	*(0.003)*	1.425	*(0.038)*
34	-6,299.24	0.299	*(0.087)*	0.260	*(0.077)*	0.619	*(0.116)*	0.543	*(0.180)*	-0.148	*(0.053)*	1.305	*(0.034)*
35	-6,271.36	0.272	*(0.085)*	0.178	*(0.063)*	0.536	*(0.110)*	0.466	*(0.134)*	-0.108	*(0.054)*	1.299	*(0.041)*
36	-6,819.55	0.051	*(0.031)*	0.260	*(0.056)*	0.721	*(0.058)*	0.553	*(0.093)*	-0.002	*(0.022)*	1.284	*(0.037)*
37	-6,273.83	0.187	*(0.062)*	0.102	*(0.053)*	0.539	*(0.097)*	0.524	*(0.107)*	-0.064	*(0.038)*	1.424	*(0.048)*
38	-5,868.86	0.154	*(0.052)*	0.335	*(0.079)*	0.661	*(0.079)*	0.465	*(0.136)*	-0.085	*(0.047)*	1.121	*(0.021)*
39	-6,719.75	0.447	*(0.179)*	0.018	*(0.142)*	0.238	*(0.182)*	0.294	*(0.096)*	-0.013	*(0.119)*	1.150	*(0.025)*
40	-6,124.44	0.147	*(0.056)*	0.207	*(0.063)*	0.550	*(0.098)*	0.415	*(0.101)*	-0.043	*(0.048)*	1.215	*(0.033)*
41	-7,016.45	0.046	*(0.066)*	0.404	*(0.068)*	0.678	*(0.065)*	0.350	*(0.094)*	0.053	*(0.071)*	1.255	*(0.039)*
42	-6,774.95	0.113	*(0.148)*	0.086	*(0.170)*	0.267	*(0.188)*	0.187	*(0.072)*	0.248	*(0.200)*	1.081	*(0.028)*
43	-6,422.34	0.147	*(0.046)*	0.321	*(0.064)*	0.728	*(0.059)*	0.526	*(0.110)*	-0.062	*(0.029)*	1.165	*(0.028)*
44	-5,670.40	0.166	*(0.062)*	0.133	*(0.073)*	0.479	*(0.114)*	0.431	*(0.108)*	-0.056	*(0.062)*	1.101	*(0.027)*
45	-6,159.79	0.158	*(0.117)*	0.343	*(0.102)*	0.519	*(0.123)*	0.196	*(0.138)*	0.151	*(0.360)*	1.212	*(0.033)*
46	-7,613.75	0.127	*(0.117)*	0.293	*(0.122)*	0.479	*(0.141)*	0.278	*(0.081)*	0.115	*(0.102)*	1.159	*(0.035)*
47	-6,184.93	0.364	*(0.124)*	0.374	*(0.128)*	0.521	*(0.147)*	0.376	*(0.112)*	-0.195	*(0.062)*	0.997	*(0.022)*
48	-6,045.89	0.063	*(0.033)*	0.724	*(0.081)*	0.830	*(0.045)*	0.260	*(0.098)*	0.052	*(0.138)*	1.212	*(0.025)*
49	-7,320.40	0.061	*(0.099)*	0.438	*(0.084)*	0.634	*(0.086)*	0.229	*(0.099)*	0.172	*(0.183)*	1.326	*(0.038)*
50	-5,774.46	0.103	*(0.031)*	0.187	*(0.059)*	0.646	*(0.085)*	0.579	*(0.114)*	-0.044	*(0.027)*	1.278	*(0.030)*
51	-5,722.65	0.074	*(0.044)*	0.357	*(0.075)*	0.647	*(0.074)*	0.396	*(0.119)*	-0.010	*(0.071)*	1.268	*(0.039)*
52	-6,297.87	0.147	*(0.058)*	0.290	*(0.063)*	0.594	*(0.094)*	0.429	*(0.111)*	-0.046	*(0.052)*	1.427	*(0.045)*

Appendix C: Table C.18. Estimated coefficients for the HYGARCH model - V_{SM}

comp	***LogL***	**ω**	**s.e.**	**α**	**s.e.**	**β**	**s.e.**	***d***	**s.e.**	***log (κ)***	**s.e.**	**GED**	**s.e.**
1	-4,176.44	0.025	*(0.020)*	0.263	*(0.075)*	0.530	*(0.099)*	0.301	*(0.099)*	0.055	*(0.111)*	1.408	*(0.038)*
2	-2,161.63	0.005	*(0.007)*	0.405	*(0.075)*	0.631	*(0.073)*	0.273	*(0.110)*	0.098	*(0.159)*	1.372	*(0.044)*
3	-4,267.11	0.014	*(0.018)*	0.236	*(0.062)*	0.516	*(0.083)*	0.310	*(0.086)*	0.076	*(0.087)*	1.378	*(0.039)*
4	-4,615.84	0.132	*(0.045)*	0.217	*(0.088)*	0.473	*(0.140)*	0.445	*(0.163)*	-0.137	*(0.080)*	1.271	*(0.039)*
5	-2,494.39	0.002	*(0.002)*	0.208	*(0.047)*	0.656	*(0.066)*	0.483	*(0.083)*	0.027	*(0.028)*	1.313	*(0.038)*
6	-729.20	0.004	*(0.005)*	-0.138	*(0.272)*	-0.039	*(0.287)*	0.155	*(0.065)*	0.379	*(0.256)*	1.241	*(0.023)*
7	-2,592.27	0.003	*(0.001)*	-0.004	*(0.048)*	0.946	*(0.019)*	0.992	*(0.059)*	-0.009	*(0.005)*	1.448	*(0.037)*
8	-3,661.10	0.000	*(0.005)*	0.456	*(0.069)*	0.742	*(0.052)*	0.364	*(0.103)*	0.081	*(0.069)*	1.278	*(0.029)*
9	-1,993.16	0.009	*(0.006)*	0.353	*(0.074)*	0.618	*(0.079)*	0.353	*(0.120)*	0.001	*(0.093)*	1.236	*(0.028)*
10	-5,077.15	0.072	*(0.040)*	0.445	*(0.082)*	0.647	*(0.079)*	0.295	*(0.141)*	0.005	*(0.159)*	1.323	*(0.034)*
11	-4,706.99	0.087	*(0.047)*	0.258	*(0.091)*	0.482	*(0.128)*	0.249	*(0.120)*	0.030	*(0.196)*	1.292	*(0.037)*
12	-5,041.58	0.060	*(0.055)*	0.054	*(0.129)*	0.269	*(0.160)*	0.204	*(0.083)*	0.182	*(0.206)*	1.317	*(0.035)*
13	-5,016.50	0.180	*(0.052)*	0.252	*(0.077)*	0.596	*(0.122)*	0.540	*(0.179)*	-0.180	*(0.063)*	1.100	*(0.015)*
14	-3,938.81	0.002	*(0.009)*	0.370	*(0.058)*	0.657	*(0.066)*	0.330	*(0.090)*	0.092	*(0.078)*	1.161	*(0.034)*
15	-3,375.56	0.011	*(0.007)*	0.510	*(0.076)*	0.750	*(0.053)*	0.413	*(0.101)*	0.006	*(0.053)*	1.220	*(0.028)*
16	-4,600.99	0.083	*(0.027)*	0.263	*(0.062)*	0.608	*(0.094)*	0.457	*(0.130)*	-0.106	*(0.048)*	1.490	*(0.050)*
17	-2,997.36	0.020	*(0.011)*	0.426	*(0.080)*	0.667	*(0.066)*	0.332	*(0.128)*	-0.023	*(0.109)*	1.226	*(0.034)*
18	-1,562.81	0.023	*(0.008)*	0.246	*(0.083)*	0.543	*(0.122)*	0.463	*(0.154)*	-0.169	*(0.071)*	1.118	*(0.028)*
19	-4,395.85	0.046	*(0.017)*	0.354	*(0.072)*	0.699	*(0.067)*	0.467	*(0.132)*	-0.063	*(0.045)*	1.427	*(0.047)*
20	-4,143.68	0.074	*(0.025)*	0.315	*(0.077)*	0.590	*(0.087)*	0.438	*(0.131)*	-0.124	*(0.063)*	1.278	*(0.034)*
21	-3,640.61	0.006	*(0.012)*	0.530	*(0.089)*	0.683	*(0.080)*	0.191	*(0.104)*	0.260	*(0.275)*	1.225	*(0.033)*
22	-4,220.44	0.232	*(0.726)*	-0.192	*(3.717)*	-0.200	*(3.693)*	0.149	*(0.084)*	0.094	*(0.355)*	1.283	*(0.034)*
23	-4,177.53	0.042	*(0.017)*	0.310	*(0.059)*	0.628	*(0.079)*	0.399	*(0.097)*	-0.036	*(0.050)*	1.144	*(0.020)*
24	-4,363.27	0.050	*(0.015)*	0.300	*(0.071)*	0.689	*(0.079)*	0.523	*(0.135)*	-0.070	*(0.034)*	1.321	*(0.023)*
25	-3,512.47	0.022	*(0.011)*	0.337	*(0.067)*	0.624	*(0.078)*	0.376	*(0.097)*	-0.007	*(0.062)*	1.244	*(0.035)*
26	-4,214.63	0.029	*(0.013)*	0.333	*(0.062)*	0.615	*(0.079)*	0.386	*(0.096)*	-0.011	*(0.050)*	1.420	*(0.045)*
27	-2,309.35	0.018	*(0.006)*	0.225	*(0.085)*	0.509	*(0.113)*	0.416	*(0.090)*	-0.056	*(0.047)*	1.184	*(0.028)*
28	-1,652.90	0.026	*(0.008)*	0.112	*(0.082)*	0.411	*(0.139)*	0.458	*(0.125)*	-0.115	*(0.063)*	1.108	*(0.020)*
29	-2,679.65	0.019	*(0.006)*	0.268	*(0.066)*	0.589	*(0.081)*	0.566	*(0.083)*	-0.051	*(0.033)*	1.161	*(0.029)*
30	-3,638.12	0.038	*(0.014)*	0.207	*(0.058)*	0.585	*(0.102)*	0.477	*(0.126)*	-0.062	*(0.046)*	1.249	*(0.035)*
31	-1,414.58	0.012	*(0.004)*	0.423	*(0.092)*	0.623	*(0.089)*	0.409	*(0.111)*	-0.100	*(0.059)*	1.222	*(0.035)*
32	-4,237.88	0.071	*(0.036)*	0.224	*(0.149)*	0.413	*(0.177)*	0.295	*(0.102)*	0.003	*(0.128)*	1.112	*(0.024)*
33	-6,165.53	0.061	*(0.026)*	0.206	*(0.053)*	0.708	*(0.061)*	0.540	*(0.100)*	-0.016	*(0.022)*	1.444	*(0.035)*
34	-4,141.78	0.069	*(0.022)*	0.278	*(0.065)*	0.567	*(0.104)*	0.401	*(0.128)*	-0.094	*(0.069)*	1.365	*(0.034)*
35	-4,267.25	0.027	*(0.021)*	0.248	*(0.067)*	0.534	*(0.098)*	0.332	*(0.098)*	0.042	*(0.088)*	1.299	*(0.040)*
36	-3,869.06	0.005	*(0.007)*	0.323	*(0.057)*	0.704	*(0.059)*	0.451	*(0.096)*	0.025	*(0.037)*	1.321	*(0.037)*
37	-4,964.24	0.057	*(0.024)*	0.151	*(0.052)*	0.563	*(0.084)*	0.488	*(0.096)*	-0.034	*(0.038)*	1.441	*(0.048)*
38	-3,481.80	0.041	*(0.016)*	0.372	*(0.090)*	0.638	*(0.089)*	0.406	*(0.158)*	-0.084	*(0.076)*	1.148	*(0.021)*
39	-4,710.41	0.126	*(0.063)*	0.242	*(0.149)*	0.404	*(0.181)*	0.218	*(0.131)*	0.043	*(0.283)*	1.223	*(0.026)*
40	-3,868.91	0.034	*(0.016)*	0.243	*(0.071)*	0.537	*(0.103)*	0.346	*(0.103)*	-0.005	*(0.074)*	1.220	*(0.033)*
41	-3,752.74	0.011	*(0.011)*	0.449	*(0.072)*	0.704	*(0.059)*	0.332	*(0.108)*	0.051	*(0.094)*	1.274	*(0.039)*
42	-4,256.63	0.060	*(0.061)*	0.037	*(0.248)*	0.164	*(0.269)*	0.103	*(0.091)*	0.553	*(0.647)*	1.107	*(0.028)*
43	-4,191.60	0.045	*(0.014)*	0.295	*(0.081)*	0.740	*(0.069)*	0.558	*(0.146)*	-0.073	*(0.030)*	1.231	*(0.030)*
44	-2,445.04	0.025	*(0.007)*	0.155	*(0.060)*	0.588	*(0.099)*	0.536	*(0.130)*	-0.085	*(0.041)*	1.110	*(0.029)*
45	-2,951.97	0.029	*(0.012)*	0.363	*(0.075)*	0.604	*(0.083)*	0.316	*(0.130)*	-0.046	*(0.123)*	1.201	*(0.031)*
46	-4,462.80	0.077	*(0.035)*	0.323	*(0.123)*	0.493	*(0.147)*	0.289	*(0.114)*	0.000	*(0.130)*	1.193	*(0.037)*
47	-3,528.25	0.353	*(0.082)*	-0.676	*(0.172)*	-0.619	*(0.199)*	0.221	*(0.092)*	-0.265	*(0.262)*	1.017	*(0.020)*
48	-4,728.84	0.025	*(0.016)*	0.733	*(0.080)*	0.830	*(0.047)*	0.200	*(0.101)*	0.159	*(0.240)*	1.288	*(0.028)*
49	-6,078.77	0.081	*(0.101)*	0.499	*(0.119)*	0.624	*(0.117)*	0.130	*(0.141)*	0.398	*(0.702)*	1.338	*(0.038)*
50	-3,021.73	0.024	*(0.007)*	0.201	*(0.066)*	0.664	*(0.087)*	0.598	*(0.131)*	-0.065	*(0.030)*	1.294	*(0.031)*
51	-3,511.85	0.017	*(0.012)*	0.396	*(0.076)*	0.641	*(0.074)*	0.337	*(0.111)*	0.029	*(0.094)*	1.274	*(0.038)*
52	-3,854.37	0.027	*(0.018)*	0.375	*(0.070)*	0.613	*(0.091)*	0.318	*(0.139)*	0.027	*(0.137)*	1.463	*(0.041)*

Appendix C: Table C.19. Estimated coefficients for the HYGARCH model - V_{KMV}

comp	*LogL*	ω	s.e.	α	s.e.	β	s.e.	*d*	s.e.	*log (κ)*	s.e.	GED	s.e.
1	-4,215.18	0.028	*(0.020)*	0.251	*(0.072)*	0.531	*(0.098)*	0.315	*(0.099)*	0.041	*(0.100)*	1.412	*(0.038)*
2	-2,176.81	0.005	*(0.007)*	0.403	*(0.074)*	0.637	*(0.071)*	0.283	*(0.109)*	0.089	*(0.146)*	1.369	*(0.043)*
3	-4,222.65	0.014	*(0.018)*	0.234	*(0.063)*	0.509	*(0.084)*	0.303	*(0.086)*	0.081	*(0.091)*	1.378	*(0.039)*
4	-4,561.49	0.123	*(0.043)*	0.225	*(0.090)*	0.475	*(0.138)*	0.434	*(0.158)*	-0.127	*(0.081)*	1.269	*(0.038)*
5	-2,485.72	0.002	*(0.002)*	0.210	*(0.047)*	0.656	*(0.065)*	0.483	*(0.083)*	0.026	*(0.028)*	1.315	*(0.039)*
6	-714.28	0.005	*(0.005)*	-0.137	*(0.285)*	-0.043	*(0.300)*	0.152	*(0.068)*	0.375	*(0.278)*	1.243	*(0.023)*
7	-2,564.05	0.003	*(0.001)*	-0.004	*(0.048)*	0.947	*(0.018)*	0.993	*(0.059)*	-0.009	*(0.005)*	1.449	*(0.037)*
8	-3,610.04	0.000	*(0.005)*	0.458	*(0.069)*	0.741	*(0.052)*	0.361	*(0.103)*	0.083	*(0.071)*	1.279	*(0.029)*
9	-1,936.04	0.009	*(0.006)*	0.353	*(0.075)*	0.616	*(0.080)*	0.352	*(0.123)*	-0.002	*(0.095)*	1.237	*(0.028)*
10	-5,068.29	0.067	*(0.036)*	0.426	*(0.076)*	0.649	*(0.077)*	0.319	*(0.134)*	-0.004	*(0.126)*	1.325	*(0.034)*
11	-4,657.30	0.083	*(0.044)*	0.257	*(0.091)*	0.482	*(0.127)*	0.249	*(0.118)*	0.033	*(0.191)*	1.291	*(0.037)*
12	-5,072.22	0.062	*(0.053)*	0.056	*(0.126)*	0.277	*(0.157)*	0.213	*(0.082)*	0.163	*(0.189)*	1.314	*(0.035)*
13	-5,058.39	0.175	*(0.051)*	0.246	*(0.076)*	0.608	*(0.120)*	0.556	*(0.176)*	-0.166	*(0.060)*	1.098	*(0.015)*
14	-3,926.48	0.002	*(0.008)*	0.364	*(0.057)*	0.660	*(0.065)*	0.343	*(0.090)*	0.082	*(0.070)*	1.160	*(0.034)*
15	-3,321.35	0.011	*(0.007)*	0.510	*(0.076)*	0.749	*(0.054)*	0.414	*(0.101)*	0.005	*(0.053)*	1.220	*(0.028)*
16	-4,585.20	0.079	*(0.026)*	0.264	*(0.061)*	0.610	*(0.092)*	0.456	*(0.127)*	-0.100	*(0.047)*	1.493	*(0.050)*
17	-2,930.95	0.020	*(0.011)*	0.426	*(0.080)*	0.667	*(0.066)*	0.333	*(0.128)*	-0.025	*(0.108)*	1.226	*(0.034)*
18	-1,462.08	0.022	*(0.007)*	0.245	*(0.083)*	0.538	*(0.123)*	0.458	*(0.153)*	-0.172	*(0.072)*	1.121	*(0.028)*
19	-4,434.24	0.044	*(0.018)*	0.363	*(0.068)*	0.690	*(0.065)*	0.443	*(0.124)*	-0.054	*(0.049)*	1.426	*(0.047)*
20	-4,168.05	0.075	*(0.024)*	0.308	*(0.076)*	0.597	*(0.087)*	0.458	*(0.131)*	-0.126	*(0.057)*	1.276	*(0.034)*
21	-3,596.79	0.007	*(0.013)*	0.552	*(0.096)*	0.687	*(0.085)*	0.166	*(0.110)*	0.321	*(0.370)*	1.229	*(0.034)*
22	-4,229.46	0.224	*(0.415)*	-0.207	*(2.203)*	-0.220	*(2.179)*	0.133	*(0.082)*	0.173	*(0.404)*	1.289	*(0.034)*
23	-4,173.88	0.040	*(0.016)*	0.312	*(0.057)*	0.635	*(0.077)*	0.400	*(0.098)*	-0.033	*(0.049)*	1.153	*(0.020)*
24	-4,354.28	0.050	*(0.015)*	0.293	*(0.071)*	0.688	*(0.080)*	0.524	*(0.137)*	-0.069	*(0.033)*	1.324	*(0.023)*
25	-3,464.66	0.021	*(0.011)*	0.334	*(0.067)*	0.622	*(0.078)*	0.375	*(0.096)*	-0.004	*(0.061)*	1.241	*(0.035)*
26	-4,242.63	0.033	*(0.014)*	0.321	*(0.063)*	0.602	*(0.083)*	0.384	*(0.096)*	-0.015	*(0.051)*	1.429	*(0.044)*
27	-2,299.06	0.019	*(0.007)*	0.224	*(0.083)*	0.511	*(0.113)*	0.423	*(0.094)*	-0.063	*(0.048)*	1.186	*(0.028)*
28	-1,654.24	0.028	*(0.008)*	0.112	*(0.081)*	0.421	*(0.142)*	0.480	*(0.134)*	-0.137	*(0.062)*	1.113	*(0.020)*
29	-2,658.32	0.019	*(0.006)*	0.270	*(0.067)*	0.589	*(0.081)*	0.564	*(0.084)*	-0.050	*(0.033)*	1.161	*(0.029)*
30	-3,606.38	0.033	*(0.013)*	0.210	*(0.058)*	0.593	*(0.099)*	0.479	*(0.122)*	-0.052	*(0.045)*	1.244	*(0.034)*
31	-1,431.73	0.013	*(0.005)*	0.408	*(0.093)*	0.616	*(0.092)*	0.422	*(0.117)*	-0.110	*(0.060)*	1.219	*(0.035)*
32	-4,190.87	0.075	*(0.037)*	0.220	*(0.152)*	0.406	*(0.182)*	0.294	*(0.105)*	-0.006	*(0.133)*	1.113	*(0.024)*
33	-6,141.89	0.057	*(0.025)*	0.206	*(0.053)*	0.716	*(0.061)*	0.548	*(0.102)*	-0.014	*(0.022)*	1.447	*(0.035)*
34	-4,144.31	0.074	*(0.024)*	0.277	*(0.066)*	0.561	*(0.107)*	0.400	*(0.132)*	-0.104	*(0.071)*	1.370	*(0.034)*
35	-4,211.22	0.022	*(0.019)*	0.249	*(0.068)*	0.533	*(0.098)*	0.325	*(0.095)*	0.054	*(0.088)*	1.299	*(0.040)*
36	-3,822.03	0.005	*(0.007)*	0.324	*(0.058)*	0.704	*(0.059)*	0.453	*(0.096)*	0.024	*(0.037)*	1.320	*(0.037)*
37	-4,960.04	0.057	*(0.024)*	0.154	*(0.052)*	0.564	*(0.084)*	0.486	*(0.096)*	-0.033	*(0.038)*	1.437	*(0.048)*
38	-3,433.77	0.039	*(0.015)*	0.376	*(0.091)*	0.639	*(0.089)*	0.400	*(0.157)*	-0.079	*(0.078)*	1.146	*(0.020)*
39	-4,704.13	0.109	*(0.056)*	0.261	*(0.137)*	0.427	*(0.167)*	0.222	*(0.124)*	0.058	*(0.258)*	1.238	*(0.026)*
40	-3,849.44	0.034	*(0.016)*	0.241	*(0.072)*	0.532	*(0.105)*	0.343	*(0.104)*	-0.005	*(0.077)*	1.220	*(0.033)*
41	-3,734.41	0.009	*(0.011)*	0.456	*(0.072)*	0.708	*(0.058)*	0.330	*(0.106)*	0.057	*(0.094)*	1.279	*(0.039)*
42	-4,330.18	0.061	*(0.063)*	0.029	*(0.244)*	0.158	*(0.265)*	0.107	*(0.088)*	0.529	*(0.596)*	1.106	*(0.028)*
43	-4,184.17	0.041	*(0.013)*	0.288	*(0.079)*	0.743	*(0.068)*	0.559	*(0.143)*	-0.065	*(0.028)*	1.237	*(0.031)*
44	-2,384.60	0.025	*(0.007)*	0.157	*(0.061)*	0.581	*(0.100)*	0.528	*(0.131)*	-0.089	*(0.043)*	1.110	*(0.029)*
45	-2,855.09	0.029	*(0.012)*	0.363	*(0.077)*	0.597	*(0.086)*	0.308	*(0.134)*	-0.047	*(0.135)*	1.201	*(0.031)*
46	-4,571.17	0.122	*(0.051)*	0.289	*(0.142)*	0.445	*(0.176)*	0.288	*(0.129)*	-0.064	*(0.150)*	1.202	*(0.037)*
47	-3,470.84	0.322	*(0.078)*	-0.685	*(0.162)*	-0.627	*(0.188)*	0.214	*(0.088)*	-0.213	*(0.258)*	1.014	*(0.021)*
48	-4,730.83	0.025	*(0.016)*	0.722	*(0.080)*	0.824	*(0.047)*	0.207	*(0.101)*	0.151	*(0.226)*	1.294	*(0.028)*
49	-6,063.90	0.082	*(0.101)*	0.502	*(0.122)*	0.622	*(0.121)*	0.122	*(0.142)*	0.436	*(0.773)*	1.339	*(0.038)*
50	-2,972.21	0.025	*(0.007)*	0.197	*(0.069)*	0.670	*(0.090)*	0.612	*(0.139)*	-0.070	*(0.030)*	1.293	*(0.030)*
51	-3,465.52	0.017	*(0.011)*	0.394	*(0.076)*	0.641	*(0.074)*	0.339	*(0.112)*	0.028	*(0.095)*	1.273	*(0.038)*
52	-3,866.52	0.027	*(0.017)*	0.368	*(0.067)*	0.618	*(0.089)*	0.332	*(0.137)*	0.016	*(0.119)*	1.466	*(0.041)*

Appendix C: Table C.20. Estimated coefficients for the HYGARCH model - V_{Proxy}

comp	*LogL*	ω	s.e.	α	s.e.	β	s.e.	*d*	s.e.	*log (κ)*	s.e.	GED	s.e.
1	-4,135.72	0.023	*(0.018)*	0.273	*(0.071)*	0.554	*(0.092)*	0.319	*(0.098)*	0.045	*(0.095)*	1.415	*(0.039)*
2	-2,083.05	0.005	*(0.006)*	0.354	*(0.069)*	0.616	*(0.074)*	0.309	*(0.106)*	0.066	*(0.117)*	1.360	*(0.043)*
3	-4,246.51	0.013	*(0.017)*	0.233	*(0.058)*	0.528	*(0.081)*	0.323	*(0.086)*	0.068	*(0.079)*	1.384	*(0.040)*
4	-4,589.63	0.127	*(0.041)*	0.196	*(0.080)*	0.488	*(0.137)*	0.480	*(0.168)*	-0.132	*(0.069)*	1.266	*(0.039)*
5	-2,375.43	0.002	*(0.002)*	0.176	*(0.044)*	0.645	*(0.066)*	0.494	*(0.080)*	0.027	*(0.025)*	1.312	*(0.038)*
6	-547.69	0.000	*(0.001)*	0.401	*(0.076)*	0.612	*(0.083)*	0.294	*(0.092)*	0.155	*(0.109)*	1.205	*(0.021)*
7	-2,534.51	0.004	*(0.002)*	-0.001	*(0.050)*	0.938	*(0.023)*	0.981	*(0.065)*	-0.012	*(0.006)*	1.445	*(0.037)*
8	-3,580.16	0.000	*(0.005)*	0.440	*(0.067)*	0.739	*(0.053)*	0.378	*(0.102)*	0.074	*(0.063)*	1.277	*(0.029)*
9	-1,910.06	0.007	*(0.006)*	0.336	*(0.077)*	0.589	*(0.088)*	0.331	*(0.120)*	0.031	*(0.112)*	1.221	*(0.028)*
10	-5,068.04	0.064	*(0.036)*	0.437	*(0.077)*	0.656	*(0.075)*	0.313	*(0.135)*	0.003	*(0.132)*	1.319	*(0.034)*
11	-4,723.68	0.094	*(0.048)*	0.253	*(0.087)*	0.488	*(0.125)*	0.266	*(0.125)*	0.003	*(0.181)*	1.290	*(0.037)*
12	-5,052.00	0.059	*(0.052)*	0.052	*(0.124)*	0.275	*(0.154)*	0.213	*(0.081)*	0.165	*(0.186)*	1.322	*(0.035)*
13	-5,073.97	0.176	*(0.052)*	0.251	*(0.080)*	0.614	*(0.120)*	0.558	*(0.182)*	-0.168	*(0.061)*	1.099	*(0.016)*
14	-3,959.57	0.002	*(0.009)*	0.353	*(0.058)*	0.654	*(0.066)*	0.344	*(0.089)*	0.085	*(0.070)*	1.158	*(0.034)*
15	-3,382.88	0.012	*(0.008)*	0.507	*(0.079)*	0.740	*(0.056)*	0.404	*(0.103)*	0.005	*(0.059)*	1.223	*(0.028)*
16	-4,628.91	0.081	*(0.026)*	0.262	*(0.061)*	0.606	*(0.093)*	0.452	*(0.126)*	-0.098	*(0.048)*	1.495	*(0.051)*
17	-2,975.99	0.019	*(0.011)*	0.423	*(0.080)*	0.666	*(0.066)*	0.332	*(0.129)*	-0.018	*(0.111)*	1.235	*(0.035)*
18	-1,554.86	0.020	*(0.007)*	0.246	*(0.084)*	0.529	*(0.124)*	0.430	*(0.145)*	-0.134	*(0.077)*	1.109	*(0.028)*
19	-4,498.50	0.043	*(0.018)*	0.364	*(0.068)*	0.696	*(0.064)*	0.447	*(0.122)*	-0.049	*(0.048)*	1.415	*(0.047)*
20	-4,192.62	0.074	*(0.025)*	0.313	*(0.076)*	0.595	*(0.087)*	0.445	*(0.128)*	-0.119	*(0.060)*	1.282	*(0.034)*
21	-3,632.15	0.005	*(0.013)*	0.512	*(0.091)*	0.669	*(0.084)*	0.189	*(0.103)*	0.269	*(0.280)*	1.223	*(0.033)*
22	-4,253.58	0.046	*(0.016)*	0.557	*(0.091)*	0.752	*(0.040)*	0.392	*(0.118)*	-0.096	*(0.061)*	1.283	*(0.033)*
23	-4,161.14	0.038	*(0.015)*	0.322	*(0.057)*	0.647	*(0.075)*	0.404	*(0.099)*	-0.031	*(0.048)*	1.150	*(0.020)*
24	-4,349.53	0.052	*(0.016)*	0.298	*(0.071)*	0.681	*(0.082)*	0.518	*(0.137)*	-0.073	*(0.035)*	1.319	*(0.023)*
25	-3,525.72	0.022	*(0.011)*	0.331	*(0.067)*	0.616	*(0.079)*	0.369	*(0.096)*	-0.001	*(0.064)*	1.247	*(0.036)*
26	-4,173.58	0.030	*(0.013)*	0.318	*(0.061)*	0.608	*(0.082)*	0.392	*(0.096)*	-0.015	*(0.049)*	1.443	*(0.045)*
27	-2,202.69	0.016	*(0.006)*	0.217	*(0.079)*	0.509	*(0.106)*	0.407	*(0.086)*	-0.050	*(0.047)*	1.210	*(0.029)*
28	-1,624.75	0.015	*(0.005)*	0.187	*(0.074)*	0.532	*(0.113)*	0.464	*(0.115)*	-0.049	*(0.051)*	1.079	*(0.018)*
29	-2,710.64	0.019	*(0.006)*	0.249	*(0.065)*	0.579	*(0.080)*	0.552	*(0.082)*	-0.046	*(0.032)*	1.146	*(0.029)*
30	-3,699.49	0.036	*(0.015)*	0.205	*(0.060)*	0.570	*(0.104)*	0.455	*(0.122)*	-0.049	*(0.052)*	1.249	*(0.035)*
31	-1,248.96	0.011	*(0.005)*	0.342	*(0.117)*	0.508	*(0.135)*	0.328	*(0.098)*	-0.054	*(0.084)*	1.227	*(0.035)*
32	-4,270.66	0.065	*(0.036)*	0.221	*(0.146)*	0.414	*(0.174)*	0.291	*(0.101)*	0.022	*(0.131)*	1.113	*(0.024)*
33	-6,047.59	0.060	*(0.027)*	0.216	*(0.051)*	0.682	*(0.063)*	0.502	*(0.096)*	-0.014	*(0.026)*	1.446	*(0.035)*
34	-4,136.18	0.072	*(0.023)*	0.275	*(0.064)*	0.570	*(0.105)*	0.414	*(0.133)*	-0.105	*(0.064)*	1.370	*(0.034)*
35	-4,286.07	0.021	*(0.019)*	0.247	*(0.066)*	0.537	*(0.097)*	0.330	*(0.094)*	0.056	*(0.085)*	1.295	*(0.040)*
36	-3,867.90	0.004	*(0.007)*	0.312	*(0.056)*	0.701	*(0.060)*	0.452	*(0.095)*	0.028	*(0.036)*	1.307	*(0.037)*
37	-5,019.83	0.058	*(0.024)*	0.144	*(0.051)*	0.571	*(0.084)*	0.501	*(0.098)*	-0.033	*(0.036)*	1.438	*(0.049)*
38	-3,529.72	0.038	*(0.015)*	0.376	*(0.088)*	0.639	*(0.087)*	0.395	*(0.151)*	-0.064	*(0.079)*	1.146	*(0.021)*
39	-4,659.78	0.106	*(0.055)*	0.267	*(0.141)*	0.429	*(0.169)*	0.218	*(0.126)*	0.064	*(0.270)*	1.240	*(0.027)*
40	-3,855.35	0.034	*(0.016)*	0.238	*(0.070)*	0.536	*(0.104)*	0.351	*(0.105)*	-0.007	*(0.074)*	1.225	*(0.033)*
41	-3,756.21	0.008	*(0.011)*	0.440	*(0.071)*	0.707	*(0.058)*	0.338	*(0.108)*	0.056	*(0.090)*	1.271	*(0.039)*
42	-4,336.45	0.042	*(0.057)*	0.063	*(0.229)*	0.198	*(0.249)*	0.105	*(0.087)*	0.569	*(0.601)*	1.091	*(0.027)*
43	-4,217.42	0.044	*(0.014)*	0.274	*(0.085)*	0.747	*(0.073)*	0.582	*(0.153)*	-0.069	*(0.029)*	1.240	*(0.031)*
44	-2,546.33	0.028	*(0.008)*	0.133	*(0.059)*	0.562	*(0.107)*	0.533	*(0.131)*	-0.083	*(0.044)*	1.114	*(0.029)*
45	-3,030.69	0.030	*(0.013)*	0.343	*(0.074)*	0.595	*(0.085)*	0.319	*(0.130)*	-0.040	*(0.122)*	1.211	*(0.031)*
46	-4,575.01	0.095	*(0.040)*	0.283	*(0.123)*	0.463	*(0.154)*	0.295	*(0.115)*	-0.017	*(0.125)*	1.177	*(0.036)*
47	-3,586.61	0.361	*(0.091)*	-0.712	*(0.171)*	-0.664	*(0.195)*	0.189	*(0.092)*	-0.180	*(0.327)*	1.024	*(0.021)*
48	-4,758.13	0.026	*(0.017)*	0.713	*(0.084)*	0.816	*(0.049)*	0.209	*(0.103)*	0.145	*(0.227)*	1.289	*(0.028)*
49	-6,046.20	0.079	*(0.100)*	0.488	*(0.121)*	0.612	*(0.123)*	0.122	*(0.139)*	0.440	*(0.759)*	1.343	*(0.038)*
50	-3,142.65	0.025	*(0.008)*	0.178	*(0.063)*	0.661	*(0.088)*	0.600	*(0.130)*	-0.059	*(0.028)*	1.300	*(0.031)*
51	-3,572.81	0.018	*(0.012)*	0.394	*(0.076)*	0.648	*(0.073)*	0.346	*(0.116)*	0.025	*(0.095)*	1.280	*(0.039)*
52	-3,880.63	0.026	*(0.017)*	0.366	*(0.067)*	0.615	*(0.089)*	0.328	*(0.135)*	0.025	*(0.124)*	1.451	*(0.041)*

Appendix C: Table C.21. Estimated coefficients for the FIEGARCH model - Equity

comp	***LogL***	**ω**	**s.e.**	**β**	**s.e.**	***d***	**s.e.**	**γ_1**	**s.e.**	**γ_2**	**s.e.**	**GED**	**s.e.**
1	-7,088.72	1.255	*(0.209)*	0.528	*(0.150)*	0.595	*(0.057)*	-0.055	*(0.017)*	0.146	*(0.031)*	1.272	*(0.029)*
2	-6,503.58	0.851	*(0.221)*	0.593	*(0.122)*	0.600	*(0.054)*	-0.060	*(0.014)*	0.122	*(0.026)*	1.359	*(0.043)*
3	-6,839.58	0.998	*(0.228)*	0.714	*(0.095)*	0.587	*(0.068)*	-0.036	*(0.010)*	0.092	*(0.018)*	1.392	*(0.037)*
4	-7,145.20	1.301	*(0.318)*	0.309	*(0.132)*	0.638	*(0.043)*	-0.112	*(0.020)*	0.230	*(0.034)*	1.238	*(0.036)*
5	-6,706.93	1.048	*(0.211)*	0.776	*(0.079)*	0.509	*(0.075)*	-0.039	*(0.010)*	0.112	*(0.021)*	1.279	*(0.033)*
6	-7,571.39	1.253	*(0.289)*	0.212	*(0.174)*	0.669	*(0.050)*	-0.053	*(0.016)*	0.200	*(0.043)*	1.226	*(0.023)*
7	-7,361.29	1.231	*(0.223)*	0.670	*(0.106)*	0.616	*(0.054)*	-0.051	*(0.013)*	0.090	*(0.020)*	1.449	*(0.036)*
8	-7,130.97	1.197	*(0.230)*	0.349	*(0.188)*	0.680	*(0.053)*	-0.079	*(0.019)*	0.131	*(0.030)*	1.303	*(0.032)*
9	-6,702.65	1.187	*(0.140)*	0.605	*(0.127)*	0.502	*(0.063)*	-0.056	*(0.016)*	0.127	*(0.029)*	1.231	*(0.029)*
10	-6,570.94	0.901	*(0.274)*	0.242	*(0.183)*	0.696	*(0.051)*	-0.064	*(0.017)*	0.157	*(0.032)*	1.265	*(0.032)*
11	-6,443.30	0.953	*(0.136)*	0.730	*(0.086)*	0.503	*(0.055)*	-0.063	*(0.014)*	0.080	*(0.020)*	1.291	*(0.035)*
12	-6,210.64	0.581	*(0.214)*	0.509	*(0.120)*	0.626	*(0.046)*	-0.115	*(0.022)*	0.107	*(0.021)*	1.350	*(0.034)*
13	-6,577.46	1.178	*(0.174)*	-0.235	*(0.213)*	0.636	*(0.046)*	-0.056	*(0.023)*	0.211	*(0.033)*	1.054	*(0.017)*
14	-6,279.74	0.855	*(0.241)*	0.714	*(0.087)*	0.603	*(0.056)*	-0.060	*(0.014)*	0.090	*(0.020)*	1.174	*(0.035)*
15	-5,750.01	0.739	*(0.254)*	0.110	*(0.198)*	0.647	*(0.051)*	-0.078	*(0.024)*	0.247	*(0.049)*	1.233	*(0.029)*
16	-5,542.59	0.454	*(0.171)*	0.674	*(0.101)*	0.548	*(0.065)*	-0.049	*(0.012)*	0.111	*(0.023)*	1.449	*(0.046)*
17	-6,333.06	0.838	*(0.193)*	0.595	*(0.124)*	0.590	*(0.057)*	-0.074	*(0.017)*	0.113	*(0.025)*	1.227	*(0.034)*
18	-5,967.30	0.725	*(0.244)*	0.643	*(0.112)*	0.590	*(0.060)*	-0.077	*(0.017)*	0.113	*(0.025)*	1.075	*(0.029)*
19	-5,359.13	0.279	*(0.212)*	0.317	*(0.189)*	0.631	*(0.055)*	-0.070	*(0.021)*	0.166	*(0.038)*	1.370	*(0.044)*
20	-5,622.76	0.440	*(0.169)*	0.791	*(0.065)*	0.425	*(0.079)*	-0.035	*(0.010)*	0.135	*(0.019)*	1.258	*(0.032)*
21	-5,743.69	0.489	*(0.124)*	0.467	*(0.152)*	0.540	*(0.053)*	-0.115	*(0.025)*	0.109	*(0.028)*	1.258	*(0.036)*
22	-5,763.35	0.559	*(0.242)*	-0.185	*(0.146)*	0.641	*(0.046)*	-0.132	*(0.021)*	0.288	*(0.028)*	1.256	*(0.033)*
23	-6,166.72	0.893	*(0.221)*	0.653	*(0.105)*	0.559	*(0.056)*	-0.044	*(0.014)*	0.122	*(0.026)*	1.048	*(0.019)*
24	-6,069.70	0.808	*(0.232)*	-0.093	*(0.222)*	0.697	*(0.040)*	-0.105	*(0.025)*	0.206	*(0.039)*	1.238	*(0.022)*
25	-5,307.09	0.225	*(0.216)*	0.389	*(0.187)*	0.650	*(0.057)*	-0.055	*(0.018)*	0.144	*(0.035)*	1.263	*(0.036)*
26	-6,005.16	0.719	*(0.234)*	0.581	*(0.113)*	0.595	*(0.051)*	-0.082	*(0.016)*	0.134	*(0.025)*	1.388	*(0.039)*
27	-6,207.16	0.840	*(0.255)*	0.476	*(0.138)*	0.606	*(0.052)*	-0.064	*(0.017)*	0.175	*(0.033)*	1.179	*(0.027)*
28	-5,932.66	0.770	*(0.172)*	0.721	*(0.103)*	0.451	*(0.082)*	-0.038	*(0.014)*	0.146	*(0.029)*	1.066	*(0.019)*
29	-5,707.50	0.146	*(0.387)*	0.355	*(0.126)*	0.614	*(0.050)*	-0.086	*(0.019)*	0.297	*(0.033)*	1.128	*(0.026)*
30	-6,331.08	0.762	*(0.271)*	0.641	*(0.106)*	0.601	*(0.060)*	-0.061	*(0.013)*	0.137	*(0.025)*	1.241	*(0.035)*
31	-6,375.07	1.087	*(0.250)*	0.220	*(0.152)*	0.605	*(0.045)*	-0.104	*(0.021)*	0.262	*(0.040)*	1.221	*(0.036)*
32	-6,212.25	0.862	*(0.205)*	0.619	*(0.094)*	0.535	*(0.053)*	-0.085	*(0.018)*	0.141	*(0.027)*	1.133	*(0.024)*
33	-7,832.38	1.530	*(0.306)*	0.782	*(0.072)*	0.626	*(0.064)*	-0.031	*(0.008)*	0.079	*(0.016)*	1.431	*(0.037)*
34	-6,301.83	0.951	*(0.135)*	0.689	*(0.102)*	0.404	*(0.078)*	-0.040	*(0.014)*	0.177	*(0.029)*	1.297	*(0.032)*
35	-6,242.09	0.870	*(0.072)*	0.578	*(0.137)*	0.550	*(0.070)*	-0.113	*(0.022)*	0.122	*(0.026)*	1.343	*(0.043)*
36	-6,790.60	0.969	*(0.226)*	0.740	*(0.061)*	0.556	*(0.044)*	-0.074	*(0.012)*	0.102	*(0.017)*	1.304	*(0.034)*
37	-6,234.83	0.775	*(0.118)*	0.810	*(0.047)*	0.380	*(0.059)*	-0.101	*(0.014)*	0.112	*(0.018)*	1.465	*(0.047)*
38	-5,832.07	0.613	*(0.223)*	0.279	*(0.178)*	0.643	*(0.046)*	-0.124	*(0.027)*	0.150	*(0.031)*	1.163	*(0.022)*
39	-6,696.17	1.361	*(0.184)*	0.506	*(0.166)*	0.620	*(0.058)*	-0.064	*(0.018)*	0.127	*(0.035)*	1.183	*(0.025)*
40	-6,100.42	0.691	*(0.208)*	0.618	*(0.111)*	0.589	*(0.052)*	-0.074	*(0.017)*	0.116	*(0.025)*	1.225	*(0.032)*
41	-7,006.11	1.230	*(0.216)*	0.607	*(0.133)*	0.658	*(0.057)*	-0.047	*(0.014)*	0.084	*(0.024)*	1.270	*(0.039)*
42	-6,757.56	1.108	*(0.211)*	0.502	*(0.171)*	0.623	*(0.058)*	-0.068	*(0.019)*	0.111	*(0.032)*	1.094	*(0.028)*
43	-6,404.74	0.937	*(0.188)*	0.770	*(0.077)*	0.512	*(0.065)*	-0.050	*(0.011)*	0.092	*(0.019)*	1.189	*(0.028)*
44	-5,668.88	0.586	*(0.235)*	0.658	*(0.108)*	0.528	*(0.066)*	-0.041	*(0.013)*	0.153	*(0.029)*	1.091	*(0.027)*
45	-6,147.07	0.819	*(0.194)*	0.533	*(0.176)*	0.603	*(0.071)*	-0.049	*(0.016)*	0.112	*(0.031)*	1.227	*(0.031)*
46	-7,595.10	1.569	*(0.215)*	0.105	*(0.175)*	0.622	*(0.046)*	-0.120	*(0.025)*	0.208	*(0.035)*	1.169	*(0.034)*
47	-6,146.26	0.733	*(0.224)*	0.123	*(0.169)*	0.659	*(0.042)*	-0.114	*(0.022)*	0.175	*(0.032)*	1.027	*(0.022)*
48	-6,024.10	0.734	*(0.167)*	-0.086	*(0.207)*	0.628	*(0.047)*	-0.110	*(0.025)*	0.192	*(0.041)*	1.239	*(0.026)*
49	-7,300.79	1.360	*(0.222)*	0.265	*(0.247)*	0.668	*(0.061)*	-0.064	*(0.019)*	0.148	*(0.039)*	1.346	*(0.038)*
50	-5,757.63	0.564	*(0.191)*	0.766	*(0.077)*	0.449	*(0.078)*	-0.064	*(0.012)*	0.147	*(0.023)*	1.291	*(0.028)*
51	-5,702.90	0.756	*(0.220)*	0.223	*(0.192)*	0.640	*(0.050)*	-0.104	*(0.024)*	0.191	*(0.043)*	1.292	*(0.039)*
52	-6,277.59	0.986	*(0.195)*	0.554	*(0.101)*	0.563	*(0.051)*	-0.088	*(0.014)*	0.148	*(0.023)*	1.443	*(0.040)*

Appendix C: Table C.22. Estimated coefficients for the FIEGARCH model - V_{SM}

comp	*LogL*	ω	s.e.	β	s.e.	*d*	s.e.	γ_1	s.e.	γ_2	s.e.	GED	s.e.
1	-4,169.44	-0.564	*(0.192)*	0.675	*(0.121)*	0.555	*(0.075)*	-0.025	*(0.011)*	0.113	*(0.025)*	1.402	*(0.036)*
2	-2,161.64	-1.608	*(0.156)*	0.745	*(0.096)*	0.469	*(0.088)*	-0.030	*(0.010)*	0.114	*(0.023)*	1.373	*(0.044)*
3	-4,258.10	-0.845	*(0.274)*	0.567	*(0.148)*	0.644	*(0.066)*	-0.021	*(0.010)*	0.116	*(0.024)*	1.373	*(0.035)*
4	-4,595.82	-0.071	*(0.062)*	0.215	*(0.196)*	0.559	*(0.077)*	-0.084	*(0.020)*	0.253	*(0.036)*	1.268	*(0.037)*
5	-2,493.10	-1.729	*(0.265)*	0.648	*(0.127)*	0.626	*(0.066)*	-0.021	*(0.009)*	0.123	*(0.027)*	1.302	*(0.034)*
6	-716.23	-2.450	*(0.265)*	0.197	*(0.138)*	0.622	*(0.051)*	-0.013	*(0.016)*	0.247	*(0.045)*	1.251	*(0.021)*
7	-2,598.77	-1.610	*(0.247)*	0.617	*(0.152)*	0.650	*(0.068)*	-0.018	*(0.010)*	0.097	*(0.027)*	1.432	*(0.035)*
8	-3,647.25	-0.884	*(0.256)*	0.089	*(0.227)*	0.691	*(0.057)*	-0.054	*(0.017)*	0.198	*(0.040)*	1.298	*(0.029)*
9	-1,986.39	-1.463	*(0.219)*	0.079	*(0.265)*	0.714	*(0.050)*	-0.021	*(0.015)*	0.187	*(0.046)*	1.248	*(0.028)*
10	-5,074.79	0.145	*(0.216)*	0.123	*(0.240)*	0.666	*(0.061)*	-0.048	*(0.017)*	0.164	*(0.034)*	1.326	*(0.031)*
11	-4,690.97	-0.045	*(0.096)*	0.837	*(0.065)*	0.372	*(0.079)*	-0.051	*(0.012)*	0.072	*(0.017)*	1.306	*(0.037)*
12	-5,009.50	-0.009	*(0.179)*	0.555	*(0.118)*	0.576	*(0.051)*	-0.104	*(0.020)*	0.110	*(0.022)*	1.345	*(0.035)*
13	-4,993.68	0.196	*(0.183)*	-0.462	*(0.188)*	0.671	*(0.049)*	-0.032	*(0.022)*	0.217	*(0.034)*	1.104	*(0.017)*
14	-3,918.63	-0.671	*(0.234)*	0.714	*(0.100)*	0.588	*(0.066)*	-0.041	*(0.012)*	0.092	*(0.022)*	1.179	*(0.033)*
15	-3,365.14	-0.806	*(0.247)*	0.007	*(0.199)*	0.647	*(0.053)*	-0.056	*(0.024)*	0.263	*(0.049)*	1.233	*(0.028)*
16	-4,600.03	-0.193	*(0.173)*	0.676	*(0.106)*	0.552	*(0.067)*	-0.035	*(0.012)*	0.109	*(0.024)*	1.479	*(0.048)*
17	-2,981.18	-1.065	*(0.089)*	0.752	*(0.247)*	0.765	*(0.074)*	-0.020	*(0.018)*	0.133	*(0.030)*	1.246	*(0.037)*
18	-1,548.38	-1.811	*(0.148)*	0.712	*(0.101)*	0.434	*(0.081)*	-0.057	*(0.016)*	0.139	*(0.028)*	1.131	*(0.029)*
19	-4,391.49	-0.340	*(0.235)*	0.272	*(0.211)*	0.662	*(0.057)*	-0.051	*(0.018)*	0.162	*(0.039)*	1.428	*(0.046)*
20	-4,148.18	-0.396	*(0.131)*	0.801	*(0.069)*	0.357	*(0.092)*	-0.024	*(0.011)*	0.146	*(0.020)*	1.257	*(0.032)*
21	-3,611.71	-0.901	*(0.165)*	0.389	*(0.179)*	0.581	*(0.053)*	-0.096	*(0.023)*	0.135	*(0.032)*	1.251	*(0.033)*
22	-4,207.82	-0.362	*(0.253)*	-0.201	*(0.155)*	0.647	*(0.054)*	-0.087	*(0.021)*	0.300	*(0.031)*	1.299	*(0.034)*
23	-4,166.70	-0.367	*(0.222)*	0.688	*(0.113)*	0.574	*(0.063)*	-0.032	*(0.011)*	0.101	*(0.022)*	1.154	*(0.020)*
24	-4,348.79	-0.142	*(0.223)*	-0.644	*(0.083)*	0.735	*(0.041)*	-0.076	*(0.024)*	0.253	*(0.040)*	1.334	*(0.026)*
25	-3,498.03	-0.950	*(0.242)*	0.261	*(0.219)*	0.681	*(0.055)*	-0.045	*(0.018)*	0.163	*(0.039)*	1.260	*(0.036)*
26	-4,206.00	-0.256	*(0.241)*	0.472	*(0.162)*	0.625	*(0.057)*	-0.054	*(0.014)*	0.151	*(0.033)*	1.440	*(0.044)*
27	-2,304.93	-1.520	*(0.265)*	0.311	*(0.171)*	0.621	*(0.055)*	-0.029	*(0.015)*	0.221	*(0.041)*	1.188	*(0.028)*
28	-1,641.12	-1.745	*(0.218)*	0.459	*(0.179)*	0.550	*(0.077)*	-0.020	*(0.016)*	0.202	*(0.039)*	1.123	*(0.019)*
29	-2,672.94	-1.612	*(0.388)*	0.271	*(0.112)*	0.616	*(0.047)*	-0.051	*(0.018)*	0.335	*(0.037)*	1.147	*(0.027)*
30	-3,626.98	-0.780	*(0.241)*	0.652	*(0.114)*	0.569	*(0.070)*	-0.042	*(0.012)*	0.141	*(0.027)*	1.261	*(0.036)*
31	-1,402.63	-2.051	*(0.235)*	0.138	*(0.158)*	0.585	*(0.051)*	-0.062	*(0.020)*	0.291	*(0.043)*	1.233	*(0.035)*
32	-4,210.48	-0.358	*(0.214)*	0.556	*(0.123)*	0.548	*(0.059)*	-0.073	*(0.019)*	0.158	*(0.031)*	1.129	*(0.023)*
33	-6,162.74	0.591	*(0.262)*	0.787	*(0.080)*	0.597	*(0.071)*	-0.019	*(0.007)*	0.080	*(0.017)*	1.449	*(0.035)*
34	-4,145.18	-0.377	*(0.203)*	0.246	*(0.194)*	0.614	*(0.055)*	-0.023	*(0.015)*	0.197	*(0.037)*	1.348	*(0.030)*
35	-4,245.49	-0.384	*(0.142)*	0.852	*(0.052)*	0.393	*(0.079)*	-0.060	*(0.011)*	0.095	*(0.016)*	1.323	*(0.041)*
36	-3,845.20	-0.637	*(0.170)*	0.763	*(0.064)*	0.491	*(0.050)*	-0.065	*(0.013)*	0.101	*(0.018)*	1.341	*(0.035)*
37	-4,931.51	-0.061	*(0.144)*	0.748	*(0.059)*	0.464	*(0.051)*	-0.090	*(0.014)*	0.109	*(0.018)*	1.465	*(0.047)*
38	-3,460.64	-0.827	*(0.226)*	0.206	*(0.205)*	0.611	*(0.057)*	-0.081	*(0.021)*	0.202	*(0.040)*	1.162	*(0.020)*
39	-4,698.37	-0.007	*(0.169)*	0.404	*(0.246)*	0.630	*(0.064)*	-0.037	*(0.017)*	0.122	*(0.042)*	1.242	*(0.027)*
40	-3,851.59	-0.594	*(0.201)*	0.624	*(0.120)*	0.568	*(0.059)*	-0.060	*(0.017)*	0.123	*(0.027)*	1.227	*(0.032)*
41	-3,747.93	-0.549	*(0.211)*	0.238	*(0.253)*	0.671	*(0.060)*	-0.043	*(0.019)*	0.153	*(0.043)*	1.279	*(0.038)*
42	-4,249.50	-0.298	*(0.173)*	0.560	*(0.168)*	0.550	*(0.075)*	-0.043	*(0.016)*	0.123	*(0.035)*	1.110	*(0.027)*
43	-4,187.64	-0.398	*(0.202)*	0.744	*(0.108)*	0.558	*(0.081)*	-0.024	*(0.009)*	0.087	*(0.021)*	1.231	*(0.028)*
44	-2,442.96	-1.249	*(0.234)*	0.724	*(0.095)*	0.515	*(0.071)*	-0.017	*(0.010)*	0.135	*(0.026)*	1.104	*(0.028)*
45	-2,942.37	-0.974	*(0.225)*	0.481	*(0.202)*	0.649	*(0.066)*	-0.027	*(0.014)*	0.116	*(0.035)*	1.215	*(0.030)*
46	-4,459.65	-0.140	*(0.202)*	0.190	*(0.187)*	0.578	*(0.055)*	-0.056	*(0.022)*	0.233	*(0.041)*	1.187	*(0.036)*
47	-3,501.49	-0.697	*(0.182)*	0.391	*(0.177)*	0.578	*(0.068)*	-0.064	*(0.017)*	0.152	*(0.033)*	1.030	*(0.020)*
48	-4,712.49	-0.072	*(0.169)*	-0.185	*(0.225)*	0.656	*(0.050)*	-0.080	*(0.023)*	0.184	*(0.040)*	1.314	*(0.028)*
49	-6,066.54	0.763	*(0.150)*	0.377	*(0.245)*	0.562	*(0.081)*	-0.051	*(0.019)*	0.148	*(0.040)*	1.350	*(0.038)*
50	-3,007.63	-1.006	*(0.156)*	0.821	*(0.068)*	0.373	*(0.096)*	-0.049	*(0.011)*	0.148	*(0.022)*	1.309	*(0.030)*
51	-3,499.21	-0.569	*(0.221)*	0.211	*(0.203)*	0.637	*(0.053)*	-0.076	*(0.021)*	0.197	*(0.045)*	1.291	*(0.038)*
52	-3,847.47	-0.337	*(0.216)*	0.298	*(0.189)*	0.628	*(0.057)*	-0.045	*(0.016)*	0.177	*(0.035)*	1.472	*(0.038)*

Appendix C: Table C.23. Estimated coefficients for the FIEGARCH model - V_{KMV}

comp	*LogL*	ω	s.e.	β	s.e.	*d*	s.e.	γ_1	s.e.	γ_2	s.e.	GED	s.e.
1	-4,207.96	-0.573	*(0.199)*	0.663	*(0.124)*	0.570	*(0.074)*	-0.026	*(0.011)*	0.112	*(0.025)*	1.405	*(0.036)*
2	-2,175.62	-1.590	*(0.157)*	0.742	*(0.095)*	0.473	*(0.085)*	-0.032	*(0.011)*	0.114	*(0.023)*	1.373	*(0.044)*
3	-4,213.77	-0.859	*(0.271)*	0.568	*(0.148)*	0.641	*(0.066)*	-0.021	*(0.010)*	0.117	*(0.024)*	1.373	*(0.036)*
4	-4,540.84	-0.108	*(0.065)*	0.205	*(0.191)*	0.566	*(0.074)*	-0.087	*(0.021)*	0.258	*(0.036)*	1.266	*(0.037)*
5	-2,484.10	-1.699	*(0.260)*	0.655	*(0.124)*	0.620	*(0.066)*	-0.023	*(0.009)*	0.123	*(0.027)*	1.304	*(0.034)*
6	-701.66	-2.442	*(0.255)*	0.202	*(0.139)*	0.613	*(0.052)*	-0.015	*(0.016)*	0.247	*(0.045)*	1.252	*(0.021)*
7	-2,570.44	-1.604	*(0.243)*	0.617	*(0.151)*	0.645	*(0.068)*	-0.020	*(0.010)*	0.097	*(0.027)*	1.432	*(0.035)*
8	-3,595.70	-0.868	*(0.250)*	0.091	*(0.227)*	0.689	*(0.056)*	-0.057	*(0.017)*	0.197	*(0.040)*	1.299	*(0.029)*
9	-1,929.25	-1.509	*(0.216)*	0.078	*(0.265)*	0.710	*(0.050)*	-0.023	*(0.016)*	0.187	*(0.046)*	1.248	*(0.028)*
10	-5,064.97	0.039	*(0.235)*	0.138	*(0.233)*	0.677	*(0.060)*	-0.046	*(0.016)*	0.161	*(0.033)*	1.327	*(0.031)*
11	-4,641.23	-0.076	*(0.099)*	0.830	*(0.067)*	0.382	*(0.077)*	-0.051	*(0.012)*	0.072	*(0.018)*	1.305	*(0.037)*
12	-5,039.51	-0.021	*(0.188)*	0.546	*(0.119)*	0.586	*(0.050)*	-0.104	*(0.021)*	0.110	*(0.022)*	1.343	*(0.034)*
13	-5,036.32	0.266	*(0.184)*	-0.428	*(0.201)*	0.670	*(0.050)*	-0.036	*(0.022)*	0.213	*(0.034)*	1.100	*(0.017)*
14	-3,906.23	-0.720	*(0.249)*	0.702	*(0.103)*	0.601	*(0.065)*	-0.041	*(0.012)*	0.094	*(0.022)*	1.178	*(0.033)*
15	-3,310.92	-0.875	*(0.248)*	0.008	*(0.199)*	0.647	*(0.053)*	-0.056	*(0.024)*	0.262	*(0.049)*	1.233	*(0.028)*
16	-4,583.81	-0.211	*(0.175)*	0.675	*(0.107)*	0.558	*(0.066)*	-0.035	*(0.011)*	0.108	*(0.024)*	1.483	*(0.048)*
17	-2,915.10	-1.103	*(0.089)*	0.752	*(0.251)*	0.765	*(0.075)*	-0.020	*(0.009)*	0.133	*(0.030)*	1.246	*(0.037)*
18	-1,447.94	-1.866	*(0.144)*	0.720	*(0.100)*	0.424	*(0.082)*	-0.056	*(0.016)*	0.139	*(0.027)*	1.133	*(0.029)*
19	-4,428.72	-0.363	*(0.240)*	0.270	*(0.210)*	0.665	*(0.057)*	-0.052	*(0.018)*	0.162	*(0.038)*	1.428	*(0.045)*
20	-4,172.52	-0.391	*(0.138)*	0.788	*(0.069)*	0.375	*(0.087)*	-0.023	*(0.011)*	0.148	*(0.020)*	1.255	*(0.032)*
21	-3,566.83	-0.797	*(0.134)*	0.433	*(0.164)*	0.546	*(0.052)*	-0.103	*(0.024)*	0.124	*(0.030)*	1.258	*(0.034)*
22	-4,217.57	-0.392	*(0.253)*	-0.220	*(0.154)*	0.653	*(0.054)*	-0.082	*(0.021)*	0.296	*(0.031)*	1.305	*(0.034)*
23	-4,163.12	-0.376	*(0.223)*	0.684	*(0.115)*	0.582	*(0.063)*	-0.032	*(0.011)*	0.098	*(0.022)*	1.163	*(0.020)*
24	-4,340.25	-0.152	*(0.221)*	-0.645	*(0.084)*	0.742	*(0.041)*	-0.077	*(0.023)*	0.246	*(0.040)*	1.337	*(0.025)*
25	-3,449.92	-1.062	*(0.250)*	0.261	*(0.220)*	0.686	*(0.055)*	-0.041	*(0.017)*	0.163	*(0.039)*	1.256	*(0.036)*
26	-4,234.27	-0.247	*(0.242)*	0.490	*(0.158)*	0.625	*(0.057)*	-0.053	*(0.014)*	0.147	*(0.032)*	1.447	*(0.044)*
27	-2,294.78	-1.506	*(0.262)*	0.326	*(0.168)*	0.616	*(0.055)*	-0.030	*(0.014)*	0.220	*(0.040)*	1.190	*(0.028)*
28	-1,643.39	-1.738	*(0.215)*	0.460	*(0.179)*	0.545	*(0.077)*	-0.022	*(0.016)*	0.204	*(0.040)*	1.127	*(0.019)*
29	-2,651.49	-1.616	*(0.388)*	0.268	*(0.112)*	0.615	*(0.047)*	-0.051	*(0.018)*	0.337	*(0.038)*	1.147	*(0.027)*
30	-3,594.26	-0.846	*(0.253)*	0.637	*(0.117)*	0.583	*(0.068)*	-0.043	*(0.012)*	0.142	*(0.028)*	1.257	*(0.036)*
31	-1,419.41	-2.062	*(0.233)*	0.152	*(0.157)*	0.580	*(0.051)*	-0.065	*(0.020)*	0.291	*(0.044)*	1.231	*(0.035)*
32	-4,164.11	-0.371	*(0.204)*	0.573	*(0.119)*	0.534	*(0.060)*	-0.073	*(0.019)*	0.157	*(0.031)*	1.130	*(0.023)*
33	-6,138.57	0.576	*(0.261)*	0.789	*(0.079)*	0.595	*(0.071)*	-0.020	*(0.007)*	0.079	*(0.017)*	1.453	*(0.035)*
34	-4,148.36	-0.371	*(0.197)*	0.234	*(0.198)*	0.612	*(0.056)*	-0.025	*(0.016)*	0.195	*(0.037)*	1.349	*(0.030)*
35	-4,190.06	-0.436	*(0.153)*	0.832	*(0.057)*	0.421	*(0.075)*	-0.059	*(0.011)*	0.097	*(0.017)*	1.321	*(0.041)*
36	-3,798.70	-0.658	*(0.169)*	0.767	*(0.063)*	0.486	*(0.051)*	-0.065	*(0.013)*	0.102	*(0.018)*	1.340	*(0.035)*
37	-4,926.96	-0.070	*(0.143)*	0.747	*(0.060)*	0.463	*(0.051)*	-0.091	*(0.014)*	0.110	*(0.018)*	1.461	*(0.047)*
38	-3,412.37	-0.887	*(0.234)*	0.186	*(0.206)*	0.619	*(0.056)*	-0.081	*(0.021)*	0.203	*(0.040)*	1.162	*(0.020)*
39	-4,691.66	-0.016	*(0.171)*	0.379	*(0.251)*	0.643	*(0.062)*	-0.038	*(0.017)*	0.123	*(0.042)*	1.257	*(0.027)*
40	-3,832.35	-0.614	*(0.201)*	0.627	*(0.119)*	0.566	*(0.060)*	-0.060	*(0.016)*	0.124	*(0.027)*	1.227	*(0.032)*
41	-3,728.86	-0.596	*(0.208)*	0.251	*(0.252)*	0.665	*(0.059)*	-0.044	*(0.018)*	0.151	*(0.042)*	1.282	*(0.038)*
42	-4,321.65	-0.260	*(0.177)*	0.548	*(0.172)*	0.562	*(0.073)*	-0.047	*(0.016)*	0.120	*(0.035)*	1.110	*(0.027)*
43	-4,179.69	-0.448	*(0.214)*	0.736	*(0.113)*	0.580	*(0.081)*	-0.021	*(0.009)*	0.084	*(0.021)*	1.236	*(0.029)*
44	-2,382.95	-1.320	*(0.236)*	0.713	*(0.099)*	0.523	*(0.070)*	-0.016	*(0.010)*	0.134	*(0.026)*	1.103	*(0.027)*
45	-2,846.06	-1.067	*(0.222)*	0.473	*(0.207)*	0.648	*(0.067)*	-0.025	*(0.013)*	0.116	*(0.035)*	1.214	*(0.029)*
46	-4,566.48	-0.092	*(0.154)*	0.320	*(0.169)*	0.505	*(0.060)*	-0.070	*(0.022)*	0.220	*(0.039)*	1.196	*(0.036)*
47	-3,442.35	-0.772	*(0.197)*	0.334	*(0.183)*	0.608	*(0.064)*	-0.068	*(0.018)*	0.153	*(0.034)*	1.030	*(0.020)*
48	-4,713.36	-0.088	*(0.173)*	-0.212	*(0.224)*	0.669	*(0.050)*	-0.081	*(0.023)*	0.181	*(0.039)*	1.320	*(0.028)*
49	-6,051.58	0.745	*(0.150)*	0.365	*(0.249)*	0.565	*(0.081)*	-0.050	*(0.019)*	0.148	*(0.041)*	1.351	*(0.038)*
50	-2,958.44	-1.042	*(0.141)*	0.842	*(0.063)*	0.334	*(0.101)*	-0.050	*(0.011)*	0.148	*(0.022)*	1.306	*(0.030)*
51	-3,452.55	-0.615	*(0.221)*	0.212	*(0.203)*	0.636	*(0.053)*	-0.077	*(0.021)*	0.197	*(0.045)*	1.289	*(0.038)*
52	-3,859.21	-0.322	*(0.225)*	0.285	*(0.193)*	0.640	*(0.056)*	-0.045	*(0.016)*	0.175	*(0.035)*	1.475	*(0.037)*

Appendix C: Table C.24. Estimated coefficients for the FIEGARCH model - V_{Proxy}

comp	*LogL*	ω	s.e.	β	s.e.	*d*	s.e.	γ_1	s.e.	γ_2	s.e.	GED	s.e.
1	-4,128.29	-0.615	*(0.198)*	0.684	*(0.117)*	0.556	*(0.076)*	-0.026	*(0.010)*	0.112	*(0.024)*	1.408	*(0.037)*
2	-2,080.51	-1.634	*(0.153)*	0.811	*(0.074)*	0.429	*(0.093)*	-0.031	*(0.010)*	0.106	*(0.020)*	1.362	*(0.044)*
3	-4,237.51	-0.853	*(0.272)*	0.622	*(0.130)*	0.635	*(0.067)*	-0.018	*(0.009)*	0.107	*(0.022)*	1.377	*(0.036)*
4	-4,569.81	-0.088	*(0.064)*	0.253	*(0.194)*	0.556	*(0.080)*	-0.084	*(0.019)*	0.246	*(0.035)*	1.262	*(0.037)*
5	-2,377.76	-1.765	*(0.274)*	0.714	*(0.098)*	0.596	*(0.067)*	-0.021	*(0.008)*	0.115	*(0.024)*	1.296	*(0.033)*
6	-533.12	-2.581	*(0.320)*	0.188	*(0.145)*	0.665	*(0.050)*	-0.003	*(0.015)*	0.241	*(0.045)*	1.218	*(0.021)*
7	-2,541.30	-1.513	*(0.212)*	0.840	*(0.070)*	0.543	*(0.092)*	-0.010	*(0.006)*	0.065	*(0.015)*	1.432	*(0.035)*
8	-3,566.58	-0.906	*(0.265)*	0.131	*(0.220)*	0.690	*(0.057)*	-0.051	*(0.017)*	0.198	*(0.040)*	1.297	*(0.029)*
9	-1,902.04	-1.576	*(0.228)*	0.293	*(0.221)*	0.676	*(0.054)*	-0.018	*(0.014)*	0.171	*(0.042)*	1.234	*(0.029)*
10	-5,064.36	0.048	*(0.234)*	0.124	*(0.236)*	0.677	*(0.060)*	-0.048	*(0.017)*	0.162	*(0.034)*	1.322	*(0.031)*
11	-4,707.19	-0.038	*(0.093)*	0.850	*(0.061)*	0.355	*(0.084)*	-0.051	*(0.012)*	0.070	*(0.017)*	1.305	*(0.037)*
12	-5,018.90	-0.025	*(0.186)*	0.552	*(0.116)*	0.582	*(0.050)*	-0.104	*(0.020)*	0.110	*(0.022)*	1.352	*(0.034)*
13	-5,051.39	0.257	*(0.186)*	-0.445	*(0.193)*	0.672	*(0.049)*	-0.032	*(0.022)*	0.217	*(0.034)*	1.102	*(0.017)*
14	-3,938.74	-0.707	*(0.248)*	0.712	*(0.099)*	0.594	*(0.066)*	-0.041	*(0.011)*	0.095	*(0.022)*	1.178	*(0.033)*
15	-3,372.02	-0.830	*(0.248)*	0.056	*(0.196)*	0.639	*(0.053)*	-0.060	*(0.023)*	0.258	*(0.048)*	1.237	*(0.029)*
16	-4,627.77	-0.191	*(0.175)*	0.678	*(0.105)*	0.554	*(0.067)*	-0.036	*(0.011)*	0.109	*(0.024)*	1.484	*(0.049)*
17	-2,960.55	-1.072	*(0.084)*	0.774	*(0.174)*	0.757	*(0.062)*	-0.072	*(0.018)*	0.130	*(0.029)*	1.253	*(0.037)*
18	-1,539.19	-1.823	*(0.146)*	0.770	*(0.086)*	0.413	*(0.086)*	-0.056	*(0.014)*	0.125	*(0.024)*	1.123	*(0.030)*
19	-4,492.56	-0.296	*(0.235)*	0.270	*(0.208)*	0.660	*(0.056)*	-0.056	*(0.019)*	0.163	*(0.039)*	1.418	*(0.045)*
20	-4,196.64	-0.376	*(0.136)*	0.795	*(0.068)*	0.368	*(0.088)*	-0.026	*(0.011)*	0.146	*(0.020)*	1.260	*(0.032)*
21	-3,598.77	-0.777	*(0.135)*	0.432	*(0.161)*	0.553	*(0.050)*	-0.107	*(0.024)*	0.122	*(0.029)*	1.255	*(0.034)*
22	-4,244.73	-0.369	*(0.259)*	-0.216	*(0.155)*	0.659	*(0.054)*	-0.085	*(0.021)*	0.289	*(0.031)*	1.295	*(0.034)*
23	-4,150.20	-0.373	*(0.223)*	0.689	*(0.113)*	0.580	*(0.063)*	-0.032	*(0.011)*	0.097	*(0.022)*	1.160	*(0.020)*
24	-4,335.29	-0.156	*(0.222)*	-0.593	*(0.099)*	0.732	*(0.041)*	-0.083	*(0.024)*	0.249	*(0.040)*	1.330	*(0.026)*
25	-3,510.79	-1.015	*(0.242)*	0.285	*(0.215)*	0.682	*(0.056)*	-0.042	*(0.017)*	0.158	*(0.039)*	1.262	*(0.036)*
26	-4,165.94	-0.278	*(0.245)*	0.507	*(0.156)*	0.627	*(0.058)*	-0.050	*(0.013)*	0.143	*(0.031)*	1.460	*(0.044)*
27	-2,199.58	-1.635	*(0.274)*	0.359	*(0.178)*	0.643	*(0.057)*	-0.019	*(0.012)*	0.196	*(0.039)*	1.212	*(0.028)*
28	-1,607.34	-1.728	*(0.217)*	0.615	*(0.144)*	0.530	*(0.086)*	-0.014	*(0.012)*	0.164	*(0.034)*	1.097	*(0.019)*
29	-2,702.91	-1.616	*(0.395)*	0.309	*(0.120)*	0.621	*(0.049)*	-0.049	*(0.018)*	0.316	*(0.037)*	1.134	*(0.027)*
30	-3,686.55	-0.753	*(0.238)*	0.648	*(0.114)*	0.566	*(0.068)*	-0.044	*(0.012)*	0.143	*(0.028)*	1.262	*(0.036)*
31	-1,237.28	-2.264	*(0.249)*	0.139	*(0.168)*	0.610	*(0.053)*	-0.046	*(0.019)*	0.272	*(0.042)*	1.236	*(0.035)*
32	-4,242.95	-0.342	*(0.214)*	0.591	*(0.111)*	0.538	*(0.060)*	-0.071	*(0.018)*	0.154	*(0.029)*	1.131	*(0.023)*
33	-6,045.44	0.531	*(0.255)*	0.791	*(0.079)*	0.586	*(0.073)*	-0.018	*(0.007)*	0.080	*(0.017)*	1.449	*(0.035)*
34	-4,139.72	-0.362	*(0.200)*	0.227	*(0.197)*	0.613	*(0.055)*	-0.027	*(0.016)*	0.200	*(0.037)*	1.351	*(0.030)*
35	-4,264.69	-0.379	*(0.150)*	0.844	*(0.053)*	0.407	*(0.075)*	-0.060	*(0.011)*	0.096	*(0.017)*	1.317	*(0.041)*
36	-3,844.47	-0.613	*(0.162)*	0.800	*(0.057)*	0.461	*(0.053)*	-0.063	*(0.012)*	0.098	*(0.017)*	1.325	*(0.035)*
37	-4,986.48	-0.062	*(0.147)*	0.754	*(0.059)*	0.466	*(0.052)*	-0.089	*(0.014)*	0.109	*(0.018)*	1.463	*(0.047)*
38	-3,507.97	-0.838	*(0.238)*	0.208	*(0.206)*	0.619	*(0.057)*	-0.077	*(0.019)*	0.202	*(0.041)*	1.160	*(0.021)*
39	-4,647.50	-0.043	*(0.173)*	0.400	*(0.242)*	0.638	*(0.062)*	-0.037	*(0.017)*	0.122	*(0.041)*	1.259	*(0.027)*
40	-3,838.87	-0.614	*(0.199)*	0.648	*(0.113)*	0.559	*(0.060)*	-0.058	*(0.016)*	0.121	*(0.026)*	1.230	*(0.032)*
41	-3,752.43	-0.597	*(0.215)*	0.359	*(0.237)*	0.663	*(0.062)*	-0.038	*(0.017)*	0.134	*(0.040)*	1.272	*(0.038)*
42	-4,327.76	-0.250	*(0.176)*	0.592	*(0.159)*	0.552	*(0.075)*	-0.044	*(0.015)*	0.115	*(0.033)*	1.095	*(0.027)*
43	-4,213.33	-0.403	*(0.208)*	0.760	*(0.104)*	0.559	*(0.085)*	-0.023	*(0.009)*	0.084	*(0.020)*	1.239	*(0.029)*
44	-2,545.05	-1.230	*(0.223)*	0.725	*(0.095)*	0.492	*(0.075)*	-0.023	*(0.011)*	0.146	*(0.027)*	1.105	*(0.028)*
45	-3,021.55	-0.966	*(0.226)*	0.533	*(0.186)*	0.634	*(0.069)*	-0.026	*(0.013)*	0.113	*(0.033)*	1.224	*(0.030)*
46	-4,571.09	-0.080	*(0.192)*	0.282	*(0.174)*	0.553	*(0.057)*	-0.059	*(0.021)*	0.223	*(0.040)*	1.167	*(0.035)*
47	-3,561.17	-0.649	*(0.160)*	0.505	*(0.166)*	0.533	*(0.078)*	-0.058	*(0.016)*	0.138	*(0.032)*	1.037	*(0.019)*
48	-4,740.40	-0.063	*(0.175)*	-0.200	*(0.223)*	0.667	*(0.050)*	-0.081	*(0.023)*	0.183	*(0.039)*	1.316	*(0.028)*
49	-6,033.97	0.734	*(0.150)*	0.395	*(0.242)*	0.560	*(0.082)*	-0.049	*(0.018)*	0.144	*(0.040)*	1.354	*(0.038)*
50	-3,127.45	-0.934	*(0.152)*	0.836	*(0.064)*	0.358	*(0.098)*	-0.050	*(0.011)*	0.146	*(0.022)*	1.318	*(0.031)*
51	-3,559.51	-0.565	*(0.222)*	0.226	*(0.201)*	0.633	*(0.053)*	-0.080	*(0.021)*	0.197	*(0.044)*	1.297	*(0.039)*
52	-3,873.38	-0.327	*(0.220)*	0.347	*(0.181)*	0.626	*(0.057)*	-0.046	*(0.015)*	0.169	*(0.034)*	1.459	*(0.038)*

Appendix C: Table C.25. Estimated coefficients for the FIAPARCH model - Equity

comp	*LogL*	ω	s.e.	*d*	s.e.	*α*	s.e.	*β*	s.e.	*γ*	s.e.	*δ*	s.e.	GED	s.e.
1	-7,088.34	0.234	*(0.051)*	0.436	*(0.077)*	0.214	*(0.054)*	0.583	*(0.072)*	0.325	*(0.094)*	1.429	*(0.188)*	1.294	*(0.030)*
2	-6,501.80	0.123	*(0.045)*	0.375	*(0.067)*	0.257	*(0.055)*	0.568	*(0.070)*	0.382	*(0.087)*	1.722	*(0.165)*	1.369	*(0.044)*
3	-6,839.26	0.215	*(0.052)*	0.353	*(0.067)*	0.188	*(0.053)*	0.512	*(0.070)*	0.376	*(0.104)*	1.604	*(0.163)*	1.404	*(0.039)*
4	-7,143.20	0.175	*(0.048)*	0.386	*(0.066)*	0.293	*(0.063)*	0.560	*(0.081)*	0.492	*(0.080)*	1.600	*(0.097)*	1.249	*(0.035)*
5	-6,699.14	0.183	*(0.046)*	0.431	*(0.071)*	0.158	*(0.052)*	0.546	*(0.080)*	0.261	*(0.074)*	1.677	*(0.174)*	1.300	*(0.036)*
6	-7,588.63	0.255	*(0.082)*	0.370	*(0.069)*	0.227	*(0.067)*	0.508	*(0.090)*	0.281	*(0.053)*	1.821	*(0.136)*	1.224	*(0.023)*
7	-7,362.08	0.223	*(0.053)*	0.347	*(0.065)*	0.223	*(0.049)*	0.533	*(0.070)*	0.480	*(0.113)*	1.604	*(0.146)*	1.460	*(0.037)*
8	-7,132.83	0.118	*(0.046)*	0.319	*(0.072)*	0.371	*(0.052)*	0.622	*(0.058)*	0.596	*(0.153)*	1.690	*(0.149)*	1.310	*(0.034)*
9	-6,699.33	0.267	*(0.064)*	0.331	*(0.069)*	0.321	*(0.064)*	0.578	*(0.074)*	0.531	*(0.147)*	1.192	*(0.226)*	1.238	*(0.030)*
10	-6,573.91	0.159	*(0.045)*	0.407	*(0.086)*	0.310	*(0.059)*	0.626	*(0.081)*	0.212	*(0.082)*	1.668	*(0.226)*	1.270	*(0.034)*
11	-6,438.75	0.224	*(0.062)*	0.213	*(0.082)*	0.304	*(0.066)*	0.482	*(0.089)*	1.000	*(0.507)*	1.396	*(0.233)*	1.300	*(0.036)*
12	-6,207.12	0.078	*(0.088)*	0.168	*(0.070)*	0.093	*(0.108)*	0.231	*(0.140)*	1.000	*(0.466)*	1.686	*(0.206)*	1.357	*(0.036)*
13	-6,565.02	0.134	*(0.029)*	0.473	*(0.085)*	0.362	*(0.052)*	0.730	*(0.053)*	0.465	*(0.129)*	0.710	*(0.176)*	1.061	*(0.015)*
14	-6,275.68	0.105	*(0.035)*	0.290	*(0.073)*	0.315	*(0.048)*	0.570	*(0.064)*	1.000	*(0.389)*	1.417	*(0.232)*	1.179	*(0.035)*
15	-5,749.93	0.080	*(0.021)*	0.529	*(0.103)*	0.357	*(0.060)*	0.757	*(0.057)*	0.397	*(0.100)*	1.336	*(0.191)*	1.236	*(0.029)*
16	-5,535.35	0.124	*(0.035)*	0.320	*(0.072)*	0.346	*(0.061)*	0.585	*(0.079)*	0.402	*(0.118)*	1.562	*(0.204)*	1.462	*(0.047)*
17	-6,330.60	0.125	*(0.031)*	0.351	*(0.080)*	0.349	*(0.052)*	0.633	*(0.059)*	0.609	*(0.175)*	1.512	*(0.176)*	1.237	*(0.036)*
18	-5,967.13	0.124	*(0.049)*	0.271	*(0.099)*	0.308	*(0.054)*	0.539	*(0.085)*	1.000	*(0.448)*	1.428	*(0.208)*	1.077	*(0.029)*
19	-5,355.68	0.103	*(0.025)*	0.424	*(0.079)*	0.350	*(0.055)*	0.676	*(0.060)*	0.406	*(0.114)*	1.250	*(0.247)*	1.378	*(0.045)*
20	-5,613.89	0.104	*(0.030)*	0.427	*(0.089)*	0.332	*(0.055)*	0.643	*(0.075)*	0.233	*(0.055)*	1.649	*(0.187)*	1.276	*(0.034)*
21	-5,745.59	0.295	*(0.087)*	0.180	*(0.090)*	0.168	*(0.117)*	0.299	*(0.149)*	1.000	*(0.627)*	1.400	*(0.253)*	1.256	*(0.036)*
22	-5,758.25	0.072	*(0.018)*	0.398	*(0.085)*	0.517	*(0.054)*	0.770	*(0.036)*	0.506	*(0.108)*	1.512	*(0.154)*	1.263	*(0.034)*
23	-6,165.90	0.158	*(0.038)*	0.522	*(0.092)*	0.241	*(0.054)*	0.678	*(0.068)*	0.425	*(0.105)*	1.056	*(0.169)*	1.055	*(0.019)*
24	-6,070.70	0.095	*(0.025)*	0.546	*(0.118)*	0.323	*(0.066)*	0.763	*(0.058)*	0.503	*(0.127)*	1.183	*(0.166)*	1.243	*(0.023)*
25	-5,308.16	0.106	*(0.026)*	0.458	*(0.087)*	0.324	*(0.053)*	0.685	*(0.062)*	0.401	*(0.113)*	1.152	*(0.229)*	1.262	*(0.035)*
26	-6,002.43	0.157	*(0.039)*	0.311	*(0.063)*	0.167	*(0.059)*	0.424	*(0.086)*	0.612	*(0.146)*	1.519	*(0.141)*	1.399	*(0.042)*
27	-6,201.55	0.214	*(0.050)*	0.438	*(0.078)*	0.199	*(0.061)*	0.540	*(0.094)*	0.394	*(0.093)*	1.215	*(0.165)*	1.188	*(0.028)*
28	-5,929.68	0.186	*(0.042)*	0.465	*(0.080)*	0.214	*(0.057)*	0.590	*(0.083)*	0.316	*(0.110)*	0.874	*(0.200)*	1.068	*(0.019)*
29	-5,704.48	0.083	*(0.023)*	0.686	*(0.123)*	0.255	*(0.063)*	0.749	*(0.072)*	0.370	*(0.053)*	1.292	*(0.144)*	1.150	*(0.028)*
30	-6,331.92	0.153	*(0.031)*	0.509	*(0.079)*	0.165	*(0.045)*	0.617	*(0.071)*	0.465	*(0.077)*	1.291	*(0.154)*	1.256	*(0.037)*
31	-6,375.45	0.230	*(0.054)*	0.418	*(0.072)*	0.253	*(0.068)*	0.531	*(0.094)*	0.403	*(0.089)*	1.182	*(0.174)*	1.227	*(0.037)*
32	-6,208.75	0.207	*(0.041)*	0.412	*(0.070)*	0.230	*(0.050)*	0.575	*(0.068)*	0.682	*(0.137)*	1.114	*(0.153)*	1.143	*(0.025)*
33	-7,826.06	0.150	*(0.033)*	0.586	*(0.094)*	0.163	*(0.050)*	0.727	*(0.059)*	0.319	*(0.086)*	1.579	*(0.153)*	1.455	*(0.041)*
34	-6,295.03	0.201	*(0.056)*	0.392	*(0.081)*	0.317	*(0.067)*	0.592	*(0.093)*	0.189	*(0.070)*	1.434	*(0.256)*	1.299	*(0.032)*
35	-6,240.18	0.217	*(0.046)*	0.314	*(0.075)*	0.254	*(0.052)*	0.517	*(0.076)*	0.834	*(0.203)*	1.229	*(0.158)*	1.344	*(0.044)*
36	-6,788.47	0.109	*(0.024)*	0.461	*(0.075)*	0.271	*(0.043)*	0.666	*(0.054)*	0.684	*(0.128)*	1.395	*(0.120)*	1.319	*(0.036)*
37	-6,227.66	0.187	*(0.030)*	0.374	*(0.080)*	0.254	*(0.045)*	0.565	*(0.059)*	1.000	*(0.245)*	1.063	*(0.107)*	1.476	*(0.047)*
38	-5,834.08	0.173	*(0.040)*	0.308	*(0.080)*	0.321	*(0.057)*	0.558	*(0.075)*	0.859	*(0.202)*	1.295	*(0.148)*	1.157	*(0.021)*
39	-6,696.13	0.313	*(0.078)*	0.286	*(0.075)*	0.196	*(0.072)*	0.435	*(0.098)*	0.580	*(0.162)*	1.538	*(0.193)*	1.180	*(0.024)*
40	-6,100.05	0.169	*(0.033)*	0.411	*(0.073)*	0.232	*(0.046)*	0.583	*(0.067)*	0.628	*(0.136)*	1.235	*(0.150)*	1.236	*(0.033)*
41	-7,003.77	0.109	*(0.052)*	0.340	*(0.078)*	0.351	*(0.055)*	0.634	*(0.063)*	0.432	*(0.120)*	1.800	*(0.194)*	1.280	*(0.039)*
42	-6,759.94	0.236	*(0.068)*	0.288	*(0.069)*	0.260	*(0.073)*	0.500	*(0.096)*	0.518	*(0.130)*	1.666	*(0.196)*	1.099	*(0.028)*
43	-6,400.77	0.131	*(0.029)*	0.446	*(0.077)*	0.351	*(0.050)*	0.707	*(0.051)*	0.524	*(0.119)*	1.378	*(0.149)*	1.197	*(0.028)*
44	-5,663.73	0.175	*(0.043)*	0.430	*(0.084)*	0.151	*(0.066)*	0.515	*(0.100)*	0.236	*(0.081)*	1.533	*(0.216)*	1.106	*(0.027)*
45	-6,148.49	0.224	*(0.058)*	0.359	*(0.068)*	0.269	*(0.063)*	0.564	*(0.083)*	0.428	*(0.130)*	1.054	*(0.252)*	1.227	*(0.032)*
46	-7,596.79	0.204	*(0.074)*	0.292	*(0.064)*	0.378	*(0.080)*	0.561	*(0.089)*	0.489	*(0.126)*	1.797	*(0.136)*	1.190	*(0.036)*
47	-6,142.99	0.148	*(0.033)*	0.490	*(0.077)*	0.331	*(0.050)*	0.703	*(0.052)*	0.615	*(0.117)*	0.871	*(0.159)*	1.031	*(0.022)*
48	-6,026.17	0.106	*(0.022)*	0.361	*(0.068)*	0.582	*(0.057)*	0.786	*(0.035)*	0.575	*(0.119)*	1.140	*(0.193)*	1.236	*(0.026)*
49	-7,308.97	0.212	*(0.058)*	0.404	*(0.091)*	0.350	*(0.058)*	0.674	*(0.066)*	0.371	*(0.103)*	1.425	*(0.238)*	1.349	*(0.038)*
50	-5,751.31	0.117	*(0.024)*	0.543	*(0.087)*	0.216	*(0.050)*	0.664	*(0.063)*	0.473	*(0.091)*	1.186	*(0.164)*	1.300	*(0.029)*
51	-5,701.60	0.108	*(0.027)*	0.381	*(0.088)*	0.382	*(0.058)*	0.663	*(0.063)*	0.551	*(0.135)*	1.392	*(0.199)*	1.288	*(0.039)*
52	-6,267.69	0.132	*(0.040)*	0.281	*(0.062)*	0.317	*(0.060)*	0.524	*(0.075)*	0.645	*(0.173)*	1.600	*(0.151)*	1.456	*(0.042)*

Appendix C: Table C.26. Estimated coefficients for the FIAPARCH model - V_{SM}

comp	*LogL*	ω	s.e.	*d*	s.e.	*α*	s.e.	*β*	s.e.	*γ*	s.e.	*δ*	s.e.	GED	s.e.
1	-4,171.79	0.066	*(0.019)*	0.463	*(0.085)*	0.239	*(0.052)*	0.640	*(0.072)*	0.198	*(0.085)*	1.335	*(0.228)*	1.420	*(0.038)*
2	-2,157.96	0.016	*(0.010)*	0.336	*(0.083)*	0.326	*(0.067)*	0.586	*(0.082)*	0.195	*(0.072)*	1.850	*(0.303)*	1.379	*(0.045)*
3	-4,264.16	0.058	*(0.019)*	0.428	*(0.080)*	0.195	*(0.050)*	0.578	*(0.071)*	0.189	*(0.076)*	1.617	*(0.202)*	1.388	*(0.039)*
4	-4,594.97	0.114	*(0.031)*	0.406	*(0.080)*	0.301	*(0.065)*	0.577	*(0.089)*	0.383	*(0.081)*	1.239	*(0.213)*	1.271	*(0.037)*
5	-2,492.44	0.005	*(0.003)*	0.506	*(0.078)*	0.204	*(0.046)*	0.661	*(0.063)*	0.113	*(0.053)*	1.990	*(0.122)*	1.318	*(0.038)*
6	-726.93	0.013	*(0.005)*	0.317	*(0.071)*	0.155	*(0.119)*	0.355	*(0.157)*	0.164	*(0.049)*	1.865	*(0.177)*	1.255	*(0.023)*
7	-2,595.39	0.020	*(0.008)*	0.484	*(0.098)*	0.260	*(0.050)*	0.701	*(0.066)*	0.145	*(0.072)*	1.627	*(0.246)*	1.438	*(0.036)*
8	-3,655.48	0.012	*(0.006)*	0.499	*(0.116)*	0.357	*(0.068)*	0.759	*(0.060)*	0.242	*(0.068)*	1.868	*(0.183)*	1.298	*(0.028)*
9	-1,986.02	0.028	*(0.011)*	0.403	*(0.088)*	0.337	*(0.061)*	0.645	*(0.072)*	0.217	*(0.087)*	1.440	*(0.239)*	1.242	*(0.027)*
10	-5,076.10	0.091	*(0.029)*	0.358	*(0.090)*	0.399	*(0.063)*	0.659	*(0.079)*	0.106	*(0.082)*	1.660	*(0.286)*	1.328	*(0.033)*
11	-4,689.67	0.162	*(0.042)*	0.253	*(0.058)*	0.276	*(0.075)*	0.487	*(0.090)*	0.693	*(0.186)*	1.271	*(0.213)*	1.307	*(0.037)*
12	-5,003.52	0.047	*(0.056)*	0.139	*(0.071)*	0.061	*(0.131)*	0.175	*(0.165)*	1.000	*(0.536)*	1.793	*(0.223)*	1.362	*(0.037)*
13	-4,988.35	0.091	*(0.022)*	0.457	*(0.086)*	0.389	*(0.055)*	0.743	*(0.052)*	0.383	*(0.127)*	0.757	*(0.186)*	1.107	*(0.016)*
14	-3,922.72	0.021	*(0.012)*	0.323	*(0.071)*	0.325	*(0.049)*	0.604	*(0.066)*	0.533	*(0.126)*	1.815	*(0.170)*	1.182	*(0.034)*
15	-3,365.55	0.034	*(0.011)*	0.581	*(0.116)*	0.352	*(0.069)*	0.786	*(0.058)*	0.292	*(0.090)*	1.203	*(0.217)*	1.240	*(0.030)*
16	-4,593.24	0.073	*(0.024)*	0.329	*(0.073)*	0.333	*(0.061)*	0.588	*(0.081)*	0.306	*(0.104)*	1.663	*(0.224)*	1.499	*(0.050)*
17	-2,986.91	0.055	*(0.017)*	0.379	*(0.080)*	0.354	*(0.057)*	0.648	*(0.069)*	0.432	*(0.130)*	1.230	*(0.230)*	1.240	*(0.036)*
18	-1,544.07	0.050	*(0.019)*	0.378	*(0.075)*	0.260	*(0.072)*	0.549	*(0.093)*	0.423	*(0.105)*	1.135	*(0.214)*	1.134	*(0.030)*
19	-4,389.02	0.063	*(0.018)*	0.429	*(0.088)*	0.360	*(0.057)*	0.691	*(0.062)*	0.289	*(0.099)*	1.360	*(0.267)*	1.436	*(0.047)*
20	-4,138.95	0.058	*(0.020)*	0.365	*(0.089)*	0.355	*(0.065)*	0.605	*(0.090)*	0.174	*(0.055)*	1.723	*(0.224)*	1.274	*(0.034)*
21	-3,619.15	0.103	*(0.031)*	0.222	*(0.066)*	0.194	*(0.104)*	0.358	*(0.128)*	0.750	*(0.223)*	1.486	*(0.179)*	1.247	*(0.034)*
22	-4,204.76	0.033	*(0.009)*	0.428	*(0.095)*	0.529	*(0.063)*	0.791	*(0.036)*	0.321	*(0.083)*	1.585	*(0.178)*	1.307	*(0.035)*
23	-4,165.91	0.076	*(0.018)*	0.466	*(0.092)*	0.238	*(0.052)*	0.653	*(0.072)*	0.372	*(0.096)*	1.262	*(0.177)*	1.167	*(0.022)*
24	-4,352.56	0.057	*(0.015)*	0.505	*(0.097)*	0.340	*(0.056)*	0.736	*(0.059)*	0.301	*(0.104)*	1.224	*(0.209)*	1.336	*(0.024)*
25	-3,503.15	0.049	*(0.014)*	0.507	*(0.096)*	0.305	*(0.057)*	0.712	*(0.062)*	0.300	*(0.102)*	1.177	*(0.239)*	1.257	*(0.036)*
26	-4,206.12	0.057	*(0.021)*	0.306	*(0.061)*	0.181	*(0.079)*	0.425	*(0.105)*	0.309	*(0.083)*	1.807	*(0.177)*	1.434	*(0.045)*
27	-2,305.83	0.029	*(0.010)*	0.435	*(0.081)*	0.246	*(0.071)*	0.559	*(0.102)*	0.071	*(0.057)*	1.571	*(0.161)*	1.193	*(0.029)*
28	-1,642.23	0.047	*(0.018)*	0.450	*(0.082)*	0.222	*(0.065)*	0.556	*(0.096)*	0.100	*(0.078)*	1.163	*(0.251)*	1.120	*(0.020)*
29	-2,670.82	0.022	*(0.007)*	0.600	*(0.101)*	0.321	*(0.064)*	0.692	*(0.075)*	0.169	*(0.043)*	1.575	*(0.126)*	1.162	*(0.028)*
30	-3,625.08	0.068	*(0.017)*	0.482	*(0.081)*	0.165	*(0.051)*	0.588	*(0.083)*	0.316	*(0.069)*	1.263	*(0.191)*	1.269	*(0.037)*
31	-1,406.29	0.038	*(0.014)*	0.383	*(0.074)*	0.342	*(0.087)*	0.554	*(0.107)*	0.203	*(0.076)*	1.242	*(0.191)*	1.231	*(0.036)*
32	-4,207.90	0.108	*(0.024)*	0.453	*(0.071)*	0.238	*(0.050)*	0.608	*(0.068)*	0.537	*(0.118)*	0.982	*(0.188)*	1.137	*(0.025)*
33	-6,160.60	0.074	*(0.023)*	0.491	*(0.083)*	0.214	*(0.047)*	0.675	*(0.062)*	0.202	*(0.076)*	1.757	*(0.165)*	1.459	*(0.036)*
34	-4,140.47	0.066	*(0.022)*	0.353	*(0.078)*	0.304	*(0.066)*	0.574	*(0.093)*	0.116	*(0.060)*	1.711	*(0.251)*	1.358	*(0.034)*
35	-4,245.10	0.070	*(0.020)*	0.392	*(0.081)*	0.262	*(0.047)*	0.596	*(0.070)*	0.576	*(0.132)*	1.386	*(0.186)*	1.325	*(0.042)*
36	-3,845.64	0.047	*(0.012)*	0.454	*(0.082)*	0.293	*(0.045)*	0.677	*(0.057)*	0.675	*(0.127)*	1.271	*(0.152)*	1.351	*(0.036)*
37	-4,929.32	0.105	*(0.023)*	0.366	*(0.070)*	0.251	*(0.046)*	0.556	*(0.059)*	0.860	*(0.181)*	1.191	*(0.121)*	1.480	*(0.048)*
38	-3,459.44	0.078	*(0.024)*	0.343	*(0.086)*	0.342	*(0.073)*	0.578	*(0.092)*	0.466	*(0.104)*	1.314	*(0.243)*	1.164	*(0.021)*
39	-4,698.27	0.135	*(0.036)*	0.287	*(0.075)*	0.304	*(0.072)*	0.540	*(0.091)*	0.440	*(0.149)*	1.442	*(0.273)*	1.239	*(0.027)*
40	-3,853.36	0.083	*(0.020)*	0.427	*(0.074)*	0.214	*(0.049)*	0.580	*(0.073)*	0.484	*(0.114)*	1.200	*(0.187)*	1.234	*(0.033)*
41	-3,746.64	0.028	*(0.012)*	0.354	*(0.090)*	0.390	*(0.062)*	0.670	*(0.065)*	0.276	*(0.094)*	1.816	*(0.250)*	1.291	*(0.039)*
42	-4,249.68	0.131	*(0.040)*	0.281	*(0.072)*	0.205	*(0.103)*	0.431	*(0.135)*	0.323	*(0.102)*	1.579	*(0.263)*	1.114	*(0.027)*
43	-4,183.22	0.047	*(0.014)*	0.529	*(0.104)*	0.314	*(0.059)*	0.758	*(0.059)*	0.259	*(0.092)*	1.376	*(0.220)*	1.241	*(0.029)*
44	-2,441.23	0.037	*(0.012)*	0.490	*(0.093)*	0.178	*(0.059)*	0.598	*(0.086)*	0.114	*(0.074)*	1.487	*(0.216)*	1.112	*(0.028)*
45	-2,946.49	0.071	*(0.023)*	0.391	*(0.075)*	0.274	*(0.063)*	0.594	*(0.083)*	0.234	*(0.116)*	1.111	*(0.256)*	1.209	*(0.030)*
46	-4,456.53	0.106	*(0.039)*	0.284	*(0.065)*	0.298	*(0.131)*	0.460	*(0.149)*	0.252	*(0.091)*	1.775	*(0.181)*	1.197	*(0.037)*
47	-3,497.05	0.102	*(0.028)*	0.404	*(0.078)*	0.336	*(0.054)*	0.636	*(0.073)*	0.450	*(0.121)*	0.693	*(0.172)*	1.034	*(0.021)*
48	-4,717.25	0.076	*(0.017)*	0.359	*(0.065)*	0.547	*(0.059)*	0.767	*(0.038)*	0.470	*(0.116)*	1.084	*(0.234)*	1.305	*(0.028)*
49	-6,066.86	0.187	*(0.055)*	0.336	*(0.069)*	0.358	*(0.068)*	0.606	*(0.084)*	0.314	*(0.116)*	0.839	*(0.255)*	1.355	*(0.039)*
50	-3,005.99	0.047	*(0.013)*	0.533	*(0.090)*	0.216	*(0.053)*	0.649	*(0.071)*	0.338	*(0.079)*	1.221	*(0.175)*	1.308	*(0.031)*
51	-3,499.97	0.040	*(0.013)*	0.396	*(0.088)*	0.385	*(0.062)*	0.673	*(0.064)*	0.368	*(0.103)*	1.509	*(0.219)*	1.284	*(0.038)*
52	-3,845.33	0.047	*(0.016)*	0.371	*(0.084)*	0.354	*(0.053)*	0.636	*(0.076)*	0.305	*(0.091)*	1.599	*(0.247)*	1.468	*(0.040)*

Appendix C: Table C.27. Estimated coefficients for the FIAPARCH model - V_{KMV}

comp	*LogL*	ω	s.e.	*d*	s.e.	*α*	s.e.	*β*	s.e.	*γ*	s.e.	*δ*	s.e.	GED	s.e.
1	-4,210.51	0.065	(0.018)	0.456	(0.086)	0.232	(0.052)	0.630	(0.074)	0.198	(0.083)	1.398	(0.229)	1.424	(0.038)
2	-2,172.48	0.016	(0.010)	0.339	(0.082)	0.324	(0.066)	0.586	(0.081)	0.211	(0.073)	1.839	(0.297)	1.378	(0.045)
3	-4,219.65	0.059	(0.020)	0.426	(0.080)	0.193	(0.050)	0.575	(0.071)	0.194	(0.078)	1.600	(0.205)	1.388	(0.039)
4	-4,539.93	0.110	(0.030)	0.409	(0.079)	0.303	(0.063)	0.582	(0.087)	0.394	(0.081)	1.240	(0.209)	1.270	(0.037)
5	-2,483.34	0.005	(0.003)	0.503	(0.077)	0.207	(0.046)	0.660	(0.063)	0.124	(0.054)	1.976	(0.124)	1.321	(0.038)
6	-711.19	0.015	(0.006)	0.311	(0.069)	0.144	(0.126)	0.338	(0.163)	0.169	(0.051)	1.825	(0.183)	1.256	(0.023)
7	-2,566.92	0.020	(0.008)	0.468	(0.094)	0.262	(0.049)	0.687	(0.067)	0.159	(0.073)	1.631	(0.246)	1.439	(0.036)
8	-3,603.66	0.012	(0.006)	0.488	(0.114)	0.362	(0.067)	0.753	(0.059)	0.259	(0.070)	1.864	(0.182)	1.300	(0.028)
9	-1,928.42	0.028	(0.011)	0.395	(0.087)	0.340	(0.061)	0.640	(0.073)	0.231	(0.090)	1.430	(0.241)	1.242	(0.027)
10	-5,067.10	0.084	(0.026)	0.379	(0.093)	0.384	(0.061)	0.666	(0.077)	0.106	(0.079)	1.661	(0.272)	1.330	(0.033)
11	-4,639.91	0.157	(0.041)	0.253	(0.059)	0.275	(0.075)	0.486	(0.090)	0.694	(0.188)	1.282	(0.212)	1.307	(0.037)
12	-5,033.58	0.045	(0.055)	0.142	(0.070)	0.064	(0.129)	0.180	(0.162)	1.000	(0.524)	1.784	(0.220)	1.359	(0.036)
13	-5,030.14	0.088	(0.021)	0.467	(0.087)	0.386	(0.057)	0.750	(0.051)	0.388	(0.124)	0.734	(0.184)	1.103	(0.015)
14	-3,910.51	0.020	(0.011)	0.337	(0.072)	0.320	(0.049)	0.612	(0.065)	0.509	(0.118)	1.818	(0.165)	1.181	(0.034)
15	-3,311.21	0.033	(0.011)	0.585	(0.117)	0.349	(0.069)	0.787	(0.058)	0.295	(0.089)	1.196	(0.217)	1.241	(0.031)
16	-4,577.32	0.071	(0.023)	0.332	(0.072)	0.332	(0.060)	0.590	(0.079)	0.308	(0.104)	1.664	(0.218)	1.502	(0.051)
17	-2,920.51	0.053	(0.016)	0.381	(0.080)	0.354	(0.057)	0.649	(0.069)	0.429	(0.130)	1.224	(0.230)	1.240	(0.036)
18	-1,443.63	0.049	(0.019)	0.375	(0.075)	0.257	(0.073)	0.544	(0.094)	0.417	(0.104)	1.148	(0.217)	1.135	(0.029)
19	-4,426.96	0.063	(0.018)	0.435	(0.088)	0.356	(0.056)	0.692	(0.061)	0.292	(0.098)	1.357	(0.260)	1.437	(0.047)
20	-4,163.12	0.059	(0.020)	0.379	(0.090)	0.353	(0.063)	0.615	(0.087)	0.175	(0.055)	1.695	(0.218)	1.272	(0.034)
21	-3,571.79	0.112	(0.033)	0.171	(0.099)	0.179	(0.120)	0.303	(0.159)	1.000	(0.691)	1.467	(0.249)	1.254	(0.035)
22	-4,214.63	0.033	(0.009)	0.425	(0.095)	0.527	(0.063)	0.788	(0.037)	0.311	(0.083)	1.593	(0.181)	1.312	(0.035)
23	-4,162.87	0.073	(0.018)	0.468	(0.093)	0.241	(0.052)	0.659	(0.071)	0.368	(0.095)	1.268	(0.180)	1.174	(0.022)
24	-4,344.04	0.057	(0.015)	0.505	(0.096)	0.336	(0.055)	0.736	(0.059)	0.307	(0.104)	1.225	(0.209)	1.339	(0.024)
25	-3,455.89	0.047	(0.014)	0.517	(0.098)	0.299	(0.058)	0.716	(0.062)	0.280	(0.099)	1.188	(0.239)	1.254	(0.036)
26	-4,233.67	0.061	(0.022)	0.305	(0.060)	0.177	(0.079)	0.424	(0.104)	0.318	(0.082)	1.790	(0.177)	1.442	(0.044)
27	-2,295.01	0.032	(0.010)	0.440	(0.082)	0.246	(0.069)	0.562	(0.102)	0.087	(0.058)	1.526	(0.167)	1.195	(0.029)
28	-1,643.36	0.049	(0.018)	0.441	(0.080)	0.226	(0.067)	0.548	(0.098)	0.111	(0.078)	1.159	(0.249)	1.125	(0.020)
29	-2,649.13	0.022	(0.007)	0.603	(0.103)	0.322	(0.064)	0.696	(0.074)	0.174	(0.043)	1.566	(0.128)	1.162	(0.028)
30	-3,593.20	0.064	(0.016)	0.492	(0.083)	0.166	(0.050)	0.599	(0.082)	0.316	(0.068)	1.281	(0.188)	1.266	(0.037)
31	-1,422.12	0.045	(0.016)	0.379	(0.070)	0.314	(0.091)	0.525	(0.114)	0.229	(0.080)	1.169	(0.190)	1.230	(0.036)
32	-4,160.79	0.109	(0.025)	0.447	(0.070)	0.240	(0.050)	0.603	(0.069)	0.537	(0.119)	0.975	(0.190)	1.138	(0.025)
33	-6,136.53	0.073	(0.022)	0.495	(0.084)	0.214	(0.047)	0.679	(0.062)	0.214	(0.077)	1.740	(0.165)	1.463	(0.036)
34	-4,142.95	0.069	(0.023)	0.340	(0.077)	0.305	(0.069)	0.563	(0.098)	0.124	(0.061)	1.716	(0.254)	1.362	(0.034)
35	-4,189.87	0.064	(0.019)	0.394	(0.081)	0.261	(0.047)	0.597	(0.071)	0.556	(0.128)	1.424	(0.185)	1.323	(0.042)
36	-3,798.28	0.047	(0.012)	0.456	(0.082)	0.292	(0.045)	0.678	(0.057)	0.675	(0.127)	1.265	(0.153)	1.350	(0.036)
37	-4,924.39	0.105	(0.023)	0.360	(0.071)	0.253	(0.046)	0.553	(0.059)	0.880	(0.190)	1.190	(0.119)	1.477	(0.048)
38	-3,411.62	0.075	(0.023)	0.350	(0.087)	0.346	(0.072)	0.587	(0.090)	0.462	(0.104)	1.313	(0.242)	1.163	(0.021)
39	-4,692.24	0.126	(0.034)	0.290	(0.074)	0.307	(0.070)	0.546	(0.087)	0.432	(0.143)	1.479	(0.264)	1.254	(0.027)
40	-3,833.67	0.084	(0.020)	0.428	(0.073)	0.212	(0.049)	0.579	(0.073)	0.487	(0.114)	1.179	(0.189)	1.234	(0.033)
41	-3,727.37	0.027	(0.012)	0.351	(0.089)	0.391	(0.061)	0.672	(0.063)	0.308	(0.101)	1.795	(0.249)	1.296	(0.039)
42	-4,322.35	0.132	(0.040)	0.282	(0.072)	0.210	(0.100)	0.437	(0.131)	0.347	(0.105)	1.576	(0.259)	1.114	(0.027)
43	-4,176.46	0.045	(0.013)	0.537	(0.109)	0.302	(0.061)	0.761	(0.060)	0.236	(0.089)	1.409	(0.216)	1.245	(0.030)
44	-2,380.96	0.036	(0.012)	0.481	(0.092)	0.182	(0.060)	0.592	(0.087)	0.112	(0.074)	1.494	(0.220)	1.111	(0.028)
45	-2,849.83	0.070	(0.023)	0.386	(0.075)	0.277	(0.064)	0.591	(0.084)	0.224	(0.115)	1.117	(0.260)	1.209	(0.031)
46	-4,562.38	0.155	(0.060)	0.244	(0.058)	0.264	(0.169)	0.394	(0.187)	0.325	(0.107)	1.675	(0.199)	1.209	(0.037)
47	-3,438.48	0.097	(0.026)	0.418	(0.080)	0.334	(0.053)	0.646	(0.070)	0.456	(0.120)	0.672	(0.167)	1.033	(0.021)
48	-4,719.04	0.073	(0.016)	0.362	(0.066)	0.546	(0.059)	0.770	(0.036)	0.471	(0.116)	1.131	(0.228)	1.310	(0.028)
49	-6,059.91	0.286	(0.093)	0.146	(0.139)	0.385	(0.086)	0.501	(0.131)	1.000	(1.105)	1.481	(0.297)	1.347	(0.039)
50	-2,956.35	0.047	(0.013)	0.533	(0.090)	0.219	(0.053)	0.650	(0.070)	0.343	(0.080)	1.209	(0.176)	1.306	(0.031)
51	-3,453.27	0.039	(0.013)	0.397	(0.088)	0.384	(0.062)	0.673	(0.064)	0.375	(0.104)	1.501	(0.218)	1.283	(0.038)
52	-3,857.57	0.046	(0.015)	0.386	(0.085)	0.353	(0.051)	0.649	(0.072)	0.305	(0.092)	1.576	(0.240)	1.471	(0.040)

Appendix C: Table C.28. Estimated coefficients for the FIAPARCH model - V_{Proxy}

comp	*LogL*	ω	s.e.	*d*	s.e.	*α*	s.e.	*β*	s.e.	*γ*	s.e.	*δ*	s.e.	GED	s.e.
1	-4,130.63	0.061	*(0.018)*	0.475	*(0.089)*	0.242	*(0.052)*	0.653	*(0.071)*	0.209	*(0.084)*	1.339	*(0.230)*	1.427	*(0.039)*
2	-2,077.99	0.017	*(0.009)*	0.373	*(0.080)*	0.288	*(0.060)*	0.592	*(0.077)*	0.222	*(0.074)*	1.767	*(0.260)*	1.366	*(0.044)*
3	-4,243.40	0.056	*(0.019)*	0.439	*(0.081)*	0.191	*(0.048)*	0.587	*(0.071)*	0.189	*(0.076)*	1.613	*(0.199)*	1.393	*(0.040)*
4	-4,569.38	0.112	*(0.030)*	0.427	*(0.083)*	0.282	*(0.061)*	0.580	*(0.088)*	0.362	*(0.079)*	1.220	*(0.213)*	1.266	*(0.037)*
5	-2,373.93	0.003	*(0.002)*	0.510	*(0.075)*	0.175	*(0.045)*	0.643	*(0.065)*	0.098	*(0.052)*	2.031	*(0.115)*	1.317	*(0.038)*
6	-549.17	0.004	*(0.002)*	0.448	*(0.093)*	0.324	*(0.060)*	0.651	*(0.067)*	0.098	*(0.050)*	1.886	*(0.147)*	1.224	*(0.022)*
7	-2,537.56	0.021	*(0.009)*	0.472	*(0.095)*	0.246	*(0.048)*	0.678	*(0.069)*	0.146	*(0.072)*	1.617	*(0.248)*	1.437	*(0.036)*
8	-3,574.74	0.010	*(0.005)*	0.504	*(0.113)*	0.351	*(0.066)*	0.758	*(0.058)*	0.238	*(0.067)*	1.890	*(0.168)*	1.295	*(0.028)*
9	-1,903.08	0.034	*(0.013)*	0.436	*(0.088)*	0.297	*(0.059)*	0.637	*(0.073)*	0.196	*(0.087)*	1.285	*(0.244)*	1.227	*(0.028)*
10	-5,066.66	0.085	*(0.026)*	0.384	*(0.093)*	0.387	*(0.060)*	0.674	*(0.075)*	0.120	*(0.082)*	1.622	*(0.273)*	1.325	*(0.033)*
11	-4,705.84	0.163	*(0.042)*	0.254	*(0.059)*	0.281	*(0.073)*	0.494	*(0.087)*	0.709	*(0.191)*	1.251	*(0.213)*	1.307	*(0.037)*
12	-5,013.02	0.047	*(0.053)*	0.143	*(0.070)*	0.062	*(0.127)*	0.179	*(0.160)*	1.000	*(0.516)*	1.773	*(0.219)*	1.368	*(0.037)*
13	-5,045.48	0.084	*(0.020)*	0.476	*(0.090)*	0.387	*(0.059)*	0.759	*(0.051)*	0.371	*(0.124)*	0.726	*(0.181)*	1.106	*(0.016)*
14	-3,943.56	0.021	*(0.011)*	0.337	*(0.071)*	0.312	*(0.049)*	0.605	*(0.065)*	0.511	*(0.118)*	1.817	*(0.163)*	1.180	*(0.034)*
15	-3,371.77	0.037	*(0.012)*	0.569	*(0.108)*	0.345	*(0.064)*	0.772	*(0.058)*	0.316	*(0.092)*	1.167	*(0.218)*	1.244	*(0.031)*
16	-4,620.77	0.072	*(0.023)*	0.331	*(0.072)*	0.329	*(0.060)*	0.587	*(0.079)*	0.311	*(0.104)*	1.669	*(0.219)*	1.505	*(0.051)*
17	-2,966.20	0.053	*(0.017)*	0.361	*(0.080)*	0.351	*(0.059)*	0.632	*(0.072)*	0.422	*(0.131)*	1.305	*(0.246)*	1.247	*(0.036)*
18	-1,534.43	0.049	*(0.018)*	0.395	*(0.076)*	0.253	*(0.064)*	0.564	*(0.087)*	0.456	*(0.109)*	1.110	*(0.211)*	1.127	*(0.031)*
19	-4,490.39	0.065	*(0.018)*	0.432	*(0.086)*	0.357	*(0.056)*	0.691	*(0.060)*	0.316	*(0.102)*	1.353	*(0.257)*	1.427	*(0.047)*
20	-4,187.25	0.058	*(0.020)*	0.368	*(0.088)*	0.353	*(0.064)*	0.606	*(0.089)*	0.185	*(0.053)*	1.727	*(0.220)*	1.278	*(0.034)*
21	-3,605.78	0.098	*(0.029)*	0.184	*(0.095)*	0.209	*(0.106)*	0.344	*(0.142)*	1.000	*(0.616)*	1.459	*(0.238)*	1.250	*(0.035)*
22	-4,241.13	0.034	*(0.009)*	0.441	*(0.099)*	0.510	*(0.064)*	0.790	*(0.038)*	0.327	*(0.085)*	1.566	*(0.180)*	1.304	*(0.035)*
23	-4,149.63	0.073	*(0.017)*	0.468	*(0.092)*	0.245	*(0.052)*	0.664	*(0.070)*	0.388	*(0.098)*	1.255	*(0.177)*	1.172	*(0.022)*
24	-4,338.75	0.058	*(0.015)*	0.500	*(0.095)*	0.338	*(0.054)*	0.730	*(0.059)*	0.311	*(0.104)*	1.215	*(0.210)*	1.334	*(0.024)*
25	-3,516.42	0.050	*(0.014)*	0.516	*(0.097)*	0.296	*(0.056)*	0.714	*(0.062)*	0.296	*(0.103)*	1.157	*(0.240)*	1.260	*(0.036)*
26	-4,165.36	0.053	*(0.020)*	0.319	*(0.062)*	0.194	*(0.075)*	0.452	*(0.099)*	0.296	*(0.077)*	1.797	*(0.174)*	1.455	*(0.045)*
27	-2,200.20	0.024	*(0.008)*	0.430	*(0.078)*	0.230	*(0.067)*	0.558	*(0.097)*	0.030	*(0.054)*	1.644	*(0.157)*	1.217	*(0.029)*
28	-1,607.86	0.049	*(0.017)*	0.528	*(0.100)*	0.240	*(0.054)*	0.664	*(0.082)*	0.091	*(0.086)*	0.801	*(0.187)*	1.096	*(0.019)*
29	-2,701.93	0.024	*(0.008)*	0.600	*(0.101)*	0.294	*(0.063)*	0.684	*(0.076)*	0.172	*(0.044)*	1.562	*(0.129)*	1.148	*(0.028)*
30	-3,684.96	0.076	*(0.018)*	0.495	*(0.079)*	0.157	*(0.050)*	0.594	*(0.080)*	0.341	*(0.071)*	1.153	*(0.186)*	1.272	*(0.037)*
31	-1,242.78	0.039	*(0.014)*	0.379	*(0.070)*	0.260	*(0.090)*	0.490	*(0.118)*	0.175	*(0.076)*	1.268	*(0.193)*	1.237	*(0.036)*
32	-4,240.30	0.107	*(0.024)*	0.472	*(0.073)*	0.233	*(0.047)*	0.623	*(0.066)*	0.534	*(0.117)*	0.925	*(0.191)*	1.139	*(0.025)*
33	-6,043.33	0.074	*(0.023)*	0.473	*(0.080)*	0.218	*(0.046)*	0.661	*(0.064)*	0.187	*(0.073)*	1.770	*(0.167)*	1.458	*(0.036)*
34	-4,134.53	0.068	*(0.022)*	0.354	*(0.077)*	0.305	*(0.066)*	0.574	*(0.093)*	0.131	*(0.061)*	1.684	*(0.246)*	1.362	*(0.034)*
35	-4,264.34	0.064	*(0.019)*	0.388	*(0.080)*	0.257	*(0.048)*	0.588	*(0.072)*	0.555	*(0.129)*	1.453	*(0.183)*	1.320	*(0.042)*
36	-3,845.01	0.051	*(0.013)*	0.463	*(0.082)*	0.279	*(0.044)*	0.678	*(0.057)*	0.691	*(0.128)*	1.230	*(0.151)*	1.336	*(0.036)*
37	-4,984.34	0.104	*(0.022)*	0.374	*(0.071)*	0.250	*(0.045)*	0.563	*(0.058)*	0.859	*(0.181)*	1.191	*(0.121)*	1.479	*(0.048)*
38	-3,506.67	0.078	*(0.023)*	0.346	*(0.086)*	0.330	*(0.073)*	0.572	*(0.093)*	0.465	*(0.100)*	1.337	*(0.233)*	1.162	*(0.021)*
39	-4,647.77	0.126	*(0.033)*	0.291	*(0.075)*	0.306	*(0.069)*	0.546	*(0.087)*	0.441	*(0.145)*	1.459	*(0.266)*	1.256	*(0.027)*
40	-3,839.41	0.080	*(0.019)*	0.429	*(0.074)*	0.216	*(0.049)*	0.585	*(0.072)*	0.484	*(0.114)*	1.208	*(0.189)*	1.239	*(0.033)*
41	-3,749.95	0.026	*(0.013)*	0.353	*(0.089)*	0.378	*(0.061)*	0.662	*(0.066)*	0.285	*(0.095)*	1.837	*(0.250)*	1.287	*(0.039)*
42	-4,328.55	0.123	*(0.037)*	0.294	*(0.074)*	0.222	*(0.090)*	0.463	*(0.121)*	0.356	*(0.107)*	1.573	*(0.255)*	1.099	*(0.027)*
43	-4,208.89	0.046	*(0.013)*	0.544	*(0.111)*	0.298	*(0.062)*	0.762	*(0.061)*	0.257	*(0.090)*	1.382	*(0.215)*	1.250	*(0.030)*
44	-2,541.68	0.042	*(0.014)*	0.496	*(0.093)*	0.149	*(0.058)*	0.577	*(0.092)*	0.147	*(0.073)*	1.458	*(0.225)*	1.115	*(0.028)*
45	-3,024.26	0.073	*(0.023)*	0.403	*(0.076)*	0.258	*(0.061)*	0.595	*(0.081)*	0.258	*(0.116)*	1.081	*(0.262)*	1.220	*(0.031)*
46	-4,567.57	0.125	*(0.044)*	0.283	*(0.063)*	0.269	*(0.130)*	0.439	*(0.149)*	0.285	*(0.097)*	1.716	*(0.176)*	1.181	*(0.036)*
47	-3,556.04	0.104	*(0.029)*	0.398	*(0.077)*	0.335	*(0.055)*	0.632	*(0.076)*	0.462	*(0.120)*	0.723	*(0.176)*	1.042	*(0.022)*
48	-4,745.86	0.076	*(0.016)*	0.370	*(0.066)*	0.532	*(0.058)*	0.764	*(0.037)*	0.470	*(0.117)*	1.100	*(0.226)*	1.304	*(0.027)*
49	-6,042.35	0.298	*(0.096)*	0.143	*(0.140)*	0.374	*(0.090)*	0.487	*(0.137)*	1.000	*(1.139)*	1.478	*(0.298)*	1.350	*(0.039)*
50	-3,125.77	0.051	*(0.013)*	0.554	*(0.092)*	0.192	*(0.051)*	0.656	*(0.070)*	0.361	*(0.081)*	1.145	*(0.172)*	1.318	*(0.031)*
51	-3,559.28	0.043	*(0.014)*	0.410	*(0.092)*	0.379	*(0.061)*	0.680	*(0.063)*	0.403	*(0.108)*	1.439	*(0.225)*	1.292	*(0.039)*
52	-3,871.21	0.047	*(0.015)*	0.382	*(0.083)*	0.346	*(0.051)*	0.641	*(0.073)*	0.317	*(0.092)*	1.583	*(0.238)*	1.456	*(0.039)*

Appendix D: Table D.1. Forecasting results

	E	V_{SM}	V_{KMV}	V_{Proxy}
MSE2				
GARCH	0.0086	0.0005	0.0005	0.0005
EGARCH	0.0075	0.0005	0.0005	0.0005
IGARCH	0.0098	0.0005	0.0006	0.0006
FIGARCH	0.0083	0.0005	0.0005	0.0005
HYGARCH	0.0081	0.0005	0.0005	0.0005
FIEGARCH	**0.0075**	**0.0004**	**0.0004**	**0.0004**
FIAPARCH	0.0076	0.0005	0.0005	0.0005
MAE2				
GARCH	0.0485	0.0109	0.0110	0.0113
EGARCH	0.0435	0.0102	0.0103	0.0106
IGARCH	0.0540	0.0121	0.0122	0.0126
FIGARCH	0.0472	0.0105	0.0107	0.0110
HYGARCH	0.0466	0.0105	0.0107	0.0110
FIEGARCH	**0.0423**	**0.0098**	**0.0099**	**0.0101**
FIAPARCH	0.0427	0.0101	0.0102	0.0105
QLIKE				
GARCH	-1.5764	-3.0985	-3.0925	-3.0722
EGARCH	-1.5944	-3.1029	-3.0968	-3.0770
IGARCH	-1.5656	-3.0927	-3.0880	-3.0675
FIGARCH	-1.5860	-3.1074	-3.1022	-3.0820
HYGARCH	-1.5871	-3.1076	-3.1021	-3.0819
FIEGARCH	**-1.6004**	**-3.1141**	**-3.1080**	**-3.0879**
FIAPARCH	-1.6001	-3.1107	-3.1049	-3.0855

Appendix D: Table D.2. Forecasting results at firm-level

	E		V_{SM}		V_{KMV}		V_{Proxy}	
	Num	***%***	***Num***	***%***	***Num***	***%***	***Num***	***%***
MSE								
GARCH	0	0.00%	1	1.92%	1	1.92%	0	0.00%
EGARCH	10	19.23%	8	15.38%	8	15.38%	9	17.31%
IGARCH	0	0.00%	0	0.00%	1	1.92%	0	0.00%
FIGARCH	1	1.92%	3	5.77%	3	5.77%	3	5.77%
HYGARCH	2	3.85%	2	3.85%	2	3.85%	2	3.85%
FIEGARCH	**26**	**50.00%**	**28**	**53.85%**	**27**	**51.92%**	**27**	**51.92%**
FIAPARCH	13	25.00%	10	19.23%	10	19.23%	11	21.15%
MAE								
GARCH	0	0.00%	3	5.77%	3	5.77%	2	3.85%
EGARCH	10	19.23%	8	15.38%	8	15.38%	9	17.31%
IGARCH	0	0.00%	0	0.00%	0	0.00%	0	0.00%
FIGARCH	4	7.69%	3	5.77%	3	5.77%	3	5.77%
HYGARCH	4	7.69%	4	7.69%	3	5.77%	4	7.69%
FIEGARCH	**20**	**38.46%**	**25**	**48.08%**	**26**	**50.00%**	**24**	**46.15%**
FIAPARCH	14	26.92%	9	17.31%	9	17.31%	10	19.23%
QLIKE								
GARCH	0	0.00%	1	1.92%	1	1.92%	1	1.92%
EGARCH	12	23.08%	7	13.46%	8	15.38%	7	13.46%
IGARCH	2	3.85%	5	9.62%	5	9.62%	4	7.69%
FIGARCH	2	3.85%	3	5.77%	3	5.77%	4	7.69%
HYGARCH	1	1.92%	7	13.46%	7	13.46%	6	11.54%
FIEGARCH	**13**	**25.00%**	**17**	**32.69%**	**15**	**28.85%**	**18**	**34.62%**
FIAPARCH	22	42.31%	12	23.08%	13	25.00%	12	23.08%

Appendix E: Table E.1. SPA test - MAE

Benchmark	MAE - Equity				
	mean p-val	N° comp p-val > 0.05		N° comp max p-val	
GARCH	0.0191	5	9.62%	0	0.00%
EGARCH	0.1163	19	36.54%	3	5.77%
IGARCH	0.0005	0	0.00%	0	0.00%
FIGARCH	0.1222	14	26.92%	3	5.77%
HYGARCH	0.0810	13	25.00%	0	0.00%
FIEGARCH	0.6553	44	84.62%	30	57.69%
FIAPARCH	0.3981	28	53.85%	16	30.77%

Benchmark	MAE - V_{SM}				
	mean p-val	N° comp p-val > 0.05		N° comp max p-val	
GARCH	0.0310	7	13.46%	0	0.00%
EGARCH	0.3004	29	55.77%	12	23.08%
IGARCH	0.0033	2	3.85%	0	0.00%
FIGARCH	0.1562	19	36.54%	5	9.62%
HYGARCH	0.1545	20	38.46%	4	7.69%
FIEGARCH	0.5905	42	80.77%	23	44.23%
FIAPARCH	0.3117	28	53.85%	8	15.38%

Benchmark	MAE -V_{KMV}				
	mean p-val	N° comp p-val > 0.05		N° comp max p-val	
GARCH	0.0321	7	13.46%	0	0.00%
EGARCH	0.2985	29	55.77%	12	23.08%
IGARCH	0.0035	2	3.85%	0	0.00%
FIGARCH	0.1438	18	34.62%	4	7.69%
HYGARCH	0.1365	18	34.62%	3	5.77%
FIEGARCH	0.6026	41	78.85%	24	46.15%
FIAPARCH	0.3130	30	57.69%	9	17.31%

Benchmark	MAE -V_{proxy}				
	mean p-val	N° comp p-val > 0.05		N° comp max p-val	
GARCH	0.0241	5	9.62%	0	0.00%
EGARCH	0.3074	29	55.77%	12	23.08%
IGARCH	0.0027	1	1.92%	0	0.00%
FIGARCH	0.1504	18	34.62%	3	5.77%
HYGARCH	0.1145	17	32.69%	1	1.92%
FIEGARCH	0.6134	42	80.77%	26	50.00%
FIAPARCH	0.3181	27	51.92%	10	19.23%

Appendix E: Table E.2. SPA test - MSE2

Benchmark	MSE^2 - Equity				
	mean p-val	N° comp p-val > 0.05		N° comp max p-val	
GARCH	0.0177	5	9.62%	0	0.00%
EGARCH	0.0974	16	30.77%	2	3.85%
IGARCH	0.0017	0	0.00%	0	0.00%
FIGARCH	0.1070	13	25.00%	2	3.85%
HYGARCH	0.0844	14	26.92%	1	1.92%
FIEGARCH	0.6508	44	84.62%	30	57.69%
FIAPARCH	0.4101	29	55.77%	17	32.69%

Benchmark	MSE^2 - V_{SM}				
	mean p-val	N° comp p-val > 0.05		N° comp max p-val	
GARCH	0.0294	8	15.38%	0	0.00%
EGARCH	0.2731	27	51.92%	11	21.15%
IGARCH	0.0029	0	0.00%	0	0.00%
FIGARCH	0.1418	18	34.62%	4	7.69%
HYGARCH	0.1573	19	36.54%	4	7.69%
FIEGARCH	0.5961	43	82.69%	25	48.08%
FIAPARCH	0.3074	29	55.77%	8	15.38%

Benchmark	MSE^2 -V_{KMV}				
	mean p-val	N° comp p-val > 0.05		N° comp max p-val	
GARCH	0.0288	8	15.38%	0	0.00%
EGARCH	0.2631	25	48.08%	10	19.23%
IGARCH	0.0031	1	1.92%	0	0.00%
FIGARCH	0.1421	18	34.62%	4	7.69%
HYGARCH	0.1463	19	36.54%	3	5.77%
FIEGARCH	0.6095	44	84.62%	26	50.00%
FIAPARCH	0.3074	30	57.69%	9	17.31%

Benchmark	MSE^2 -V_{proxy}				
	mean p-val	N° comp p-val > 0.05		N° comp max p-val	
GARCH	0.0243	6	11.54%	0	0.00%
EGARCH	0.2811	25	48.08%	11	21.15%
IGARCH	0.0023	0	0.00%	0	0.00%
FIGARCH	0.1529	21	40.38%	3	5.77%
HYGARCH	0.1288	19	36.54%	2	3.85%
FIEGARCH	0.6240	43	82.69%	27	51.92%
FIAPARCH	0.3053	28	53.85%	9	17.31%

Appendix E: Table E.3. SPA test - MAE2

Benchmark	MAE^2 - Equity				
	mean p-val	N° comp p-val > 0.05		N° comp max p-val	
GARCH	0.0172	5	9.62%	0	0.00%
EGARCH	0.0978	17	32.69%	2	3.85%
IGARCH	0.0018	0	0.00%	0	0.00%
FIGARCH	0.1066	14	26.92%	2	3.85%
HYGARCH	0.0847	14	26.92%	1	1.92%
FIEGARCH	0.6502	44	84.62%	30	57.69%
FIAPARCH	0.4098	30	57.69%	17	32.69%

Benchmark	MAE^2 - V_{SM}				
	mean p-val	N° comp p-val > 0.05		N° comp max p-val	
GARCH	0.0292	8	15.38%	0	0.00%
EGARCH	0.2734	27	51.92%	11	21.15%
IGARCH	0.0030	1	1.92%	0	0.00%
FIGARCH	0.1415	18	34.62%	4	7.69%
HYGARCH	0.1576	19	36.54%	4	7.69%
FIEGARCH	0.5960	43	82.69%	25	48.08%
FIAPARCH	0.3074	28	53.85%	8	15.38%

Benchmark	MAE^2 -V_{KMV}				
	mean p-val	N° comp p-val > 0.05		N° comp max p-val	
GARCH	0.0287	8	15.38%	0	0.00%
EGARCH	0.2631	25	48.08%	10	19.23%
IGARCH	0.0034	1	1.92%	0	0.00%
FIGARCH	0.1414	17	32.69%	4	7.69%
HYGARCH	0.1452	18	34.62%	3	5.77%
FIEGARCH	0.6090	44	84.62%	26	50.00%
FIAPARCH	0.3067	30	57.69%	9	17.31%

Benchmark	MAE^2 -V_{proxy}				
	mean p-val	N° comp p-val > 0.05		N° comp max p-val	
GARCH	0.0242	5	9.62%	0	0.00%
EGARCH	0.2812	25	48.08%	11	21.15%
IGARCH	0.0024	0	0.00%	0	0.00%
FIGARCH	0.1526	20	38.46%	3	5.77%
HYGARCH	0.1281	18	34.62%	2	3.85%
FIEGARCH	0.6234	43	82.69%	27	51.92%
FIAPARCH	0.3056	28	53.85%	9	17.31%

Appendix E: Table E.4. SPA test - QLIKE

Benchmark	QLIKE - Equity				
	mean p-val	N° comp p-val > 0.05		N° comp max p-val	
GARCH	0.0172	6	11.54%	0	0.00%
EGARCH	0.1181	18	34.62%	2	3.85%
IGARCH	0.0001	0	0.00%	0	0.00%
FIGARCH	0.1127	15	28.85%	3	5.77%
HYGARCH	0.0687	11	21.15%	0	0.00%
FIEGARCH	0.6619	43	82.69%	30	57.69%
FIAPARCH	0.3819	30	57.69%	17	32.69%

Benchmark	QLIKE - V_{SM}				
	mean p-val	N° comp p-val > 0.05		N° comp max p-val	
GARCH	0.0274	6	11.54%	0	0.00%
EGARCH	0.3207	27	51.92%	13	25.00%
IGARCH	0.0024	1	1.92%	0	0.00%
FIGARCH	0.1218	16	30.77%	2	3.85%
HYGARCH	0.1152	17	32.69%	2	3.85%
FIEGARCH	0.6001	43	82.69%	25	48.08%
FIAPARCH	0.3182	30	57.69%	10	19.23%

Benchmark	QLIKE -V_{KMV}				
	mean p-val	N° comp p-val > 0.05		N° comp max p-val	
GARCH	0.0275	5	9.62%	0	0.00%
EGARCH	0.3044	27	51.92%	12	23.08%
IGARCH	0.0026	1	1.92%	0	0.00%
FIGARCH	0.1165	16	30.77%	2	3.85%
HYGARCH	0.1077	14	26.92%	2	3.85%
FIEGARCH	0.5950	41	78.85%	25	48.08%
FIAPARCH	0.3169	30	57.69%	11	21.15%

Benchmark	QLIKE -V_{proxy}				
	mean p-val	N° comp p-val > 0.05		N° comp max p-val	
GARCH	0.0211	5	9.62%	0	0.00%
EGARCH	0.3179	27	51.92%	14	26.92%
IGARCH	0.0019	1	1.92%	0	0.00%
FIGARCH	0.1280	17	32.69%	2	3.85%
HYGARCH	0.0953	14	26.92%	2	3.85%
FIEGARCH	0.5699	43	82.69%	23	44.23%
FIAPARCH	0.3089	28	53.85%	11	21.15%

Chapter 3

Forecasting CDS Implied Asset Volatility

1. Introduction

Asset volatility is one of the most relevant features in credit risk modelling and refers to a degree of fluctuation of the firm's asset returns. Although the volatility of firm's assets is one of the fundamental theoretical determinants of credit spreads and plays a key role in determining capital structure valuation, surprisingly, little is known about the behaviour of this key parameter. In particular, Choi and Richardson (2016) argue that very little is known about its cross-sectional and time-series properties. They provide the first examination of the time-series properties of firm's asset volatility using an Exponential GARCH model proposed by Nelson (1991). In that context, they observed that equity volatility is significantly more persistent and asymmetric than asset volatility for levered firms, and that the differences between equity volatility and asset volatility generally increase with leverage.

In their recent studies González-Pla and Lovreta (2019), and González-Pla and Lovreta (2020) provide two important findings on the firm's asset value process. On the one hand, these authors have examined the long-memory properties in the volatility of the firm's asset and equity returns. They have shown that firm's asset volatility undoubtedly possesses the long-memory features very similar to the process followed by equity volatility. On the other hand, they examined the in-sample fit and the out-of-sample forecasting performance of different GARCH-type volatility models. Their results point out to the outperformance of models that simultaneously account for asymmetry and fractional integration over the classical short-memory GARCH models or infinite memory IGARCH models. The main implication of these results is that not only that the long-memory is a stylized fact of the firm's volatility process but also that forecasting firm's asset volatility should be based on a model that incorporates this volatility feature.

In empirical applications, such as estimating default probabilities (and credit spreads) from a structural credit risk model, unobserved firm's asset volatility is usually treated as a constant. This assumption has consequences on the applications of structural credit risk models for CDS estimation and CDS predictions, especially when longer time periods are considered. This is because an input needed to provide an estimate of the credit spread is the expected volatility of the firm's assets over some future horizon. By way of example, pricing a CDS with 5-year maturity (the most liquid maturity of the CDS contract) in a structural model setting requires an estimate of the 5-year firm's asset volatility over the following 5-year period. This is precisely the reason why recent literature intends to jointly model the process for the firm asset value and the process for the volatility of firm's assets. Engle and Siriwardane (2018), for example provide a joint estimation of the firm's asset value process and a GARCH volatility model. However, as previous evidence undoubtedly shows, volatility of asset returns is better modeled with a long-memory GARCH-type process.

In this paper we consider an inverted analysis in which we back out the unobserved volatility of asset returns from market observable CDS spreads (i.e. CDS implied firm's asset volatility), and then use its time-series properties to forecast future firm's asset volatility for the purpose of CDS pricing. As previous findings on asset volatility already indicate, classical GARCH-type models which only incorporate short memory features may not be appropriate to model asset volatility. Therefore, the main focus of this paper is to improve the performance of structural credit risk models for the credit risk prediction by modeling the time-varying volatility of the unobserved firm asset value process as a long-memory process. From the practical point of view, the possibility to predict CDS implied volatility inherently implies the prediction of the future development of CDS spreads. Our results support this hypothesis.

The literature on implied volatility prediction has focused exclusively on equity volatility implied in equity options (see for example, Konstantinidi et al. 2008). Up to our knowledge no previous study has examined the predictability of firm's asset volatility implied in the CDS premia. To analyze the time-series properties and predictability of the CDS implied firm's asset volatility we use ARFIMA and ARIMA models. We find that the firm's asset volatility implied in the most liquid, 5-year CDS spreads has a very high degree of persistence. However, for most of the companies in the sample the degree of fractional integration is lower than 1, which implies that the process eventually mean-reverts in the very long-run. The in-sample-fit and out-of-sample forecasting performance of ARFIMA and ARIMA models shows that for most of the companies in the sample (roughly 75%) ARFIMA models perform better, independently of the selection criteria or error measure used. These results are supported for both the forecasts of firm's asset volatility, and the forecasts of credit spreads.

The rest of the paper is structured as follows. Section 2 describes the methodology applied to obtain firm's asset volatility implied in the CDS premia. Section 3 describes the data set and provides the preliminary analysis of the long-memory properties of firm's asset volatility implied in the CDS premia. Section 4 analyses in-sample fit of ARFIMA models and an ARIMA model. Section 5 provides results on the performance of out-of-sample prediction of CDS implied asset volatilities and CDS spreads implied by the structural model. Section 6 concludes.

2. Methodology

To estimate firm's asset volatility implied in the CDS premia we follow a two step approach of Forte and Lovreta (2019), Forte and Lovreta (2020) and Lovreta and Silaghi (2020). In the first step we estimate the firm's asset value using the structural model framework, and in the second we estimate implied volatility from market observable CDS spreads. In the last step, we fit ARFIMA models to the obtained time-series of CDS implied firm asset volatilities.

2.1 Structural credit risk models

This paper starts from the underlying assumptions of the models proposed initially by Black and Scholes (1973) and Merton (1974). Under the structural model setting firm's asset returns follow a geometric Brownian motion, interest rates are constant, there are no impediments to arbitrage, and a firm's capital structure can be collapsed into equity plus one issue of zero-coupon debt (with a maturity that matches the duration of the actual data). The market value of total assets (V) is assumed to evolve according to the continuous diffusion process of the following form:

$$dV = (\mu - \delta)Vdt + \sigma V dz, \qquad (1)$$

where μ is the expected rate of return on asset value, δ is the fraction of the asset value paid out to investors, σ is the asset return volatility, and z is a standard Brownian motion.

The Black– Scholes–Merton assumptions imply that the probability of default in essence depends on two factors: the size of the firm's asset value relative to the default point and the volatility of the firm's asset value. The closer the firm's asset value is to the default point, the more likely default is. On the other hand, the higher the volatility of the firm's assets, the higher the risk of a sudden deterioration in the firm's asset value. Given that debt has features similar to a short position in the put option, an increase in the volatility of the firm value process increases the value of the put option and reduces the value of the corporate risky debt. Therefore, increase in asset volatility increases the probability of default and, consequently, leads to higher credit spreads. This converts volatility of the underlying firm's assets in one of the key determinants of the price of credit sensitive instruments.

2.2 Implied asset volatility in CDS spreads

Following a structural model framework, a theoretical credit spread can be derived as the spread from issuing, at par value, a hypothetical bond with the same maturity as the CDS spread that serves as a benchmark (five years in this case).

$$p_n(V_t, 5) = \frac{c_n}{r} + e^{-5r}\left[p_n - \frac{c_n}{r}\right][1 - F_t(\tau_n)] + \left[(1-\alpha)\beta p_n - \frac{c_n}{r}\right] G_t(\tau_n), \qquad (2)$$

where $(1-\alpha)\beta$ is the recovery rate and r is the risk-free rate. In terms of CDS spread valuation, the market practice is to consider a fixed recovery rate of 40%. That is, to set $\rho = (1-\alpha)\beta$.

Following Ericsson et al. (2015), Forte and Lovreta (2019), and Forte and Lovreta (2020) we use the closed-form solution for the theoretical CDS spread, derived by equating the premium leg and the protection leg of the CDS contract:

$$CDS_t = g(V_t, \beta, \sigma) = \frac{r(1-\rho)G_t(\tau)}{1 - e^{-r\tau}[1 - F_t(\tau)] - G_t(\tau)} \qquad (3)$$

where, ρ is the exogenous recovery rate. To obtain the CDS implied firm's asset volatility we set the recovery rate to 40%. This choice for the recovery rate is consistent with the market practice and is consistent with the Moody's historical recovery rates for the senior unsecured bonds for our period under consideration. The expressions for $F_t(\tau_n)$ and $G_t(\tau_n)$ are as follows:

$$F_t(\tau) = \boldsymbol{\Phi}[h_{1t}(\tau)] + \left(\frac{V_t}{V_b}\right)^{-2a} \boldsymbol{\Phi}[h_{2t}(\tau)],$$

$$G_t(\tau) = \left(\frac{V_t}{V_b}\right)^{-a+z} \Phi[q_{1t}(\tau)] + \left(\frac{V_t}{V_b}\right)^{-a-z} \Phi[q_{2t}(\tau)],$$

with,

$$q_{1t}(\tau) = \frac{-b_t - z\sigma^2\tau}{\sigma\sqrt{\tau}}; \qquad q_{2t}(\tau) = \frac{-b_t + z\sigma^2\tau}{\sigma\sqrt{\tau}};$$

$$h_{1t}(\tau) = \frac{-b_t - a\sigma^2\tau}{\sigma\sqrt{\tau}}; \qquad h_{2t}(\tau) = \frac{-b_t + a\sigma^2\tau}{\sigma\sqrt{\tau}};$$

$$a = \frac{r - \delta - \frac{\sigma^2}{2}}{\sigma^2}; \qquad b_t = \ln\left(\frac{V_t}{V_b}\right); \qquad z = \frac{\sqrt{(a\sigma^2)^2 + 2r\sigma^2}}{\sigma^2}.$$

The expression for the theoretical credit spread can be inverted to derive the implied volatility in the observed CDS spread:

$$\sigma_{t,CDS} = g^{-1}(CDS_t | V_t, \beta) \tag{4}$$

2.3 The Leland and Toft model

In this paper we consider the Leland and Toft (1996) structural credit risk model to derive the underlying, unobservable, firm's asset values. The choice of the model is predominantly based on the feasibility for out-of-sample forecasting. Namely, this model assumes endogenous default barrier, which considerably facilitates the estimation of model parameters. Moreover, Lovreta and Silaghi (2020) have shown that the time development of the CDS implied volatility is largely not affected by choice of the structural model used to derive the unobservable firm's asset values. These authors show that the way in which the firm's asset value is derived will predominantly affect the level of CDS implied volatility, but not the way in which it changes over time. In fact, the expression (3) implies that the CDS implied firm's asset volatility depends on the structural model at hand only through the ratio of the firm's asset value (V) over the specific critical default point (V_b). Finally, an important advantage of using the smooth-pasting (SP) condition value of Leland and Toft (1996) is that these are shown to provide sensible credit spread estimates in line with those observed in the CDS market (Forte and Lovreta, 2012).

The Leland and Toft (1996) model specifies the value of debt as follows:

$$D(V;V_B,T) = \frac{C}{r} + \left(P - \frac{C}{r}\right)\left(\frac{1-e^{-rT}}{rT} - I(T)\right) + \left[(1-\alpha)V_b - \frac{C}{r}\right]J(T), \tag{5}$$

where, r is the risk-free rate and the expressions $I(T)$ and $J(T)$ are given by:

$$I(T) = \frac{1}{rT}\left[G(T) - e^{-rT}F(T)\right]$$

$$J(T) = \frac{1}{z\sigma\sqrt{T}}\left[-\left(\frac{V}{V_b}\right)^{-a+z} \Phi[q_1(T)]q_1(T) + \left(\frac{V_t}{V_b}\right)^{-a-z} \Phi[q_2(T)]q_2(T)\right]$$

For convenience, following Forte (2011), we express the specific critical point V_b as a fraction β of the nominal value of total debt P. That is, $V_b = \beta P$. Finally, under the Leland and Toft (1996) model the value of equity is given by:

$$E(V;V_b,T) = V + \frac{\vartheta C}{r}\left[1 - \left(\frac{V}{V_b}\right)^{-(a+z)}\right] - \alpha V_b \left(\frac{V}{V_b}\right)^{-(a+z)} - D(V;V_b,T), \tag{6}$$

The default barrier, V_b is defined using the smooth-pasting condition value.

$$\left.\frac{\partial E(V;V_b,T)}{\partial V}\right|_{V=V_b} = 0, \tag{7}$$

3. Data

In this paper, we use a sample of 52 non-financial companies that belong to the iTraxx Europe index, which we track during the period that spans from January 2004 to December 2016. Data on CDS spreads, as well as the data on the accounting items and market value of equity are downloaded from Datastream. For the purposes of this study, we consider only the most liquid Euro-denominated 5-year CDS contracts on senior unsecured debt.

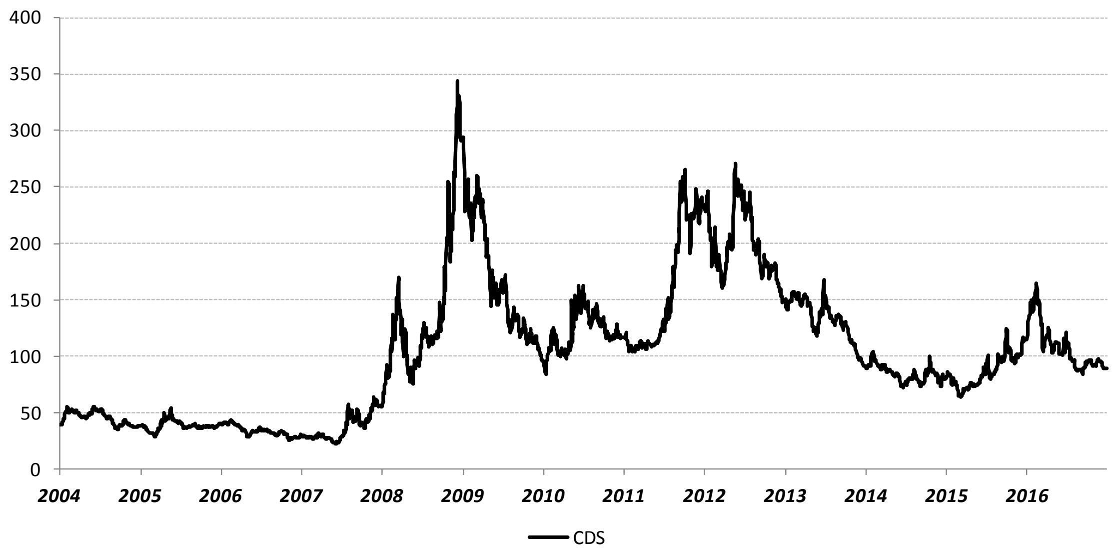
The average company in the sample has a market capitalization of €25.43 billion, a leverage of 0.52 and a historical equity volatility of 30%. Leverage is defined as the ratio of the book value of total liabilities to the proxy for the market value of the firm, historical equity volatility is the annualized standard deviation of the continuously compounded returns on equity. This main descriptive statistics for CDS spreads on a cross-sectional basis are reported in Table 1. The mean CDS spread is 104 bp, and ranges between the 30.67 bp and 389.97 bp. Time development of the cross-sectional mean of CDS spreads for our sample of companies is depicted in Figure 1.

Table 1. *Descriptive statistics - CDS spreads*

	Mean	Median	Max	Min	SD	Skew	Kurt
CDS	104.15	83.09	389.97	30.67	64.73	2.09	8.79

This table reports the main descriptive statistics for CDS spreads on a cross-sectional basis: the mean, median, standard deviation, minimum, maximum, skewness and kurtosis for a sample of 52 companies.

Figure 1. *Time development of the cross-sectional mean of CDS spreads*



3.1 Model estimation

For practical application of the Leland and Toft (1996) model, we set the maturity of the firm's aggregate debt, T, to the usual value of 6.76. Following Ericsson et al. (2015) we set the tax rate, ϑ, to 20%, realized costs of financial distress, α to 15%, The nominal amount of debt, P, is determined on the basis of accounting data on firm's short-term and long-term liabilities available in firm's balance sheets. The coupon paid to all the firm's debt holders is determined on the basis of interest expenses available in the firm's balance sheets. The payout rate δ computed as the average annualized payment of interest expenses and cash dividends divided by the value of the firm proxied by the sum of the market value of equity and the book value of total liabilities. The average payout ratio, δ, for our sample of companies (2.68%) is approximately equal to the average payout ratio of 2.65% reported in Ericsson et al. (2015).

The main descriptive statistics for the parameter estimates of the Leland and Toft (1996) model for our sample of 52 companies under analysis are shown in Table 2. The endogenous default barrier is lower than the nominal value of debt in all of the cases. Specifically, the parameter β is on average equal to 0.84, but ranges between 0.68 and 0.93. The long-term firm asset volatility that drives the estimates of the firm's asset values is equal to 13%, and ranges between the minimum of 6% and maximum of 26%. All estimated parameters are consistent with previous empirical evidence.

Table 2. *Descriptive statistics - parameter values*

	Mean	Median	Max	Min	SD	Skew	Kurt
β_{CDS}	0.84	0.83	0.93	0.68	0.05	-0.59	4.11
σ_{SPC}	0.13	0.12	0.26	0.06	0.04	0.99	5.35
δ	0.03	0.03	0.05	0.01	0.01	0.18	2.46
lev	0.52	0.51	0.86	0.27	0.14	0.26	2.60

This table reports the main descriptive statistics of the estimated parameter values on a cross-sectional basis: the mean, median, standard deviation, minimum, maximum, skewness and kurtosis for a sample of 52 companies.

3.2 CDS implied asset volatility

The time-development of the CDS implied asset volatility is depicted in Figure 2. The main descriptive statistics for CDS implied asset volatility on a cross-sectional basis for a sample of 52 companies are shown in Table 3. The mean CDS implied volatility is found to be 18%, ranging between the 6.7% and 32%. These results are in line with those reported in Lovreta and Silaghi (2020) who find the mean level of implied volatility in CDS spreads of 17.8%. As well, and in line with the previous literature on implied vs. realized equity volatility comparison, implied asset volatility is higher but less volatile than realized volatility.

Table 3. *Descriptive statistics - CDS implied asset volatility*

	Mean	Median	Max	Min	SD	Skew	Kurt
IV^{CDS}	0.18	0.18	0.32	0.07	0.06	0.21	2.53

This table reports the main descriptive statistics of CDS implied asset volatility on a cross-sectional basis: the mean, median, standard deviation, minimum, maximum, skewness, kurtosis the mean, median, standard deviation, minimum, maximum, skewness and kurtosis for a sample of 52 companies.

Figure 2. *Time development of CDS implied firm's asset volatility*

In our preliminary analysis of the CDS implied asset volatility we first conduct Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests for the presence of unit roots and Kwiatowski-Phllips-Schmidt-Shin (KPSS) test for stationarity. Test statistics for ADF, PP and KPSS test are reported in Table 4. The ADF and PP tests are designed to test the null hypothesis of unit roots against the I(0) alternative. The main summary results for the ADF test for the presence of unit roots in the CDS implied asset volatilities are presented in Table 4. The lag-length for the ADF test is selected on the basis of a downward *t-test*, i.e. starting from the maximum number of lags (p_{max}) the number of lags is reduced until the last lag of the first difference included is significant at the 5% level. The maximum number of lags is determined according to Schwert (1989) as $p_{max} = \left[12(T/100)^{1/4}\right]$, where $[\cdot]$ denotes the integer part and T is the sample size. ADF unit root tests are performed for the three possible alternatives: without constant and trend in the series, with constant and without trend, and with constant and trend. Reported ADF test statistics correspond to the model with the lowest Schwarz Information Criterion. The results on the ADF test show that for the vast majority of the companies we fail to reject the null hypothesis of unit-roots. To be precise, for 3 companies the ADF test rejects the null hypothesis of unit roots at 10% level, for 2 companies the null is rejected at 5% level, whereas for 47 companies (90% of the sample) we fail to reject the unit-root hypothesis.

Number of lags for the PP test is set to $p = \left[12(T/100)^{1/4}\right]$. PP unit root tests are performed as well for the three possible alternatives: without constant and trend in the series, with constant and without trend, and with constant and trend. Reported PP test statistics correspond to the model with the lowest Schwarz Information Criterion. In the case of the PP test the null hypothesis of unit-roots is rejected for 15 companies (28.8% of the sample): for 4 companies at 10% level, for 7 companies at 5% level and for 4 companies at 1% level. In contrast we fail to reject the null unit-root hypothesis for 37 companies (71.2% of the sample). However, Diebold and Rudebusch (1991) claim that ADF and PP tests are consistent against $I(d)$ alternatives, although perform relatively poorly in distinguishing between the $I(1)$ null hypothesis and the $I(d)$ alternative.

The KPSS test, on the contrary, is designed to test the null hypothesis of stationarity against the alternative of non-stationarity. For all of the firm-specific CDS implied asset volatility series the KPSS test rejects the null hypothesis of stationarity.

Table 4. *ADF, PP and KPSS test*

comp	ADF test					PP test				KPSS test		
	model	ADF_stat		p_val	lags	model	PP_stat		p_val	KPSS_stat		pval
1	ct	-2.6228		0.2843	21	c	-2.8034	*	0.0580	2.1642	***	<0.01
2	ct	-3.1919	*	0.0864	22	c	-3.0517	**	0.0307	2.4676	***	<0.01
3	c	-2.3741		0.1493	13	n	0.2036		0.7265	4.7340	***	<0.01
4	ct	-3.6124	**	0.0293	23	c	-3.8160	***	0.0035	3.3165	***	<0.01
5	n	-0.0494		0.6339	26	n	-0.4376		0.4917	1.6139	***	<0.01
6	n	-0.5191		0.4618	13	n	0.2135		0.7301	1.3665	***	<0.01
7	n	0.0262		0.6615	10	n	-0.4582		0.4841	2.9725	***	<0.01
8	n	-0.7549		0.3754	4	n	0.3509		0.7805	7.1915	***	<0.01
9	n	0.4839		0.8194	17	n	-0.6898		0.3993	5.6091	***	<0.01
10	c	-2.8305	*	0.0543	27	c	-3.2360	**	0.0184	1.4957	***	<0.01
11	n	0.3838		0.7925	17	n	-0.6947		0.3975	6.8168	***	<0.01
12	c	-2.3208		0.1658	23	c	-2.4507		0.1282	1.5821	***	<0.01
13	n	-0.2942		0.5442	19	c	-3.6006	***	0.0062	3.5441	***	<0.01
14	c	-1.1763		0.6613	24	n	0.5299		0.8301	8.7474	***	<0.01
15	n	-0.6210		0.4245	26	n	0.3976		0.7976	1.1137	***	<0.01
16	n	-0.9227		0.3140	15	c	-2.8481	*	0.0520	2.1775	***	<0.01
17	n	0.8269		0.8893	9	n	-1.1724		0.2225	7.8198	***	<0.01
18	n	1.1214		0.9325	22	n	-1.1902		0.2160	8.2669	***	<0.01
19	n	-0.5162		0.4629	13	c	-2.7079	*	0.0729	0.9711	***	<0.01
20	n	0.6698		0.8601	26	n	-0.6780		0.4036	2.1338	***	<0.01
21	n	-1.0586		0.2642	25	n	0.7485		0.8757	9.6311	***	<0.01
22	n	0.0840		0.6827	24	n	-0.3544		0.5221	4.9752	***	<0.01
23	n	-0.0603		0.6299	1	n	-0.6378		0.4183	2.9623	***	<0.01
24	c	-0.7426		0.8334	2	n	-1.0123		0.2812	9.8782	***	<0.01
25	n	-0.3323		0.5302	26	n	0.1423		0.7041	4.1582	***	<0.01
26	n	0.3924		0.7957	27	ct	-3.9842	***	0.0096	9.0133	***	<0.01
27	ct	-2.3236		0.4324	2	n	-0.7093		0.3921	6.8546	***	<0.01
28	n	-0.1037		0.6140	23	n	-0.4731		0.4786	1.9952	***	<0.01
29	n	0.0178		0.6585	5	n	-0.4604		0.4833	5.1593	***	<0.01
30	n	-0.0021		0.6512	0	n	-0.5568		0.4480	2.5464	***	<0.01
31	n	0.9559		0.9104	21	n	-0.6227		0.4239	4.9235	***	<0.01
32	c	-1.7687		0.3989	24	n	0.1995		0.7250	1.0952	***	<0.01
33	n	-0.1043		0.6137	19	n	-0.3782		0.5134	2.9021	***	<0.01
34	c	-2.8311	*	0.0542	23	n	0.0628		0.6750	1.1949	***	<0.01
35	n	-0.9526		0.3030	24	n	0.5504		0.8346	9.1048	***	<0.01
36	n	-0.3153		0.5364	20	c	-3.0464	**	0.0311	1.3238	***	<0.01
37	n	-0.8976		0.3232	9	n	0.2595		0.7470	1.3926	***	<0.01
38	n	-0.5829		0.4385	19	c	-2.7636	*	0.0639	5.2380	***	<0.01
39	n	0.1402		0.7033	22	n	-0.3436		0.5261	1.9338	***	<0.01
40	n	0.1611		0.7109	16	n	-0.4916		0.4719	2.8078	***	<0.01
41	n	-0.3134		0.5372	19	n	-0.1930		0.5812	2.0721	***	<0.01
42	n	-0.7434		0.3796	25	c	-2.8845	**	0.0475	2.6612	***	<0.01
43	c	-1.2997		0.6066	14	n	0.2790		0.7541	3.2713	***	<0.01
44	n	0.2874		0.7572	7	n	-0.6785		0.4034	6.4524	***	<0.01
45	n	0.3714		0.7880	5	n	-0.7242		0.3867	4.9031	***	<0.01
46	n	-0.4748		0.4780	26	c	-3.4778	***	0.0090	3.3100	***	<0.01
47	n	-0.0857		0.6205	24	n	-0.3395		0.5276	5.1265	***	<0.01
48	n	-0.1081		0.6123	26	c	-3.2321	**	0.0186	0.6541	**	0.02
49	c	-3.2793	**	0.0162	23	c	-3.2046	**	0.0200	4.2780	***	<0.01
50	n	1.2322		0.9448	25	n	-1.1085		0.2459	7.6719	***	<0.01
51	c	-1.9927		0.2997	1	n	0.0833		0.6825	1.7507	***	<0.01
52	n	0.3934		0.7960	24	c	-3.1988	**	0.0203	4.1443	***	<0.01

This table reports the ADF, PP and KPSS unit root tests on a firm-specific basis.

Preliminary evidence based on the ADF, PP and KPSS tests suggests that the choice between $I(0)$ and $I(1)$ might be too restrictive. In line with recent literature on stock market volatility, we examine a more flexible model that allows the series of implied volatility to be integrated of order d. We first estimate the memory parameter d utilizing the commonly applied semi-parametric approach of Geweke and Porter-Hudak (1983) and Robinson (1995), denoted here as GPH. The GPH approach estimates the parameter d using the following least-squares regression:

$$logI_y(\lambda_j) = \beta_0 - dlog\{4sin^2(\lambda_j/2)\} + u_j, \quad (8)$$

where $I_y(\lambda_j)$ denotes the sample periodogram for the series y evaluated at frequencies $\lambda_j = 2\pi j/T$, with $j = 1,2,\dots,T^m$. We estimate GPH statistics using differenced data following Velasco (1999) who shows that when data are differenced the estimator is consistent for $0.5 < d < 2$. Table 5 reports GPH estimates of the degree of fractional integration (d_{GPH}) for the common choice of the truncation parameter, $m = 0.5$. The tests for $d_{GPH} = 0$ are rejected for all companies in the sample. The standard error of the d_{GPH} estimates is 0.095 and is calculated using the asymptotic variance $(\pi^2/6)$. The mean d_{GPH} value is equal to 0.92, and ranges between the minimum of 0.56 to the maximum value of 1.14. That is, the degree of fractional integration is higher than 0.5 in all of the cases. For the most of companies in the sample (75% of the sample) the estimated degree of fractional integration is lower than 1 and for 13 companies (25% of the sample) is higher than 1. However, a formal test for the hypothesis that $d = 1$ (a test if the estimated $\hat{d}_{GPH}$ is statistically different from 1, $H_0: \hat{d}_{GPH} = 1$) could not be rejected for 39 companies (75% of the sample), suggesting that the implied volatility series for this sub-set of companies are in the non-stationary region. In the context of our analysis a degree of persistence $d \geq 1$ would imply non-stationarity and no mean reversion (Gil-Alana, 2008; Gil-Alana and Hualde, 2009), i.e. that the shock to the process would have a permanent effect. This would further suggest that ARIMA models would better fit this sub-sample of companies.

We also consider another common semi-parametric approach for estimating the memory parameter d: the exact local Whittle estimator (ELW) proposed by Shimotsu and Phillips (2005). The asymptotic standard error of the ELW estimate is $(4m)^{-1/2}$ and equals 0.066. The bandwidth parameter m is set in both cases to $T^{0.5}$. The results obtained for the ELW estimator d_{ELW} are shown in Table 5. The tests for $d_{ELW} = 0$ are as well rejected for all companies in the sample. The mean d_{ELW} value is slightly lower and equal to 0.89. The d_{ELW} estimates range between the minimum of 0.53 to the maximum value of 1.12, in line with the results previously obtained with the GPH estimator. In all of the cases the degree of fractional integration is higher than 0.5 as well. We fail to reject the null hypothesis H_0: $\hat{d}_{ELW} = 1$ for 26 companies. Therefore, the ELW estimates are not statistically significantly different from 1 for 50% of the sample. For the remaining 26 companies the ELW estimates show that the process has a high degree of persistence but is still mean reverting, implying that a shock in the process eventually dies in the very long-run (Gil-Alana and Robinson, 1997).

As a conclusion, the GPH and ELW estimates indicate that all CDS implied volatility series have a high degree of fractional integration. The order of integration is in general lower than 1. However, statistically, a formal hypothesis test fails to reject an integrated process for many companies. The GPH and ELW semi-parametric approaches undoubtedly show that d falls in the area $d > 0.5$. Therefore, further analysis will consider the estimation of non-stationary ARFIMA process. Following Konstantinidi et al. (2008), we estimate ARFIMA models using the first-difference of the original CDS implied volatility series.

Table 5. *GPH and ELW estimates*

comp	d_{GPH}	H_0: *d=1* *p-val*	d_{ELW}	H_0: *d=1* *p-val*
1	0.863	0.146	0.802	0.003
2	0.807	0.042	0.804	0.003
3	0.973	0.773	0.892	0.100
4	0.560	0.000	0.672	0.000
5	0.961	0.678	0.915	0.195
6	1.008	0.931	1.015	0.818
7	1.124	0.191	1.094	0.150
8	0.967	0.727	0.941	0.370
9	1.011	0.905	0.972	0.669
10	0.847	0.105	0.783	0.001
11	0.919	0.389	0.898	0.118
12	0.925	0.430	0.836	0.013
13	0.810	0.044	0.689	0.000
14	0.815	0.050	0.782	0.001
15	0.880	0.203	0.911	0.173
16	0.937	0.507	0.875	0.056
17	0.877	0.192	0.906	0.152
18	0.943	0.547	0.886	0.081
19	0.918	0.387	0.902	0.136
20	0.933	0.480	0.914	0.189
21	0.896	0.271	0.928	0.270
22	0.972	0.768	0.988	0.858
23	0.867	0.158	0.833	0.011
24	1.027	0.776	1.014	0.836
25	0.974	0.785	0.528	0.517
26	0.797	0.032	0.756	0.000
27	1.067	0.480	1.038	0.567
28	1.001	0.995	0.955	0.490
29	0.860	0.140	0.880	0.068
30	0.948	0.583	0.862	0.035
31	1.090	0.343	1.030	0.652
32	1.000	0.996	0.928	0.274
33	0.840	0.090	0.852	0.024
34	0.944	0.555	0.888	0.088
35	1.096	0.308	1.022	0.732
36	0.899	0.287	0.787	0.001
37	1.140	0.140	1.125	0.057
38	0.765	0.013	0.831	0.010
39	0.818	0.055	0.858	0.031
40	0.864	0.149	0.846	0.019
41	0.907	0.323	0.877	0.060
42	0.781	0.021	0.832	0.010
43	1.059	0.530	1.029	0.658
44	0.911	0.346	0.894	0.107
45	1.021	0.823	1.005	0.938
46	0.831	0.073	0.860	0.032
47	1.012	0.901	1.022	0.732
48	0.786	0.024	0.813	0.004
49	0.799	0.034	0.778	0.001
50	0.851	0.116	0.898	0.121
51	1.046	0.626	1.007	0.915
52	0.758	0.010	0.737	0.000

This table reports the fractional degree of persistence estimated using the Geweke and Porter-Hudak (1983), d_{GPH}, and the exact local Whittle estimator of Shimotsu and Phillips (2005), d_{ELW}.

4. In-sample fit

The first step in the CDS implied volatility prediction is to analyze in-sample fit of the long-memory models. We estimate the ARFIMA (i.e. Autoregressive Fractionally Integrated Moving Average) models originally introduced by Granger and Joyeux (1980) and Hosking (1981). The ARFIMA model allows a fractionally integrated process $I(d)$ for the volatility, as it allows that the autocorrelation function decays at a slow hyperbolic rate. The ARFIMA models extend the traditional ARIMA models by allowing intermediate values of d. These models have been extensively applied in the literature on daily stock returns (see for example, Lo, 1991; Jacobsen, 1996; Gil-Alana, 2006; among others).

To date, the literature has not examined the predictability of the CDS implied asset volatility. In contrast, there are several studies on predictability of implied equity volatility. These studies are primarily based on the use of a set of economic variables as predictors (Harvey and Whaley, 1992; Dumas et al., 1998; Guo, 2000; Gonçalves and Guidolin, 2006). A notable distinction is the study of Konstantinidi et al. (2008), who consider economic and statistical models for implied volatility in equity options, including the ARFIMA models. They show that implied volatility in equity options is highly persistent, which implies that patterns in implied volatility are predictable. Konstantinidi et al. (2008) consider $ARFIMA(p,d,q)$ models, to account for both short and long memory characteristics in implied volatility dynamics.

The ARFIMA models, replace the difference operator $(1-L)$ of the ARIMA model with the fractional difference operator $(1-L)^d$. The parameter d, is the parameter reflecting the degree of fractional integration. The $ARFIMA(p,d,q)$ model is defined as follows:

$$\Psi(L)(1-L)^d(y_t-\mu)=\Theta(L)\varepsilon_t \quad (9)$$

$$\varepsilon_t = z_t \sigma_t, \qquad z_t \sim N(0,1) \tag{10}$$

where, y_t is the time-series of observations, ε_t is the i.i.d. error term, μ is the unconditional mean, $(1-L)^d$ is the fractional difference operator, $\Psi(L) = 1 + \psi_1 L + \cdots + \psi_p L^p$ is the autoregressive polynomial and $\Theta(L) = 1 + \theta_1 L + \cdots + \theta_p L^p$ is the moving average polynomial. The fractional difference operator $(1-L)^d$ can be obtained through a Maclaurin series expansion; considering that this operator behaves as an infinite lagged decreasing series its coefficients contribute as a "long memory" effect in the ARFIMA model. ARFIMA model is stationary and invertible if $\Psi(L)$ and $\Theta(L)$ have no common roots and that their roots lie outside the unit circle, and $d \in (-0.5, 0.5)$. When setting the fractional integration parameter d to integer values, ARFIMA model is reduced to an ARIMA model when $d = 1$ and to an ARMA model when $d = 0$. Olbermann et al. (2006) analyzed the invariance of the fractional integration parameter when the process is an $ARFIMA(p, d, q)$ model where $d \in [0.5,1.5)$ and $d = d^* + 1$ with its first difference, and observed that this property holds when $\in [0.5,1.0]$. In our case, given the preliminary evidence we estimate the model in first differences of the original time-series of CDS implied asset volatilities and estimate the degree of fractional integration from the $d = d^* + 1$, where d^* is the parameter estimated from the model in first-differences. That is, in our case, y_t in Equation 9 is equal to ΔIV_t^{CDS} and μ denotes the expected value of ΔIV_t^{CDS}. We have examined different $ARFIMA(p, d, q)$ models following the approach usually considered in the literature (Cheung, 1993; Konstantinidi et al. 2008; Kasman et al. 2009). In particular, we follow the approach of Konstantinidi et al. (2008) which is based on the value that minimizes BIC criterion with the aim to avoid over-fitting. In that context, we estimated ARFIMA models with $p + q \leq 5$ obtaining that $ARFIMA(0, d, 0)$, $ARFIMA(1, d, 0)$, $ARFIMA(0, d, 1)$ and $ARFIMA(1, d, 1)$ are the models that meet these criteria. These additional results are provided in the Appendix B, Table B.1 and Table B.2. Information criteria are used to choose the ARFIMA model that provides the best fit to the data. In addition to that, we also use error measures to evaluate prediction error with ARFIMA models.

Table 6 shows the main descriptive statistics for the fractional integration parameter d^* estimated with ARFIMA model in first differences. For most of the firms the parameter d^* is statistically different from 0 with a significance of 10% in a range that varies from 57.65% to 80.77% of companies considered depending on the ARFIMA model. We observe that as we increase the number of parameters in the model the number of companies for which the parameter d^* is statistically significant decays. It is important to point that in general, d^* remains in the range $(-0.5\ ,0)$ for most of the companies, as 88.46 % to 94.23% of companies have a negative value of d^*, which is consistent with the estimates of the degree of persistence previously calculated by GPH and ELW semi-parametric methods. In cross sectional terms, the value of fractional integration parameter is negative but very close to 0, which confirms that CDS implied volatility is a highly persistent time series; when correcting this value with the first difference, the mean of the parameter d observed varies from 0.93068 to 0.95904. Table 7 details the estimated values of the fractional integration parameter d^* at the firm level with its corresponding t-stat.

Table 6. *Main descriptive statistics for the fractional integration parameter*

	ARFIMA(0,d,0)	**ARFIMA(1,d,0)**	**ARFIMA(0,d,1)**	**ARFIMA(1,d,1)**
mean (d*)	-0.04343	-0.04428	-0.04096	-0.06932
max	0.07422	0.05743	0.06114	0.02213
min	-0.10193	-0.09666	-0.09596	-0.39272
std	0.03637	0.03159	0.03123	0.07744
mean (d)	0.95657	0.95572	0.95904	0.93068
significant at 10%	42 (80.77%)	32 (61.54%)	31 (59.62%)	30 (57.69%)
significant at 5%	42 (80.77%)	29 (55.77%)	25 (48.08%)	25 (48.08%)
significant at 1%	37 (71.15%)	20 (38.46%)	19 (36.54%)	19 (36.54%)
d<0	47	49	49	46
d>0	5	3	3	6

This table reports the main descriptive statistics of the fractional integration parameter (estimated with ARFIMA models in first differences) on a cross-sectional basis: the mean (of d and d^{*}), maximum, minimum and standard deviation, for a sample of 52 companies, as well as the number of companies for which the parameter d is statistically significant at 10%, 5% and 1%, and for which is positive and negative.

Table 7. *Fractional integration parameter at the firm level*

comp	ARFIMA(0,d,0)		ARFIMA(1,d,0)		ARFIMA(0,d,1)		ARFIMA(1,d,1)	
	d	*t-stat*	*d*	*t-stat*	*d*	*t-stat*	*d*	*t-stat*
1	-0.0544	-3.9877	-0.0387	-1.6678	-0.0401	-1.7344	-0.0401	-2.0354
2	-0.0814	-6.1085	-0.0465	-2.0743	-0.0502	-2.1859	-0.0553	-3.4237
3	-0.0550	-4.0503	-0.0568	-2.5538	-0.0567	-2.5997	-0.0573	-1.7398
4	-0.0018	-0.1382	0.0574	2.5962	0.0611	2.3562	-0.2741	-4.8807
5	-0.0302	-2.2395	-0.0127	-0.5597	-0.0153	-0.6894	-0.0092	-0.5827
6	-0.0562	-4.3299	-0.0415	-2.0641	-0.0389	-1.7738	-0.0137	-0.3885
7	-0.0635	-4.7830	-0.0594	-2.8156	-0.0595	-2.8102	-0.0594	-2.7471
8	-0.0345	-2.4771	-0.0652	-2.8394	-0.0620	-2.9394	-0.0823	-2.4406
9	-0.0309	-2.3480	-0.0091	-0.4248	-0.0097	-0.4434	-0.0191	-1.3348
10	-0.0632	-4.6699	-0.0611	-2.7580	-0.0611	-2.7970	-0.0642	-1.7361
11	-0.0575	-4.4004	-0.0291	-1.3952	-0.0235	-0.9965	-0.0123	-0.3986
12	-0.0821	-5.9791	-0.0967	-4.3149	-0.0960	-4.4691	-0.1012	-1.9452
13	0.0742	4.8204	-0.0767	-2.5193	-0.0257	-1.2063	-0.1567	-3.5999
14	-0.0496	-3.4868	-0.0858	-3.4846	-0.0833	-3.7198	-0.1815	-3.4147
15	-0.0859	-6.5572	-0.0725	-3.5144	-0.0715	-3.2982	-0.0748	-3.5103
16	-0.0585	-4.2734	-0.0895	-4.1976	-0.0895	-4.4530	-0.0819	-3.9662
17	-0.0466	-3.4191	-0.0581	-2.6441	-0.0588	-2.6920	-0.0570	-2.1510
18	-0.0436	-3.0852	-0.0879	-3.8024	-0.0846	-4.0479	-0.0903	-2.9937
19	-0.0696	-5.2229	-0.0447	-2.0393	-0.0435	-1.8569	-0.0622	-4.4688
20	-0.0866	-6.5877	-0.0847	-4.2061	-0.0842	-3.8975	0.0019	0.0397
21	-0.0631	-4.7643	-0.0444	-2.0777	-0.0428	-1.8816	-0.0407	-1.5897
22	-0.0464	-3.5359	-0.0233	-1.0977	-0.0241	-1.1000	-0.0300	-1.6668
23	0.0347	2.4362	-0.0137	-0.5700	-0.0065	-0.3103	-0.0220	-0.7249
24	-0.0820	-6.1156	-0.0907	-4.2823	-0.0901	-4.4089	-0.0873	-2.5290
25	-0.0896	-6.9743	-0.0691	-3.5094	-0.0629	-2.7875	-0.0018	-0.0452
26	-0.1019	-7.8615	-0.0681	-3.3753	-0.0544	-2.1643	0.0206	0.4870
27	-0.0183	-1.4101	-0.0116	-0.5806	-0.0106	-0.4993	0.0170	0.4776
28	0.0520	3.5418	-0.0320	-1.2680	-0.0102	-0.5006	-0.0573	-1.7365
29	-0.0451	-3.3606	-0.0454	-2.1047	-0.0454	-2.1389	-0.0459	-1.4317
30	0.0117	0.8450	-0.0062	-0.2669	-0.0053	-0.2391	-0.0188	-0.4094
31	-0.0323	-2.4697	-0.0429	-2.2013	-0.0437	-2.2460	-0.0397	-1.3904
32	-0.0267	-1.9757	0.0126	0.5363	0.0074	0.3092	0.0023	0.1120
33	-0.0172	-1.2214	-0.0303	-1.2227	-0.0277	-1.2251	-0.1464	-3.1976
34	-0.0355	-2.7382	0.0192	0.8998	0.0232	0.9488	0.0164	0.6612
35	-0.0906	-6.9570	-0.0552	-2.6035	-0.0540	-2.3505	-0.0639	-2.7060
36	-0.0577	-4.2053	-0.0363	-1.5305	-0.0392	-1.6675	-0.2000	-3.9540
37	-0.0785	-6.3307	-0.0360	-1.8476	-0.0321	-1.4693	-0.0368	-1.4715
38	-0.0586	-4.4158	-0.0172	-0.7744	-0.0186	-0.7847	-0.1794	-3.6213
39	-0.0613	-4.4362	-0.0704	-3.0147	-0.0686	-3.1859	-0.0471	-3.1049
40	-0.0490	-3.6605	-0.0285	-1.3074	-0.0255	-1.0677	-0.0188	-0.6611
41	-0.0398	-2.8332	-0.0628	-2.6060	-0.0590	-2.6932	-0.1341	-2.9595
42	-0.0707	-5.1739	-0.0903	-4.1415	-0.0913	-4.3027	-0.0874	-4.5270
43	-0.0652	-5.1038	-0.0578	-3.0107	-0.0564	-2.7509	0.0221	0.4809
44	-0.0567	-4.1442	-0.0654	-2.9145	-0.0654	-2.9668	-0.1039	-2.2523
45	-0.0464	-3.3747	-0.0817	-3.8389	-0.0797	-4.0375	-0.0789	-3.2136
46	0.0424	2.7754	-0.0442	-1.1556	-0.0004	-0.0174	-0.3927	-8.6056
47	-0.0366	-2.7549	-0.0262	-1.2265	-0.0267	-1.2418	-0.0301	-0.8982
48	-0.0106	-0.7789	-0.0106	-0.4775	-0.0106	-0.4597	-0.0105	-0.1712
49	-0.0359	-2.5822	-0.0509	-2.1806	-0.0496	-2.2494	-0.1124	-2.1264
50	-0.0405	-3.0329	-0.0096	-0.4311	-0.0126	-0.5517	-0.0187	-1.0923
51	-0.0156	-1.1495	-0.0147	-0.6583	-0.0147	-0.6656	-0.0146	-0.4554
52	-0.0502	-3.6788	-0.0395	-1.7413	-0.0390	-1.6512	-0.1731	-3.0233

This table reports the the estimated values of the fractional integration parameter d^{*} at the firm level with its corresponding *t-stat*.

To evaluate which model provides the best in-sample fit to the data we use several information criteria: Akaike (AIC), Bayesian (BIC), and Hannan-Quinn (HQ) criteria. These information criterion methods are commonly used in model selection when evaluating models with different number of parameters, as they consider the contribution of goodness of fit (log-likelihood) and the complexity of the model as a penalty term. It should be highlighted that in the model selection problem models with different numbers of parameters are being compared, and in general, log-likelihood increases as the number of parameters are added into the model; thus, cross-sectional mean of ARFIMA(0,d,0) log-likelihood is smaller than the other models which have 1 or 2 additional parameters. For this reason, the penalty term which compensates the inclusion of an additional parameter has a critical effect in model selection. In addition to information criteria, Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) of the standard residuals have also been considered. The analytical formulation is as follows:

$$AIC = -2\frac{\log L}{n} + 2\frac{q}{n} \tag{11}$$

$$BIC = -2\frac{\log L}{n} + q\frac{\log n}{n} \tag{12}$$

$$HQ = -2\frac{\log L}{n} + 2\frac{q \log(\log n)}{n} \tag{13}$$

$$MAE = \frac{1}{N}\sum_{t=1}^{N}|e_t| \tag{14}$$

$$RMSE = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(e_t)^2} \tag{15}$$

where $\log L$ is the log-likelihood function, q is the number of estimated parameters, n is the number of observations and e_t contains the residuals of the estimated model in differences.

Table 8 reports the cross sectional results of the information criteria and error measures and the number of firms for which the model is selected as the best one.

Looking at the cross-sectional level (Panel A of Table 8), the $ARFIMA(1, d, 1)$ seem to provide the best fit according to the AIC, BIC and HQ criteria as well as the RMSE measure. In contrast, according to the MAE measure $ARIMA(1,1)$ seems to provide the best fit to the data. At the firm level, Panel B of Table 8, reports the number of firms for which the model is preferred by each of the measures. The AIC and BIC criteria provide similar results where $ARFIMA(0, d, 0)$ is selected by 19 (36.54%) and 21 (40.38%) of 52 firms respectively, followed by $ARIMA(1,1)$. For the HQ criteria we observe that the best selection is mainly distributed between $ARIMA(1,1)$ chosen by 15 (28.85%) firms, $ARFIMA(1, d, 1)$ chosen by 14 (26.92%) firms and $ARFIMA(0, d, 0)$ that is selected by 13 (25%) firms. Regarding error measures, RMSE measure clearly points to $ARFIMA(1, d, 1)$ as the model that fits best the data in 41 of 52 firms (78.85%). Finally, model selection by MAE measure is almost equally distributed along the estimated models (similar to HQ).

Overall, the results of the in-sample-fit of ARFIMA models vs. ARIMA model show that inclusion of the fractional differencing parameter in modeling CDS implied volatility improves the model fit on average, independently of the selection criteria or error measure used. To be specific, ARFIMA models provide a better in-sample-fit compared to ARIMA model in 67.31% (Akaike), 73.08% (Schwarz), 71.15% (HQ), 76.92% (RMSE), and 88.46% (MAE) of the cases. These results are consistent with the presence of long-memory in the CDS implied asset volatility for most of the companies in the sample.

Table 8. *Information criteria and error measures*

	ARFIMA(0,d,0)	ARFIMA(1,d,0)	ARFIMA(0,d,1)	ARFIMA(1,d,1)	ARIMA(1,1)
Panel A: Cross-section					
Log-L	16,293.70	16,295.33	16,295.18	16,297.53	16,295.52
Akaike	-9.6053648	-9.6057352	-9.6056498	-9.6064473	-9.6058467
Schwarz	-9.6040143	-9.6039350	-9.6038499	-9.6041970	-9.6040461
HQ	-9.6061651	-9.6068028	-9.6067177	-9.6077817	-9.6069139
MAE	0.0014533	0.0014767	0.0012594	0.0014408	0.0011395
RMSE	0.0021096	0.0021084	0.0021086	0.0021061	0.0021083
Panel B: Firm level					
Akaike	19	6	1	9	17
Schwarz	21	5	4	8	14
HQ	13	6	4	14	15
MAE	6	13	13	8	12
RMSE	0	3	2	41	6

This table reports the cross sectional results of the information criteria (Akaike, Schwarz, HQ) and error measures (RMSE, MAE) and the number of firms for which the model is selected as the best one.

5. Out-of-sample prediction

To conduct out-of-sample prediction of CDS implied volatilities, we select an initial subsample with a period of ten years, from January 1st, 2004 to December 31st, 2013. This time period includes 2,610 daily observations of CDS implied asset volatilities per company. We estimate the ARFIMA and ARIMA models over this period of time and generate firm's asset volatility forecasts 1 day ahead. Once we perform the initial forecasting, we generate a rolling window over the remaining out-of-sample period which spans from January 1st, 2014 to December 31st, 2016. To do that, we shift forward 1 observation (1 day) the fixed-length estimation window of 2,610 observations and re-estimate again the model parameters. This process has been repeated over the out-of-sample period, obtaining a total of 782 forecasts per model and company. We perform the estimation of ARFIMA models in first differences following the same procedure explained in the previous section. The one-step ahead forecasts are formed following Konstantinidi et al. (2008):

$$E(IV_{t+1}^{CDS}|I_t) = IV_t^{CDS} + \mu + \sum_{k=1}^{\infty} \pi_k (\Delta IV_{t-k+1}^{CDS} - \mu) \tag{16}$$

where $\pi_k = \sum_{i=0}^{k} (b_i + \phi b_{i-1})(-\theta)^{k-i}$, $b_i = \frac{\Gamma(-d+i)}{\Gamma(-d)\Gamma(i+1)}$ and $\Gamma(\cdot)$ denotes the gamma function.

To evaluate the accuracy of forecasted volatility we proceed in two steps. First, we analyze which model provides better volatility forecasts by comparing the forecasted firm's asset volatility to CDS implied volatility on the following day. Secondly, we use the forecasted firm's asset volatility to generate model CDS forecasts (i.e. using the structural credit risk model) and compare it with subsequently realized market observable CDS spreads. It is important to note that in this case, to avoid any look-ahead bias, we generate CDS forecasts on the basis of the Leland and Toft (1996) structural credit risk model using only past information on market capitalization, and accounting items (i.e. information available up to *t*).

5.1 Out-of-sample firm's asset volatility prediction

In this section we evaluate the performance of 1-day-ahead firm's asset volatility forecasts. To ease the exposition, we consider $\sigma_{R,t+1}$ as the CDS implied volatility (IV^{CDS}) at time $t+1$, and $\sigma_{F,t}$ as the one-day forecast of firm's asset volatility made a time t. To evaluate forecasting performance we use several prediction error statistics: Mean Absolute Error (MAE), Root Mean Square Error (RMSE) and Mean Correct Prediction (MCP), which are defined as follows:

$$MAE = \frac{1}{N} \sum_{t=1}^{N} \left| \sigma_{R,t+1} - \sigma_{F,t} \right| \tag{17}$$

$$RMSE = \sqrt{\frac{1}{N} \sum_{t=1}^{N} \left(\sigma_{R,t+1} - \sigma_{F,t} \right)^2} \tag{18}$$

$$MCP = \frac{1}{N} \sum_{t=1}^{N} \left\{ 1 \text{ if } sign\left(\sigma_{R,t+1}\right) = sign\left(\sigma_{F,t}\right) \text{ else } 0 \right\} \tag{19}$$

Table 9 summarizes the out-of-sample forecasting performance for each model and shows the number of firms for which a specified model performs the best according to the MAE, RMSE and MCP criteria. On average (see Panel A of Table 9), the cross-sectional mean of prediction error shows that for the RMSE criteria $ARFIMA(1,d,0)$ is the best model closely followed by $ARFIMA(0,d,0)$ and $ARFIMA(0,d,1)$. For the MAE and MCP, $ARFIMA(0,d,0)$ is the model which produces the lowest forecast error on average. In cross-section, $ARFIMA(1,d,1)$ and $ARIMA(1,1)$ are generally the models with higher prediction error according to the three forecasting error measures (RMSE, MAE and MCP).

Table 9. *Out-of-sample performance*

	ARFIMA(0,d,0)	**ARFIMA(1,d,0)**	**ARFIMA(0,d,1)**	**ARFIMA(1,d,1)**	**ARIMA(1,1)**
Panel A: Cross-section					
RMSE	0.16307	0.16306	0.16307	0.16313	0.16319
MAE	0.11643	0.11648	0.11648	0.11660	0.11655
MCP	0.72057	0.71912	0.71926	0.71705	0.71636
Panel B: Firm level					
RMSE	20	6	6	9	11
MAE	22	3	6	8	13
MCP	13	8	12	10	13

This table reports the cross sectional results of the out-of-sample prediction error measures (RMSE, MAE and MCP) and the number of firms for which the model is selected as the best one.

When taking into account the firm-level forecasting performance (see Panel B of Table 9) we observe that according to the RMSE and MAE, $ARFIMA(0,d,0)$ is the preferred model for the majority of firms. To be specific, the $ARFIMA(0,d,0)$ model provides the lowest RMSE prediction error for 20 (38.46%) companies and the lowest MAE prediction error for 22 (42.37%) companies. Overall, ARFIMA models outperform ARIMA model in 78.85% (RMSE) and 75% (MAE) of the cases. The integrated $ARIMA(1,1)$ model outperforms ARFIMA models in 21.15% (RMSE) and 25% (MAE) of the cases.

The MCP measure at the firm-level goes in the same direction, ARFIMA models outperform in 76.79% and ARIMA model in 23.21% of the cases. However, the best prediction is almost uniformly distributed through the specific ARFIMA models: model $ARFIMA(0, d, 0)$ is the best model for 13 companies, followed by $ARFIMA(0, d, 1)$ for 12 companies, $ARFIMA(1, d, 1)$ for 10 companies, and finally $ARFIMA(1, d, 0)$ for 8 companies.[29] Therefore, in terms of the MCP measure at the firm-level all the ARFIMA models seem to behave similarly. It is important to reiterate the difference among prediction error measures: RMSE and MAE provide an analytical measure of prediction error whereas MCP indicates the ability of the models to predict the correct direction of change of volatility (positive when volatility increases and negative when volatility decreases).

These results are completely in line with our previous findings that for the majority of the companies considered CDS implied firm's asset volatility is a highly persistent time-series but with the degree of fractional integration lower than 1. As a result, we observe that generally the models which minimize the forecasting error are precisely the ARFIMA models, in contrast to the integrated ARIMA model. In addition, we observe that the specific ARFIMA model which provides the best forecasting performance when modeling implied firm's asset volatility is $ARFIMA(0, d, 0)$. Table 10, Table 11 and Table 12 show the forecasting prediction error (RMSE, MAE and MCP) of firm's asset volatility implied from CDS data for each firm and model.

[29] Note that the number of companies for which the respective model outperforms other models according to the MCP adds up to 56 instead of 52. This is because for some companies two competing models are selected with the same MCP prediction error. For more details see Table 12.

Table 10. *Forecasting performance - RMSE*

comp	ARFIMA(0,d,0)	ARFIMA(1,d,0)	ARFIMA(0,d,1)	ARFIMA(1,d,1)	ARIMA(1,1)
1	0.12918	0.12919	0.12918	0.12911	0.12928
2	0.10618	0.10601	0.10605	0.10616	0.10585
3	0.14639	0.14639	0.14639	0.14642	0.14630
4	0.21081	0.21158	0.21156	0.21224	0.21063
5	0.10599	0.10598	0.10598	0.10594	0.10596
6	0.11296	0.11296	0.11296	0.11297	0.11283
7	0.12519	0.12525	0.12525	0.12530	0.12520
8	0.19877	0.19860	0.19859	0.19865	0.19908
9	0.09052	0.09051	0.09051	0.09062	0.09059
10	0.18506	0.18509	0.18509	0.18504	0.18503
11	0.14955	0.14978	0.14981	0.14979	0.14984
12	0.19377	0.19413	0.19410	0.19423	0.19466
13	0.20850	0.20697	0.20729	0.20672	0.20784
14	0.19370	0.19417	0.19412	0.19419	0.19457
15	0.11522	0.11524	0.11524	0.11531	0.11527
16	0.17819	0.17829	0.17829	0.17835	0.17847
17	0.10608	0.10606	0.10606	0.10608	0.10613
18	0.09461	0.09444	0.09444	0.09445	0.09437
19	0.17508	0.17502	0.17503	0.17544	0.17498
20	0.14494	0.14500	0.14502	0.14478	0.14481
21	0.15543	0.15541	0.15541	0.15550	0.15546
22	0.15174	0.15185	0.15184	0.15200	0.15205
23	0.15690	0.15700	0.15698	0.15701	0.15701
24	0.17171	0.17179	0.17179	0.17180	0.17205
25	0.12641	0.12649	0.12652	0.12674	0.12676
26	0.13772	0.13745	0.13740	0.13831	0.13801
27	0.08228	0.08226	0.08226	0.08230	0.08228
28	0.10962	0.10947	0.10947	0.10943	0.10956
29	0.08888	0.08886	0.08886	0.08886	0.08886
30	0.18930	0.18911	0.18913	0.18903	0.18912
31	0.07991	0.07992	0.07987	0.07995	0.07988
32	0.16412	0.16440	0.16431	0.16457	0.16445
33	0.44150	0.44196	0.44182	0.44253	0.44285
34	0.11925	0.11990	0.11988	0.11990	0.11982
35	0.18238	0.18222	0.18223	0.18222	0.18221
36	0.14519	0.14531	0.14529	0.14552	0.14589
37	0.18313	0.18303	0.18304	0.18309	0.18319
38	0.14181	0.14197	0.14196	0.14216	0.14199
39	0.19943	0.19946	0.19946	0.19994	0.19924
40	0.11954	0.11951	0.11952	0.11957	0.11957
41	0.17064	0.17066	0.17066	0.17071	0.17096
42	0.21304	0.21303	0.21301	0.21302	0.21312
43	0.17838	0.17844	0.17845	0.17852	0.17851
44	0.10362	0.10367	0.10369	0.10364	0.10381
45	0.12150	0.12160	0.12159	0.12164	0.12148
46	0.56730	0.56549	0.56616	0.56493	0.56717
47	0.14748	0.14759	0.14757	0.14760	0.14782
48	0.20896	0.20901	0.20901	0.20910	0.20903
49	0.28355	0.28345	0.28347	0.28352	0.28351
50	0.11812	0.11810	0.11809	0.11811	0.11825
51	0.13055	0.13055	0.13055	0.13043	0.13068
52	0.11940	0.11943	0.11942	0.11924	0.11953

Table 11. *Forecasting performance* - MAE

comp	ARFIMA(0,d,0)	ARFIMA(1,d,0)	ARFIMA(0,d,1)	ARFIMA(1,d,1)	ARIMA(1,1)
1	0.08948	0.08953	0.08952	0.08946	0.08963
2	0.07577	0.07564	0.07567	0.07570	0.07543
3	0.11001	0.11002	0.11002	0.11007	0.10985
4	0.15936	0.16014	0.16007	0.16084	0.15922
5	0.07571	0.07580	0.07579	0.07600	0.07604
6	0.07982	0.07979	0.07979	0.07982	0.07978
7	0.09237	0.09242	0.09242	0.09243	0.09228
8	0.14961	0.14961	0.14958	0.14961	0.14986
9	0.06577	0.06577	0.06576	0.06599	0.06584
10	0.13208	0.13214	0.13214	0.13215	0.13225
11	0.11120	0.11144	0.11145	0.11148	0.11150
12	0.13882	0.13930	0.13926	0.13946	0.13933
13	0.14620	0.14534	0.14566	0.14494	0.14623
14	0.14036	0.14073	0.14066	0.14085	0.14107
15	0.08465	0.08468	0.08469	0.08475	0.08472
16	0.13281	0.13300	0.13301	0.13308	0.13310
17	0.07758	0.07758	0.07759	0.07758	0.07763
18	0.06919	0.06912	0.06911	0.06912	0.06909
19	0.12479	0.12478	0.12478	0.12540	0.12486
20	0.10805	0.10815	0.10818	0.10782	0.10766
21	0.11430	0.11426	0.11426	0.11439	0.11407
22	0.11469	0.11472	0.11471	0.11476	0.11490
23	0.11264	0.11272	0.11270	0.11277	0.11273
24	0.12111	0.12125	0.12125	0.12129	0.12134
25	0.09039	0.09042	0.09044	0.09065	0.09066
26	0.10017	0.09989	0.09983	0.10040	0.10027
27	0.05972	0.05972	0.05972	0.05975	0.05972
28	0.07678	0.07677	0.07679	0.07671	0.07686
29	0.06609	0.06610	0.06610	0.06609	0.06616
30	0.13215	0.13203	0.13204	0.13202	0.13207
31	0.05912	0.05909	0.05905	0.05912	0.05908
32	0.12349	0.12357	0.12352	0.12366	0.12355
33	0.29034	0.29056	0.29050	0.29149	0.29103
34	0.08710	0.08775	0.08773	0.08776	0.08765
35	0.13872	0.13847	0.13846	0.13845	0.13839
36	0.10688	0.10688	0.10687	0.10712	0.10747
37	0.13170	0.13186	0.13187	0.13189	0.13203
38	0.09810	0.09831	0.09829	0.09841	0.09849
39	0.13788	0.13785	0.13786	0.13826	0.13738
40	0.09102	0.09096	0.09096	0.09094	0.09101
41	0.12712	0.12716	0.12717	0.12741	0.12732
42	0.15002	0.14996	0.14993	0.14984	0.14977
43	0.11784	0.11788	0.11789	0.11825	0.11825
44	0.07834	0.07838	0.07838	0.07835	0.07847
45	0.09219	0.09238	0.09235	0.09242	0.09210
46	0.33720	0.33707	0.33703	0.33849	0.33793
47	0.11142	0.11152	0.11150	0.11157	0.11173
48	0.14106	0.14108	0.14107	0.14110	0.14113
49	0.20917	0.20910	0.20914	0.20929	0.20901
50	0.08809	0.08813	0.08811	0.08816	0.08827
51	0.09689	0.09687	0.09687	0.09675	0.09697
52	0.08926	0.08930	0.08929	0.08920	0.08942

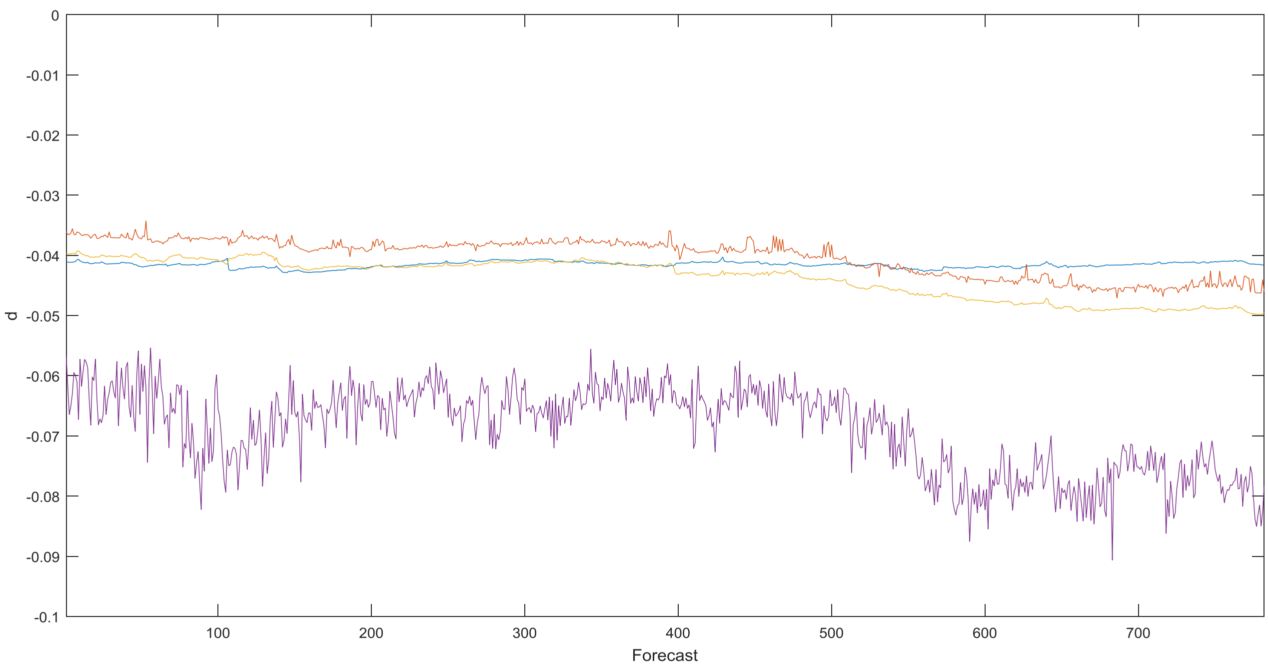
Table 12. *Forecasting performance* - MCP

comp	**ARFIMA(0,d,0)**	**ARFIMA(1,d,0)**	**ARFIMA(0,d,1)**	**ARFIMA(1,d,1)**	**ARIMA(1,1)**
1	0.71609	0.71161	0.71071	0.71609	0.69249
2	0.71161	0.71341	0.70711	0.71520	0.72143
3	0.72408	0.72408	0.72497	0.72143	0.73111
4	0.69341	0.69525	0.69064	0.70711	0.71965
5	0.72497	0.71251	0.71788	0.69892	0.69892
6	0.73373	0.73199	0.73547	0.72320	0.72143
7	0.72673	0.72408	0.72497	0.72673	0.72761
8	0.72849	0.72936	0.72849	0.72585	0.72054
9	0.72673	0.72054	0.71877	0.70348	0.71430
10	0.74240	0.74067	0.74067	0.73634	0.73111
11	0.71520	0.70981	0.70801	0.71430	0.71698
12	0.72143	0.71161	0.71430	0.69984	0.71520
13	0.71520	0.73373	0.72673	0.72936	0.70257
14	0.74326	0.73373	0.73286	0.73460	0.71251
15	0.70620	0.70348	0.70257	0.70711	0.70530
16	0.73547	0.72849	0.72585	0.72232	0.72761
17	0.74497	0.73981	0.73894	0.74412	0.73547
18	0.72143	0.71430	0.71698	0.72232	0.72320
19	0.72936	0.73721	0.73460	0.72497	0.73111
20	0.73199	0.72849	0.72673	0.73721	0.73286
21	0.72585	0.73111	0.73111	0.72408	0.75096
22	0.71877	0.70711	0.70257	0.71965	0.70166
23	0.71071	0.69156	0.69341	0.69892	0.69892
24	0.70439	0.70166	0.70166	0.69801	0.70891
25	0.69709	0.69064	0.69064	0.68599	0.68971
26	0.72673	0.73981	0.74067	0.71609	0.72408
27	0.70801	0.70891	0.70891	0.69984	0.71430
28	0.70530	0.72232	0.72497	0.71609	0.71520
29	0.72054	0.72761	0.72761	0.72497	0.71520
30	0.68599	0.69525	0.69433	0.71251	0.69433
31	0.72143	0.71161	0.71341	0.70257	0.72408
32	0.73721	0.73894	0.73981	0.73894	0.73111
33	0.68693	0.68971	0.68413	0.68693	0.68693
34	0.71161	0.67944	0.68319	0.67944	0.68506
35	0.71071	0.72054	0.71877	0.72232	0.71698
36	0.73981	0.74583	0.74755	0.74583	0.73111
37	0.72320	0.71965	0.72143	0.71788	0.72673
38	0.74497	0.73981	0.74669	0.72497	0.72673
39	0.72232	0.72054	0.72320	0.71877	0.72054
40	0.72408	0.72408	0.72585	0.72673	0.71877
41	0.70891	0.71071	0.70981	0.70257	0.71698
42	0.72936	0.72761	0.72673	0.72673	0.72054
43	0.71609	0.72143	0.72232	0.71609	0.71161
44	0.71520	0.71609	0.71609	0.71698	0.71161
45	0.75011	0.71965	0.71965	0.71430	0.72585
46	0.70981	0.71430	0.71430	0.71609	0.71609
47	0.70348	0.69709	0.69892	0.70801	0.70620
48	0.71430	0.72497	0.72320	0.71965	0.70530
49	0.71788	0.72849	0.72849	0.72054	0.71965
50	0.72673	0.71698	0.72761	0.72232	0.71341
51	0.72232	0.72761	0.72936	0.71965	0.70620
52	0.73721	0.73894	0.73808	0.73286	0.73460

We compare all the firm's asset volatility forecasts to the benchmark random walk model using the Diebold and Mariano (1995) test modified by Harvey et al. (1997). The Diebold and Mariano (1995) test is a widely used test designed to compare predictive accuracy of two competing forecasts, with the null hypothesis that the two competing forecasts perform equally well. The predictive accuracy of each model is measured on the basis of a particular loss function. In our case, and following the literature, we use the widely used mean squared error. The results of the Diebold and Mariano (1995) test show that the $ARFIMA(0, d, 0)$ outperforms the benchmark random walk model for 31 company (60%); the $ARFIMA(1, d, 0)$ for 30 companies (58%); the $ARFIMA(0, d, 1)$ for 27 companies (52%); the $ARFIMA(1, d, 1)$ for 28 companies (54%); and the ARIMA model for 23 companies (44%). This confirms the usefulness of forecasting the firm's asset volatility, at least in a statistical sense.

In addition, we further examine the time-varying behavior of the fractional integrating parameter from rolling-window estimations. For each estimation window of 2,610 observations, we calculate the cross-sectional mean of the fractional integrating parameter for each model and for our sample of 52 companies. The values observed in the time-development of the fractional integrated parameter are fully consistent with the cross-sectional mean reported in the in-sample estimation (Table 7). Figure 3 shows the evolution of the cross-sectional mean of the estimated fractional integrating parameter with the $ARFIMA(0, d, 0)$, $ARFIMA(1, d, 0)$, $ARFIMA(0, d, 1)$, and $ARFIMA(1, d, 1)$ models which correspond to the blue, green, red and purple lines, respectively. We can observe that $ARFIMA(0, d, 0)$ provides a stable estimation of the parameter d^* along the forecasting period with the mean of about -0.04. The $ARFIMA(0, d, 1)$ and $ARFIMA(1, d, 0)$ models provide starting values of -0.035 and -0.04, respectively at the beginning of the period. However, their time development slowly decreases resulting in the values of approximately -0.04 and -0.045 at the end of the forecasting period. Although the evolution of the parameter d^* along the forecasting period is very similar for both models, the estimate of d^* with $ARFIMA(1, d, 0)$ is constantly below the $ARFIMA(0, d, 1)$ by a margin of approximately -0.005. Finally, the time development of the cross-sectional mean of fractional integrating parameter with $ARFIMA(1, d, 1)$ is more unstable and volatile along the forecasting period compared to other models. The estimated value ranges from -0.055 to -0.09 and small movements in the rolling window result in large variability in the estimated order of integration. In general, as additional autoregressive parameters are added to the model, the cross-sectional mean of the fractional integrating parameters turns out to be more fluctuating and unpredictable. As we have observed in the in-sample estimation (see Table 7), as we add complexity to the model, fractional integrating parameter becomes significant for a less number of companies.

Figure 3. *Time deveolopment of the fractional integrated parameter*



5.2 Out-of-sample CDS prediction

In this section we empirically test the performance of ARFIMA and ARIMA models by comparing the credit spreads obtained using the out-of-sample forecasts of firm's asset volatility with the market observable CDS spreads. We calculate the 1-day-ahead forecasts of the credit spread using the forecasts of firm's asset volatility, Equation 4 and the structural model of Leland and Toft (1996). To avoid the look-ahead bias we rely only on the past information on market capitalization and accounting items. The main objective and implication of the paper is to achieve an improvement of the empirical performance of structural credit risk models by providing more precise estimates of credit spreads which are based on time-varying volatility with long-memory features.

As in the previous section, we calculate RMSE, MAE and MCP as the standard forecast error measures. In Equations 17 to 18, we consider $CDS_{R,t+1}$ as the market observable CDS at time $t+1$, and $CS_{F,t}$ as the one-day forecast of credit spread calculated using the forecast of firm's asset volatility made a time t and using historical information on market capitalization and accounting items. The summary of the main results, in the cross-section and at the firm-level are provided in Table 13.

Table 13. *Out-of-sample performance*

	ARFIMA(0,d,0)	**ARFIMA(1,d,0)**	**ARFIMA(0,d,1)**	**ARFIMA(1,d,1)**	**ARIMA(1,1)**
Panel A: Cross-section					
RMSE	3.06225	3.06348	3.06361	3.06343	3.06594
MAE	1.62369	1.63115	1.62961	1.63989	1.63106
MCP	0.62673	0.62455	0.62471	0.62449	0.62321
Panel B: Firm level					
RMSE	24	5	3	10	10
MAE	31	4	3	0	14
MCP	22	7	5	15	3

This table reports the cross sectional results of the out-of-sample prediction error measures (RMSE, MAE and MCP) and the number of firms for which the model is selected as the best one.

As it could be observed, in terms of the out-sample performance, all the main indicators of the difference between the model and market CDS spreads indicate the outperformance of models in which the degree of persistence is estimated rather than imposed to 1. Specifically, ARFIMA models outperform ARIMA model in 80.77% (RMSE), 73.08% (MAE) and 94.23% (MCP) of the cases. Therefore, the economic performance of ARFIMA vs. ARIMA model provides even stronger support to modeling firm's asset volatility as a long-memory process. Out of all the considered ARFIMA models, the $ARFIMA(0, d, 0)$ seems to provide the best out-of-sample forecasts of CDS spreads. Not only that in the cross-section the forecast errors are lowest for the $ARFIMA(0, d, 0)$ according to the three considered measures, but also, at the firm level this model is supported for the highest number of companies. These results support even more our previous findings on the degree of fractional integration of the firm's asset volatilities. Table 14, Table 15 and Table 16 show the forecasting prediction error (RMSE, MAE and MCP) of credit spreads calculated on the basis of forecasted firm's asset volatility for each firm and model.

We also compare the credit spread forecasts to the benchmark random walk model. For that purpose, as in the previous section we employ the Diebold and Mariano (1995) test modified by Harvey et al. (1997) using the mean squared error as a loss function. The results of the Diebold and Mariano (1995) test show that the $ARFIMA(0, d, 0)$ outperforms the benchmark random walk model for 25 companies (48.08%); the $ARFIMA(1, d, 0)$ for 20 companies (38.46%); the $ARFIMA(0, d, 1)$ for 18 companies (34.61%); the $ARFIMA(1, d, 1)$ for 22 companies (42.31%); and the ARIMA model for 12 companies (23.08%).[30] This confirms the usefulness of

[30] For some companies with lower liquidity of the CDS spreads, the daily change is equal to 0. This goes in favor of the random walk model. When these days are not taken into account the performance of ARFIMA models improves as expected.

forecasting firm's asset volatility on the basis of the long-memory process, in an economical sense.

Table 14. *Credit spread forecasting performance - RMSE*

comp	ARFIMA(0,d,0)	ARFIMA(1,d,0)	ARFIMA(0,d,1)	ARFIMA(1,d,1)	ARIMA(1,1)
1	2.67098	2.67247	2.67231	2.67068	2.67958
2	3.06478	3.05020	3.05349	3.05116	3.03384
3	1.60945	1.60964	1.60962	1.60924	1.61071
4	1.82597	1.84128	1.83906	1.85475	1.82660
5	2.31896	2.32223	2.32138	2.32536	2.32906
6	7.59135	7.59770	7.59985	7.61104	7.61104
7	3.84808	3.84867	3.84867	3.84967	3.84746
8	2.07972	2.07818	2.07797	2.07773	2.08596
9	3.09832	3.11443	3.11390	3.11642	3.12597
10	1.10297	1.10327	1.10323	1.10300	1.10336
11	0.90136	0.90241	0.90244	0.90113	0.90117
12	0.86779	0.87085	0.87054	0.87094	0.87430
13	2.81876	2.83933	2.83464	2.82736	2.83624
14	1.35908	1.37095	1.36961	1.36152	1.36101
15	0.65788	0.65805	0.65810	0.65924	0.65872
16	1.19042	1.19130	1.19042	1.19125	1.19083
17	1.52314	1.52319	1.52342	1.52322	1.52278
18	7.38517	7.38106	7.38119	7.38275	7.37887
19	1.18409	1.18177	1.18181	1.18093	1.18156
20	0.97693	0.97644	0.97627	0.97712	0.97990
21	0.47372	0.47436	0.47436	0.47501	0.47407
22	1.34540	1.34909	1.34864	1.35310	1.35011
23	3.32507	3.31579	3.31522	3.31580	3.32124
24	5.14660	5.14314	5.14304	5.15041	5.15890
25	1.25974	1.25868	1.25849	1.25592	1.25535
26	2.75430	2.75161	2.75181	2.77381	2.76825
27	1.82002	1.81973	1.81968	1.82130	1.81900
28	6.05520	6.07130	6.06152	6.08077	6.07716
29	2.28280	2.28225	2.28225	2.28248	2.28209
30	7.05736	7.05114	7.05152	7.05716	7.05637
31	2.40532	2.40614	2.40661	2.40852	2.40525
32	1.18737	1.18964	1.18878	1.18879	1.18717
33	20.85182	20.87436	20.86701	20.85178	20.89047
34	1.38140	1.39769	1.39726	1.39777	1.39455
35	1.01641	1.02247	1.02224	1.02253	1.02272
36	1.63208	1.63422	1.63357	1.63898	1.64775
37	1.03253	1.03643	1.03650	1.03672	1.03988
38	0.77521	0.78237	0.78111	0.78227	0.78385
39	2.61077	2.61047	2.61054	2.61086	2.62247
40	0.72875	0.72734	0.72713	0.72640	0.72670
41	5.02454	5.03154	5.02946	5.02732	5.02765
42	3.98482	3.99477	3.99594	3.99758	3.98574
43	1.38018	1.38187	1.38234	1.39239	1.39156
44	1.00815	1.00900	1.00929	1.00832	1.01096
45	1.23363	1.25051	1.25054	1.25014	1.23770
46	21.78267	21.69388	21.72722	21.64600	21.73288
47	1.73870	1.73825	1.73831	1.73851	1.73765
48	1.56647	1.56532	1.56512	1.55822	1.56734
49	3.01709	3.02543	3.02408	3.01537	3.02458
50	3.47634	3.47164	3.47259	3.47415	3.47579
51	0.84462	0.84384	0.84413	0.84567	0.84763
52	0.96254	0.96348	0.96348	0.96978	0.96711

Table 15. *Credit spread forecasting performance* - MAE

comp	**ARFIMA(0,d,0)**	**ARFIMA(1,d,0)**	**ARFIMA(0,d,1)**	**ARFIMA(1,d,1)**	**ARIMA(1,1)**
1	1.48466	1.48297	1.48307	1.48605	1.47951
2	1.44215	1.43479	1.43589	1.44364	1.43327
3	0.88363	0.88383	0.88379	0.88487	0.87464
4	1.02345	1.04189	1.03939	1.07451	1.03456
5	1.09744	1.09507	1.09518	1.10148	1.10204
6	4.72146	4.72283	4.72327	4.72746	4.72982
7	1.99437	1.99375	1.99385	1.99382	1.99717
8	1.10301	1.11106	1.10975	1.11117	1.09727
9	1.70173	1.72212	1.72089	1.73419	1.73185
10	0.63536	0.63503	0.63497	0.63542	0.63532
11	0.49316	0.50015	0.50139	0.50092	0.50064
12	0.49258	0.50110	0.50044	0.50251	0.49818
13	1.61853	1.67981	1.65836	1.66891	1.66931
14	0.76502	0.78082	0.77812	0.80936	0.78912
15	0.36375	0.36376	0.36380	0.36427	0.36410
16	0.60127	0.60599	0.60674	0.60794	0.61017
17	0.88880	0.88900	0.88899	0.88952	0.88981
18	3.15203	3.15595	3.15742	3.15863	3.15317
19	0.65951	0.65923	0.65914	0.66357	0.65851
20	0.54943	0.54993	0.55003	0.55019	0.54910
21	0.24916	0.24974	0.24977	0.25132	0.24504
22	0.76734	0.76715	0.76661	0.77494	0.76823
23	1.96803	1.97157	1.96990	1.97053	1.97489
24	2.90934	2.91067	2.91141	2.91509	2.93224
25	0.52027	0.52167	0.52226	0.52573	0.52494
26	1.44879	1.45205	1.45435	1.47809	1.46670
27	1.03139	1.03109	1.03095	1.03180	1.03121
28	3.58532	3.62134	3.60899	3.62262	3.62420
29	1.31678	1.31693	1.31693	1.31702	1.31751
30	3.60792	3.60383	3.60391	3.61157	3.60501
31	1.40794	1.41418	1.41818	1.42113	1.42746
32	0.65065	0.66443	0.66022	0.66699	0.66402
33	10.29346	10.31671	10.30884	10.34524	10.31668
34	0.76114	0.79293	0.79256	0.79262	0.78684
35	0.61463	0.61752	0.61739	0.61761	0.61237
36	0.97030	0.97192	0.97104	0.99186	0.98579
37	0.41186	0.44029	0.44002	0.44026	0.44356
38	0.41079	0.41930	0.41636	0.42729	0.42197
39	1.29601	1.29383	1.29422	1.30437	1.26970
40	0.42117	0.42158	0.42153	0.42291	0.42112
41	3.09908	3.11410	3.11083	3.12224	3.09825
42	2.40046	2.40958	2.41058	2.41089	2.39906
43	0.82310	0.82409	0.82450	0.83463	0.83430
44	0.62414	0.62372	0.62376	0.62385	0.62364
45	0.76928	0.78627	0.78705	0.78610	0.77199
46	10.29910	10.33721	10.30766	10.48021	10.33229
47	1.06917	1.07000	1.06973	1.07294	1.07070
48	0.81405	0.81593	0.81618	0.81867	0.81643
49	1.15963	1.17205	1.16981	1.20534	1.15777
50	2.02102	2.01751	2.01773	2.02483	2.02733
51	0.49659	0.49693	0.49708	0.49993	0.49899
52	0.54249	0.54488	0.54496	0.55721	0.54749

Table 16. *Credit spread forecasting performance* - MCP

comp	**ARFIMA(0,d,0)**	**ARFIMA(1,d,0)**	**ARFIMA(0,d,1)**	**ARFIMA(1,d,1)**	**ARIMA(1,1)**
1	0.61588	0.60755	0.60922	0.61088	0.61255
2	0.64085	0.64251	0.64251	0.63419	0.63585
3	0.60256	0.59923	0.59923	0.60589	0.60423
4	0.62087	0.63752	0.63252	0.64085	0.63585
5	0.66248	0.67081	0.66748	0.67580	0.67081
6	0.75570	0.75237	0.75403	0.75570	0.75237
7	0.63419	0.63252	0.63252	0.63585	0.62919
8	0.63585	0.62753	0.62586	0.65416	0.63752
9	0.64750	0.65083	0.65583	0.62919	0.63419
10	0.58425	0.57926	0.57926	0.58592	0.57926
11	0.51601	0.50435	0.49770	0.48937	0.49104
12	0.61088	0.60090	0.60256	0.60589	0.60423
13	0.68745	0.64917	0.66581	0.65583	0.65583
14	0.63419	0.61754	0.63252	0.60922	0.62087
15	0.50102	0.50102	0.50269	0.50435	0.50435
16	0.45941	0.48771	0.48438	0.48604	0.47273
17	0.64750	0.64584	0.64417	0.64750	0.64417
18	0.73572	0.72407	0.72407	0.72241	0.73406
19	0.59590	0.60090	0.59923	0.59091	0.59257
20	0.61088	0.61088	0.60755	0.60589	0.60755
21	0.50602	0.50768	0.50768	0.51434	0.50435
22	0.52433	0.51601	0.51933	0.49770	0.49936
23	0.67247	0.67414	0.67414	0.67746	0.67247
24	0.71076	0.70410	0.70410	0.70576	0.69744
25	0.56927	0.56927	0.56927	0.57093	0.57260
26	0.65749	0.65749	0.64917	0.64251	0.64417
27	0.62753	0.61588	0.61088	0.62586	0.63252
28	0.74238	0.74738	0.75070	0.74405	0.74238
29	0.64085	0.64085	0.64085	0.63252	0.64085
30	0.70909	0.70576	0.70243	0.70410	0.70243
31	0.65250	0.65749	0.64750	0.65250	0.65083
32	0.57426	0.57759	0.57093	0.59590	0.59091
33	0.71575	0.71242	0.71741	0.71908	0.71242
34	0.63252	0.63252	0.63585	0.62919	0.63086
35	0.54097	0.53431	0.53764	0.53764	0.54097
36	0.59257	0.59091	0.59257	0.57093	0.57260
37	0.42945	0.44110	0.43944	0.44776	0.43777
38	0.55262	0.55762	0.55429	0.56428	0.55928
39	0.60755	0.60922	0.60755	0.60090	0.60922
40	0.56927	0.58425	0.58425	0.57926	0.58592
41	0.74571	0.74738	0.74405	0.75736	0.75070
42	0.75403	0.76236	0.76402	0.76069	0.75403
43	0.67081	0.66581	0.66581	0.66581	0.66581
44	0.68745	0.68912	0.68912	0.68912	0.68079
45	0.66748	0.66248	0.65915	0.65915	0.66748
46	0.73239	0.72574	0.73239	0.72574	0.73239
47	0.68412	0.66415	0.66415	0.67247	0.66581
48	0.54930	0.54597	0.54597	0.53764	0.53598
49	0.52766	0.52766	0.53265	0.52100	0.52766
50	0.71242	0.71408	0.71408	0.69744	0.69577
51	0.64417	0.61255	0.61421	0.63419	0.62087
52	0.58758	0.58092	0.58425	0.59424	0.59091

6. Conclusions

In this paper we have analyzed the predictability of CDS implied firm's asset volatility using a sample of 52 non-financial firms using ARFIMA and ARIMA models. We find evidence of long-memory in the CDS implied asset volatility for most of the companies in the sample. The results of the in-sample-fit of ARFIMA models vs. ARIMA model show that inclusion of the fractional differencing parameter in modeling CDS implied volatility improves the model fit on average, independently of the selection criteria or error measure used. We have tested the statistical and economical implications of our findings in an out-of-sample prediction. First, we conduct an out-of-sample prediction of firm's asset volatilities and contrast it with the CDS implied asset volatilities. In line with our in-sample findings, ARFIMA models on average outperform ARIMA model. Second, we conduct out-of-sample prediction of credit spreads on the basis of the Leland and Toft (1996) structural credit risk model using only historical data. We find that CDS spreads are predictable, and for an important number of companies including the fractional integrating parameter substantially improves the forecasting performance. A practical implication of results provided in this paper is that a possibility to predict the CDS spread development would allow adopting appropriate trading positions to achieve abnormal returns.

References

Baillie, R. T., Bollerslev, T., Mikkelsen, H. O., 1996. Fractionally integrated generalized autoregressive conditional heteroskedasticity, *Journal of Econometrics*, vol. 74(1), 3–30

Baillie, R. T., 1996. Long memory process and fractional integration in econometrics, *Journal of Econometrics*, vol. 73(1), 5–59

Black, F., and Scholes, M., 1973. The pricing of options and corporate liabilities. *Journal of Political Economy*, vol. 81(3), 637-654

Cheung, Y.W. 1993. Tests for fractional integration: a Monte Carlo investigation. *Journal of Time Series Analysis*, vol. 14, 331–345.

Choi, J. and Richardson, M. 2016. The volatility of a firm's assets and the leverage effect, *Journal of Financial Economics*, vol. 121(2), 254–277

Diebold, F. and G. Rudebusch. 1991. On the power of Dickey-Fuller tests against fractional alternatives. *Economics Letters*, vol. 35, 155-160;

Diebold, F.X., and Mariano, R.S. 1995. Comparing predictive accuracy, *Journal of Business and Economic Statistics*, vol. 13, 253-263

Dumas, B., Fleming, J., and Whaley, R.E. 1998. Implied volatility functions: empirical tests. *Journal of Finance*, vol. 53, 2059-2016

Engle, R.F., and Siriwardane, E.N., 2018, Structural GARCH: The volatility-leverage connection, *The Review of Financial Studies*, vol. 31(2), 449–492

Forte, S. 2011. Calibrating structural models: a new methodology based on stock and credit default swap data, *Quantitative Finance*, vol. 11, no. 12, 1745–59

Forte, S., and Lovreta, L., 2012. Endogenizing exogenous default barrier models: the MM algorithm. *Journal of Banking and Finance*, vol. 36, 1639-1652

Forte, S., and Lovreta, L., 2019. Volatility discovery: can the CDS market beat the equity options market? *Finance Research Letters*, vol. 28, 107-111.

Forte, S., and Lovreta, L., 2020. Credit Default Swaps, leverage effect, and the cross-sectional predictability of equity and firm asset volatility, Working paper

Geweke, J. and Porter-Hudak, S. 1983. The estimation and application of long memory time series models. *Journal of Time Series Analysis*, vol. 4, 221–238

Gil-Alana, L.A., Robinson, P.M., 1997. Testing of unit roots and other nonstationary hypotheses in macroeconomic time series. *Journal of Econometrics*, vol. 80, 241–268.

Gil-Alana, L.A. 2006. Fractional integration in daily stock market indexes. *Review of Financial Economics*, vol. 15, 28–48

Gil-Alana, L.A. 2008. Fractional integration and structural breaks at unknown periods of time. *Journal of Time Series Analysis*, vol. 29(1), 163-185.

Gil-Alana, L.A., and Hualde, J. 2009. Fractional integration and cointegration: an overview and an empirical application. Palgrave Handbook of Applied Econometrics, Part III 2, 1190–1219.

Glover, B. 2016. The expected cost of default. *Journal of Financial Economics*, vol. 119, 284-299

Granger, C., Joyeux, R. 1980. An introduction to long memory time series models and fractional differencing. *Journal of Time Series Analysis*, vol. 1, 15–39

Gonçalves, S., and Guidolin, M. 2006. Predictable dynamics in the S&P 500 index options implied volatility surface. *Journal of Business*, vol. 79, 1591-1635

González-Pla, F. and Lovreta, L. 2019. Persistence in firm's asset and equity volatility, *Physica A: Statistical Mechanics and its Applications*, vol. 535, 122265

González-Pla, F. and Lovreta, L. 2020. Modeling and forecasting firm-specific volatility: the role of asymmetry and long-memory, *Working Paper*

Guo, D., 2000. Dynamic volatility trading strategies in the currency option market. *Review of Derivatives Research,* vol. 4, 133-154.

Harvey, C.R., and Whaley, R.E. 1992. Market volatility prediction and the efficiency of the S&P 100 Index option market. *Journal of Financial Economics*, vol. 31, 43-73

Harvey, D., Leybourne, S., and Newbold, P. 1997. Testing the equality of prediction mean squared error, *International Journal of Forecasting*, vol. 13, 281–291

Hosking, J.R.M., 1981. Fractional differencing. *Biometrika*, vol. 68, 165–176

Jacobsen, B., 1996. Long-term dependence in stock returns. *Journal of Empirical Finance*, vol. 3, 393–417

Kasman, A., Kasman, S., and Torun, E. 2009. Dual long memory property in returns and volatility: Evidence from the CEE countries' stock markets. *Emerging Markets Review*, vol, 10(2), 122–139

Konstantinidi, E., Skiadopoulos, G., and Tzagkaraki, E., 2008. Can the evolution of implied volatility be forecasted? Evidence from European and US implied volatility indices. *Journal of Banking and Finance*, vol. 32(11), 2401-2411

Leland, H.E. and Toft, K.B. 1996. Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads. *The Journal of Finance*, vol. 51(3), 987-1019

Lo, A.W. 1991. Long-term memory in stock market prices. *Econometrica*, vol. 59, 1279–1313

Lovreta, L., and Silaghi, L., 2020. The surface of implied firm's asset volatility. *Journal of Banking and Finance*, vol. 112, 105253

Merton R.C. 1974. On the pricing of corporate debt: the risk structure of interest rates. *Journal of Finance*, vol. 29(2), 449-470

Nelson, D. B. 1991. Conditional heteroskedasticity in asset returns: a new approach, *Econometrica*, vol. 59(2): 347–70

Olbermann, B.P., Lopes, S.R.C. and Reisen, V.A. 2006. Invariance of the first difference in ARFIMA models. *Computational Statistics*, vol. 21, 445–461

Robinson, P. 1995. Log periodogram regression of time series with long range dependence. *Annals of Statistics*, vol. 23, 1048–1072

Schwert, G.W. 1989. Tests for unit roots: a Monte Carlo investigation. *Journal of Business and Economic Statistics*, vol. 7 (2), 147‑159

Shimotsu, K. and Phillips, P.C.B. 2005. Exact local Whittle estimation of fractional integration. *Annals of Statistics*, vol. 33, 1890–1933

Appendix

Appendix A. Table A1. Firm's specific CDS spreads and LT parameter estimates

No	Company	CDS	Asset Value	β	Long-term volatility	CDS Implied volatility	delta	leverage
1	AB Volvo	123.82	345,701.49	0.8379	0.1076	0.1370	0.0234	0.6479
2	BMW AG	75.16	115,658.31	0.8915	0.0739	0.0974	0.0101	0.7187
3	Michelin SCA	98.88	21,164.04	0.8175	0.1458	0.1862	0.0209	0.5121
4	Continental AG	216.52	31,475.52	0.8128	0.1568	0.2254	0.0203	0.4646
5	Daimler AG	90.64	172,653.77	0.8744	0.0873	0.1092	0.0139	0.7126
6	Peugeot SA	246.79	54,609.82	0.9115	0.0577	0.0671	0.0110	0.8571
7	Renault SA	173.95	65,144.18	0.8780	0.0875	0.1096	0.0129	0.7396
8	Valeo SA	138.97	9,934.51	0.8419	0.1218	0.1543	0.0179	0.5854
9	Deutsche Lufthansa AG	132.21	24,543.63	0.8967	0.0748	0.0952	0.0186	0.7551
10	Kingfisher PLC	110.19	9,864.03	0.7655	0.1621	0.2613	0.0250	0.3829
11	Koninklijke Philips NV	57.17	37,799.57	0.7984	0.1644	0.2306	0.0185	0.3847
12	LVMH SE	53.39	70,604.28	0.7539	0.1890	0.2929	0.0170	0.2876
13	Marks & Spencer Group PLC	139.31	11,274.80	0.7680	0.1737	0.2571	0.0352	0.4034
14	Kering SA	127.20	27,022.31	0.8226	0.1383	0.2125	0.0236	0.4725
15	Sodexo SA	55.47	16,122.72	0.8490	0.1175	0.1717	0.0201	0.4859
16	BAT PLC	54.85	62,881.64	0.8037	0.1472	0.2742	0.0373	0.2702
17	Carrefour SA	71.56	57,373.42	0.8685	0.0998	0.1300	0.0160	0.6125
18	Casino Guichard SA	133.76	26,232.03	0.9304	0.0675	0.0888	0.0296	0.7338
19	Diageo PLC	50.46	44,810.47	0.8290	0.1375	0.2668	0.0323	0.2744
20	Danone SA	48.69	44,085.86	0.8125	0.1472	0.2448	0.0196	0.3434
21	Henkel & Co KGaA AG	43.90	19,715.08	0.8429	0.1199	0.1807	0.0241	0.4225
22	Imperial Tobacco Group PLC	95.40	36,358.35	0.8252	0.1290	0.2068	0.0334	0.4621
23	J Sainsbury PLC	114.61	12,808.58	0.8132	0.1293	0.1806	0.0293	0.5497
24	Tesco PLC	87.91	49,327.05	0.8140	0.1319	0.1991	0.0219	0.4873
25	Unilever NV	30.67	70,653.33	0.8219	0.1218	0.1817	0.0433	0.4005
26	BP PLC	60.12	183,065.75	0.7975	0.1284	0.1780	0.0243	0.4909
27	E.ON SE	59.81	129,436.75	0.8796	0.0920	0.1016	0.0260	0.6658
28	EDP Energias de Portugal SA	185.17	35,264.97	0.9319	0.0711	0.0905	0.0385	0.7205
29	Iberdrola SA	105.00	74,123.48	0.8657	0.1173	0.1482	0.0270	0.5925
30	Repsol SA	144.22	50,133.90	0.8498	0.1248	0.1782	0.0333	0.5360
31	RWE AG	64.04	95,785.10	0.9116	0.0684	0.0670	0.0296	0.7535
32	Akzo Nobel NV	65.08	19,999.85	0.8135	0.1506	0.2049	0.0301	0.4286
33	Anglo American PLC	172.28	41,439.86	0.6787	0.2637	0.2973	0.0251	0.3908
34	BAE Systems PLC	79.77	26,968.93	0.8230	0.1144	0.1569	0.0271	0.5563
35	Bayer AG	50.17	81,393.15	0.8237	0.1469	0.1991	0.0268	0.4031
36	Saint Gobain SA	105.23	43,246.14	0.8394	0.1300	0.1592	0.0256	0.5727
37	Investor AB	64.09	117,686.77	0.7945	0.1631	0.2337	0.0475	0.3501
38	Linde AG	47.57	31,999.70	0.8592	0.1195	0.1632	0.0246	0.4719
39	Rolls-Royce Holdings PLC	67.14	21,870.66	0.7987	0.1407	0.1915	0.0055	0.5042
40	Siemens AG	50.40	131,670.35	0.8145	0.1378	0.1831	0.0181	0.4829
41	Stora Enso OYJ	219.25	12,376.26	0.8464	0.1222	0.1675	0.0367	0.5999
42	UPM Kymmene OYJ	192.74	13,555.55	0.8126	0.1467	0.2164	0.0377	0.4937
43	BT Group PLC	82.68	43,841.32	0.8449	0.1193	0.1523	0.0368	0.5276
44	Deutsche Telekom AG	66.98	131,629.95	0.9147	0.0872	0.1124	0.0409	0.5948
45	Orange SA	69.10	102,741.09	0.9020	0.0951	0.1147	0.0445	0.5895
46	Hellenic Telecommunication:	389.97	11,747.57	0.8301	0.1574	0.2188	0.0285	0.5435
47	Koninklijke KPN NV	83.50	31,677.04	0.8752	0.1207	0.1556	0.0427	0.5148
48	Pearson PLC	61.20	11,396.80	0.7892	0.1463	0.2334	0.0347	0.3685
49	STMicroelectronics NV	80.77	10,930.29	0.6949	0.2377	0.3193	0.0218	0.2795
50	Telefonica SA	125.09	140,855.90	0.8901	0.1022	0.1589	0.0435	0.5356
51	Wolters Kluwer NV	59.38	10,132.34	0.8567	0.1195	0.1835	0.0242	0.4528
52	WPP PLC	93.48	25,022.13	0.8509	0.0943	0.1438	0.0179	0.5852

Appendix B. Table B.1. BIC in ARFIMA(p,d,q) estimates with $p + q \leq 3$

comp	(0,d,0)	(0,d,1)	(1,d,0)	(1,d,1)	(0,d,2)	(2,d,0)	(0,d,3)	(3,d,0)	(1,d,2)	(2,d,1)
1	-9,84347	-9,84126	-9,84128	-9,83908	-9,83985	-9,83981	-9,83808	-9,83799	-9,83778	-9,83775
2	-10,50850	-10,50700	-10,50720	-10,50790	-10,50650	-10,50680	-10,50520	-10,50540	-10,50620	-10,50610
3	-9,42827	-9,42588	-9,42588	-9,42348	-9,42363	-9,42363	-9,42130	-9,42130	-9,42121	-9,42454
4	-8,51625	-8,51681	-8,51706	-8,52309	-8,51479	-8,51504	-8,51357	-8,51410	-8,52157	-8,52180
5	-10,17270	-10,17050	-10,17060	-10,17130	-10,17010	-10,17050	-10,16860	-10,16880	-10,17030	-10,17020
6	-10,87460	-10,87250	-10,87250	-10,87050	-10,87100	-10,87120	-10,86880	-10,86880	-10,86870	-10,86880
7	-10,13670	-10,13430	-10,13430	-10,13190	-10,13200	-10,13200	-10,13050	-10,13060	-10,13230	-10,12950
8	-9,66642	-9,66480	-9,66484	-9,66263	-9,66259	-9,66262	-9,66048	-9,66034	-9,66007	-9,66024
9	-10,40890	-10,40700	-10,40700	-10,40640	-10,40500	-10,40500	-10,40260	-10,40260	-10,40400	-10,40400
10	-9,01522	-9,01283	-9,01283	-9,01042	-9,01066	-9,01065	-9,00876	-9,00876	-9,00803	-9,00803
11	-9,39956	-9,39818	-9,39808	-9,39598	-9,39625	-9,39647	-9,39451	-9,39439	-9,39408	-9,39419
12	-9,27483	-9,27263	-9,27263	-9,27026	-9,27015	-9,27026	-9,26788	-9,26787	-9,26799	-9,26799
13	-8,55720	-8,56364	-8,56556	-8,56553	-8,56284	-8,56492	-8,56508	-8,56407	-8,56322	-8,56319
14	-9,37013	-9,36871	-9,36871	-9,36732	-9,36632	-9,36631	-9,36514	-9,36559	-9,36676	-9,36659
15	-10,02110	-10,01890	-10,01890	-10,01650	-10,01660	-10,01660	-10,01470	-10,01490	-10,01430	-10,01430
16	-9,38687	-9,38557	-9,38550	-9,38333	-9,38342	-9,38345	-9,38128	-9,38131	-9,38146	-9,38151
17	-10,18430	-10,18200	-10,18200	-10,17950	-10,18000	-10,18000	-10,17850	-10,17860	-10,17790	-10,17790
18	-10,55580	-10,55510	-10,55510	-10,55270	-10,55280	-10,55270	-10,55040	-10,55260	-10,55050	-10,55050
19	-9,52448	-9,52268	-9,52269	-9,52352	-9,52030	-9,52030	-9,51795	-9,51791	-9,51859	-9,51913
20	-9,47565	-9,47326	-9,47326	-9,47223	-9,47372	-9,47362	-9,47136	-9,47141	-9,47136	-9,47141
21	-9,96387	-9,96186	-9,96185	-9,95948	-9,95951	-9,95956	-9,95826	-9,95826	-9,95734	-9,95734
22	-9,40014	-9,39828	-9,39833	-9,39636	-9,39634	-9,39635	-9,39435	-9,39428	-9,39409	-9,39407
23	-8,58431	-8,58363	-8,58376	-8,58144	-8,58175	-8,58153	-8,57888	-8,58016	-8,58027	-8,57934
24	-9,60574	-9,60342	-9,60342	-9,60098	-9,60124	-9,60122	-9,59942	-9,59938	-9,59960	-9,60009
25	-10,05300	-10,05130	-10,05120	-10,05040	-10,05130	-10,05120	-10,04900	-10,04950	-10,04890	-10,04910
26	-9,28821	-9,28759	-9,28723	-9,28737	-9,28822	-9,28750	-9,28605	-9,28861	-9,29139	-9,29082
27	-10,25820	-10,25580	-10,25580	-10,25380	-10,25430	-10,25440	-10,25220	-10,25280	-10,25190	-10,25210
28	-10,06950	-10,07120	-10,07200	-10,07020	-10,07140	-10,07060	-10,06990	-10,06980	-10,07010	-10,06970
29	-9,70918	-9,70679	-9,70679	-9,70439	-9,70456	-9,70456	-9,70349	-9,70358	-9,70466	-9,70450
30	-9,11638	-9,11425	-9,11426	-9,11190	-9,11187	-9,11187	-9,10947	-9,10947	-9,10947	-9,10947
31	-10,83040	-10,82810	-10,82810	-10,82560	-10,82640	-10,82640	-10,82590	-10,82580	-10,82460	-10,82500
32	-9,53379	-9,53239	-9,53263	-9,53105	-9,52999	-9,53266	-9,53148	-9,53196	-9,53236	-9,53221
33	-8,07332	-8,07103	-8,07105	-8,07110	-8,07070	-8,07111	-8,06938	-8,06935	-8,06928	-8,06924
34	-9,86209	-9,86269	-9,86284	-9,86046	-9,86061	-9,86047	-9,85956	-9,85933	-9,85928	-9,85902
35	-9,61142	-9,61033	-9,61039	-9,60811	-9,60808	-9,60808	-9,60586	-9,60575	-9,60837	-9,60612
36	-9,40401	-9,40191	-9,40197	-9,40161	-9,40128	-9,40152	-9,39934	-9,39948	-9,39976	-9,39977
37	-9,14453	-9,14467	-9,14470	-9,14230	-9,14230	-9,14230	-9,14002	-9,13993	-9,13990	-9,14073
38	-9,89619	-9,89522	-9,89541	-9,89348	-9,89373	-9,89379	-9,89133	-9,89140	-9,89174	-9,89179
39	-9,32846	-9,32611	-9,32613	-9,32610	-9,32627	-9,32609	-9,32408	-9,32427	-9,32559	-9,32568
40	-9,64923	-9,64729	-9,64725	-9,64496	-9,64518	-9,64542	-9,64483	-9,64500	-9,64318	-9,64344
41	-9,22707	-9,22503	-9,22508	-9,22381	-9,22380	-9,22380	-9,22140	-9,22141	-9,22139	-9,22141
42	-8,90976	-8,90779	-8,90775	-8,90555	-8,90588	-8,90592	-8,90378	-8,90390	-8,90359	-8,90366
43	-9,88446	-9,88215	-9,88214	-9,88101	-9,88122	-9,88127	-9,87893	-9,87894	-9,87893	-9,87894
44	-10,34810	-10,34580	-10,34580	-10,34370	-10,34340	-10,34340	-10,34170	-10,34170	-10,34100	-10,34130
45	-10,31190	-10,31080	-10,31080	-10,30840	-10,30840	-10,30840	-10,30600	-10,30600	-10,30600	-10,30760
46	-7,31183	-7,31061	-7,31153	-7,33074	-7,31654	-7,32934	-7,30591	-7,32982	-7,32922	-7,32904
47	-9,67613	-9,67384	-9,67385	-9,67136	-9,67171	-9,67173	-9,66953	-9,66956	-9,66987	-9,66987
48	-9,50674	-9,50435	-9,50435	-9,50195	-9,50271	-9,50280	-9,50072	-9,50071	-9,50143	-9,50058
49	-8,65526	-8,65305	-8,65306	-8,65111	-8,65086	-8,65084	-8,64844	-8,64875	-8,64933	-8,64922
50	-9,73561	-9,73396	-9,73409	-9,73298	-9,73315	-9,73320	-9,73078	-9,73086	-9,73163	-9,73091
51	-9,85530	-9,85290	-9,85290	-9,85050	-9,85059	-9,85060	-9,84814	-9,84875	-9,84888	-9,84887
52	-10,07220	-10,06990	-10,06990	-10,06930	-10,06750	-10,06760	-10,06540	-10,06540	-10,06780	-10,06790

This table reports the Schwarz or Bayesian Information Criterion (BIC) for different ARFIMA(p,d,q) models with $p + q \leq 3$ by firm level.

Appendix B. Table B.2. BIC in ARFIMA(p,d,q) estimates with $p+q>3$ and $p+q\leq 5$

comp	**(0,d,4)**	**(3,d,1)**	**(1,d,3)**	**(2,d,2)**	**(4,d,0)**	**(0,d,5)**	**(5,d,0)**	**(4,d,1)**	**(3,d,2)**	**(2,d,3)**	**(1,d,4)**
1	-9,83567	-9,83576	-9,83584	-9,83804	-9,83586	-9,83355	-9,83346	-9,83346	-9,83565	-9,83565	-9,83355
2	-10,50560	-10,50390	-10,50400	-10,50410	-10,50530	-10,50350	-10,50300	-10,50290	-10,50310	-10,50140	-10,50150
3	-9,41891	-9,41890	-9,41885	-9,42209	-9,41890	-9,41668	-9,41672	-9,41651	-9,42535	-9,42535	-9,41651
4	-8,51271	-8,51999	-8,52120	-8,51839	-8,51994	-8,51030	-8,52136	-8,51754	-8,51885	-8,51978	-8,52177
5	-10,16620	-10,16790	-10,16810	-10,16770	-10,17040	-10,16920	-10,16810	-10,16800	-10,16550	-10,16550	-10,16910
6	-10,86700	-10,86660	-10,86650	-10,86670	-10,86700	-10,86480	-10,86470	-10,86620	-10,86600	-10,86590	-10,86610
7	-10,12930	-10,13100	-10,13110	-10,13010	-10,12950	-10,12760	-10,12800	-10,12880	-10,13020	-10,12790	-10,12900
8	-9,65897	-9,65859	-9,65871	-9,65810	-9,65890	-9,65668	-9,65667	-9,65666	-9,65818	-9,65744	-9,65667
9	-10,40050	-10,40020	-10,40050	-10,40170	-10,40050	-10,39980	-10,40010	-10,40000	-10,39940	-10,39940	-10,40010
10	-9,00637	-9,00639	-9,00565	-9,00593	-9,00638	-9,00410	-9,00409	-9,00705	-9,00705	-9,00701	-9,00556
11	-9,39272	-9,39279	-9,39141	-9,39441	-9,39276	-9,39046	-9,39046	-9,39054	-9,39095	-9,39096	-9,39061
12	-9,26548	-9,26553	-9,26553	-9,26555	-9,26608	-9,26452	-9,26420	-9,26372	-9,26308	-9,26561	-9,26316
13	-8,56278	-8,56267	-8,56093	-8,56251	-8,56250	-8,56045	-8,56036	-8,56025	-8,56626	-8,56304	-8,56030
14	-9,36355	-9,36439	-9,36445	-9,36449	-9,36395	-9,36119	-9,36156	-9,36149	-9,36251	-9,36205	-9,36222
15	-10,01420	-10,01410	-10,01400	-10,01350	-10,01430	-10,01250	-10,01270	-10,01230	-10,01520	-10,01510	-10,01220
16	-9,37963	-9,37923	-9,37909	-9,37860	-9,37984	-9,37719	-9,37745	-9,37745	-9,37821	-9,37627	-9,37724
17	-10,17680	-10,17660	-10,17660	-10,17980	-10,17680	-10,17440	-10,17440	-10,17440	-10,17710	-10,17630	-10,17440
18	-10,55060	-10,55020	-10,55040	-10,55760	-10,55030	-10,54990	-10,55030	-10,55180	-10,54900	-10,54980	-10,55340
19	-9,51979	-9,51901	-9,51895	-9,51550	-9,51977	-9,51755	-9,51738	-9,51738	-9,51555	-9,51751	-9,51772
20	-9,46902	-9,46902	-9,46895	-9,46987	-9,46903	-9,46674	-9,46665	-9,46664	-9,46662	-9,46750	-9,46722
21	-9,95586	-9,95603	-9,95606	-9,95563	-9,95609	-9,95373	-9,95372	-9,95412	-9,95369	-9,95370	-9,95373
22	-9,39215	-9,39195	-9,39201	-9,39249	-9,39217	-9,39032	-9,39026	-9,38989	-9,39664	-9,39642	-9,38989
23	-8,57800	-8,57779	-8,57778	-8,57942	-8,57793	-8,57839	-8,57909	-8,57688	-8,57704	-8,57704	-8,57684
24	-9,59723	-9,59716	-9,59718	-9,59867	-9,59720	-9,59486	-9,59485	-9,59482	-9,59488	-9,59479	-9,59484
25	-10,04800	-10,04830	-10,04790	-10,04760	-10,04780	-10,04560	-10,04550	-10,04550	-10,04600	-10,04510	-10,04610
26	-9,28798	-9,28622	-9,28745	-9,29743	-9,28622	-9,28806	-9,29130	-9,29155	-9,30424	-9,30535	-9,29064
27	-10,25670	-10,25270	-10,25200	-10,26140	-10,25690	-10,25440	-10,25470	-10,25460	-10,25620	-10,25550	-10,25440
28	-10,06770	-10,06770	-10,06780	-10,06780	-10,06780	-10,06530	-10,06540	-10,06530	-10,06680	-10,06640	-10,06530
29	-9,70307	-9,70327	-9,70347	-9,70405	-9,70301	-9,70221	-9,70221	-9,70148	-9,70176	-9,70176	-9,70159
30	-9,10704	-9,10708	-9,10708	-9,10834	-9,10759	-9,10588	-9,10601	-9,10581	-9,10572	-9,10571	-9,10577
31	-10,82400	-10,82560	-10,82560	-10,82270	-10,82380	-10,82260	-10,82310	-10,82340	-10,82350	-10,82350	-10,82350
32	-9,53044	-9,52997	-9,53015	-9,53048	-9,53020	-9,52831	-9,52781	-9,52781	-9,52813	-9,52813	-9,52817
33	-8,06716	-8,06696	-8,06706	-8,06700	-8,06696	-8,06536	-8,06529	-8,06947	-8,06511	-8,06480	-8,07023
34	-9,85791	-9,85740	-9,85754	-9,85762	-9,85798	-9,85593	-9,85635	-9,85997	-9,85523	-9,85903	-9,86006
35	-9,60718	-9,60393	-9,60598	-9,60569	-9,60671	-9,60505	-9,60500	-9,60468	-9,60717	-9,60563	-9,60493
36	-9,39852	-9,39781	-9,39756	-9,39782	-9,39807	-9,39750	-9,39694	-9,39628	-9,39544	-9,39544	-9,39536
37	-9,13870	-9,13771	-9,13750	-9,13867	-9,13924	-9,13694	-9,13729	-9,13699	-9,13754	-9,13691	-9,13611
38	-9,88972	-9,88954	-9,88910	-9,88996	-9,88976	-9,88785	-9,88772	-9,88762	-9,88762	-9,88762	-9,88769
39	-9,32171	-9,32335	-9,32331	-9,32336	-9,32192	-9,32049	-9,32059	-9,32101	-9,32089	-9,32092	-9,32119
40	-9,64332	-9,64321	-9,64294	-9,64309	-9,64354	-9,64168	-9,64182	-9,64307	-9,64097	-9,64051	-9,64278
41	-9,21901	-9,21928	-9,21937	-9,21908	-9,21907	-9,21660	-9,21853	-9,21711	-9,22449	-9,21818	-9,21711
42	-8,90161	-8,90157	-8,90146	-8,90547	-8,90170	-8,89973	-8,89995	-8,90009	-8,90363	-8,90085	-8,89943
43	-9,87656	-9,87654	-9,87646	-9,87650	-9,87656	-9,87415	-9,87434	-9,87416	-9,87415	-9,87413	-9,87543
44	-10,34050	-10,33970	-10,33980	-10,34440	-10,34020	-10,33660	-10,33780	-10,33780	-10,33640	-10,33630	-10,33810
45	-10,30370	-10,30360	-10,30520	-10,30520	-10,30360	-10,30490	-10,30460	-10,30320	-10,30420	-10,30280	-10,30290
46	-7,32309	-7,33125	-7,32752	-7,32595	-7,32846	-7,32551	-7,32657	-7,32895	-7,32896	-7,32875	-7,32570
47	-9,66767	-9,66753	-9,66754	-9,66745	-9,66766	-9,66534	-9,66535	-9,66531	-9,66509	-9,66509	-9,66537
48	-9,49848	-9,50074	-9,50080	-9,50083	-9,49849	-9,49581	-9,49667	-9,49846	-9,49845	-9,49845	-9,49846
49	-8,64777	-8,64718	-8,64734	-8,65068	-8,64778	-8,64543	-8,64539	-8,64549	-8,64828	-8,64828	-8,64502
50	-9,72850	-9,72854	-9,72837	-9,72855	-9,72850	-9,72713	-9,72687	-9,72621	-9,72616	-9,72616	-9,72628
51	-9,84705	-9,84684	-9,84572	-9,85363	-9,84706	-9,84533	-9,84487	-9,84474	-9,85124	-9,85124	-9,84501
52	-10,06400	-10,06560	-10,06550	-10,06580	-10,06420	-10,06330	-10,06320	-10,06240	-10,06310	-10,06310	-10,06260

This table reports the Schwarz or Bayesian Information Criterion (BIC) for different ARFIMA(p,d,q) models with $p+q>3$ and $p+q\leq 5$ by firm level.

Future Work

This thesis provides a better understanding of firm's asset volatility with relevant contributions in long memory properties, asymmetry and its modeling and forecasting. However, it is important to mention that in our analysis we use a sample of non-financial companies which belong to iTraxx Europe Index that includes the most liquid CDS referencing European investment-grade companies. Further research could analyze firm's asset volatility considering high-yield companies, or mid or small capitalization companies. Another possible line of research could be the analysis of financial institutions as a high leveraged companies and its temporary response in presence of structural breaks. In our results we observed that the differences in persistence between firm's asset and equity volatility are negatively correlated with idiosyncratic volatility, and would be interesting to perform similar analysis in other markets and geographical areas. In addition to that, another future line of research is to incorporate fractionally integrated models of asset volatility directly in a structural credit risk model setting. Finally, from the practical point of view, as our results are showing, the possibility to predict CDS implied volatility inherently implies the prediction of the future development of CDS spreads. Consequently, a possibility to predict the CDS spread development would allow adopting appropriate trading positions to achieve abnormal returns. The future line of research could analyze different trading strategies over different time horizons.