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Essays in Asset Pricing, Expectations and Monetary Policy

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Preface

The central theme throughout this thesis is the role of imperfect information in shaping subjective expectations about future macroeconomic variables and their effects on macroeconomic outcomes, policies and asset prices. I have started to think about this topic while trying to understand standard rational expectation (RE) models and their implications for macroeconomic policy. I soon realized that one of the obstacles I was facing was in fact the central assumption of the RE theory, the fact that economic agents have perfect and full information about everything in the economy. This seemed odd, at first, especially given that in reality not even the greatest economists possess this knowledge, let alone average individuals in the economy that have, at best, basic economic knowledge. Later on, when working on models with departures from the RE hypothesis, I understood that the mathematical simplicity that RE models come with should not be taken for granted and things can become less straightforward when moving away from it. Nevertheless, the real world is anything but simple and hopefully this approach of modeling realistically expectations will yield a deeper understating of the macroeconomy and how policies should be designed.

Expectations have been a long-standing subject in macroeconomics and finance and are crucial for every aspect of decision making. The modeling of expectations is of particular interest in financial markets where asset prices today depend mainly on agents' expectations about future prices and cash flows. Furthermore, markets are at times driven by sentiment or animal spirits which can have a ripple effect throughout the whole economy and thus understating the mechanism of such cycles is crucial for designing appropriate macroeconomic policies. In this thesis, I aim to contribute to the research studying how expectation-driven asset price cycles influence the economy, how policy should respond to it and the influence of subjective beliefs on asset price dynamics. Chapter 1 explores the implications of imperfect information on agents' decision making in the context of investing in stock prices and its implications for the real economy. It further analyses the optimal response of monetary policy in such economies. Chapter 2 brings empirical evidence on the influence of expectations on asset price dynamics while Chapter 3 studies the nature of heterogeneity in expectations and its implications for asset price cycles. A summary of each chapter is presented in the reminder.

Chapter 1 studies how stock price wealth effects impact the real economy and whether monetary policy should take into account asset price cycles when agents have imperfect knowledge about the structure of the economy. The chapter first shows that when agents do not possess full information, stock price wealth effects appear naturally in this framework: sentiment swings in expectations, unrelated to fundamental developments in the economy, influence asset prices which are interpreted by agents as movements in their net wealth. Subsequently, agents adjust their consumption decision accordingly which influences aggregate demand. This result is in contrast with the RE framework where only dividends affect stock prices. This new channel is introduced in a limited asset market participation framework which can closely reproduce quantitatively the high volatility of the US stock market and the dynamics of the business cycle. Using the estimated model, I evaluate two regimes in which the central bank responds to asset prices: transparency vs non-transparency. Under transparency agents understand the full systematic component of monetary policy rules, including the response to stock prices, and internalize this information into their expectations while under non-transparency the reverse is true. The first policy implication arising from this evaluation is that the central bank should respond symmetrically to asset prices that is, not only in periods of stock market crashes (standard documented Fed Put). The second takeaway is that by responding transparently and symmetrically, monetary policy can bring considerable welfare gains compared to the non-transparency regime.

Chapter 2 investigates the determinants of movements in Price Dividend (PD) ratios and brings additional empirical evidence on the expectations shocks that drive stock prices in the model economy from Chapter 1. The current paradigm argues that PD ratio varies almost entirely because of discount rate shocks. Firstly, the chapter shows that the distribution of subjective capital gain expectations can be well characterized by two factors: mean sentiment and disagreement which account for over 95% of the variability of the distribution. Using this insight, I augment a standard asset pricing structural vector-autoregressive (SVAR) model with the distribution of survey capital gain expectations and identify jointly standard and sentimental discount rate shocks.

The latter is understood as a shock to agents' capital gain expectations that does not affect contemporaneously dividends. The results show that sentimental discount rate shocks account for between 30 and 50% of the variation of the PD ratio.

Chapter 3, which is based on a paper with Pau Belda, proposes a model of heterogeneous subjective beliefs that can help explain the distribution of survey capital gains expectations and the dynamics of stock prices. Recent empirical evidence suggests that there is considerable heterogeneity in agents' capital gains expectations with some agents being extremely optimistic while others pessimistic. Moreover, these beliefs are persistent over time meaning that optimists remain optimists and pessimists remain pessimists, without interchanging. We interpret and model this fact as agents having different persistent long-term capital gain expectations in their mental model while learning in the short-run about stock price fluctuations. This framework is consistent with a large number of empirical facts regarding disagreement, subjective expectations and asset price cycles. Furthermore, it allows us to shed light also on the nature of stock market cycles and, in particular, on the contribution of optimists/pessimists in driving booms and busts in asset prices and trading. We argue that optimists are key in understating disagreement and trading and that, in general, the latter two variables are a consequence of expectation driven asset price cycles and not the other way around.

Contents

Chapter 1

The Fed Put and Monetary Policy: An Imperfect Knowledge Approach

Abstract

This paper argues that a central bank can increase macroeconomic stability by reacting explicitly to stock prices and therefore rationalizes the observed empirical evidence of the "Fed put". Waves of optimism/pessimism, unrelated to fundamental developments in the economy affect asset prices and aggregate demand through wealth effects which appear due to imperfect knowledge of the economy. Monetary policy can dampen these effects by influencing agents' expectations about the future path of interest rates and therefore eliminate the non-fundamental effects of belief-driven asset price cycles. Reacting symmetrically and transparently to stock prices increases welfare significantly and brings efficiency gains compared to the standard Fed put policy. Announcing an interest rate increase of 12 basis points for every 100% increase in stock prices accomplishes this goal.

1.1 Introduction

How do asset prices affect the real economy and what is the proper response of central banks to asset price cycles? This paper argues that stock prices affect aggregate demand and influence cyclical fluctuations through consumption wealth effects which appear due to agents' imperfect information about the structure of the economy. Consistent with recent evidence, agents extrapolate past returns and display slow and persistent movements in expectations. Booms and busts in asset prices driven by sentiment swings affect stock prices and the financial position of market participants which translate via consumption-wealth effects into changes in aggregate demand. In this environment, monetary policy can increase macroeconomic stability and efficiency by managing long-term interest rate expectations by responding explicitly and transparently to asset prices. This policy is not accompanied by increased interest rate volatility but on the contrary: eliminating the non-fundamental effects of asset cycles on the real economy reduces macroeconomic volatility and via the Taylor rule also the variability of interest rates. Crucial for this result is the assumption that agents understand and internalize into their expectations the response of central banks to asset prices. If on the contrary, the central bank acts in a discretionary manner and does not communicate the reaction to asset prices transparently, the gains are insignificant. Moreover, the central bank does not have to respond linearly to stock price movements in order to implement the optimal policy within the class of simple Taylor rules considered. Responding symmetrically only after stock market returns surpass a certain threshold can achieve the same result. In the case of the US economy, reacting symmetrically to stock prices only when quarterly returns exceed 7% in absolute value maximizes macroeconomic stability and welfare.

During the 1987 stock market crash, the aggressive easing of monetary policy that helped the recovery of the economy and reflate asset prices has become to be known as the Greenspan put. The implied promise that the Fed will step in and help the financial markets, if needed, has continued over the years and has been relabelled as the Fed put. Recent evidence suggests that, although not explicitly, the Fed does indeed take into account the stance of the stock market when setting interest rates and moreover the main channel that they consider important is the consumption wealth effect: increases in stock prices make consumers feel wealthier and as a result adjust their consumption decisions accordingly.[1] The empirical evidence also shows that the marginal propensity to consume (MPC) out of (unrealized) capital gains can be as high as 20%.[2] Given the high volatility of stock prices, these effects can have a large impact on aggregate demand. Nevertheless, most of the research concerning the optimal response of monetary policy to asset prices does not take into account the actual dynamics of stock prices or the consumption wealth effect as the main channel through which stock prices affect the real economy. The present paper tries to fill this gap.

At the core of the analysis is the realistic assumption that agents have imperfect knowledge about the determination of macroeconomic variables. Agents are internally rational, in the sense of Adam and Marcet (2011), maximizing their utility given their system of beliefs. The asset pricing literature has pointed out that survey measures of expectations are positively correlated with actual prices while actual returns tend to display a negative correlation.[3] Rational expectations (RE) models have the opposite prediction, namely agents expect lower returns at the top of the cycle. In a lab experiment, Galí et al. (2021) show that agents' asset price beliefs are not consistent with rational expectations and propose that adaptive expectations fit better the experimental data. Moreover, the high volatility of stock prices relative to fundamentals, which has become known as the volatility puzzle, poses additional difficulty for RE macro-finance models.[4] This work builds on this evidence and specifies the belief system of the agents as extrapolative, using constant gain learning to update their beliefs about variables exogenous to their decision making. When stock prices depart from their fundamental value due to sentiment/expectation swings, consumption-wealth effects appear naturally in this framework since agents interpret their asset position as real wealth and modify the consumption decision accordingly. The proposed theory links directly the volatility puzzle with stock price wealth effects.

The stock price consumption wealth effect is incorporated in a quantitative Limited Asset Market Participation (LAMP) New Keynesian model where agents are heterogeneous with respect to their participation in the stock market

[1]See Cieslak and Vissing-Jorgensen (2021) and the literature review section
[2]See Di Maggio et al. (2020)
[3]See Greenwood and Shleifer (2014) and Adam et al. (2017)
[4]See Shiller (1981)

and have homogeneous imperfect information about macroeconomic variables. The economy is hit by three shocks: supply (cost push), monetary policy and a sentiment shock which affects the beliefs of the agents on their expected capital gains. The latter will operate as a demand shock influencing stock prices and aggregate demand via the consumption wealth effect. The model is estimated on US data by targeting a standard set of business cycle and financial moments. Although not explicitly targeted, the model is able to capture remarkably well the dynamics of survey expectations regarding capital gains, inflation and interest rates and the joint dynamics of the real economy and financial markets.

The quantitative model is used to evaluate the welfare properties of monetary policy under two regimes: responding to stock prices explicitly and transparently vs responding to stock prices without agents understanding the full systematic component of monetary policy. By transparency, it is understood that agents take into account the reaction of policy to stock prices when forming expectations about future interest rates. The reverse is true under non-transparency. Under each regime the central bank can respond linearly to stock prices or non-linearly using the Fed put or the Fed put-call. The former is a policy of responding to stock prices only during periods of negative returns (after a certain threshold) while the latter adds a response also during stock price increases. I show that by reacting transparently to stock prices (linearly or by using the put-call strategy with a threshold of 7%) monetary policy can increase welfare by 0.14% on average per period.[5] The optimal response implies increasing interest rates by 12 basis points for every 100% deviation of stock prices from their long run trend. The welfare improvement is not accomplished through pricking the bubble but instead from disconnecting sentiment driven cycles from the real economy. This result emphasizes the key mechanism through which stock prices targeting influences the economy in this environment, namely through managing long-term interest rate expectations and linking them to stock prices.

The rest of the paper is organized as follows. Section II reviews the literature on monetary policy and stock price targeting. Section III studies the origin of stock price wealth effects in a simple endowment economy. Section IV incorporates the mechanism from the previous section in a quantitative LAMP-NK model with homogeneous imperfect information and evaluates the quantitative

[5]If agents do not internalize the reaction of monetary policy to stock prices the gains are insignificant

performance on US data. Section V studies the macroeconomic stability and welfare properties of stock price targeting. Lastly, section VI concludes.

1.2 Related Literature

This research contributes to the literature that analyses stock price targeting and monetary policy. The seminal papers of Bernanke and Gertler (2000) and Bernanke and Gertler (2001) use a model with credit market frictions that features a financial accelerator effect in which exogenous shocks have an amplified effect on the economy. Using a calibrated version of the model they argue that targeting stock prices has no gain and that a central bank is better off, in terms of macroeconomic stability, by sticking to a flexible-inflation targeting regime. In reaching this conclusion their model does not take into account key financial facts like excess volatility of stock prices or market expectations. Carlstrom and Fuerst (2007) analyze the implications of stock price targeting on equilibrium determinacy and conclude that a central bank targeting explicitly stock prices raises the risk of inducing real indeterminacy in the system. Bullard, Schaling, et al. (2002) reach a similar conclusion. Cecchetti et al. (2000), using the same model as Bernanke and Gertler (2001), conclude that central banks can derive some benefit by reacting to stock prices. The main difference between Cecchetti et al. (2000) and Bernanke and Gertler (2000) and Bernanke and Gertler (2001) is the assumption about the nature of the shock. In Cecchetti et al. (2000) the central bank knows that the swings in stock prices are non-fundamental and, with this knowledge, reacting to stock prices can increase economic performance. In the papers described above and in most of the literature, the effect of stock prices on the economy either come from the supply side, as in Bernanke and Gertler (2001), or from the central bank reacting explicitly to stock price deviation in the Taylor rule. Nisticò (2012) develops a NK model with OLG households that features a direct demand effect of stock prices on output in the IS equation.[6]

[6]The effect appears due to the fact that in each period a fraction of households who own financial wealth die and are replaced by newcomers with zero stock holdings. Therefore, increases in stock prices in period t (which forecast higher financial wealth next period) generate higher consumption due to the desire of households to intertemporally smooth consumption. Once next period arrives, some households will not be affected by the higher financial wealth (since they were replaced with newcomers who do not hold any) and therefore the increase in aggregate consumption seems higher than granted by the increase in financial wealth. In this sense stock prices affect consumption although this stock wealth effect is

The author concludes that targeting stock price growth increases macroeconomic stability. Bask (2012) also argues for stock price targeting in a model with both fundamental and technical traders.

In a follow up paper, Airaudo et al. (2015), using the same model as Nisticò (2012) analyze the stability and learnability of the model and conclude that, if the stock-wealth effect is sufficiently strong, reacting to stock prices increases the policy space for which the equilibrium is both determinate and learnable.

Winkler (2019) introduces learning in a monetary model with financial frictions, similar to the one in Bernanke et al. (1999), and finds that the effects of shocks are amplified when agents learn about stock prices. The author also finds that by including a reaction to stock price growth in the Taylor rule improves macroeconomic stability. Adam et al. (2017) build a real model of the economy in which agents learn about stock price behavior and which is quantitatively able to reproduce the joint behavior of stock prices and the business cycle. Airaudo (2016) studies asset prices in a monetary model in which agents have long-horizon learning and finds the existence of a wealth effect. The issue of stability is then analyzed in the context of the central bank responding to stock prices and finds that reacting to stock prices increases the stability of the economy. Eusepi and Preston (2018a) show that imperfect knowledge about the structure of the economy generates wealth effects arising from long-term bond holdings. In their framework, the steady state level of long-term bonds influences the magnitude of this effect and the effects of monetary policy. Agents have perfect knowledge of how prices of long-term bonds are determined but there exists a wedge between their forecast of the quantity of future bonds and the future level of taxes which make bonds net wealth giving rise to the wealth effect.

Cieslak and Vissing-Jorgensen (2021) analyze Federal Open Market Committee (FOMC) transcripts and conclude that the FED officials pay attention to asset prices and perceive the stock market as influencing the economy mainly through a consumption-wealth effect. They show that stock prices decreases between 2 consecutive FOMC meetings is one of the best predictors of subsequent federal rate cuts. Case et al. (2005) and Case et al. (2013) and more recently Chodorow-Reich et al. (2021) bring empirical evidence for the existence of this

artificially generated by the assumption of households being replaced with 0 financial wealth ones.

wealth effect. The magnitude of this effect is not insignificant either. Di Maggio et al. (2020) show that unrealised capital gains lead to MPC ranging from 20% for the bottom 50% of the wealth distribution to 3% for the top 30%.

1.3 Wealth effects in endowment economies

This section lays out a basic endowment economy in which it is shown that incomplete information about stock prices fundamentally changes the equilibrium of the economy. In this environment, stock prices affect the endogenous variables due to a wedge between actual stock prices and their expected discounted sum of dividends.

Consider a flexible price endowment economy populated by a continuum of households, indexed by i, who maximize their utility by choosing how much to consume, C_t^i, save in bonds, B_t^i and invest in a risky asset, S_t^i. The risky asset is a claim to an exogenous stream of dividends, D_t. For simplicity assume that $D_t \sim \mathcal{N}(\mu, \sigma^2)$. Specifically, the problem of a typical household i is

$$\begin{aligned} &\max_{C_t^i, B_t^i, S_t^i} E_0^{\mathscr{P}_i} \sum_{t=0}^{\infty} \delta^t \frac{(C_t^i)^{1-\sigma}}{1-\sigma} \\ &s.t. \quad P_t C_t^i + B_t^i + S_t^i Q_t \leq B_{t-1}^i(1+i_{t-1}) + S_{t-1}^i(Q_t + D_t) \\ &\qquad 0 \leq S_t^i \leq S^H, \forall\, t \end{aligned} \tag{1.1}$$

where P_t is the aggregate price index, i_t is the nominal interest rate (set exogenously by the monetary authority) and Q_t is the ex-dividend price of the risky asset. The expectation is taken over the subjective probability measure $\mathscr{P}_i$ which is household specific and different than the rational expectation hypothesis, denoted by E. Furthermore there is a central bank following a Taylor type rule $i_t = \phi_\pi \pi_t$ which is common knowledge among all the agents in the economy. The FOCs are

$$\frac{1}{1+i_t} = \delta E_t^{\mathscr{P}_i} \Big\{ \Big(\frac{C_{t+1}^i}{C_t^i} \Big)^{-\sigma} \frac{1}{1+\pi_{t+1}} \Big\}, \tag{1.2}$$

$$Q_t = \delta E_t^{\mathscr{P}_i} \Big\{ \Big(\frac{C_{t+1}^i}{C_t^i} \Big)^{-\sigma} \frac{(Q_{t+1}+D_{t+1})}{1+\pi_{t+1}} \Big\}. \tag{1.3}$$

Letting $\mathscr{W}_t^i = B_{t-1}^i(1+i_{t-1}) + S_{t-1}^i(Q_t + D_t)$ and after imposing a transversality condition, the intertemporal BC becomes

$$\frac{\mathscr{W}_t^i}{P_t} = E_t^{\mathscr{P}_i} \sum_{j=0}^{\infty} \delta^j \left(\frac{C_{t+j}^i}{C_t^i}\right)^{-\sigma} C_{t+j}^i. \tag{1.4}$$

Equation 1.4 says that the discounted sum of future consumption equals real wealth.

In equilibrium, all agents are identical although they do not know this to be true. This will prove to be essential to the pricing of the risky asset and for the existence of the wealth effect. Given that agents have the same preferences, constraints and beliefs they will make the same decisions. Equilibrium implies

$$\begin{aligned} \int_0^1 B_t^i \, di &= 0, \\ \int_0^1 C_t^i \, di &= C_t = d_t, \\ \int_0^1 S_t^i \, di &= 1. \end{aligned} \tag{1.5}$$

Aggregating equation (1.4), imposing $E^{\mathscr{P}_i} = E^{\mathscr{P}}$ and applying the equilibrium condition (1.5) yields

$$q_t + d_t = E_t^{\mathscr{P}} \sum_{j=0}^{\infty} \delta^j \left(\frac{C_{t+j}}{C_t}\right)^{-\sigma} C_{t+j}. \tag{1.6}$$

where q_t and d_t are the risky asset real price and real dividends.

Before turning to the case of imperfect information, it is useful for comparison purposes to derive first the optimal consumption decision and equilibrium under RE.

1.3.1 Rational Expectations

First, it will be imposed that agents have RE: $E^{\mathscr{P}} = E$. Given this, the FOC with respect to stock prices can be substituted forward to arrive at

$$q_t = E_t \sum_{j=1}^{\infty} \delta^j \left(\frac{C_{t+j}}{C_t}\right)^{-\sigma} d_{t+j}. \tag{1.7}$$

Given this, equation (1.6) becomes

$$E_t \sum_{j=0}^{\infty} \delta^j \left(\frac{C_{t+j}}{C_t}\right)^{-\sigma} d_{t+j} = E_t \sum_{j=0}^{\infty} \delta^j \left(\frac{C_{t+j}}{C_t}\right)^{-\sigma} C_{t+j}. \tag{1.8}$$

Applying a first-order approximation around a non-stochastic steady state yields

$$E_t \sum_{j=0}^{\infty} \delta^j \tilde{d}_{t+j} = E_t \sum_{j=0}^{\infty} \delta^j \tilde{C}_{t+j}. \tag{1.9}$$

Using the fact that $E_t(\tilde{C}_{t+k}) = \tilde{C}_t + \frac{1}{\sigma} E_t \sum_{j=0}^{k-1} (i_{t+j} - \pi_{t+j+1})$ we arrive at the optimal consumption rule for the household.

Lemma 1. *Optimal consumption decision under RE*

$$\tilde{C}_t = (1-\delta) E_t \sum_{j=0}^{\infty} \delta^j \tilde{d}_{t+j} - \frac{1}{\sigma} \delta E_t \sum_{j=0}^{\infty} \delta^j (i_{t+j} - \pi_{t+j+1}). \tag{1.10}$$

Equation (1.10) highlights the standard transmission mechanism of monetary policy which operates through the inter-temporal substitution of consumption which is influenced by the whole future path of real interest rates. Notice that the equilibrium condition for the goods market: $\tilde{C}_t = \tilde{d}_t$ has not been imposed yet. Imposing this condition, using the interest rate rule and the process for dividends yields the unique RE equilibrium condition.

Proposition 1. *RE Equilibrium*

$$\pi_t = -\frac{\sigma}{\phi_\pi} \tilde{d}_t \tag{1.11}$$

Similarly to Eusepi and Preston (2018a), inflation is a linear function of the endowment process. Stock prices or beliefs about stock prices do not influence the real economy. Anticipating the next section, this will not be the case under imperfect knowledge and the reason will soon be clear.

1.3.2 Imperfect Knowledge: Learning

In deriving the optimal decision (1.10) we have used the fact that the price of the risky asset, q_t, can be written as the discounted sum of dividends, as in equation (1.7). Indeed, under RE this is true. Under imperfect knowledge, equation (1.3) cannot be iterated forward since this would imply that any agent would know that either he is the marginal agent forever or that all the other agents in the economy share his beliefs, preferences and constraints.

Since agents have imperfect knowledge about the economy, even if agents know that the other agents share their preferences and constraints but have different beliefs, agent i would not be able to apply the Law of Iterated Expectations (LIE) to his FOC since

$$\begin{aligned} q_t &= \delta E_t^{\mathscr{P}_i}\left\{\left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\sigma}(q_{t+1}+d_{t+1})\right\} \\ &= \delta E_t^{\mathscr{P}_i}\left\{\left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\sigma}\left(d_{t+1}+\delta E_{t+1}^{\mathscr{P}_{mg}}\left\{\left(\frac{C_{t+2}^{mg}}{C_{t+1}^{mg}}\right)^{-\sigma}(d_{t+2}+q_{t+2})\right\}\right\}\right. \end{aligned} \tag{1.12}$$

and $E_t^{\mathscr{P}_i}E_{t+1}^{\mathscr{P}_{mg}} \neq E_t^{\mathscr{P}_i}$. Here $\mathscr{P}^{mg}$ is the subjective probability measure of the marginal agent which is not known by agent i at time t. The marginal agent is the agent with the highest valuation of the asset which will determine the price of the asset in that period.[7] Therefore, in this environment, the optimality condition for stock prices is of the one-step ahead form which after log-linearization becomes

$$\tilde{q}_t = (1-\delta)E_t^{\mathscr{P}}(\tilde{d}_{t+1}) + \delta E_t^{\mathscr{P}}(\tilde{q}_{t+1}) + \sigma(\tilde{C}_t - E_t^{\mathscr{P}}\tilde{C}_{t+1}) \tag{1.13}$$

where the expectation regarding stock prices follows an updating equation that will next be derived from agents' belief system

$$E_t^{\mathscr{P}}(\tilde{q}_{t+1}) = E_{t-1}^{\mathscr{P}}(\tilde{q}_t) + \lambda(\tilde{q}_{t-1} - E_{t-1}^{\mathscr{P}}(\tilde{q}_t)).^{8} \tag{1.14}$$

Using the previous results, the assumption of the Average Marginal Agent described in Appendix A results in the optimal decision of the household under imperfect knowledge.

[7] See Adam and Marcet (2011) for more details

[8] See Appendix A for a detailed discussion of the consistency of this result in the context of long-horizon learning

Lemma 2. *Optimal consumption decision under Imperfect Knowledge*

$$\begin{aligned}\tilde{C}_t \approx & (1-\delta)E_t^{\mathscr{P}}\sum_{j=0}^{\infty}\delta^j\tilde{d}_{t+j} - \frac{1}{\sigma}\delta E_t^{\mathscr{P}}\sum_{j=0}^{\infty}\delta^j(i_{t+j}-\pi_{t+j+1}) \\ & \underbrace{+\delta\tilde{q}_t - (1-\delta)\left[E_t^{\mathscr{P}}\sum_{j=1}^{\infty}\delta^j\tilde{d}_{t+j} - \frac{\delta}{1-\delta}E_t^{\mathscr{P}}\sum_{j=0}^{\infty}\delta^j(i_{t+j}-\pi_{t+j+1})\right]}_{\textit{Wealth Effect=0 in RE}}.\end{aligned} \tag{1.15}$$

The first line from (1.15) is the standard transmission mechanism as also found under RE (see equation (1.10)). The second line represents a new channel through which stock prices and beliefs about stock prices affect the consumption decision of the household. The second channel is the difference between actual stock prices and the discounted sum of future dividends. Under RE these terms would sum exactly to 0 since stock prices are exactly equal to the discounted sum of dividends. Under learning, there is no reason for this to be the case. Since beliefs influence stock prices and vice versa, stock prices may drift away from their perceived fundamental value therefore causing agents to feel wealthier and increase consumption. In the current framework stock price wealth effects appear because agents do not have perfect knowledge about the economy and how stock price are actually determined. That people do not have perfect knowledge about how stock prices are determined should not surprise anyone. What is interesting is that this lack of knowledge is the principal determinant of stock price wealth effects.

In order to determine the learning equilibrium I will assume the following:

1. similarly to RE, agents have perfect knowledge about the dividend process, therefore $E^{\mathscr{P}}d_{t+j} = \mu$
2. agents know the interest rate rule, therefore $E^{\mathscr{P}}i_{t+j} = \phi_\pi E^{\mathscr{P}}\pi_{t+j}$
3. agents think that inflation and stock prices follow an unobserved component model

$$\begin{aligned}x_t &= \beta_t^x + \epsilon_t \\ \beta_t^x &= \beta_{t-1}^x + \psi_t\end{aligned} \tag{1.16}$$

where $x = (\tilde{q}, \pi)'$.

Denoting by $\hat{\beta}_{t-1} = (\hat{\beta}^{\pi}_{t-1}, \hat{\beta}^{q}_{t-1})$ period t subjective expectations, agents use the following optimal recursive algorithm to update their beliefs

$$\hat{\beta}_t = \hat{\beta}_{t-1} + \lambda(x_t - \hat{\beta}_{t-1}) \tag{1.17}$$

where λ is the constant gain coefficient which governs the speed at which agents incorporate new information into current beliefs.[9] Given these assumptions the expectations of real interest rates from equation (1.15) can be evaluated as

$$E^{\mathscr{P}}_t \sum_{j=0}^{\infty} \delta^j (i_{t+j} - \pi_{t+j+1}) = \phi_\pi \pi_t + \frac{\delta\phi_\pi - 1}{1-\delta} \beta^{\pi}_{t-1}. \tag{1.18}$$

Substituting the forecasts, (1.18), into the consumption equation (1.15), applying assumption 1 and market clearing in the goods market, $\tilde{C}_t = \tilde{d}_t$, gives the data-generating process or the actual law of motion for inflation.

Proposition 2. *Learning Equilibrium*

$$\pi_t = \frac{\delta\sigma}{\phi_\pi} \beta^{q}_{t-1} - \left(\frac{\sigma}{\phi_\pi} - \frac{(1-\sigma)(\delta\phi_\pi - 1)}{(1-\delta)\phi_\pi} \right) \beta^{\pi}_{t-1} - \frac{\sigma}{\phi_\pi} \tilde{d}_t. \tag{1.19}$$

The learning equilibrium is fundamentally different from the RE counterpart. The first term in the above equation is totally absent from the RE equilibrium relation. Beliefs about stock prices influence directly inflation in equilibrium through a stock price wealth effect. Eusepi and Preston (2018a) reach a similar conclusion for the case of long-term bonds, although in that case agents are assumed to know the pricing map and learn about taxes and long term bonds.

1.4 Monetary Policy and Stock Prices: Quantitative Evaluation

This section describes a heterogeneous agents New Keynesian model with learning in which agents hold subjective beliefs about the variables which are exogenous

[9] As it is usually done in the learning literature, in order to avoid the simultaneity formation of beliefs and equilibrium variables, agents form expectations at period t using information from the previous period

to their decision making (from the point of view of an individual agent). There are two types of consumers and the only source of heterogeneity between them is the fact that a constant fraction, $\mathscr{O}$, of the agents is assumed not to participate in the stock market. This assumption is in line with the empirical evidence on US stock market participation. Notice that while some of the agents are excluded from saving in stocks all agents have access to the bond market and can smooth consumption by investing in a riskless asset. In essence, the model is a limited asset market participation New Keynesian model (LAMP-NK) with homogeneous imperfect information. The economy is comprised of households, final goods producers, intermediary goods producers, a mutual fund and a central bank conducting monetary policy.

1.4.1 Households

The economy is populated by a continuum of infinitely lived consumers indexed by i who choose consumption, C_t^i, labor, N_t^i, bond holdings, B_t^i, stock holdings in a mutual fund, S_t^i, and receive income in form of dividends, D_t and wages, W_t. The mutual fund is introduced to abstract from the portfolio choice of the households and its problem will be described in a later section. Let $i = S^U, S^C$ denote the agents who do/do not participate in the stock market. The problem of the household is to maximize utility subject to a standard budget constraint

$$\begin{aligned} &\max_{C_t^i, N_t^i, B_t^i, S_t^i} E_0^{\mathscr{P}} \sum_{t=0}^{\infty} \delta^t \left[\frac{(C_t^i)^{1-\sigma}}{1-\sigma} - \frac{(N_t^i)^{1+\phi}}{1+\phi} \right] \\ &\quad s.t. \quad P_t C_t^i + B_t^i + S_t^i Q_t \leq B_{t-1}^i (1 + i_{t-1}) + W_t N_t^i + S_{t-1}^i (Q_t + D_t) \\ &\qquad 0 \leq S_t^i \leq S^H, \forall\, t \end{aligned} \tag{1.20}$$

where P_t is the aggregate price index, i_t is the nominal interest rate, Q_t is the ex-dividend price of the mutual fund share, W_t is the nominal wage and D_t is the nominal dividend paid by the mutual fund. Short-selling is not allowed and there is an upper bound for stock holdings, S^H, which can be bigger than 1.[10] The optimality conditions of the household problem are

[10] It has been assumed that the upper bound on stock holdings is never reached.

$$\frac{(N_t^i)^{\phi}}{(C_t^i)^{-\sigma}} = w_t, \tag{1.21}$$

$$\frac{1}{1+i_t} = \delta E_t^{\mathscr{P}} \Big\{ \Big(\frac{C_{t+1}^i}{C_t^i} \Big)^{-\sigma} \frac{1}{1+\pi_{t+1}} \Big\}, \tag{1.22}$$

$$Q_t = \delta E_t^{\mathscr{P}} \Big\{ \Big(\frac{C_{t+1}^i}{C_t^i} \Big)^{-\sigma} \frac{(Q_{t+1}+D_{t+1})}{1+\pi_{t+1}} \Big\}, \tag{1.23}$$

where π_{t+1} is the inflation rate between t and $t+1$ and $w_t = \frac{W_t}{P_t}$ is the real wage. Equation (1.21) determines the consumption and labor decision, equation (1.22) is the Euler equation and equation (1.23) is the asset pricing equation. Also notice that equation (1.23) holds with equality as long as $S_t^i \in [0, S^H)$.

The only difference from the standard household problem is the operator $E_t^{\mathscr{P}}$. The expectations of the households are determined using the subjective probability measure $\mathscr{P}$ which assigns probabilities to the variables the household is trying to forecast. I proceed in deriving the consumption decision of the household following the anticipated utility framework of Preston (2005).[11] The intertemporal budget constraint of the household reads

$$\frac{\mathscr{W}_t^i}{P_t} \approx E_t^{\mathscr{P}} \sum_{j=0}^{\infty} \delta^j \Big(\frac{C_{t+j}^i}{C_t^i} \Big)^{-\sigma} \big[C_{t+j}^i - w_{t+j}^{\frac{1+\phi}{\phi}} (C_{t+j}^i)^{\frac{-\sigma}{\phi}} \big] \tag{1.24}$$

where $\mathscr{W}_t^i = B_{t-1}^i(1+i_{t-1}) + S_{t-1}^i(Q_t + D_t)$ represents nominal wealth at time t. Log-linearization of equation (A.3) around a steady state characterized by $\pi = 0$, $S = 1$, $C = Y$ yields

$$\tilde{w}_t^i = (1-\delta) E_t^{\mathscr{P}} \Big\{ \sum_{j=0}^{\infty} \delta^j \Big[-r_{t+j}^N + \sigma \tilde{c}_t^i + \Delta_r \, \tilde{c}_{t+j}^i - \frac{\Delta_i}{1-\delta} \tilde{w}_{t+j} \Big] \Big\}. \tag{1.25}$$

[11] A large body of the literature uses the *Euler Equation* approach to introduce learning in DSGE models. This approach entails that after solving the model using the RE assumption, expectations are replaced mechanically with some subjective expectations. This approach implies that agents are mixing two probability measures, the RE measure, on the one hand, and the subjective one. Furthermore, the stock market wealth effect is not present under this approach since agents implicitly know the mapping from dividends to prices. See Preston (2005) for a detailed discussion of this issue and Eusepi and Preston (2018b) for a comparison between these two approaches.

and $w\ \tilde{w}_t^i = (1+i)b_{t-1}^i + q(\tilde{S}_{t-1}^i + \tilde{q}_t) + d(\tilde{S}_{t-1}^i + \tilde{d}_t)$ where $w = \frac{d}{1-\delta}$. In the above expression any variable $\tilde{x}$ denotes percentage deviation of real variables from their steady-state values while Y, q and d represent steady-state values of aggregate output, real stock price and real dividends.
Log-linearization of the Euler equation (1.22) yields

$$\tilde{c}_t^i = E_t^{\mathscr{P}} \tilde{c}_{t+1}^i - \frac{1}{\sigma}(i_t - E_t^{\mathscr{P}_i} \pi_{t+1}) \tag{1.26}$$

which can be rewritten as

$$E_t^{\mathscr{P}}(\tilde{c}_{t+k}^i) = \tilde{c}_t^i + \frac{1}{\sigma} E_t^{\mathscr{P}_i} \big[\sum_{j=0}^{k-1} i_{t+j} - \pi_{t+j+1} \big]. \tag{1.27}$$

Substitution of equation (1.27) in the linearized budget constraint (1.25) and rearranging results in the decision rule of the household[12]

$$\tilde{c}_t^i = \Delta_i \tilde{w}_t^i + \Delta_w \sum_{j=0}^{\infty} \delta^j E_t^{\mathscr{P}}(\tilde{w}_{t+j}) - \frac{\delta}{\sigma} \Delta_r \sum_{j=0}^{\infty} \delta^j E_t^{\mathscr{P}}(i_{t+j} - \pi_{t+j+1}). \tag{1.28}$$

Equation (1.28) makes clear that the consumption decision of the household today depends not only on the next period's output and interest rate (as dictated by the standard Euler equation) but on the whole future path of wages, inflation and interest rates, as well as on the current wealth. Therefore, the agent will need to form expectations/forecasts for all future π, $\tilde{w}$ and i using the subjective probability measure $\mathscr{P}$. The next proposition presents the optimal consumption decision for the two types of agents.

Proposition 3. *The log-linearized aggregate consumption decisions at time t for households participating in the stock market (U) and excluded from trading*

[12] See appendix for expressions of the composite parameters and details about derivation of equation (1.28)

stocks (C) are given by

$$\tilde{c}_t^U = \Delta_i\Big[\frac{(1+i)}{w}b_{t-1}^U + \tilde{S}_{t-1}^U + \delta\tilde{q}_t + (1-\delta)\tilde{d}_t\Big] + \Delta_w\sum_{j=0}^{\infty}\delta^j E_t^{\mathscr{P}}(\tilde{w}_{t+j}) - \frac{\delta}{\sigma}\Delta_r\sum_{j=0}^{\infty}\delta^j E_t^{\mathscr{P}}(i_{t+j}-\pi_{t+j+1}). \tag{1.29}$$

$$\tilde{c}_t^C = \Delta_i\Big[(1+i)b_{t-1}^C\Big] + \Delta_w\sum_{j=0}^{\infty}\delta^j E_t^{\mathscr{P}}(\tilde{w}_{t+j}) - \frac{\delta}{\sigma}\Delta_r\sum_{j=0}^{\infty}\delta^j E_t^{\mathscr{P}}(i_{t+j}-\pi_{t+j+1}). \tag{1.30}$$

Notice that the only difference between the optimal consumption decisions of the two types of households is given by the first term from both equations, namely the asset position at time t.

1.4.2 Firms

Intermediate goods producers

There is a continuum of firms indexed by j which produce differentiated goods using the Cobb-Douglas production function with labor input $N_t(j)$

$$Y_t(j) = N_t(j)^{1-\alpha}. \tag{1.31}$$

Firms are subject to nominal rigidities when setting prices. Following Calvo (1983), each firm cannot reset its price in a given period with probability θ. The problem of the firm is to maximize profits subject to the demand function

$$\begin{aligned} \max_{P_t^*} \quad & \sum_{k=0}^{\infty}\theta^k E_t^{\mathscr{P}_j}\{Q_{t,t+k}(P_t^* Y_{t+k/t} - \psi_{t+k}(Y_{t+k/t}))\} \\ s.t. \quad & Y_{t+k/t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} Y_{t+k} \end{aligned} \tag{1.32}$$

where $Y_{t+k/k}$ denotes output in period $t+k$ for a firm that last reset price in period t, $\psi_t()$ is the cost function and $Q_{t,t+k} = \delta^k\left(\frac{Y_{t+k}}{Y_t}\right)^{-\sigma}\frac{P_t}{P_{t+k}}$ is the stochastic discount factor for nominal profits.[13] Notice that compared to the RE framework, the stochastic discount factor is a function of aggregate output and not of

[13] It is assumed implicitly that households and firms share the same belief $\mathscr{P}$

consumption. This is because firms do not know the problem of the households or of the mutual fund and therefore, it makes possible for firms to hold subjective beliefs about aggregate outcomes.

The solution to the profit maximization problem yields the optimal price setting decision of the firm

$$P_t^* = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{k=0}^{\infty} (\theta\delta)^k E_t^{\mathscr{P}_j} \left[Y_{t+k}^{1-\sigma} P_{t+k}^{\epsilon} MC_{t+k/k} \right]}{\sum_{k=0}^{\infty} (\theta\delta)^k E_t^{\mathscr{P}_j} \left[Y_{t+k}^{1-\sigma} P_{t+k}^{\epsilon-1} \right]} \tag{1.33}$$

where $MC_{t+k/k}$ is the real marginal cost of a firm which last updated prices in period t. After log-linearization the previous relation becomes

$$p_t^* = (1 - \delta\theta) \sum_{k=0}^{\infty} (\theta\delta)^k E_t^{\mathscr{P}_j} \left\{ \frac{\alpha}{1 - \alpha + \epsilon\alpha} \tilde{y}_{t+k} + \frac{1 - \alpha}{1 - \alpha + \epsilon\alpha} (\tilde{w}_{t+k} + \epsilon_{t+k}^u) + p_{t+k} \right\} \tag{1.34}$$

where ϵ_{t+k}^u is an exogenous process interpreted as a cost-push shock.

Final goods producers

The consumption good in this economy is produced by perfectly competitive firms which use intermediary goods as inputs in their CES production function:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{1-\epsilon}} \tag{1.35}$$

Profit maximization yields the following demand for intermediary goods:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} Y_t \tag{1.36}$$

where $P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}$ is the aggregate price index.

1.4.3 Mutual Fund

For the sake of simplicity, the analysis abstracts from the portfolio choice of the households and instead assumes the existence of a mutual fund which holds all the intermediary firms in this economy and issues shares with nominal price Q_t which are sold in a perfectly competitive market to the household sector. The asset pricing equation of the mutual fund is given by:

$$Q_t = \delta E_t^{\mathscr{P}} \Big\{ \Big(\frac{Y_{t+1}}{Y_t} \Big)^{-\sigma} \frac{(Q_{t+1} + D_{t+1})}{1 + \pi_{t+1}} \Big\} \tag{1.37}$$

which in equilibrium will be the same as the asset pricing equation of the households.

1.4.4 Central Bank

The monetary authority sets the interest rate by following a Taylor rule

$$i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \epsilon_t^i \tag{1.38}$$

where ϵ_t^i is a stochastic process with zero mean which can be interpreted as a monetary policy shock.

1.4.5 Equilibrium

Defining aggregate consumption of the two types of households as

$$C_t^C = \int_0^1 C_t^{i,C} di, \; C_t^U = \int_0^1 C_t^{i,U} di,$$

and aggregate labour as

$$N_t^C = \int_0^1 N_t^{i,C} di, \; N_t^U = \int_0^1 N_t^{i,U} di,$$

the equilibrium conditions are

$$\begin{aligned} &\int_0^1 B_t^i \, di = 0, \\ &\int_0^1 C_t^i \, di = C_t = \mathscr{O} C_t^C + (1 - \mathscr{O}) C_t^U = Y_t, \\ &\int_0^1 N_t(j) \, dj = \int_0^1 N_t^i \, di = \mathscr{O} N_t^C + (1 - \mathscr{O}) N_t^U, \\ &S_t^U = 1. \end{aligned} \tag{1.39}$$

First equation is the bond market clearing condition which assumes that bonds are in 0 net supply. The next two equations are the good market and labor market clearing conditions and finally the last equation requires clearing in the equity market.

On the supply side, since producers of intermediate goods are identical, the fraction of firms that will re-optimize each period $(1-\theta)$ will choose the same price, p_t^*. This fact combined with the definition of the aggregate price level (see Final goods producers section) results in the following aggregate law of motion for inflation

$$\pi_t = (1-\theta)(p_t^* - p_{t-1}). \tag{1.40}$$

Aggregating the household decision rule (1.28) and combining it with the market clearing condition (1.39) results in the demand block of the model, the IS equation

$$\tilde{y}_t = \Delta_i \, \mathcal{O}(\delta \tilde{q}_t + (1-\delta)\tilde{d}_t) + \Delta_w \sum_{j=0}^{\infty} \delta^j E_t^{\mathscr{P}}(\tilde{w}_{t+j}) - \frac{\delta}{\sigma}\Delta_r \sum_{j=0}^{\infty} \delta^j E_t^{\mathscr{P}}(i_{t+j} - \pi_{t+j+1}). \tag{1.41}$$

Equation (1.41) implies that not only current and future wages and real interest rates affect output today but also current stock prices. Agents do not internalize the fact that their pricing equation is determining stock prices today but instead they hold subjective beliefs about its evolution, therefore creating an equity channel effect: an increase in the equity prices today makes the consumers feel wealthier which affects aggregate consumption and output. As discussed in the previous section this stock price wealth effect appears because of the difference between actual stock prices and their fundamental value, determined by the discounted sum of dividends. If the same economy would be studied under the Euler Equation approach, then there would be no equity channel effect.

Combining the law of motion of inflation (1.40) with the pricing equation of the firms (1.34) results in the supply block of the model, the Phillips Curve equation

$$\begin{aligned}\pi_t =&\Theta_y \sum_{k=0}^{\infty}(\theta\delta)^k E_t^{\mathscr{P}} \tilde{y}_{t+k} + \Theta_w \sum_{k=0}^{\infty}(\theta\delta)^k E_t^{\mathscr{P}} \tilde{w}_{t+k} \\ &+ \sum_{k=0}^{\infty}(\theta\delta)^k E_t^{\mathscr{P}} u_{t+k} + (1-\theta)\delta \sum_{k=0}^{\infty}(\theta\delta)^k E_t^{\mathscr{P}} \pi_{t+k+1}.\end{aligned} \tag{1.42}$$

Log-linearization of equation (1.23) yields the low of motion of stock prices:

$$\tilde{q}_t = (1-\delta)E_t^{\mathscr{P}}(\tilde{d}_{t+1}) + \delta E_t^{\mathscr{P}}(\tilde{q}_{t+1}) - (i_t - E_t^{\mathscr{P}}(\pi_{t+1})). \tag{1.43}$$

Given optimal prices, firms supply the desired output which determines the amount of labor

$$N_t = Y_t^{1/(1-\alpha)} e^{d_t} \tag{1.44}$$

which is obtained by aggregating the individual production technologies. The last term captures price dispersion and is given by $d_t = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)$. Wages are determined by the optimality condition of the households

$$w_t = \frac{N_t^{\phi}}{Y_t^{\sigma}}. \tag{1.45}$$

Finally, real dividends are given by the profits of the firms

$$D_t = Y_t - w_t N_t. \tag{1.46}$$

1.4.6 Agents' model of learning

The subjective belief system of the agents can be characterized by the probability space $(\Omega, \mathscr{P})$ with a typical element $\omega \in \Omega$, $\omega = \{Y_t, P_t, Q_t, D_t, W_t, u_t, \epsilon^i\}$. As in Eusepi and Preston (2018a) the belief model includes the variables (exogenous from the point of view of the individual agents) that agents need to forecast in order to make optimal consumption decision today. These are output, inflation, stock prices dividends and wages.

It is assumed that agents believe that output, inflation, wages, dividends and equity prices follow an unobserved component model

$$\begin{aligned} z_t &= \beta_t + \zeta_t \\ \beta_t &= \rho\beta_{t-1} + \vartheta_t \end{aligned} \tag{1.47}$$

where $z_t = (\tilde{y}_t, \tilde{\pi}_t, \tilde{q}_t, \tilde{d}_t, \tilde{w}_t)'$, $\rho \in (0,1]$, $\zeta_t \sim N(0, \sigma_\zeta^2 I_5)$ and $\vartheta_t \sim N(0, \sigma_\vartheta^2 I_5)$. Agents have also knowledge of the Taylor rule that the central bank is following and uses it to forecast interest rates. As usually done in the learning literature, agents have full knowledge of the exogenous shocks.[14] The optimal filter for $E^{\mathscr{P}}(\beta_t/g^{t-1}) = \hat{\beta}_t$ is the Kalman filter and optimal updating implies the following recursion of beliefs

$$\hat{\beta}_t = \rho\, \hat{\beta}_{t-1} + \lambda(z_t - \rho\hat{\beta}_{t-1}) + e_3\, \epsilon_t^\beta \tag{1.48}$$

where $\hat{\beta}_t = [\hat{\beta}_t^y, \hat{\beta}_t^\pi, \hat{\beta}_t^q, \hat{\beta}_t^d, \hat{\beta}_t^w]'$, $g^{t-1} = \{g_{t-1}, g_{t-2} \dots g_1\}$ denotes information up to time t, $e_3 = (0,0,1,0,0)'$, ϵ_t^β, is a shock to stock price beliefs (sentiment shock) and λ is the steady state Kalman gain which controls the speed of learning.[15] Adam et al. (2017) show that survey data regarding price expectations are captured well by an extrapolative updating equation of the form (1.48). Nagel and Xu (2019) call this "learning with fading memory" and links it to the theoretical biology literature which models memory decay in organisms.

It follows from (1.47) that the agents' forecasts/beliefs about output, inflation, equity prices, dividends and wages are given by

$$E_t^{\mathscr{P}} z_{t+k} = \rho^{k-1}\hat{\beta}_{t-1} \tag{1.49}$$

where beliefs, $\hat{\beta}_t$, are updated each period according to (1.48).

Belief system (1.47) together with the optimal filtering rule implies that agents learn about the long-run conditional means of the variables in the economy. As argued in Eusepi et al. (2018) the belief system proposed is less restrictive than might be thought since usually the drift term drives the largest deviations from rational expectations predictions.

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[14] There is no reason for agents to know these shocks and how they affect the other variables. In fact, if we assume that agents do not have perfect knowledge about the exogenous shocks the dynamics of the model would be quite different. For example, if agents do not understand how monetary policy affects the economy and just observe an increase/decrease in interest rates then the IRF of output would exhibit the same hump shape response that we observe in the empirical VARs, e.g. Christiano et al. (2005). This is not related to the wealth effect so I prefer to stick to the status-quo in the literature on this issue.

[15] The resulting equation of belief updating is optimal given the assumption of agents observing the transitory component with a lag. In that case, ϵ_t^q represents the new information about the transitory component. For further details and derivation see Appendix 6 from Adam et al. (2017)

1.4.7 Full linearized model and learning dynamics

Equilibrium equations (1.41), (1.42), (1.43), (1.38), (1.45) and (1.46) together with the learning scheme represented by equations (1.48) and (1.49) fully characterize the dynamics of this economy. Substituting agents' subjective forecasts (1.49) into equilibrium conditions, results in the following system of equations

$$\begin{aligned} A\, Z_t &= B\,(\hat{\beta}_t^Z + e_3 \epsilon_t^{\beta}) + C\,\epsilon_t \\ \hat{\beta}_t^Z &= \rho\,\hat{\beta}_{t-1}^Z + \lambda(Z_{t-1} - \hat{\beta}_{t-1}^Z) \end{aligned} \tag{1.50}$$

where

$$Z_t = (\tilde{y}_t, \pi_t, \tilde{q}_t, i_t, d_t, \tilde{w}_t)',$$

$$\hat{\beta}_{t-1}^Z = (\hat{\beta}_{t-1}^y, \hat{\beta}_{t-1}^{\pi}, \hat{\beta}_{t-1}^q, \hat{\beta}_{t-1}^i, \hat{\beta}_{t-1}^d, \hat{\beta}_{t-1}^w)',$$

$$\epsilon_t = (u_t, \epsilon_t^i)',$$

$$e_3 = (0, 0, 1, 0, 0, 0)'.$$[16]

The stock market and the real output gap are determined simultaneously in equilibrium. Suppose that in period t agents are hit by a wave of optimism which causes stock prices to increase in the same period. This in turn triggers the stock price wealth effect and increases output contemporaneously via the IS equation. The central bank reacts to this increase in output by increasing interest rates. The increase in interest rate has two effects. Firstly by the intertemporal substitution channel of monetary policy it lowers consumption and output today. Secondly, it affects negatively stock prices which through the stock price wealth effect might further decrease output. If monetary policy does not react strongly enough, the increase in interest rates might not be sufficient to counteract the initial increase in stock prices which will trigger a positive revision in stock price beliefs which reinforces further the rise in stock prices. The system is self-referential in the sense of Marcet and Sargent (1989): prices affect beliefs which influence prices therefore resulting in a positive feedback

[16]see Appendix B for details of matrices A, B and C.

loop. Policy can play an important role in breaking or further accommodating this positive feedback loop.

1.4.8 Estimation of the model

This sections starts by calibrating/estimating the parameters of the model on US quarterly data. Using standard values found in the literature, the elasticity of substitution among goods, ϵ, is to 6 and the Frisch elasticity of labor-supply, ϕ, to 0.75 following the recommendation from Chetty et al. (2011). From the supply side, the share of labor, α, equals 1/3 and the probability of not being able to adjust prices, θ, is set to 2/3 implying an average duration of keeping prices fixed of 3 quarters. The Taylor rule response to output-gap is set to 0.5/4 and the one for inflation to 1.5. The response of the central bank to the stock-price gap is set to 0 for now but its effect on financial stability will be discussed in a later section. As a benchmark, three exogenous shocks will drive the dynamics of the model: cost push shocks, u_t, equity belief shocks, ϵ_t^β and monetary policy shocks, ϵ^i. These shocks follow AR(1) processes:

$$x_t = \rho_x x_{t-1} + \xi_t^x \tag{1.51}$$

where $x \in \{u, \epsilon^\beta, \epsilon^i\}$ and $\xi_t^x \sim \mathcal{N}(0, \sigma^x)$. Sentiment shocks are assumed to be $i.i.d$, $\rho_\beta = 0$.

The risk aversion parameter, σ is set to 1 and the discount factor, δ to 0.9928. The stock ownership is set to 0.47 which represents the average stock ownership over the period 1989-2019 according to the Survey of Consumer Finances. The calibration is summarized in the following table.

Calibrated	Symbol	Value
Discount factor	δ	0.9928
Risk aversion coef.	σ	1
Frisch labor supply elasticity	$\frac{1}{\phi}$	0.75
Elasticity of substitution	ϵ	6
Prob. of not adjusting price	θ	2/3
Share of labor	α	0.25
Taylor-rule coef. of inflation	ϕ_π	1.5
Taylor-rule coef. of output	ϕ_y	0.5/4
Equity Share Ownership	$1-\mathcal{O}$	0.47

TABLE 1.1: Calibrated Parameters

The rest of the parameters: standard deviation of cost push shock, σ^u, standard deviation of belief shock, σ^β, standard deviation of monetary policy shock, σ^{ϵ_i}, persistence of cost-push shock, ρ_u, persistence of monetary policy shock, ρ_{ϵ_i}, kalman gain coefficient, λ and autoregressive coefficient of beliefs, ρ, are jointly estimated using the method of simulated moments (MSM) to match a set of eight business cycle and financial moments.

Defining $\theta = (\sigma^u, \sigma^\beta, \sigma^{\epsilon_i}, \rho_u, , \rho_{\epsilon_i}, \lambda, \rho)$ as the vector of parameters to be estimated, the MSM estimator is given by

$$\hat{\theta} = \arg\min_{\theta} \; [\hat{S} - S(\theta)]' \, \hat{\Sigma} \, [\hat{S} - S(\theta)] \tag{1.52}$$

where $\hat{S}$ is the vector of empirical moments to be matched, $S(\theta)$ is the model moments counterpart and $\hat{\Sigma}$ is a weighting matrix.[17] The vector of empirical moments is given by

$$\hat{S} = \left[\, \hat{\sigma}(y), \hat{\sigma}(\pi), \hat{\rho}_{y,\pi}, \hat{E}(P/D), \hat{\sigma}(P/D), \hat{\rho}(P/D), \hat{\sigma}(r^e), \hat{\sigma}(r^f) \,\right]' \tag{1.53}$$

where

$\hat{\sigma}(y)$: standard deviation of the business cycle component of real output,

[17] I use the inverse of the estimated variance-covariance matrix of the data moments, $\hat{S}$. The latter is obtained using a Newey-West estimator and the delta method as in Adam et al. (2016). For further details on the estimation of $\hat{\Sigma}$ please refer to online appendix of that paper.

$\hat{\sigma}(\pi)$: standard deviation of business cycle component of inflation rate,

$\hat{\rho}_{y,\pi}$: correlation between inflation and business cycle component of output,

$\hat{E}(P/D)$: average of the Price Dividend ratio of stock market index,

$\hat{\sigma}(P/D)$: standard deviation of Price Dividend ratio,

$\hat{\rho}(P/D)$: persistence of the PD ratio,

$\hat{\sigma}(r^e)$: standard deviation of rate of return of the stock market index,

$\hat{E}(r^f)$: average real short term interest rate,

$\hat{\sigma}(r^f)$: standard deviation of real short term interest rate. [18]

The model is estimated on quarterly US data for the post-war period 1955Q1-2018Q4. The data for the business cycle statistics are obtained from the FRED database: the inflation rate is measured as the % change in the CPI for all urban consumers [CPIAUCSL], output as real GDP [GDPC1] and the fed funds rate [FEDFUNDS] is used for the short term nominal interest rate. The real interest rate is obtained by subtracting the ex-post inflation rate from the nominal short term interest rate. Data on real stock market prices and dividends are obtained from Robert Shiller webpage. Since data is monthly, quarterly variables have been obtained by selecting end of period values.[19]

Table 2.1 summarizes the estimated parameters while Table 1.3 shows the data moments and the model implied counterparts.

[18] The business cycle component is extracted using Hodrick-Prescott filter with a smoothing parameter of 1600 for quarterly data

[19] Averaging monthly variables instead of taking end of period does not change the results; the correlation between the two series is 0.997

Estimated	Symbol	Value
Std. cost push shock	σ^u	0.0013
Std. equity belief shocks	σ^{β_q}	0.0623
Std. MP shocks	σ^{ϵ_i}	0.0007
Autoregressive coef. cost push shock	ρ_u	0.9539
Autoregressive coef. MP shocks	ρ_{β_q}	0.9685
Kalman gain	λ	0.0011
Autoregressive coef. beliefs	ρ	0.99

TABLE 1.2: Estimated Parameters

Business Cycle	Symbol	Data Moment	Learning Model Moment	Learning Model t-ratio	RE Model
Std. dev. of output	$\sigma(y)$	1.45	1.47	-0.39	0.27
Std. dev. of inflation	$\sigma(\pi)$	0.54	0.45	1	0.29
Correlation output/inflation	$\rho_{y,\pi}$	0.29	0.26	0.36	-1
Financial Moments					
Average PD ratio	E (P/ D)	154	154	-0.38	138
Std. dev. of PD ratio	$\sigma(P/D)$	63	65	-0.34	9
Auto-correlation of PD ratio	$\rho(P/D)$	0.99	0.96	0.57	0.05
Std. dev. of equity return (%)	$\sigma(r^e)$	6.02	6.05	0.04	9
Std. dev. real risk free rate (%)	$\sigma(r^f)$	0.72	0.8	0.59	0.0017
Non Targeted moments					
Average Equity Return (%)	$E(r^e)$	1.78	0.9	1.92	0.73
Average real risk free rate (%)	$E(r^f)$	0.32	0.78	-3.5	0.72
volatility ratio stock prices/output	$\sigma(Q)/\sigma(y)$	6.7	5.2	2	23
corr. Stock Prices/ output	$\rho(Q,y)$	0.5	0.45	0.53	1
Consumption Wealth Effect	dy/dQ	[0.03-0.2]	0.09		0
corr. Survey Expect./ PD ratio	$\rho(PD_t, E_t(r^e_{t,t+4}))$	0.74	0.45		-1
Std. dev. Expected Returns(%)	$\sigma(E_t(r^e_{t,t+4}))$	2.56	1.8		

TABLE 1.3: Model implied moments. Data moments are computed over the period 1955Q1: 2018Q3. Moments have been computed as averages over 1000 simulations, each of 260 time periods. Subjective expectations are measured by the UBS Gallup survey for own portfolio returns for the period 1998Q2-2007Q3. t-ratios are defined as (data moment-model moment)/ $S.E$ of data moment. The RE model moments are computed using the parameters estimated using the learning model

The consumption wealth effect in the model economy is 0.09 meaning that for every 1% increase in stock market wealth consumption responds by 0.09%. The magnitude delivered by the estimation is well within the bounds usually found in the empirical literature. As mentioned in the introduction, Di Maggio et al. (2020) find that the consumption wealth effect from unrealized capital gains ranges between 3 and 20%. The model matches well business cycle moments, the volatility of financial variables and the persistence of the PD ratio. Although not targeted in the estimation, the model delivers a stock market which is 5.2 times more volatile than the real economy at the business cycle frequency and which has a 0.45 correlation coefficient with the output-gap. Figure A.9 from appendix D shows the business cycle component of real GDP and the US stock market represented by the S&P 500 index. The stock market is 6.7 times more volatile than the real economy.

The model implied beliefs about stock prices are positively correlated with the PD ratio and are several orders of magnitude less volatile than actual realised returns, replicating the survey evidence. Similar to other findings from the learning literature, the model is not able to match the equity premium since although prices are very volatile this volatility is not priced since it comes from subjective beliefs.

Using the estimated and calibrated parameters, Figure 1.1 presents one simulation of 260 periods from the learning model for the actual stock price and the ex-post rational price. The figure shows that while the rational price (P^*) does not fluctuate much, actual stock prices experience booms and busts of magnitudes of up to 100 % in absolute values. The figure can be directly compared to Figure 1 in Shiller (1981) which is the evidence of excess volatility compared to fundamentals. Figure 1.2 shows one random simulation for the PD ratio.

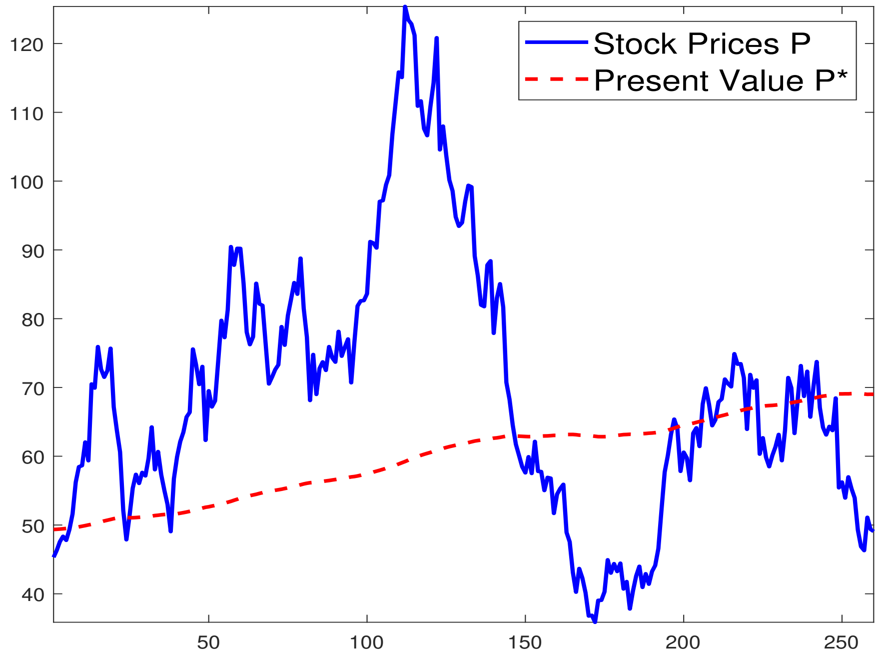


FIGURE 1.1: Stock Prices and Discounted Dividends. The figure presents one simulation of 260 periods for the time series of stock prices and the corresponding present value of discounted dividends or ex-post rational price in the language of Shiller (1981). Similar to that study, it has been assumed that the end value for the rational price, P^*, is the sample average of the real stock price. Given that, the rest of the time series for the rational price can be backed out by the following recursion $P_t^* = \delta P_{t+1}^* + D_t$ where D_t is the real dividend at time t.

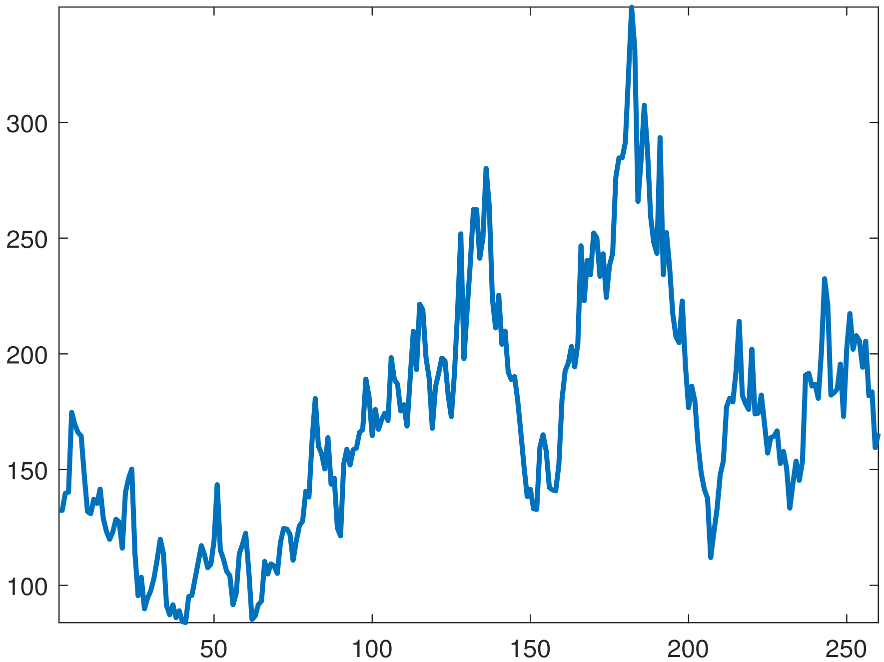


FIGURE 1.2: One simulation of the PD ratio

1.4.9 Do Sentiment Shocks matter?

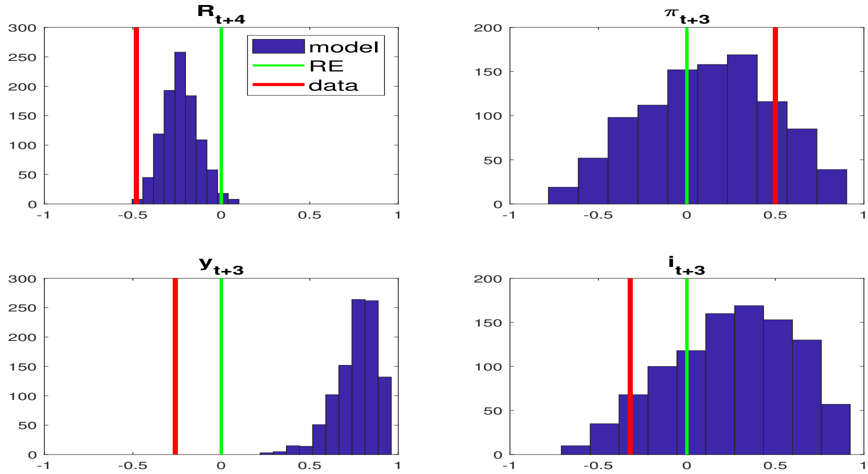
The model estimated in the previous sections matches remarkably well the dynamics (especially volatility) of the stock market and its joint behavior with survey expectations. Responsible for this success is the combination of learning and sentiment shocks. Imperfect information has the role of creating a direct effect of stock prices on output via the consumption wealth channel while sentiment shocks have the objective of creating realistic dynamics of stock price expectations which affect stock prices and via the before mentioned channel, the real economy. Table 1.4 re-estimates the model without sentiment shocks while keeping all the other ingredients as before. The model fails in matching financial and expectation moments, fact attested by the large t-ratios of the moments. This shows that in the present framework, sentiment shocks are crucial for replicating the dynamics of the financial market.

Business Cycle	Symbol	Data Moment	Learning Model Moment	Learning Model t-ratio	RE Model
Std. dev. of output	$\sigma(y)$	1.45	0.62	5.5	0.27
Std. dev. of inflation	$\sigma(\pi)$	0.54	0.29	3.4	0.29
Correlation output/inflation	$\rho_{y,\pi}$	0.29	8.6	-3.2	-1
Financial Moments					
Average PD ratio	E (P/ D)	154	134	1.33	133
Std. dev. of PD ratio	$\sigma(P/D)$	63	11	4.8	9
Auto-correlation of PD ratio	$\rho(P/D)$	0.99	0.84	3.2	0.05
Std. dev. of equity return (%)	$\sigma(r^e)$	6.02	0.79	12	9
Std. dev. real risk free rate (%)	$\sigma(r^f)$	0.72	0.78	0.7	0.0017
Non Targeted Moments					
volatility ratio stock prices/output	$\sigma(Q)/\sigma(y)$	6.7	1.05	7.2	23
corr. Stock Prices/ output	$\rho(Q,y)$	0.5	0.99	3.74	1
Average Equity Return (%)	$E(r^e)$	1.78	0.76	2.23	0.73
Average real risk free rate (%)	$E(r^f)$	0.32	0.75	3.4	0.72
Consumption Wealth Effect	dy/dQ	[0.03-0.2]	0.09		0
Std. dev. Expected Returns(%)	$\sigma(E_t(r^e_{t,t+4}))$	2.56	1.72		
corr. Survey Expect./ PD ratio	$\rho(PD_t, E_t(r^e_{t,t+4}))$	0.74	-0.99		-1

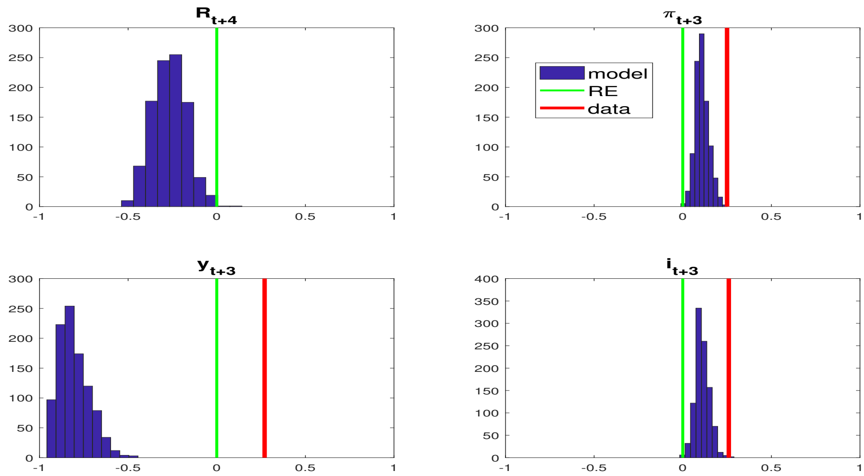
TABLE 1.4: Model implied moments excluding Sentiment Shocks; moments have been computed as averages over 10.000 simulations, each of 260 time periods. Subjective expectations are measured by the UBS Gallup survey for own portfolio returns. Survey data covers the period 1998Q1-2007Q3. Re-estimated parameter vector: $\hat{\theta} = (\sigma^u, \sigma^{\epsilon_i}, \rho_u, \rho_{\epsilon_i}, \lambda, \rho) = (0.0009, 0.0076, 0.9864, 0.0684, 0.9976, 0.0154)$

1.4.10 Survey Expectations vs Model Beliefs

Coibion and Gorodnichenko (2012) and Coibion and Gorodnichenko (2015) bring evidence in favor of information rigidity in expectation formation and show that aggregate forecasts of inflation and other macroeconomic variables exhibit under-reaction described by a positive relation between forecast revisions and forecast errors. Let $FR^x_{t,h} = x_{t+h/t} - x_{t+h/t-1}$ and $FE^x_{t,h} = x_{t+h} - x_{t+h/t}$ denote the forecast revision and forecast error for variable x at time t and horizon h.



(A) $\rho(PD_t, FE^x_{t,h})$



(B) $\rho(FE^x_{t,h}, FR^x_{t,h})$

FIGURE 1.3: Forecast Error Predictability. The figure shows the correlation coefficient for forecast errors with the PD ratio and the revision in beliefs for 1 year ahead expected capital gains and three quarters ahead inflation, output growth and interest rates. Expected capital gains are measured by the US Gallup survey (own portfolio) which covers the period 1998Q1-2007Q3. Similar to Coibion and Gorodnichenko (2015) the 3 quarters ahead also includes the nowcast. The survey data for the macroeconomic variables comes from SPF and covers the period 1981Q1-2016Q4. The series for the revision of beliefs is not available for the US Gallup survey. The model statistics are computed over 1000 simulations each of 260 time periods

Using the survey of professional forecasters (SPF) for inflation, output growth and interest rates and the US Gallup survey for expected capital gains, Figure 1.3 presents the correlation coefficients between the forecast error and PD ratio and between forecast errors and revisions in beliefs for both model and data. Under RE both of these coefficients would be 0.

Panel (a) shows the correlation coefficient between the PD ratio and the forecast errors for each of the four variables considered. When stock prices are high agents tend to systematically over-predict future capital gains, fact reproduced by the model (top-left figure). For the forecast errors of inflation and interest rates the model also produces reasonable ranges for the correlation coefficients. Nevertheless, for output growth the model generates a positive relation while the data suggests the opposite. This result is due to the fact that in the model the agents learn about the output-gap while in survey data agents predict directly output growth.

Following Coibion and Gorodnichenko (2015) panel (b) presents the correlation between the forecast errors and revision in beliefs. The model replicates the positive (under reaction) correlation for inflation and interest rates although the magnitude is smaller in the model. For output, the model delivers again a wrong sign of the coefficient for the reason outlined above. It is useful to compare these results to the ones in Winkler (2019). The model presented there is able to reproduce the patterns of forecast errors for output and other variables but fails in the case of inflation. The model outlined here delivers the opposite result: it matches well inflation and other variables and fails regarding the subjective output forecast error dynamics. The mechanism through which stock prices affect the real economy is nevertheless different (supply vs demand) and that could explain, at least partially, the difference in results.

1.5 Stock Price Targeting and Macroeconomic Stability

Stock price booms and busts driven by market sentiment affect the real economy through a consumption wealth effect. Since these wealth effects are reflected in output and inflation dynamics, monetary policy could in principle, by responding to just two macroeconomic variables influence or eliminate the non-fundamental effect of stock prices on the real economy. Compared to the RE assumption, in the current economic environment, agents have imperfect information about the structural relations between the real economy and stock prices. To this end, the Taylor rule is augmented with a lagged response to stock prices: $i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_q \tilde{q}_{t-1}$.[20] To investigate the effect of monetary policy on the wealth effect, figure 1.4 plots the magnitude of the wealth effect when the central bank targets only one variable at the time: inflation, output-gap or stock prices.

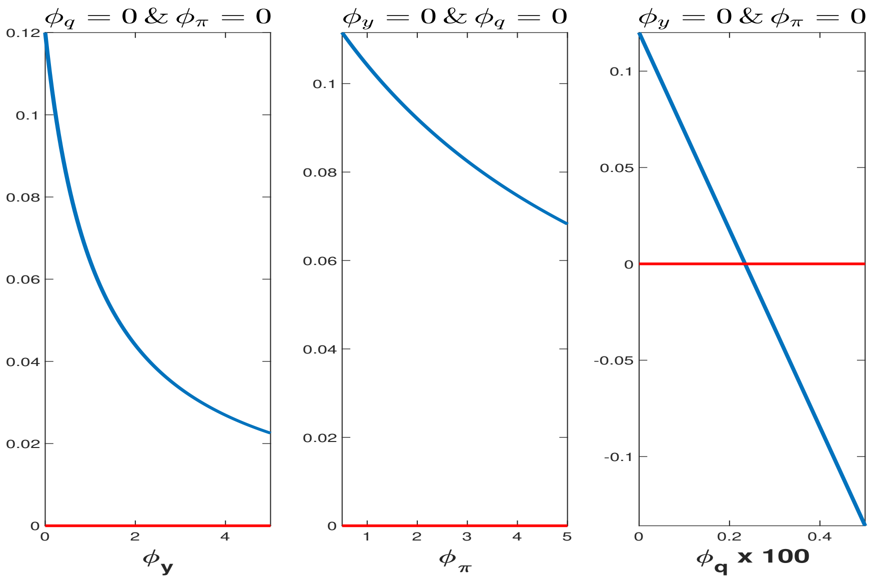


FIGURE 1.4: Stock Price Wealth Effects and Monetary Policy; each panel presents the magnitude of the wealth effects as a function of the central bank response to output, inflation and stock prices while keeping the other coefficients fixed at 0. The Taylor rule is of the following type: $i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_q \tilde{q}_{t-1}$

To the extent that the central bank would want to eliminate the effects of the fluctuations of stock prices on output then the only possibility under this simple Taylor rule would be to include an explicit response to stock prices into the monetary policy reaction function. Figure 1.4 shows that responding stronger to inflation or output has a smaller effect on the stock price wealth effect than by responding directly to asset prices. In fact, no matter how strong the central bank responds to inflation or output it would not manage to totally neutralize the effects of stock prices on output. The reason for this dynamics lies in the

[20] Appendix A.4 presents the results with a contemporaneous response to stock prices and none of the qualitative conclusions change.

fact that agents do not internalize the relation between stock prices and output and as a consequence, the extra-volatility of stock prices with respect to the real economy would not be internalized if the central bank responds just to output and inflation.

Responding to stock prices might on the other hand introduce additional volatility in the economy which might destabilize the system. Figure 1.5 shows that this is not necessarily the case for small enough reactions to stock prices and even less when there is in place a reaction to output.[21]

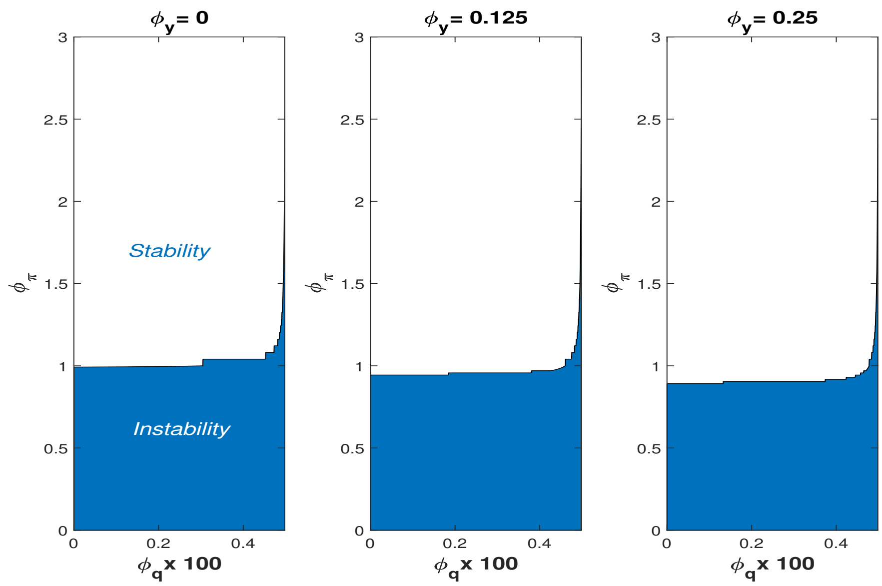


FIGURE 1.5: E-Stability and Monetary Policy. The figure presents the stability (white) and instability (blue) regions for different combinations of Taylor rule coefficients. Each panel plots the e-stability regions for different combinations of inflation (Y axis) and stock price (X axis) reaction coefficients while keeping the output reaction fixed. The Taylor rule is of the following type: $i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_q \tilde{q}_{t-1}$. The stability of the system is given by the eigenvalues of the matrix $A^{-1}B$. Following Evans and Honkapohja (2012), the dynamical system is e-stable if the largest eigenvalue of the previous matrix has the real part smaller than 1.

[21] The maximum considered stock price reaction coefficient is 0.05 implying an increase of 1% in the interest rate as a result of a 20% increase in stock prices. The lack of instability due to responding to stock prices is in line with the findings of Nisticò (2012) and Airaudo et al. (2015) and Airaudo (2016).

To better understand the dynamics of the stock market and its effects on the economy, figure 1.6 reports the IRFs from a 1% shock in stock prices in the RE model and a 1% shock in the beliefs of agents about the stock prices in the imperfect information model. These two shocks would have a similar effect in a RE model but in the learning model, where agents hold subjective beliefs about the stock market and where these beliefs have a high degree of persistence, the effects of these shocks have very different implications for the dynamics of the stock market and the real economy.

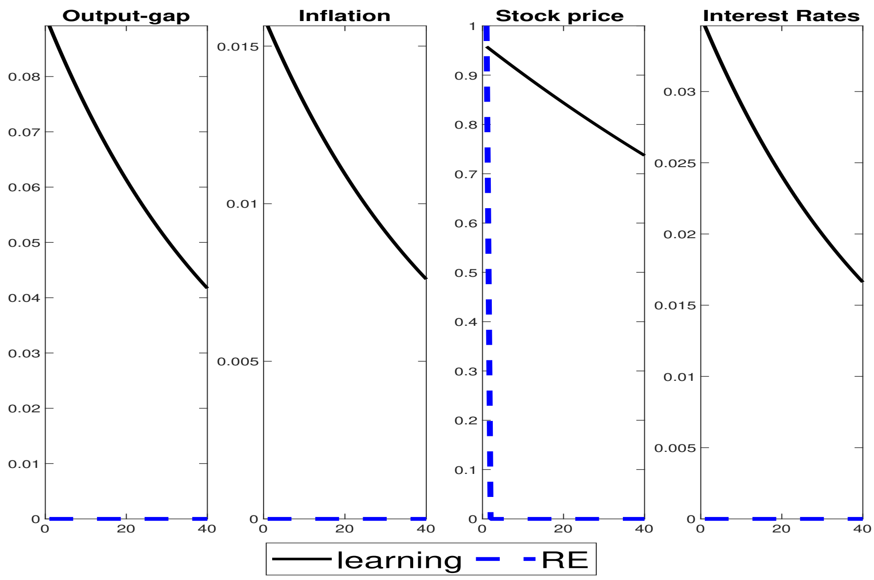


FIGURE 1.6: Stock Price vs Belief Shocks. The figure presents the IRF of selected endogenous variables with respect to stock price shocks in RE and shocks to beliefs under the learning framework. Both shocks have an impact magnitude of 1% and persistence 0.

The equity shock increases stock prices contemporaneously and then returns to 0 without affecting any other variable. Belief or sentiment shocks, on the other hand, although being *i.i.d*, have a persistent effect on stock prices. This is because of the persistence of beliefs which translates into further increases in prices, therefore justifying the initial beliefs. This rise in stock prices is then transmitted through wealth effects on the rest of the economy. Output, inflation and interest rates closely follow the dynamics of the stock market. Although the central bank does not target stock-prices directly, the interest rates rise as a response of increases in inflation and output-gap. This shows that waves of optimism/ pessimism can affect the real economy without any change in fundamentals.

Since monetary policy is effective in influencing the magnitude of the stock price wealth effect (see figure 1.4) it is natural to ask how would the dynamics of the economy change if monetary policy would include a dedicated response to asset prices. Figure 1.7 answers this question by plotting the IRFs to sentiment shocks for different stock price reaction coefficients.

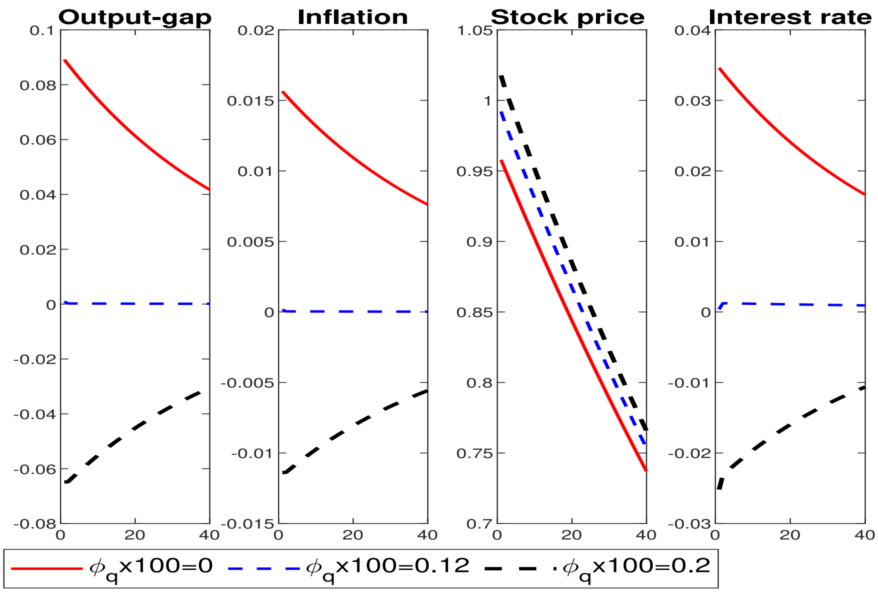


FIGURE 1.7: IRFs to Sentiment Shocks. The figure presents the IRF to a 1 % *i.i.d* shock in equity price beliefs for different reaction coefficients to stock prices. The Taylor rule is of the following form: $i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_q \tilde{q}_{t-1}$

The red line denotes the response in the case in which the central bank does not target explicitly stock prices. As the response of the monetary authority to stock prices increases, the effect of sentiment shocks on the economy decreases and is even reversed for large enough coefficients. The blue dotted line shows that the central bank can approximately neutralize the effects of sentiment shocks on the economy by picking a reaction coefficient to stock prices around 0.0012. This response implies that the central bank commits to raising interest rates by 12 b.p. for every 100% increase in stock prices from their steady-state value. Reacting too strongly to stock prices (black line) has the effect of reversing the effects of sentiment shocks and causing a recession. While it is true that reacting strongly to stock prices is undesirable since it can cause an economic recession which in turn has to be accommodated by reversing the increase in interest rates, it is not the case that the response to stock prices should be absent. The key is to fine-tune the response of monetary policy to stock prices such that the real economy is isolated from the effects of belief-driven asset price cycles.

A central bank reacting to stock prices and *communicating clearly this policy* can disrupt the effects of sentiment shocks on the economy by influencing agents' expectations on stock prices and interest rates: agents take into account a possible rate increase which, given the persistence of beliefs, is internalized as a persistent interest rate increase; this results in an adjustment of the intertemporal consumption decision counteracting the positive effect on the real economy of the initial optimism wave. The inclusion of stock price targeting in the Taylor rule does not, necessarily, create additional interest volatility in the economy since the sole fact that the central bank *threatens* to not tolerate large stock price swings is enough to influence agents' sentiment and consequently the booms and busts would not materialize in the first place.

1.5.1 Monetary Policy and Welfare

What is the appropriate response of monetary policy in the face of real effects of swings in the stock market and does an explicit response to stock prices improve macroeconomic stability or welfare? The current section tries to answer these questions using the model developed in this paper.

To analyze what are the implications of stock-price targeting on macroeconomic stability and welfare one would need to specify a welfare criterion under which different monetary policy rules can be examined. The literature on monetary theory uses a second-order approximation of the lifetime utility of the representative agent as a criterion of welfare.[22] The resulting criterion, average welfare loss per period, is an increasing function of the volatility of output and inflation. In the current framework stock prices play an important

[22] See Rotemberg and Woodford (1999) and Galí (2015)

role as a source of output and inflation variation and therefore the standard welfare function does not apply in this economy.

To this end, it is assumed that the central bank maximizes welfare under the equilibrium probability measure and not under the subjective one held by the agents. Therefore the central bank (social planner) assumes a paternalistic objective for the agents. The learning literature has also adopted this approach although, compared to RE, here we are dealing with two types of beliefs: subjective vs model or objective beliefs.[23] Adopting a criterion of this form implies that we are assuming that the model or objective beliefs is what matters for the overall welfare of the agents. This mustn't necessarily be the case but for now I will also adopt this assumption which is standard in the literature.[24] The following lemma presents the welfare function describing the economy presented in section IV.

Lemma 3. *Up to a second-order approximation and ignoring terms independent of policy the expected utility in the TANK model with homogeneous imperfect information is proportional to* $\sum_{t=0}^{\infty} -\mathscr{L}$ *where*

$$\mathscr{L} = \frac{\epsilon}{\psi} var(\pi_t) + \Upsilon_1 \, var(\tilde{y}_t) + \Upsilon_2 \, var(\tilde{q}_t) + \Upsilon_3 \, E(\tilde{y}_t \tilde{q}_t) \tag{1.54}$$

is the average expected welfare loss per period measured as a fraction of steady-state consumption.

Proof. See Appendix A.3 □

Lemma 3 shows that in the current framework the welfare of the agents depends also on the variability of stock prices and the correlation of stock prices with output.

Stock price targeting rules take the form of Taylor rules augmented with a dedicated response to stock prices. The first rule that is considered is a standard linear and symmetric response to lagged stock prices (rule I in table

[23] See Eusepi and Preston (2018b)

[24] See Kahneman et al. (1997). Models with subjective beliefs potentially open the door to the interesting exploration of the importance of subjective beliefs to welfare and on the role of *remembered* utility. This line of research has been almost exclusively left to psychology or to the applied and experimental economists. One notable exception is Caines and Winkler (2021) which evaluates the welfare implications of monetary policy rules both using objective and subjective expectations and find that the implications for optimal monetary policy differ depending on which measure of expectations one chooses.

1.5). The empirical evidence shows that the Fed intervenes mostly when stock prices decrease while booms are left to their own. Since stock price wealth effects appear both in booms and busts, the following two non-linear monetary policy rules will be considered. Consistent with the empirical evidence, the first rule implies that the central bank reacts only when lagged stock prices drop under a certain threshold, Q^-, which is the standard Fed put documented in the literature (rule II in table 1.5). In the second rule, in addition to reacting to stock price decreases, the central bank also reacts when the stock market increases above a specified threshold, Q^+, which is labeled the fed put-call rule (rule III in table 1.5). These rules are summarized in the following table.

I. Linear	$i_t = 1.5\ \pi_t + \phi_y\ \tilde{y}_t + \phi_q\ \tilde{q}_{t-1}$
II. Fed put	$i_t = 1.5\ \pi_t + \phi_y\ \tilde{y}_t + \phi_q\ \tilde{q}_{t-1}\mathbb{1}_{\tilde{q}_{t-1}<Q^-}$
III. Fed put-call	$i_t = 1.5\ \pi_t + \phi_y\ \tilde{y}_t + \phi_q\ \tilde{q}_{t-1}(\mathbb{1}_{\tilde{q}_{t-1}<Q^-} + \mathbb{1}_{\tilde{q}_{t-1}>Q^+})$

Table 1.5: Monetary Policy rules under stock price targeting; $\mathbb{1}_{\tilde{q}_t<Q^-}$ is an indicator functions taking a value of one if the condition $\tilde{q}_t < Q^-$ is satisfied and 0 otherwise

The impulse response analysis from the previous section regarding the implications of responding explicitly to stock prices has been performed under the assumption that agents fully understand that monetary policy is responding to stock prices. Therefore, when forming expectations of interest rates agents would have internalized that given the current level of the stock market and their beliefs, the central bank would adjust accordingly the level of interest rates accordingly to the Taylor rule followed. In reality, the Fed does not react explicitly to stock prices although there is increasing evidence that it does intervene when needed. As a result it is reasonable to assume that agents might not internalize this reaction of the monetary authority to stock prices. To investigate the implications of this lack of information I assume (as before) that agents fully understand that the central bank responds to inflation and output but do not take into account the reaction to stock prices. In reality, the central bank is responding to stock price deviations even though agents do not internalize this fact. For clarity, the following table summarizes the information set of the central bank and agents under transparency vs non-transparency.

	Transparency	Non-Transparency
Central Bank	$i_t = 1.5\ \pi_t + \phi_y\ \tilde{y}_t + F(\tilde{q}_{t-1})$	$i_t = 1.5\ \pi_t + \phi_y\ \tilde{y}_t + F(\tilde{q}_{t-1})$
Agents	$i_t = 1.5\ \pi_t + \phi_y\ \tilde{y}_t + F(\tilde{q}_{t-1})$	$i_t = 1.5\ \pi_t + \phi_y\ \tilde{y}_t$

TABLE 1.6: Information set under Transparency vs Non-Transparenc. The first line of the table shows the actions of the central bank while the second line shows how agents understand the central bank is responding to macroeconomic variables which is used to forecast future interest rates. $F(\tilde{q}_{t-1})$ denotes the central banks' response to stock prices which can take one of the options presented in table 1.5.

1.5.2 Non-Transparency

Figure 1.8 plots the welfare costs implied by monetary rules considered in table 1.5 for different thresholds regarding the response to stock prices.[25] The figure makes clear that even under non-transparency reacting during periods with positive and negative returns (Fed put-call) is more efficient than reacting just when stock prices decrease (Fed put), no matter the threshold level chosen.

[25]Notice that the the Fed put-call policy with a threshold of 0 (red line in the fist panel in figure 1.8 is equivalent to the linear Taylor rule

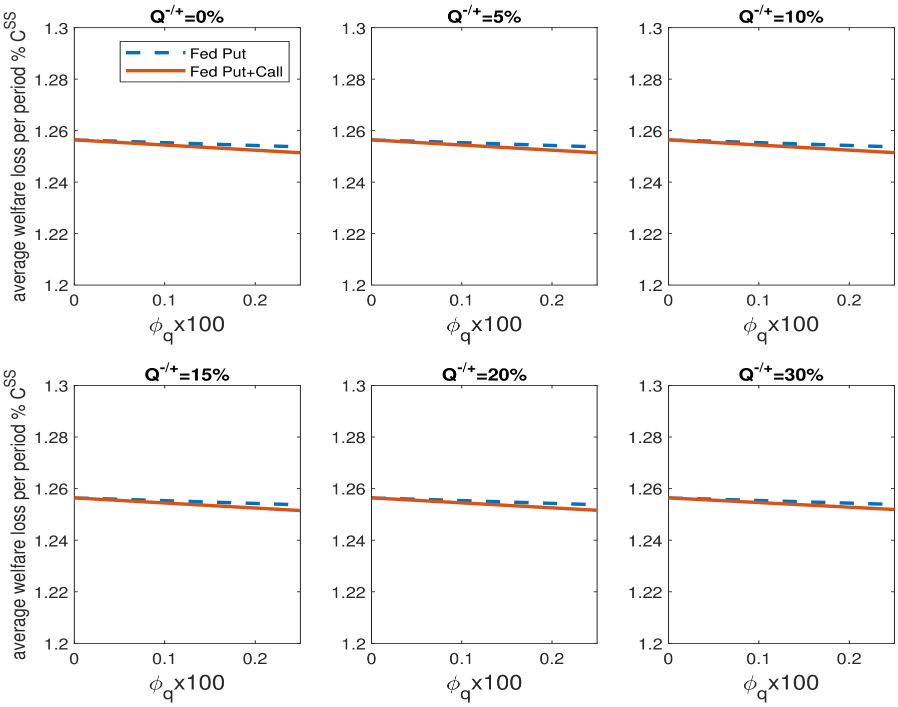


FIGURE 1.8: Welfare Implications of Fed put and call under Non-Transparency. It has been assumed that $Q^- = -Q^+$ for the Fed put-call rule. Non-transparency implies that although the central bank is reacting to stock prices using either of the two nonlinear rules considered, agents do not internalize this fact and form beliefs regarding interest rates using the systematic component of the Taylor rule concerning only output and inflation: $i_t = 1.5\ \pi_t + 0.125\ \tilde{y}_t$. The Fed put is specified as $i_t = 1.5\ \pi_t + 0.125\ \tilde{y}_t + \phi_q\ \tilde{q}_{t-1}\mathbb{1}_{\tilde{q}_{t-1}<Q^-}$ while the Fed put-call is $i_t = 1.5\ \pi_t + 0.125\ \tilde{y}_t + \phi_q\ \tilde{q}_{t-1}(\mathbb{1}_{\tilde{q}_{t-1}<Q^-} + \mathbb{1}_{\tilde{q}_t>Q^+})$.

Figure 1.8 also reveals that a threshold between 5 and 10% attains the highest efficiency gain for both policies considered.[26]The welfare costs implied from responding symmetrically to stock prices are minimal for small enough stock price reaction coefficients (of the order 0.005 or smaller). The main conclusion arising from this exercise is that although there are welfare gains from responding symmetrically to stock prices under non-transparency, they are likely to be quantitatively small.

[26]See also Figure A.8 from appendix D

1.5.3 Transparency

Moving to the case in which the central bank communicates transparently its policies in such a way that agents internalize this information into their belief system, figure 1.9 presents the implied welfare loss for different policy parameters in case of linear response to stock prices (rule I from table 1.5). Each line in the figure corresponds to a different output coefficient in the Taylor rule for different values of the stock price reaction coefficients. Notice first that no matter the response to output, including a reaction to stock prices is always optimal but the benefit decreases the higher the reaction to output. Nevertheless, reacting too strongly to output variations decreases welfare since it worsens the output-inflation trade-off in the case of cost-push shocks.[27]

The shape of the welfare loss as a function of the stock price targeting parameter has a U shape: reacting too strongly to stock prices can in fact decrease welfare by introducing additional volatility in the economy. For the baseline parametrization of the Taylor rule ($\phi_y = 0.125$, red line in figure 1.9) including a dedicated coefficient to stock prices of 0.12% in the Taylor rule increases welfare by 0.14% on average per period. This reaction implies increasing interest rates by 12 basis points for every 100% deviation of stock prices from their long-run trend. Figure 1.9 from appendix D, repeats this exercise for the case in which the economy is solely hit by sentiment shocks and shows that the 0.14% welfare gain comes from counteracting the inefficiencies arising from the waves of optimism/pessimism about capital gains.

[27] A cost-push shock has the effect of increasing inflation contemporaneously. If the central bank responds strongly enough to inflation deviations, interest rate rise and output gap decreases which counteracts the initial increase in inflation. If at the same time the monetary authority reacts to output gap deviations then the interest rate will not increase as much and the initial impact of the cost push shock will dominate. The overall impact would be a less negative output gap and higher inflation. The optimal policy in standard New-Keynesian models (see for example Galí (2015) chapter 5) is to accommodate the cost-push shock by allowing a negative output-gap.

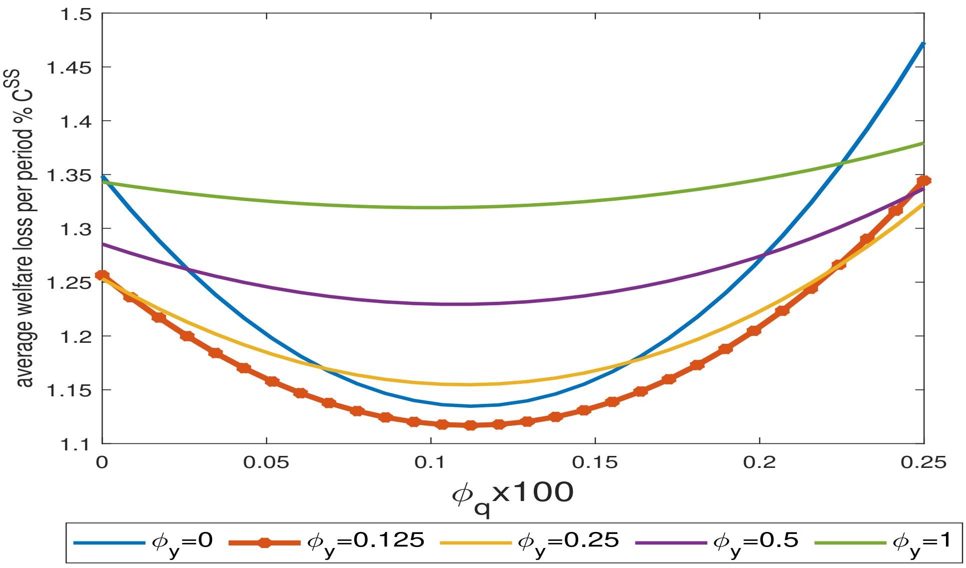


FIGURE 1.9: Welfare Maps: Linear response to asset prices. The figure shows the average welfare loss per period as defined in equation (1.54) for different combinations of Taylor rule coefficients for output and stock prices while keeping the inflation reaction coefficient fixed at 1.5. Welfare losses have been computed as averages over 200 independent simulations, each one including 260 time periods using the estimated parameters from section IV.H

Figure 1.10 decomposes the sources of the welfare gains by plotting the standard deviations of output, inflation and stock prices together with the co-movement between stock prices and output. Including a dedicated reaction to stock prices reduces both the volatility of inflation and output up to a certain point displaying the same U shape dynamics as the welfare losses. Stock price volatility increases monotonically but the magnitude is relatively small. The penultimate panel shows that responding to stock prices breaks the link between stock prices and output by reducing their co-movement generated by the stock price wealth effect.

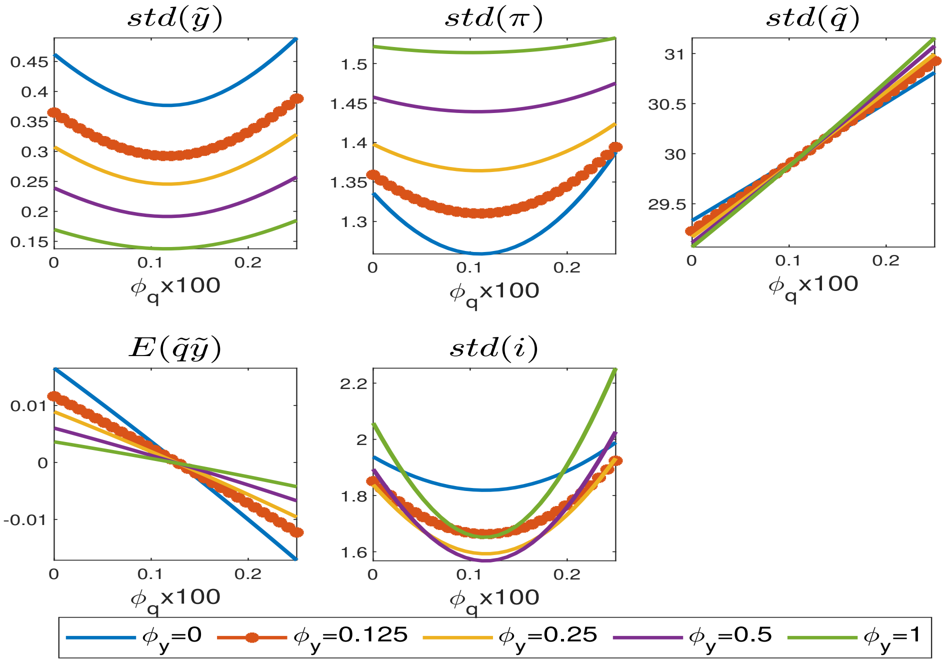


FIGURE 1.10: Influence of Monetary policy on Macroeconomic Volatility under Rule 1; implied volatility of output, inflation, stock prices, co-movement of output with stock prices and interest rates for different combinations of policy parameters. The Taylor rule is specified as $i_t = 1.5\ \pi_t + \phi_y\ \tilde{y}_t + \phi_q \tilde{q}_{t-1}$.

In the case of the non-linear response to stock prices under transparency, the subjective expectations of agents for the response of monetary policy to stock prices can be computed as follows. Notice that under agents' belief system, stock prices are distributed according to $\tilde{q}_{t+k} \overset{\mathscr{P}}{\sim} \mathscr{N}(\mu^q_{t+k}, \left(\sigma^q_{t+k}\right)^2)$ with

$$\begin{aligned} \mu^q_{t+k} &= \rho^k \beta^q_{t-1} \\ \left(\sigma^q_{t+k}\right)^2 &= \left(\sigma^q_0\right)^2 \rho^{2k} + \left(\sigma^q_\zeta\right)^2 + \left(\sigma^q_\vartheta\right)^2 \frac{1-\rho^{2k}}{1-\rho^2} \end{aligned} \tag{1.55}$$

where σ^q_0 is the prior subjective belief about the volatility of stock prices, σ^q_ϑ and σ^q_ζ the volatilities of the permanent and transitory components of stock prices under agents' belief system. The expectation of the response of monetary policy using the Fed put policy under the subjective probability measure can be computed as

$$
\begin{aligned}
E_t^{\mathscr{P}}(Q_{t+k}^{put}) =& E_t^{\mathscr{P}}(\tilde{q}_{t+k}\ \mathbb{1}_{\tilde{q}_{t+k}<Q^-}) \\
=& \int_{-\infty}^{Q^-} \tilde{q}_{t+k} f(\tilde{q}_{t+k}) dq = \int_{-\infty}^{Q^-} \mu_{t+k}^q + (\tilde{q}_{t+k} - \mu_{t+k}^q) dq \\
=& \mu_{t+k}^q \Phi\left(\frac{Q^- - \mu_{t+k}^q}{\sigma_{t+k}^q}\right) - \sigma_{t+k}^q \phi\left(\frac{Q^- - \mu_{t+k}^q}{\sigma_{t+k}^q}\right)
\end{aligned} \tag{1.56}
$$

where $\phi()$ and $\Phi()$ are the pdf and cdf of the standard normal distribution. Similarly for the Fed call policy

$$
E_t^{\mathscr{P}}(Q_{t+k}^{call}) = \mu_{t+k}^q \left(1 - \Phi\left(\frac{Q^+ - \mu_{t+k}^q}{\sigma_{t+k}^q}\right)\right) + \sigma_{t+k}^q \phi\left(\frac{Q^+ - \mu_{t+k}^q}{\sigma_{t+k}^q}\right). \tag{1.57}
$$

Figure 1.11 shows the welfare losses arising in the case of transparency for the three monetary policy rules included in table 1.5. Notice that the welfare costs implied by the linear Taylor rule (blue line) are almost identical to the ones in which the central bank starts responding only after stock prices increases/decrease by 7% (yellow line). In reality, a linear policy as described by rule I would imply a continuous adjustment of interest rates in line with asset price movements which could introduce undesirable volatility in the economy through other channels not taken into account in the present framework. The previous result shows that the central bank can reap the welfare benefits by responding symmetrically only to large asset price movements as long as this policy is communicated transparently to the agents. A central bank following the Fed Put policy rule (red dotted line) increases welfare up to a certain point and requires a stronger response to asset prices compared to a symmetrical policy ($\phi_q = 0.0014$ compared to 0.0012). Nevertheless, such a policy cannot eliminate all of the inefficiencies arising from belief driven asset price cycles since it does not take into account the effect that stock price booms have on aggregate demand.

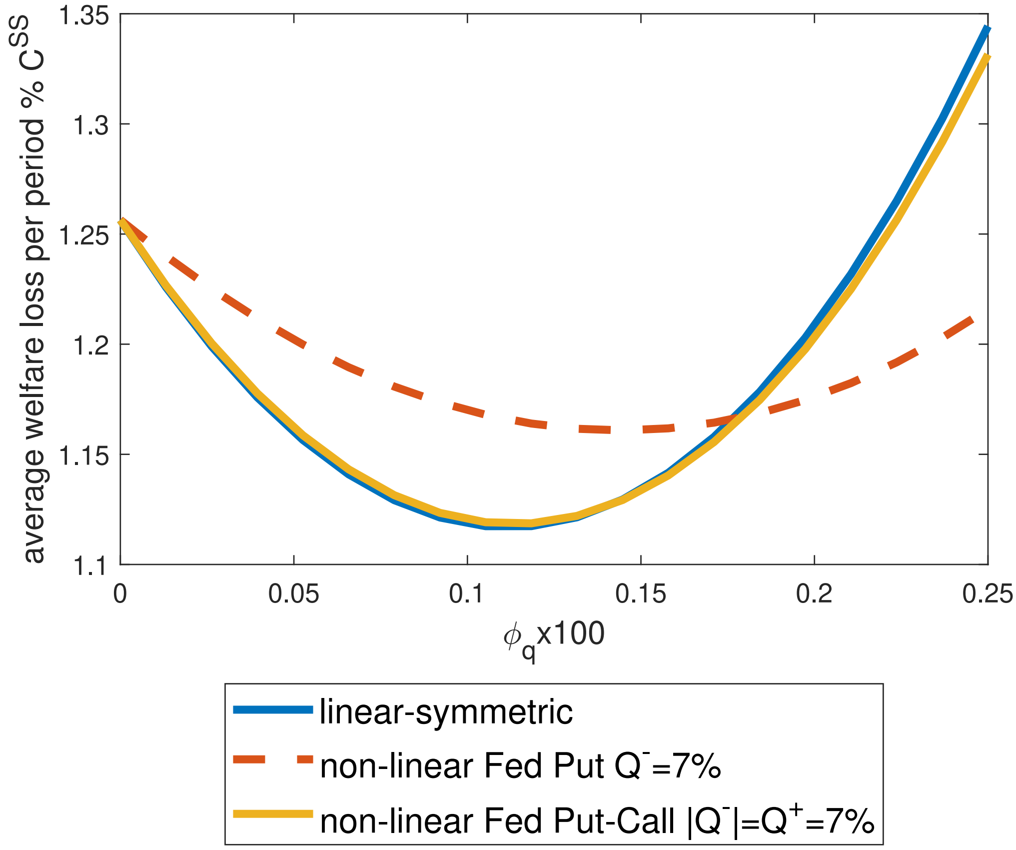


FIGURE 1.11: Welfare costs under Transparency and non-linear response to stock prices. The figure presents the CEV (in %) for the monetary policy rules from table 1.5 under the case of Transparency for different reaction coefficients for stock prices (ϕ_q).

Table 1.7 summarises the welfare gains arising from responding to stock prices under the two information scenarios. It is clear from the table below that announcing transparently the reaction to stock prices is several orders of magnitude more efficient than reacting under non-transparency (0.14% vs 0.002). The benefits of responding to stock prices when agents do not internalize this reaction (column A) are at most limited. This confirms that the management of agents' expectations about capital gains, interest rates and the link between these two is crucial for successfully counteracting the inefficiencies arising from booms and busts in asset prices.

CEV(%) gain / **Monetary Rules**	A. Non-Transparency	B. Transparency
I. Linear $i_t = 1.5\ \pi_t + \frac{0.5}{4}\ \tilde{y}_t + \phi_q\ \tilde{q}_{t-1}$	0.002	0.14
II. Fed put $i_t = 1.5\ \pi_t + \frac{0.5}{4}\ \tilde{y}_t + \phi_q\ \tilde{q}_{t-1} \mathbb{1}_{\tilde{q}_{t-1}<Q^-}$	0.001	0.09
III. Fed put-call $i_t = 1.5\pi_t + \frac{0.5}{4}\tilde{y}_t + \phi_q\tilde{q}_{t-1}(\mathbb{1}_{\tilde{q}_{t-1}<Q^-} + \mathbb{1}_{\tilde{q}_{t-1}>Q^+})$	0.002	0.14

TABLE 1.7: Welfare gains from stock price targeting. The values in the table represent CEV differences from the case in which the central bank does not include any response to stock prices ($\phi_q = 0$). The welfare gains have been computed under the optimized Taylor rules under transparency: under rule I and III $\phi_q = 0.0011$ while for rule II $\phi_q = 0.00145$; $Q^+ = |Q^-| = 7\%$.

1.5.4 (Un)conventional Monetary Policy

The previous analysis considered standard monetary policy which follows a simple Taylor rule when setting interest rates. During the last decade, central banks have adopted several non-conventional policies partly due to the constraint of the zero lower bound on interest rates. Since in this current framework expectations play a key role in determining equilibrium values of macroeconomics variables one natural question that might be asked is whether monetary policy can eliminate the non-efficiency arising from booms and busts in asset prices through other types of policies. The one that is considered here is Odyssean forward guidance (O-FG)[28] in which the central bank announces a change in policy in the future. Consider the following scenario under two alternative policies: in one the central bank is conducting monetary policy via a Taylor rule with optimal response on stock prices while in the second one the central bank does not respond to stock prices initially but announces a change of policy starting next period in which it

[28]see Campbell et al. (2012) for a discussion on the different types of forward guidance

includes stock prices in its monetary policy strategy. The following figure plots the impulse-response functions from an expectation shock of 1% under the two scenarios.

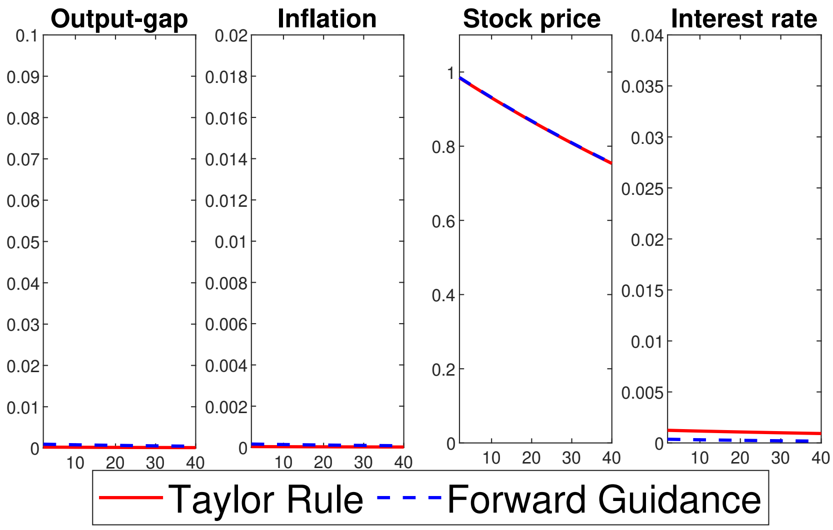


FIGURE 1.12: IRFs from a 1% *i.id* sentiment shock under Taylor rule and Odyssean forward guidance

The effect on the economy under the two alternative policies is almost indistinguishable except for the evolution of interest rates which increase more under the Taylor rule regime for the reason that the central bank reacts initially to the increase in stock prices. Both policies manage efficiently to eliminate the effect on the real economy via consumption wealth effects arising from sentiment shocks. The main takeaway from this exercise is that monetary policy operates through expectations about future reaction on asset prices and not by reacting to asset prices today.

1.6 Conclusions

The interaction between monetary policy and stock prices has been a long-standing subject both among academic economists and market professionals. Recent evidence suggests that the Fed is responding to stock prices and that the main channel (considered by policy makers) through which stock prices affect the real economy is given by consumption wealth effects. The empirical literature has also found that these effects can have a sizable magnitude ranging from

3 to 20%. Given this evidence, this paper first shows in a simple endowment economy, how stock prices influence the consumption decision of the agents in the case of imperfect information. Departures of stock prices from the expected discounted sum of dividends give rise to a consumption wealth effect through which stock prices influence aggregate demand. The result links directly the volatility puzzle with stock price wealth effects.

The mechanism is embedded in a quantitative model with homogeneous imperfect information where agents differ only regarding their participation in the equity market. The model is estimated on US data using two standard shocks, cost-push and monetary policy and a sentiment shock which affects agents' beliefs about future capital gains. Quantitatively, the model does remarkably well in matching the financial market and the dynamics of survey expectations while producing a smooth business cycle.

The estimated model is used to study the implications of responding to stock price on macroeconomic stability and welfare. By targeting stock prices the monetary authority does not introduce additional volatility in the economy and furthermore is especially efficient in counteracting the effects that sentiment swings have on the real economy via the consumption wealth effect. The results show that if the central bank announces explicitly and transparently a 12 bp increase in interest rates for every 100% increase in stock prices from the long run average, welfare improves by 0.14% on average per period. If on the contrary, the central bank reacts to stock prices in a non-transparent manner, the gains are limited. Odyssean forward guidance is equally efficient in eliminating the effects of sentiment driven asset price cycles reinforcing the key transmission channel through which monetary policy operates in this framework, namely by creating a link between future capital gains and interest rates.

Central banks can increase macroeconomic stability and welfare by responding explicitly to stock prices and can counteract the effects of asset price movements on the real economy by shutting down the wealth effect channel of stock prices. The present analysis is mostly limited to standard monetary policy strategies (e.g. Taylor rules) and further research is needed in order to understand the effects and interactions of non-conventional policies like forward guidance and quantitative easing which have become the norm in the last years. Furthermore, booms and busts in other asset classes (e.g. housing) produce similar effects on the real economy. The inclusion of these in the current framework and the interplay between cycles in multiple asset classes is left for further research.

Chapter 2

Sentimental Discount Rate Shocks

Abstract

This paper argues that the price-dividend ratio variability is explained in a large proportion by shocks affecting the subjective distribution of capital gain expectations: sentimental discount rate shocks affecting average beliefs explain 30% and disagreement shocks up to 20%. Using an estimated FAVAR model identified with sign and short-run restrictions, this paper shows that in contrast to discount rate shocks, sentiment shocks produce a hump-shape response in the PD ratio and introduce additional persistence into the impulse-response functions. These shocks played an important role during the 2002 dot-com bubble by driving the boom and subsequent bust in asset prices.

2.1 Introduction

A central question in asset pricing concerns the determinants of the stock market variability. The Campbell-Shiller price-dividend (PD) ratio decomposition shows that the PD ratio varies because of two main components: dividends and discount rates. The current paradigm argues that the vast majority of the variability of stock prices can be attributed to discount rates while dividends play an insignificant role.[1] In this paper I bring evidence that what is usually attributed to pure discount rate shocks hides instead shocks to the subjective distribution of future capital gains. Moreover, these shocks explain around 50% of the variability of the PD ratio.

This paper contributes to the literature on the determinants of stock price variability by incorporating the distribution of survey expectations on expected capital gains in a structural vector auto-regressive (VAR) framework. It first shows that the distribution of subjective capital gain expectations can be accurately captured by two factors: average sentiment and disagreement which explain over 95% of the variance of the distribution. Using these two factors in a standard asset pricing VAR that includes the PD ratio and dividend growth, I identify four sources of stock price variability: dividend growth, discount rate, disagreement and sentiment shocks. The latter is to be understood as a shock to the agents' beliefs about future capital gains. It operates as a typical discount rate shock but with two important differences: *i)* the pattern of subsequent decrease in discount rates implied by the Campbell-Shiller decomposition is more persistent compared to a standard discount rate shock *ii)* produces a hump-shaped impulse-response of the PD ratio. The identification of the sentimental discount rate shocks (SDR) is based on the findings from the literature of asset pricing with subjective beliefs and imposes that a shock to return expectations can have a contemporaneous effect on prices while a discount rate shock cannot affect on impact beliefs. I show that SDR shocks contribute between 30 and 50% to the variability of the PD ratio for the period 1999M1-2007M11 depending on the identification method used. Disagreement shocks play an important role in driving average sentiment suggesting that there exists a strong interdependence between disagreement and average subjective beliefs.

The literature on the determinants of asset price movements is vast.

[1] See Cochrane (2011)

Cochrane (2008) shows that PD ratio variability can be explained almost 100% by subsequent discount rate movements. Dividends vary much less compared to stock prices as famously pointed out by Shiller (1981) and have little explanatory power for the PD ratio. More recently, De La O and Myers (2021) using analyst forecasts on dividends and earnings show that short-term subjective expected dividend growth and earnings account for between 70-90% of the PD ratio variability with subjective expectations of returns playing an insignificant part. Using the same data and a similar Campbell-Shiller decomposition, Bordalo et al. (2020) bring evidence that expectations about long-term growth in earnings are an important source of price level variability. In reaching this result they impose a constant discount rate but as the same authors point out subjective expectations of returns and earning are positively correlated which could explain part of their results. In contrast, Adam et al. (2017) employ a quantitative asset pricing model of subjective beliefs and argue that variation in subjective capital gain expectations can explain the excess volatility puzzle and the predictability of returns while matching a wide variety of stylized facts about stock prices. The results from the current paper bring additional empirical evidence in this direction by showing that variation in subjective capital gain expectations is an important source of the PD ratio variability.

The rest of the paper is organized as follows. Section II describes the survey data concerning the distribution of expected capital gains and its determinants. Section III presents the main results regarding the contribution of (sentimental) discount rates to the PD ratio variability. Section IV explores several robustness checks including an alternative identification of disagreement shocks and lastly section V concludes.

2.2 Factors of Subjective Beliefs Distribution

The data describing the distribution of subjective capital gain expectations is obtained from the Gallup survey which reports investors subjective beliefs about stock returns over the next 12 months. The series are monthly and spans the period 1999M2-2007M10. I use the Gallup survey to measure subjective own portfolio return expectations due to the large number of respondents each period (around 700) which in principle should come with more reliability in measuring the mean and dispersion of beliefs. The other time series used are the monthly PD ratios for the $S\&P$ 500 index and the dividend growth series.[2]

The statistics characterizing the distribution of beliefs are obtained in two steps. First, since the number of survey respondents is varying over time, the distribution is reduced to percentiles by computing the mean at each point in time for the corresponding group. I choose 10 percentiles to obtain a balanced panel of the subjective return distribution. Figure 2.1 shows the evolution of different sentiment groups (percentiles).

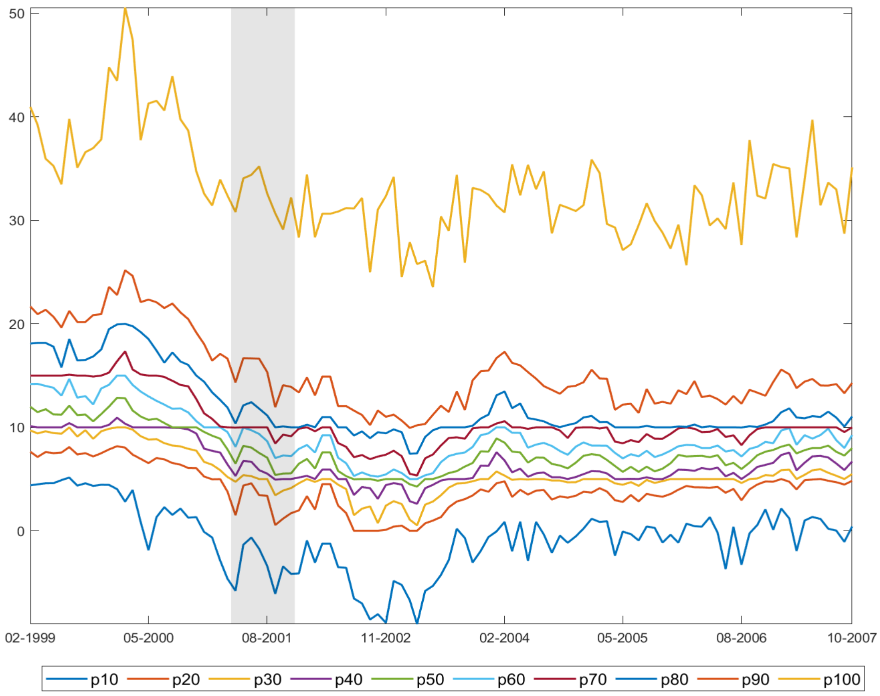


FIGURE 2.1: Distribution of Expected 1Y return, Gallup survey data on own portfolio returns 1999M2-2007M10. Each line denotes the mean for the corresponding percentile; shaded bands denote NBER recessions

In the second step, the distribution of subjective expectations on capital gains is summarized using two statistics: the mean and a disagreement index, DI. The former has been widely used in the asset pricing literature to capture the average sentiment of agents. For the latter I will measure the dispersion of

[2] Source: Robert Shiller webpage

beliefs as the difference between the 10% most optimistic agents and the 10% most pessimistic ones. A similar measure was used to measure disagreement on the bond market yields by Giacoletti et al. (2018). Atmaz and Basak (2018) argue that disagreement in the stock market is an important determinant of stock price variability and proposes a model in which agents' beliefs can be well characterized by the average sentiment and a measure of dispersion.

Moreover, the choice of the two statistics is not arbitrarily. A principal-component (PC) analysis performed on the distribution of beliefs reveals that the first two components explain 95% of the distribution of beliefs with the first component accounting for 86% while the rest being attributed to the second. The 1^{st} PC is highly correlated with mean beliefs ($> 99\%$) while the second with the disagreement index proposed to capture dispersion (correlation 91%). Figure 2.2 shows the first two principal components and the mean and disagreement index.

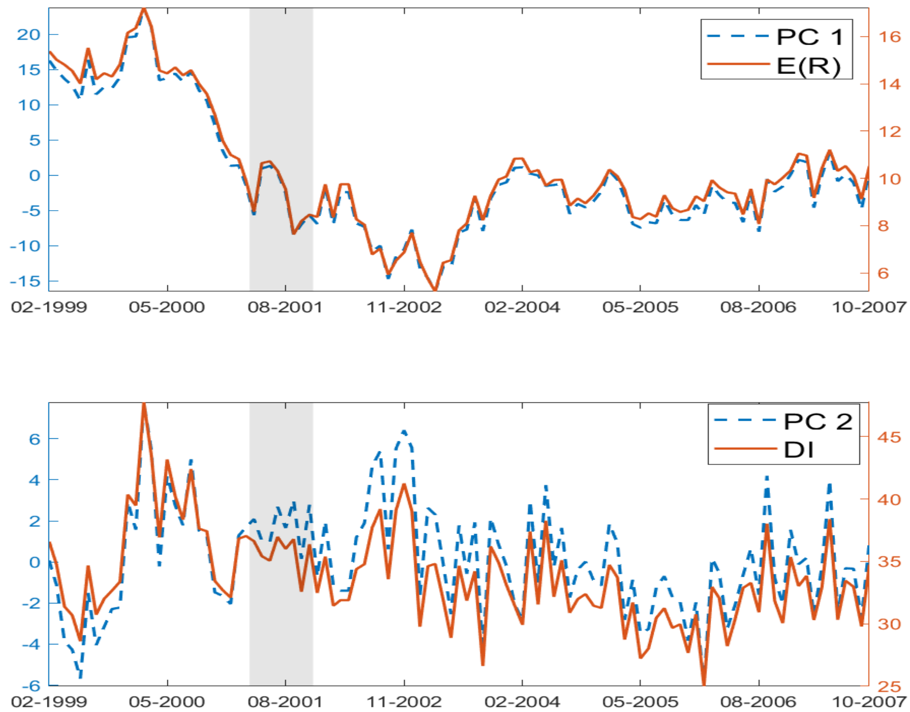


FIGURE 2.2: Principal components and Moments of the Subjective Expected Return Distribution

2.3 (Sentimental) Discount Rate Shocks

To fix notations, let R_t denote the total return of the risky asset at time t. Then

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{(\frac{P_{t+1}}{D_{t+1}} + 1)\Delta D_{t+1}}{\frac{P_t}{D_t}} \tag{2.1}$$

where D_t is the dividend paid by the asset and ΔD_{t+1} denotes the gross dividend growth rate. Linearising

$$r_{t+1} = \rho \, pd_{t+1} - pd_t + \Delta d_{t+1} \tag{2.2}$$

where $pd_t \equiv log(P_t/D_t)$, $\rho = \frac{\overline{PD}}{1+\overline{PD}}$ and $\overline{PD}$ denotes the long term mean of the PD ratio. Small letters denote variables in logs. Iterating forward equation (2.2) results in the Campbell-Shiller decomposition

$$pd_t \approx \underbrace{\sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j}}_{\text{Cash Flow}} - \underbrace{\sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}}_{\text{Discount Rates}}. \tag{2.3}$$

The above equation is an accounting identity and must hold at all points in time and in expected terms.

2.3.1 Classical View: Discount Rate Shocks

Cochrane (2008) and Cochrane (2011) shows that discount rates explain the majority of PD ratio variability while cash flows are insignificant in explaining the movement in the PD ratio. Discount rate shocks are defined as shocks to returns that do not affect contemporaneously dividend growth. There are pure price increases due to changes in discounting. I identify these shocks in a simple bi-variate VAR containing dividend growth and the PD ratio using short-run restrictions to identify the discount rate shocks (DR) as a shock that changes the PD ratio and does not affect dividend growth contemporaneously.[3] Formally, letting y_t be the vector of endogenous variables, the VAR(p) can be written as

$$\mathbf{y_t} = c + \sum_{l=1}^{p} B_l \mathbf{y_{t-l}} + u_t \quad \text{for } 1 \leq t \leq T \tag{2.4}$$

[3]Returns will not be included in the VAR to avoid imposing restrictions on the relation between the shocks; shocks to returns are a combination of the shocks on dividend growth and PD ratio according to equation 2.2.

where $\mathbf{y_t}$ is a $n \times 1$ vector of variables, B_l are $n \times n$ matrices of coefficients to be estimated, and u_t is a $n \times 1$ vector of residuals with covariance matrix Σ. The model in 2.4 can be rewritten in a more compact form as

$$\mathbf{y_t} = B_+ x_t + u_t \quad \text{for } 1 \leq t \leq T \tag{2.5}$$

where $B_+ = (B_1 \dots B_p \; c)$ and $x_t^{'} = (y_{t-1}^{'} \dots y_{t-p} \; 1)$. The dimensions of B_+ and x_t are $n \times (np+1)$ and $(np+1) \times 1$. Letting P denote the lower Cholesky factor of Σ, by premultiplying equation 2.5 by P^{-1} and taking the transpose we arrive at the structural representation of the VAR model:

$$\mathbf{y_t}^{'} A_0 = x_t^{'} A_+ + \epsilon_t^{'} \quad \text{for } 1 \leq t \leq T \tag{2.6}$$

where $A_0 = (P^{-1})^{'}$, $A_+ = B_+^{'}(P^{-1})^{'}$ and $\epsilon_t^{'} = u_t^{'}(P^{-1})^{'}$. Notice that $\mathbf{E}[\epsilon_t \epsilon_t^{'}] = I$. The matrices (A_0, A_+) are the structural parameters while B_+ and Σ are the reduced-form parameters. Defining Q as any orthonormal matrix of dimension $n \times n$, the structural parameters (A_0, A_+) and $(A_0 Q, A_+ Q)$ are observationally equivalent in the sense that they produce the same reduced form VAR representation.

Identification 1: Discount Rate Shocks
DR shocks affect the PD ratio and do not impact dividend growth contemporaneously
Implementation: Cholesky factorization with ordering $y_t = (\Delta D, pd)$

Figure 2.3 shows the impulse response function from a DR shock and the decomposition from equation 2.3.

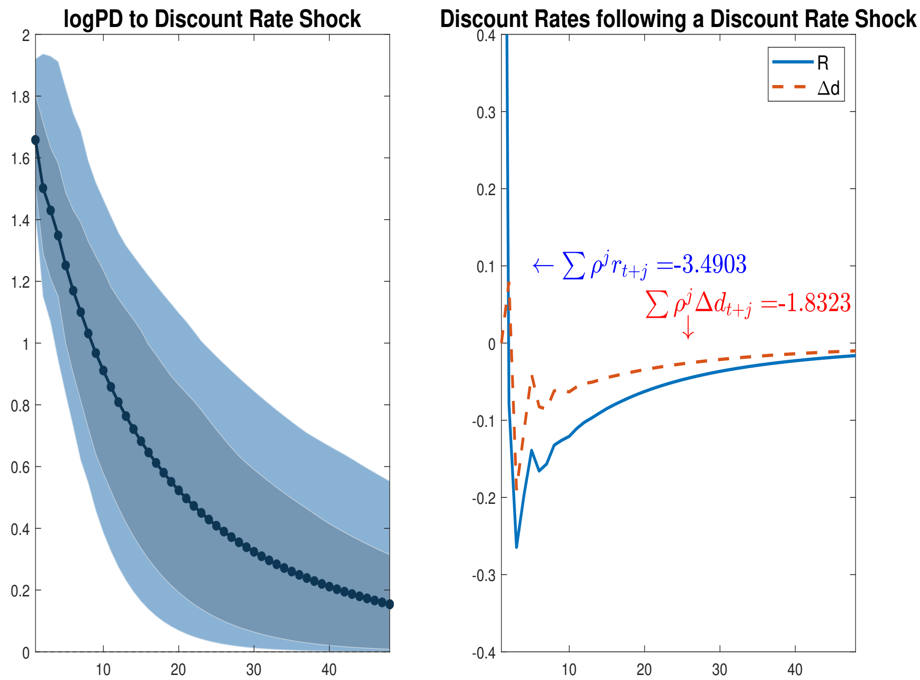


FIGURE 2.3: IRF to DR shock and implied subsequent returns (eq.3). 90% and 68% confidence bands

A pure discount rate shock increases the PD ratio contemporaneously by increasing prices without any move in dividends. The right panel shows what drives this increase in the PD ratio. Returns jump at the time of the shock and then turn negative in the next period and remain below 0 before returning to the long-run mean. Overall the discounted sum of returns is negative which sustains the initial price increase. The cash flow part has the wrong sign given the initial price increase which implies that dividends play no role and that all variation is due to discounting.

Figure 2.4 shows the contributions of the two shocks to the variance of the endogenous variables. Discount rate shocks explain over 95% of the movements in PD ratios while cash-flows shocks have limited explanatory power.

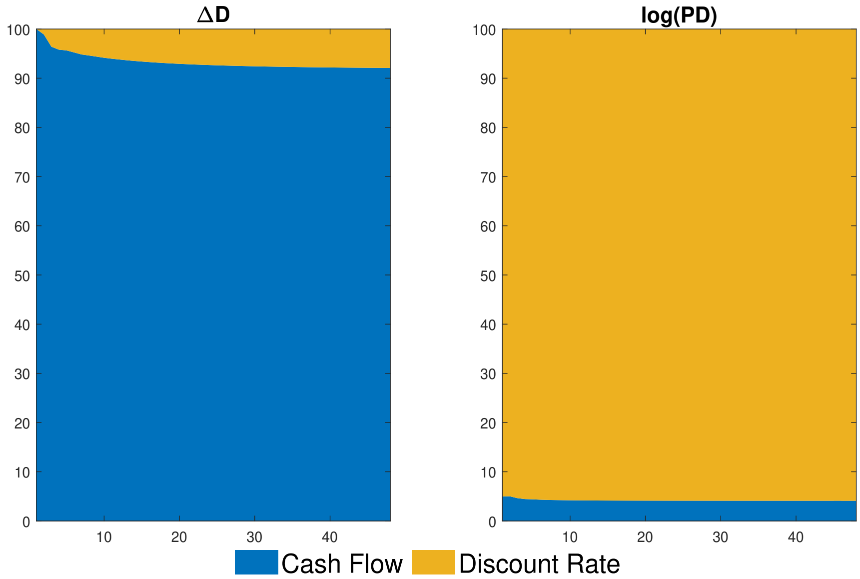


FIGURE 2.4: Variance Decomposition of discount rate and dividend growth shocks

The question that is at the core of this paper is whether what is attributed to discount rate shocks is actually hiding other sources of variation. The literature on learning and belief formation suggests that agents do not form expectations according to the rational expectation (RE) paradigm and that swings in agents' sentiment can have a considerable impact on market prices.

2.3.2 Sentimental Discount Rate Shocks

The previous analysis is simple yet instructive to the sources of PD ratio variability: PD ratios vary because of changes in discount rates. The theoretical asset pricing literature has come with different explanations regarding the sources of these variations: changes in risk aversion, long-run risks and animal spirits among others. The channel that will be investigated here is the one associated with animal spirits or dynamics of the agents' subjective beliefs. As mentioned previously, market sentiment will be measured by the distribution of subjective (survey) expectations which can be summarized by two statistics (factors): average beliefs and disagreement. The model will therefore include 4 variables, $y_t = (\Delta d, DI, E_t^{\mathcal{P}}(R_{t+1}), pd)'$ where $E_t^{\mathcal{P}}(R_{t+1})$ is the average subjective capital gain expectations and DI is the disagreement index. Since average beliefs and the disagreement index capture the two principal components of the subjective expected return distribution, the model is in fact a factor augmented VAR (FAVAR) model. Compared to the usual FAVAR models used in the literature, in the present framework we know exactly what are these two factors which will help in the identification of shocks and the interpretation of results.

For the identification of the subjective expectation shock (an exogenous increase in $E_t^{\mathcal{P}}(R_{t+1})$) the quantitative asset pricing literature can help in determining the effects and dynamics of such shocks. Bayesian RE models in which part of investors extrapolate past returns as in Barberis et al. (2015) or the internally rational asset pricing framework of Adam et al. (2017) have in common that in equilibrium, subjective beliefs or sentiment affect contemporaneously prices while the reverse does not hold. Formally, letting $\beta_t = E_t^{\mathcal{P}}(R_{t+1})$ the equilibrium price is

$$\begin{aligned} P_t &= F(\beta_t) \\ \beta_t &= S(\beta_{t-1}, P_{t-1}; *) + \epsilon_t \end{aligned} \tag{2.7}$$

where $F()$ represents the equilibrium function mapping subjective beliefs to prices, $S()$ the law of motion of expectations and ϵ_t a sentiment shock. Notice that while the sentiment shock affects current beliefs and prices, current stock prices do not affect contemporaneously beliefs. Average survey beliefs adjust slowly compared to prices and are well characterized by a constant gain learning updating equation with a small gain parameter, as shown for example in Adam et al. (2017). Given this slow-moving behavior of subjective expectations and the quantitative performance of asset pricing models in replicating jointly survey expectations and stock prices, it will be assumed that an exogenous increase in stock prices will not affect contemporaneously agents' beliefs and disagreement. The reverse is not true. A sentiment shock, understood as a innovation to subjective beliefs, should not affect dividend growth on impact but can affect prices at the time the shock hits, in line with the theoretical predictions discussed before.

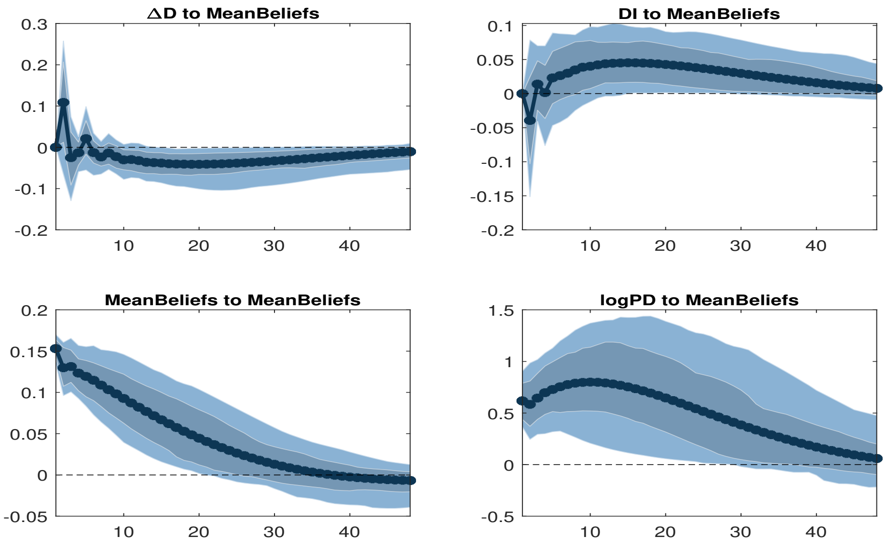
A discount rate shock (shock in the *pd* equation) influences prices contemporaneously without having any effect on dividend growth, Δd. Disagreement shocks do not influence on impact dividend growth but can influence prices and average subjective beliefs. This assumption is in line with the theoretical model proposed by Atmaz and Basak (2018). These assumptions on the effects of shocks on the endogenous variables can be accommodated by short-run restrictions imposed on the VAR model given by equation 2.6 which is accomplished by the following identification strategy.

Identification 2: Sentimental Discount Rate (SDR) Shocks
SDR affect agents' subjective beliefs and can influence contemporaneously market prices without having any effect, on impact, on disagreement and dividend growth.
Implementation: Cholesky factorization with ordering $y_t = (\Delta d, DI, E_t^{\mathscr{P}}(R_{t+1}), pd)'$ [a]

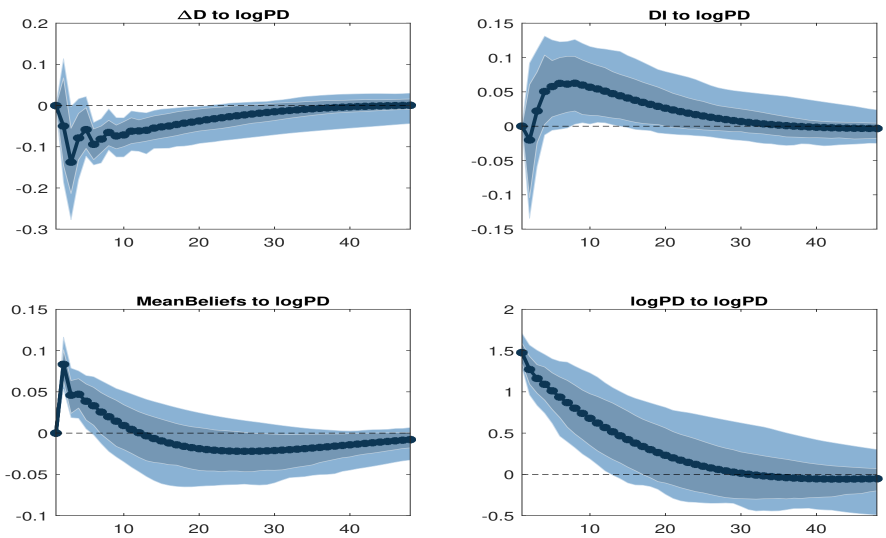
[a]The estimation uses 2 lags following the indication from the BIC and AIC information criterion. The results are robust to varying the lag order from 1 to 6

Notice that under identification 2 sentiment or discount rate shocks do not affect contemporaneously disagreement. This assumption will be relaxed in the next section but will not have any sizable impact on the main message of the paper.

Figure (2.5) presents the impulse response functions to sentiment and discount rate shocks.



(A) Sentimental Discount Rate Shock



(B) Discount Rate Shock

FIGURE 2.5: Impulse Response functions to SDR shocks (panel a) and DR shocks (panel b). Median IRFs together with 90% and 68% bootstrap confidence bands

The main difference between the two shocks is the effect on the PD ratio and the persistence of the responses. Standard DR shock increase the PD ratio on impact after which it decreases monotonically to the long-term mean. In contrast, the SDR shock produces a hump-shaped response in the PD ratio which together with subjective expectations show higher persistence over time.

2.3.3 PD Ratio Decomposition

The two main shocks of interest have different a effect on prices but it could still be the case that the large part of the PD variability is explained by standard DR shocks. Figure 2.9 shows the main result of the paper: *sentimental discount rate shocks account for around 40% of the variability of the PD ratio in the long run as much as standard discount rate shocks*. This implies that animal spirits or swings in investors' sentiment, as measured by subjective capital gain expectations, contribute significantly to the boom and busts in asset prices. This evidence is in line with the predictions from models such as Adam et al. (2017) or Adam and Merkel (2019) where agents' subjective capital gain expectations drive to a large extent asset price movements. Under this identification strategy disagreement shocks have a limited effect on the PD ratio and seem to operate mainly through average sentiment explaining around 20% of the short-run variation of average beliefs.

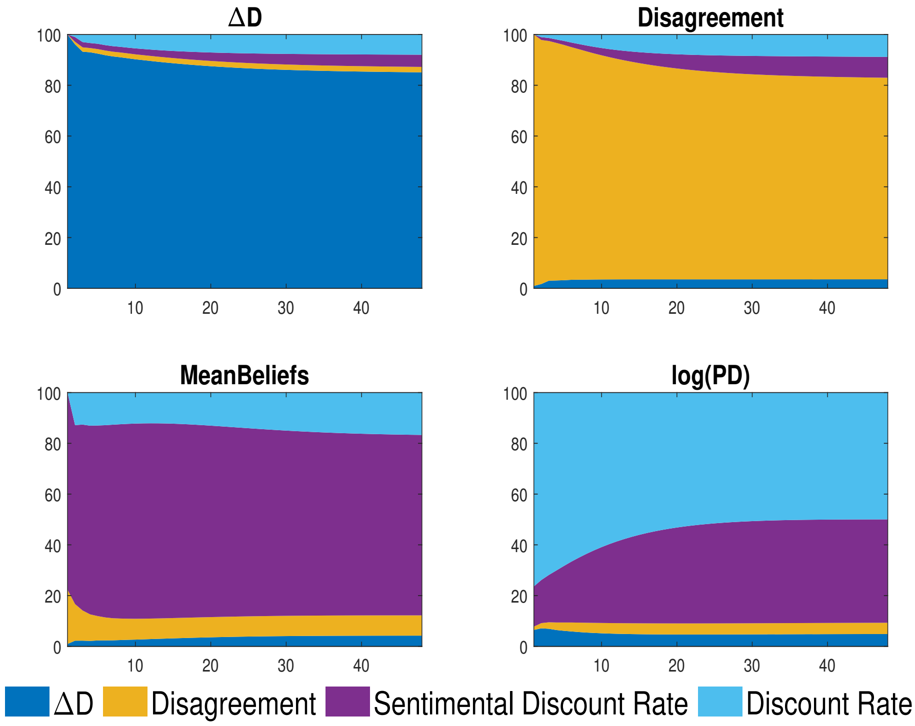


FIGURE 2.6: Variance Decomposition of shocks under Identification 2: each colour show the contribution (in %) of each one of the four shocks corresponding to $y_t = (\Delta d, DI, E_t^{\mathcal{P}}(R_{t+1}), pd)$

It is instructive to back out returns from IRFs of PD ratios using equation (2.2) to compute the pattern of subsequent discount rates following the shocks. Figure 2.7 presents the two sources of variation from the Campbell-Shiller decomposition for discount rate and sentimental discount rate shocks conditional on assuming that the magnitude of the two shocks is such that it produces the same initial effect on prices.

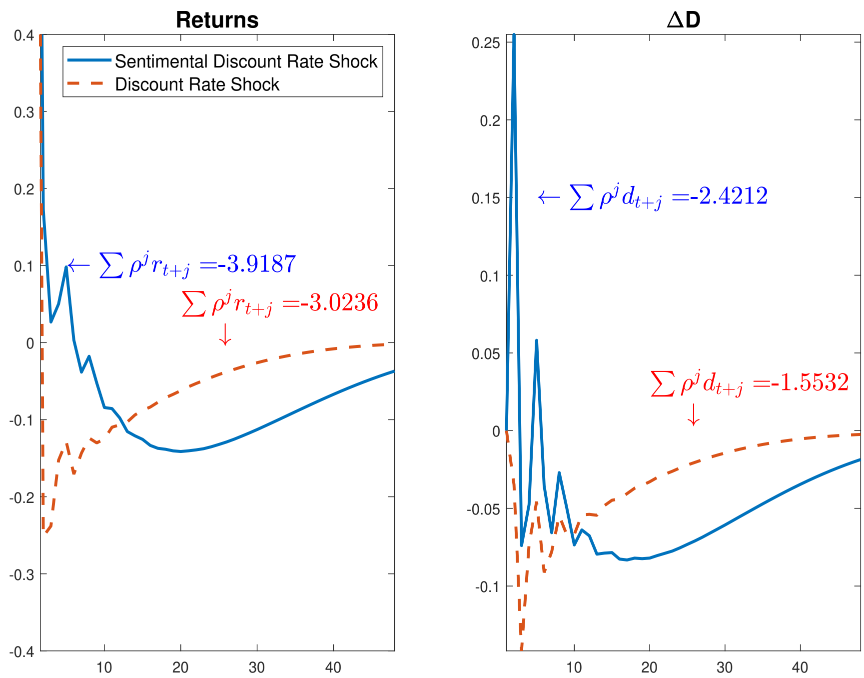


FIGURE 2.7: Implied returns and dividends from (S)DR Shocks conditional on same impact effect on the PD ratio

Compared to standard discount rate shocks, left panel from figure 2.7 shows that sentiment shocks produce a more persistent increase in returns following the shock and larger decrease in later periods before returning to the long-term mean. Both shocks increase prices on impact due to lower cumulative future discount rates, the difference being the inverse hump-shaped pattern of discount rates from sentimental discount rate shocks.

Turning to the historical decomposition from Figure 2.8 we notice that during the 2000 boom in asset prices sentimental discount rate shocks were one of the main determinants of the rapid increase in prices. Nevertheless, after the collapse in prices from 2002 the role of SDR shocks diminishes over time. This suggests that these shocks contribute the most at the top of asset price cycles and consequently during the bust.

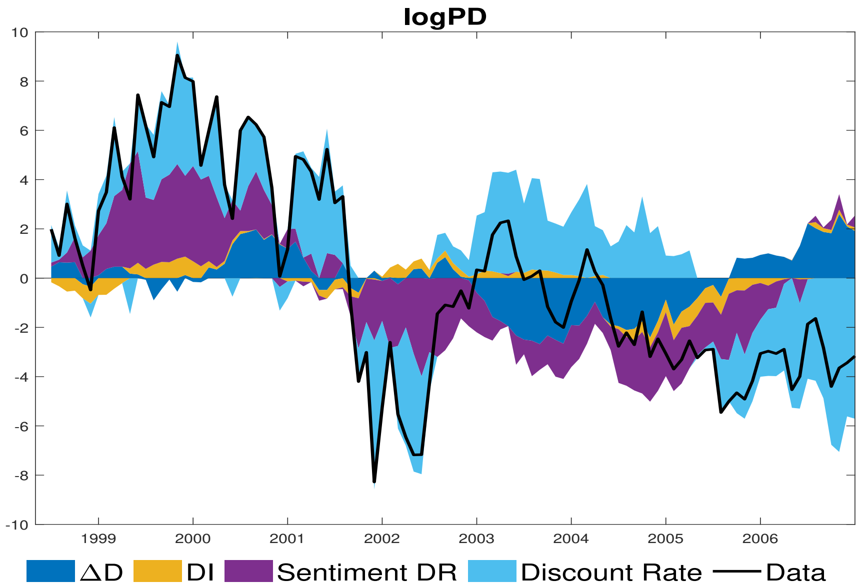


FIGURE 2.8: Historical Decomposition of the PD ratio

2.4 Robustness

This section considers several robustness checks to the previous identification strategy. Firstly, it explores if the identification of sentiment shocks captures in fact shocks in the real economy at a business cycle frequency. Secondly, it proposes an alternative identification strategy that adopts an agnostic view on the effects of sentiment and discount rate shocks on disagreement.

2.4.1 Business cycle fluctuations

Sentimental discount rate shocks as identified in the previous section could in principle capture the business cycle movement in the real economy which influences agents' expectations about the stock market and the PD ratio. To explore this possible scenario, the cycle component of real GDP, denoted by $\tilde{y}$, will be added to previous analysis. Since the estimation is using data at a monthly frequency, the output-gap series is not available and instead the Brave-Butters-Kelley (BBK) index of the cycle component of real GDP will be used.[4] This index is constructed from 500 time series of real economic activity and quarterly GDP growth. I assume that none of the identified previous shocks

[4]See Brave et al. (2019)

can influence contemporaneously the business cycle component of real GDP while leaving unrestricted the effect of a shock to the real economy on the other variables. This is accomplished using the following ordering in the FAVAR identified with zero short-run restrictions: $y_t = (BBK, \Delta d, DI, E_t^{\mathscr{P}}(R_{t+1}), pd)'$. Figure 2.9 shows the explained variance of each type of shock.

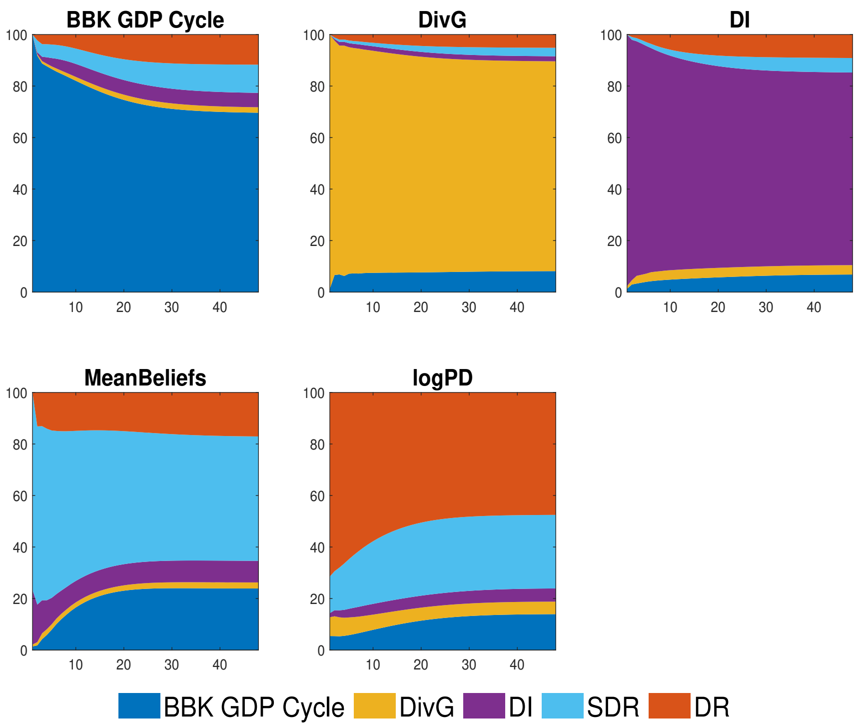


FIGURE 2.9: Variance Decomposition of the contributions of shocks to $y_t = (\tilde{y}, \Delta d, DI, E_t^{\mathscr{P}}(R_{t+1}), pd)$

First notice that shocks to the business cycle component of GDP explain the majority of the variation in the BBK index but also a sizable share in subjective beliefs (around 20% in the long run), suggesting that part of the previous contribution of subjective beliefs to PD ratio variability could be in fact captures by business cycle shocks. Indeed, the contribution of sentiment shocks (light blue area in figure 2.9) to variations in the PD ratio is slightly reduced compared to the results arising under identification 2, part of the variability being now related to business cycles movements. Nevertheless, SDR still represents an important share of PD ratio variability explaining around 30% of the total variance of prices while discount rate shocks explain the largest share of approximately 45%.

2.4.2 Agnostic identification of disagreement shocks

In the baseline identification strategy it has been assumed that neither the discount rate nor the sentiment shocks have a contemporaneous effect on disagreement. Indeed, some theoretical papers argue that this is indeed the case although the evidence on this is almost absent and the literature has not yet reached a consensus. To explore the implications of relaxing this assumption, a mixed identification strategy combining zero and sign restrictions will be employed.[5] The identification restrictions are summarized in Table 2.1 while the periods on which the restrictions have been imposed are presented in Table 2.2. The model is estimated with Bayesian techniques using an uninformative normal-diffuse prior.

	SDR	DR	Disagreement	Dividend	GDP
Mean Beliefs	+	0			
logPD	+	+			
DI			+		
ΔD	0	0	0	+	
GDP cycle	0	0	0	0	+

Table 2.1: Identification Restrictions
"+" denotes a positive impact of the shock from the column on the endogenous variable on the corresponding row; the entries with "0" represent short-run restrictions.

[5]Following the methodology developed by Arias et al. (2018). This implies imposing joint restrictions on the parameters of the matrices A_0 and A_+ defined in equation 2.6

	SDR	DR	Disagreement	Dividend	GDP
Mean Beliefs	3	1			
logPD	1	3			
DI			3		
ΔD	1	1	1	3	
GDP cycle	1	1	1	1	3

Table 2.2: Periods on which Restrictions have been imposed

The identification strategy imposes that all shocks have a significant impact on the variable where it originates for at least 1 quarter (3 periods) while the zero restrictions are imposed only on impact. Furthermore, the identification also imposes that a positive SDR shock has a non-negative effect on impact on the PD ratio (column 1 row 3 in Table 2.2). As argued previously, the effects of the shocks on the disagreement index are left unrestricted. The contributions of the shocks to the variance of the endogenous variables are presented in Figure 2.10.

Sentimental/Discount Rate shocks contribute equally to the variability of PD ratio, both accounting for around 40% of the explained long-run variance while disagreement shocks account for approximately 10%. What changes under this alternative identification strategy is the contribution of disagreement shocks to the dynamics of mean beliefs and vice-versa. Disagreement shocks account for 70% of the short-run variance of average sentiment, decreasing to around 40% in the long run while sentiment shocks account for 30% from the disagreement index variability. These results suggest that average subjective beliefs and disagreement are tightly interconnected.

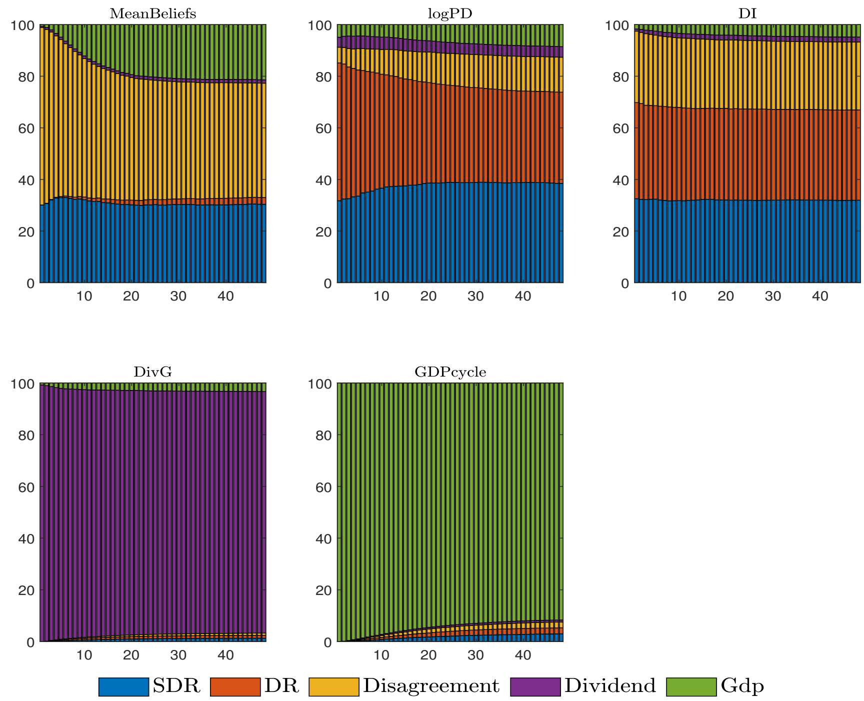


FIGURE 2.10: Variance Decomposition of $y_t = (\tilde{y}, \Delta d, DI, E_t^{\mathscr{P}}(R_{t+1}), pd)$ based on the identifying restrictions from Tables 2.1 and 2.2

2.5 Conclusions

The vast majority of the empirical asset pricing literature suggests that PD ratios vary due to movements in discount rates. This paper argues that a large share of these movements is associated with shocks affecting the subjective distribution of capital gain expectations that can explain up to 50% of PD ratio variability: 34% due to shocks to average subjective capital gain expectations and up to 10% attributed to disagreement shocks.

The distribution of survey data on capital gain expectations can accurately be characterized by two factors explaining over 95% of its variability: average sentiment and disagreement. Using these factors, I augment a standard asset pricing VAR with the subjective distribution of beliefs and identify standard and sentimental discount rate shocks (SDR) with the latter to be understood as shocks to agents' subjective average beliefs. The identification is inspired by the theoretical predictions of asset pricing models with subjective beliefs and assumes that SDR shocks can affect contemporaneously prices and beliefs while standard discount rate shocks do not have a short-run effect on beliefs due to their slow-moving dynamics. Sentiment shocks behave in a similar way to standard discount rate shocks but with several important differences: sentimental discount rate shocks produce a hump shape response of the PD ratio and introduce more persistence in the impulse response functions. Using a historical decomposition of the PD ratio, the results show that the 2002 dot-com boom in the stock market has been predominantly fueled by the identified sentimental discount rate shocks.

The main result of the paper is robust when considering business cycle shocks and under an alternative identification strategy that remains agnostic to the nature of disagreement shocks. Under this identification method, disagreement shocks play a more important role in driving average beliefs suggesting a tight connection between average sentiment and subjective disagreement.

Chapter 3

The Anatomy of Stock Market Cycles: Heterogeneous Expectations, Price Volatility and Trading

with *Pau Belda*

Abstract

This chapter shows that a model of learning about capital gains with heterogeneous expectations can jointly explain several old and new facts about stock prices, portfolio adjustments and survey expectations. Our key innovation is to model the whole distribution of expectations in a way consistent with many survey stylized facts: perpetual disagreement, procyclical expectations/disagreement and forecast error predictability. Using this model we replicate hard-to-reconcile facts regarding market volatility, expected returns, disagreement and trading. A typical boom would follow this sequence: i) an income or sentiment shock make investors more willing to invest in equities, driving up prices ii) the initial price increase make all investors more optimistic, reinforcing the cycle iii) however, certain conservative investors are reluctant to be drawn by such optimism iv) this heterogeneous reaction of expectations raises disagreement and trading. Therefore, disagreement and trading appear as a consequences of a bullish market and not as a driving force.

3.1 Introduction

The purpose of this paper is to provide a simple asset pricing model with heterogeneous beliefs that can replicate jointly the distribution of subjective beliefs, equilibrium prices and quantities. This framework allows us to shed light also on the nature of stock market cycles and, in particular, on the contribution of optimists/pessimists in driving booms and busts in asset prices and trading.

We suggest that the archetype of a stock market cycle is characterized by the following sequence of events. At some point, an exogenous factor (e.g. particular news (an "expectation shock") or extraordinary incomes (a "wage shock")) makes some investors more willing to invest in the stock market. That generates a rise in prices which turn all investors more optimistic amplifying the initial increase in prices. Nevertheless, not all investors react equally to the rise in prices due to their different expectation formation process (some are more conservative than others, turning only moderately more optimistic as these higher prices materialize). Thus, the heterogeneous reaction of expectations to prices increases disagreement and trading. Note then, disagreement and trading are, in general, a consequence of a price boom and not the other way around.

An increasing amount of the recent asset-pricing literature has emphasized the importance of understating how investors form beliefs and the implications for asset pricing. One of the reasons for this change of direction away from standard rational expectation (RE) models is the evidence coming from survey data on agents' expectations that shows significant departures from the RE hypothesis.[1] The above quote is taken from the latest NBER asset pricing program agenda for future research which clearly points out to the importance of incorporating realistic belief systems in asset pricing models. Moreover, recent empirical evidence has shown that subjective beliefs are characterized by considerable heterogeneity and persistence over time and across agents. Understating how heterogeneous expectations evolve and their implications for asset pricing is therefore crucial for building asset pricing models that match the survey evidence on subjective belief dynamics. We seek to contribute to the latter and present several empirical facts about the distribution of subjective beliefs on capital gain expectations and propose a framework in which we model explicitly the heterogeneity of expectations in line with survey evidence.

[1] See Greenwood and Shleifer (2014) and Adam and Nagel (2022)

The proposed model is an extension of the work of Adam et al. (2017) to accommodate for several sources of heterogeneity in capital gains expectations. Giglio et al. (2021) show that investors' subjective stock price expectations are directly reflected in portfolio allocations and that agents' beliefs are characterized by large and persistent heterogeneity over time and across agents.[2] We incorporate this new stylized fact about the heterogeneity of agents' beliefs in our model by allowing agents to hold different long-run levels of capital gain expectations while learning in the short-run about stock price dynamics. We interpret the former as heterogeneity in beliefs about long-run cash flows or fundamentals and the latter as short-run expected returns. We depart from the rational expectations (RE) assumption and instead assume that agents are internally rational with belief systems heterogeneous in mean, persistence and learning speed which we calibrate closely to match the distribution of subjective beliefs from survey data.[3] As it turns out, this specification is sufficient to capture a wide variety of the anatomy of US stock price cycles and subjective beliefs: persistent and pro-cyclical subjective disagreement, co-movement in beliefs among different sentiment groups, co-movement between trading/disagreement, pro-cyclical capital gains expectations and forecast-error predictability.

Our framework also allows us to investigate the contribution of different sentiment groups to booms and busts in price cycles. In particular, we want to answer the following question: given the observed empirical distribution of beliefs, prices and quantities, to what degree do optimists and pessimists drive booms and busts? We argue, through the lens of our model, that the positive correlation between disagreement and prices that we observe in booms can only be driven by optimists becoming more optimistic and not pessimistic agents adjusting their beliefs upward. Moreover, the decrease in this correlation during busts and normal periods is strongly driven by the beliefs of pessimists. This has implications for policy to the degree that belief-driven asset price cycles introduce inefficient dynamics to the real economy.[4] In this regard, managing capital gain expectations for the most optimistic agents is crucial for leaning

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[2] Moreover, this heterogeneity cannot be explained by standard factors such as wealth, age, gender or past returns.

[3] see Adam and Marcet (2011)

[4] Belief driven asset price cycles can impact the real economy through multiple channels: see Ifrim (2021) for demand side inefficient wealth effects and Winkler (2020) for supply side with financial frictions

against the wind policies in reducing the inefficiencies created by belief-driven asset price cycles.

To the best of our knowledge, this is the first paper to jointly replicate quantitatively the distribution of subjective beliefs, price dynamics and quantities in the context of the stock market. We model closely the beliefs of our agents based on available evidence from survey data and from the empirical evidence provided by Giglio et al. (2021) on which we will expand in the next section. The literature on belief heterogeneity and asset pricing is vast. Nevertheless, most of the literature has not provided yet a realistic quantitative evaluation. Atmaz and Basak (2018) provide, for example, a theoretical model of heterogeneous beliefs that is able to replicate several of the stylized facts observed in the data. In contrast to that framework in which agents possess beliefs about fundamentals (dividends) for which survey data is limited, we work with expectations on expected return which allows us to compare directly the model with survey data and evaluate the quantitative performance of the model. On a similar note, Martin and Papadimitriou (2019) develop a model with heterogeneous beliefs about probabilities of good/bad news in which sentiment is another source of risk fully internalized by agents and which stimulates speculation and volatility. For a comprehensive review on the literature on heterogeneous beliefs about asset prices see Simsek (2021).

The rest of the paper is organized as follows. Section 2 presents several stylized facts regarding the empirical survey distribution of beliefs and the dynamics of subjective beliefs across different levels of sentiment, section 3 lays out the theoretical asset pricing model, section 4 the quantitative performance and the mechanism through which heterogeneous beliefs drive asset price cycles and lastly section 5 concludes.

3.2 Stylized Facts about Heterogeneous Expectations

Our preferred data is the Gallup survey on future stock market return expectations on individual investors for the period 1998Q2-2007Q4.[5] We choose this survey for the simple reason that it includes the most number of respondents per

[5]Quarterly observations are obtained from averaging over the corresponding monthly data

period (around 700) which in principle should bring more reliability in capturing the heterogeneous dynamics of expectations. We first split the distribution of beliefs into sentiment groups based on the level of optimism/pessimism of individual investors regarding future returns. Specifically, we order the distribution of beliefs across agents at each point in time in three subgroups ranked by their level of optimism and compute averages for each group. Although our data is not a panel, the evidence from Giglio et al. (2021) shows that beliefs are persistent over time meaning that optimists remain optimists and pessimists remain pessimists, without interchanging. Given this fact, we argue that the mean of each sentiment group captures reasonably well the heterogeneity of expectations of each group and proceed with this caveat in mind. Figure 3.1 presents the evolution over time of the sentiment groups with S_1 being the most pessimistic, S_2 the average investors and S_3 representing the sentiment group of agents with the most optimistic beliefs.

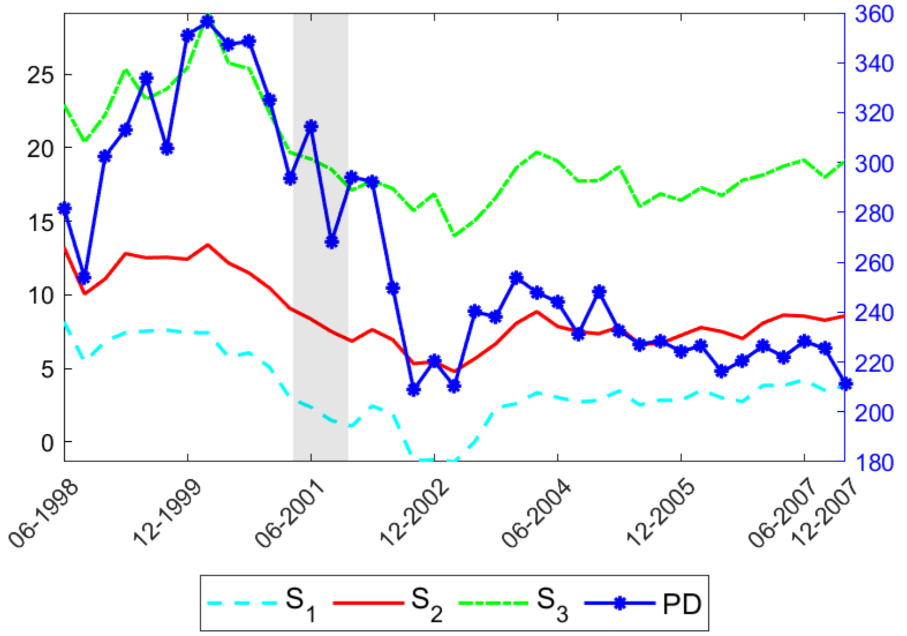

FIGURE 3.1: Dynamics of Sentiment Groups. Each sentiment group represents the average return expectation at each point in time across agents depending on the position in the distribution (eg. S_1 represents the average of the beliefs between 0 and $\frac{1}{3}$ percentiles); shaded bars denote NBER reccesions

At the top of the dot-com bubble, optimists were expecting as high as 30% yearly returns while pessimists only 7%. Sentiment groups are highly correlated across each other (0.8-0.95) and with prices although the return expectations of optimists (S_3) has a larger magnitude compared to the ones of pessimists(S_1), 0.9 compared to 0.5.

Our preferred measure of disagreement/dispersion of beliefs is defined by the difference between the beliefs held by the most optimistic/ pessimistic groups.[6] For three sentiment groups, this measure is defined as $DI_{33}^{33} = S_3 - S_1$. Figure 3.2 presents the evolution of disagreement together with the PD ratio. Disagreement about future stock returns tends to be high near the top of the price cycle and highly correlated with the PD ratio (0.7). Moreover, subjective beliefs are characterized by persistent positive disagreement with a mean of approximately 16%, in line with the evidence from Giglio et al. (2021) on the existence of individual fixed effects in the cross-section of beliefs.

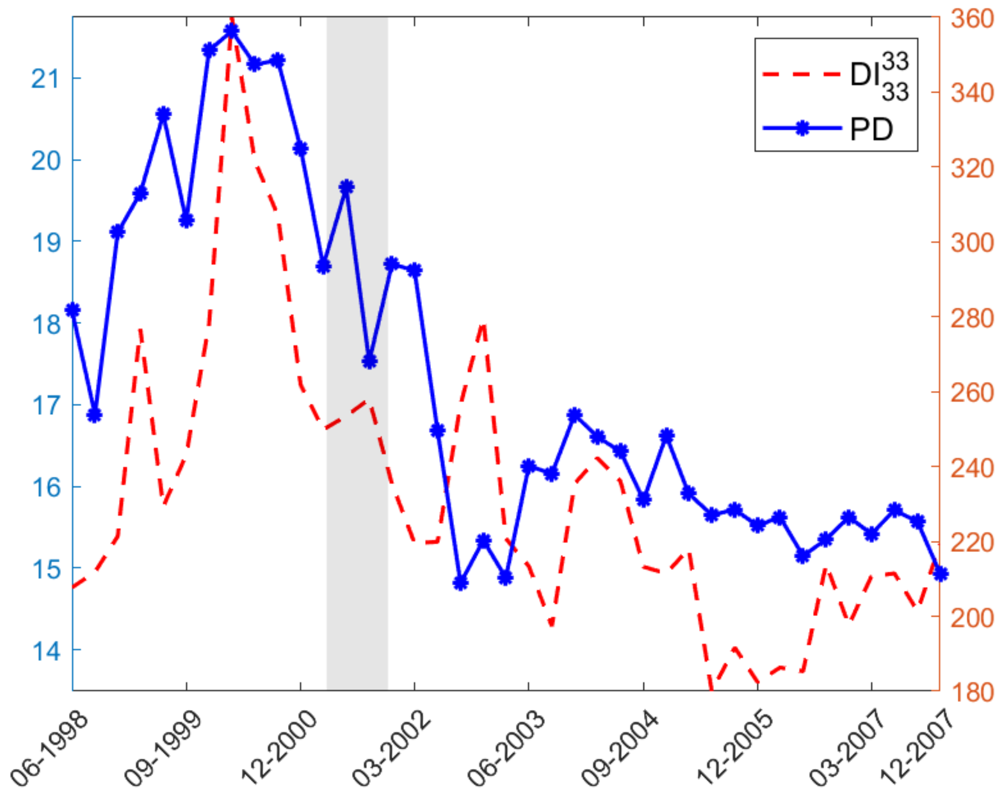

FIGURE 3.2: Disagreement and PD ratio

[6]A similar measure has been used by Giacoletti et al. (2018) to measure disagreement in bond markets.

The next figure shows disagreement computed both as the inter-group standard deviation and as the difference between the 90th and 10th percentile (DI_{10}^{10}). These measures behave very similarly to our benchmark specification with correlation coefficients higher than 0.9. This suggests that the dynamics of disagreement is not sensitive on the exact measure used but instead is fundamentally rooted into the data.

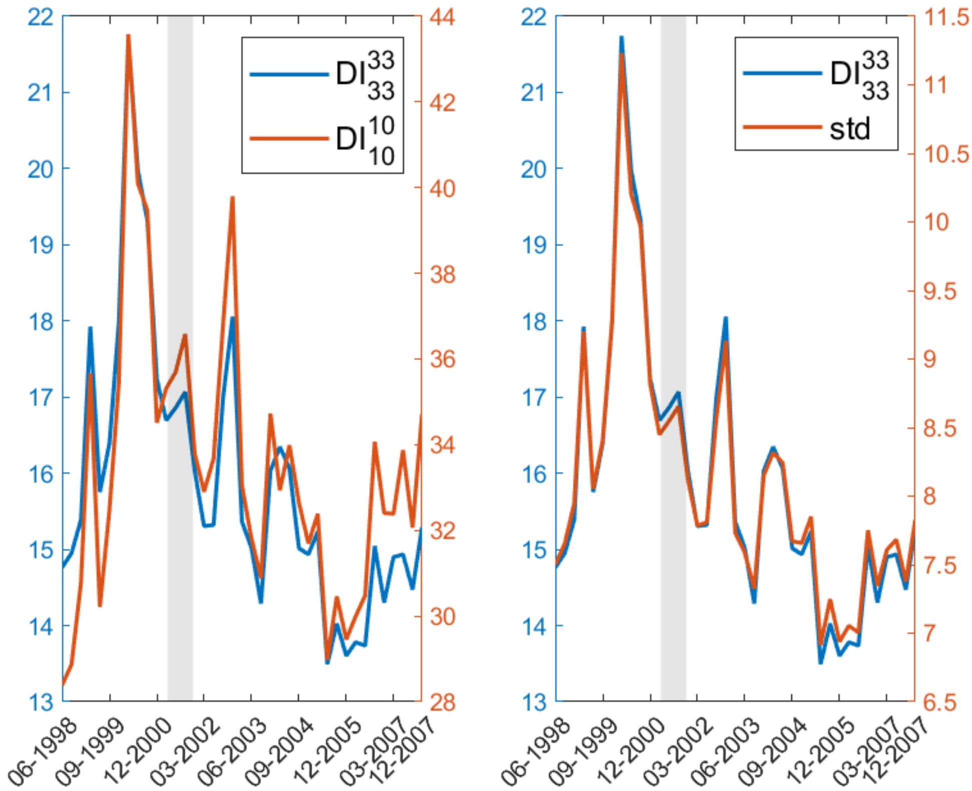

Figure 3.3: Alternative measures of disagreement

Several studies have pointed out that average expectations about stock mark market returns fail to pass standard RE tests.[7] It is possible, in principle, that some sentiment groups could behave more in line with the RE hypothesis than the others. We check this possibility by applying the RE test proposed in Adam et al. (2017) for each sentiment group.

The test implies running the following two regressions

[7] see Adam et al. (2017) and Greenwood and Shleifer (2014) among others

$$\begin{aligned} \mathbb{SE}_t &= \boldsymbol{a} + \boldsymbol{c}\, PD_t + u_t + \mu_t \\ R_{t,t+n} &= a + c\, PD_t + \epsilon_t \end{aligned} \tag{3.1}$$

where $\mathbb{SE}_t$ represents survey expectations regarding future returns at time t, PD_t is the Price Dividend ratio and $R_{t,t+n}$ is the realized return between t and $t+n$. Moreover, u_t and ϵ_t represent variations in survey expectations and returns due to other factors than the PD ratio and μ_t captures measurement error in survey expectations which is assumed to be uncorrelated with the previous two exogenous variations. The RE test is basically a test of equality between c and $\boldsymbol{c}$. Results from table C.1 indicate that the RE hypothesis with respect to survey expectations on capital gains is rejected at the 1% significance level for each one of the three sentiment groups.[8]

			p-value
	c	$\boldsymbol{c}$	H_0: c= $\boldsymbol{c}$
p_{0-33}	0.0576***	-0.2421 ***	0.0000
p_{33-66}	0.0545***	-0.2415***	0.0000
p_{66-100}	0.0809***	-0.2423***	0.0000

TABLE 3.1: RE Tests across different sentiment groups; p_{0-33} denotes the sentiment group which expectations lie between between the 0 and 1/3 percentile. The data in each group is aggregated by taking the average of that particular group at each point in time. Data used for this particular test is the Gallup UBS survey data for expected stock market return of all individuals. Estimates are based on asymptotic theory and have been adjusted for small sample bias. *** denotes significance at the 1% level.

Since expectations of different sentiment groups fail at passing standard RE tests and given the previous evidence pointing to the extrapolative nature

[8]The results are unchanged if instead of three sentiments groups we consider two or four, see Appendix 1 for results on RE tests based on different partitions of the distribution of subjective returns.

of beliefs, we estimate the following equation of belief formation for each group of expectations

$$\beta_t^i = (1-\rho_i)(1-g^i)\bar{\beta}^i + \rho_i\beta_{t-1}^i + g^i(lnP_{t-1} - lnP_{t-2} - \rho_i\beta_{t-1}^i) + \varepsilon_t^i \tag{3.2}$$

where β_t^i is the subjective expectation of sentiment group $i \in \{1,2,3\}$ regarding real capital gain, ρ_i governs the persistence of the process, g^i the speed at which past errors are included in future beliefs and $\bar{\beta}_i$ a measure of heterogeneity among sentiment groups.[9] We interpret the latter as perceived long-term heterogeneity in the fundamental/cash-flow value of the asset. This specification allows us to capture several sources of heterogeneity in the belief formation process of different sentiment groups. We estimate the parameters by NLS for each sentiment band individually and present the results in the following table.[10]

Sentiment group i	1	2	3
g^i	0.014	0.02	0.03
	(0.0025)	(0.0006)	(0.007)
ρ_i	0.9	0.9	0.91
	(0.0013)	(4.4e-5)	(0.0013)
$\bar{\beta}^i$ (in %)	-0.5	1	4.8
	(0.14)	(0.11)	(0.5)

Table 3.2: Estimated Learning Parameters. Parameters have been estimated by non-linear least squares; bootstrap standard errors in parentheses calculated by a sieve bootstrap method over 1000 simulations using AR(p) innovations with order p chosen by the AIC criterion.

[9] To be consistent with the theoretical asset pricing model from the next section we transform the UBS survey return expectations into price growth using the following identity: $R_{t+1} = \frac{P_{t+1}}{P_t} + \beta^d \frac{D_t}{P_t}$ where β^d is the expected quarterly dividend growth which we set equal to 1.0048. The resulting nominal capital gain data is transformed into real series by subtracting SPF inflation forecasts.

[10] Appendix C presents the bootstrap distributions of these estimated parameters

Table 3.2 shows that the speed of learning (g^i) is increasing with the sentiment band with optimists (S^3) having the highest learning parameter. Using the same survey data as us, Adam et al. (2015) show that the constant gain parameter is inversely related with the experience of investors with low experience investors having the largest parameter. According to this evidence, the optimist investors are mostly characterized by low experience while the reverse is true for pessimists. On the other side, the persistence is similar among these groups and the measure of long-term heterogeneity increases in optimism as expected. Figure 3.4 shows the fit for each sentiment band.

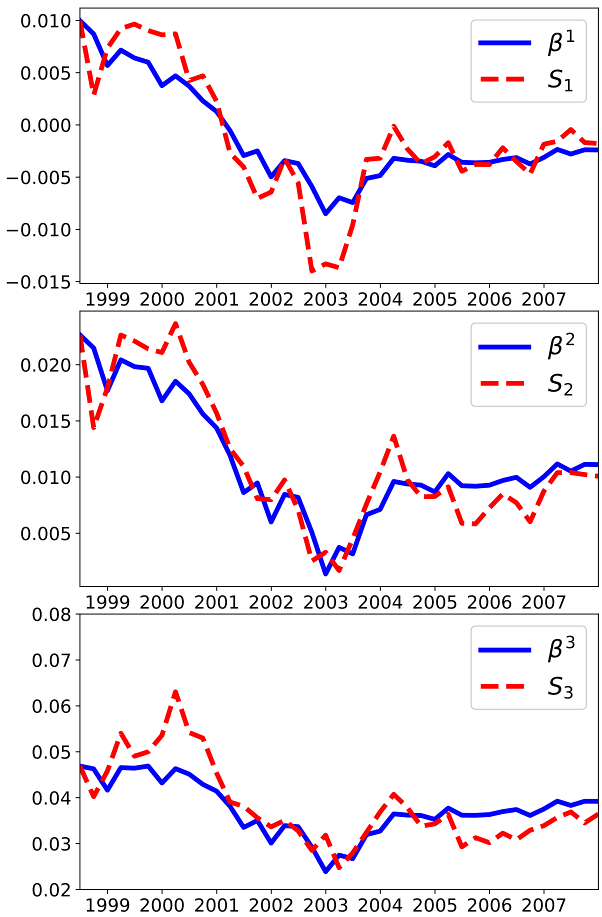

FIGURE 3.4: Model fit from equation 3.2. The equation has been estimated by non-linear least squares by minimizing for each sentiment group $\sum(S_i - \beta^i)^2$

We summarize these stylized facts regarding the distribution of subjective beliefs together with several others pointed out in the asset pricing literature in the following table.[11]

Fact	Statistic	Value
1. Persistence of expectations	ρ	0.90
2. Procyclicality of capital gains expectations	$corr(PD, \beta_t)$	0.82
	$corr(PD_t, \beta_t^3)$	0.86
	$corr(PD_t, \beta_t^1)$	0.7
3. Perpetual disagreement	$\mathbb{E}(DI)$	0.04
	$\sigma(DI_t)$	0.0044
4. Disagreement led by i) optimists	$corr(DI_t, S_t^3)$	0.73
ii) pesimists	$corr(DI_t, S_t^1)$	0.36
4. Disagreement procyclicality	$corr(DI_t, PD_t)$	0.72
5. Comovement disagreement-trading	$corr(DI_t, TV_t)$	0.41
6. Correlation among sentiment groups	$corr(S^1, S^2)$	0.95
	$corr(S^1, S^3)$	0.87
	$corr(S^2, S^3)$	0.95

TABLE 3.3: Facts on the Heterogeneity of Subjective Expectations

3.3 An asset pricing model with realistic expectations

We present in this section a simple asset pricing model with heterogeneous beliefs consistent with the empirical evidence from the previous section. Consider an endowment economy populated by M types of agents, $i \in [1, M]$, who solve the following utility maximization problem

[11] See for example Adam et al. (2015) and Adam et al. (2017)

$$
\begin{aligned}
&\max_{\{C_t^i, S_t^i\}_{t=0}^{\infty}} \mathbb{E}_0^{\mathscr{P}_i} \sum_{t=0}^{\infty} \delta^t \frac{(C_t^i)^{1-\gamma}}{1-\gamma} \\
s.t.& \\
&C_t^i + P_t S_t^i \leq (P_t + D_t) S_{t-1}^i + W_t^i \\
&\underline{S} \leq S_t^i \leq \bar{S}
\end{aligned}
\tag{3.3}
$$

where C denotes consumption, W income (wages) that agents receive, S the amount of stock holdings in the risky asset with price P that pays exogenous dividend D. $\mathscr{P}_i$ represents the probability measure of agents of type i. We assume that the risky asset, which we interpret as stocks, is in fixed supply $S^s > 0$. The share of each agent in the population is equal to μ_i with $\sum_{i=1}^{M} \mu_i = 1$.

Exogenous processes

Following Adam et al. (2017) we specify in a similar way the exogenous processes for dividend growth and wage-dividend ratio such that to obtain empirical plausible processes for dividends, consumption and consumption to dividend ratio.

1. Dividends: grow at a constant rate a with *iid* growth innovations $ln\varepsilon_t^D$ to be described further below

$$lnD_t = lna + lnD_{t-1} + ln\varepsilon_t^D. \tag{3.4}$$

2. Wage-dividend ratio: follow an AR(1) process with persistence p, mean $1 + WD$ and innovation $ln\varepsilon_t^W$

$$ln\left(1 + \frac{W_t^i}{D_t}\right) = (1-p)ln(1+WD) + p\, ln\left(1 + \frac{W_{t-1}^i}{D_{t-1}}\right) + ln\varepsilon_t^{W,i}. \tag{3.5}$$

where innovations are given by the following exogenous processes

$$\begin{pmatrix} ln\varepsilon_t^D \\ ln\varepsilon_t^{W,i} \end{pmatrix} \sim \mathscr{N}\left(-\frac{1}{2}\begin{pmatrix} \sigma_D^2 \\ \sigma_W^2 \end{pmatrix}, \begin{pmatrix} \sigma_D^2 & \sigma_{DW} \\ \sigma_{DW} & \sigma_W^2 \end{pmatrix}\right), \tag{3.6}$$

$$\begin{pmatrix} ln\varepsilon_t^{W,i} \\ ln\varepsilon_t^{W,-i} \end{pmatrix} \sim \mathcal{N}\left(-\frac{1}{2}\begin{pmatrix}\sigma_W^2 \\ \sigma_W^2\end{pmatrix}, \begin{pmatrix}\sigma_D^2 & \sigma_{WW} \\ \sigma_{WW} & \sigma_W^2\end{pmatrix}\right). \tag{3.7}$$

Agents' Belief System

Agents are endowed with full knowledge of the law of motions for dividends and wages given by equations 3.4 and 3.5. However, we endow agents with imperfect knowledge regarding how stock prices evolve and the exact mapping from dividends and prices.

The empirical evidence presented in the previous section and in Giglio et al. (2021) shows that heterogeneity in beliefs across investors is large and persistent over time. Furthermore, the difference between optimists and pessimists cannot be explained by variables such as wealth, past returns or experience. In light of this evidence, we adopt the view that agents' beliefs differ in terms of the long-term mean of capital gains which we interpret as subjective heterogeneity on the fundamentals of the asset.

Investors from sentiment group i possess the following belief system about stock prices

$$\begin{aligned} lnP_t &= lnP_{t-1} + lnb_t^i + ln\varepsilon_t^{P,i} \\ lnb_t^i &= (1-\rho_i)\bar{\beta}_i + \rho_i lnb_{t-1}^i + ln\nu_t \end{aligned} \tag{3.8}$$

where b_t^i represents the permanent price growth component and $\varepsilon_t^{P,i}$ a transitory innovation. The permanent component, b_t^i, follows an autoregressive process with persistence ρ_i and mean $\bar{\beta}_i$. The latter represents the perceived long-term mean of stock price return of sentiment group i. Innovations $ln\varepsilon_t^P$ and $ln\nu_t$ are jointly normal and uncorrelated. The noisy price component is comprised of two independent components

$$ln\varepsilon_t^{P,i} = ln\varepsilon_t^{P1,i} + ln\varepsilon_t^{P2,i}. \tag{3.9}$$

where $ln\varepsilon_t^{Pj,i} \sim \mathcal{N}\left(\frac{-\sigma^{\epsilon^{Pj}}}{2}, \left(\sigma^{\epsilon^{Pj}}\right)^2\right)$ with $j = 1, 2$. We assume further that only $ln\varepsilon_t^{P1,i}$ is observed at time t.

The permanent price growth component, b_t, is unobserved and is estimated optimally by the internally rational agents using the available information from price signals. Given their belief system from equation 3.8, the optimal posterior distribution of the permanent component of prices is

$$lnb_t^i \sim \mathcal{N}(ln\beta_t^i, \sigma^2) \tag{3.10}$$

where σ^2 is the steady-state variance of the posterior and β_t^i is the conditional mean. The latter is evolving according to the steady-state Kalman filter

$$ln\beta_t^i = (1-\rho_i)(1-g^i)\bar{\beta}_i + \rho_i ln\beta_{t-1}^i + g^i(lnP_{t-1} - lnP_{t-2} - \rho_i ln\beta_{t-1}^i) + g^i ln\varepsilon_t^{P1,i} \tag{3.11}$$

where g^i represents the steady-state Kalman gain. This is the exact law of motion used to fit the dynamics of survey expectations from section 2. The shock $ln\varepsilon_t^{P1,i}$ will be interpreted as a sentiment shock to the beliefs of agents from group i.

Equilibrium

Equilibrium in this economy with internally rational agents and heterogeneous belief systems is defined as follows:

1. Given each sentiment group's belief system (equation 3.8) agents solve optimally problem 3.3
2. Beliefs for each sentiment group i are updated optimally according to equation 3.8
3. Markets clear
    - Goods market: $\sum_{i=1}^{M} \mu_i C_t^i = D_t S^s + \sum_{i=1}^{M} \mu_i W_t^i$
    - Stock market: $\sum_{i=1}^{M} \mu_i S_t^i = S^s$

The market clearing condition for stocks will determine endogenously the price-dividend ratio $\frac{P_t}{D_t}$.

Model Solution Technique: We solve the model using the PEA approach proposed by Belda (2022). The idea is to numerically approximate the stock policy function via a function grounded on economic theory. One of the advantages of this approach is that it allows for a closed-form solution for the equilibrium P/D ratio. It reads as

$$\frac{P_t}{D_t} = \frac{\sum_{i=1}^{M} \mu^i \chi^i \beta_t^i (\frac{W_t^i}{D_t} + S_{t-1}^i)}{\bar{S} - \sum_{i=1}^{M} \mu^i \chi^i \beta_t^i S_{t-1}^i} \tag{3.12}$$

where χ^i is the only parameter of the approximation function[12]. Thus, equilibrium prices depend on the distribution of expectations, wages and stock holdings across agents.

3.4 Quantitative Analysis

In this section, we evaluate the quantitative performance of the model in replicating the stylized facts about the heterogeneity of beliefs and stock market cycles.

We start by calibrating the model parameters. We assume that there are three types of agents in our model, $M = 3$ and set their share μ_i equal to $\frac{1}{3}$. Since our model is an extension of the one from Adam et al. (2017) we approach the calibration of most of the parameters in a similar way except for the parameters concerning the dynamics of the three sentiment groups (ρ^i, g^i and S^i) which are set according to the empirical evidence presented in the previous section. We calibrate the stock supply of stocks, S^s such that to obtain a reasonable average price-dividend ratio while the parameter for the covariance of income shocks, σ_{WW}, implies a correlation of around 0.3 among these shocks. Table 3.4 gathers the calibrated parameters in our model.

[12]In Appendix B, we summarize the algorithm to estimate χ^i.

Parameter	Symbol	Value
Discount factor	δ	0.995
Mean dividend growth	a	1.0048
Dividends growth standard deviation	σ_D	0.0167
Wage-dividends shocks standard deviation	σ_W	0.0167
Covariance (wage-dividend, dividend)	σ_{WD}	0.000351
Covariance wage-dividends agents	σ_{WW}	0.009
Persistence wage-dividend process	p	0.96
Average consumption-dividend ratio	$1{+}WD$	23
Std of transitory component	$\sigma^{\epsilon^{P1}} = \sigma^{\epsilon^{P2}}$	0.04
Risk aversion parameter	γ	2
Stock Supply	S^s	3.3
Expectations persistence	ρ^i	Table 2
Learning speed	g^i	Table 2
Long-run view on asset fundamental value	S^i	Table 2

TABLE 3.4: Benchmark calibration. This table reports the values of the model parameters used for the quantitative analysis.

We introduce the quantitative performance in table 3.5 for three specifications of the model. The first one (column 4) represents our benchmark calibration with heterogeneous income and information shocks, in the second one (column 5) we shut off information shocks ($ln\varepsilon_t^{P1,i} = 0$), while the third specification (column 6) assumes homogeneous wages ($\varepsilon_t^{W,i} = \varepsilon_t^W$). On top of the statistics regarding the heterogeneity of expectations from table 3.3 we present also stylized facts about the trading behavior (panel III) and aggregate stock market behavior (panel IV).

			Model		
Fact	Statistic	US data	Bench.	w/o Sent. Shock	w/o het. W
I. Expectation Heterogenity					
Expectations persistence	$corr(\beta_t, \beta_{t-1})$	0.90	0.91	0.91	0.88
Correlation among sentiment groups	$corr(\beta_t^1, \beta_t^2)$	0.96	0.87	1	0.49
	$corr(\beta_t^1, \beta_t^3)$	0.87	0.86	1	0.46
	$corr(\beta_t^2, \beta_t^3)$	0.95	0.87	1	0.46
Expectations procyclicality	$corr(PD_t, \beta_t)$	0.82	0.66	0.66	0.41
	$corr(PD_t, \beta_t^3)$	0.86	0.66	0.66	0.31
	$corr(PD_t, \beta_t^1)$	0.7	0.66	0.66	0.34
II. Disagreement					
Disagreement driven by beliefs	$corr(DI_t, \beta_t^3)$	0.73	0.94	0.99	0.88
	$corr(DI_t, \beta_t^1)$	0.36	0.63	0.99	0
Perpetual disagreement	$\mathbb{E}(DI_t)$	0.04	0.04	0.04	0.04
	$\sigma(DI_t)$	0.0044	0.0047	0.0037	0.0032
Disagreement procyclicality	$corr(DI_t, PD_t)$	0.72	0.53	0.39	0.17
III. Trading					
Comovement disagreement-trading	$corr(DI_t, TV_t)$	0.41	0.24	0.26	0.36
Trading driven by beliefs	$\hat{\beta}(\lvert\Delta S_t^1\rvert, \lvert\Delta\beta_t^1\rvert)$	0.2*	0.2	0.15	0.04
	$\hat{\beta}(\lvert\Delta S_t^2\rvert, \lvert\Delta\beta_t^2\rvert)$	0.2	0.012	-0.01	0.14
	$\hat{\beta}(\lvert\Delta S_t^3\rvert, \lvert\Delta\beta_t^3\rvert)$	0.2	0.047	0.02	0.25
IV. Stock Prices					
Mean Price-Dividend	$\mathbb{E}(PD_t)$	154.86	173	173	159
Price-Dividend volatility	$\sigma(PD_t)$	64.42	55	55	13
Price-Dividend persistence	$\rho(PD_t, PD_{t-1})$	0.98	0.96	0.96	0.96
Mean returns	$\mathbb{E}(r_t)$	1.89	1.015	1.015	1.01
Returns volatility	$\sigma(r_t)$	7.70	9.2	9.1	3.8

TABLE 3.5: Model quantitative performance. The table reports the statistics from the model together with the US data for period 1973:I-2019:IV for prices and returns and 1998:II-2007:IV for expectations-related and trading statistics. Model implied statistics are obtained via a long simulation with T=10.000 periods; $\hat{\beta}(Y, X)$ denotes the OLS regression coefficient between Y and X; *estimate from Giglio et al. (2021)

The benchmark calibration captures well all of the stylized facts including the heterogeneity of expectations, nature of disagreement, trading behavior and the excess volatility of the stock price cycles. Our model produces highly correlated beliefs among sentiment groups and positive co-movement between expectations and prices. Expectations shocks contribute in bringing down the co-movement between beliefs as can be seen when comparing with the calibration excluding sentiment shocks (column 5). The mean and volatility of disagreement match exactly the ones observed in the data and reproduces the positive correlation with prices. Moreover, similarly to the data, expectations of optimists exhibit a stronger correlation with disagreement compared to the pessimist group. As argued in the next section, the positive co-movement between prices and disagreement is driven largely by optimists becoming more optimistic, increasing trading, prices and disagreement. Panel III shows that disagreement is also positively related to trading and that changes in beliefs do not lead to trading, consistent with the empirical evidence presented in Giglio et al. (2021). Finally, panel IV documents that our model replicates closely aggregate stock market volatility and persistence.

Figure 3.5 plots one simulation arising from the calibrated model. Notice that although different sentiment groups have persistent different beliefs, stock holdings vary across agents and there is not only one group holding the largest/smallest amount of stocks. Instead, agents with the largest/smallest equity holdings alternate among sentiment groups over time.

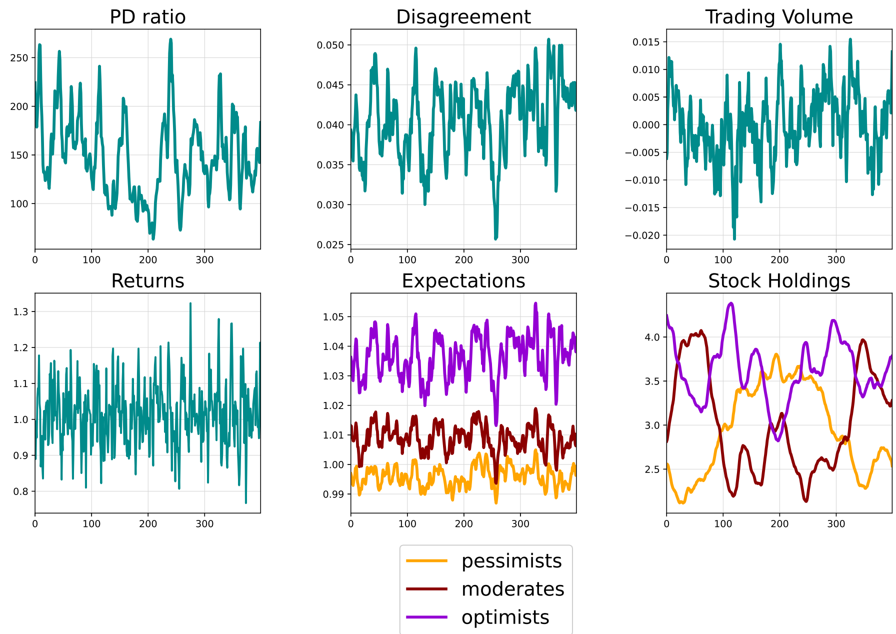

FIGURE 3.5: Simulation of 400 periods based on the benchmark model

3.4.1 Dissecting stock market dynamics

In this section, we highlight the key mechanisms behind to joint evolution of prices, trading and expectations. The cycle starts with an exogenous factor (e.g. particular news (the "expectations shock") or extraordinary incomes (the "wage shock")) that make some investors more willing to invest in the stock market. This generates a rise in prices which turns all investors more optimistic creating amplification over time. Nevertheless, not all investors react equally to the rise in prices due to their different expectation formation process (some are more conservative than others, turning only moderately more optimistic once these higher prices materialize). Thus, the heterogeneous reaction of expectations to prices increases disagreement and trading. Note then, disagreement and trading are, in general, a consequence of a price boom and not the other way around.[13]

Mechanism 1: learning and stock market volatility

From the equilibrium P/D ratio (equation 3.12), it follows

$$\frac{P_{t-1}}{P_{t-2}} = f_1\Big(\{\beta^i_{t-1}, \beta^i_{t-2}\}_{i=1}^{M}, \cdot\Big) \tag{3.13}$$

and from the expectations law of motion (equation 3.11) is clear that

$$\beta^i_t = f_2\Big(\frac{P_{t-1}}{P_{t-2}}, \cdot\Big). \tag{3.14}$$

Other things equal, these two equations constitute a feedback loop that produces endogenous price cycles as a result of self-fulfilling prophecies. An increase in optimism would rise stock demand and prices which would confirm the initial optimistic expectations (or even overcome them, rescaling the process upwards). This feedback loop is a mechanism capable of replicating the high observed volatility of stock prices.[14]

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[13] In the particular case of information shocks, they drive disagreement up and trading up initially. Observe, though, that in any case disagreement is causing the stock price boom.

[14] See Adam et al. (2016) for a detailed analysis. The main difference with respect to that paper is that in the present framework equilibrium prices depends on another endogenous variable, the stock holdings distribution.

Mechanism 2: heterogeneous expectations and disagreement

Based on survey evidence, we introduce idiosyncratic long-run expectations which are characterized by two parameters: the long-run view β^i and its weight on current expectations ρ^i. However, according to survey data, only β^i is significantly different among investors and therefore we focus here on it. Imposing $\rho^i = \rho$ and $g^i = g$ and the same initial conditions $\beta_0^i = \beta_0$, the expectations law of motion can be rewritten as

$$ln\beta_t^i = (1-\rho)(1-g)\beta^i \sum_{j=0}^{t-1} \tilde{\rho}^j + g \sum_{j=0}^{t-1} \tilde{\rho}^j ln \frac{P_{t-j}}{P_{t-1-j}} + \tilde{\rho}^{t-1} ln\beta_0 \tag{3.15}$$

where $\tilde{\rho} = \rho(1 - g\rho)$. It follows

$$ln\beta_t^i - ln\beta_t^m = (\beta^i - \beta^m)(1-\rho)(1-g)\frac{1-\tilde{\rho}^t}{1-\tilde{\rho}} \tag{3.16}$$

where $ln\beta_t^m$ represents the beliefs of agent $m \neq i$. Since $\tilde{\rho} < 0$, $\tilde{\rho}^t$ goes to zero relatively quickly. Thus, disagreement among investors i and m would be almost constant, reflecting their perpetual differences in long-run views up to a scale. Altogether, heterogeneous long run expectations produce perpetual disagreement as the one documented in surveys.

However, the idiosyncratic β^i does not explain the dynamics of disagreement. In particular, in the data, we observe a positive covariance between prices and disagreement. To explain these non-random movements in disagreement we need additional heterogeneity in the expectations formation process. As in the data, consider the case of heterogeneous learning speed g^i. In this case, disagreement between investor i and m can be written as:

$$\begin{aligned} ln\beta_t^i - ln\beta_t^m =& (1-\rho)\Big(\beta^i \frac{(1-\rho^t(1-g^i\rho)^t)(1-g^i)}{1-\rho(1-g^i\rho)} \\ & - \beta^m \frac{(1-\rho^t(1-g^m\rho)^t)(1-g^j)}{1-\rho(1-g^m\rho)}\Big) \\ & + \sum_{j=0}^{t-1} ln\frac{P_{t-j}}{P_{t-1-j}}\rho^j\Big(g^i(1-g^i\rho)^j - g^m(1-g^m\rho)^j\Big) \\ & + ln\beta_0(\rho^{t-1}(1-g^i\rho)^{t-1} - \rho^{t-1}(1-g^m\rho)^{t-1}) \\ \approx & \, c(\beta^i - \beta^m) + \sum_{j=0}^{t-1} ln\frac{P_{t-j}}{P_{t-1-j}}\rho^j\Big(g^i(1-g^i\rho)^j - g^m(1-g^m\rho)^j\Big) \end{aligned} \tag{3.17}$$

where $c(\beta^i - \beta^m)$ is a constant, increasing in the difference of long-run views. Hence, the element determining the sign of the comovement between disagreement and price growth is the parenthesis inside summation from the last line. It turns out that

$$\frac{\partial g^i(1-g^i\rho)^j}{\partial g^i} \begin{cases} > 0 \text{ if } j < 1/g\rho - 1 \\ \leq 0 \text{ otherwise} \end{cases}$$

In other words, for relatively recent periods (low j), the higher the learning speed the larger the disagreement. That would reverse for higher j, but at that point ρ^j becomes very close to zero, almost canceling this effect. Hence,

$$g^i > g^m \Rightarrow ln\beta_t^i - ln\beta_t^m \approx \underset{(+)}{f}\Big(ln\frac{P_{t-j}}{P_{t-1-j}}, \cdot\Big).$$

Returning to the quantitative model and noting that optimistic investors have higher learning speeds than pessimistic investors ($g^3 > g^1$), an exogenous increase in the beliefs of the optimists (β^3) would imply an increase in price and, via the above equation, in disagreement producing a positive co-movement among these variables. The impulse response analysis from figure 3.6 illustrates this mechanism. Notice that an increase in pessimists' expectations increases prices but generates a negative co-movement with disagreement. In Appendix C we report an equivalent shock to disagreement coming from different sources: a positive information shock to optimists and a negative shock to pessimists. In both cases, cases disagreement widens. However, the effects on aggregate prices are the opposite: when optimists become more optimistic, mean expectations and prices go up; when pessimists become more pessimist (driving up disagreement), mean expectations and prices decrease.

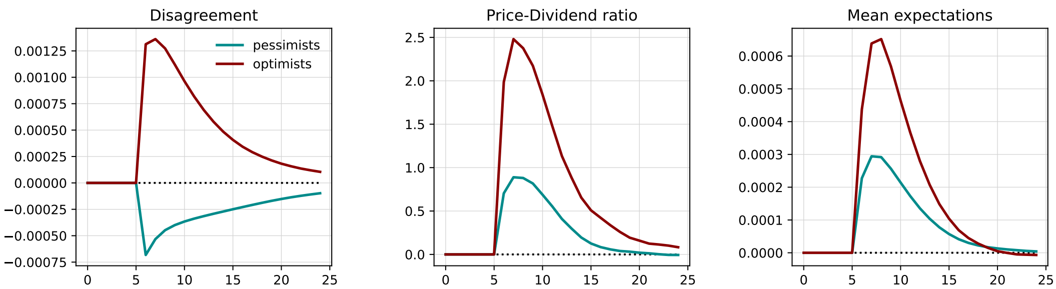

FIGURE 3.6: Responses to a positive information shock. The graph show the GIRFs of different variables to a positive information shock hitting either the optimists or the pessimists.

The effect of heterogeneous ρ is similar to that of heterogeneous g. Altogether, the different heterogeneity layers on the expectations formation process allow to capture salient features of surveys. First, different long-run views β^i gives rise to a perpetual disagreement: optimists investors stay always more optimistic than pessimists. In line with the evidence reported by Giglio et al. (2021), this basic disagreement is modeled as an individual fixed effect, an idiosyncratic parameter unrelated to investors' features. Second, investors extrapolate news at different intensities g^i: some react faster, others are more conservative. This difference relates disagreement to price dynamics; in good times, disagreement will tend to rise, in line with the procyclicality observed in the data.

Mechanism 3: multiple trading motives

Trading is an aggregate property of the model that requires a time-varying heterogeneity among agents.[15] The model includes three idiosyncratic features: wage shocks, information shocks and expectations formation parameters.[16] Thus, agents trade in the stock market to insure against income risk (fundamental motive) or because of their different views about the future evolution of stock prices (speculative motive).

Formally, the change in stock holdings of investor i can be characterized by combining the stock policy function, equilibrium prices and the expectations formation equation

$$\Delta S_t^i = f\Big(\{\beta_{t-1}^i, \varepsilon_t^{P,i}, \varepsilon_t^{W,i}, S_{t-1}^i\}_{i=1}^M\Big). \tag{3.18}$$

It is clear that idiosyncratic fundamental and non-fundamental shocks and endogenously evolving variables for all investors determine the trading of any agent i. Conditional on the wealth levels, the more investors disagree and the more they are hit by different income shocks the higher the volume of trading.

[15] Notice that a constant heterogeneity (for instance, in terms of long run views) would generate inequality (other things equal, the most optimist would hold more stocks) but not trading.

[16] The distribution of stock holdings is time-varying, capturing nothing but the joint dynamics of the three aforementioned variables.

Simulated Impulse Response Functions

To further illustrate the role of heterogeneous long-run views as well as the effects of shocks, we present the responses of the main variables to changes in these variables. Figure 3.7 shows that a permanent increase in the optimist's long-run expectations implies a permanent rise in their expectations level. Following this burst of optimism, prices (and mean expectations) go up and, via learning, that optimism spills over the expectations of the other groups, reinforcing their effect on prices. However, the effect across groups is unequal: the impact on optimists' expectations is much larger and their propensity to invest out of wealth increases at a faster rate compared to the ones of the other two groups. This explains also why stock holdings of pessimists and moderates decrease although their return expectations increase: since prices go up (driven by optimists' expectations), their wealth increases sufficiently rapidly to counterbalance the desire to accumulate more equity. Optimists, on the other hand, experience a rapid increase in expectations (driving up disagreement) and acquire more stocks reducing their consumption along the way. Hence, trading increases to accommodate the stock holdings in line with the expectations distribution. Finally, the rise in prices gives rise to a temporary spike in returns due to capital gains.

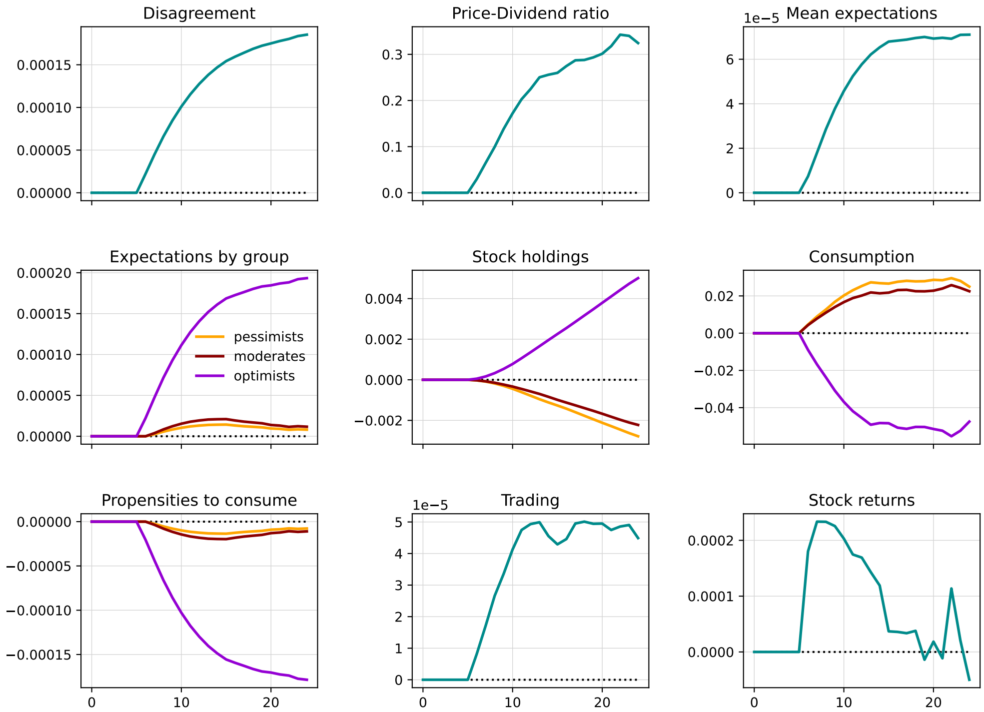

FIGURE 3.7: Responses to a long run optimism shock to optimists. The graph show the GIRFs of different variables to a permanent increase in β^3 in period 5. Periods are quarters. IRFs are computed following these steps: i) simulate the model T =10.000 periods across U=100 different shock realizations; ii) introduce a shock to the variable/parameter in a particular period p and compute new TxU series; iii) repeat ii) at different P=5 points, to tackle possible nonlinearities; iv) compute the differences between shocked and unshocked series at each P and U; v) average the differences across points and realizations.

A transitory shock to optimists' wages represents an inflow that allows them to raise simultaneously consumption and stock holdings. This additional stock demand by optimists is met by a reduction in stock holdings of the other groups (for similar reasons explained in the context of the previous shock) together with a rise in their consumption. Prices increase and via learning, mean expectations also adjust upward. Given the different extrapolative speeds, the adjustment in expectations is heterogeneous, with optimists becoming relatively more optimistic. That raises disagreement giving rise to additional trading. Thus, a positive fundamental shock to one agent increases prices and trading; as result, expectations also react, which sustains the effect over time. Appendix C shows the IRFs of an aggregate wage shock. The dynamics are very similar except that initially that shock raises the stock market participation of pessimists and moderates who end up increasing their stock holdings by trading with optimists.

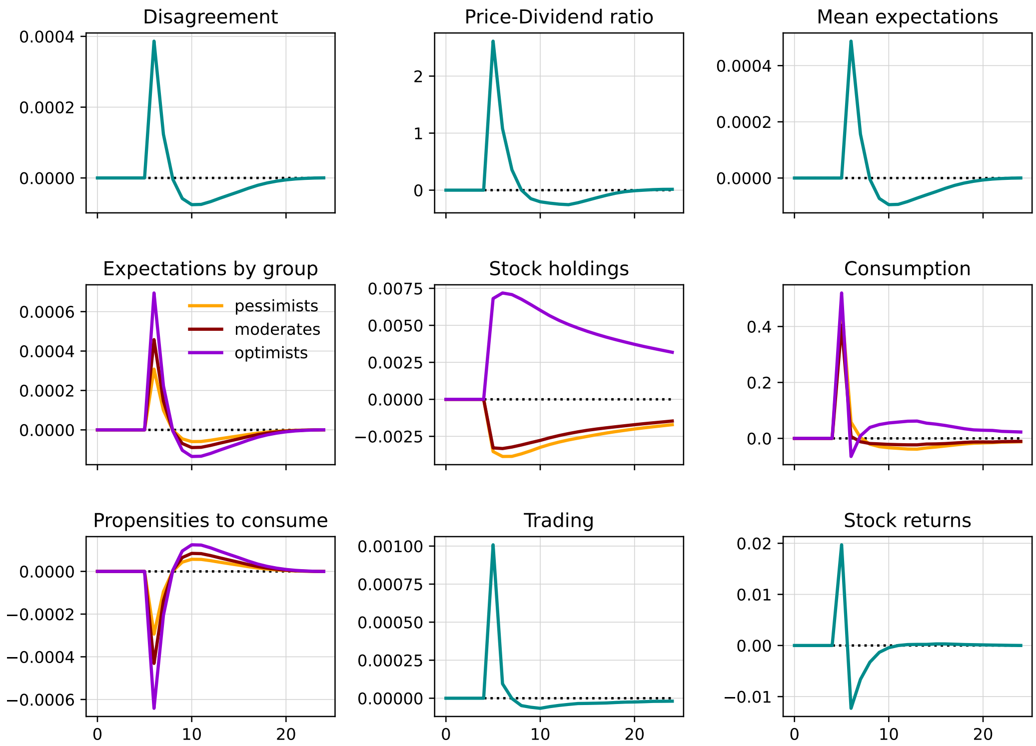

FIGURE 3.8: Responses to a shock to optimists wages $ln\varepsilon_t^{w,3}$. The graph shows the GIRFs of different variables to a wage shock to group 3. Periods are quarters. IRFs are computed as in figure 3.7

.

On the other hand, figure 3.9 pictures the responses to an information shock (i.e., a non-fundamental shock) to optimists. Similarly to the income shock, optimists desire to hold more equities. Nonetheless, they have to give up on some consumption. The reason is that the spike in optimists' stock demand is simply due to their higher optimism. This higher stock demand raises prices and, given the beliefs of the other agents, generates higher disagreement and trading. In other words, it is only the fact that optimists become *relatively* more optimistic that triggers trading.[17] Higher prices make the other agents adjust upwards their beliefs. However, since these other agents are less extrapolative that does not close disagreement but widens it, amplifying the effects over time. Altogether, the responses to a non-fundamental shock turn out to be similar in terms of signs (except for consumption) but remarkably more persistent than the responses to a wage shock.

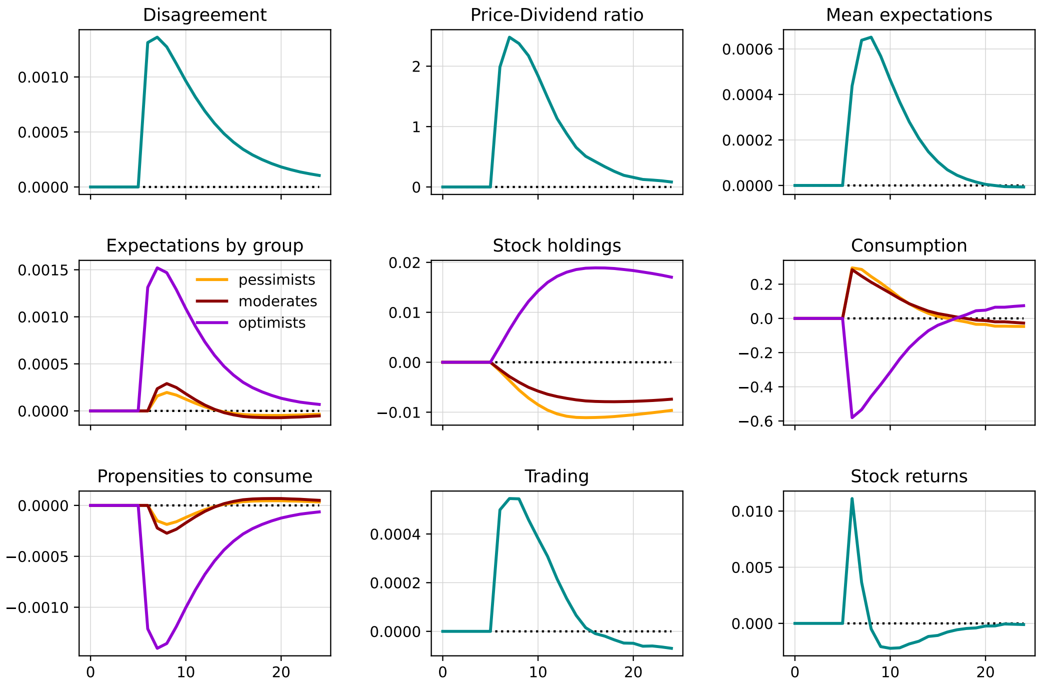

FIGURE 3.9: Responses to an information shock to optimists $ln\varepsilon_t^{P,3}$. The graph show the GIRFs of different variables to an information shock to group 3. Periods are quarters. IRFs are computed as in figure 3.7

3.5 Conclusions

In this paper, we present a quantitative model that jointly replicates the empirical dynamics of stock prices, trading and the heterogeneity of expectations. We place our emphasis on a realistic way of modeling expectations, that allows

[17] In other words, an aggregate optimism shock that does not change disagreement would have changed prices without triggering any trading.

for different layers of heterogeneity. In particular, we point out the role of heterogeneous long-run expectations and the different learning speeds at which agents adapt their expectations to new information. This way of modeling expectations captures salient features of recent survey evidence such as high and permanent disagreement and the procyclical nature of both individual expectations and disagreement which we first document using available survey data on expected returns.

We show that an otherwise simple asset pricing model with internally rational agents and heterogeneous beliefs delivers a remarkable quantitative performance across a wide variety of stylized facts regarding the joint dynamics of prices, heterogeneous expectations and trading patterns. The good quantitative performance legitimates the use of the model to shed some additional light on the mechanics of stock market booms. In particular, we point out that disagreement and trading always emerges as a result of a rise in prices and that the positive co-movement between disagreement and the PD ratio is driven mostly by optimistic investors.

Finally, we point that, although the model replicates the joint movement of expectations and trading, it is completely unable to generate an amount of trading comparable to that of the real world. We conjecture that it is related to the type of agents we are modeling (retail investors), characterized by infrequent trading and which account for a rather small fraction of total trading volume. Thus, important improvements can be obtained by taking into account a different population of agents ("institutional investors"), which engage in more quantitatively significant trading operations.

Appendix A

Appendix: Chapter 1

A.1 Consistency of one-step ahead forecasts

Equation (4) from section III holds with equality from the perspective of the agent only under the Rational Expectation assumption. Under imperfect information agent will not have knowledge of the fact that he will be the marginal agent forever and therefore cannot substitute with equality the FOC 2 in the budget constraint to obtain equation 4. Letting, λ_t denote the lagrange multiplier associated with FOC with respect to stock prices and assuming the agent knows that he will be the marginal agent in the bond market (equation (1) holding with equality) the intertemporal budget constraint becomes

$$\frac{\mathscr{W}_t^i}{P_t} = E_t^{\mathscr{P}_i} \sum_{j=0}^{\infty} \delta^j \Big(\frac{C_{t+j}^i}{C_t^i}\Big)^{-\sigma} C_{t+j}^i + A_t^i. \tag{A.1}$$

where

$$A_t^i = \sum_{j=1}^{\infty} \delta^j E_t^{\mathscr{P}} E_{t+1}^{\mathscr{P}} \dots E_{t+j-1}^{\mathscr{P}} \Big(\frac{C_{t+j}^i}{C_t^i}\Big)^{-\sigma} \frac{\lambda_{t+j}^i}{\prod_{s=0}^{j}(1+\pi_{t+s})} \tag{A.2}$$

is the term collecting all the Lagrange multipliers λ which take into account that the agent does not know that he will be marginal in all future periods. In steady state $\lambda = 0$ and $A = 0$. Specifically,

$$\lambda_{t+j}^i = \delta \left[E_t^{\mathscr{P}_{mg}} \left(\Big(\tfrac{C_{t+j}^{mg}}{C_{t+j-1}^{mg}}\Big)^{-\sigma} (P_{t+j} + D_{t+j}) \right) - E_t^{\mathscr{P}_i} \left(\Big(\tfrac{C_{t+j}^{i}}{C_{t+j-1}^{i}}\Big)^{-\sigma} (P_{t+j} + D_{t+j}) \right) \right] S_{t+j}^i$$

is the perceived error of agent i with respect to the marginal agent valuation. If

A_t^i is sufficiently small up to a first order approximation then we can describe accurately the optimal consumption decision of the agent by equation 1.15 as if the agent knows he is the marginal agent. I call this the *Average Marginal Agent* assumption.

Average Marginal Agent (AMA) Assumption: *up to a first order approximation* $A_t^i \approx 0$

Notice that the AMA assumption is in line with the equilibrium actual law of motion since all agents who have access to the equity market are identical. Moreover, knowledge of this assumption is not enough, from the perspective of the individual agent, to recover the exact mapping from dividends to prices. As a result, in this environment the agent cannot apply the Law of Iterated Expectations when forming beliefs and therefore the linearized FOC with respect to stock prices is of one-step ahead form

$$\tilde{q}_t = (1-\delta)E_t^{\mathscr{P}}(\tilde{d}_{t+1}) + \delta E_t^{\mathscr{P}}(\tilde{q}_{t+1}) + \sigma(\tilde{C}_t^i - E_t^{\mathscr{P}}\tilde{C}_{t+1}^i). \tag{A.3}$$

.

This result is a mix between the *long-horizon learning* approach of Preston (2005) and the *Euler Equation* approach. Under the Average Marginal Agent assumption the optimal consumption decision under long-horizon learning given by equation 1.15 is consistent under Internal Rationality with the one step ahead pricing equation A.3.

A.2 Model Derivation Details

Demand Side

Replacing Q_t in the budget constraint with equation (1.23) and rearranging, I obtain

$$\mathscr{W}_t^i = (P_tC_t^i - W_tN_t^i) + B_t^i + \delta S_t^i E_t^{\mathscr{P}}\left\{\frac{A_{t+1}}{A_t}\left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\sigma}\frac{(Q_t+D_t)}{1+\pi_{t+1}}\right\} \tag{A.4}$$

where $\mathscr{W}_t^i = B_{t-1}^i(1+i_{t-1}) + S_{t-1}^i(Q_t+D_t)$ represents wealth at time t. Adding and subtracting $\delta E_t^{\mathscr{P}}\left\{\frac{A_{t+1}}{A_t}\left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\sigma}\frac{(B_t^i(1+i_t))}{1+\pi_{t+1}}\right\}$ from the RHS, equation (A.4) becomes

$$\begin{aligned}\mathscr{W}_t^i &= (P_t C_t^i - W_t N_t^i) + \delta E_t^{\mathscr{P}}\Big\{\frac{A_{t+1}}{A_t}\Big(\frac{C_{t+1}^i}{C_t^i}\Big)^{-\sigma}\frac{\mathscr{W}_{t+1}^i}{1+\pi_{t+1}}\Big\} \\ &+ B_t^i\Big(1 - \delta E_t^{\mathscr{P}}\Big\{\frac{A_{t+1}}{A_t}\Big(\frac{C_{t+1}^i}{C_t^i}\Big)^{-\sigma}\frac{1+i_t}{1+\pi_{t+1}}\Big\}\Big) \\ &= (P_t C_t^i - W_t N_t^i) + \delta E_t^{\mathscr{P}}\Big\{\frac{A_{t+1}}{A_t}\Big(\frac{C_{t+1}^i}{C_t^i}\Big)^{-\sigma}\frac{\mathscr{W}_{t+1}^i}{1+\pi_{t+1}}\Big\}\end{aligned} \tag{A.5}$$

where the second equality follows from the Euler equation of the household. Substituting forward for $\mathscr{W}_{t+1}$ I obtain

$$\mathscr{W}_t^i = E_t^{\mathscr{P}}\sum_{j=0}^{\infty}\delta^j\frac{A_{t+j}}{A_t}\Big(\frac{C_{t+j}^i}{C_t^i}\Big)^{-\sigma}\frac{(P_{t+j}C_{t+j}^i - W_{t+j}N_{t+j}^i)}{\prod_{s=0}^{j}(1+\pi_{t+s})} \tag{A.6}$$

where I have imposed the following transversality condition

$$\lim_{j\to\infty} E_t^{\mathscr{P}}\frac{A_{t+j}}{A_t}\Big(\frac{C_{t+j}^i}{C_t^i}\Big)^{-\sigma}\frac{\mathscr{W}_{t+j}^i}{\prod_{s=0}^{j}(1+\pi_{t+s})} = 0. \tag{A.7}$$

Dividing equation (A.6) by P_t leads to the following expression for the real wealth

$$\frac{\mathscr{W}_t^i}{P_t} = E_t^{\mathscr{P}}\sum_{j=0}^{\infty}\delta^j\frac{A_{t+j}}{A_t}\Big(\frac{C_{t+j}^i}{C_t^i}\Big)^{-\sigma}\big[C_{t+j}^i - w_{t+j}^{\frac{1+\phi}{\phi}}(C_{t+j}^i)^{\frac{-\sigma}{\phi}}\big]. \tag{A.8}$$

The steady state (SS) of the model corresponds to the RE SS and is given by

$$\begin{aligned}Y &= \Big((1-\alpha)\frac{\epsilon-1}{\epsilon}\Big)^{\frac{1-\alpha}{\sigma(1-\alpha)+\alpha+\phi}} \\ w &= Y^{\sigma+\frac{\phi}{1-\alpha}} \\ d &= Y - Y^{\sigma+\frac{1+\phi}{1-\alpha}} \\ q &= \frac{\delta}{1-\delta}d.\end{aligned} \tag{A.9}$$

At the SS equation A.8 becomes

$$\begin{aligned}q + d &= \sum_{j=0}^{\infty}\delta^j(Y - w^{\frac{1+\phi}{\phi}}Y^{\frac{-\sigma}{\phi}}) \\ \frac{q}{\delta} &= \frac{Y^{\sigma+\frac{1+\phi}{1-\alpha}}}{1-\delta}.\end{aligned} \tag{A.10}$$

Applying a first order Taylor approximation to the IBC around a non-stochastic steady state yields

$$\begin{aligned}\tilde{w}_t^{\,i} =&\frac{\delta}{q}E_t^{\mathscr{P}}\Big\{\sum_{j=0}^{\infty}\delta^j\Big[-(Y-Y^{\sigma+\frac{1+\phi}{1-\alpha}})r^N_{t+j}-\sigma(Y-Y^{\sigma+\frac{1+\phi}{1-\alpha}})\,(\tilde{c}^i_{t+j}-\tilde{c}_t^{\,i})\\ &+(Y+\frac{\sigma}{\phi}Y^{\sigma+\frac{1+\phi}{1-\alpha}})\,\tilde{c}^i_{t+j}-\frac{1+\phi}{\phi}Y^{\sigma+\frac{1+\phi}{1-\alpha}}\tilde{w}_{t+j}\Big]\Big\}\\ =&(1-\delta)E_t^{\mathscr{P}}\Big\{\sum_{j=0}^{\infty}\delta^j\Big[-r^N_{t+j}+\sigma\tilde{c}^i_t+(\frac{Y+\frac{\sigma}{\phi}Y^{\sigma+\frac{1+\phi}{1-\alpha}}}{Y-Y^{\sigma+\frac{1+\phi}{1-\alpha}}}-\sigma)\,\tilde{c}^i_{t+j}\\ &-\frac{1+\phi}{\phi}\frac{Y^{\sigma+\frac{1+\phi}{1-\alpha}}}{Y-Y^{\sigma+\frac{1+\phi}{1-\alpha}}}\tilde{w}_{t+j}\Big]\Big\}.\end{aligned} \tag{A.11}$$

where $r^N_{t+j}=a_t-a_{t+j}$. Moving from the first line to the third made use of (A.9). Log-linearization of the Euler equation (1.22) yields

$$\tilde{c}^i_t=E_t^{\mathscr{P}}\tilde{c}^i_{t+1}-\frac{1}{\sigma}(i_t-E_t^{\mathscr{P}}\pi_{t+1}-r^N_{t+1}) \tag{A.12}$$

which can be rewritten as

$$E_t^{\mathscr{P}}(\tilde{c}^i_{t+k})=\tilde{c}^i_t+\frac{1}{\sigma}E_t^{\mathscr{P}}\big[\sum_{j=0}^{k-1}i_{t+j}-\pi_{t+j+1}-r^N_{t+j}\big]. \tag{A.13}$$

Substituting equation A.13 in A.11, rearranging and using the fact that

$$\sum_{j=0}^{\infty}\delta^j\sum_{k=0}^{j-1}R_t=\frac{\delta}{1-\delta}\sum_{j=0}^{\infty}\delta^jR_t$$

for any variable R_t, yields

$$\tilde{c}^i_t=\Delta_i\tilde{w}^i_t+\Delta_w\sum_{j=0}^{\infty}\delta^jE_t^{\mathscr{P}}(\tilde{w}_{t+j})-\frac{\delta}{\sigma}\Delta_r\sum_{j=0}^{\infty}\delta^jE_t^{\mathscr{P}}(i_{t+j}-\pi_{t+j+1}-\Gamma^rr^N_{t+j}). \tag{A.14}$$

where

$$\Delta_i = \frac{Y - Y^{\sigma+\frac{1+\phi}{1-\alpha}}}{Y + \frac{\sigma}{\phi}Y^{\sigma+\frac{1+\phi}{1-\alpha}}}$$
$$\Delta_w = (1-\delta)\frac{1+\phi}{\phi}\frac{Y^{\sigma+\frac{1+\phi}{1-\alpha}}}{Y + \frac{\sigma}{\phi}Y^{\sigma+\frac{1+\phi}{1-\alpha}}},$$
$$\Delta_r = \frac{(1-\sigma)Y + \sigma\frac{1+\phi}{\phi}Y^{\sigma+\frac{1+\phi}{1-\alpha}}}{Y + \frac{\sigma}{\phi}Y^{\sigma+\frac{1+\phi}{1-\alpha}}},$$
$$\Gamma^r = 1 + \frac{1-\delta}{\delta}\sigma\frac{Y - Y^{\sigma+\frac{1+\phi}{1-\alpha}}}{(1-\sigma)Y + \sigma\frac{1+\phi}{\phi}Y^{\sigma+\frac{1+\phi}{1-\alpha}}}. \tag{A.15}$$

Evaluating expectations in equation A.14 using the PLM of agents and applying the equilibrium conditions results in the demand side of the model

$$\tilde{y}_t - \Delta_i\delta\tilde{q}_t - \Delta_i(1-\delta)\tilde{d}_t - \Delta_w\tilde{w}_t + \frac{\delta}{\sigma}\Delta_r i_t = \frac{\delta}{1-\rho\delta}\Delta_w\hat{\beta}^w_{t-1} - \frac{\delta^2}{\sigma(1-\delta\rho)}\Delta_r\phi_y\hat{\beta}^y_{t-1}$$
$$- \frac{\delta^2}{\sigma(1-\delta\rho)}\Delta_r\phi_q\hat{\beta}^q_{t-1} - \frac{\delta^2}{\sigma(1-\delta\rho)}\Delta_r(\phi_\pi - \frac{1}{\delta})\hat{\beta}^\pi_{t-1}$$
$$- \frac{\delta}{\sigma}\frac{\delta\rho_i}{1-\delta\rho_i}\delta_r\epsilon^i_t + \frac{\delta}{\sigma}\Delta_r\Gamma_r(1-\rho_a)\frac{\delta\rho_a}{1-\delta\rho_a}a_t. \tag{A.16}$$

Supply Side

The solution to the profit maximization problem yields the optimal price setting decision of the firm

$$P^*_t = \frac{\epsilon}{\epsilon-1}\frac{\sum_{k=0}^{\infty}(\theta\delta)^k E^{\mathscr{P}}_t\left[\frac{A_{t+k}}{A_t}Y^{1-\sigma}_{t+k}P^{\epsilon}_{t+k}MC_{t+k/k}\right]}{\sum_{k=0}^{\infty}(\theta\delta)^k E^{\mathscr{P}}_t\left[\frac{A_{t+k}}{A_t}Y^{1-\sigma}_{t+k}P^{\epsilon-1}_{t+k}\right]} \tag{A.17}$$

where

$$MC_{t+k/k} = \frac{1}{1-\alpha}\frac{W_{t+k}}{P_{t+k}}\left(\frac{P^*_t}{P_{t+k}}\right)^{\frac{-\epsilon\alpha}{1-\alpha}}Y^{\frac{\alpha}{1-\alpha}}_{t+k}e^{\epsilon^u_{t+k}}. \tag{A.18}$$

In the above equation ϵ^u_{t+k} is a shock to the the marginal costs of the firm and will be interpreted as a cost-push shock.

Log-linearization around a 0 inflation steady state and noting that at SS $\frac{\epsilon-1}{\epsilon} = \frac{1}{1-\alpha}Y^{\sigma+\frac{\phi+\alpha}{1-\alpha}}$ yields the pricing decision rule of the firms

$$p_t^* = (1-\delta\theta)\sum_{k=0}^{\infty}(\theta\delta)^k E_t^{\mathscr{P}}\Big\{\frac{\alpha}{1-\alpha+\epsilon\alpha}\tilde{y}_{t+k} + \frac{1-\alpha}{1-\alpha+\epsilon\alpha}(\tilde{w}_{t+k}+\epsilon^u_{t+k}) + p_{t+k}\Big\}. \tag{A.19}$$

Substracting p_{t-1} from both sides and taking into account that in equilibrium $\pi_t = (1-\theta)(p_t^* - p_{t-1})$ results in the equation for inflation

$$\begin{aligned}\pi_t =& \frac{(1-\theta\delta)(1-\theta)}{\theta}\frac{\alpha}{1-\alpha+\epsilon\alpha}\sum_{k=0}^{\infty}(\theta\delta)^k E_t^{\mathscr{P}}\tilde{y}_{t+k}\\ &+\frac{(1-\theta\delta)(1-\theta)}{\theta}\frac{1-\alpha}{1-\alpha+\epsilon\alpha}\sum_{k=0}^{\infty}(\theta\delta)^k E_t^{\mathscr{P}}\tilde{w}_{t+k}\\ &+\sum_{k=0}^{\infty}(\theta\delta)^k E_t^{\mathscr{P}}u_{t+k} + (1-\theta)\delta\sum_{k=0}^{\infty}(\theta\delta)^k E_t^{\mathscr{P}}\pi_{t+k+1}\end{aligned} \tag{A.20}$$

where $u_{t+k} = \frac{(1-\theta\delta)(1-\theta)}{\theta}\frac{1-\alpha}{1-\alpha+\epsilon}\epsilon^u_{t+k}$ is an exogeneous AR(1) process with persistence ρ_u and zero mean.

Evaluating the expectations using agents PLM results in the demand block of the model, the Phillips curve

$$\pi_t - \Theta_y\tilde{y}_t - \Theta_w\tilde{w}_t = \Theta_{\beta^y}\hat{\beta}^y_{t-1} + \Theta_{\beta^w}\hat{\beta}^w_{t-1} + \Theta_{\beta^\pi}\hat{\beta}^\pi_{t-1} + \Theta_u u_t \tag{A.21}$$

where

$$\begin{aligned}\Theta_y =& \frac{(1-\theta\delta)(1-\theta)}{\theta}\frac{\alpha}{1-\alpha+\epsilon\alpha}\\ \Theta_w =& \frac{(1-\theta\delta)(\theta)}{1-\theta}\frac{1-\alpha}{1-\alpha+\epsilon\alpha}\\ \Theta_{\beta^y} =& \frac{(1-\theta\delta)(1-\theta)}{\theta}\frac{\alpha}{1-\alpha+\epsilon\alpha}\frac{\theta\delta}{1-\theta\delta\rho}\\ \Theta_{\beta^w} =& \frac{(1-\theta\delta)(1-\theta)}{\theta}\frac{1-\alpha}{1-\alpha+\epsilon\alpha}\frac{\theta\delta}{1-\theta\delta\rho}\\ \Theta_{\beta^\pi} =& \frac{(1-\theta)\delta}{1-\theta\delta\rho}\\ \Theta_u =& \frac{(1-\theta\delta)(1-\theta)}{\theta}\frac{1-\alpha}{1-\alpha+\epsilon\alpha}\frac{1}{1-\theta\delta\rho_u}\end{aligned} \tag{A.22}$$

Asset Prices

Log-linearization of the FOC wrt to stock holding yields the asset pricing equation

$$\tilde{q}_t = (1-\delta)\hat{\beta}^d_{t-1} + \delta\hat{\beta}^q_{t-1} - (i_t - \hat{\beta}^\pi_{t-1}) \tag{A.23}$$

where ϵ^q_t is a stochastic process with persistence ρ_q and can be interpreted as a equity market fad.

Equilibrium

Labor is demand determined and is obtained by log-linearization of the production function

$$\tilde{n}_t = \frac{\tilde{y}_t}{1-\alpha}. \tag{A.24}$$

Wages come from the FOC wrt to labor from the households problem which after loglinearization becomes

$$\tilde{w}_t = \phi\tilde{n}_t + \sigma\tilde{y}_t \tag{A.25}$$

Dividends are given are given by the profits of the firms

$$D_t = Y_t - W_t N_t \tag{A.26}$$

which after log-linearization becomes:

$$\tilde{d}_t = \frac{Y}{d}\tilde{y}_t - \frac{WN}{d}(\tilde{n}_t + \tilde{w}_t). \tag{A.27}$$

using the expressions for labor and wages, the above equation can be rewritten only as a function of $\tilde{y}_t$

$$\tilde{d}_t = \psi_d\,\tilde{y}_t. \tag{A.28}$$

where $\phi_d = \frac{Y}{d} - \frac{WN}{d}(\sigma + \frac{1+\phi}{1-\alpha})$

Belief System

Let $z_t = (\tilde{y}_t, \tilde{\pi}_t, \tilde{q}_t, \tilde{d}_t, \tilde{w}_t)'$. Agents think that z_t follows an unobserved component model

$$\begin{aligned} z_t &= \beta_t + \zeta_t \\ \beta_t &= \rho\beta_{t-1} + \vartheta_t \end{aligned} \tag{A.29}$$

where β_t is the permanent component. Agents have perfect knowledge about interest rates and about the shock process. Agents form expectations at time t using information up to $t-1$. Denoting these time t expectations by β_{t-1}, agents update their beliefs following the recursion

$$\hat{\beta}_t = \rho\,\hat{\beta}_{t-1} + \lambda(z_t - \hat{\beta}_{t-1}). \tag{A.30}$$

Given that agents forecast $E_t^{\mathscr{P}} z_{t+k} = \rho^{k-1}\hat{\beta}_t$ we can evaluate the subjective expectations necessary to compute the Actual Law of Motion (ALM) as

$$
\begin{aligned}
\sum_{j=1}^{\infty}\delta^j E_t^{\mathscr{P}}(\tilde{w}_{t+j}) &= \sum_{j=1}^{\infty}\delta^j \rho^{j-1}\hat{\beta}_{t-1}^{w} = \frac{\delta}{1-\rho\delta}\hat{\beta}_{t-1}^{w}, \\
\sum_{j=0}^{\infty}\delta^j E_t^{\mathscr{P}}(i_{t+j}) &= \sum_{j=0}^{\infty}\delta^j E_t^{\mathscr{P}}(\phi_{pi}\pi_{t+j} + \phi_y\tilde{y}_{t+j} + \phi_q\tilde{q}_{t+j} + \epsilon_{t+j}^{i} \\
&= i_t + \frac{\delta}{1-\delta\rho}(\phi_{pi}\hat{\beta}_{t-1}^{\pi} + \phi_y\hat{\beta}_{t-1}^{y} + \phi_q\hat{\beta}_{t-1}^{q}) + \frac{\delta\rho_i}{1-\delta\rho_i}\epsilon_t^{i} \\
\sum_{j=0}^{\infty}\delta^j E_t^{\mathscr{P}}(\pi_{t+j+1}) &= \frac{1}{1-\delta\rho}\hat{\beta}_{t-1}^{\pi} \\
\sum_{j=0}^{\infty}\delta^j E_t^{\mathscr{P}}(r_{t+j}^{N}) &= \sum_{j=0}^{\infty}\delta^j E_t^{\mathscr{P}}(a_t - a_{t+j}) \\
&= \sum_{j=0}^{\infty}\delta^j E_t^{\mathscr{P}}((1-\rho_a)\rho_a^{j-1}a_t) = \frac{(1-\rho_a)\delta}{1-\delta\rho_a}a_t \\
\sum_{j=0}^{\infty}(\delta\theta)^j E_t^{\mathscr{P}}(\tilde{y}_{t+j}) &= \tilde{y}_t + \frac{\theta\delta}{1-\theta\delta\rho}\hat{\beta}_{t-1}^{y} \\
\sum_{j=0}^{\infty}(\delta\theta)^j E_t^{\mathscr{P}}(\tilde{w}_{t+j}) &= \tilde{w}_t + \frac{\theta\delta}{1-\theta\delta\rho}\hat{\beta}_{t-1}^{w} \\
\sum_{j=0}^{\infty}(\delta\theta)^j E_t^{\mathscr{P}}(\tilde{u}_{t+j}) &= \frac{\theta\delta}{1-\theta\delta\rho}\tilde{u}_t \\
\sum_{j=0}^{\infty}\delta\theta)^j E_t^{\mathscr{P}}(\pi_{t+j+1}) &= \frac{1}{1-\delta\rho\theta}\hat{\beta}_{t-1}^{\pi}
\end{aligned}
\tag{A.31}
$$

System in State-Space form

The system of equations determining $\tilde{y}_t, \pi_t, \tilde{q}_t, i_t, d_t$ and $\tilde{w}_t$ can be written in a compact form

$$A\,Z_t = B\,\hat{\beta}_{t-1}^{Z} + C\,\epsilon_t \tag{A.32}$$

where

$$Z_t = (\tilde{y}_t, \pi_t, \tilde{q}_t, i_t, d_t, \tilde{w}_t)',$$

$$\hat{\beta}^Z_{t-1} = (\hat{\beta}^y_{t-1}, \hat{\beta}^\pi_{t-1}, \hat{\beta}^q_{t-1}, \hat{\beta}^i_{t-1}, \hat{\beta}^d_{t-1}, \hat{\beta}^w_{t-1})',$$

$$\epsilon_t = (\tilde{a}_t, \tilde{u}_t, \epsilon^q_t, \epsilon^i_t)',$$

$$A = \begin{pmatrix} 1 & 0 & -\Delta_i\delta & \frac{\Delta_r\delta}{\sigma} & -\Delta_i(1-\delta) & -\Delta_w \\ -\Theta_y & 1 & 0 & 0 & 0 & -\Theta_w \\ 0 & 0 & 1 & 1 & 0 & 0 \\ -\phi_y & -\phi_\pi & -\phi_q & 1 & 0 & 0 \\ \psi_d & 0 & 0 & 0 & 1 & 0 \\ -(\sigma + \frac{\phi}{1-\alpha}) & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$B = \begin{pmatrix} -\frac{\delta^2\Delta_r\phi_y}{\sigma(1-\delta\rho)} & -\frac{\delta^2\Delta_r}{\sigma(1-\delta\rho)}(\phi_\pi - \frac{1}{\delta}) & -\frac{\delta^2\Delta_r\phi_q}{\sigma(1-\delta\rho)} & 0 & 0 & \frac{\Delta_w\delta}{1-\rho\delta} \\ \Theta_{\beta^y} & \Theta_{\beta^\pi} & 0 & 0 & 0 & \Theta_{\beta^w} \\ 0 & 1 & \delta & 0 & 1-\delta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$C = \begin{pmatrix} \frac{\delta\Gamma^R\Delta_r}{\sigma}(1-\rho_a)\frac{\delta}{1-\delta\rho_a} & 0 & 0 & -\frac{\delta^2\Delta_r}{\sigma(1-\delta\rho_i)} \\ 0 & \Theta_u & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

A.2.1 Lagged response to stock prices

The interest rule is

$$i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_q \tilde{q}_{t-1} + \epsilon^i_t. \tag{A.33}$$

Given this response of monetary policy the forecast of interest rates is given by

$$\begin{aligned}\sum_{j=0}^{\infty} \delta^j E^{\mathscr{P}}_t(i_{t+j}) =& \sum_{j=0}^{\infty} \delta^j E^{\mathscr{P}}_t(\phi_\pi \pi_{t+j} + \phi_y \tilde{y}_{t+j} + \phi_q \tilde{q}_{t+j-1}) + \epsilon^i_{t+j} \\ =& i_t + \frac{\delta}{1-\delta\rho}(\phi_\pi \hat{\beta}^\pi_{t-1} + \phi_y \hat{\beta}^y_{t-1}) + \delta\phi_q \tilde{q}_t + \frac{\delta^2\phi_q}{1-\delta\rho}\hat{\beta}^q_{t-1} + \frac{\delta\rho_i}{1-\delta\rho_i}\epsilon^i_t\end{aligned} \tag{A.34}$$

The IS equation becomes

$$\begin{aligned}&\tilde{y}_t - (\Delta_i\delta - \frac{\delta^2\phi_q}{\sigma}\Delta_r)\tilde{q}_t - \Delta_i(1-\delta)\tilde{d}_t - \Delta_w \tilde{w}_t + \frac{\delta}{\sigma}\Delta_r i_t = \frac{\delta}{1-\rho\delta}\Delta_w \hat{\beta}^w_{t-1} - \\ &\frac{\delta^2}{\sigma(1-\delta\rho)}\Delta_r \phi_y \hat{\beta}^y_{t-1} - \frac{\delta^3}{\sigma(1-\delta\rho)}\Delta_r \phi_q \hat{\beta}^q_{t-1} - \frac{\delta^2}{\sigma(1-\delta\rho)}\Delta_r(\phi_\pi - \frac{1}{\delta})\hat{\beta}^\pi_{t-1} \\ &- \frac{\delta}{\sigma}\frac{\delta\rho_i}{1-\delta\rho_i}\Delta_r \epsilon^i_t.\end{aligned} \tag{A.35}$$

The system of equations determining $\tilde{y}_t, \pi_t, \tilde{q}_t, i_t, d_t$ and $\tilde{w}_t$ can be written in a compact form

$$A\,Z_t = B\,\hat{\beta}^Z_{t-1} + D\,Z_{t-1} + C\,\epsilon_t \tag{A.36}$$

where

$$Z_t = (\tilde{y}_t, \pi_t, \tilde{q}_t, i_t, d_t, \tilde{w}_t)',$$

$$\hat{\beta}^Z_{t-1} = (\hat{\beta}^y_{t-1}, \hat{\beta}^\pi_{t-1}, \hat{\beta}^q_{t-1}, \hat{\beta}^i_{t-1}, \hat{\beta}^d_{t-1}, \hat{\beta}^w_{t-1})',$$

$$\epsilon_t = (\tilde{u}_t, \epsilon^i_t)',$$

$$A = \begin{pmatrix} 1 & 0 & -(\Delta_i\delta - \frac{\delta^2\phi_q}{\sigma}\Delta_r)\delta & \frac{\Delta_r\delta}{\sigma} & -\Delta_i(1-\delta) & -\Delta_w \\ -\Theta_y & 1 & 0 & 0 & 0 & -\Theta_w \\ 0 & 0 & 1 & 1 & 0 & 0 \\ -\phi_y & -\phi_\pi & 0 & 1 & 0 & 0 \\ \psi_d & 0 & 0 & 0 & 1 & 0 \\ -(\sigma + \frac{\phi}{1-\alpha}) & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$B = \begin{pmatrix} -\frac{\delta^2\Delta_r\phi_y}{\sigma(1-\delta\rho)} & -\frac{\delta^2\Delta_r}{\sigma(1-\delta\rho)}(\phi_\pi - \frac{1}{\delta}) & -\frac{\delta^3\Delta_r\phi_q}{\sigma(1-\delta\rho)} & 0 & 0 & \frac{\Delta_w\delta}{1-\rho\delta} \\ \Theta_{\beta^y} & \Theta_{\beta^\pi} & 0 & 0 & 0 & \Theta_{\beta^w} \\ 0 & 1 & \delta & 0 & 1-\delta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$C = \begin{pmatrix} 0 & -\frac{\delta^2 \Delta_r}{\sigma(1-\delta\rho_i)} \\ \Theta_u & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix},$$

$$D = \begin{pmatrix} & & \underset{2\text{x}6}{0} & & & \\ 0 & 0 & \phi_q & 0 & 0 & 0 \\ & & \underset{3\text{x}6}{0} & & & \end{pmatrix},$$

where $\underset{a\text{x}b}{0}$ denotes a matrix of zeros of dimension $a \,\text{x}\, b$.

A.3 Welfare Approximation

Assuming the steady state is efficient under RE equalizes the consumption and labor decision of the two agents. This is ensured by a tax subsidy on sales by the fiscal authority which is rebated back to firms as a lump sum transfer conditional on a balanced budget. This ensures that profits are zero at the steady state but not otherwise since markups will vary over time. At steady steady

$$\begin{gathered} C^C = C^U = C \\ N^C = N^U = N \\ Y = N^{1-\alpha} \\ w = N^{\phi} Y^{\sigma} \\ \frac{V'(N)}{U'(C)} = w = (1-\alpha)\frac{Y}{N} \end{gathered} \tag{A.37}$$

Following Bilbiie, 2008 assume the social planner is maximizing a weighted average of the utility of the agents $U_t(\cdot) = \mathscr{O}\, U^C(C_t^C, N_t^C) + (1-\mathscr{O})\, U^U(C_t^U, N_t^U)$. Up to a second order approximation the utility of type j can be written as

$$\begin{aligned}\hat{U}_t^j(\cdot) &= U^j(C_t^j, N_t^j) - U(C,N) \\ &\approx U_C C\, \left(\hat{c}_t^j + \frac{1-\sigma}{2}(\hat{c}_t^j)^2\right) - V_N N\, \left(\hat{n}_t^j + \frac{1+\phi}{2}(\hat{n}_t^j)^2\right) + t.i.p + H.O.T\end{aligned} \tag{A.38}$$

where the hat variables denote log deviation from the flexible price RE equilibrium which given the absence of fluctuations in the natural output (e.g. TFP) coincides with the steady state of the model. Explicitly, $\hat{c}_t = log(C_t) - log(C)$, $t.i.p$ denotes terms independent of policy and $H.O.T$ higher order terms (greater than 2). In equilibrium $\hat{c}_t = \hat{y}_t$ and $\hat{n}_t = \frac{1}{1-\alpha}\hat{y}_t + d_t$ where $d_t = log \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di$[1]. Given this and aggregating across agents

$$\begin{aligned}\hat{U}_t(\cdot) \approx U_C C\, &\left[\hat{c}_t + \frac{1-\sigma}{2}\left(\mathscr{O}\,(\hat{c}_t^C)^2 + (1-\mathscr{O})(\hat{c}_t^U)^2\right)\right] \\ - V_N N &\left[\hat{n}_t + \frac{1+\phi}{2}\left(\mathscr{O}\,(\hat{n}_t^C)^2 + (1-\mathscr{O})(\hat{n}_t^U)^2\right)\right] + H.O.T\end{aligned} \tag{A.39}$$

Using the last equation from A.37 we can write $\frac{V_N N}{U_C C} = (1-\alpha)$. The linear terms from the utility approximation boil down to

$$U_C C\ (\hat{c}_t) - V_N N\, (\hat{n}_t) = -U_C C\ [(1-\alpha)d_t] + H.O.T \tag{A.40}$$

Regarding the quadratic terms we can also rewrite them in terms of output-gaps and stock prices. First notice that from Proposition 1 we have

$$\hat{c}_t^U - \hat{c}_t^C = \Delta_i \left[\delta\hat{q}_t + (1-\delta)\hat{d}_t\right] = \Delta_i \left[\delta\hat{q}_t + (1-\delta)\psi_d\, \tilde{y}_t\right].$$

Using the previous relation together with goods market clearing and FOC with respect to labor for the two types of agents we obtain the following

[1] see Galí, 2015 pag 87

$$\begin{aligned}
\tilde{y}_t &= \mathscr{O}\tilde{c}_t^C + (1-\mathscr{O})\tilde{c}_t^U \\
\tilde{y}_t &= \mathscr{O}\tilde{c}_t^C + (1-\mathscr{O})\left\{\Delta_i\left[\delta\tilde{q}_t + (1-\delta)\tilde{d}_t\right] + \tilde{c}_t^C\right\} \\
\tilde{c}_t^C &= \tilde{y}_t - (1-\mathscr{O})\Delta_i\left[\delta\tilde{q}_t + (1-\delta)\psi_d\,\tilde{y}_t\right] \\
\tilde{c}_t^C &= [1-(1-\mathscr{O})\Delta_i(1-\delta)\psi_d]\tilde{y}_t - (1-\mathscr{O})\Delta_i\delta\tilde{q}_t \\
&= \Upsilon_{cy}^C\,\tilde{y}_t - \Upsilon_{cq}^C\,\tilde{q}_t \\
\tilde{n}_t^C &= \frac{\tilde{w}_t - \sigma\,\tilde{c}_t^C}{\phi} = \frac{\left(\frac{\phi}{1-\alpha}+\sigma\right)\tilde{y}_t + \phi d_t - \sigma\,\tilde{c}_t^C}{\phi} \\
&= \left(\frac{1}{1-\alpha}+\frac{\sigma}{\phi}\right)\tilde{y}_t - \frac{\sigma}{\phi}\left(\Upsilon_{cy}^C\,\tilde{y}_t - \Upsilon_{cq}^C\tilde{q}_t\right) + d_t \\
&= \left(\frac{1}{1-\alpha}+\frac{\sigma}{\phi}(1-\Upsilon_{cy}^C)\right)\tilde{y}_t + \frac{\sigma}{\phi}\Upsilon_{cq}^C\tilde{q}_t + d_t \\
&= \Upsilon_{ny}^C\,\tilde{y}_t + \Upsilon_{nq}^C\,\tilde{q}_t + d_t \qquad \text{(A.41)} \\
\tilde{c}_t^U &= \Delta_i\left[\delta\tilde{q}_t + (1-\delta)\psi_d\,\tilde{y}_t\right] + \Upsilon_y^C\,\tilde{y}_t - \Upsilon_q^C\tilde{q}_t \\
&= []1 + \mathscr{O}\Delta_i(1-\delta)\psi_d]\,\tilde{y}_t + \mathscr{O}\Delta_i\delta\tilde{q}_t \\
&= \left[\Delta_i(1-\delta)\psi_d + \Upsilon_{cy}^C\right]\tilde{y}_t + (\Delta_i\delta - \Upsilon_{cq}^C)\tilde{q}_t \\
&= \Upsilon_{cy}^U\,\tilde{y}_t + \Upsilon_{cq}^U\,\tilde{q}_t \\
\tilde{n}_t^U &= \frac{\tilde{w}_t - \sigma\,\tilde{c}_t^U}{\phi} = \frac{\left(\frac{\phi}{1-\alpha}+\sigma\right)\tilde{y}_t + \phi d_t - \sigma\,\tilde{c}_t^U}{\phi} \\
&= \left(\frac{1}{1-\alpha}+\frac{\sigma}{\phi}\right)\tilde{y}_t - \frac{\sigma}{\phi}\left(\Upsilon_{cy}^U\,\tilde{y}_t + \Upsilon_{cq}^U\tilde{q}_t\right) + d_t \\
&= \left[\frac{1}{1-\alpha}+\frac{\sigma}{\phi}(1-\Upsilon_{cy}^U)\right]\tilde{y}_t - \frac{\sigma}{\phi}\Upsilon_{cq}^U\tilde{q}_t + d_t \\
&= \Upsilon_{ny}^U\,\tilde{y}_t - \Upsilon_{nq}^U\,\tilde{q}_t + d_t.
\end{aligned}$$

Using these last results we can derive the quadratic terms for consumption and labor in terms of output gaps and stock prices

$$(\tilde{c}_t^C)^2 = (\Upsilon_{cy}^C)^2\tilde{y}_t^2 + (\Upsilon_{cq}^C)^2\tilde{q}_t^2 - 2\,\Upsilon_{cy}^C\Upsilon_{cq}^C\tilde{y}_t\,\tilde{q}_t + H.O.T \qquad \text{(A.42)}$$

$$(\tilde{c}_t^U)^2 = (\Upsilon_{cy}^U)^2\tilde{y}_t^2 + (\Upsilon_{cq}^U)^2\tilde{q}_t^2 + 2\,\Upsilon_{cy}^U\Upsilon_{cq}^U\tilde{y}_t\,\tilde{q}_t + H.O.T \qquad \text{(A.43)}$$

$$(\tilde{n}_t^C)^2 = (\Upsilon_{ny}^C)^2\tilde{y}_t^2 + (\Upsilon_{nq}^C)^2\tilde{q}_t^2 + 2\,\Upsilon_{ny}^C\Upsilon_{nq}^C\tilde{y}_t\,\tilde{q}_t + H.O.T \qquad \text{(A.44)}$$

$$(\tilde{n}_t^U)^2 = (\Upsilon_{ny}^U)^2\tilde{y}_t^2 + (\Upsilon_{nq}^U)^2\tilde{q}_t^2 - 2\,\Upsilon_{ny}^U\Upsilon_{nq}^U\tilde{y}_t\,\tilde{q}_t + H.O.T. \qquad \text{(A.45)}$$

The aggregate *per-period* approximation of the welfare function is then, up to a second order approximation

$$\hat{U}_t(\cdot) \approx -U_C C\left[(1-\alpha)d_t + \Upsilon_1\,\tilde{y}_t^2 + \Upsilon_2\,\tilde{q}_t^2 + \Upsilon_3\,\tilde{q}_t\tilde{y}_t\right] \tag{A.46}$$

where

$$\begin{aligned}
\Upsilon_1 &= \left[\frac{1+\phi}{2}(1-\alpha)\left(\mathscr{O}(\Upsilon_{ny}^C)^2 + (1-\mathscr{O})(\Upsilon_{ny}^U)^2\right) - \frac{1-\sigma}{2}\left(\mathscr{O}(\Upsilon_{cy}^C)^2 + (1-\mathscr{O})(\Upsilon_{cy}^U)^2\right)\right]\\
\Upsilon_2 &= \left[\frac{1+\phi}{2}(1-\alpha)\left(\mathscr{O}(\Upsilon_{nq}^C)^2 + (1-\mathscr{O})(\Upsilon_{nq}^U)^2\right) - \frac{1-\sigma}{2}\left(\mathscr{O}(\Upsilon_{cq}^C)^2 + (1-\mathscr{O})(\Upsilon_{cq}^U)^2\right)\right]\\
\Upsilon_3 &= \Big[\,\big[(1+\phi)(1-\alpha)\left(\mathscr{O}\Upsilon_{nq}^C\Upsilon_{ny}^C - (1-\mathscr{O})\Upsilon_{nq}^U\Upsilon_{ny}^U\right) +\\
&\quad (1-\sigma)\left(\mathscr{O}\Upsilon_{cq}^C\Upsilon_{cy}^C - (1-\mathscr{O})\Upsilon_{cq}^U\Upsilon_{cy}^U\right)\big]\,.
\end{aligned} \tag{A.47}$$

The price dispersion term, $(1-\alpha)\,d_t$, can be rewritten using the arguments from Galí, 2015 as $(1-\alpha)\,d_t \approx \frac{\epsilon}{\psi}\,\pi_t^2$ where $\psi = \frac{(1-\theta)(1-\delta\theta)}{\theta}\frac{1-\alpha}{1-\alpha+\alpha\epsilon}$.

The average welfare loss per period in terms of steady steady consumption is

$$\mathscr{L} = \frac{\epsilon}{\psi}var(\pi_t) + \Upsilon_1\,var(\tilde{y}_t) + \Upsilon_2\,var(\tilde{q}_t) + \Upsilon_3\,E(\tilde{y}_t\tilde{q}_t) \tag{A.48}$$

A.4 Contemporaneous Response to stock prices

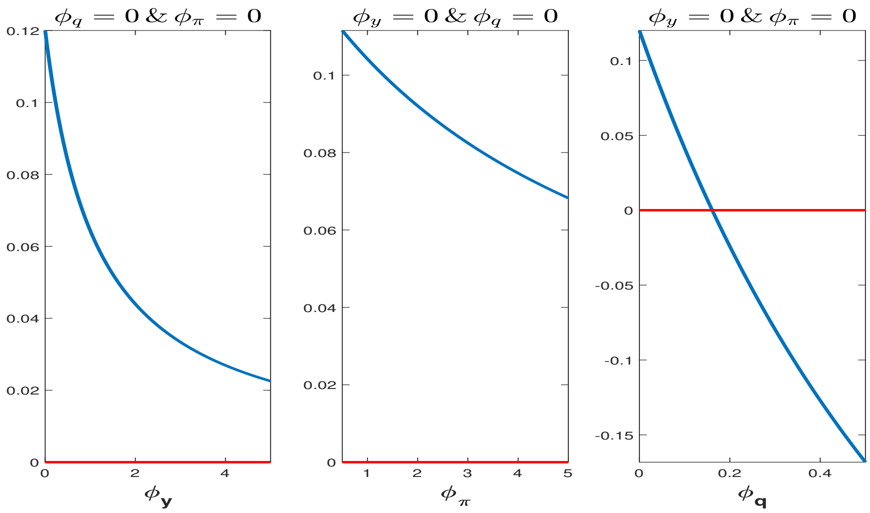

FIGURE A.1: Stock Price Wealth Effects and Monetary Policy
Each panel presents the magnitude of the wealth effects as a function of the central bank response to output, inflation and stock prices while keeping the other coefficients fixed at 0. The Taylor rule is of the following type: $i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_q \tilde{q}_t$

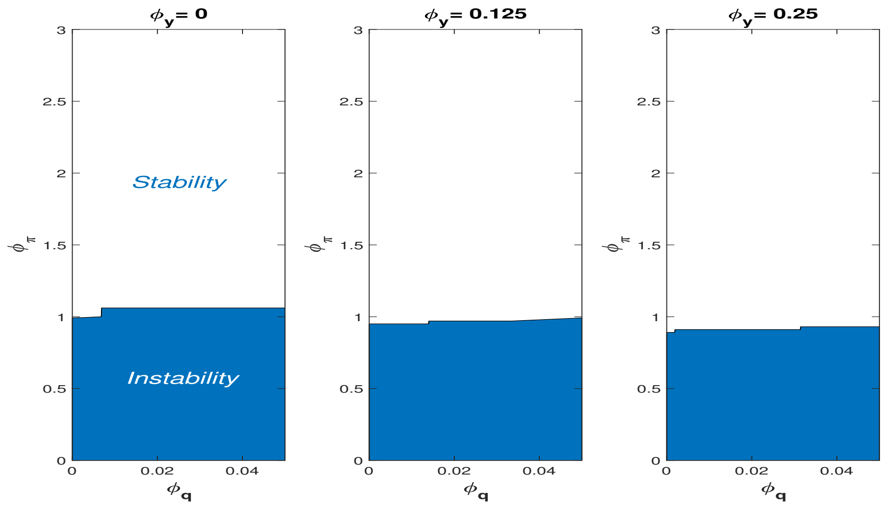

FIGURE A.2: E-Stability and Monetary Policy. The figure present the stability (white) and instability (blue) regions for different combinations of Taylor rule coefficients. Each panel plots the e-stability regions for different combinations of inflation (Y axis) and stock price (X axis) reaction coefficients while keeping the output reaction fixed. The Taylor rule is of the following type: $i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_q \tilde{q}_t$. The stability of the system is given by the eigenvalues of the matrix $A^{-1}B$. Following Evans and Honkapohja (2012), the dynamical system is e-stable if the largest eigenvalue of the previous matrix has the real part smaller than 1.

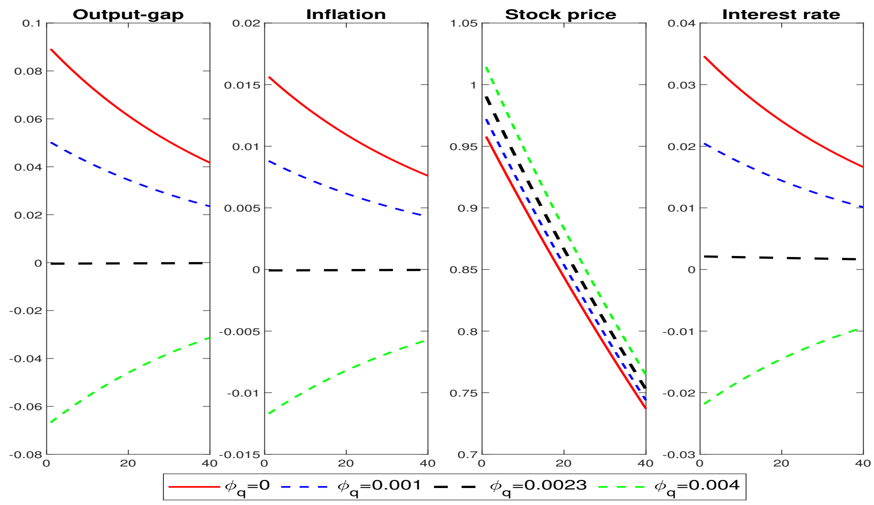

FIGURE A.3: IRFs to Sentiment Shocks
The figure presents the IRF to a 1 % *i.i.d* shock in equity price beliefs for different reaction coefficients to stock prices. The Taylor rule is of the following form: $i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_q \tilde{q}_t$

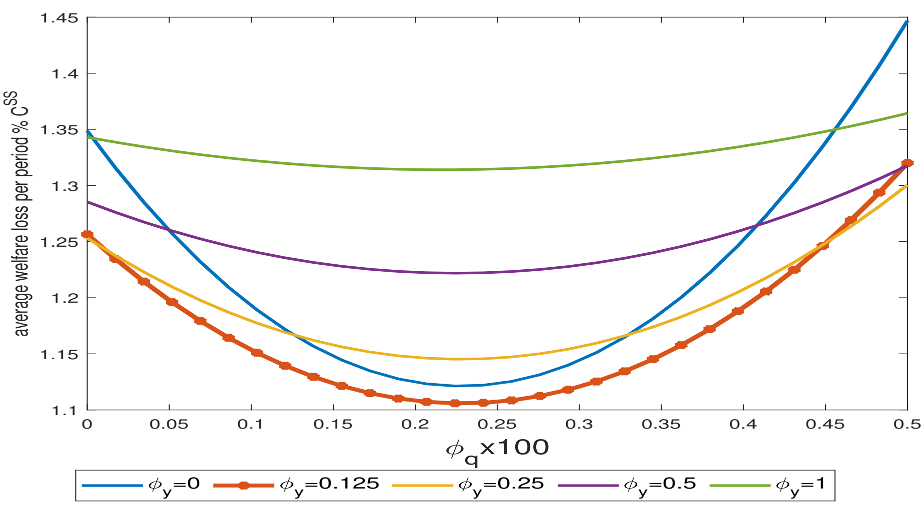

FIGURE A.4: Welfare Maps. The figure shows the average welfare loss per period as defined in equation (1.54) for different combinations of Taylor rule coefficients for output and stock prices while keeping the inflation reaction coefficient fixed at 1.5. Welfare losses have been computed as averages over 200 independent simulations, each one including 260 time periods using the estimated parameters from section IV.H

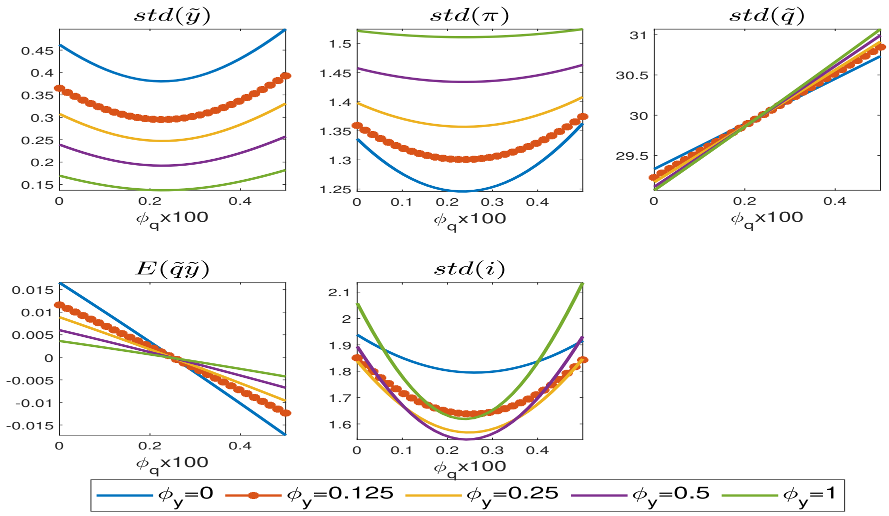

FIGURE A.5: Influence of Monetary policy on Macroeconomic Volatility. Implied volatility of output, inflation, stock prices, co-movement of output with stock prices and interest rates for different combinations of policy parameters. The Taylor rule is specified as $i_t = 1.5\ \pi_t + \phi_y\ \tilde{y}_t + \phi_q \tilde{q}_t$.

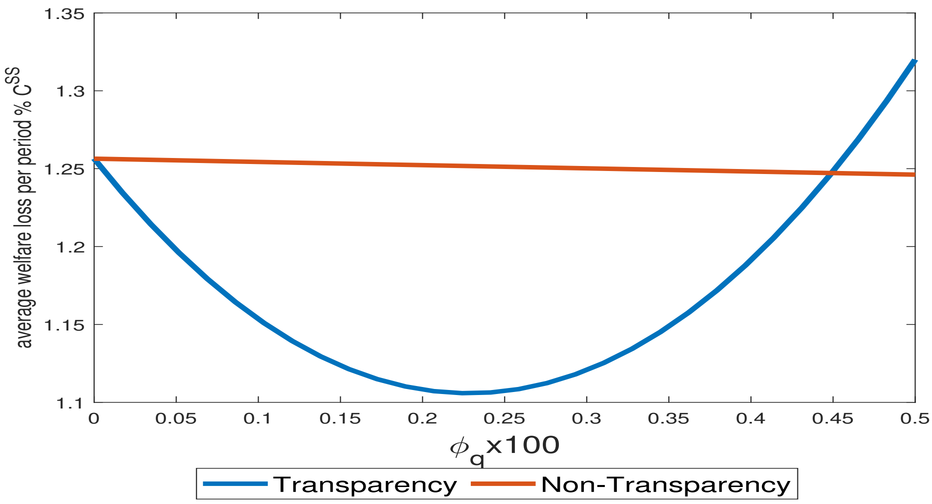

FIGURE A.6: (Non) Transparency of stock price targeting. Transparency implies that agents internalize the reaction to stock prices while in the non-transparency scenario agents only take into account the response to output and inflation in the Taylor Rule. The latter is specified as $i_t = 1.5\ \pi_t + 0.125\ \tilde{y}_t + \phi_q \tilde{q}_t$.

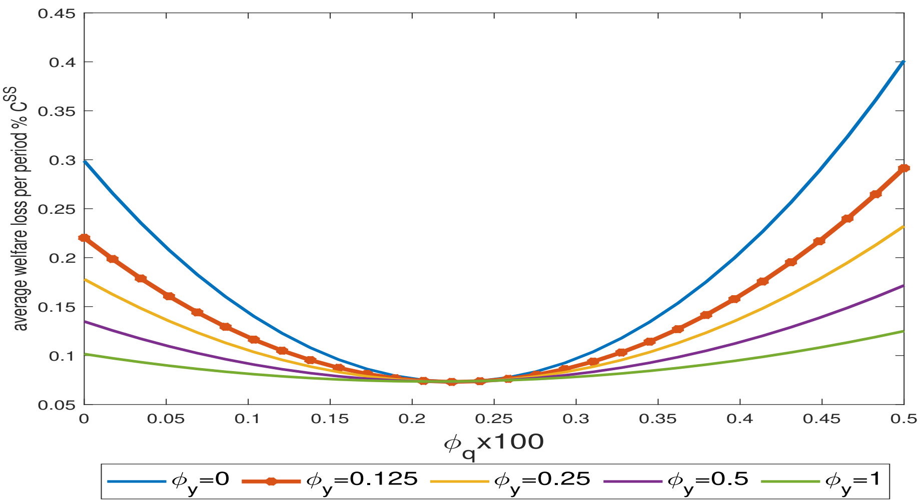

FIGURE A.7: Welfare maps when the economy is hit only by Sentiment Shocks. The figure shows the average welfare loss per period for different policy parameters for output and stock prices in the case the only source of variation in the economy is given by Sentiment Shocks. The volatility of sentiment shocks is the one estimated in section chapter 1 section IV

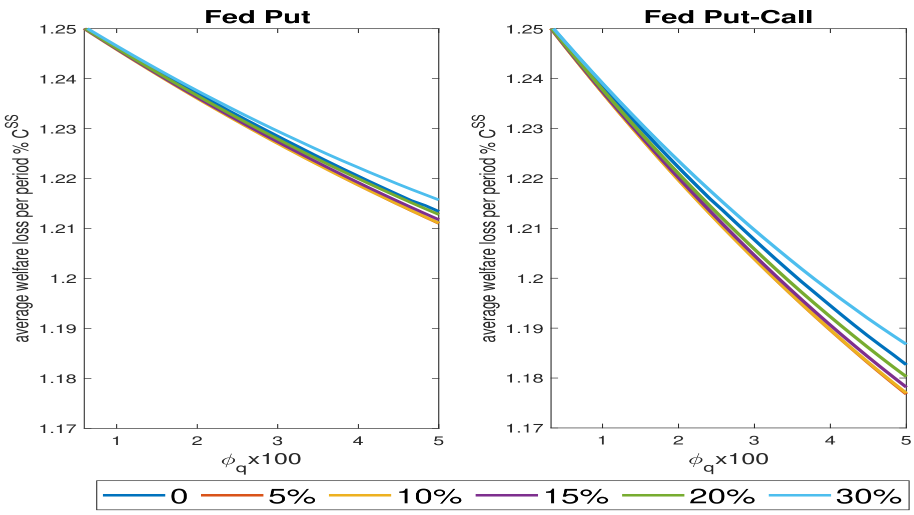

FIGURE A.8: Welfare Implications of Fed Put and Call under Non-Transparency; $Q^- = -Q^+$

A.5 Additional Figures

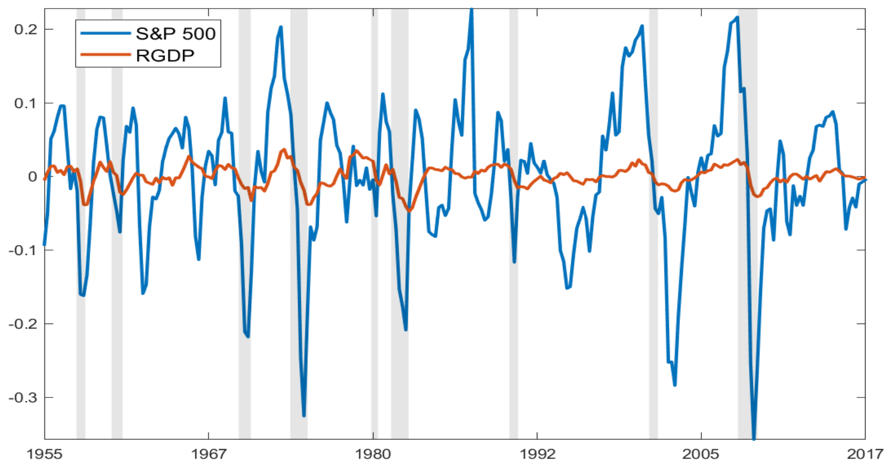

FIGURE A.9: Real and Financial Volatility at business cycle frequency; HP-filtered quarterly data; shaded bands denote NBER recessions

Appendix B

Appendix: Chapter 2

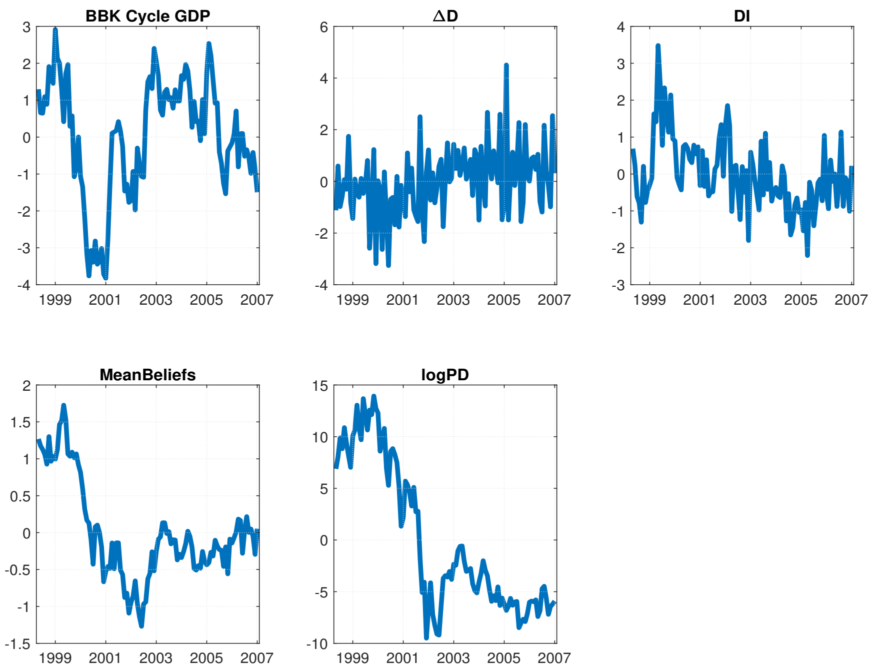

FIGURE B.1: Time series 1999M2-2007M10 used in the VARs

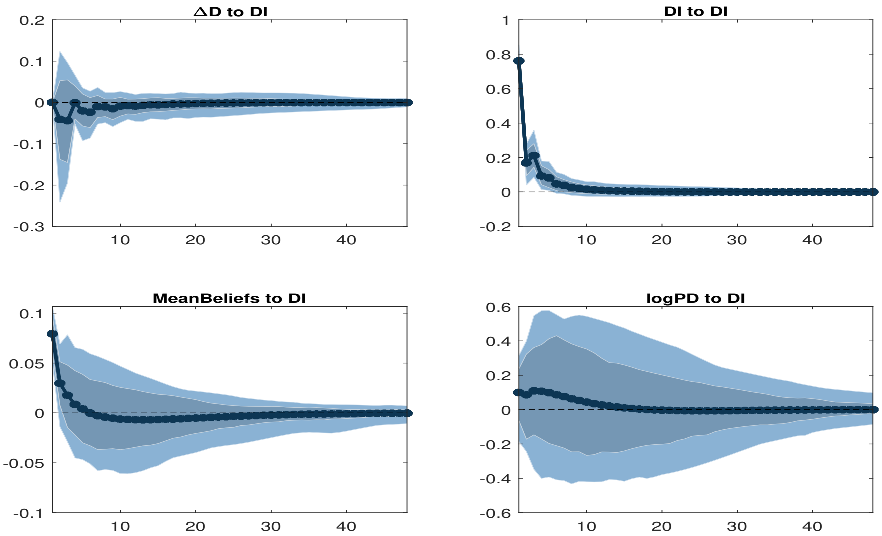

FIGURE B.2: IRFs to a 1σ Disagreement shock

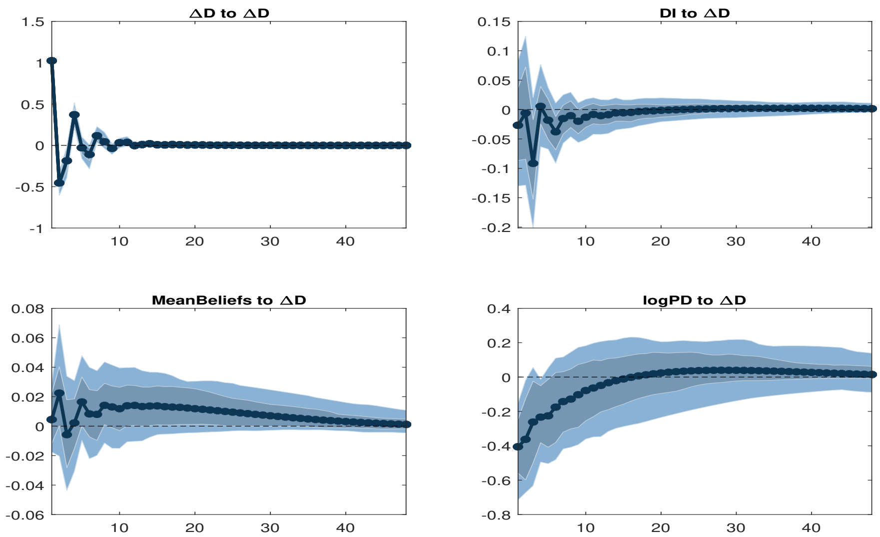

FIGURE B.3: RFs to a 1σ dividend growth shock

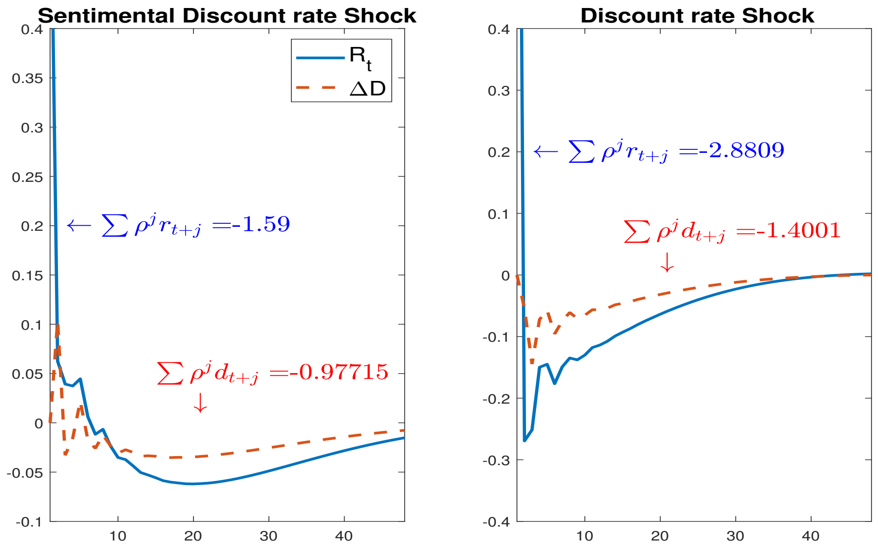

FIGURE B.4: PD decomposition from 1σ (S)DR shocks

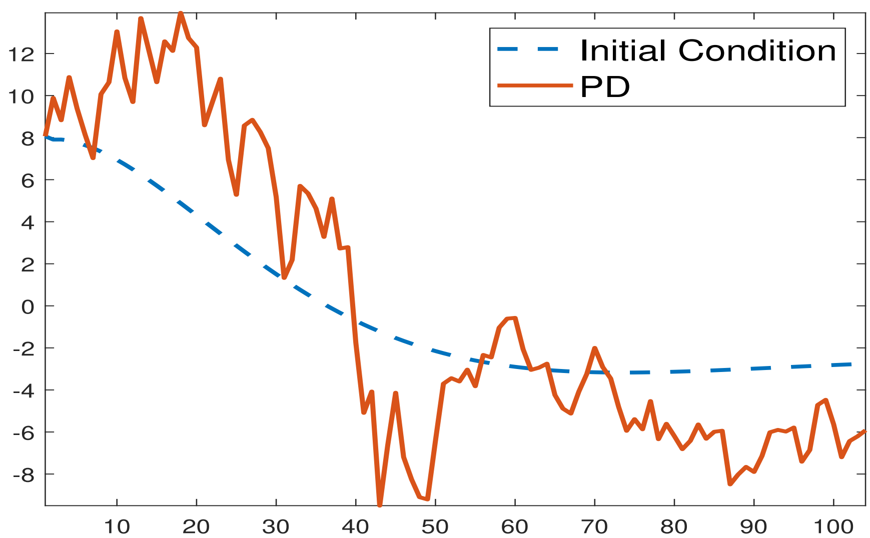

FIGURE B.5: The Influence of initial condition in HD

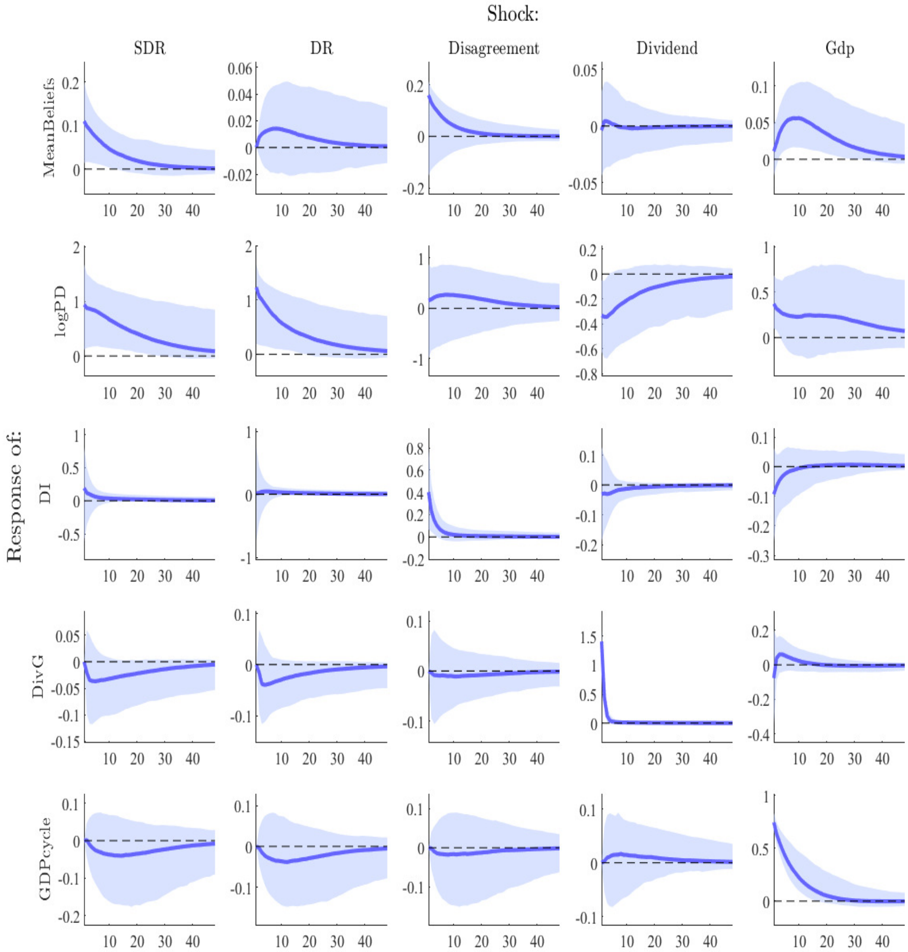

FIGURE B.6: Impulse Response functions from the Identification with short-run and zero restriction from Tables 2.1 and 2.2

Appendix C

Appendix: Chapter 3

C.1 Additional Results

	c	***c***	*p-value* H_0: c= ***c***
	2 Sentiment groups		
p_{0-50}	0.0546 ***	-0.2421***	0.0000
p_{50-100}	0.0744 ***	-0.2419 ***	0.0000
	3 Sentiment groups		
p_{0-33}	0.0576***	-0.2421 ***	0.0000
p_{33-66}	0.0545***	-0.2415***	0.0000
p_{66-100}	0.0809***	-0.2423***	0.0000
	4 Sentiment groups		
p_{0-25}	0.0591	-0.2421	0.0000
p_{25-50}	0.0501	-0.2422	0.0000
p_{50-75}	0.0621	-0.2420	0.0000
p_{75-100}	0.0867	-0.2421	0.0000

TABLE C.1: RE Tests across different sentiment groups; p_{0-50} denotes the sentiment group which expectations lies between between the 0 and 50th percentile. The data in each group is aggregated by taking the average of that particular group. Data used for this particular test is the Gallup UBS survey data for expected stock market return of all individuals. Estimates have are based on asymptotic theory and have been adjusted for small sample bias

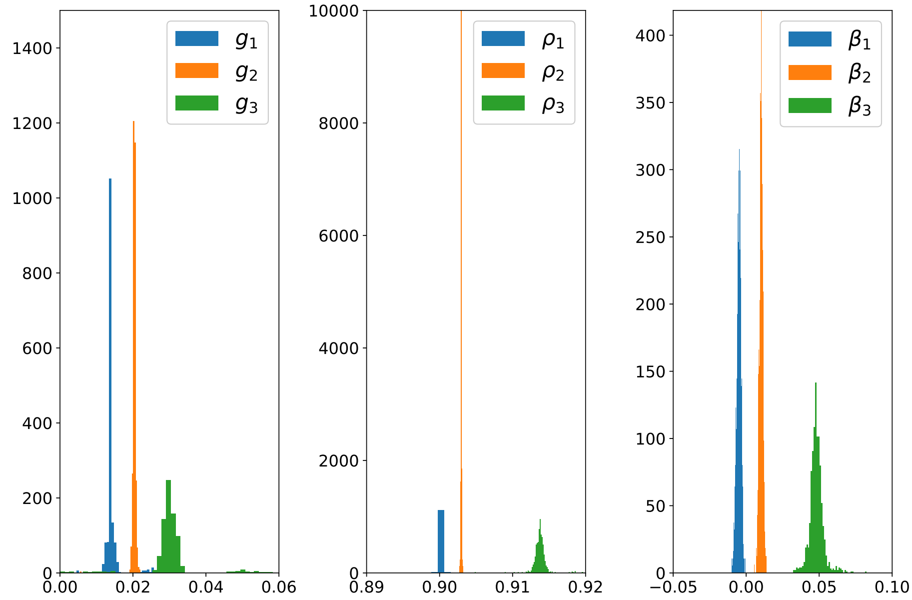

FIGURE C.1: Bootstrap densities of estimated parameters from equation 3.2

C.2 Model Solution Strategy

The concavity of the objective function and the convexity set guarantee the sufficiency of the first order conditions for an interior optimal plan. The optimal condition for the household plan is given by the Euler equation:

$$(CD_t^i)^{-\gamma} = \delta \mathbb{E}_t^{\mathscr{P}} \left(\left(\frac{D_{t+1}}{D_t} \right)^{1-\gamma} \frac{(PD_{t+1}+1)}{PD_t} (CD_{t+1}^i)^{-\gamma} \right) = \delta \mathscr{E}(X_t^i) \qquad \text{(C.1)}$$

where X_t^i are the state variables. The problem is that this Euler Equation includes an unknown conditional expectation. To solve the model, it must be computed somehow. The Parameterized Expectations Algorithm (PEA) is one of the alternatives. PEA consists of replacing the conditional expectation $\mathscr{E}(X_t^i)$ by some parametric function ψ (Marcet, 1988). The choice of the approximating functions ψ is not obvious and not unique. Popular possibilities are polynomials, splines, neural networks, etc. We follow the approach outlined by Belda, 2022: use approximating functions rooted in economic theory. Among the advantages of that approach is the possibility of getting closed-form solutions. Altogether, we follow the next steps

1. Approximate the consumption policy using a

$$CD_t^i = CD(S_{t-1}^i, PD_t, WD_t, \beta_t^i) = B(\beta_t^i)\Big(WD_t + (PD_t+1)S_{t-1}^i\Big) \tag{C.2}$$

$$B_t^i = B(\beta_t^i) = 1 - \chi^i \beta_t^i \tag{C.3}$$

where χ^i is an unknown parameter which will be estimated via PEA to be discussed below. The consumption policy function is linear in wealth and the propensity to consume depends negatively on expectations.

2. Obtain the stock holdings policy function by plugging the consumption policy in the budget constraint:

$$S_t^i = (1 - B_t^i)\frac{\Big(WD_t^i + (PD_t + 1)S_{t-1}^i\Big)}{PD_t}. \tag{C.4}$$

3. Determine market-clearing prices by adding individual demands, equating them to the aggregate supply and solving for prices. In this case,

$$\frac{P_t}{D_t} = \frac{\sum_{i=1}^{M} \mu_i (1 - B_t^i)(S_{t-1}^i + \frac{W_t^i}{D_t})}{S^s - \mu_i \sum_{i=1}^{M} (1 - B_t^i) S_{t-1}^i}. \tag{C.5}$$

The only unknown at this point is the parameter χ^i from equation C.3. To obtain this parameter we make use of PEA on the first order condition of the agent which we rewrite as

$$(CD_t^i)^{-\gamma}\delta^{-1} = \mathbb{E}_t^{\mathscr{P}}\left(\Big(\frac{D_{t+1}}{D_t}\Big)^{1-\gamma}\frac{(PD_{t+1}+1)}{PD_t}(CD_{t+1}^i)^{-\gamma}\right). \tag{C.6}$$

The PEA algorithm involves the following steps:

1. Draw a series of the exogenous processes for a large T.

2. For a given $\chi \in \mathbb{R}^n$, recursively compute the series of the endogenous variables.

3. Minimize the Euler Equation square residuals

$$G(\chi) = \underset{\chi}{\text{argmin}}\left[\left(\Big(\frac{D_{t+1}^{\mathscr{P}}}{D_t}\Big)^{1-\gamma}\frac{(PD_{t+1}^{\mathscr{P}}+1)}{PD_t}(CD(\chi)_{t+1}^{i,\mathscr{P}})^{-\gamma}\right) - \frac{(CD(\chi)_t^i)^{-\gamma}}{\delta}\right]^2 \tag{C.7}$$

Note the interior of the expectation must be computed according to investor's beliefs. Since investors know the process for dividends and wage-dividends, the only problematic objects are PD_{t+1} and CD_{t+1}. Using agents subjective price model

$$\beta_{t+1}^{i,\mathscr{P}} = \beta_t^i \nu_{t+1} \Rightarrow \left(\frac{P_{t+1}}{P_t}\right)^{\mathscr{P}} = \beta_t^i \nu_{t+1} \varepsilon_{t+1}^p \Rightarrow \left(\frac{P_{t+1}}{D_{t+1}}\right)^{\mathscr{P}} = \left(\frac{P_{t+1}}{P_t}\right)^{\mathscr{P}} \frac{D_t}{D_{t+1}} \frac{P_t}{D_t}$$

In turn, expected consumption reads

$$CD_{t+1}^{i,\mathscr{P}} = (1 - \chi \beta_{t+1}^{i,\mathscr{P}}) \left[WD_{t+1}^i + \left(\left(\frac{P_{t+1}}{D_{t+1}}\right)^{\mathscr{P}} + 1 \right) S_t \right]$$

4. Find a fixed point $\chi = G(\chi)$. For that, update χ following

$$\chi^{j+1} = \chi^j + d(G(\chi^j) - \chi^j) \tag{C.8}$$

where j iteration number and d the dampening parameter.

C.3 Aggregate Shocks

In this section we report the responses of the model main variables to simultaneous equivalent shocks on investors wages (figure C.2) and transitory information (figure C.3).

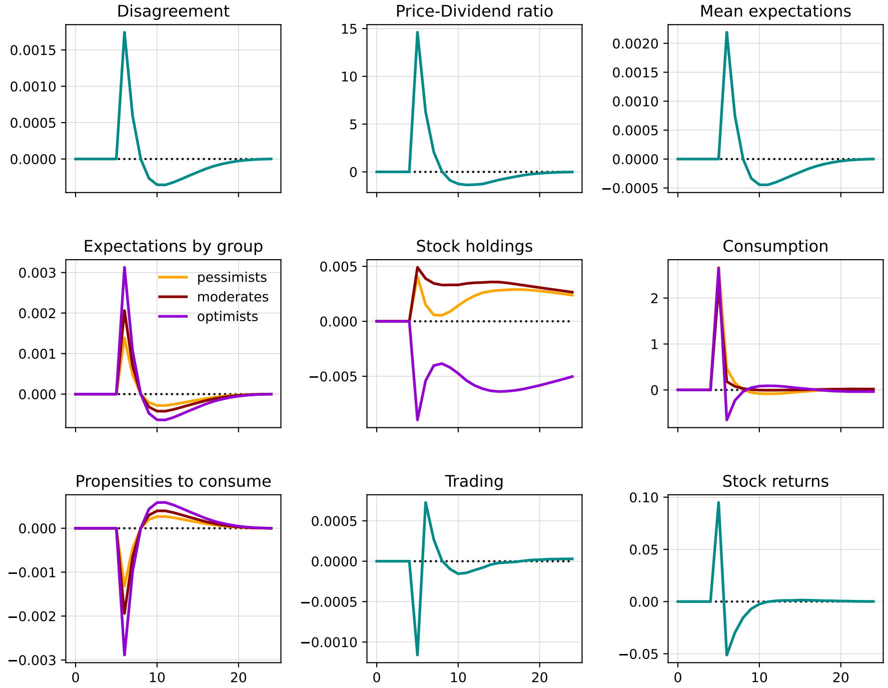

FIGURE C.2: Responses to a general wage shock. The graph show the GIRFs of different variables to an equivalent wage shock enjoyed by all investors. group 3. Periods are quarters.

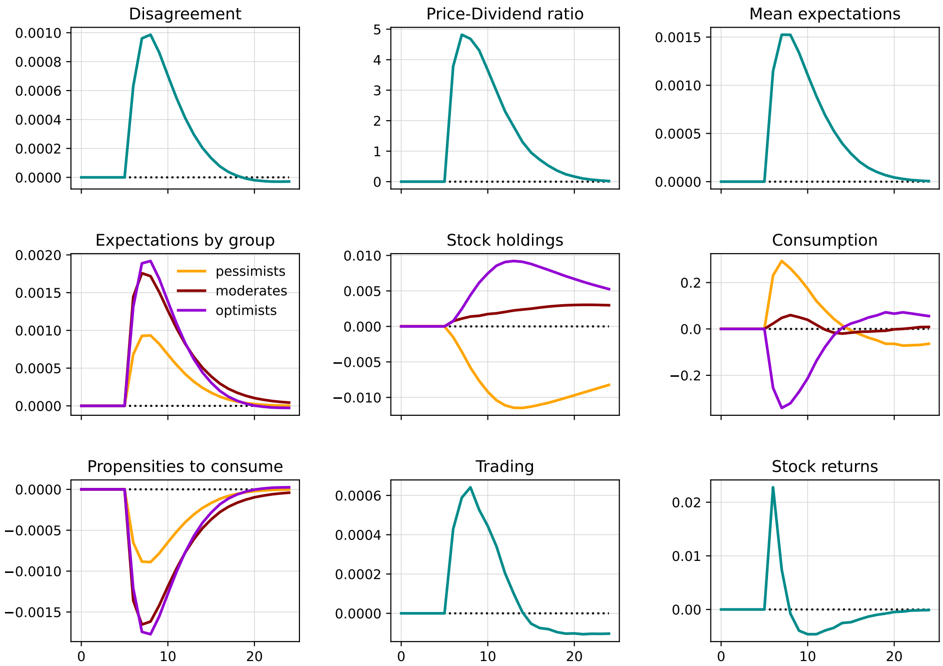

FIGURE C.3: Responses to a general information shock. The graph show the GIRFs of different variables to an information equally received by all investors. Periods are quarters.

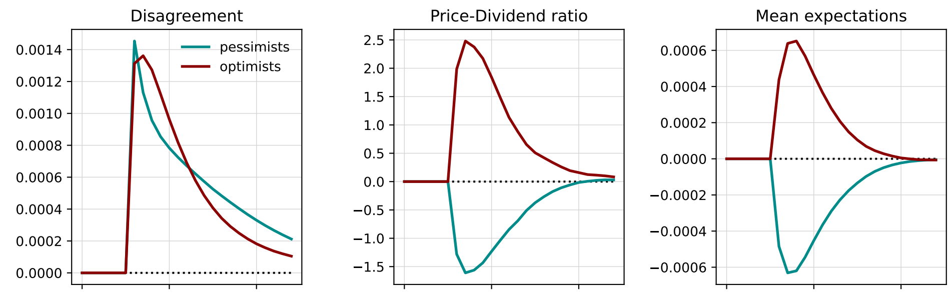

FIGURE C.4: Responses to a disagreement shock. The graph show the GIRFs of different variables to a positive information shock hitting the optimists and a negative shock hitting the pessimists.

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