Essays in Safe Assets

Trends, Drivers and Implications for Financial Stability

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Badira hainbat pertsona bidai luze honen gazi-gozoak nirekin batera bizi izan dituztenak, eta momentu zailenetan eskutik heldu eta askatu ez nautenak.

Bihotz-biothez zuei dedikatua.

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Abstract

This dissertation consists of three chapters centred on safe assets, shedding light on different aspects of the growing literature on the subject. Chapter 1 introduces the set of assets that falls under the safe category, and further documents the trends of safe assets in certain industrialised economies, finding the demand/supply factors behind the observed increase in their amount. Financial intermediaries play a key role in supplying (private) safe assets. Chapter 2 is devoted to assessing in a theoretical framework the determinants of private safe assets supply and its implications on economic stability. The tension between the private supply of safe assets and the quality of financial intermediaries' investment unveils important inefficiencies that call for policy intervention. The increase in the foreign sector's safe asset demand has been significant in the analysed countries, and Chapter 3 focusses on understanding the real effects of capital flows that search for safety. The preliminary evidence, while only suggestive raises important questions that are rationalised in a model.

Resum

Aquesta tesi tracta dels actius segurs, i a través dels seus 3 capítols contribueix a la creixent literatura de la matèria. El Capítol 1 introdueix el conjunt d'actius que formen la categoria d'actius segurs, i analitza les tendències que aquesta tipologia d'actius segueixen en certes economies industrialitzades, profunditzant en els factors d'oferta i demanda darrera l'increment observat. Els intermediaris financers juguen un paper fonamental en l'oferta (privada) d'actius segurs, al Capítol 2 és on es desenvolupa, des d'una perspectiva teòrica, què determina l'oferta d'actius segurs i les seves implicacions en l'estabilitat econòmica. Les tensions entre l'oferta privada d'actius segurs i la qualitat de les inversions dels intermediaris financers, ens descobreix ineficiències importants que exigeixen ser regulades. L'increment en els països analitzats de la demanda exterior d'actius segurs ha estat significativa; el Capítol 3 es centra en l'enteniment dels efectes reals provocats pels moviments de capital que busquen invertir en actius segurs. L'evidència preliminar, generen qüestions rellevants que estan explicades al model.

Preface

Safe assets, i.e., assets deemed to maintain a stable value over time, have been the object of recent policy and research debates. This class of assets plays an important role in the economy; they cater to the demand for store of value since they are immune to adverse macroeconomic shocks (Caballero and Farhi (2018)), for collateral useful in financial transactions (Gorton and Ordoñez (2022))), and even provide transaction services (Dang et al. (2017)). The global financial crisis, however, unveiled their contribution to the build-up of important economic and financial fragilities, and reinvigorated the empirical and theoretical research on the subject. This thesis contains three chapters that contribute to the growing literature on this topic from three different angles.

In Chapter 1, safe assets' characteristics and trends are studied. First, the set of financial instruments that falls under the safety category is discussed. Second, Dmitry Kuvshinov, Björn Richter, Victoria Vanasco, and I track the evolution of safe assets across 15 industrialised economies in recent decades. We document the fast-growing level of safe assets, and further disentangle their demand and supply composition. We highlight the key role of the financial sector on both the demand and the supply side of safe assets, compare the private supply by financial intermediaries to the public supply by the government and further show the expansion of the international demand that search for safety in these countries – not only the US but also the rest of the industrialized countries.

In Chapter 2, I study the determinants of private safe asset supply and its macroeconomic implications in a theoretical framework. The model builds on the documented tension between financial intermediaries' risk sharing activities and the quality of their investments due to a moral hazard problem. The interplay between these two determinants shapes the supply curve of safety and provides an important insight: along the upward-sloping curve, the risk-sharing activity intensifies, jeopardising the incentives to enhance the quality of the investment, deteriorating the expected output and amplifying economic volatility. Thus, the real costs of safe asset supply are particularly acute in the current environment in which safe assets are scarce, an their price is high. Incomplete markets, i.e., the lack of a full set of state-contingent claims, hinders financial intermediaries' ability to cope with the informational friction, and further explain important sources of inefficiencies that call for policy intervention. However, the model highlights the need to understand the interaction between different forms to ensure safety for an adequate and effective policy response.

As documented in Chapter 1, capital flows that search for safety have increased in recent decades, suggesting that there is a growing global need for safe assets. Chapter 3 focusses on evaluating the relationship between foreign safe assets positions and the real economic activity and further rationalises the preliminary findings through the lenses of a model. Based on a larger sample of industrialise economies, Dmitry Kuvshinov, Björn Richter, Victoria Vanasco and I analyse the cyclical variation of safe asset positions and find that changes in foreign sector's net safe asset position (demand side) are mapped almost one-to-one with changes in privately issued safe liabilities (supply side). In addition, we find that an increase in the net supply of safe assets by financial intermediaries or an increase in the net demand by the foreign sector is associated with lower GDP growth over the following years, a strong correlation that holds even when controlling for the credit growth it generates. This preliminary evidence, while only suggestive, it raises important questions: Is there something intrinsic in the foreign demand for safety? Alternatively, is private supply hindering economic growth? Would a public supply increase ameliorate the burden? How is this pattern affected by the business cycle? Motivated by these questions, I propose a two-country model that studies the determinants of safe assets demand and supply at the international level and study the interplay with the real economy.

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Chapter 1

Trends in Safe Assets

1.1 What is a safe asset?

Although the answer seems obvious, defining safe assets is challenging. At the highest level of generality, a safe asset is an asset that is immune to any source of risk, preserving its value [1] even after adverse macroeconomic shocks. This notion of safety, while useful in theoretical frameworks, is delusive in practice because even the highest rated assets are not genuinely risk free. Hence, safe cannot mean *absolutely* safe, but rather *relatively* safe, always in relation to other assets.

Despite being more pragmatic, the latter concept leaves room for ambiguity. First, (relative) safety is not a binary characteristic, so that different financial instruments can be safe even if in different degrees. Second, safety is not an intrinsic characteristic of an asset so that the composition of the safest assets can change over time. Therefore, this class of assets comprises a varied and dynamic set of assets that are deemed to be safe "enough" to be suitable for storing value (e.g., Caballero et al. (2017)), as collateral in a wide range of financial transactions

[1] At this point, value refers to both, intrinsic/fundamental and market value. Absent any friction, the concept of value is unambiguous, and the distinction between intrinsic and market value is redundant. With frictions, however, liquidity risk might matter, opening a gap between the two values. I will later make the distinction clear.

rendering insurance against sudden consumption needs (Diamond and Dybvig (1983)) or investment opportunities (Holmström and Tirole (1998)) and even for providing transaction services (e.g., Gorton (2017)).

There is, however, a wide range of risks, and the sensitivity towards each depends on the aforementioned functions they cater to. For instance, an asset is a good store of value if it has minimal inflation and exchange rate risk, retaining value over time, even if it is not fully liquid. On the other hand, an asset serves as good collateral when it has a deep and liquid market. In any case, the asset's fundamental risk must be low, i.e., stable payoffs across states of nature, so that its value has some immunity to the arrival of information. Consequently, debt or debt-like contracts with low default risk and limited loss exposure (often achieved through collateral) dominate the category.

There are two sources of supply: public supply by governments and private supply by financial intermediaries. While the public supply is backed by taxes, the private supply is backed by a diversified pool of loans and financial assets. Taxing power gives governments with a comparative advantage in issuing safe assets; however, financial intermediaries can also manufacture private (imperfect) substitutes. Hence, fiscal capacity of sovereigns, constraints in the financial sector, and the level of financial development determine the capacity of a country to produce safe assets. Therefore, the supply of safe assets, public and private, has historically been concentrated in small number of industrialised economies, most predominantly, but not exclusively, in the US.

Safe assets are associated with a convenience yield, i.e., a non-pecuniary return due to the safety and/or liquidity attributes, which lowers their pecuniary return. Therefore, this class of assets is traded at a premium, i.e., *safety premium.* [2] Investors are willing to pay a higher price than that implied by the discounted value of an asset's cash flow due to the future service flows associated with this

[2] The literature often uses the safety/liquidity premium and convenience yield interchangeably. I follow Gorton (2017) who argues that "the specialness of safe assets implies the existence of non-pecuniary returns (called the "convenience yield") investors receive from holding safe debt, and so the pecuniary return is lower than it otherwise would be".

class of assets. In this respect, the demand that search for safety does not consider riskier assets as substitutes, and this lack of substitutability implies that the price of safety is partially determined by the supply and demand for safety, pointing to certain degree of market segmentation. This discontinuity in the pricing between safe and risky assets is not captured in standards asset pricing models.

1.2 The determinants of safe assets

While safe asset status is not directly observable, estimating the safety premium allows for the empirical identification of this class of assets. The burgeoning empirical literature has shed light on the type of financial instruments that fall into this category.

In a seminal paper, Krishnamurthy and Vissing-Jorgensen (2012) document that US Treasury securities carry a safety and liquidity premium, which is declining in the total supply of Treasuries. The safety attribute is not only associated with short-term but also long-term Treasuries. In addition, Krishnamurthy and Vissing-Jorgensen (2015) finds that US banks' net short-term debt (e.g., deposits, MMF shares, commercial paper) also satisfies the special demand for safe and liquid debt, a finding that is supported by other studies (e.g., Sunderam (2015), Greenwood et al. (2015)). Not only commercial banks, but also shadow banks – i.e., the unregulated segment of financial intermediaries – can issue debt with safety attributes. For instance, Gissler et al. (2020) shows that shadow banks have crowded out traditional banks' supply of safe assets. Kacperczyk et al. (2021) studies the determinants and fragility of private safe assets (e.g., certificates of deposit) using European data, finds that while T-bills always enjoy a safety premium, the private supply (credit) loses its safety status during periods of market stress, irrespective of its maturity. [3]

On the theory side, the literature has focussed on understanding the deter-

[3] While recent literature has claimed that non-financial corporate debt is "safe", the size of the market for this debt is very small relative to the size of the total market in question. So we take a conservative approach, and abstract from this segment.

minants of those safe assets to assess potential sources of financial and economic fragilities. These studies, however, focus on different aspects and roles of safe assets. On their role as store of value, He et al. (2019) highlights that an asset is safe if others expect it to be safe. [4] Due to these strategic complementarities, safety relies on a coordination component as opposed to (exclusively) on the income process backing the asset. In this aspect, the relative fundamentals, instead of the absolute fundamentals, and debt size are important determinants of safety, explaining why government bonds, especially in the US and Germany, are a quintessential source of supply. [5] Brunnermeier et al. (2022), instead, posits that when markets are incomplete, governments bonds are a desirable hedging instrument as they can be sold after an adverse shock, while private alternatives are subject to uninsurable idiosyncratic risk.

Financial intermediaries' role in supplying safe stores of value, through either securitization or maturity transformation, has been extensively studied in the literature. The securitization process favoured the surge of novel financial instruments with safety attributes (e.g., highly rated asset-back securities) by enhancing risk-sharing though the assembly of loans into diversified pools and designating different classes of claimants on the pool's cash flow, where senior tranches are likely to have high credit quality (Gennaioli et al. (2012), Diamond (2020), Segura and Villacorta (2020)). On the other hand, safety can also be ensured by making debt short-term, that is, granting seniority through an early exit option (e.g., Stein (2012), Hanson et al. (2015)).

Another strand of the theoretical literature studies the collateral role of safe assets, for which liquidity considerations matter. [6] In the presence of informational

[4] In the same vein, Brunnermeier et al. (2022) draw the attention of two key characteristics of a safe asset. The first is the good friend analogy. A safe asset is like a good friend – valuable and liquid when one needs it. The second characteristic is the safe asset tautology, that is, a safe asset is safe when it is perceived to be safe so that in times of crisis investors flock to it.

[5] The model is useful to assess the ongoing proposal of Eurobonds. They argue that issuing a common Eurobond, it could compete with US bonds in size, and that all countries benefit from investors' need for a safe asset, as opposed to just one country (Germany) which is the de facto safe asset in the absence of a coordinated security design.

[6] Liquidity measures the difference between the market and the fundamental value of an asset to capture the asset's ability to convert future cash flows into current resources. In a perfect capital market environment, liquidity is a redundant concept as illustrated by the Arrow-Debreu

frictions, "information insensitivity" becomes a salient feature of safe assets (see Gorton (2017)). This builds on the premise that no agent has incentives to acquire private information about the value of the security, so safe assets can be transacted without much analysis or concern for adverse selection (Gorton and Pennacchi (1990), Vi et al. (2020), Moreira and Savov (2017)). Therefore, financial intermediaries' ability to share and distribute risk, to ameliorate financial frictions inherent in credit or secondary markets, or to cope with market incompleteness determine their supply level.

This introduction to the safe asset literature, while brief, is informative of the type of instruments that constitute the safe asset universe, in addition to the factors driving those assets into this selective group. For a more exhaustive review of this literature, see Gorton (2017). In the next section, we track the evolution of safe assets positions in the different sectors in the economy (e.g., households, financial corporations, etc.), evaluating the main trends of safe asset demand and supply in recent decades.

1.3 Trends in safe asset supply and demand

In this section, we use the international financial accounts data in Diebold and Richter (2021) to construct a new cross-country dataset of safe assets and liabilities of different institutional sectors. We restrict our analysis to 15 OECD countries, namely, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, Norway, Spain, Sweden, UK, and the US.

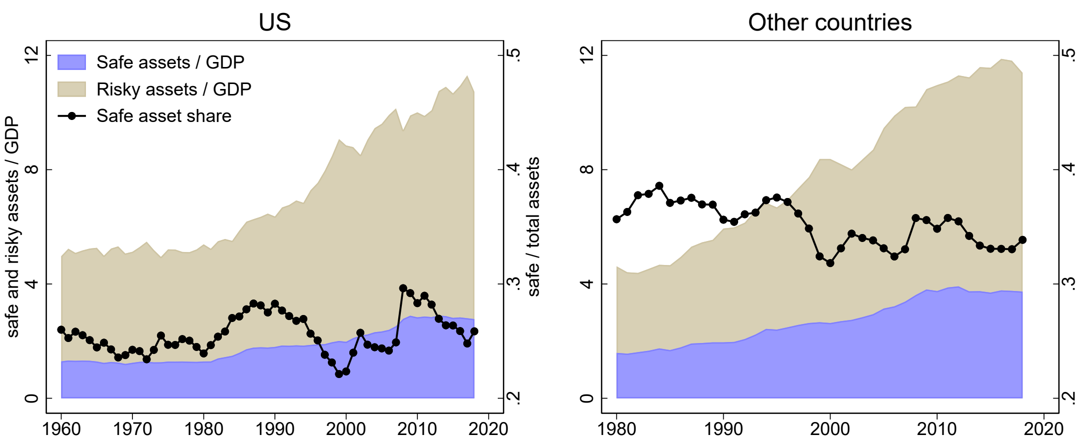
This database provides information about different instruments on financial balance sheets for the following sectors: general government, central bank, financial institutions (both banks and non-banks), the real sector (households, Non-Profit Institutions Serving Households (NPISH), and non-financial firms), and rest of the

complete state-contingent framework in which every asset is liquid. Hence, liquidity matters in the presence of financial frictions, and depending on the friction under consideration liquidity takes one form or the other.

world (i.e., the foreign sector within each the country). Broadly consistent with the characterization of safe assets from the previous section, and in line with Gorton et al. (2012), we define safe assets as bonds issued by governments and financial institutions in these economies. This class of assets comprises currency, gold, special drawing rights (SDRs), deposits, money market mutual funds' shares and bonds issued by governments and financial corporations within each country. This leaves non-financial bonds, loans, shares, insurance and pension assets, derivatives and stock options and other accounts receivable as risky assets. Refer to the appendix for further details on the instrument.

The relevant facts about the trends in safe assets positions are outlined next. Most studies have focussed on the US, so we pursue a comparison between the US and the rest of the countries in the sample.

Figure 1.1: Safe and risky assets relative to GDP

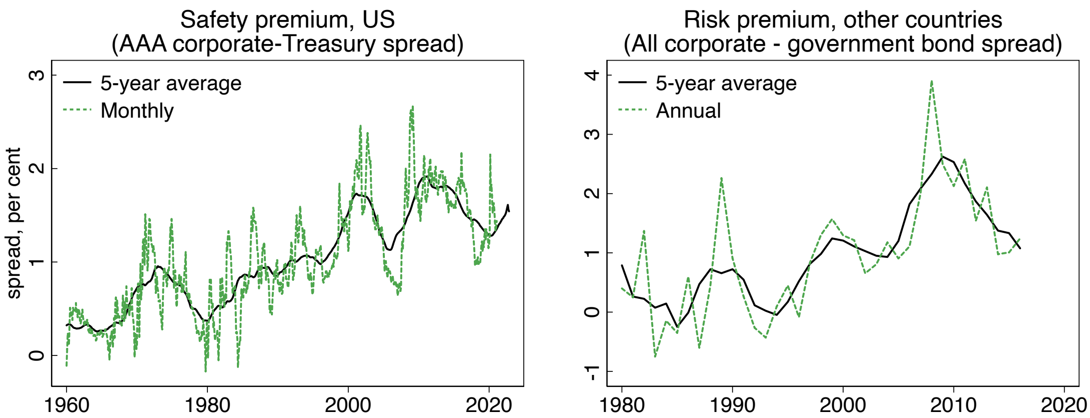


Notes: These graphs plot the domestic financial liabilities as a fraction of the country's GDP, and its decomposition according to the risk category based on the definition provided earlier. The graph on the right-hand side plots the unweighted average of the countries in the sample, excluding the US. The safe asset share is the fraction of the total financial assets that are safe.

Figure 1.1 plots the evolution of total financial assets as a fraction of GDP. There has been a significant increase in the amount of total assets as a fraction of GDP in recent decades, and this trend is common to all the countries in the sample (see Figure 1.9 in the appendix for a country breakdown). For instance, the US's total financial assets as a fraction of GDP has doubled since the late 1980s, from 5 to over 11. On average, the trends and magnitudes are similar in the rest of the countries in our sample.

This figure further shows that assets under both risk categories have increased in tandem, equally contributing (in relative terms) to the expansion of the total financial assets. Hence, the safe assets share, i.e., the fraction of the total financial assets that are safe under the provided definition, has stayed relatively constant. Gorton et al. (2012) documented this fact for the US. We extend the analysis to a larger set of countries and a longer horizon and find that the stability of the safe asset share holds on average, even if it gravitates at a higher level. This aggregate measure, however, hides important cross-country differences.

Figure 1.2: Safety and risky premium

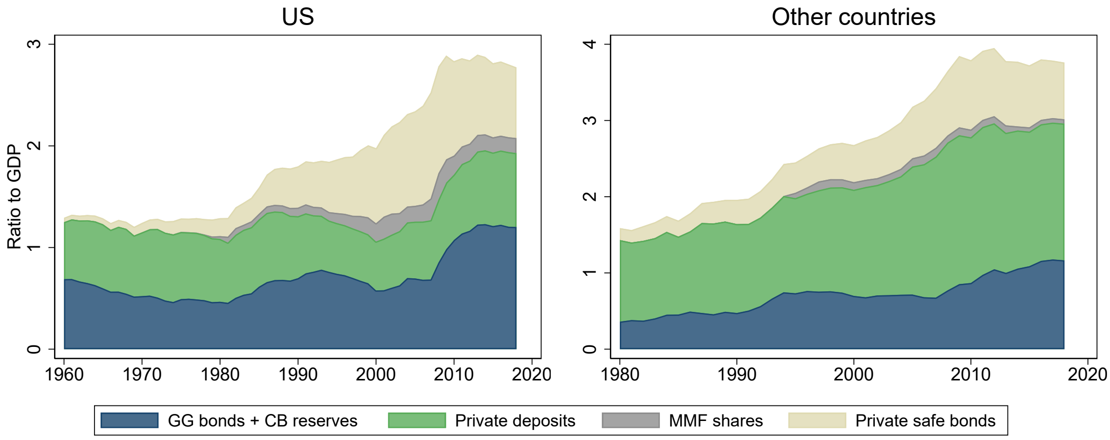


Notes: These graphs are building using the same data and methodology described in Kuvshinov and Zimmermann (2021). The left-hand side figure plots the spread between the AAA corporate bonds and the treasury yields for the US, which Krishnamurthy and Vissing-Jorgensen (2012) claims that captures the safety premium.The right-hand side figure plots the spread between between the 10-year corporate bond and government bond yields an unweighted averages of 17 countries.

The stability in the safe assets shares contrasts with the steady increase in the risky premium starting in the late 1990s, when the raise in quantities became most pronounced (see Figure 1.2). In light of these developments, a growing body of literature has claimed that the demand outstripped the supply, generating a structural shortage of safe assets (see e.g.Caballero and Krishnamurthy (2009), Bernanke et al. (2011), Caballero et al. (2017), Del Negro et al. (2019)). Therefore, understanding the supply/demand forces behind the observed increase across countries is of prime importance.

Let us first focus on the supply side. Figure1.3 shows a breakdown by issuer and financial instrument, which allows us to tract both sources of supply, namely, public supply by governments and central banks and private supply by the financial sector.

Figure 1.3: Supply composition: safe liabilities by issuer and instrument



Notes: In the appendix, a more detailed description of each of the plotted variables is provided. The graph on the right-hand side plots the unweighted average of the countries in the sample, excluding the US.

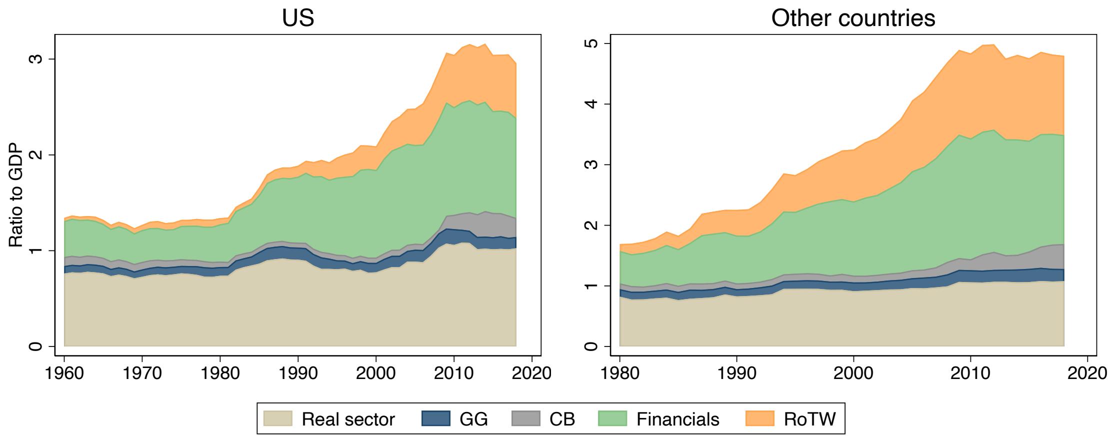
Figure 1.3 outlines some differences between the US and other countries. Traditionally, government debt and publicly insured deposits in commercial banks have been the dominant forms of safe-asset supply (and in some cases this remains true). But cross-country differences in the composition of supply have emerged in the wake of recent developments.

In the US, the pre-crisis increase in supply was driven by financial bonds. This observation is consistent with that of Gorton et al. (2012) who further argued that financial innovation enabled the expansion of the safe category to include commercial paper, money market funds, repurchase agreements and highly rated asset-backed securities. These financial instruments became the motor of the growth in that period. Extending this analysis after 2012, however, reveals that in the aftermath of the global financial crisis, that trend was partially reversed; there has been a switch from financial to government bonds. In this aspect, Figure 1.11 in the appendix illustrates that the public safe assets share in the US, i.e., the fraction of total supply that is issued by governments and central banks, has reverted to the level it was prior to the onset of the boom in the late 1980s.

Outside the US, private bonds have gained momentum, yet, on average, there has not been such a remarkable shift in the supply composition compared to the US: there is much more of a steady increase in the supply by all sectors and instruments. In addition, deposits are much more important, in terms of both size and growth. There is some substitution between private and public bonds in crisis, but it is more modest than in the US. The public safe asset share shows heterogeneous dynamics across countries prior to the global financial crisis (see Figure 1.11 in the appendix), but the increase in the share during the crisis was almost unanimous (except in Sweden). In most cases, the soaring public debt, and the deceleration in the private supply both contributed to the tilt towards public supply.

Related to the demand side, Figure 1.4 plots the safe asset holdings by sector.

Figure 1.4: Demand composition: safe assets by holder



Notes: The real sector comprises households, non-profit institutions serving households (NPISH), and non-financial firms. GG stands for general government and CB for Central Banks. Financials comprises all financial institutions in the economy except the central bank. RoTW stands for rest of the world, which includes foreign sector's safe asset holding within each country.

The role of the financial sector and the foreign sector is salient, with both sector's demand largely explaining most of the growth. On the contrary, the safe asset holdings by the real sector have stayed relatively constant, consistent with the observation pointed out of Gourinchas and Jeanne (2012). The increase in central banks' safe asset demand, amid a lax monetary policy environment, has been noticeable but still accounts for a small fraction of the total. Figure 1.12 in the appendix shows that this pattern largely holds across the different countries. Let us further explain these points.

First, the financial sector does not only play an important role in issuing safe assets, but it also holds an increasing number of safe financial instruments in the asset side. This suggests that a non-negligible fraction of the outstanding amount of safe assets stays within the financial sector. Hence, this sector's gross position partially captures the increase in size and complexity of the financial sector. [7] Previous literature has highlighted a confluence of factors consistent with this observation. Collateral has become instrumental in many financial transactions for the purposes of mitigating counterparty risk in derivatives and settlement systems. The collateral in these cases is required to be extremely safe, thus, it also drives the demand for safety (Gorton and Ordoñez (2022)). Safe and liquid assets are integral to prudential regulations, influencing, at least in part, the number of safe assets on banks' balance sheets (Iorgova et al. (2012)).

Second, post-crisis, the upward trend of financial intermediaries' holding of safe assets became smoother, or in many cases abruptly declined, in parallel with their safe asset supply dynamics. During this period, the generalized monetary policy response to the crisis led to an expansion of central banks' balance sheet, including their holdings of safe assets, partially compensating for the shrink of the private financial sector. Still, the magnitude of central bank's holdings accounts for a small fraction of the total. See Figure 1.12 in the appendix for further details.

Third, the foreign sector's holdings have experienced the biggest increase in relative terms. Financial integration has surely contributed to this development.

[7] The financial balance sheet data we are using is partially consolidated. Thus, the gross position capture the lower bound of the increase in size of the financial sector.

Thus, it is important to assess if the increase in foreign demand has followed suit an increase in foreign supply (domestic demand for foreign safe assets), or if, alternatively, the foreign sector was effectively searching for safety within these countries.

To shed light on this issue, and to further evaluate how the financial sector's supply of safe assets backed by risky assets evolved, let us focus next on the evolution of net safe asset supply. Net safe asset supply is defined by:

$$Net\ safe\ supply_{i,j,t} = Safe\ Liabilities_{i,j,t} - Safe\ Assets_{i,j,t}$$

where i, j, t are country, sector and year indexes. A positive net safe asset supply captures the safe liabilities that are backed by risky assets. A negative net safe asset supply captures the safe assets funded with risky financial liabilities.

Figure 1.5 and Figure 1.6 show some differences with respect to the previous analysis that are worth mentioning. When taking net positions into account, the increase in safe assets halves, highlighting the importance of the safe assets that stay in the financial system. Although the private share drops significantly, the financial sector still plays an important role as a net supplier of safe assets. In addition, the private net supply is more cyclical compared to the gross supply, and the substitution with the public supply becomes even more pronounced during crisis. This observation which is consistent with Kacperczyk et al. (2021), who claims that the economic distress taints the ability of the private sector to produce safe assets. See figure 1.13 and figure 1.14 in the appendix for a country decomposition.

On the other hand, the foreign sector is a net demander in the analysed countries and period. This sector is the largest contributor to the rise in safe asset's demand in most countries. The role of the US in attracting capital flows that search for safety has been largely studied in the literature. The early literature, brought to light by Bernanke (2005), claims that the "saving glut" in Asia, most notably in China, had created severe and persistent imbalances, where the capital inflows to the US from other countries significantly increased, pushing long-term interest

Figure 1.5: Net supply and demand for the US

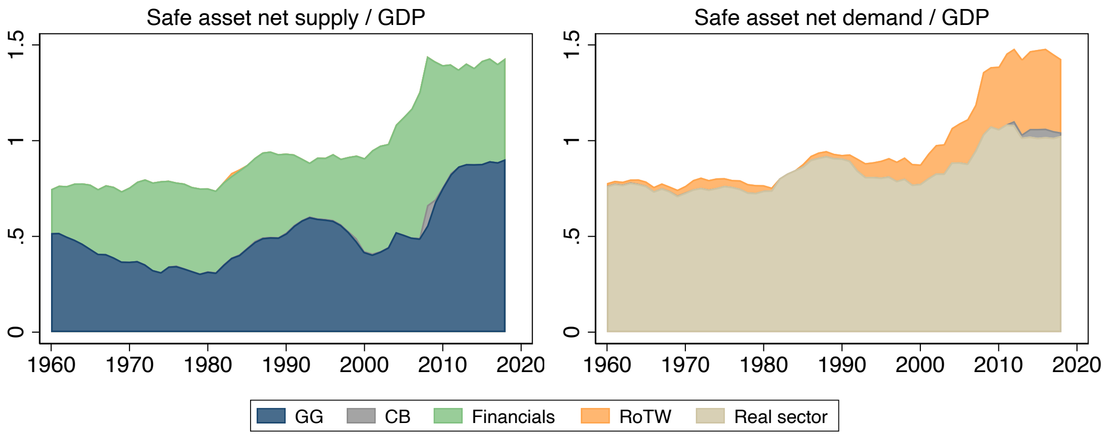
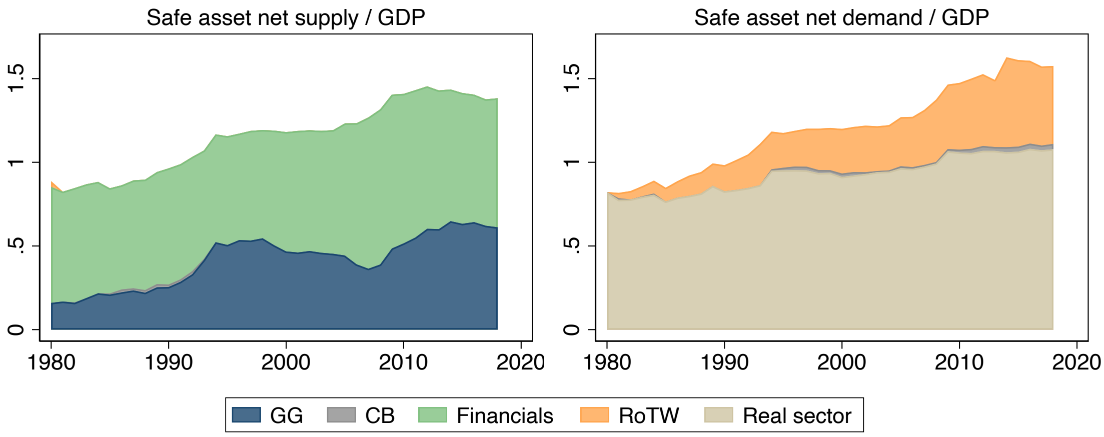


Figure 1.6: Net supply and demand for the rest of the countries



Notes: Net supply and demand have been calculated following the procedure introduced above.

rates to historically low levels. It was later discovered that a large share of these flows was invested in assets perceived to be safe (Caballero and Krishnamurthy (2009), Bernanke et al. (2011)). Ever since, the literature on US safe assets has proliferated. However, this phenomenon does not seem to be exclusive to the US. Here we find that other industrialised countries might also play an important role in supplying safety to the rest of the world. [8]

[8]Even if the presented foreign net supply might partially capture the interactions between the countries in the sample, the fact that they all have a positive net foreign demand indicates that countries outside the sample might play a role (not necessarily emerging markets).

1.4 Conclusion

This cross-country study of safe asset demand and supply shows that the increase in the (gross) amount of safe assets is mainly explained by financial intermediaries' supply, even if there has been a tilt towards public supply in the aftermath of the crisis, and the financial and foreign sector's demand. This trend partially captures the increase in size of the financial system, as a rising number of safe assets stays within the financial sector. The upward trend in financial intermediaries' net safe asset supply is more modest and it becomes more cyclical, while the substitution with the public supply becomes more pronounced. The increase in the net demand is largely accounted for the foreign sector's holdings, suggesting that the international demand for the US's safe assets, as well as for the rest of the countries in our sample, has expanded over recent decades.

Appendix

Diebold and Richter (2021) extend the data in table *720. Financial balance sheets - non consolidated-SNA 2008* provided by the OECD, by digitalizing the early publications of the series that were in paper. This variables are measured in national currency and current prices. Next, the OECD's description of the instruments considered is provided:

1. **Monetary gold and SDRs**

   a. **Monetary gold:** Monetary gold is gold to which the monetary authorities (or others who are subject to the effective control of the monetary authorities) have title and is held as a reserve asset.

   b. **Special Drawing Rights (SDRs):** Special Drawing Rights (SDRs) are international reserve assets created by the International Monetary Fund (IMF) and allocated to its members to supplement existing reserve assets.

2. **Currency and deposits**

   a. **Currency:** Currency consists of notes and coins that are of fixed nominal values and are issued or authorized by the central bank or government.

   b. **Transferable deposits:** Transferable deposits comprise all deposits that (i) are exchangeable for bank notes and coins on demand at par and without penalty or restriction; and (ii) are directly usable for making payments by cheque, draft, giro order, direct debit/credit, or other direct payment facility. Transferable deposits should be cross-classified according to whether they are denominated in domestic currency or in foreign currencies; and whether they are liabilities of resident institutions or the rest of the world. Can be disentangled between Inter-bank positions and other transferable deposits.

c. **Other deposits**: Other deposits comprise all claims, other than transferable deposits, that are represented by evidence of deposit. Typical forms of deposits that should be included under this classification are savings deposits (which are always non-transferable), fixed-term deposits and non- negotiable certificates of deposit.

3. **Debt securities:** Debt securities are negotiable instruments serving as evidence of a debt. They include bills, bonds, negotiable. certificates of deposit, commercial paper, debentures, asset- backed securities, and similar instruments normally traded in the financial markets.

   a. Short-term debt securities include those securities that have an original maturity of one year or less. Securities with a maturity of one year or less should be classified as short-term even if they are issued under long-term facilities such as note issuing facilities.

   b. Long-term debt securities include those securities that have an original maturity of more than one year. Claims with optional maturity dates, the latest of which is more than one year away, and claims with indefinite maturity dates should be classified as long-term.

4. **Money market fund shares /units:** Money market funds are investment funds that invest only or primarily in short-term money market securities such as Treasury bills, certificates of deposit and commercial paper.

Based on Gorton et al. (2012)'s definition, we can measure safe liabilities precisely. For safe assets, we only have the total for bonds, so we have to take out the non-financials, and split the safe bonds between government and financials. We do this based on a mix of sectoral liabilities and US data, which has the detailed split. In particular, to break bond assets into public safe bonds, private safe bonds, and private risky bonds, we rely on the apportioning method. Broadly speaking, we split total country-specific bond liabilities into risky, private safe, and public safe, and then adjust the liability shares by the preferences of each sector (e.g., the fact that the central bank holds almost exclusively public bonds) using the more detailed data from the use flow of funds accounts (FFA). The step by step procedure is as follows:

1. Calculate the relative size of bonds issued by (i) public entities (ii) financials, and (iii) non-financials. This we do for each country and year.
2. Calculate the "adjustment factors" of how much more of each type of bonds each sector holds relative to total liabilities. For example, households hold relatively more public bonds, and banks hold relatively more financial bonds. This is for US data only.
3. Split the bond assets using the liability ratios in 1. multiplied by the adjustment shares in 2.

An example of this methods is illustrated next:

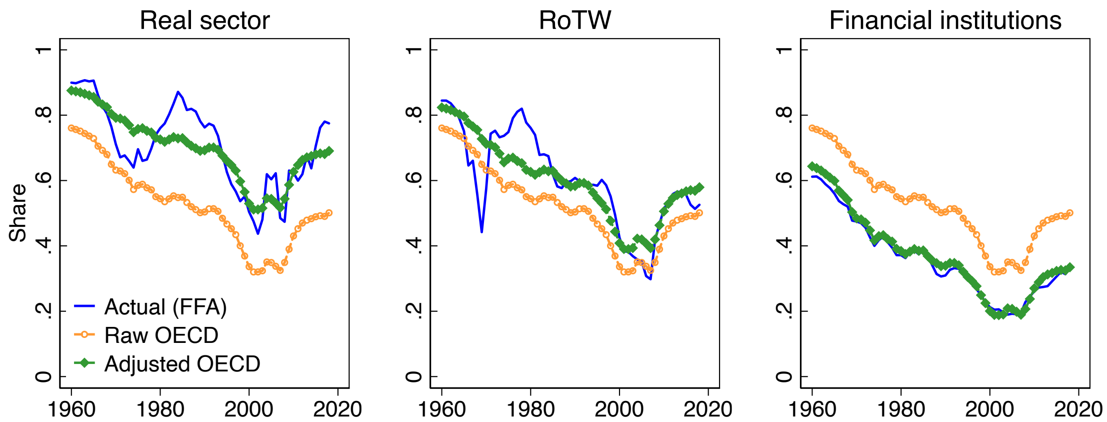
$$\begin{aligned}\text{Government bond holdings of the real sector}_{i,t} &= \frac{\text{Government bond liabilties}_{i,t}}{\text{Total bond liabilities}_{i,t}} \\ &\times \text{Real sector preference,} \\ \text{where Real sector preference} &= \left(\frac{\text{Real sector government bond assets}}{\text{real sector total bond assets in US}}\right) \\ &\times \left(\frac{\text{Government bond liabilities}}{\text{total bond liabilities in US}}\right)^{-1}\end{aligned}$$

Above, i and t are country and year indices, so we multiply the share in bond liabilities by the relative preference of the real sector for government bonds estimated from US data for 1960-2020.

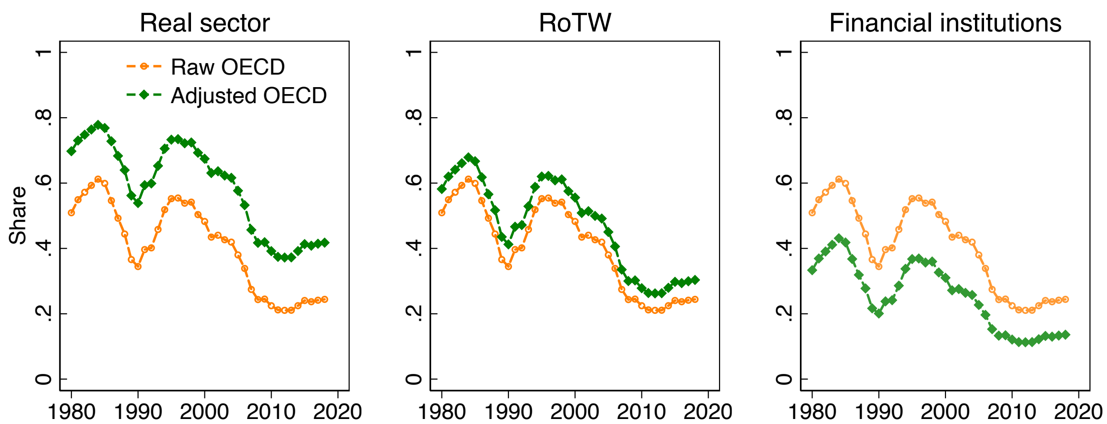
Figure (1.7) illustrates the results. It shows the three main sectors that demand safe assets: private real sector (households, NPISH, and non-financials), foreign sector, and the financial sector. It then plots the share of public safe bonds relative to total bonds, for each of these sectors, in actual FFA data, raw OECD data on sectoral liabilities, and OECD liability share adjusted for sectoral preferences based on FFA data. If sectoral preferences for specific bond types are relatively stable across countries and time, this method will work relatively well. Figures (1.7) plots these series for the US and shows that the apportioning method (green diamond) applied to raw OECD data (orange circles) gives a reasonably close match to actual holdings data (solid blue line). Figure (1.7) shows how this method is applied to adjust the ratios for Sweden. For example, in the left panel we adjust the real sector public bond share up, because we know from US data that the real sector holds relatively more public bonds compared to other sectors of the economy. We apply this procedure to all countries, and all five main economic sectors (real, financial, government, central bank, rest of the world).

Figure 1.7: Share of public bonds in total bond assets held by each sector

(a) United States:

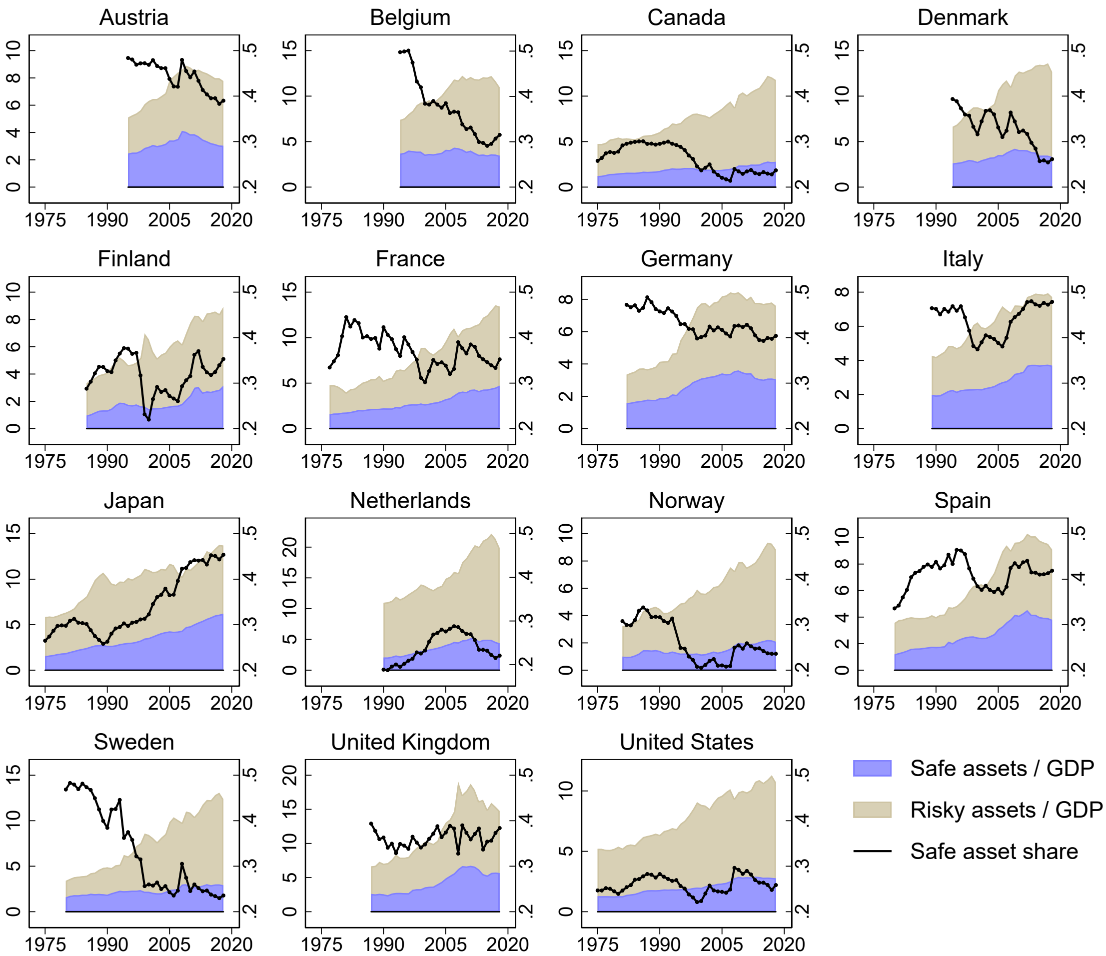


(b) Sweden:



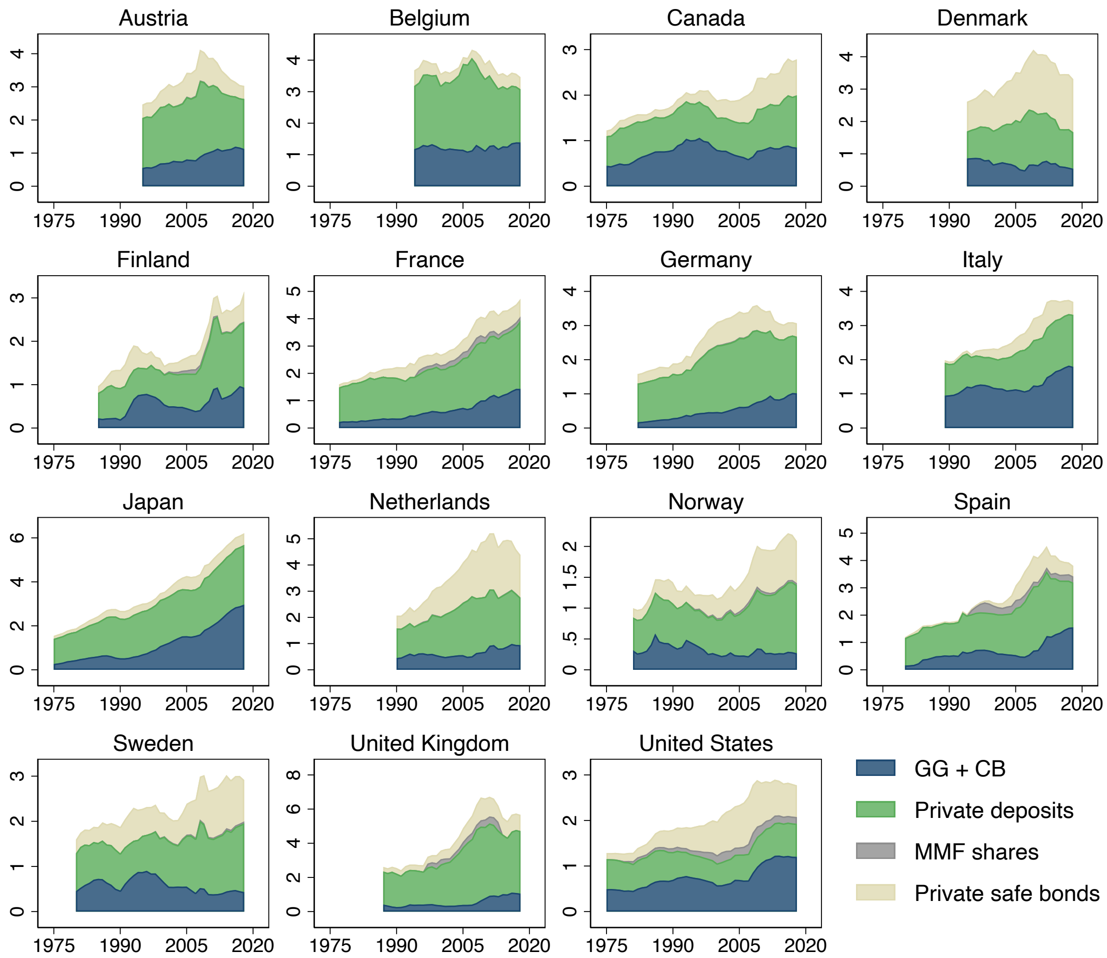
Figures

Figure 1.9: Safe and risky assets as a fraction of GDP by country



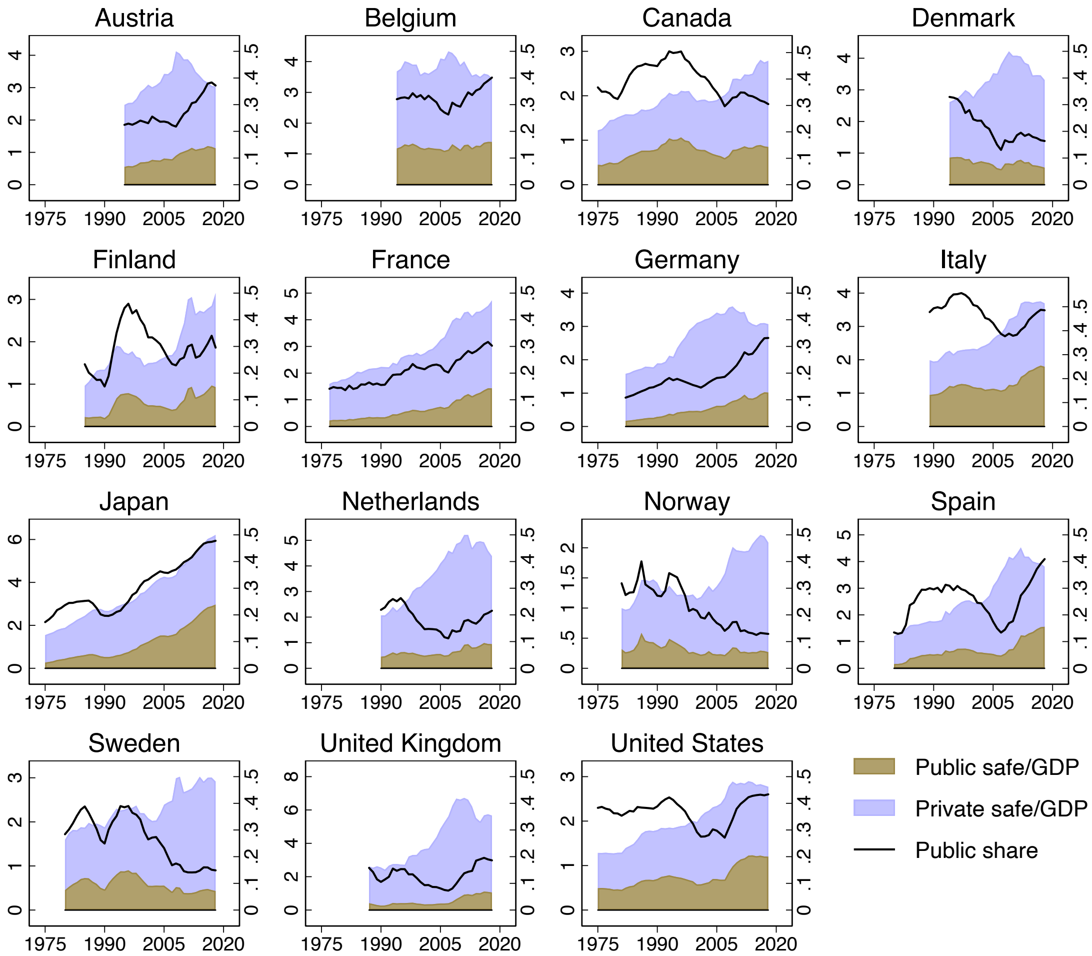
Notes: These graphs plot the domestic financial liabilities as a fraction of the country's GDP, and its decomposition according to the risk category based on the definition provided earlier. The safe asset share captures the fraction of the total financial assets that are safe.

Figure 1.10: Supply composition: safe liabilities by issuer & instrument and country



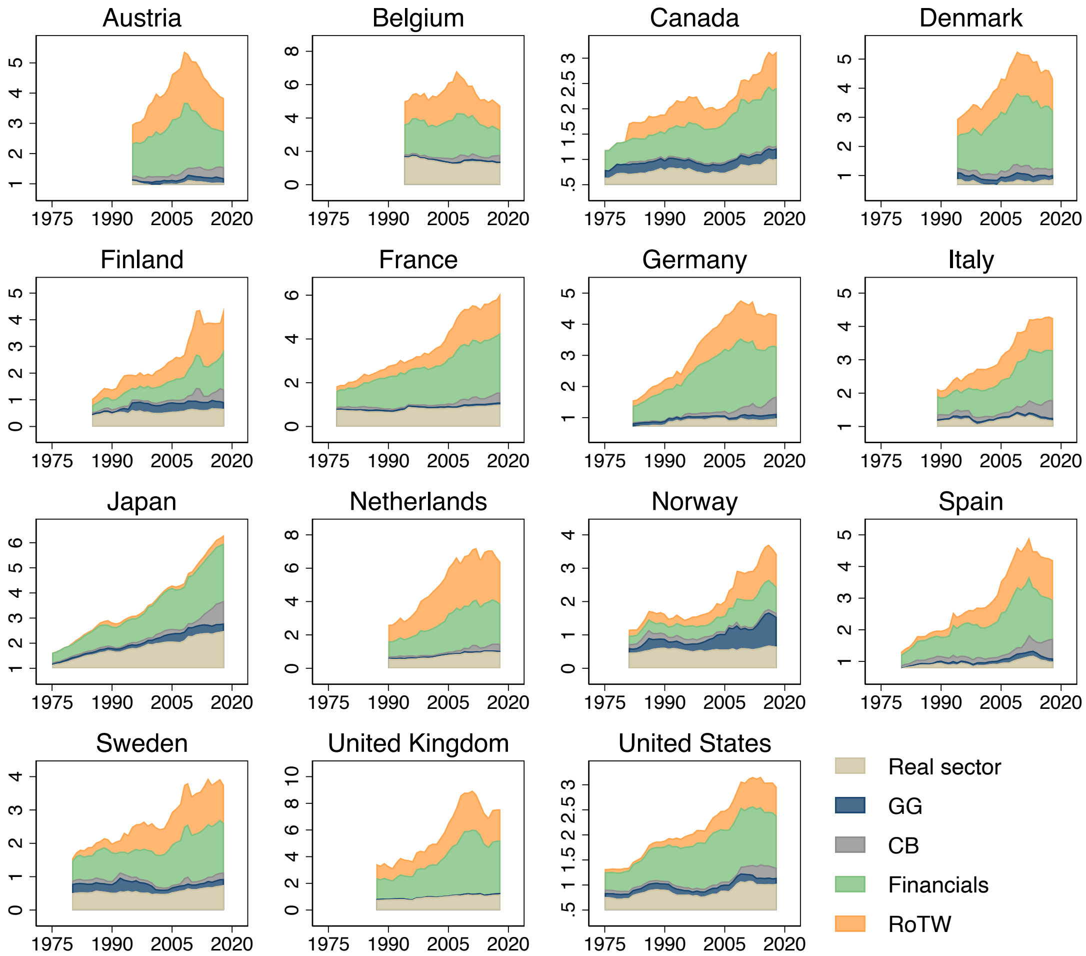
Notes: These graphs plot the safe liabilities issued by domestic government, central banks, and financial institutions. The public supply comprises bonds issued by the general government and Central Bank's reserves. The private supply comprises deposits, money market mutual funds' shares, and bonds issued by financial institutions. The public share captures the fraction of the total supply that is public.

Figure 1.11: Supply composition: public and private by country



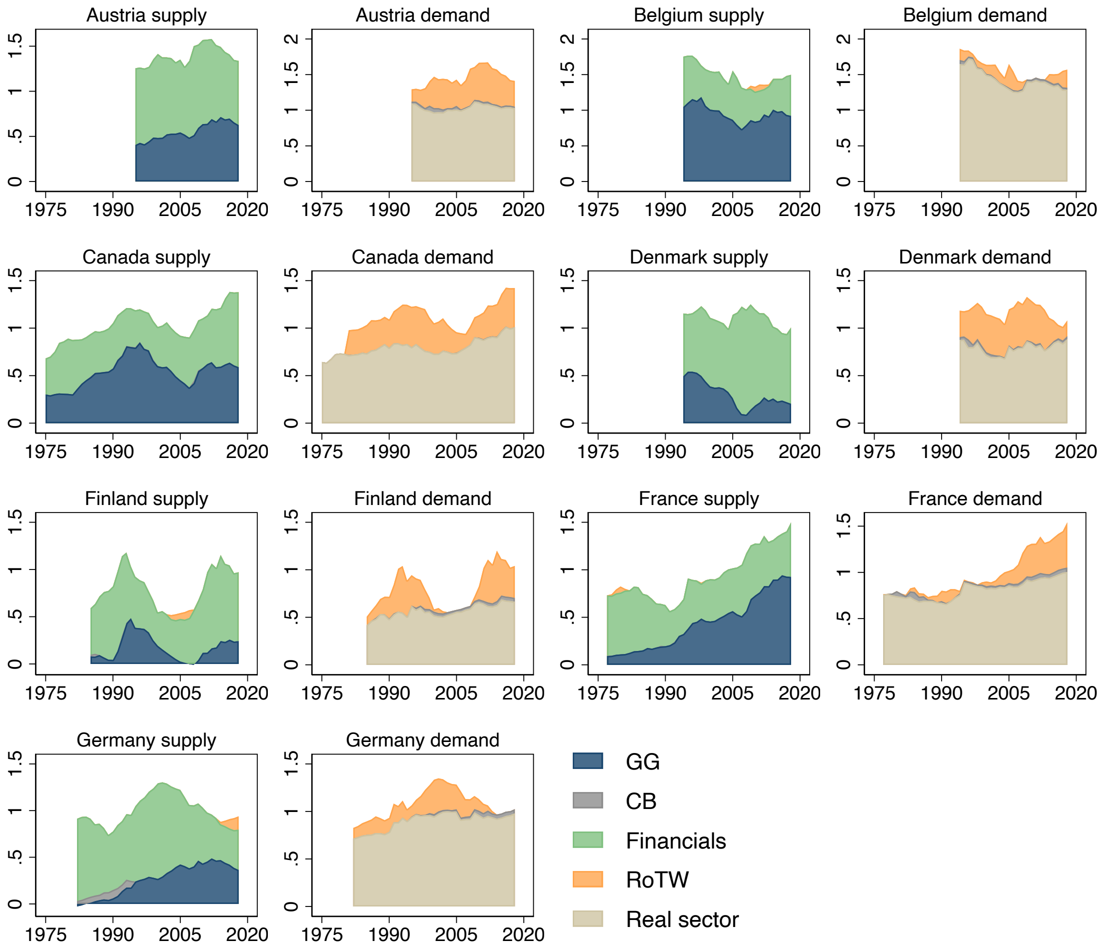
Notes: The public supply comprises bonds issued by the general government and Central Bank's reserves. The private supply comprises deposits, money market mutual funds' shares, and bonds issued by financial institutions. The public share captures the fraction of the total supply that is public.

Figure 1.12: Demand composition: safe asset by holder and country



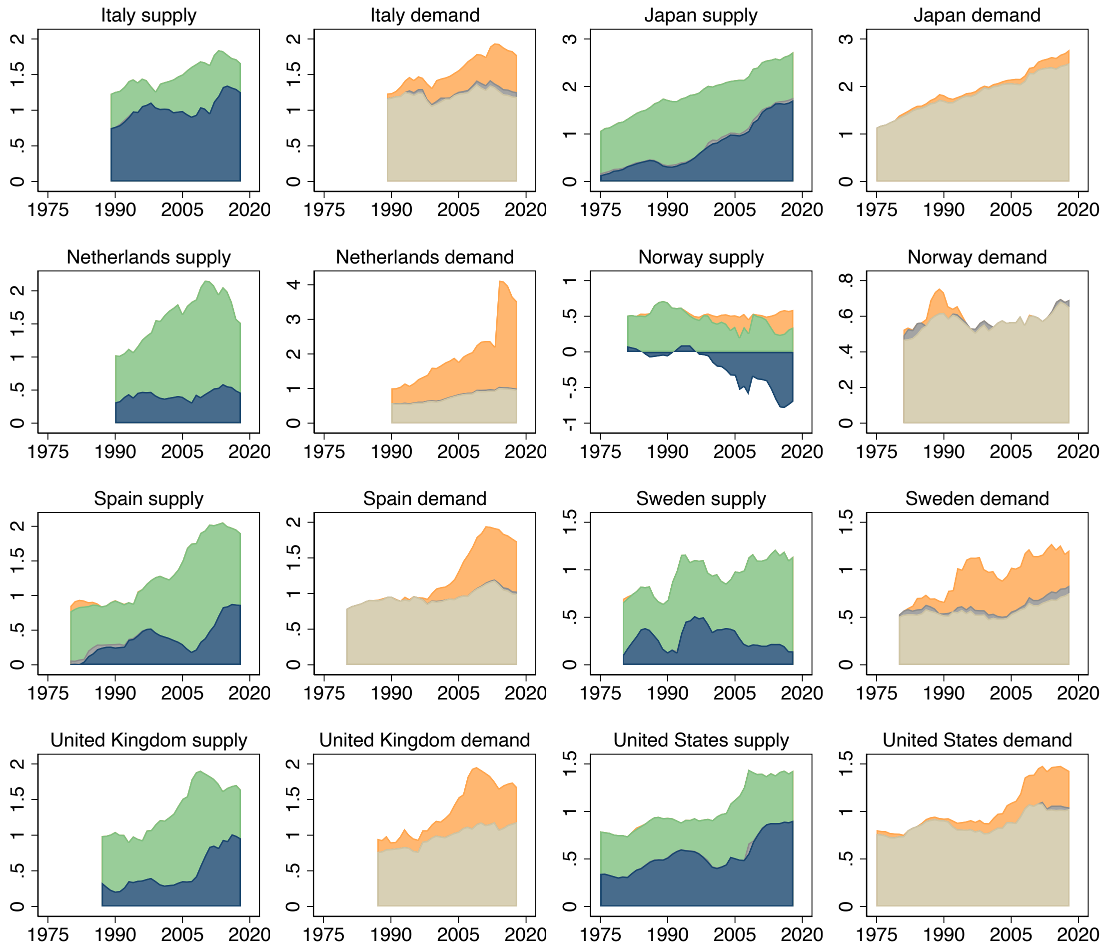
Notes: The real sector comprises households, Non-Profit Institutions Serving Households (NPISH), and non-financial firms. GG stand for General Goverment and CB for Central Banks. Financials comprises all financial institutions in the economy except the Central Bank. RoTW stands for rest of the world, which includes the foreigners' safe asset holding within each country.

Figure 1.13: Net safe supply and demand by country: A–G



Notes: Net supply and demand have been calculated following the procedure introduced above.

Figure 1.14: Net safe supply and demand by country: I–U



Notes: Net supply and demand have been calculated following the procedure introduced above.

Chapter 2

Private Safe-Asset Supply and Economic Instability

2.1 Introduction

There is growing academic and policy interest in safe assets, i.e., assets deemed to maintain a stable value over time and to be immune to adverse macroeconomic shocks. Traditionally, government debt and publicly insured deposits in commercial banks have been the quintessential forms of safe asset supply. However,the high global demand for safety has encouraged the private sector, especially the US financial sector, to manufacture private substitutes, e.g., senior tranches of asset-backed securities. [1]

The rise in private safe asset supply by financial intermediaries has been under severe scrutiny; evidence suggests that it contributed to the deterioration of lending standards in the run-up to the financial crisis, and financial intermediaries' incentives to properly monitor and screen borrowers has been called into question. [2] However,

[1] See e.g., Caballero and Krishnamurthy (2009), Bernanke et al. (2011), Krishnamurthy and Vissing-Jorgensen (2015).

[2] See e.g., Berndt and Gupta (2009), Dell'Ariccia et al. (2012), Keys et al. (2010), Mian and Sufi (2009).

the interplay between private safe asset supply and the quality of the investment that financial institutions intermediate [3] is still not well understood, which raises important questions: How is the private supply of safe assets determined? What are its implications on economic stability? Is the private supply efficient? Or, is there room for policy intervention?

To address these positive and normative questions, I develop a theoretical framework that explores jointly assets' risk and quality transformations by financial intermediaries in an environment, in which there is a strong demand for safety [4]. The model builds on the tension between intermediaries' risk-sharing and quality-enhancing incentives (e.g., screening, monitoring, etc.) due to a moral hazard problem, and finds that private safe asset supply factors negatively affect aggregate productivity and volatility. An analysis of the normative properties of the model reveals a novel source of inefficiency that calls for policy intervention and highlights the need to understand the interaction between different forms to ensure safety for an adequate and effective policy response.

I consider a two-date economy populated by infinitely-risk averse households and risk-neutral bankers who have access to a risky investment project whose productivity depends on two factors: the state of nature (aggregate and idiosyncratic state) and an unobservable effort exerted by the owner of the project, that improves the quality of the investment – increases the expected output and reduces its risk. The main focus lies in understanding the interplay between bankers' investment/effort activity with their ability to transform the risky output of their projects into safe financial claims (safe debt), catering to households' demand for safety.

To meet their debt obligations in every contingency, bankers need to insure against their idiosyncratic risk; they can sell financial claims backed by their risky project's output and buy in exchange a diversified pool of claims sold by other

[3] The theory of financial intermediation has extensively emphasised the role of financial intermediaries in monitoring and screening borrowers in the process of lending. In this way, they are better than markets at resolving informational problems among others, enhancing the quality of the investments they intermediate.

[4] For simplicity I define safe assets as non-contingent claims, yielding a stable payoff across states of nature.

bankers. Hence, bankers' debt capacity, and the amount of safe assets in the economy, positively depends on the risk-sharing level (quantity of claims traded) and effort level (quality of claims traded). The unobservability of effort, however, causes the standard trade-off between risk-sharing and effort due to a moral hazard problem: bankers' incentives to exert effort depend on their *skin-in-the-game* – the output retained to gain exposure to the productivity increase induced by their effort. Thus, bankers will optimally decide on the amount of output to sell in financial markets, balancing the benefits of risk-sharing and the costs on incentive provision.

How does the negative relationship between the two inputs of safe assets shape their supply curve? The latter monotonically increases with the price of safety (the inverse of safe interest rate), but the positive slope hides an important insight: the risk-sharing and effort mix changes along the curve. That is, as the price of safe assets increases, or the safe interest rate decreases, bankers' optimal fraction of output sold increases, improving the risk-sharing while reducing the skin-in-the-game, which jeopardizes their effort incentives.

This collective drop in effort is detrimental to economic stability. The quality of individual projects deteriorates, which reduces future expected output. In addition, when the marginal product of effort is higher in bad states of the economy, the collective drop in effort, not only decreases aggregate investment's productivity but it further amplifies its volatility. This negative real effects of the private supply of safe assets are particularly acute in the current environment where there is a scarcity of safe assets. [5] Hence, understanding the normative properties of the model is of prime importance.

If markets were (exogenously) complete [6] and bankers have no restrictions on the type of financial claims they can sell, the output sold for each unit of safe asset issued is minimised, ameliorating the burden of the moral hazard. That is achieved

[5] See, e.g., Caballero and Farhi (2018), Del Negro et al. (2019), Krishnamurthy and Vissing-Jorgensen (2012)

[6] Due to the informational friction, the model is not endogenously complete, in the same vein as in Biais et al. (2021).

by selling a higher fraction of the project in those aggregate states in which the expected output of the project is lower. Therefore, the output bankers' sell is not subject to aggregate risk, so after diversification, that output is safe, delimiting the amount of safe assets in the economy. Despite the cost of supplying safe assets, the competitive equilibrium is constraint-efficient.

This type of financial claims, however, is not used in practice, suggesting that the aforementioned environment is not a good representation of reality, even if it is a useful benchmark. To focus on widely observed market mechanisms, I assume that financial claims, can be contingent on the output (i.e., idiosyncratic state), but not on the economic conditions (i.e., aggregate state) [7]. The timing of the sale becomes important, and two relevant cases are carefully analysed: (1) when trade occurs before the aggregate shock is realized (*ex-ante risk-sharing*) and (2) when trade occurs at an intermediate date after a public signal of the aggregate state, but before the idiosyncratic risk is realized (*ex-post risk-sharing*).

In the first case, the claim bankers' sale is subject to aggregate risk; only the output in the worst aggregate state contributes to the build-up of safe assets. [8] The constraint in the claim space has non-trivial consequences. First, it affects the feasibility set in a relevant manner, as the security that efficiently deals with the moral hazard problem becomes unattainable, exacerbating the real costs of a private safe asset supply. Second, the decoupling of the output sold and the safe assets in the economy causes an inefficiency; bankers do not fully internalise that when they collectively increase their effort level they reduce aggregate volatility which boost safe assets in the economy. Hence, the competitive equilibrium is not constraint-efficient taking both the informational friction and financial contract constraint into account, and the supply of safe assets is inefficiently high, and effort inefficiently low.

[7] For instance, banks in the securitization process, sell a fraction of their loans, and that fraction will not vary (ex-ante) on the future economic conditions.

[8] Similar to the securitization process, safety is ensured through diversification by the assembly of financial claims into diversified pools, and through seniority by tranching the cash flows designating different classes of claimants.This process closely resembles that described in Gennaioli et al. (2012).

Despite the inability to trade state-contingent claims, bankers can trade after a public signal about the underlying economic fundamentals (second case). This alternative, ensure safety by trading contingent on the public signal, and using the proceeds to repay their debt. Following Stein (2012), I assume that there is a set of investors who have resources on this intermediate date, but their opportunity cost of purchasing bankers' financial claims is high, such that bankers need to sell their assets at a discount (which is endogenous in the model). Therefore, strategically postponing the risk-sharing activity circumvents some of the limitations posed by market incompleteness, but it carries a cost: fire-sales. Bankers do not internalize the effect of their collective effort on prices and this pecuniary externalities, coupled with the binding borrowing constraint due to the informational friction, creates an inefficiency that goes in the same direction as the one described above.

It is tentative to conclude, that policy makers should restrict the two forms of risk sharing. But before doing so, it is important to assess the interaction (if any) of these two alternatives. For a realistic parametrization of the model both, ex-ante and ex-post risk sharing, coexist. In particular, ex-ante risk sharing is complemented with ex-post risk sharing in the bad economic states. Bankers' ex-ante risk sharing yields a higher safe output in the good state, then ex-post risk sharing only happens in bad states, ameliorating the cost imposed by the (exogenous) market incompleteness while limiting the fire-sales cost.

Both inefficiencies described are still present, and reinforce each other amplifying the original one. In addition, the fraction of safe assets backed by ex-post risk sharing is inefficiently high, as opposed to ex-ante risk sharing. Policy responses aiming to ameliorate theses inefficiencies, however, must take a holistic approach and take into account that both forms are jointly determined. Otherwise, the policy can have unintended consequences, e.g., a policy targeting individual financial instruments could affect the form in which safe assets are produced, instead of affecting the debt level, which could potentially exacerbate the original inefficiency.

The outline of the paper is as follows. The contribution to the literature is discussed next. In Section 2.2, the model is introduced, and in the following sections, the equilibrium of the model is characterised and its efficiency is assessed in environments with different frictions. Section 2.3, assumes that markets are (exogenously) complete, Section 2.3.1 solves the model in a frictionless economy and Section 2.3.2 introduces an informational friction (non-observability of effort). Section 2.4 restrict the type of financial claims that can be traded, and analyses different forms of risk-sharing: Section 2.4.1 examines ex-ante risk-sharing, Section 2.4.2 examines ex-post risk-sharing and Section 2.4.3 examines jointly ex-ante and ex-post risk sharing. Finally, Section 2.5 concludes.

2.1.1 Literature Review

This study relates to several strands in the literature. First, it contributes to the literature on the private supply of safe assets. This paper is closely related to those centred on the manufacturing of safe assets through diversification [9] (e.g., Gennaioli et al. (2012), Diamond (2020), Segura and Villacorta (2020), and through asset sales that are contingent on economic conditions and costly in downturns (*fire-sales*) (e.g., Stein (2012), Hanson et al. (2015)). I contribute to this literature by examining the interaction between the different forms of manufacturing privatively safe assets, focusing on the interplay between safe asset supply by financial intermediaries and the quality of their investment. This quality channel, is closely related to the one described in Segura and Villacorta (2020), who focus on securitization; thus, their normative analysis is different.

The information-insensitive characteristic of safe assets has also been widely discussed in the literature (see Gorton (2017) for a detailed discussion). An essential feature of safe securities is that all investors are symmetrically informed (or ignorant) about their payoffs, and, therefore can trade them without fear of adverse selection, i.e., they are liquid. This builds on the premise that no agent has incentives to obtain private information about their value; thus, any factor that triggered an

[9] Diversification plays a crucial role in much of financial intermediation theory (e.g., Diamond (1984)); however, its role is to reduce the cost of asymmetric information rather than to meet a demand for safe assets as in my model.

abrupt asset-quality disclosure would set up the stage for a financial crisis (Dang et al. (2017), Gorton and Ordoñez (2022), Moreira and Savov (2017)). Unlike these papers, which focus on the information production of already existing assets, I focus on the information gathering of assets at their origination stages, which is conductive to better resources allocation, enhancing the investment quality. Thus, I emphasise a complementary source of fragility and, to keep things simple, I define safe assets as a completely riskless asset (binary characteristic), which is a sufficient condition to ensure that such assets are also liquid (Gorton and Pennacchi (1990)).

This paper also relates to the literature that centres on trade in secondary financial markets and the impaired incentives of the security issuer ascribed to informational frictions. In my model, the hidden action causes a moral hazard problem broadly similar to the one in Hartman-Glaser et al. (2012), Chemla and Hennessy (2014), Hébert (2018), Vanasco (2017), Biais et al. (2021). As in the aforementioned papers, cash flow retention is the disciplinary and signalling device to enhance incentives, as opposed to other banking paper that highlight the role of capital in enhancing incentives (e.g., Martinez-Miera and Repullo (2010)). As opposed to the optimal security design focus, the objective of this paper is to understand how the trade-off between risk-sharing and incentive provision, shapes the supply curve of safe assets, and to assess its efficiency in an environment in which markets are not complete.

Furthermore, the paper contributes to various ongoing debates. For instance, it sheds light on the debate on how the investment and financing decisions of financial intermediaries may be linked. The existence of a safety premium explains why corporations that can issue safe liabilities are highly levered, as debt becomes a cheap source of funding. This deviation from the Modigliani-Miller Theorem has important implications: financial intermediaries need to reduce asset-side risk to support greater amounts of safe debt, which is not necessarily in line with increasing the value of the firm. In this particular case, diversification fosters the production of safe liabilities; in turn, it reduces the quality of the investment and the economic value generated by the firm compared to an alternative firm that does not contemplate the production of safe debt.

2.2 Model set-up

The model has two dates ($t \in \{0, 1\}$), one perishable good that can be used for consumption or investment, and two groups of agents – both with a mass equal to one– that differ in their endowment and risk preferences: (i) infinitely risk-averse agents are endowed with $w < 1$ units of the perishable good at $t = 0$ and enjoy a utility given by $U^d = \min_z \left\{c^d_{1z}\right\}$, and (ii) risk-neutral agents are endowed with $1-w$ units of the perishable good at $t = 0$ and enjoy a utility given by $U^j = \mathbb{E}[c^j_{1z}]$, where c^i_{1z} is agent i's (where $i \in \{d, j\}$) consumption at $t = 1$ and state $z \in \Omega \neq \emptyset$.

Investment project. Each agent has access to a constant-return-to-scale investment project, whose stochastic productivity depends on two factors: the state of nature (*exogenous factor*) and the effort exerted by the owner of the project (*endogenous factor*), denoted by e^i for owner $i \in \{d, j\}$. The state of nature comprises the realization of two sources of risk: (i) aggregate risk, which is common to all projects and indexed by $\omega \in \{\mathbf{g}ood, \mathbf{b}ad\}$, and (ii) idiosyncratic risk, which is specific to each project (*iid* across projects) and indexed by $\iota \in \{\mathbf{s}uccess, \mathbf{f}ail\}$. The state of nature is indexed by $z = \langle\omega, \iota\rangle$. Conditional on the ω aggregate state that occurs with probability π_ω, each unit of investment at $t = 0$ yields the following payoff structure at $t = 1$:

$$A_\iota = \begin{cases} A & \text{with probability} \quad \theta_\omega(e^i) \\ 0 & \text{with probability} \quad 1 - \theta_\omega(e^i) \end{cases}$$

where $\theta_\omega : [0, \overline{e}] \to (0, 1)$ denotes the success probability for $\omega \in \{g, b\}$, which increases with good economic conditions, $\theta_g(.) > \theta_b(.)$ for $\forall e \in [0, \overline{e}]$, and owner's effort, $\theta'_\omega(.) > 0$, and $\theta''_\omega(.) = 0$ for $\omega \in \{g, b\}$ and $\forall e \in [0, \overline{e}]$. While increasing the likelihood of obtaining a high output, exerting effort entails (non-pecuniary) cost per unit of investment beard by the owner of the project and given by $c(e^j)$ where $c : [0, \overline{e}] \to \mathbb{R}_+$, such that $c'(.), c''(.) > 0$ and $c'''(.) = 0$ for $\forall e \in (0, \overline{e}]$, $c(0) = 0$, $c'(0) = 0$. In addition, $c'(\overline{e}) > \mathbb{E}[\theta'_\omega(\overline{e})] A$ is assumed to focus on the interesting and realistic case in which the upper bound of effort is never binding.

Financial markets and informational friction. In this economy, gains from trade stem from risk sharing to enhance the allocation of risk according to the heterogeneous risk preferences. The unobservability of effort, [10] however, yields a standard moral hazard problem when financial claims backed by the project's output are sold. This friction will play a crucial role in the model, as it will be clear soon, and it is the key foundation of the following lemma that simplifies the exposure of the problem.

Lemma 1 *In any equilibrium with unobservable effort, infinitely risk-averse agents invest all their resources in financial claims sold by risk-neutral agents who invest in their own project their resources along with the resources raised from infinitely risk-averse agents.*

Similar to standard problems with hidden actions, owner's incentives to exert effort are guided by the exposure to the effect of their actions, i.e., exposure to the output of their project. Paradoxically, infinitely risk-averse agents have no incentives to exert effort given that they value stochastic output in the worst-case scenario, which is independent of their effort. In contrast, risk-neutral agents value the increase in the expected output of the fraction of the project that they retain –not sold–. They are more efficient in dealing with the moral hazard problem, having a comparative advantage in managing the investment technology and attracting infinitely risk averse agents' resources.

Notice, however, that the lemma is silent about the type of financial claim issued by risk-neutral agents, henceforth *bankers*. To be consistent across sections, I will assume, without loss of generality, that bankers issue safe debt which is optimally bought by infinitely risk-averse agents, henceforth *depositors*. In turn, depositors chose the safe debt contract that is cheapest. The main focus of the model is understanding the interplay between bankers' investment/effort activity

[10] If the mapping between effort and output were completely deterministic, outside investors would have no difficulty in inferring each agent's effort from the observed output; thus, it could be indirectly contracted upon, since output would itself be observable and verifiable. But effort is not observable, and since the realized output is only a noisy signal of this hidden action, it is not contractible either.

and their ability to transform the risky output of their project into safe financial claims, catering to depositors' demand for safety. In such a risk transformation, selling financial claims backed by the project's output is essential. In subsequent chapters, I will analyse the risk-sharing activity under different financial frictions. To make a coherent comparison across these sections, the following assumption will hold throughout the paper.

Assumption 1 *In any of the proposed environments, the fraction of investment demanding safety is lower than the fraction of safe output in the economy:*

$$w \leq min\left\{\frac{\theta_b(0)}{\mathbb{E}\left[\theta_\omega(0)\right]}, \frac{\theta_b(e^{max})}{\mathbb{E}\left[\theta_\omega(e^{max})\right] - c(e^{max})}\right\}$$

where e^{max} *is implicitly determined in* $\mathbb{E}\left[\theta'_\omega\right] A - c'(e^{max}) = 0$.

The assumption places the focus on the interesting case in which the source of scarcity of safe assets is related to the informational friction in the model. In addition, it ensures that the solution is always interior, which is a more realistic representation of reality.

2.3 (Exogenously) complete financial markets

Bankers can trade state-contingent financial claims backed by their projects' output at a price $p_z(\hat{e}^j)$, which is a function of the market belief about seller j's effort, denoted by $\hat{e}^j$, such that $p_z : [0, \overline{e}] \rightarrow \mathbb{R}$. Buyers and sellers have limited liability.

Bankers' optimization problem. As I focus on symmetric equilibria, and all bankers are identical at $t = 0$ when the decisions are made, I drop the subscript j when redundant to reduce notation. Bankers take the price functions of state contingent claims $\{p_z(\hat{e})\}$ and the price of safe debt q^s as given, and choose the investment level (k), the face value of debt (d), the financial claims to buy and sell

$(\{b_z^{-j}, s_z\})$ for $z = \langle \omega, \iota \rangle$ and $-j \neq j$, and the effort level (e) to maximize

$$\mathbb{E}_e \Big[\, \Big(A_\iota \; - \; c(e) \Big) \, k \; - \; s_z \, \Big] \; + \; \mathbb{E}_{e^m} \left[b_z^m \right] \; - \; d \tag{2.1}$$

where $\mathbb{E}_e\,[.]$ denotes the expectation operator for effort level e. Notice that the idiosyncratic shock is the only source of heterogeneity among bakers once risks materialize. Consequently, gains from trade arises to insure against idiosyncratic risk so that they are willing to buy (if any) a well-diversified portfolio. Hence, the portfolio choice is irrelevant in this setting (how much to buy from each banker), instead, the assembly of a large number of financial claims is important for exploiting the law of large numbers. For simplicity, I assume that they buy the same amount from each banker, $b_z^m = b_z^{-j}$ for $\forall(-j)$, thus $e^m = \int \hat{e}^{-j} d(-j)$.

The maximization problem is subject to the budget constraint

$$k \;\; = \;\; (1-w) \;\; + \;\; q^s d \;\; + \;\; \sum_z \Big[\; p_z(\hat{e}) s_z \;\; - \;\; p_z(e^m) b_z^m \; \Big] \tag{2.2}$$

The amount of resources available for investment is determined by bankers' internal resources, external resources raised through debt, in addition to the net proceeds obtained from buying and selling financial claims. However, due to the limited liability, there is a limit on the financial claims bankers can sell given by

$$0 \leq s_z \leq A_\iota k \tag{2.3}$$

which implies that $s_{\omega f} = 0$ for $\omega \in \{g, b\}$.

Bankers are risk-neutral and do not care about risk per se, but getting rid of the idiosyncratic risk is essential to boost their debt capacity, as they need to ensure that their debt obligations are repaid in every contingency. Hence, only the lowest realization of the output – *safe collateral* – can be pledged to issue safe debt. Considering the limited liability constraint, the borrowing constraint boils down to

$$d \;\; \leq \;\; \min_\omega \Big\{ \; \theta_\omega(e^m) b_{\omega s}^m \;\; + \;\; (1 - \theta_\omega(e^m)) \, b_{\omega f}^m \; \Big\} \tag{2.4}$$

Since the diversified portfolio might only be subject to aggregate risk (if any), it yields a positive output even in the worst-case scenario. Importantly, the safe collateral depends on the collective effort level exerted by all bankers; this is, a higher effort by all bankers increases the return of the financial claims traded; thus increasing the amount of safe output after diversification. Therefore, effort does not affect the quality of the safe assets, as safety is a binary characteristic, instead, effort affects the amount of safe assets in the economy.

While effort is not observable, bankers' incentives to exert effort can be perfectly inferred from their other observable choices. Therefore, the incentive compatibility constraint is given by

$$\hat{e} = e^* \equiv \underset{e' \in [0,\overline{e}]}{max} \ \mathbb{E}_{e'} \left[\ \Big(A_\iota \ - \ c(e') \Big) \ k \ - \ s_z \ \right] \ + \ \mathbb{E}_{e^m} \left[b_z^m \right] \ - \ d \tag{2.5}$$

Equilibrium definition. A symmetric competitive equilibrium comprises a vector of pricing functions $\langle q^s, \{p_z(e)\}\rangle$, bankers' allocation $\langle k^*, d^*, \{s_z^*, b_z^*\}, e^*\rangle$, and depositors' allocation $\left\langle d^{d*} \right\rangle$ such that the following conditions hold:

1. Bankers' choices are the solution to (2.1) subject to (2.2)-(2.5)

2. Depositors chose the safe debt with the lowest price

3. Markets for state contingent claims clear at competitive prices $\{p_z(e)\}$

4. Markets for safe debt clear at competitive price q^s

5. Belief consistency: $e^* = \hat{e}$

Next, I will characterize the equilibrium under exogenously complete markets. First, I consider bankers' optimal decisions for given asset prices—partial equilibrium. Then, I show how bankers' decisions shape the supply curve of safe assets and the market clearing prices. But before, let me introduce the following preliminary result, which will simplify the subsequent analysis.

Lemma 2 *The price $p_z(e)$ can be disentangled in two terms:*

$$p_{\langle \omega,s\rangle}(e) = q_\omega \times \theta_\omega(e) \quad and \quad p_{\langle \omega,f\rangle}(e) = q_\omega \times (1-\theta_\omega(e)) \quad for\ \omega \in \{g,b\}$$

where the first term is the ω state price, and the second term is the (expected) output at state ω per unit invested at $t = 0$. Thus, the price of safe debt equals $q^s = q_g + q_b$.

Based on a no-arbitrage condition, the price of financial claims equals the present value of (expected) future output in a given state ω. Bankers' internalizes that when their effort (based on market belief) increases, the expected output of the financial claim and its price will also increase according to the upgrade in the expected output.

In addition, given that nothing refrains depositors from directly buying the financial claims in secondary markets, the price of debt equals the cost of the replicating portfolio, i.e., the diversified portfolio with no aggregate risk. Consequently, the debt level is determined by the borrowing constraint, and given that bankers do not (directly) earn any profit from issuing debt, they are indifferent with the amount of the debt to issue, i.e., its level is not determined by the optimality conditions of bankers.

Instead, the issuance of debt in equilibrium, thus, the supply of safe assets in the economy, will depend on the quantity (risk-sharing level) and quality (effort level) of the financial claims sold by bankers. Hence, understanding the determinants of the latter two variables will be the objective in the remainder of the section. To understand the cost of the informational friction, I will examine a setting in which effort is observable and compared it to a setting in which effort is not observable.

2.3.1 Endogenously complete: observable effort

When effort is observable, constraint (2.5) is redundant, and the optimal effort level is given by

$$\mathbb{E}\left[\theta'_{\omega}(e^{o})\right] A - c'(e^{o}) = 0 \tag{2.6}$$

Effort is set at the level that maximizes the expected output net of the effort cost, and it is decoupled from other endogenous variables. The objective function is a linear function of the sale of financial claims; therefore, trade is positive only when the market price is at least equal to the bankers' reservation price.

Lemma 3 *When markets are exogenously complete and effort is observable, ω state-contingent price functions are given by*

$$q_{\omega} = q^{r} \pi_{\omega} \text{ for } \omega \in \{g, b\} \tag{2.7}$$

where q^{r} represents the inverse of the Lagragian multiplier of the budget constraint.

Therefore, selling financial claims backed by their project's output has a zero NPV for bankers. This implies that bankers are indifferent to the amount of sale. Hence, in this setting, risk sharing is not costly; consequently, the optimal level is attainable. The following lemma summarizes the supply curve of safe assets, and the equilibrium prices.

Proposition 4 *When markets are complete and effort is observable, the supply curve is flat at the equilibrium price*

$$q^{s} = q^{r} = \frac{1}{\mathbb{E}\left[\theta_{\omega}(e^{0})\right] - c(e^{0})} \tag{2.8}$$

where a^{o} is determined in equation (2.6), and the equilibrium debt $d^{0} = \frac{w}{\mathbb{E}[\theta_{\omega}(e^{0})]A - c(e^{0})}$.

The prices of safe and risky assets equal the ratio between the aggregate investment and the expected (net) output from that investment, this is, to the inverse of the gross return. The risky premium is zero, as the agents holding the risk are risk neutral. Hence, absent any friction, both agents earn the same return on their investment. Most importantly, absent any friction, issuing safe debt, i.e., supplying safe assets, is not in conflict with output maximizing decisions, in line with Modigliani-Miller Theorem.

2.3.2 Endogenously incomplete: unobservable effort

When effort is not observable, its optimal level (e^*) is determined by the incentive compatibility constraint:

$$\mathbb{E}\left[\ \theta'_\omega(e^*) \times \left(Ak \ - \ s_{\omega s}\right)\ \right] - \ c(e^*)k \ = \ 0 \tag{2.9}$$

The optimal level of effort depends negatively on the project's output sold; therefore, bankers' effort is not decoupled from their other choices. The unobervability of effort creates a moral hazard problem when bankers share the productivity gain (or increase in expected output) induced by their effort with a third party. Thus, the incentives to exert effort will depend on the exposure to such productivity gain, and such exposure is achieved by retaining (at least partially) their project's output –*skin-in-the-game.*

While effort is not observable, the sold fraction of the project's income is. Thus, the market infers the effort level of the seller based on their skin-in-the-game, and the market prices their financial claims accordingly. This indirect effect is present in the optimality condition of the sale. In particular, $s^*_{\omega f} = 0$ and $s^*_{\omega s}$, is implicitly determined in

$$\underbrace{\underbrace{\frac{1}{q^r}}_{\text{return on investment}} \times \underbrace{\left[q_\omega \theta_\omega(\hat{e}) - \left(-\frac{\partial e}{\partial s_{\omega s}}\right) \sum_\omega q_\omega \theta'_\omega s^*_{\omega s}\right]}_{\text{investment increase}}}_{\text{output increase}} - \underbrace{\pi_\omega \theta_\omega(e^*)}_{\text{forgone output}} = 0 \tag{2.10}$$

where $\frac{\partial e}{\partial s_\omega} = -\frac{\pi_\omega \theta'_\omega}{c''(e)k} < 0$, and $\frac{1}{q^r}$ is Lagrangian multiplier of the budget constraint stated in (2.2). The investment level is $k = (1 - w) + \sum_\omega q_\omega \theta_\omega\left(\hat{e}\right) s_{\omega s}$, and $e^* = \hat{e}$

are determined in equation (2.9).

Equation (2.10) encapsulates the main trade-off present in the model. When selling financial claims, bankers can use the proceeds to increase investment by $q_\omega \theta_\omega(e)$, earning a return of $\frac{1}{q^r}$. However, the increase in investment is partially offset by the effect of selling on prices through effort; selling financial claims backed by the project's income decreases bankers' skin-in-the-game and signals a lower effort to the market. Consequently, the price of the claim drops reducing the proceeds from the sale, partially offsetting the investment increase. Hence, the marginal cost of selling is bigger than the forgone expected payoffs, $\pi_\omega \theta_\omega(e)$, due to the latter cost on effort. Under assumption (1), which ensures that the solution is interior, bankers will optimally decide to hold some skin-in-the-game on the benefit of incentive provision.

The optimal holding of bankers' skin-in-the-game ultimately determine the risk sharing level (in equilibrium $b_z = s_z$) and the optimal effort level, both being inputs in the safe asset manufacturing. An important insight from the model is that risk sharing and effort are substitutes; higher risk sharing requires bankers to sell a higher fraction of their income, reducing their skin-in-the-game and the incentives to exert effort, *ceteris paribus.*

This prompts an important question: How does the interplay between risk sharing and effort affect the safe asset supply? To answer this question, let us first understand the equilibrium in the state-contingent financial markets (as a function of the price for safe q^s and risky q^r claims), and the focus on the supply curve of safe assets.

Lemma 5 *When markets are exogenously complete and effort is not observable, in equilibrium $s_z = b_z$ and $\theta_g(e)s_{gs} = \theta_b(e)s_{bs}$, so that the ω state-contingent price functions are given by*

$$q_\omega = q^r \pi_\omega + \rho_\omega(e) \times (q^s - q^r) \tag{2.11}$$

where $\rho_\omega(e) \equiv \frac{\frac{\pi_\omega \theta'_\omega(e)}{\theta_\omega(e)}}{\mathbb{E}\left[\frac{\theta'_\omega(e)}{\theta_\omega}\right]}$ *such that* $\rho_g + \rho_b = 1$.

Demand for state-contingent claims is exclusively driven by demand for safe assets; the diversified pool just needs to ensure equal output across states, i.e., it is not subject to aggregate risk, to maximize the skin-in-the-game per unit of safe collateral. Since $\theta_g(a^*) > \theta_b(a^*)$, to ensure the same level of safe output across states requires $s^*_{gs} < s^*_{bs}$; this is, the retention rate is higher in the good aggregate state than in the bad aggregate state.

Notice that the equilibrium price of the financial claims is higher than the reservation price for bankers ($q_\omega > q^r \pi_\omega$) when the there is a positive deviation between the prices of safe and risky assets, i.e., $q^s - q^r > 0$. Therefore, when selling financial claims backed by their project's output, bankers (indirectly) benefit from the safety premium. In fact, $\rho_\omega(e)$ precisely captures the relative cost of selling in state ω out of the total cost just mentioned.

The following proposition combines the aforementioned lemma and banker's first order conditions, to understand the supply curve of safe assets per unit of investment.

Lemma 6 *When markets are exogenously complete and effort is not observable, bankers issue safe debt only when* $q^s - q^r > 0$*, and its level is determined in*

$$\left(\frac{q^s}{q^r} - 1\right) - \left(-\frac{\partial e^*}{\partial d}\right) \times \mathbb{E}\left[\frac{\theta'_\omega(e^*)}{\theta_\omega(e^*)}\right] d^* = 0 \tag{2.12}$$

where e^* *is implicitly determined in*

$$\mathbb{E}\left[\theta'_\omega(e^*)\right] A - \mathbb{E}\left[\frac{\theta'_\omega(e^*)}{\theta_\omega(e^*)}\right] d^* - c'(e^*) = 0 \tag{2.13}$$

so that $\frac{\partial e^*}{\partial d} < 0$ *and* q^r *is implicitly determined in*

$$\frac{1}{q^r} = \frac{\mathbb{E}\left[\theta_\omega(e^*)\right] A - c(e^*) - d^*}{1 - q^s d^*}.$$

First, the supply of safe assets is positive if and only if the gap between the price of safe and risky claims is positive. This result is in stark contrast to the previous section, where we learned that in the absence of any friction, the risky premium is zero (see proposition (4)). In this environment, the price difference has to be positive, and the magnitude of that difference is the so-called *safety premium*. Bear in mind that this is not the standard risky premium, where the deviation from the safe rate captures the market-wide required compensation for bearing risk. Instead, it captures a scarcity rent derived from the constrained safe assets supply due to the moral hazard problem.

The safety premium compensates for the distortive effect of supplying safe assets on effort incentives. In particular, each infinitesimal increase in effort increases the expected net output by $\mathbb{E}\left[\frac{\theta'_\omega(e^*)}{\theta_\omega(e^*)}\right] d^*$, which coincides with the additional term on the optimality condition of effort in (2.13) compared to the previous case's (2.6). In addition, even if bankers do not directly internalize the effect of their effort on the build-up of safe assets, market prices give the correct incentives in this aspect, as the value of the claims sold map one to one in aggregate with the safe assets in the economy. The following corollary builds on proposition (6) and provides an important insight about the supply curve.

Corollary 7 *The supply curve is upward sloping* $\left(\frac{dd}{dq^s} > 0\right)$*, however, along the curve, the fraction of the project's output sold monotonically increases* $\left(\frac{ds_{\omega s}}{dq^s} > 0\right)$ *for* $\omega \in \{g, b\}$ *while the effort level monotonically decreases* $\left(\frac{de}{dq^s} < 0\right)$.

An increase in the price of safe claims increases the supply of safe assets; in the first instance, it increases the safety premium, thus, the marginal benefit of issuing safe debt. The upward sloping supply curve, however, hides an important characteristic of safe assets manufacturing: the risk-sharing and effort mix changes along the curve. When moving up in the supply curve, the risk sharing becomes higher, i.e., the output sold by bankers increases, which reduces their skin-in-the-game and their optimal level of effort.This collective deterioration of effort reduces the productivity of the aggregate investment, and unintentionally dampens the safe asset supply. The risk-sharing effect will always dominate the effort effect, so that absent any other friction, the slope will be upward sloping. The latter result is derived from the optimality conditions that are shaped by the moral hazard problem inherent in the model, and not by a specific assumption or function form. The following proposition describe the equilibrium prices and the corresponding output distribution.

Proposition 8 *When markets are complete, there is a unique symmetric competitive equilibrium, where the price of safe assets and risky assets are give by*

$$q^s = \frac{w}{d^*} > q^r = \frac{1-w}{\mathbb{E}\left[\theta_\omega(e^*)\right] A - c(e^*) - d^*}$$

where d^* is determined in equation (2.12) and e^* in equation (2.13)

In equilibrium, the safety premium is positive so that depositors earn a lower return on their investment than bankers. Hence, the informational friction ultimately causes an output redistribution on detriment to depositors. The comparison with the previous section highlights additional costs associated with the informational friction, which prevent the competitive market from achieving the first best. In particular, the information friction undermines safe asset production in two ways: (i) optimal risk sharing is not possible, as bankers find it optimal to retain a fraction of their project's output to maintain effort incentives, and (ii) effort incentives are distorted due to the moral hazard problem.

So even if costly, is the private supply efficient compared to a planer that take into account the incentive problem? The following proposition summarizes the answer to the posed question.

Proposition 9 *When markets are complete, the competitive equilibrium is constraint-efficient for the following Pareto weights*

$$W^d = \frac{q^s}{q^s + q^r} \quad \textit{and} \quad \left(1 - W^d\right) = \frac{q^r}{q^s + q^r}$$

depositor's and banker's, respectively.

Manufacturing safe assets is costly, due to its adverse effect on productivity. However, this proposition suggests that such a cost is minimal, as the competitive outcome lies in the efficient frontier. Therefore, the competitive equilibrium under (exogenously) complete markets displays an efficient way to manufacture safe assets, given the informational friction. The cornerstone of such efficiency relates to agents' ability to trade claims that are state-contingent, minimizing the cost of risk-sharing on effort. Next, let us consider how the outcome changes when the exogenous market completeness assumption is relaxed.

2.4 (Exogenously) incomplete financial markets

In this section, I focus on a more realistic environment, by restricting the type of financial claims that bankers can trade to centre on widely observed market mechanisms. In particular, through this section I assume that the financial claims that bankers trade, can be contingent on the output (i.e., idiosyncratic state), but not on the economic conditions (aggregate state). For instance, banks when they securitize their loans, they sell a fraction of the loans, but the fraction sold will not vary (ex-ante) based on the future economic conditions. [11]

In the new environment, the timing of the sale, thus risk sharing, is important. In the subsequent sections two relevant cases will be carefully analysed: (1) when trade occurs at $t = 0$ before the aggregate shock is realized – *ex-ante risk-sharing* – and (2) when trade occurs after a perfect and public signal about the aggregate state (but before the the idiosyncratic risk is realized) – *ex-post risk-sharing*.

Before analysing these two alternatives, let me point out that Lemma (1) still holds. Moreover, it is further complemented by the following one:

Lemma 10 *In any equilibrium with (exogenously) incomplete markets and unob-*

[11] Due to the fact that there are only two possible outputs, and that one of them is zero, the type of security (e.g., debt or equity contract) traded is irrelevant here. The focus is not on how to distribute output across idiosyncratic states to enhance effort incentives, but rather on understanding how different risk sharing alternatives cope with the aggregate state.

servable effort, bankers will issue safe debt that will be bought by depositors.

In contrast to the previous section, where markets are (exogenously) complete, bankers have a comparative advantage not only in managing the effort and investment technology but also in the risk-sharing activity. As it will be soon become clear, the optimal risk-sharing requires bearing some risks, and since the reservation value of such risk is higher for bankers than for depositors, they will directly issue safe debt.

2.4.1 Ex-ante risk-sharing

Bankers can trade at $t = 0$ financial claims backed by their project's output, and these claims can be contingent on the idiosyncratic state but not on the aggregate state. The price of these claims is $p_\iota(\hat{e})$, which is a function of the market belief about seller's effort, denoted by $\hat{e}$, such that $p_\iota : [0, \overline{e}] \to \mathbb{R}$. I assume that buyers and sellers have limited liability.

Bankers' optimization problem. The only difference with respect to the previous section, is that the feasibility set attainable by bankers is a subset of the feasibility set under a complete market environment. In particular, $s_\iota = s_{g\iota} = s_{b\iota}$ for $\iota \in \{s, f\}$. This affects the borrowing constraint as follows:

$$d \leq \theta_b(e^m) b_s^m \tag{2.4'}$$

where the limited liability constraint and the fact that b_f^m is in zero supply/demand has been include. Notice that the expected output yield by the diversified portfolio is $\mathbb{E}\left[\theta_\omega(e^m)\right] b_s^m$, of which the output in the worst-case scenario only serves as safe collateral to boost their debt capacity. Hence, even if the latter is a well-diversified portfolio, in which the idiosyncratic risk has faded away, the portfolio is subject to aggregate risk. The present case resembles the securitization process, as its two main ingredients – pooling and tranching – are captured here. Pooling denotes to the assembly of loans into diversified pools. Tranching denotes the designation of different classes of claimants on the cash flow of the pool, which differ in the seniority and, thus, credit quality. For further details on the effect on the rest of the equations, refer to the appendix.

Equilibrium definition. A symmetric competitive equilibrium comprises a vector of pricing functions $\langle q^s, \{p_\iota(e)\}\rangle$, bankers' allocation $\langle k^a, d^a, \{s_\iota^a, b_\iota^a\}, e^a\rangle$, and depositors' allocation $\left\langle d^d \right\rangle$ such that the following conditions hold:

1. Bankers' choices are the solution to (2.1') subject to (2.2')-(2.4'), and
2. Depositors chose the safe debt with the lowest price
3. Markets for financial claims clear at competitive prices $\{p_\iota(e)\}$
4. Markets for safe debt clear at competitive price q^s
5. Belief consistency: $e^a = \hat{e}$

Equilibrium characterization

The following lemma amends lemma (2), and adds the market clearing price for financial claims at $t = 0$.

Lemma 11 *The price $p_\iota(e)$ can be disentangled as follows*

$$p_s(e) = q \times \mathbb{E}\left[\theta_\omega(e)\right] \quad \textit{and} \quad p_f(e) = q \times \mathbb{E}\left[1 - \theta_\omega(e)\right]$$

where the first term is the discount factor, and second term is the (expected) future output per unit invested at $t = 0$.

When markets are exogenously incomplete and effort is not observable, in equilibrium $s_\iota = b_\iota$, so that the discount factor is given by

$$q = \kappa(e^m)q^s + \left(1 - \kappa(e^m)\right) q^r \tag{2.14}$$

where $\kappa(e) \equiv \frac{\theta_b(e)}{\mathbb{E}[\theta_\omega(e)]} < 1$, e^m the collective effort level.

The discount factor is a weighted average of the (shadow) price of risky and safe claims, i.e., the safe fraction of output embed in the claims traded ($\kappa(e^m)$)) has a price of q^s while the risky fraction $(1-\kappa(e^m))$ has a price of q^r. Crucially, the fraction of safe output depends on the collective effort level.

Similarly, effort is determined by the incentive compatibility constraint in equation (2.9), for the particular case $s_\iota = s_{\omega\iota}$ for $\omega \in \{g, b\}$, and it is a function of the fraction of output bankers sell (s_ι). The optimality condition where s_ι is determined is given by

$$\underbrace{\underbrace{\frac{1}{q^r}}_{\text{return on investment}} \times \underbrace{\left[q\mathbb{E}\left[\theta_\omega(e^a)\right] - \left(-\frac{\partial e}{\partial s_s}\right) q\mathbb{E}\left[\theta'_\omega(e^a)\right] s_s^a\right]}_{\text{investment increase}}}_{\text{output increase}} - \underbrace{\mathbb{E}\left[\theta_\omega(e^a)\right]}_{\text{forgone output}} = 0$$

where $\frac{\partial e}{\partial s_s} = -\frac{\mathbb{E}[\theta'_\omega(e)]}{c''(e)} < 0$. The same intuition follows with the particularity that in this case the claims sold yield output in both aggregate states, which is incorporated in the discount factor.

In equilibrium, due to perfect competition among financial intermediaries, they earn zero direct profits by issuing safe debt. But they capture the benefit of issuing debt indirectly when selling financial claims. So taking into account the equilibrium condition $s_\iota = b_\iota$, let's analyse the supply curve of safe assets described in the following proposition.

Lemma 12 *When markets are exogenously incomplete, effort is not observable and trade is only possible at* $= 0$*, only when the safety premium is positive bankers issue safe debt, and its level is determined by*

$$\left(\frac{q^s}{q^r} - 1\right) - \left(-\frac{\partial e^a}{\partial d}\right) \times \left[\frac{\mathbb{E}\left[\theta'_\omega(e^a)\right]}{\kappa(e^m)\mathbb{E}\left[\theta_\omega(e^a)\right]}\right] d^a = 0 \tag{2.15}$$

where $\kappa(e^m) \equiv \frac{\theta_b(e^m)}{\mathbb{E}[\theta_\omega(e^m)]}$ *and* e^a *is implicitly determined in*

$$\mathbb{E}\left[\theta'_\omega(e^a)\right] A - \left[\frac{\mathbb{E}\left[\theta'_\omega(e^a)\right]}{\kappa(e^m)\mathbb{E}\left[\theta_\omega(e^a)\right]}\right] d^a - c'(e^a) = 0 \tag{2.16}$$

so that $\frac{\partial e^a}{\partial d} < 0$ *and* q^r *is implicitly determined in*

$$\frac{1}{q^r} = \frac{\mathbb{E}\left[\theta_\omega(e^a)\right] A - c(e^a) - d^a}{1 - q^s d^a}.$$

Several differences from the previous case are notable. First, limiting the contract space has an adverse effect on the risk-sharing and effort trade-off. In fact, since $\theta_g(e)s_s \neq \theta_g(e)s_s$, the moral hazard problem is exacerbated: bankers sell $\frac{1}{\theta_b(e)}$ units of output in both states instead of $\frac{1}{\theta_\omega}$ of the previous section ($\frac{1}{\theta_b} \geq \frac{1}{\theta_\omega}$), for each unit of safe debt issued. This is a direct and mechanical result of exogenously restricting the contract space bankers can write, reducing the feasibility set of choices in a manner that is relevant in this context. Here an interesting question emerges: Does this restriction have further implications in bankers' optimization problem?

Equation (2.15) suggests that it does, and it is through $\frac{\partial e^a}{\partial d}$. In particular, in this new environment, bankers do not correctly internalize the effect of their effort in the build out of safe assets, as they do not take into consideration the full effect of their collective effort on prices/discount factor (through $\kappa(a^m)$). In other words, bankers do internalize the fact that by increasing their effort, they increase the quality of the financial claims sold, which contributes to the buildup of safe assets, but they do not take into account that the fraction of the safe output embedded in the claims increases in line with their collective effort. In essence, the difference with respect to the previous case lies in the fact that the financial claim sold is subject to aggregate risk and only a fraction of their expected output accounts as safe output.

Social welfare discussion

The optimality condition of the social planner, that is subject to the same constraint as the market, and for the specific weight $W^d = \frac{q^s}{q^s+q^r}$, is given by

$$\left(\frac{q^s}{q^r} - 1\right) - \left(-\frac{de}{dd}\right) \times \left[\frac{\mathbb{E}\left[\theta'_\omega(e^{sp})\right]}{\kappa(e^{sp})\mathbb{E}\left[\theta_\omega(e^{sp})\right]}\right] d^{sp} = 0 \tag{2.17}$$

where $\frac{de}{dd} \neq \frac{\partial e}{\partial d}$ if $\kappa'(e) \neq 0$. The direction of the inefficiency (if any) will depend on the sign of the $\kappa'(e)$, as described in the following proposition.

Proposition 13 *When* $\frac{\theta'_g(e)}{\theta_g(e)} = \frac{\theta'_b(e)}{\theta_b(e)}$ *the competitive equilibrium is efficient. When* $\frac{\theta'_g(e)}{\theta_g(e)} \neq \frac{\theta'_b(e)}{\theta_b(e)}$ *the competitive equilibrium is not efficient, such that:*

1. *If* $\frac{\theta'_g(e)}{\theta_g(e)} < \frac{\theta'_b(e)}{\theta_b(e)}$*, the debt level issued is inefficiently high and the effort is inefficiently low. Then, expected value and volatility of output is too low and too high, respectively.*

2. *If* $\frac{\theta'_g(e)}{\theta_g(e)} > \frac{\theta'_b(e)}{\theta_b(e)}$*, the debt level issued is inefficiently low, the effort is inefficiently high. Then, expected value and volatility of output is high low and too low, respectively.*

The inefficiency increases with the debt level.

Insofar, I have been silent about this aspect of the effort technology. But, when effort's marginal product (in relative terms) varies across aggregate states, effort not only affects expected output, but it further affects the volatility of the aggregate investment, and the fraction of the investment that is going to be safe. In this case, the competitive equilibrium is not efficient since bankers underestimate or overestimate – depending on whether the first of second condition holds – the cost of manufacturing safe assets on output.

At a first glance, it might not seem obvious why such pecuniary externality leads to a divergence between private and socially optimal outcomes. After all, pecuniary externalities by themselves need not lead to violations of the standard welfare theorems. The informational friction, which induces the scarcity of safe assets and the safety premium, is responsible for the welfare effect of the pecuniary externality. In fact, when they collectively reduce their effort, this could affect to the incentive compatibility constraint (reflected in $\frac{\partial e}{\partial d}$), which ultimately determines the debt bankers issue, and the safe assets in the economy.

Therefore, understanding which of these cases is likely to prevail in equilibrium is necessary to assess the adequate corrective policy measure. So, lets rationalize the effort technology. The theory of financial intermediation has extensively emphasis the role of financial intermediaries in monitoring and screening borrowers in the process of lending, so that they are better than markets at resolving informational problems among others, enhancing the quality of the investment they intermediate. Hence, this effort broadly capture screening and monitoring intensity. When bankers' dedicate more time and effort to screen their borrowers, they can increase the probability of financing a better quality borrowers among the heterogeneous candidates. Granting loans to a higher quality pool, increases banker's expected output especially in bad economic states where bad quality borrowers disproportionally default (in line with the cleansing effect). On the other hand, effort could also capture the monitoring intensity. During bad economic states, monitoring is particularly valuable, as the perverse incentives of borrowers increase. Hence, assumption $\frac{\theta_g'(e)}{\theta_g(e)} < \frac{\theta_b'(e)}{\theta_b(e)}$ is more realist.

This collective drop in effort is detrimental to economic stability. The quality of individual projects deteriorates which implies that banker's claim on their project's output becomes riskier. In addition, when the marginal product of effort is higher in bad states of the economy, the collective drop in effort, not only decreases aggregate investments' productivity but it further amplifies its volatility. Market incompleteness exacerbates the real costs of safe asset supply, as the debt level is inefficiently, so that the expected out level and its volatility is inefficiently low and higher, respectively.

Without assessing how this form to manufacture safe assets interact with other alternative, it is too early to conclude that a policy restricting the issuance of debt is effective. Let's the ex-post risk sharing next this case next.

2.4.2 Ex-post risk-sharing

In this section, bankers can only trade at $t = 1$, after a public signal revealing the aggregate state has realized but before the realization of the idiosyncratic state. [12] Resources at this intermediate date can be storage until the final date at no cost. Bankers can sell claims backed by their project's output, and such claims can be contingent on the idiosyncratic state but not on the aggregate state (which does not make a difference here). The price of these claims conditional on the ω realization is given by $p_{\iota|\omega}(\hat{e})$, which is a function of the market belief about the seller's effort at this intermediate date. But given that there is not additional information there is not change in the beliefs about effort since $t = 0$. Those financial claims are bought by late investors, as bankers do not have resource at this intermediate date to do so.

Late-investors. There is a continuum of this type of investor, with mass equal to one, who are endowed with $w^{li}_{1\omega}$ units of the perishable good in the state ω, and enjoy a utility given by $U^{li} = \mathbb{E}\left[c^{li}_{1\omega}\right]$. They are born at this intermediate date, thus, it is not possible to write a contract with them before hand. They have access to the following technology: at the intermediate date (begin of date $t = 1$), they choose the level of investment $k^{li}_{1|\omega}$ which yields $F(k^{li}_{1|\omega})$ units of output at the final date, where $F : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, such that $F'(.) > 0$ and $F''(.) \leq 0$ for $\forall k^{li}_{1|\omega} \in R_+$, and $F'(0) = 0$ and $F'(w^{li}_{1\omega}) \geq 1$.

Banker's optimization problem. At $t = 0$, bankers the price functions of financial claims $\left\{p_{\iota|\omega}(\hat{e})\right\}$ where $\hat{e}$ is expected effort at this intermediate date (but depends on the same information as in $t = 0$), and decide on the investment level

[12]The signal could be imperfect, and still the results would not qualitatively change.

(k), the face value of debt (d), and the actual effort level (e) to maximize:

$$\mathbb{E}_e\Big[\ \Big(A_\iota\ -\ c(e)\Big)\ \Big]\,k\ -\ \Big[\theta_\omega(e) - p_{s|\omega}(\hat{e})\Big]\,\hat{s}_{s|\omega}\ -\ d \tag{2.18}$$

were $\hat{s}_{s|\omega}$ is the expected output sale at the intermediate date and ω state. The maximization problem is subject to the budget constraint

$$k\ =\ (1-w)\ +\ q^s d \tag{2.19}$$

The amount of resources available for investment is determined by bankers' internal resources and external resources raised through debt. In this particular case, the borrowing constraint boils down to

$$d\ \leq\ \underset{\omega}{min}\Big\{\ p_{s|\omega}(\hat{e})\hat{s}_{s|\omega}\ \Big\} \tag{2.20}$$

The safe collateral is given by the proceeds of future sales – the minim across states. It depends on the market belief about bankers' effort and on the expected amount of claims sold.

The incentive compatibility constraint is given by

$$\hat{e} = e^p \equiv \underset{e'\in[0,\bar{e}]}{max}\ \ \mathbb{E}_{e'}\Big[\ \Big(A_\iota\ -\ c(e')\Big)\ \Big]\,k\ -\ \Big[\theta_\omega(e') - p_{s|\omega}(\hat{e})\Big]\,\hat{s}_{s|\omega}\ -\ d \tag{2.21}$$

At $t = 1$ bankers choose the amount of output to sell $s_{\iota|\omega}$ and the limited liability condition still holds.

$$0 \leq s_{s|\omega} \leq Ak$$

At this date, the choice of $s_{s|\omega}$ is taken after the rest of the variables are fixed; this is, bankers' might face incentives to deviate from their initial promises. However, if they default in their debt all their output is sized.

Late-investors' maximization problem. Late investors only value future consumption, thus, they will invest all of their endowment. They face different investment opportunities: they can invest their endowment on the productive technology they have access to $(k^{li}_{1|\omega})$, they can buy financial claims sold by bankers

$(b_{1|\omega})$, or alternatively, they will invest the rest on the storage technology $(w^{li}_{\omega} - k^{li}_{1|\omega} - p_{1|\omega}(\hat{e})b_{1|\omega})$. They choose their investment strategy to maximize

$$F(k^{li}_{1|\omega}) + \theta_{\omega}(\hat{e})b_{1|\omega} + w^{li}_{\omega} - k^{li}_{1|\omega} - p_{1|\omega}(\hat{e})b_{1|\omega} \tag{2.22}$$

Equilibrium definition. A symmetric competitive equilibrium comprises a vector of pricing functions $\left\langle q^s, \left\{p_{\iota|\omega}(e)\right\}\right\rangle$, bankers' allocation $\left\langle k^p, d^p, \left\{s^p_{\iota|\omega}\right\}, e^p\right\rangle$, and depositors' allocation $\left\langle d^d \right\rangle$ such that the following conditions hold:

1. Bankers' choices are the solution to (2.18) subject to (2.19)-(2.21).
2. Depositors chose the safe debt with the lowest price
3. Late investors' choices are the solution to (2.22) subject to the nonnegativity constraint
4. Markets for financial claims at the intermediate date clear at competitive prices $\left\{p_{\iota|\omega}(e)\right\}$
5. Markets for safe debt clear at competitive price q^s
6. Belief consistency: $e^p = \hat{e}$

Equilibrium characterization

The following lemma amends lemma (2), and adds the market clearing price for financial claims at $t = 1$.

Lemma 14 *The price $p_{\iota}(e)$ can be disentangled as follows*

$$p_{s|\omega}(e) = q_{1|\omega} \times \theta_{\omega}(e) \quad \textit{and} \quad p_{f|\omega}(e) = q_{1|\omega} \times (1 - \theta_{\omega}(e)) \quad \textit{for} \quad \forall \omega \in \{g, b\}$$

where the first term is the discount factor, and second term is the (expected) future output for each unit invested at $t = 1$.

The equilibrium discount factor is given by

$$q_{1|\omega} = \frac{1}{F'\left(k^{li}_{1|\omega}\right)} < 1 \quad \textit{for} \quad \forall\omega \in \{g, b\} \tag{2.23}$$

where $k^{li}_{1|\omega} \leq w^{li}_{1\omega}$.

Notice, that when $q_{1|\omega} < 1$, bankers sell their claims at a discount, i.e., fire-sale, so that they do not benefit from the transaction itself. In addition, the discount is endogenous in the model and depends on the sales relative to the resources available. In this environment, however, the early liquidation of the project is necessary to meet their debt obligations. But given that selling is costly for banker, it is important to analyse whether bankers have the incentives to pay their debt obligations, or if instead, they have incentives to deviate from their initial promises. Due to the fact that output is observable, and it is seized fully depositors in case they default in their debt, bankers do not have incentives to not fulfil their obligations. The amount of the sale is determined by the borrowing constraint

$$s_{s|\omega} = \frac{d}{q_{1|\omega}\theta_\omega(\hat{e})}$$

So, the level of debt will ultimately determined the amount of posterior sales. Due to the simplifying assumption in which the signal to the aggregate state is perfect, the optimality conditions of bankers are very similar to the case in which markets were (exogenously) complete, but taking into account the cost of fire-sales. Let's analysed how the latter affect the supply curve of safe assets.

Lemma 15 *When markets are exogenously incomplete, effort is not observable and trade is only possible at* $t = 1$*, only when the safety premium is positive bankers issue safe debt, and its level is determined in*

$$\left(\frac{q^s}{q^r} - \mathbb{E}\left[\frac{1}{q_{1|\omega}}\right]\right) - \left(-\frac{\partial e^p}{\partial d}\right) \times \mathbb{E}\left[\frac{\theta'_\omega(e^p)}{q_{1|\omega}\theta_\omega(e^p)}\right] d^p = 0 \tag{2.24}$$

where e^* is implicitly determined in

$$\mathbb{E}\left[\theta'_\omega(e^p)\right] A - \mathbb{E}\left[\frac{\theta'_\omega(e^p)}{q_{1|\omega}\theta_\omega(e^p)}\right] d^p - c'(e^p) = 0 \tag{2.25}$$

so that $\frac{\partial e}{\partial d} < 0$ and q^r is implicitly determined in

$$\frac{1}{q^r} = \frac{\mathbb{E}\left[\theta_\omega(e^p)\right] A - c(e^p) - \mathbb{E}\left[\frac{1}{q_{1|\omega}}\right] d^p}{1 - q^s d^p}$$

This lemma highlights the main differences with respect to the previous cases. First, the fact that the sale occurs after the realization of the aggregate state, enables the possibility to make the sale choice conditional on newly available information about the aggregate state. Notice, that in this aspect postponing the sale decision helps to circumvent until certain extend the cost of restricting the contract set. In fact, for the particular case in which $q_{1|\omega} = 1$ for $\omega \in \{g, b\}$, this problem is exactly equivalent to the (exogenously) complete environment benchmark. Otherwise, the problem differs, and the difference will depend on the discount factor. Nonetheless, the output sold is such that $q_{1|g}\theta_g(e)s_{s|g} = q_{1|b}\theta_b(e)s_{s|b}$, which for the reasonable condition $q_{1|g} \geq q_{1|b}$, implies $\theta_g(e)s_{s|g} \leq \theta_b(e)s_{s|b}$, instead of holding with equality as previously.

However, postponing the sale decision has a cost for bankers, that of selling assets at a discount. This reduced the benefit captured by issuing safe debt, while increases its cost as more units of output have to be sold for each unit of debt issued. Both of this effect reinforce each other, and reduces both the optimal debt issued and the bankers' incentives to exert effort.

The following proposition describe the equilibrium prices, and the corresponding output distribution.

Proposition 16 *When markets are incomplete, there is a unique symmetric com-*

petitive equilibrium, where the price of safe assets and risky assets are give by

$$q^s = \frac{w}{d^p} > q^r = \frac{1-w}{\mathbb{E}\left[\theta_\omega(e^p)\right]A - c(e^p) - \mathbb{E}\left[\frac{1}{q_{1|\omega}}\right]d^p}$$

where $q_{1|\omega}$ *is determined in*

$$q_{1|\omega} = \frac{1}{F'\left(w^{li}_{1\omega} - d^p\right)} < 1 \quad \textit{for} \quad \forall \omega \in \{g, b\} \tag{2.26}$$

and d^p *is determined in equation (2.12),* e^p *in equation (2.13).*

After outlining the main costs and benefits with respect to the ex-ante risk-sharing environment, the natural question to ask is whether this choices are efficient or not. The aspect is discussed in the following section.

Welfare discussion

The optimality condition of the social planner, that is subject to the same constraint as the market, and for the specific weight $W^d = \frac{q^s}{q^s+q^r}$ is given by

$$\left(\frac{q^s}{q^r} - \mathbb{E}\left[\frac{1+\varepsilon_\omega}{q_\omega}\right]\right) - \left(-\frac{de}{dd}\right) \times \mathbb{E}\left[\frac{\theta'_\omega(e)}{q_{1|\omega}\theta_\omega(e)}\right] d = 0$$

where $\frac{de}{dd} = \frac{\partial e}{\partial d}(1+\varepsilon_\omega)$ and $\varepsilon_\omega \equiv -\frac{\partial q_{1|\omega}}{\partial d}\frac{d}{q_{1|\omega}} = \frac{-F''()}{F'^2()}\frac{d}{q_{1|\omega}} > 0$. The latter term captures potential change in prices in the wake of an increase in debt. Bankers, however, do not take into account the effect of their actions on prices, giving rise to a welfare reducing pecuniary externality. The following proposition describes how the inefficiency affects allocations.

Proposition 17 *The competitive equilibrium is not efficient when* $F''(.) < 0$*, such that the debt level issued is inefficiently high, while the effort is inefficiently low. Then, expected value and volatility of output is too low and too high, respectively.*

The debt level is too high exacerbating the effect on real outcomes. In a similar vein, this pecuniary externality turns out to be welfare reducing as it interact with the informational friction inherent in the economy. In this particular case, the direction of the inefficiency is unambiguous under the condition that $F''(.) < 0$, similar to Stein (2012).

2.4.3 Ex-ante and ex-post risk-sharing

After studying each alternative separately, in this section I put both pieces together. The objective lies in understanding which of the two prevails in equilibrium, and in evaluating their interaction (if any). On the benefit of conciseness, I will skip all the details already explained in the previous sections to focus on the novel insights. A detailed description can be found in the Appendix in section 2.5. In this environment, the borrowing constraint is given by

$$d \leq \min_{\omega} \left\{ \underbrace{\theta_{\omega}(e^m) b_s^m}_{\text{Ex-ante risk-sharing}} + \underbrace{p_{s|\omega}(\hat{e}) \hat{s}_{s|\omega}}_{\text{Ex-post risk-sharing}} \right\} \tag{2.27}$$

Therefore, safe collateral can be obtained, either by buying a diversified pool of financial claims, or by liquidating early the project at a cost. The following lemma describes banker's optimal choice between the two alternatives.

Lemma 18 *When markets are exogenously incomplete, effort is not observable and trade can take place at $t = 0$ and $t = 1$, the optimal form to ensure safety:*

- *If $\frac{1}{F'(w^{li}_{1\omega})} < \underline{q_{1|b}}$, only ex-ante risk-sharing prevails and the supply of safe assets is described in lemma 12.*

- *If $\underline{q_{1|b}} \leq q_{1|b} < 1$ ex-ante risk-sharing is combined with ex-post risk sharing in the bad economic state and the supply of safe assets is described lemma 21 in the appendix.*

This lemma highlight an important insights: Ex-ante risk-sharing is the prefer form to ensure safety for certain parameter values, while ex-post risk sharing never prevails alone. However, ex-post risk-sharing in the bad state may become a complement that helps to circumvent the limitations of market incompleteness from which ex-ante risk sharing suffer from, while limiting the fire-sale costs.

The lemma provides a rational to understand why and when different (private) financial instruments with safety attributes coexist in equilibrium. The empirical evidence has claimed that different privately issued liabilities cater to the demand for safety (Gorton et al. (2012), Kacperczyk et al. (2021), Krishnamurthy and Vissing-Jorgensen (2015)), and these different options rely either on ex-ante or ex-post risk-sharing (or a combination of the two). For instance, and as already mentioned, securitization relies on ex-ante risk sharing; where the backing collateral represent the senior tranch of a diversified pool of loans. Other alternatives, instead, require additional actions contingent on the state of the economy (e.g., Repos, or covered bonds). This analysis reveals, how these different instruments may interact to supply safe assets.

For a realistic parametrization of the model, each of these alternatives are interrelated. The following lemma further characterize the proportion of each in the build-up of safe collateral.

Lemma 19 *When ex-ante and ex-post risk-sharing coexist in equilibrium, then*

$$\underbrace{\overbrace{\theta_g(e)s_s}^{\textit{Good aggregate state}}}_{d} = \overbrace{\underbrace{\theta_b(e)s_s}_{(1-\sigma(e))d} + \underbrace{q_{1|b}\theta_b(e)s_{s|\omega}}_{\sigma(e)d}}^{\textit{Bad aggregate state}}$$

where $\sigma(e) \equiv \frac{\theta_g(e)-\theta_b(e)}{\theta_g(e)}$ *and* $q_{1|b} = \frac{1}{F'(w_{1b}^{li}-\sigma(e)d)}$.

This lemma highlights a new dimension to the problem, namely, the proportion of the safe asset issuance that is backed through ex-ante versus ex-post risk sharing. As already anticipated, in the good state, the safe output of the diversified pool are abundant and enough to meet bankers' debt obligation. In the bad state instead, the safe output from the diversified portfolio is not enough, and an additional fraction of the project is sold to fill the gap.

In particular, the fraction of the safe debt backed by asset sales in the bad state is given by $\sigma(e)$, which it is a decreasing function of effort for the relevant case in which $\frac{\theta_b'(e)}{\theta_b(e)} > \frac{\theta_g'(e)}{\theta_g(e)}$. This condition, further implies that along the supply curve of safe assets, as the optimal effort decreases, the fraction of debt backed by ex-post risk sharing increases.

Welfare discussion

We can conclude that under certain parametrization of the model, ex-ante and ex-post risk-sharing are jointly determined. The following proposition describes the efficiency for these parameter values.

Proposition 20 *The competitive equilibrium with incomplete markets and unobservable effort is not constraint-efficient: the debt level issued is inefficiently high, while the effort is inefficiently low for $F''(.)$ and $\frac{\theta_b'(e)}{\theta_b(e)} > \frac{\theta_g'(e)}{\theta_g(e)}$. The fraction of ex-post risk sharing is inefficiently high, while the fraction of ex-ante risk sharing is inefficiently low.*

Equation 2.41 in the appendix clearly illustrates the different sources of inefficiencies. The inefficiencies are not solved, and in fact reinforce each other, amplifying the original inefficiencies and the cost of supplying safe assets in the real economy.

Policy responses aiming to ameliorate the aforementioned inefficiencies,however, must follow a holistic approach and take into account that both forms are jointly determined. Otherwise, the policy can have unintended consequences, e.g., policy targeting to individual financial instruments could affect the form in which safe assets are produced, instead of affecting their level, which could potentially exacerbate the original inefficiency.

2.5 Conclusion

I study the determinants of private safe asset supply and its macroeconomic implications in a theoretical framework. The model builds on the documented tension between financial intermediaries' risk sharing activities and the quality of their investments due to a moral hazard problem. The interplay between these two determinants shapes the supply curve of safety and provides an important insight: along the upward-sloping curve, the risk-sharing activity intensifies, jeopardising the incentives to enhance the quality of the investment, deteriorating the expected output and amplifying economic volatility. Thus, the real costs of safe asset supply are particularly acute in the current environment in which safe assets are scarce. Incomplete markets, i.e., the lack of a full set of state-contingent claims, hinders financial intermediaries' ability to cope with the informational friction, and further explain important sources of inefficiencies that call for policy intervention. However, the model highlights the need to understand the interaction between different forms to ensure safety for an adequate and effective policy response.

Appendix

Risk preferences. In line with previous studies that focused on the supply side of safe assets, the demand for safety services is modelled in reduced form. In particular, I follow Gennaioli et al. (2012), Caballero and Farhi (2018) and Segura and Villacorta (2020), in which demand for safety arises from a subset of investors who are infinitely risk averse. Several studies in the same vein but with a different approach model savers' preferences for safe assets by introducing a convenience yield associated to this class of assets (e.g., Krishnamurthy and Vissing-Jorgensen (2015), Stein (2012)), in the spirit of the traditional approach to demand for money in terms of transactions costs or "money in the utility function". Alternatively, some studies introduce a cost for participating in risky equity markets, which effectively leads to a preference for safe assets/debt in the economy (Carletti et al. (2020)). Although different, all of these approaches are built on the same premise: the recognition of a segmentation between risky and safe assets, which causes a pricing discontinuity around the zero risk bound, i.e., the safety premium. The demand that aims to satisfy the especial services that safe assets provide, does not consider risky assets as substitutes, and this lack of substitutability implies that the price of safety is, at least partly, determined only by the supply of and demand for safety services. The existence of a safety premium has been well documented in the empirical literature (e.g., Krishnamurthy and Vissing-Jorgensen (2012), Del Negro et al. (2019), Kacperczyk et al. (2021)).

Investment technology. The investment project broadly represents the real investment undertaken by financial intermediaries when granting loans to the real sector. The idiosyncratic risk at the intermediary level reflects the fact that intermediaries cannot fully diversify idiosyncratic risk with the loans they grant, which may result from the (unmodelled) frictions that force them to specialize along some geographic or industrial segment of the loan issuance market. Consistent with this assumption, the empirical literature that exploits bank-firm loan-level data finds idiosyncratic variation at the bank-level

In addition, the each project yields a zero output with a positive probability. This stark assumption excludes the equilibrium type in which bankers hold safe collateral to issue safe assets, to focus on the case in which bankers are forced to trade in secondary markets to obtain the safe collateral. The latter is motivated by recent the developments related to the alternative forms to manufacture safe assets based on securitization and other forms of financial innovation.

Lastly, the fact that only two idiosyncratic states are considered, coupled with the fact that one of the outcomes is zero, implies that the type of securities backed by the cash flows of the investment technology is irrelevant. I am abstracting from security design at this stage to focus on how different risk-sharing alternatives cope with the aggregate investment.

Effort technology. Bankers' effort partially capture bankers screening activity: the costly information acquisition pursued by some lenders when granting loans with the purpose of learning about the (unobserved) value of borrowers' project, beyond the information conveyed by the market price. Investing time and effort in the screening technology improves the quality of the loans originated, which is conductive to better capital allocation. The screening technology assumed in this paper aims to capture financial intermediaries' comparative advantage in loan origination. The focus of this study is on understanding the interaction between effort incentives and trade in secondary markets in which financial claims backed by bankers' project's output (i.e., granted loans) are traded. Consequently, information asymmetries in the primary credit market are not explicitly modelled; instead, the positive effect of screening on the likelihood of a good outcome is modeled in reduced form. But let's micro found this channel in order to understand the effect of screening.

Fixed aggregate investment. It is assumed that agents in this economy inelastically invest their funds to transfer them to future dates. By doing so, I assume that, in equilibrium, there is no quantity effect, and that all adjustments occur through prices. This simplifying assumption avoids distraction from the quality channel to produce safe assets, which is the main focus of the paper.

Mathematical appendix

Exogenously complete financial markets

Endogenously incomplete: unobservable effort

Each financial claims is sold in different market based on their quality. The price in each market can be disentangled in two components: (i) ω state contingent Arrow-Debreu price which is common across markets within the same aggregate state, and (ii) a market specific component that depends on the probability of the idiosyncratic state occurring given the effort level and aggregate state ω:

$$p_z(e) = q_\omega \times Prob. \left[\iota | e, \omega\right]$$

where q_ω is the ω Arrow-Debreu state contingent price. If q_ω is not common across markets within the same aggregate state, there would be an arbitrage opportunity. And if the market specific component does not reflect the expected output per unit of investment, demand and supply in that market would not equate.

In addition, all purely financial transactions yield a zero NPV, thus, there is a trivial multiplicity of equilibrium. Without loss of generality, all financial transactions that do not yield positive gains are disregarded for clarity reasons. Therefore, without loss of generality financial claims sold are exclusively backed by bankers' project's output, and due to limited liability, then $0 \leq s_{\omega,s} \leq Ak$ and $s_{\omega,f} = 0$ (hence, so is $b_{\omega,f} = 0$).

Infinitely risk averse maximize $U^h = \min_z \left\{c^h_{1z}\right\} - c(e^n)k$ subject to:

(i) Budget constraint at $t = 0$: $k^h \leq w + \sum_\omega q_\omega \left[\theta_\omega(\hat{e}) s^h_{\omega s} - \theta_\omega(e^m) b^{hm}_{\omega s}\right]$

(ii) Feasibility constraint at $t = 1$ and state $z = \langle \omega, \iota \rangle$:

$$c^h_{1,z} \leq A_\iota k^h - s^h_z + \theta_\omega(e^m) b^{hm}_{\omega s}$$

(iii) Non-negativity constraint for $k^h, c^h_{1z}, \left\{s^h_{\omega\iota}\right\}$, and for $e^h \in [0, \overline{e}]$

(iv) Limited liability financial claims: $s^h_{\omega\iota} \leq A_\iota k^h$

(v) Incentive compatibility constraint: $\hat{e}^h = e^{h*} \equiv \underset{e' \in [0,\bar{e}]}{\text{argmax}} \ \min_z \left\{ c^h_{1z} \right\} - c(e')k^h$

The value of consumption in the worst case scenario is independent of infinitely risk averse investors' effort, consequently, their optimal effort level (and the one expected by the market) equals zero, $\hat{e}^h = e^{h*} = 0$. Their project is subject to idiosyncratic risk, thus, with positive probability it yields a zero output. Hence, if they invest in their project, they will fully sell the corresponding cash flow in financial markets, $s^h_{\omega\iota} = A_\iota k^h$, and buy in exchange a portfolio that is riskless, $\theta_g(e^m)b^{hm}_{gs} = \theta_b(e^m)b^{hm}_{bs}$. Hence, Their utility is given by

$$U^h = \frac{w + \left[\sum_\omega q_\omega \theta_\omega(0)A - 1 \right] k^h}{q_g + q_r}$$

Therefore, the necessary condition for a positive investment in their project is $\sum_\omega q_\omega \theta_\omega(0)A \geq 1$.

Risk neutral investors maximize $U = \mathbb{E}_e [c_{1z}] - c(e)k$ subject to:

(i) Budget constraint at $t = 0$: $k \leq (1 - w) + \sum_\omega q_\omega [\theta_\omega(\hat{e})s_{\omega s} - \theta_\omega(e^m)b^m_{\omega s}]$

(ii) Feasibility constraint at $t = 1$ and state $z = \langle \omega, \iota \rangle$:

$$c_{1,z} \leq A_\iota k - s_z + \theta_\omega(e^m)b^m_{\omega s}$$

(iii) Non-negativity constraint for $k, c_{1z}, \{s_{\omega\iota}\}$, and for $e \in [0, \bar{e}]$

(iv) Limited liability financial claims: $s_{\omega\iota} \leq A_\iota k$

(v) Incentive compatibility constraint: $\hat{e} = e^* \equiv \underset{a' \in [0,\bar{e}]}{\text{argmax}} \ \mathbb{E}_{e'} [c_{1z}] - c(e')k$

Since their utility is monotonically increasing in c_{1z} for $\forall z$, the budget constraint and feasibility constraint hold with equality. Therefore, $\theta_\omega(e^m)b^m_{\omega s} \geq 0$ for $\forall \omega$, which implies that short selling of these financial claims is not feasible.

From the incentive compatibility constraint, the optimal effort level is

$$\mathbb{E}\left[\theta'_{\omega} \times (Ak - s_{\omega s})\right] - c'(e)k = 0 \tag{2.28}$$

where $\theta''_{\omega} = 0$ and $c''(e) > 0$ so that the second order conditions holds. In addition, $\overline{e}$ is such that the upper bound of e is never binding. The constraint $s_{\omega\iota} \leq A_{\iota}k$ further ensures that $e^* \geq 0$, so that the solution is interior. Hence, the first-order approach (FOA) will be apply.

Therefore, the Langrangian of the problem is given by

$$\begin{aligned}
\mathcal{L} \ =& \mathbb{E}\left[\left[\theta_{\omega}(e)A - c(e)\right]k - \theta_{\omega}(e)s_{\omega s} + \theta_{\omega}(e^m)b^m_{\omega s} - \lambda_{\omega}\left[s_{\omega s} - Ak\right]\right] \\
& - \frac{1}{q^r}\left[k - (1-w) - \sum_{\omega} q_{\omega}\left[\theta_{\omega}(\hat{e})s_{\omega s} - \theta_{\omega}(e^m)b^m_{\omega s}\right]\right] \\
& - \eta\left[\mathbb{E}\left[\theta'_{\omega} \times (Ak - s_{\omega s})\right] - c'(\hat{e})k\right]
\end{aligned}$$

The Kuhn-tucker conditions for $\omega \in \{g, b\}$ are given by

$$\begin{aligned}
k: &\quad \mathbb{E}\left[\theta_{\omega}(e)\right]A - c(e) + \mathbb{E}\left[\lambda_{\omega}\right]A - \frac{1}{q^r} - \eta\left(\mathbb{E}\left[\theta'_{\omega}\right]A - c'(e)\right) \leq 0 \\
b^m_{\omega}: &\quad \left[\pi_{\omega} - \frac{1}{q^r}q_{\omega}\right]\theta_{\omega}(\hat{e}^m) \leq 0 \\
s_{\omega}: &\quad -\pi_{\omega}\theta_{\omega}(e) - \pi_{\omega}\lambda_{\omega} + \frac{1}{q^r}q_{\omega}\theta_{\omega}(\hat{e}) + \eta\pi_{\omega}\theta'_{\omega} \leq 0 \\
\hat{e}: &\quad \frac{1}{q^r}\sum_{\omega} q_{\omega}\theta'_{\omega}s_{\omega s} + \eta c''(e)k = 0 \\
&\quad \lambda_{\omega} \geq 0 \text{ and } \lambda_{\omega}\left[s_{\omega s} - Ak\right] = 0 \\
&\quad \frac{1}{q^r} \geq 0 \text{ and } \frac{1}{q^r}\left[k - (1-w) - \sum_{\omega} q_{\omega}\left[\theta_{\omega}(\hat{e})s_{\omega s} - \theta_{\omega}(e^m)b^m_{\omega s}\right]\right] = 0 \\
&\quad \eta \geq 0 \text{ and } \eta\left[\mathbb{E}\left[\theta'_{\omega} \times (Ak - s_{\omega s})\right] - c'(e)k\right] = 0
\end{aligned}$$

The Lagrangian multiplier on the budget constraint, denoted by $\frac{1}{q^r}$, represents the expected rate of return of their wealth, this is, the amount by which risk neutral investors' utility would increase if they would have an additional unit of wealth relaxing their budget constraint. They will choose the investment opportunity with the highest expected return, and such opportunity cost determines the inverse of

q^r. In particular, they face two investment alternatives: (i) either they invest in the financial claims sold by others, or/and (ii) they invest resource and effort in their project and sell financial claims on their cash flows (if any). Let's analyse each of this cases to find the necessary conditions required for each of these cases to prevail in equilibrium.

If risk neutral investors only invest in the financial claims sold by other agents, then $k, \{s_\omega\}$ and e equal zero, and

$$b^m_{\omega s} \in \begin{cases} 0 & \text{if } \frac{\pi_\omega}{q_\omega} < \frac{\pi_{\omega^-}}{q_{\omega^-}} \\ \frac{1 - w - q_{w^-}\theta_{\omega^-}(e^m)b^m_{\omega^- s}}{\theta_\omega(\hat{e}^i)q_\omega} & \text{if } \frac{\pi_\omega}{q_\omega} = \frac{\pi_{\omega^-}}{q_{\omega^-}} \\ \frac{1-w}{\theta_\omega(e^m)q_\omega} & \text{if } \frac{\pi_\omega}{q_\omega} > \frac{\pi_{\omega^-}}{q_{\omega^-}} \end{cases}$$

where $\omega, \omega^- \in \{g, b\}$ such that $\omega \neq \omega^-$. If this option prevails, then $\frac{1}{q^r} = \max_\omega \left\{\frac{\pi_\omega}{q_\omega}\right\}$.

If risk neutral investors invest all their resource in their project, then, $k = 1 - w + \sum_s q_\omega \theta_\omega(\hat{e}) s_{\omega s}$. Therefore, both investment level and also $e = \hat{e}$, which is implicitly determined in equation (2.28), depend on $\{s_\omega\}$. Notice that a necessary condition for $\{s_{\omega s} > 0\}$ is that $\frac{1}{q^r} > \frac{\pi_\omega}{q_\omega}$. This is the case because

$$\eta = -\frac{\frac{1}{q^r}\sum_\omega q_\omega \theta'_\omega s_{\omega s}}{c''(e)k} < 0$$

due to the moral hazard problem. Then, in case of an interior solution ($\lambda_\omega = 0$), $s_{\omega s}$ is implicitly determined in

$$\frac{1}{q^r} \times \left[q_\omega \theta_\omega(e) - \left(\frac{\sum_\omega q_\omega \theta'_\omega s_{\omega s}}{c''(e)k}\right) \times \pi_\omega \theta'_\omega\right] - \pi_\omega \theta_\omega(e) = 0 \tag{2.29}$$

Otherwise, if the constraint is binding ($\lambda_\omega > 0$), then $s_{\omega s} = Ak$.

If this option prevails, the expected return risk neutral investors earned is

$$\frac{1}{q^r} = \frac{\mathbb{E}\left[\theta_\omega(e)\,Ak - c(e)k - \theta_\omega(e)s_{\omega s}\right]}{k - \sum_\omega q_\omega \theta_\omega(e) s_{\omega s}} \tag{2.30}$$

where $k, \{s_\omega\}$ and e are determined through equations (2.28) - (2.29).

The necessary condition for a positive investment in their technology is that the return determined in equation (2.30) is (weakly) bigger than $\max_{\omega}\left\{\frac{\pi_\omega}{q_\omega}\right\}$. Otherwise, risk neutral investors will only invest in the financial claims sold by other agents.

Equilibrium characterization

In order to find the equilibrium of this model, first an investment strategy is assume for both agents, and then it is checked whether there are incentives to deviate from that assumed strategy. So let's first assume that infinitely risk averse agents invest in their technology, while bankers purchase financial claims sold by households. Then, $\sum_\omega q_\omega \theta_\omega(0)A = 1$, and depending on the parameter values there are different types of equilibriums:

1. If $w \leq \frac{\theta_b(0)}{\mathbb{E}[\theta_\omega(0)]}$, recession state cash flows are abundant, and not only households but also bankers buy them which implies $\frac{\pi_g}{q_g} = \frac{\pi_b}{q_b}$ so that $b_{bs} = \frac{1-w-q_g\theta_g(0)b_{gb}}{\theta_b(0)q_b} \geq 0$. Hence, in equilibrium $q_\omega = \frac{\pi_\omega}{\mathbb{E}[\theta_\omega(0)]A}$.

   To proof whether at the equilibrium q_ω investing in financial claims issued by infinitely risk averse agents is consistent with bankers' optimal behaviour, the opportunity cost of investing in their technology needs to be calculated. From their optimality conditions if $\eta = \frac{\pi_\omega}{q_\omega} = \mathbb{E}\left[\theta_\omega(0)\right]A$, then $s_{\omega s} = 0$, but

   $$\mathbb{E}\left[\theta_\omega\left(e^{max}\right)\right]A - c\left(e^{max}\right) > \mathbb{E}\left[\theta_\omega(0)\right]A$$

   where e^{max} is the level that maximizes the net investment payoff. Hence, there is a contradiction as bankers' opportunity cost is higher.

2. if $w > \frac{\theta_b(0)}{\mathbb{E}[\theta_\omega(0)]}$, recession state cash flows are scarce and only depositors buy them. Hence, $b_{gs} = \frac{1-w}{q_g}$ and in equilibrium

   $$q_g = \frac{1-w}{(\varepsilon_g - \varepsilon_r)A} \quad \text{and} \quad q_r = \frac{w}{\theta_b(0)A} - \frac{1-w}{(\theta_g(0)-\theta_b(0))A}$$

   From their optimality conditions if $\eta = \frac{\pi_g}{q_g} = \frac{\pi_g(\theta_g(0)-\theta_b(0))A}{1-w} > \frac{\pi_r}{q_r}$, then $s_{gs} = 0$, but $s_{bs} > 0$ if $k > 0$. The expected return on their investment technology

$$\frac{\mathbb{E}\left[\theta_\omega\left(a_1^*\right)\right]A - \pi_r\theta_r(a_1^*)(s_{bs}^*/k^*) - c(a_1^*)}{1 - q_b\theta_r(a_1^*)(s_{bs}^*/k^*)} \geq \frac{\pi_g\theta_g(a_2^*)A - c(a_2^*)}{1 - q_b\theta_b(a_2^*)A} > \frac{\pi_g}{q_g}$$

where $a_1^* = f\left(\left(\frac{s_{bs}}{k}\right)^*, 0\right)$ and $a_2^* = f(A, 0)$. The range of optimal diversification level is in $\left(\frac{s_r}{k}\right)^* \in (0, A]$, and if interior, it is determined at the level that maximizes the expected return of their investment technology. Hence, the expected return is at least as high as when $s_{bs} = Ak$, which in turn easy to proof that it is higher than their outside option. Therefore, the assumed strategy cannot be an equilibrium.

Hence, the proposed strategy cannot be an equilibrium since the state-contingent prices are too high. Therefore, the latter have to decrease, which desincentives infinitely risk averse agents from investing in their own project. Instead, bankers invest in their technology and depositors in turn purchase financial claims issued by bankers, thus, $k = 1$. Then, depending on the parameter values:

(a) If $w \leq \overline{w}$ [13] the safe collateral constraint is not binding ($\lambda_b = 0$).

The equilibrium price functions are

$$q_g = q^r\pi_g + \rho(e)\left(q^s - q^r\right)$$
$$q_b = q^r\pi_b + (1 - \rho(e))\left(q^s - q^r\right)$$

where $\rho(e) \equiv \frac{\frac{\pi_g\theta_g'(e)}{\theta_g(e)}}{\frac{\pi_g\theta_g'(e)}{\theta_g(e)} + \frac{\pi_b\theta_b'(e)}{\theta_b(e)}} < 1$. The amount of safe assets in the economy, $d = \theta_\omega(\hat{e})s_{\omega s}$ for $\forall\omega$, is implicitly determined as a function of q^s in the

[13]where $\overline{w} = \frac{1}{1+\Psi(\underline{e})} < 1$ where $\underline{e}$ is determined in $\pi_g\theta_g'\left(\frac{\theta_g(\underline{e})-\theta_b(\underline{e})}{\theta_g(\underline{e})}\right)A - c'(\underline{e}) = 0$ and

$$\Psi(\underline{e}) \equiv \left(\frac{1}{1+\xi(\underline{e})}\right)\times\left(\frac{\pi_g\left[\theta_g(\underline{e}) - \theta_b(\underline{e})\right]A - c(\underline{e})}{\theta_b(\underline{e})A}\right) > 0 \text{ where } \xi(\underline{e}) \equiv \frac{\frac{\mathbb{E}^2\left[\frac{\theta_\omega'}{\theta_\omega(e)}\right]}{c''(e)}\theta_b(\underline{e})A}{1 - \frac{\mathbb{E}\left[\frac{\theta_\omega'}{\theta_\omega(\underline{e})}\right]}{c''(e)}\rho'(\underline{e})\mathbb{E}\left[\frac{\theta_g(\underline{e})-\theta_b(\underline{e})}{\theta_\omega(\underline{e})}\right]\theta_b(\underline{e})A} > 0.$$

following system of equations:

$$\begin{aligned}
&\mathbb{E}\left[\theta'_\omega\right] A-\mathbb{E}\left[\frac{\theta'_\omega}{\theta_\omega(e)}\right] d-c'(e)=0\\
&\left(\frac{q^s}{q^r}-1\right)-\left(-\frac{de}{dd}\right)\times\mathbb{E}\left[\frac{\theta'_\omega}{\theta_\omega(e)}\right] d-=0\\
&q^r=\frac{1-q^s d}{\mathbb{E}\left[\theta_\omega(e)\right] A-c(e)-d}
\end{aligned}$$

where $\frac{de}{dd}=\left(\frac{\mathbb{E}\left[\frac{\theta'_\omega}{\theta_\omega(e)}\right]}{c''(e)-\mathbb{E}\left[\frac{\theta'^2_\omega}{\theta^2_\omega(e)}\right] d}\right)$.

Supply curve. It is upward sloping but changes the inputs.

$$\begin{aligned}
&\frac{dd^s(q^s)}{dq^s}=-\frac{H_{q^s}+H_{q^r}\frac{\partial q^r}{\partial q^s}}{H_d+H_e\frac{\partial e}{\partial d}+H_{q^r}\left[\frac{\partial q^r}{\partial d}+\frac{\partial q^r}{\partial e}\frac{\partial e}{\partial d}\right]}>0\\
&H_d<0 \text{ and } H_e>0\\
&H_{q_s}>0 \text{ and } H_{q_r}<0\\
&\frac{\partial e}{\partial d}=\frac{-\mathbb{E}\left[\frac{\theta'_\omega}{\theta_\omega(e)}\right]}{c''(e)-\mathbb{E}\left[\frac{\theta'^2_\omega}{\theta^2_\omega(e)}\right] d}\overset{\text{from FOC}}{<}0\\
&\frac{\partial q^r}{\partial d}=\frac{-\left(q^s-q^r\right)}{\mathbb{E}\left[\theta_\omega(e)\right] A-c(e)-d}<0\\
&\frac{\partial q^r}{\partial q^s}=\frac{-d}{\mathbb{E}\left[\theta_\omega(e)\right] A-c(e)-d}<0\\
&\frac{\partial q^r}{\partial e}=\frac{-q^r\left(\mathbb{E}\left[\theta'_\omega(e)\right] A-c'(e)\right)}{\mathbb{E}\left[\theta_\omega(e)\right] A-c(e)-d}<0\\
&\frac{\partial q^r}{\partial d}+\frac{\partial q^r}{\partial e}\frac{\partial e}{\partial d}\overset{\text{from FOC}}{=}0
\end{aligned}$$

Demand curve. Households devote all their resources to buy safe debt. Hence, $d^h=\frac{w}{q^s}$ which is downward sloping:

$$\frac{\partial d^s(q^s)}{\partial q^s}=-\frac{w}{q^{s2}}<0$$

The equilibrium prices of safe and risky assets, which further determine the consumption allocation of infinitely risk averse and risk neutral investors are given by

$$q^s = \frac{w}{d} \quad \text{and } q^r = \frac{1-w}{\mathbb{E}\left[\theta_\omega\left(e\right)\right]A - c(e) - d}$$

where d and e are implicitly determined in the aforementioned system of equations.

(b) If $w > \overline{w}$ the safe collateral constraint is binding ($\lambda_r > 0$). The pricing functions are given by

$$\begin{aligned} q_g &= \frac{1-w}{\pi_g\left[\theta_g(a) - \theta_r(a)\right]A - c(a)} \left[\frac{\pi_g + \left[\frac{\pi_g[\theta_g(a)-\theta_r(a)]A - c(a)}{\theta_r(a)A} \frac{w}{1-w}\right] \frac{\gamma}{\theta_r(a)} \frac{\pi_g\gamma}{\theta_g(a)} \frac{\theta_r(a)A}{c''(a)}}{1 + \left[\frac{\gamma}{\theta_r(a)} - \frac{\gamma}{\theta_g(a)}\right] \frac{\pi_g\gamma}{\theta_g(a)} \frac{\theta_r(a)A}{c''(a)}} \right] \\ q_r &= \frac{1-w}{\pi_g\left[\theta_g(a) - \theta_r(a)\right]A - c(a)} \left[\frac{\left[\frac{\pi_g[\theta_g(a)-\theta_r(a)]A - c(a)}{\theta_r(a)A} \frac{w}{1-w}\right] \left[1 - \frac{\pi_g\gamma^2}{\theta_g^2(a)} \frac{\theta_r(a)A}{c''(a)}\right] - \pi_g}{1 + \left[\frac{\gamma}{\theta_r(a)} - \frac{\gamma}{\theta_g(a)}\right] \frac{\pi_g\gamma}{\theta_g(a)} \frac{\theta_r(a)A}{c''(a)}} \right] \end{aligned}$$

The amount of safe assets in the economy, and the effort level is given by

$$\begin{aligned} s_{bs} &= A \\ s_{gs} &= \frac{\theta_b(e)}{\theta_g(e)} A \\ \pi_g \theta_g' &\left[1 - \frac{\theta_b(e)}{\theta_{bg}(e)}\right] A - c'(e) = 0 \\ q^r &= \frac{\pi_g\left[\theta_g\left(e\right) - \theta_b\left(e\right)\right]A - c(e)}{1 - q^s\theta_b\left(e\right)A} \end{aligned}$$

Supply curve. The supply curve is flat. Any increase in q^s is fully offset by an increase in q^r. **Demand curve**. Is the same as in the previews case. **The equilibrium prices** of safe and risky assets, which further determine the consumption allocation of infinitely risk averse and risk neutral investors are given by

$$q^s = \frac{w}{d} \quad \text{and } q^r = \frac{1-w}{\pi_g\left[\theta_g\left(e\right) - \theta_b\left(e\right)\right]A - c(e)}$$

where e is implicitly determined in the aforementioned equation.

Social Planner

The social planner decides on the consumption of infinitely risk averse investors (c_z^d) and risk neutral investors (c_z^b) to maximize the expected utility of the two groups of investors (with respective Pareto-efficient weights W^d and W^b):

$$U^{sp} = W^d \left[\underset{\omega}{min} \left\{ c_{1z}^d \right\} - c(e^d)k^d \right] + W^b \left[\mathbb{E}_e \left[c_{1z}^b \right] - c(e^b)k^b \right]$$

The social planner commits to seize all investment from agents and, in turn, make transfers (possibly) on their output realization. The maximization is subject to the following constraints:

(i) The resource constraint at $t = 0$ and $t = 1$

$$k^d + k^b \leq 1$$

(ii) Feasibility constraint (assuming each agent is only subject to their own idiosyncratic risk if any)

$$\sum_{h,b} \left(\theta_\omega(e^i)c_{\omega s} + (1 - \theta_\omega(e^i))c_{\omega f} \right) = \theta_\omega(e^b)Ak^b + \theta_\omega(e^d)A_\iota k^d$$

(ii) The participation constraint for each of the groups:

$$\underset{\omega}{min} \left\{ c_{2\omega}^d \right\} \geq 0$$
$$\mathbb{E}_e \left[c_{2z}^b \right] - c(e)k^b \geq \left(\mathbb{E}_{e^{max}} \left[A \right] - c(e^{max}) \right) (1 - w)$$

where e^{max} is implicitly determined in $\mathbb{E} \left[\theta_\omega' \right] A - c'(e^{max}) = 0$

(i) Incentive compatibility constraint:

$$e^h = \underset{e' \in [0,\bar{e}]}{argmax} \underset{\omega}{min} \left\{ c_{2\omega}^d \right\} - c(e')k$$
$$e^b = \underset{e' \in [0,\bar{e}]}{argmax} \mathbb{E}_{e'} \left[c_{2z}^b \right] - c(e')k^b$$

(iv) Non-negativity constraint for c_{tz}^j for each group of agent j, date t and state z

Notice that the burden of the moral hazard is lower for risk neutral investors than for infinitely risk averse investors. The former are more productive in managing the effort and investment technology, thus, $k^b = 0$ and $k^b = 1$. Therefore, only the transfer (t_z) from risk neutral to infinitive risk averse investors can be positive.

$$\begin{aligned}\mathcal{L} \ &= W^d \underline{c}^d + (1 - W^d)\left[\mathbb{E}\left[\theta_\omega(e)\left(A - t_{\omega s}\right)\right] - c(e)\right] - \mathbb{E}\left[\mu_\omega\left[\underline{c}^h - \theta_\omega(e)t_{\omega s}\right]\right] \\ &- \mathbb{E}\left[\lambda_\omega\left[t_{\omega s} - A\right]\right] - \eta\left[\mathbb{E}\left[\theta'_\omega\right]A - \mathbb{E}\left[\theta'_\omega t_{\omega s}\right] - c'(e)\right]\end{aligned}$$

The Kunh-tucker conditions are given by

$$\begin{aligned}\underline{c}^d: \quad & W^d - \mathbb{E}\left[\mu_\omega\right] \leq 0 \\ t_{\omega s}: \quad & -\left(1 - W^h\right)\pi_\omega\theta_\omega(e) + \pi_\omega\mu_\omega\theta_\omega(e) - \pi_\omega\lambda_\omega - \eta\pi_\omega\theta'_\omega \leq 0 \\ e: \quad & \mathbb{E}\left[\mu_\omega\theta'_\omega t_{\omega s}\right] + \eta c''(e) = 0 \\ & \mu_\omega \geq 0 \text{ and } \mu_\omega\left[\underline{c}^h - \theta_\omega(e)t_{\omega s}\right] = 0 \\ & \lambda \geq 0 \text{ and } \lambda_\omega\left[t_{\omega s} - A\right] = 0 \\ & \eta \geq 0 \text{ and } \eta\left[\mathbb{E}\left[\theta'_\omega\right]A - \mathbb{E}\left[\theta'_\omega t_{\omega s}\right] - c'(e)\right] = 0\end{aligned}$$

These optimality conditions are equivalent to the competitive equilibrium's optimality conditions when $\frac{\pi_\omega\mu_\omega}{(1-W^d)} = \frac{q_\omega}{q^r}$, which further implies $W^d = \frac{q^s}{q^s+q^r}$.

Endogenously complete: observable effort

Risk neutral investors maximize $U = \mathbb{E}_e\left[c_{1z}\right] - c(e)k$ subject to:

(i) Budget constraint at $t = 0$: $k \leq (1 - w) + \sum_\omega q_\omega\left[\theta_\omega(e)s_{\omega s} - \theta_\omega(e^m)b^m_{\omega s}\right]$

(ii) Feasibility constraint at $t = 1$ and state $z = \langle\omega, \iota\rangle$:

$$c_{1,z} \leq A_\iota k - s_{\omega\iota} + \theta_\omega(e^m)b^m_{\omega s}$$

(iii) Non-negativity constraint for $k, c_{1z}, \{s_{\omega\iota}\}$, and for $e \in [0, \bar{e}]$

(iv) Limited liability financial claims: $s_{\omega\iota} \leq A_\iota k$

Since their utility is monotonically increasing in c_{1z} for $\forall z$, the budget constraint and feasibility constraint hold with equality. Therefore, $\theta_\omega(e^m)b^m_{\omega s} \geq 0$ for $\forall\omega$, which implies that short selling of these financial claims is not feasible. Therefore, the Langrangian of the problem is given by

$$\begin{aligned}\mathcal{L} =&\mathbb{E}\left[[\theta_\omega(e)A - c(e)]\, k - \theta_\omega(e)s_{\omega s} + \theta_\omega(e^m)b^m_{\omega s} - \lambda_\omega \left[s_{\omega s} - Ak\right]\right] \\ &-\frac{1}{q^r}\left[k - (1-w) - \sum_\omega q_\omega \left[\theta_\omega(e)s_{\omega s} - \theta_\omega(e^m)b^m_{\omega s}\right]\right]\end{aligned}$$

The Kuhn-tucker conditions for $\omega \in \{g, b\}$ are given by

$$\begin{aligned}
k: &\qquad \mathbb{E}\left[\theta_\omega(e)\right] A - c(e) + \mathbb{E}\left[\lambda_\omega\right] A - \frac{1}{q^r} \leq 0 \\
s_\omega: &\qquad -\pi_\omega\theta_\omega(e) - \pi_\omega\lambda_\omega + \frac{1}{q^r} q_\omega\theta_\omega(e) \leq 0 \\
e: &\qquad \mathbb{E}\left[\theta'_\omega\right] Ak - \mathbb{E}\left[\theta'_\omega s_{\omega s}\right] - \frac{1}{q^r}\sum_\omega q_\omega\theta'_\omega s_{\omega s} - c'(e)k \leq 0 \\
&\qquad \lambda_\omega \geq 0 \text{ and } \lambda_\omega \left[s_{\omega s} - Ak\right] = 0 \\
&\frac{1}{q^r} \geq 0 \text{ and } \frac{1}{q^r}\left[k - (1-w) - \sum_\omega q_\omega\left[\theta_\omega(\hat{e})s_{\omega s} - \theta_\omega(e^m)b^m_{\omega s}\right]\right] = 0 \\
&\qquad \eta \geq 0 \text{ and } \eta\left[\mathbb{E}\left[\theta'_\omega \times (Ak - s_{\omega s})\right] - c'(e)k\right] = 0
\end{aligned}$$

Let's simplify the analysis of the partial equilibrium, and assume that the conditions for k and $s_{\omega s}$ to be positive hold. When effort is observable, the effort level is implicitly determined in

$$\mathbb{E}\left[\theta'_\omega\right] Ak + \sum_\omega \left(\frac{q_\omega}{q^r} - \pi_\omega\right)\theta'_\omega s_{\omega s} - c'(e)k = 0 \tag{2.31}$$

So the effort level depends on claim sold as long as $q_\omega \neq q^r\pi_\omega$. The optimal financial claims sold is given by

$$s_{\omega s} = \begin{cases} 0 & \text{if } q_\omega < q^r\pi_\omega \\ [0, Ak] & \text{if } q_\omega = q^r\pi_\omega \\ Ak & \text{if } q_\omega > q^r\pi_\omega \end{cases}$$

In the last case, $\lambda_\omega > 0$, otherwise $\lambda_\omega = 0$.

Equilibrium characterization

Depending on the parameter values:

(a) If $w \leq \frac{\theta_b(e)}{\mathbb{E}[\theta_\omega(e)]}$ where e is implicitly determined in (2.32), then the equilibrium price functions are

$$q_\omega = q^r \pi_\omega$$

The amount of safe assets in the economy, $d \leq \theta_\omega(e) s_{\omega s}$ for $\forall \omega$, where $s_\omega \in \left[\frac{d}{\theta_\omega(e)}, Ak\right]$. Effort is implicitly determined in

$$\mathbb{E}\left[\theta'_\omega\right] A - c'(e) = 0 \tag{2.32}$$

The supply curve is flat at the equilibrium safety price (which is independent of debt). **Demand curve.** is the same as before. **The equilibrium prices** of safe and risky assets are given by

$$q^s = q^r = \frac{1}{\mathbb{E}\left[\theta_\omega(e)\right] A - c(e)}$$

where effort is determined in equation (2.32).

(b) If $w > \frac{\theta_b(e)}{\mathbb{E}[\theta_\omega(e)]}$ where e is implicitly determined in (2.32), then

$$q_g = q^r \pi_g$$
$$q_r = q^r \pi_g + (q^s - q^r)$$

The amount of safe assets in the economy, and the effort level is given by

$$s_{bs} = A \text{ and } s_{gs} = \frac{\theta_b(e)}{\theta_g(e)} A \text{ such that } d = \theta_b(e) A$$
$$\mathbb{E}\left[\theta'_\omega\right] A + \left(\frac{q^s - q^r}{q^r}\right) \theta'_b A - c'(e) = 0$$
$$q^r = \frac{\pi_g \left[\theta_g(e) - \theta_b(e)\right] A - c(e)}{1 - q^s d}$$

Supply curve. It is upward sloping, $\frac{dd^s(q^s)}{dq^s} > 0$, since e is a positive function of q^s. **Demand curve**. remains the same.**The equilibrium prices** of safe and risky assets, which further determine the consumption allocation of infinitely risk averse and risk neutral investors are given by

$$q^s = \frac{w}{d} \quad \text{and } q^r = \frac{1-w}{\pi_g \left[\theta_g\left(e\right) - \theta_b\left(e\right)\right] A - c(e)}$$

where e is implicitly determined in the aforementioned equation.

Social Planner

The social planner problem is equivalent to the previous section's without the incentive compatibility constraint.

$$\begin{aligned}\mathcal{L} \; =& W^d \underline{c}^d + (1-W^d)\left[\mathbb{E}\left[\theta_\omega(e)\left(A - t_{\omega s}\right)\right] - c(e)\right] - \mathbb{E}\left[\mu_\omega\left[\underline{c}^h - \theta_\omega(e)t_{\omega s}\right]\right] \\ & - \mathbb{E}\left[\lambda_\omega\left[t_{\omega s} - A\right]\right]\end{aligned}$$

The Kunh-tucker conditions are given by

$$\begin{aligned}\underline{c}^d : \quad & W^d - \mathbb{E}\left[\mu_\omega\right] \leq 0 \\ t_{\omega s} : \quad & -\left(1 - W^h\right)\pi_\omega\theta_\omega(e) + \pi_\omega\mu_\omega\theta_\omega(e) - \pi_\omega\lambda_\omega \leq 0 \\ e : \quad & (1-W^d)\mathbb{E}\left[\theta'_\omega \times \left(A - s_{\omega s}\right)\right] + \mathbb{E}\left[\mu_\omega\theta'_\omega t_{\omega s}\right] - c'(e) \leq 0 \\ & \mu_\omega \geq 0 \text{ and } \mu_\omega\left[\underline{c}^d - \theta_\omega(e)t_{\omega s}\right] = 0 \\ & \lambda \geq 0 \text{ and } \lambda_\omega\left[t_{\omega s} - A\right] = 0\end{aligned}$$

What is clear from the optimality conditions is that for certain Pareto weights the competitive equilibrium is efficient. When $\frac{q^\omega}{q^r} = \frac{\mu_\omega}{(1-W^d)}$, then $\frac{\mu_g}{(1-W^d)} = \pi_g$ and $\frac{\mu_b}{(1-W^d)} = \pi_b + \frac{q^s - q^r}{q^r}$, so that $\frac{\mathbb{E}[\mu_\omega]}{1-W^d} = \frac{q^s}{q^r}$ which implies that $W^d = \frac{q^s}{q^s + q^r}$.

Exogenously incomplete market environment

Ex-ante risk sharing

Risk neutral investors maximize $U = \mathbb{E}_e\left[c_{1z}\right] - c(e)k$ subject to:

(i) Budget constraint at $t = 0$: $k \leq (1-w) + q^s d + q\left[\mathbb{E}\left[\theta_\omega(\hat{e})\right]s_s - \mathbb{E}\left[\theta_\omega(e^m)\right]b_s\right]$

(ii) Feasibility constraint at $t = 1$ and state $z = \langle \omega, \iota \rangle$:

$$c_{1,z} \leq A_\iota k - s_\iota + \theta_\omega(e^m) b_s^m - d$$

(iii) Non-negativity constraint for $k, c_{1z}, \{s_\iota\}$, and for $e \in [0, \overline{e}]$

(iv) Limited liability financial claims: $s_\iota \leq A_\iota k$

(v) Incentive compatibility constraint: $\hat{e} = e^* \equiv \underset{a' \in [0,\overline{e}]}{\text{argmax}} \; \mathbb{E}_{e'} [c_{1z}] - c(e')k$

Since their utility is monotonically increasing in c_{1z} for $\forall z$, the budget constraint and feasibility constraint hold with equality. Therefore, $\theta_\omega(e^m) b_s^m \geq 0$ for $\forall \omega$, which implies that short selling of financial claims is not feasible.

From the incentive compatibility constraint, the optimal effort level is

$$\mathbb{E} [\theta'_\omega \times (Ak - s_s)] - c'(e)k = 0 \tag{2.33}$$

where $\theta''_\omega = 0$ and $c''(e) > 0$ so that the second order conditions holds. In addition, $\overline{e}$ is such that the upper bound of e is never binding. The constraint $s_{\omega\iota} \leq A_\iota k$ further ensures that $e^* \geq 0$, so that the solution is interior.

Therefore, the Langrangian of the problem is given by

$$\begin{aligned}
\mathcal{L} \; =& \mathbb{E} \left[[\theta_\omega(e) A - c(e)] \, k - \theta_\omega(e) s_s + \theta_\omega(e^m) b_s^m - d - \lambda \Big[s_s - Ak \Big] \right] \\
& - \frac{1}{q^r} \left[k - (1 - w) - q^s d - q \left[\mathbb{E} \left[\theta_\omega(\hat{e}) \right] s_s - \mathbb{E} \left[\theta_\omega(e^m) \right] b_s^m \right] \right] \\
& - \mu \left[d - \theta_b(e^m) b_s \right] - \eta \left[\mathbb{E} \left[\theta'_\omega \times (Ak - s_s) \right] - c'(\hat{e}) k \right]
\end{aligned}$$

The Kuhn-tucker conditions for $\omega \in \{g, b\}$ are given by

$$\begin{aligned}
k: &\quad \mathbb{E}\left[\theta_\omega(e)\right]A - c(e) + \lambda A - \frac{1}{q^r} - \eta\left(\mathbb{E}\left[\theta'_\omega\right]A - c'(e)\right) \leq 0 \\
d: &\quad -1 + \frac{q^s}{q^r} - \mu \leq 0 \\
b_s^m: &\quad \left[1 - \frac{q}{q^r}\right]\mathbb{E}\left[\theta_\omega(\hat{e})\right] + \mu\theta_b(\hat{e}^m) \leq 0 \\
s_s: &\quad -\mathbb{E}\left[\theta_\omega(e)\right] + \frac{q}{q^r}\mathbb{E}\left[\theta_\omega(\hat{e})\right] - \lambda + \eta\mathbb{E}\left[\theta'_\omega\right] = 0 \\
\hat{e}: &\quad \frac{1}{q^r}q\mathbb{E}\left[\theta'_\omega\right]s_s + \eta c''(e)k = 0
\end{aligned}$$

$$\lambda_\omega \geq 0 \text{ and } \lambda_\omega\left[s_{\omega s} - Ak\right] = 0$$

$$\frac{1}{q^r} \geq 0 \text{ and } \frac{1}{q^r}\left[k - (1-w) - q^s d - q\left[\mathbb{E}\left[\theta_\omega(\hat{e})\right]s_s - \mathbb{E}\left[\theta_\omega(e^m)\right]b_s^m\right]\right] = 0$$

$$\mu \geq 0 \text{ and } \mu\left[d - \theta_b(e^m)s_s\right] = 0$$

$$\eta \geq 0 \text{ and } \eta\left[\mathbb{E}\left[\theta'_\omega \times (Ak - s_s)\right] - c'(e)k\right] = 0$$

Then, based on the Lagragian multiplier $\frac{1}{q^r}$, the optimal level of debt is given by

$$d \in \begin{cases} 0 & \text{if } q^s < q^r \text{ then } \mu_\omega = 0 \forall\omega \\ \left[0, \theta_b\left(\hat{e}^m\right)b_s^m\right] & \text{if } q^s = q^r \text{ then } \mu_\omega = 0\forall\omega \\ \theta_b\left(\hat{e}^m\right)b_s^m & \text{if } q^s > q^r \text{ then } \mathbb{E}\left[\mu_\omega\right] > 0 \end{cases}$$

Therefore, a necessary condition for a positive supply of debt is $q^s \geq q^r$. This is the supply of debt will be positive if the price spread is non-negative. Depending on these prices there are different cases:

Case I: $q^r \geq q^s \implies \mu = 0$ where $q^r = max\left\{q, \frac{1}{\mathbb{E}[\theta_\omega(e)]A - c(e)}\right\}$ where e is implicitly determined in $\mathbb{E}\left[\theta'_\omega(e)\right] - c'(e) = 0$

Then, $s_s = 0$ as the market does not compensate for the indirect cost of selling financial claims on incentives provision. Then, $d \in [0, \theta_b(a^m)b_s]$ for

$q^r = q^s$, and $d = 0$ otherwise. Regarding b_s^m

$$b_s^m \in \begin{cases} 0 & \text{if } q > \dfrac{1}{\mathbb{E}\left[\theta_\omega(e^{max})\right] A - c(e^{max})} \\ \left[0, \dfrac{(1-w)+q^s d}{q\mathbb{E}\left[\theta_\omega(e^m)\right]}\right] & \text{if } q = \dfrac{1}{\mathbb{E}\left[\theta_\omega(e^{max})\right] A - c(e^{max})} \\ \dfrac{(1-w)+q^s d}{q\mathbb{E}\left[\theta_\omega(e^m)\right]} & \text{if } q < \dfrac{1}{\mathbb{E}\left[\theta_\omega(e^{max})\right] A - c(e^{max})} \end{cases}$$

where k is determined in the budget constraint, so that in the latter case $k = 0$, and $e = e^{max}$ except in the latter case in which $e = 0$.

Case II: $q^r < q^s \implies \mu > 0 \implies d = \theta_b\left(e^m\right) b_s^m$ where b_s^m

$$b_s^m \in \begin{cases} 0 & \text{if } q > \kappa(e^m)q^s + (1-\kappa(e^m))q^r \\ \left[0, \dfrac{(1-w)+q^s d}{q\mathbb{E}\left[\theta_\omega(e^m)\right]}\right] & \text{if } q = \kappa(e^m)q^s + (1-\kappa(e^m))q^r \\ \dfrac{(1-w)+q^s d}{q\mathbb{E}\left[\theta_\omega(e^m)\right]} & \text{if } q < \kappa(e^m)q^s + (1-\kappa(e^m))q^r \end{cases}$$

Their investment level, $k = 1 - w + \sum_s q_\omega \theta_\omega(\hat{e}) s_{\omega s}$ and also $e = \hat{e}$, which is implicitly determined in equation (2.33), depend on s_s. Notice that a necessary condition for $s_s > 0$ is that $\frac{1}{q^r} > \frac{1}{q}$. This is the case because

$$\eta = -\frac{\frac{1}{q^r} q\mathbb{E}\left[\theta'_\omega\right] s_s}{c''(e)k} < 0$$

due to the moral hazard problem. Then, in case of an interior solution ($\lambda = 0$), s_s is implicitly determined in

$$\frac{1}{q^r}\left[q\mathbb{E}\left[\theta_\omega(\hat{e})\right] + \left(-\frac{\mathbb{E}\left[\theta'_\omega\right]}{c''(e)k}\right) q\mathbb{E}\left[\theta'_\omega\right] s_s\right] - \mathbb{E}\left[\theta_\omega(e)\right] = 0 \tag{2.34}$$

Otherwise, if the constraint is binding ($\lambda > 0$), then $s_s = Ak$.

If this option prevails, the expected return risk neutral investors earned is

$$\frac{1}{q^r} = \frac{\mathbb{E}\left[\theta_\omega\left(e\right) Ak - c(e)k - \theta_\omega(e) s_s\right]}{k - q\mathbb{E}\left[\theta_\omega(e)\right] s_s} \tag{2.35}$$

where $k, \{s_\omega\}$ and e are determined through equations (2.28) - (2.29).

Equilibrium characterization

Depending on the parameter values:

(a) If $w \leq \tilde{w}$ [14] the safe collateral constraint is not binding ($\lambda = 0$).

The equilibrium price functions are

$$q = q^r(1 - \kappa(e)) + q^s \kappa(e)$$

where $\kappa(e) \equiv \frac{\theta_b(e)}{\mathbb{E}[\theta_\omega(e)]} < 1$. The amount of safe assets in the economy, $d = \theta_b(\hat{e}) s_s$, is implicitly determined as a function of q^s in the following system of equations:

$$\begin{aligned} &\mathbb{E}\left[\theta'_\omega\right] A - \mathbb{E}\left[\frac{\theta'_\omega}{\theta_b(e)}\right] d - c'(e) = 0 \\ &\left(\frac{q^s - q^r}{q^r}\right) - \left(-\frac{\partial e}{\partial d}\right) \times \mathbb{E}\left[\frac{\theta'_\omega}{\theta_b(e)}\right] d = 0 \\ &q^r = \frac{1 - q^s d}{\mathbb{E}\left[\theta_\omega(e)\right] A - c(e) - d} \end{aligned}$$

where $\frac{\partial e}{\partial d} = \frac{\mathbb{E}\left[\frac{\theta'_\omega}{\theta_b(e)}\right]}{c''(e) - \mathbb{E}\left[\frac{\theta'^2_\omega}{\kappa(e)\mathbb{E}[\theta^2_\omega(e)]}\right] d} \neq \frac{de}{dd}$ when $\kappa'(e) \neq 0$.

Supply curve. Upward sloping but changes the inputs.

$$\frac{dd^s(q^s)}{dq^s} = -\frac{H_{q^s} + H_{q^r}\frac{\partial q^r}{\partial q^s}}{H_d + H_e\frac{\partial e}{\partial d} + H_{q^r}\left[\frac{\partial q^r}{\partial d} + \frac{\partial q^r}{\partial e}\frac{\partial e}{\partial d}\right]} > 0$$

$H_d < 0$ and $H_{q_s} > 0$ and $H_{q_r} < 0$

$$\frac{de}{dd} = \frac{-\mathbb{E}\left[\frac{\theta'_\omega}{\theta_b(e)}\right]}{c''(e) - \mathbb{E}\left[\frac{\theta'_\omega \theta'_b}{\theta^2_b(e)}\right] d} < 0 \text{ for a wide rage of parameters}$$

[14] where $\tilde{w} = \frac{1}{1+\tilde{\Psi}} < 1$ where $\tilde{\Psi} \equiv \left(\frac{1}{1+\tilde{\xi}}\right) \times \left(\frac{\pi_g[\theta_g(0) - \theta_b(0)]A}{\theta_b(0)A}\right) > 0$ and $\tilde{\xi} \equiv \frac{\frac{\theta_b(0)A}{c''(0)}}{\mathbb{E}^{-2}\left[\frac{\theta'_\omega}{\theta_b(0)}\right] - \frac{\theta^2_b(0)A}{c''(0)\mathbb{E}[\theta_\omega(0)]}} > 0$.

$$\frac{\partial q^r}{\partial d} = \frac{-\left(q^s - q^r\right)}{\mathbb{E}\left[\theta_\omega(e)\right]A - c(e) - d} < 0$$

$$\frac{\partial q^r}{\partial q^s} = \frac{-d}{\mathbb{E}\left[\theta_\omega(e)\right]A - c(e) - d} < 0$$

$$\frac{\partial q^r}{\partial e} = \frac{-q^r\left(\mathbb{E}\left[\theta'_\omega(e)\right]A - c'(e)\right)}{\mathbb{E}\left[\theta_\omega(e)\right]A - c(e) - d} < 0$$

$$\frac{\partial q^r}{\partial d} + \frac{\partial q^r}{\partial e}\frac{\partial e}{\partial d} \overset{\text{from FOC}}{=} 0$$

Demand curve. Households devote all their resources to buy safe debt. Hence, $d^h = \frac{w}{q^s}$ which is downward sloping:

$$\frac{\partial d^s(q^s)}{\partial q^s} = -\frac{w}{q^{s2}} < 0$$

The equilibrium prices of safe and risky assets, which further determine the consumption allocation of infinitely risk averse and risk neutral investors are given by

$$q^s = \frac{w}{d} \quad \text{and } q^r = \frac{1-w}{\mathbb{E}\left[\theta_\omega\left(e\right)\right]A - c(e) - d}$$

where d and e are implicitly determined in the aforementioned system of equations.

(b) If $w > \overline{w}$ the safe collateral constraint is binding ($\lambda_r > 0$). The amount of safe assets in the economy, and the effort level is given by

$$s_s = A$$

$$e = 0$$

$$q^r = \frac{\pi_g\left[\theta_g\left(0\right) - \theta_b\left(0\right)\right]A}{1 - q^s\theta_b\left(0\right)A}$$

Supply curve. The supply curve is constant at $\theta_b(0)s_b$. Any increase in q^s is fully offset by an increase in q^r. **Demand curve**. Is the same as in the previews case. **The equilibrium prices** of safe and risky assets, which further determine the consumption allocation of infinitely risk averse and risk neutral investors are given by

$$q^s = \frac{w}{d} \quad \text{and } q^r = \frac{1-w}{\pi_g\left[\theta_g\left(0\right) - \theta_b\left(0\right)\right]A}.$$

Social Planner The social planner decides on the consumption of infinitely risk averse investors (c_z^d) and risk neutral investors (c_z^b) to maximize the expected utility of the two group of investors (with respective Pareto-efficient weights W^h and W^b):

$$U^{sp} = W^h \left[\underset{\omega}{min} \left\{ c_{2z}^{ra} \right\} \right] + \left(1 - W^h\right) \left[\mathbb{E}_e \left[c_{2z}^{rn} \right] - c(e) k_0^{rn} \right]$$

The planner commits to seize all investment from agents and, in turn, make payments (possibly) contingent on their output realization. The maximization is subject to the following constraints:

(i) The resource constraint at $t = 0$ and $t = 1$

$$k \leq 1$$

(ii) Feasibility constraint

$$c_{1\omega}^d + \theta_\omega(e) c_{1\omega s}^b = \theta_\omega(e) A k$$

(ii) The participation constraint for each of the groups:

$$\underset{\omega}{min} \left\{ c_{2\omega}^{ra} \right\} \geq 0$$

$$\mathbb{E}_e \left[c_{2z}^{rn} \right] - c(e) \geq \left[\mathbb{E}_{e^{max}} \left[A_\iota \right] - c(e^{max}) \right] (1 - w)$$

where e^{max} is implicitly determined in $\mathbb{E} \left[\theta'_\omega \right] A - c'(e^{max}) = 0$

(i) Incentive compatibility constraint:

$$e = \underset{e' \in [0, \bar{e}]}{argmax} \mathbb{E}_{e'} \left[c_{2z} \right] - c(e') k$$

(iv) Non-negativity constraint for c_{tz}^i for each group of agent i, date t and state z.

(v) the transfer function can only be contingent on the idiosyncratic state: t_ι

where $z^h + z^b = \mathbb{E} \left[\theta_\omega(e) \right] t_s$ and given the preferences of both groups, the efficient distribution $z^h = \theta_b(e) t_s$ and $z^b = \pi_g \left(\theta_g(e) - \theta_b(e) \right) t_s$.

Then, the Langrangian of risk neutral investors' problem at $t = 0$ is given by

$$\begin{aligned}\mathcal{L} \quad =& W^b \left[\mathbb{E}_e \left[\theta_\omega(e)\right](A - t_s) - c(e) + \pi_g \left(\theta_g(e) - \theta_b(e)\right) t_s\right] + W^h U^h \\ & -\mu \left[U^h - \theta_b(e) t_s\right] - \lambda \left[t_s - A\right] - \eta \left[\mathbb{E}\left[\theta'_\omega\right](A - t_s) - c'(e)\right]\end{aligned}$$

The Kunh-tucker conditions are given by

$$\begin{aligned}
U^h: & \quad W^h - \mu \leq 0 \\
t_s: & \quad -W^b \mathbb{E}\left[\theta_\omega(e)\right] + W^b \pi_g \left(\theta_g(e) - \theta_b(e)\right) + \mu \theta_b(e) + \eta \mathbb{E}\left[\theta'_\omega\right] - \lambda \leq 0 \\
e: & \quad W^b \pi_g \left(\theta'_g - \theta'_b\right) t_s + \mu \theta'_b t_s + \eta c''(e) \leq 0 \\
& \quad \mu \geq 0 \text{ and } \mu \left[U^{ra} - \theta_b(e) t_s\right] = 0 \\
& \quad \lambda \geq 0 \text{ and } \lambda \left[t_s - A\right] = 0
\end{aligned}$$

$$\left(\frac{W^h - W^b}{W^b}\right)\theta_b(e) - \frac{\mathbb{E}\left[\theta'_\omega\right]}{c''(e)}\left[\mathbb{E}\left[\theta'_\omega\right] + \left(\frac{W^h - W^b}{W^b}\right)\theta'_b\right] t_s = 0$$

In this case the competitive equilibrium is not efficient. To see this point, substitute $W^h = \frac{q^s}{q^r + q^s}$, in addition to the equilibrium pricing function:

$$\left(\frac{q}{q^r} - 1\right)\mathbb{E}\left[\theta_\omega(e)\right] - \frac{\mathbb{E}\left[\theta'_\omega\right]}{c''(e)}\left[\frac{q}{q^r}\mathbb{E}\left[\theta'_\omega\right] + \left(\frac{q^s}{q^r} - 1\right)\kappa'(e)\mathbb{E}\left[\theta_\omega(e)\right]\right] t_s = 0$$

where $\kappa'(e) = \frac{\pi_g \theta_g}{\mathbb{E}[\theta'_\omega(e)]}\left(\frac{\theta'_b}{\theta_b(e)} - \frac{\theta'_g}{\theta_g(e)}\right)$. The term in blue highlights the additional term that it is not present in the competitive equilibrium outcome. The direction of the inefficiency depends on the sign of $\kappa'(e)$, which at the same time depends on:

$$\kappa'(e) \begin{cases} > 0 & \text{if } \dfrac{\theta'_b}{\theta_b(e)} > \dfrac{\theta'_g}{\theta_g(e)} \\ = 0 & \text{if } \dfrac{\theta'_b}{\theta_b(e)} = \dfrac{\theta'_g}{\theta_g(e)} \\ < 0 & \text{if } \dfrac{\theta'_b}{\theta_b(e)} < \dfrac{\theta'_g}{\theta_g(e)} \end{cases}$$

The sign of $\kappa'(e)$ will determine if the debt issuance is inefficiently high or low. In particular, if $\kappa'(e) > 0$ ($\kappa'(e) < 0$) the issuance of debt is inefficiently high (low), while effort level inefficiently low (high), and only when $\kappa'(e) = 0$, the competitive equilibrium is efficient.

Ex-post risk sharing

Late investors maximize $U^{li} = \mathbb{E}_\omega \left[c^{li}_{2z}\right]$ subject to :

(i) Budget constraint at $t = 1$ and state ω: $k^{li}_{1\omega} + q_{1\omega}\theta_\omega(\hat{e})b_{1\omega s} \leq w^{li}_1$

(ii) Feasibility constraint at $t = 1$ and state ω: $\mathbb{E}_\omega \left[c^{li}_{2z}\right] \leq F(k^{li}_{1\omega}) + \theta_\omega(\hat{e})b_{1\omega s}$

(iii) Non-negativity constraint for k^{li}_ω, $b_{1\omega s}$ and c^{li}_{2z}

The Langrangian of the late investors' problem is given by

$$\mathcal{L}^{li} \quad = F(k^{li}_{1\omega}) + \theta_\omega(\hat{e})b_{1\omega s} - \eta\left[k^{li}_{1\omega} + q_{1\omega}\theta_\omega(\hat{e})b_{1\omega s} - w^{li}_1\right]$$

The Kuhn-tucker conditions for $\omega \in \{g, b\}$ are given by

$$k^{li}_\omega : \qquad F'(k^{li}_{1\omega}) - \eta \leq 0$$

$$b_{1\omega\iota} : \qquad \theta_\omega(\hat{e}) - \eta q_{1\omega}\theta_\omega(\hat{e}) \leq 0$$

$$\eta \geq 0 \text{ and } \eta\left[k^{li}_{1\omega} + q_{1\omega}\theta_\omega(\hat{e})b_{1\omega s} - w^{li}_{1\omega}\right] = 0$$

Depending on the parameter values there are three cases:

Case I: If $F'(0) < \frac{1}{q_{1\omega}}$ then $k^{1l}_\omega = 0$ and $b_{1\omega} = w^{li}_1$

Case II: If $F'(w^{li}_{1\omega}) \geq \frac{1}{q_{1\omega}}$ then $k^{1l}_\omega = w^{li}_{1\omega}$ and $b_{1\omega s} = 0$

Case III: Otherwise, $F'\left(k^{li}_\omega\right) = q_{1\omega}$ and $b_{1\omega s} = \frac{w^{li}_{1\omega} - k^{li}_\omega}{q_{1\omega}\theta_\omega(\hat{e})}$

Since, $F'(w^{li}_{1\omega}) > 1$, a necessary condition for $b_{1\omega}$ to be positive is $q_{1\omega} < 1$. This participation constraint, implies that risk-neutral agents need to sell their asset at a discount from their fundamental value.

Risk neutral investors maximize $U = \mathbb{E}_e\left[c_{2z}\right] - c(e)k$ subject to :

(i) Budget constraint at $t = 0$: $k \leq (1 - w) + q^s d$

(ii) Feasibility constraint at $t = 2$ and state $z = \langle \omega, \iota \rangle$:

$$c_{2z} \leq A_\iota k - s_{1\omega\iota} + q_{1\omega}\theta_\omega(\hat{e})s_{1\omega s} - d$$

(iii) Non-negativity constraint for k and c_{1z}, and $e \in [0, \overline{e}]$

(iv) Limited liability state contingent claims: $0 \leq s_{1\omega\iota} \leq A_\iota k$

(v) Incentive compatibility constraint:$\hat{e} = e^* \equiv \underset{e' \in [0,\overline{e}]}{\text{argmax}}\ \mathbb{E}_{e'}\left[c_{1z}\right] - c(e')k$

Following backward induction, let's start with the decision of asset sales that risk neutral investors face at $t = 1$. If their debt level is positive, then, there are two cases:

- Default in their debt, but then their assets are seized and they earn 0.
- Do not default in their debt, selling $s_{1\omega} \geq \frac{d}{q_{1\omega}\theta_\omega(\hat{e})}$, and earning $\theta_\omega(e)\left(Ak - s_{1\omega\iota}\right) \geq 0$.

Risk neutral investors are better off not defaulting. Hence, ex-post deviations are not optimal, so that the promise of asset sales at $t = 1$ serve as safe collateral to issue safe debt, this is, $\hat{s}_{1\omega} = s_{1\omega}$ at $t = 0$.

Then, from the incentive compatibility constraint, the optimal effort level is determined at $t = 0$ and is implicitly determined in

$$\mathbb{E}\left[\theta'_\omega(e)\left(Ak - s_{1\omega s}\right)\right] - c'(e)k = 0 \tag{2.36}$$

The market pins down their beliefs about effort based on the real incentives to exert effort, and price the financial claims sold by risk neutral investors at any time accordingly.

Then, the Langrangian of risk neutral investors' problem at $t = 0$ is given by

$$\begin{aligned}\mathcal{L} \; =& \mathbb{E}\left[\theta_\omega(e)\left(Ak - s_{1\omega s}\right)\right] + \mathbb{E}\left[q_{1\omega}\theta_\omega(\hat{e})s_{1\omega s}\right] - d - c(e)k \\ & - \frac{1}{q^r}\left[k - (1-w) - q^s d\right] - \eta\left[\mathbb{E}\left[\theta'_\omega(e)\left(Ak - s_{1\omega s}\right)\right] - c'(e)k\right] \\ & - \mathbb{E}\left[\mu_\omega\left[d - q_{1\omega}\theta_\omega\left(\hat{e}\right)s_{1\omega s}\right] - \lambda_\omega\left[s_{1\omega s} - Ak\right]\right]\end{aligned}$$

The Kunh-tucker conditions are given by

$$\begin{aligned}
k: \quad & \mathbb{E}\left[\theta_\omega(e)\right]A - c(e) - \frac{1}{q^r} + \mathbb{E}\left[\lambda_\omega\right]A - \eta\mathbb{E}\left[\theta'_\omega(e)A - c'(e)\right] \leq 0 \\
d: \quad & -1 + \frac{q^s}{q^r} - \mathbb{E}\left[\mu_\omega\right] \leq 0 \\
s_{1\omega s}: \quad & -\pi_\omega\theta_\omega(e) + \pi_\omega(1+\mu_\omega)q_{1\omega}\theta_\omega(\hat{e}) - \pi_\omega\lambda_\omega + \eta\pi_\omega\theta'_\omega \leq 0 \\
\hat{e}: \quad & \mathbb{E}\left[\left(1+\mu_\omega\right)q_{1\omega}\theta'_\omega s_{1\omega s}\right] + \eta c''(e)k = 0 \\
& \frac{1}{q^r} \geq 0 \text{ and } \frac{1}{q^r}\left[k + q\mathbb{E}\left[\theta_\omega\left(e^m\right)\right]b - (1-w) - q^s d - q\mathbb{E}\left[\theta_\omega(\hat{e})\right]s\right] = 0 \\
& \lambda_\omega \geq 0 \text{ and } \lambda_\omega\left[s_s + s_{1\omega s} - Ak\right] = 0 \\
& \mu_\omega \geq 0 \text{ and } \mu_\omega\left[d - \theta_\omega\left(e^m\right)b_s - q_{1\omega}\theta_\omega\left(\hat{e}\right)s_{1\omega s}\right] = 0 \\
& \eta \geq 0 \text{ and } \eta\left[\mathbb{E}\left[\theta'_\omega(e)\left(Ak - s_{1\omega s}\right)\right] - c'(e)k\right]
\end{aligned}$$

Then, the optimal level of debt is given by

$$d \in \begin{cases} 0 & \text{if } q^s < q^r \text{ then } \mu_\omega = 0 \forall\omega \\ \left[0, \underset{\omega}{min}\left\{q_{1\omega}\theta_\omega\left(\hat{e}\right)s_{1\omega}\right\}\right] & \text{if } q^s = q^r \text{ then } \mu_\omega = 0\forall\omega \\ \underset{\omega}{min}\left\{q_{1\omega}\theta_\omega\left(\hat{e}\right)s_{1\omega}\right\} & \text{if } q^s > q^r \text{ then } \mathbb{E}\left[\mu_\omega\right] > 0 \end{cases}$$

Therefore, a necessary condition for a positive supply of debt is $q^s \geq q^r$. This is, the supply of debt will be positive if the price margin is non-negative. Depending on both prices there are different cases:

Case I: $q^r \geq q^s \implies \mu_\omega = 0\forall\omega$ where $q^r = \frac{1}{\mathbb{E}[\theta_\omega(e)]A - c(e)}$ and e is implicitly determined in $\mathbb{E}\left[\theta'_\omega(e)\right] - c'(e) = 0$.

Then, $s_\omega = 0$ as the market does not compensate for the indirect cost of selling financial claims on incentives provision nor the cost of fire sales.

Case II: $q^r < q^s \implies \mathbb{E}\left[\mu_\omega\right] > 0 \implies d = \underset{\omega}{min}\left\{q_{1\omega}\theta_\omega\left(\hat{e}\right)s_{1\omega}\right\}$

Due to fire-sale cost ($q_{1\omega} \leq 1$), the amount of financial claims sold is minimal. The optimal debt level is given by $d = q_{1\omega}\theta_\omega(\hat{e})s_{\omega s}$ for $\forall\omega$ ($\mu_\omega > 0$). In addition, this form of late asset sales also create and incentive problem, so that

$$\eta = -\frac{\mathbb{E}\left[(1+\mu_\omega)\, q_{1\omega}\theta'_\omega s_{1\omega s}\right]}{c''(e)k} < 0$$

due to the moral hazard problem. Then, if it is an interior solution ($\lambda_\omega = 0$), $s_{1\omega}$ is implicitly determined in

$$\pi_\omega\left(\mu_\omega - \frac{1-q_{1\omega}}{q_{1\omega}}\right) - \left(\frac{\mathbb{E}\left[(1+\mu_\omega)\, q_{1\omega}\theta'_\omega s_{1\omega s}\right]}{c''(e)k}\right)\frac{\pi_\omega\theta'_\omega}{q_{1\omega}\theta_\omega(e)} = 0 \tag{2.37}$$

Otherwise, if the constraint is binding ($\lambda_\omega > 0$), then $s_{1\omega s} = Ak$.

The expected return risk neutral investors earned is

$$q^r = \frac{1-q^s d}{\mathbb{E}\left[\theta_\omega(e)\right]Ak - c(e)k - \mathbb{E}\left[\frac{1}{q_{1\omega}}\right]d}$$

where k, $\{s_{1z}\}$ and e are determined through equations (2.36) - (2.37).

Equilibrium

Depending on the parameter values:

(a) If $w \leq \tilde{w}$ [15] the safe collateral constraint is not binding ($\lambda_\omega = 0$).

The amount of safe assets in the economy, $d = q_{1\omega}\theta_\omega(\hat{e})s_{1\omega s}$ for $\forall\omega$, is implicitly

[15] where $\overline{w} = \frac{1}{1+\Psi(\underline{e})} < 1$ where $\underline{e}$ is determined in $\mathbb{E}\left[\theta'_\omega - \frac{\theta_\omega}{q_{1\omega}\theta_\omega(e)}\theta_b(e)\right]A - c'(e) = 0$ and

$$\Psi(\underline{e}) \equiv \left(\frac{1}{1+\xi(\underline{e})}\right)\times\left(\frac{\left(\mathbb{E}\left[\theta_\omega(e)\right] - \frac{1}{F'(w^{li}-\theta_b(\underline{e})A)}\theta_b(\underline{e})\right)A - c(\underline{e})}{\theta_b(\underline{e})A}\right) > 0 \text{ where } \xi(\underline{e}) \equiv \frac{\frac{\mathbb{E}^2\left[\frac{\theta'_\omega}{q_\omega\theta_\omega(e)}\right]}{c''(e)}\theta_b(\underline{e})A}{1 - \frac{\mathbb{E}\left[\frac{\theta'_\omega}{q_{1\omega}\theta_\omega(e)}\right]}{c''(e)}x\theta_b(\underline{e})A} > 0.$$

determined as a function of q^s in the following system of equations:

$$\mathbb{E}\left[\theta'_\omega\right] A - \mathbb{E}\left[\frac{\theta'_\omega}{q_{1\omega}\theta_\omega(e)}\right] d - c'(e) = 0$$

$$\left(\frac{q^s}{q^r} - \mathbb{E}\left[\frac{1}{q_{1\omega}}\right]\right) - \left(\frac{\mathbb{E}\left[\frac{\theta'_\omega(e)}{q_\omega\theta_\omega(e)}\right] d}{c''(e) - \mathbb{E}\left[\frac{\theta'^2_\omega(e)}{q_\omega\theta^2_\omega(e)}\right]}\right) \times \mathbb{E}\left[\frac{\theta'_\omega(e)}{q_\omega\theta_\omega(e)}\right] d = 0$$

$$q^r = \frac{1 - q^s d}{\mathbb{E}\left[\theta_\omega(e)\right] A - c(e) - \mathbb{E}\left[\frac{1}{q_{1\omega}}\right] d}$$

$$q_{1\omega} = \frac{1}{F'(w^{li}_\omega - d)}$$

Supply curve. Upward sloping but changes the inputs.

$$\frac{dd^s(q^s)}{dq^s} = -\frac{H_{q^s} + H_{q^r}\frac{\partial q^r}{\partial q^s}}{H_d + H_e\frac{\partial e}{\partial d} + \sum_\omega H_{q_{1\omega}}\frac{\partial q_{1\omega}}{\partial d} + H_{q^r}\left[\frac{\partial q^r}{\partial d} + \frac{\partial q^r}{\partial e}\frac{\partial e}{\partial d} + \sum_\omega \frac{\partial q^r}{\partial q_{1\omega}}\frac{\partial q_{1\omega}}{\partial d}\right]} > 0$$

$H_d < 0$ and $H_e > 0$

$H_{q_s} > 0$ and $H_{q_r} < 0$ and $H_{q_{1\omega}} > 0$

$$\frac{\partial e}{\partial d} = \frac{-\mathbb{E}\left[\frac{\theta'_\omega}{q_{1\omega}\theta_\omega(e)}\right]}{c''(e) - \mathbb{E}\left[\frac{\theta'^2_\omega}{q_{1\omega}\theta^2_\omega(e)}\right] d} \overset{\text{from FOC}}{<} 0$$

$$\frac{\partial q^r}{\partial d} = \frac{-\left(q^s - \mathbb{E}\left[\frac{q^r}{q_{1\omega}}\right]\right)}{\mathbb{E}\left[\theta_\omega(e)\right] A - c(e) - \mathbb{E}\left[\frac{1}{q_{1\omega}}\right] d} < 0$$

$$\frac{\partial q^r}{\partial q^s} = \frac{-d}{\mathbb{E}\left[\theta_\omega(e)\right] A - c(e) - \mathbb{E}\left[\frac{1}{q_{1\omega}}\right] d} < 0$$

$$\frac{\partial q^r}{\partial e} = \frac{-q^r\left(\mathbb{E}\left[\theta'_\omega(e)\right] A - c'(e)\right)}{\mathbb{E}\left[\theta_\omega(e)\right] A - c(e) - \mathbb{E}\left[\frac{1}{q_{1\omega}}\right] d} < 0$$

$$\frac{\partial q^r}{\partial q_{1\omega}} = \frac{-q^r\left(\frac{1}{q^2_{1\omega}}\right) d}{\mathbb{E}\left[\theta_\omega(e)\right] A - c(e) - \mathbb{E}\left[\frac{1}{q_{1\omega}}\right] d} < 0$$

$$\frac{\partial q_{1\omega}}{\partial d} = \frac{F''(w^{li}_\omega - d)}{F'^2(w^{li}_\omega - d)} < 0$$

$$\frac{\partial q^r}{\partial d} + \frac{\partial q^r}{\partial e}\frac{\partial e}{\partial d} \overset{\text{from FOC}}{=} 0$$

$$\frac{\partial q^r}{\partial d} + \frac{\partial q^r}{\partial e}\frac{\partial e}{\partial d} + \sum_\omega \frac{\partial q^r}{\partial q_{1\omega}}\frac{\partial q_{1\omega}}{\partial d} > 0$$

Demand curve. Households devote all their resources to buy safe debt.

Hence, $d^h = \frac{w}{q^s}$ which is downward sloping:

$$\frac{\partial d^s(q^s)}{\partial q^s} = -\frac{w}{q^{s2}} < 0$$

The equilibrium prices of safe and risky assets, which further determine the consumption allocation of infinitely risk averse and risk neutral investors are given by

$$q^s = \frac{w}{d} \quad \text{and } q^r = \frac{1-w}{\mathbb{E}\left[\theta_\omega(e)\right]Ak - c(e)k - \mathbb{E}\left[\frac{1}{q_{1\omega}}\right]d}$$

where d and e are implicitly determined in the aforementioned system of equations.

(b) If $w > \overline{w}$ the safe collateral constraint is binding ($\lambda_r > 0$). The amount of safe assets in the economy, and the effort level is given by

$$s_{bs} = A$$
$$s_{gs} = \frac{q_{1g}\theta_b(e)}{q_{1b}\theta_g(e)}A$$
$$\mathbb{E}\left[\theta'_\omega - \frac{\theta_\omega}{q_{1\omega}\theta_\omega(e)}\theta_b(e)\right]A - c'(e) = 0$$

Supply curve. The supply curve is a vertical line. Any increase in q^s is fully offset by an increase in q^r. **Demand curve**. Is the same as in the previews case. **The equilibrium prices** of safe and risky assets, which further determine the consumption allocation of infinitely risk averse and risk neutral investors are given by

$$q^s = \frac{w}{d} \quad \text{and } q^r = \frac{1-w}{\mathbb{E}\left[\theta_\omega(e)\right]Ak - c(e)k - \mathbb{E}\left[\frac{1}{q_{1\omega}}\right]d}$$

where e is implicitly determined in the aforementioned equation.

Social Planner

$$U^{sp} = W^d \left[\underset{\omega}{min} \left\{ c^d_{2z} \right\} \right] + W^b \left[\mathbb{E}_e \left[c^b_{2z} \right] - c(e) k^b_0 \right]$$

subject to the following constraints:

(i) The resource constraint at $t = 0$ and $t = 1$

$$k_0 \leq 1$$
$$k^{li}_{1\omega} + k^s_{1\omega} \leq w^{li}$$

(ii) Feasibility constraint

$$c^d_{2\omega} \leq k^s_{1\omega}$$
$$c^d_{2\omega} + \theta_\omega(e) c^b_{2\omega s}(1 - \theta_\omega(e)) c^b_{2\omega f} + c^{li}_{2\omega} \leq \theta_\omega(e) A k_0 + F(k^{li}_1) + k^s_{1\omega}$$

(ii) The participation constraint for each of the groups:

$$\underset{\omega}{min} \left\{ c^d_{2\omega} \right\} \geq 0$$
$$\mathbb{E}_e \left[c^b_{2z} \right] - c(e) \geq \left(\mathbb{E}_{e^{max}} \left[A_\iota \right] - c(e^{max}) \right) (1 - w)$$
$$c^{li}_{2\omega} \geq F(w^{li})$$

where e^{max} is implicitly determined in $\mathbb{E} \left[\theta'_\omega \right] A - c'(e^{max}) = 0$.

(iii) Incentive compatibility constraint:

$$e^* \equiv \underset{e' \in [0, \overline{e}]}{argmax} \mathbb{E}_{e'} \left[c_{2z} \right] - c(e') k$$
$$k^{s*}_{1\omega} \equiv \underset{k^{s'}_{1\omega} \in \left[0, w^{li}\right]}{argmax} F(w^{li} - k^{s'}_{1\omega}) + \frac{1}{q_{1\omega}} k^{s'}_{1\omega}$$

where $\frac{1}{q_{1\omega}}$ is the return obtained by investing those resources in an alternative investment opportunity. This is, of the safe capital in the intermediate date, such that $k^s_\omega \leq q_{1\omega} \theta_\omega(e) t_{\omega s}$, and $t_{1\omega s}$ is a transfer to late investors.

(iv) Non-negativity constraint for c^i_{tz} for each group of agent i, date t and state z.

From the incentive compatibility constraint, $\mathbb{E} \left[\theta'_\omega \times (c_{\omega s} - c_{\omega f}) \right] - c'(e) = 0$.

Thus, to minimize the burden of the moral hazard problem, positive transfers (if any) to risk neutral agents will be to the success, i.e. $c^b_{2\omega f} = 0$. For the second incentive compatibility constraint, we have that

$$q_{1\omega} = \frac{1}{F'(w^{li} - k^{s'}_{1\omega})}$$

where

$$\frac{\partial q_{1\omega}}{\partial t_{1\omega}} = -\frac{\frac{(-F''(.))}{F'^2(.)} q_{1\omega}\theta_\omega(e)}{1 + \frac{(-F''(.))}{F'^2(.)}\theta_\omega t_\omega} < 0$$

$$\frac{\partial q_{1\omega}}{\partial e} = -\frac{\frac{(-F''(.))}{F'^2(.)} q_{1\omega}\theta'_\omega(e) t_{1\omega}}{1 + \frac{(-F''(.))}{F'^2(.)}\theta_\omega t_\omega} < 0$$

Lets focus in the interesting case in which $\underline{c^d_2} > 0$. For, $k^s_{1\omega} < w^{li}_1$, the Langrangian of risk neutral investors' problem at $t = 1$ is given by

$$\begin{aligned}\mathcal{L}_0 \; =& W^d \underline{c^d_2} + W^b \left[\mathbb{E}_e\left[\theta_\omega(e) \times (A - t_{\omega s})\right] - c(e)\right] - \eta\left[\mathbb{E}\left[\theta'_\omega \times (A - t_{\omega s})\right] - c'(e)\right] \\ & - \mathbb{E}\left[\mu_\omega\left[\underline{c^d_2} - q_{1\omega}\theta_\omega(e)t_{\omega s}\right] + \lambda_\omega\left[t_{1\omega s} - A\right]\right]\end{aligned}$$

The Kunh-tucker conditions are given by

$$\begin{aligned}
\underline{c^d_2}: \quad & W^d - \mathbb{E}\left[\mu_\omega\right] \leq 0 \\
t_{\omega s}: \quad & -W^b\pi_\omega\theta_\omega(e) + \pi_\omega\mu_\omega\left(q_{1\omega}\theta_\omega(e) + \frac{\partial q_{1\omega}}{\partial t_{1\omega}}\theta_\omega(e)t_{1\omega}\right) + \eta\pi_\omega\theta'_\omega - \pi_\omega\lambda_\omega \leq 0 \\
e: \quad & 0 + \eta c''(e) + \mathbb{E}\left[\mu_\omega\left(q_{1\omega}\theta'_\omega + \frac{\partial q_{1\omega}}{\partial e}\theta_\omega(e)\right)\right] t_{1\omega} = 0 \\
& \lambda_\omega \geq 0 \text{ and } \lambda_\omega\left[t_{1\omega s} - A\right] = 0 \\
& \eta \geq 0 \text{ and } \eta\left[\mathbb{E}\left[\theta'_\omega \times (A - t_{\omega s})\right] - c'(e)\right] = 0 \\
& \mu_\omega \geq 0 \text{ and } \mu_\omega\left[\underline{c^d_2} - q_{1\omega}\theta_\omega(e)t_{\omega s}\right] = 0 \\
& \lambda_\omega \geq 0 \text{ and } \lambda_\omega\left[t_{1\omega s} - A\right] = 0
\end{aligned}$$

After some algebra

$$\frac{W^d}{W^b} - \mathbb{E}\left[\frac{1 + \varepsilon^d_\omega}{q_{1\omega}}\right] - \left(-\frac{de}{dd}\right) \times \mathbb{E}\left[\frac{\theta'_\omega}{q_{1\omega}\theta_\omega(e)}\right] d = 0$$

where $\varepsilon^d_\omega \equiv -\frac{\partial q_{1\omega}}{\partial d}\frac{d}{q_{1\omega}}$ and $\frac{de}{dd} = -\frac{\mathbb{E}\left[\frac{\theta'_\omega(1+\varepsilon^d_\omega)}{q_{1\omega}\theta_\omega(e)}\right]}{c''(e)-\mathbb{E}\left[\frac{\theta'^2_\omega}{q_{1\omega}\theta^2_\omega(e)}\right]}$. Clearly, if $\varepsilon^d_\omega = 0$, the competitive equilibrium is Pareto efficient for the weights $\frac{W^d}{W^b} = \frac{q^s}{q^r}$. Otherwise, the competitive equilibrium is not efficient such that $d^{sp} < d$, and $e^{sp} > e$.

Ex-ante and ex-port risk-sharing

Putting the above two options together, the Langrangian of risk neutral investors' problem at $t = 0$ is given by

$$
\begin{aligned}
\mathcal{L} =& \mathbb{E}\left[\theta_\omega(e)\left(Ak - s_s - s_{1\omega s}\right)\right] + \mathbb{E}\left[\theta_\omega(e^m)\right] b_s + \mathbb{E}\left[q_{1\omega}\theta_\omega(\hat{e})s_{1\omega s}\right] - d - c(e)k \\
& - \frac{1}{q^r}\left[k + q\mathbb{E}\left[\theta_\omega\left(e^m\right)\right] b_s - (1-w) - q^s d - q\mathbb{E}\left[\theta_\omega(\hat{e})\right] s_s\right] \\
& - \mathbb{E}\left[\lambda_\omega\left[s_s + s_{1\omega s} - Ak\right] + \mu_\omega\left[d - \theta_\omega\left(e^m\right) b_s - q_{1\omega}\theta_\omega\left(\hat{e}\right) s_{1\omega s}\right]\right] \\
& - \eta\left[\mathbb{E}\left[\theta'_\omega(e)\left(Ak - s_s - s_{1\omega s}\right)\right] - c'(e)k\right]
\end{aligned}
$$

The Kunh-tucker conditions are given by

$$
\begin{aligned}
k: &\quad \mathbb{E}\left[\theta_\omega(e)\right] A - c(e) - \frac{1}{q^r} + \mathbb{E}\left[\lambda_\omega\right] A - \eta\left(\mathbb{E}\left[\theta'_\omega(e)A\right] - c'(e)\right) \leq 0 \\
d: &\quad -1 + \frac{q^s}{q^r} - \mathbb{E}\left[\mu_\omega\right] \leq 0 \\
b_s: &\quad \mathbb{E}\left[\theta_\omega(e^m)\right] - \frac{q}{q^r}\mathbb{E}\left[\theta_\omega\left(e^m\right)\right] + \mathbb{E}\left[\mu_\omega\theta_\omega\left(e^m\right)\right] \leq 0 \\
s_s: &\quad -\mathbb{E}\left[\theta_\omega(e)\right] + \frac{q}{q^r}\mathbb{E}\left[\theta_\omega(\hat{e})\right] - \mathbb{E}\left[\lambda_\omega\right] + \eta\mathbb{E}\left[\theta'_\omega(e)\right] \leq 0 \\
s_{1\omega s}: &\quad -\pi_\omega\theta_\omega(e) + \pi_\omega(1+\mu_\omega)q_{1\omega}\theta_\omega(\hat{e}) - \pi_\omega\lambda_\omega + \eta\pi_\omega\theta'_\omega(e) \leq 0 \\
\hat{e}: &\quad \mathbb{E}\left[(1+\mu_\omega)q_{1\omega}\theta'_\omega s_{1\omega s}\right] + \frac{q}{q^r}\mathbb{E}\left[\theta'_\omega\right] s_s + \eta c''(e)k = 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{q^r} \geq 0 \text{ and } \frac{1}{q^r}\left[k + q\mathbb{E}\left[\theta_\omega\left(e^m\right)\right] b - (1-w) - q^s d - q\mathbb{E}\left[\theta_\omega(\hat{e})\right] s\right] = 0 \\
\lambda_\omega \geq 0 \text{ and } \lambda_\omega\left[s_s + s_{1\omega s} - Ak\right] = 0 \\
\mu_\omega \geq 0 \text{ and } \mu_\omega\left[d - \theta_\omega\left(e^m\right) b_s - q_{1\omega}\theta_\omega\left(\hat{e}\right) s_{1\omega s}\right] = 0 \\
\eta \geq 0 \text{ and } \eta\left[\mathbb{E}\left[\theta'_\omega(e)\left(Ak - s_s - s_{1\omega s}\right)\right] - c'(e)k\right] = 0
\end{aligned}
$$

Then, based on the Lagragian multiplier $\frac{1}{q^r}$, the optimal level of debt is

$$d \in \begin{cases} 0 & \text{if } q^s < q^r \text{ then } \mu_\omega = 0 \forall \omega \\ \left[0, \underset{\omega}{min}\left\{\theta_\omega\left(e^m\right)b + q_{1\omega}\theta_\omega\left(\hat{e}\right)s_{1\omega}\right\}\right] & \text{if } q^s = q^r \text{ then } \mu_\omega = 0 \forall \omega \\ \underset{\omega}{min}\left\{\theta_\omega\left(e^m\right)b + q_{1\omega}\theta_\omega\left(\hat{e}\right)s_{1\omega}\right\} & \text{if } q^s > q^r \text{ then } \mathbb{E}\left[\mu_\omega\right] > 0 \end{cases}$$

Therefore, a necessary condition for a positive supply of debt is $q^s \geq q^r$. This is, the supply of debt will be positive if the price margin is non-negative. Depending on both prices there are different cases:

Case I: $q^r \geq q^s \implies \mu_\omega = 0 \forall \omega$ where $q^r = max\left\{q, \frac{1}{\mathbb{E}[\theta_\omega(e^{max})]A - c(e^{max})}\right\}$ and e^{max} is implicitly determined in $\mathbb{E}\left[\theta'_\omega(e^{max})\right] - c'(e^{max}) = 0$

Then, $s = s_\omega = 0$ as the market does not compensate for the indirect cost of selling financial claims on incentives provision. Then, $d \in [0, \theta_b(a^m)b_s]$ for $q^r = q$, and $d = 0$ otherwise.

$$b_s \in \begin{cases} 0 & \text{if } q > \dfrac{1}{\mathbb{E}\left[\theta_\omega(e^{max})\right]A - c(e^{max})} \\ \left[0, \dfrac{(1-w) + q^s d}{q\mathbb{E}\left[\theta_\omega(e^m)\right]}\right] & \text{if } q = \dfrac{1}{\mathbb{E}\left[\theta_\omega(e^{max})\right]A - c(e^{max})} \\ \dfrac{(1-w) + q^s d}{q\mathbb{E}\left[\theta_\omega(e^m)\right]} & \text{if } q < \dfrac{1}{\mathbb{E}\left[\theta_\omega(e^{max})\right]A - c(e^{max})} \end{cases}$$

where k is determined in the budget constraint, so that in the latter case $k = 0$, and $e = e^{max}$ except in the latter case in which $e = 0$.

Case II: $q^r < q^s \implies \mathbb{E}\left[\mu_\omega\right] > 0 \implies d = \underset{\omega}{min}\left\{\theta_\omega\left(e^m\right)b_s^m + q_{1\omega}\theta_\omega\left(\hat{e}\right)s_{1\omega}\right\}$.

Lets understand the backing collateral if it is through ex-ante risk sharing ($b_\iota^m > 0$), or ex-post risk sharing ($s_{1\omega\iota} > 0$).

Investment level is determined in the budget constraint and $e = \hat{e}$ in the incentive compatibility constraint, so that both depend on the asset sold at $t = 0$ and at $t = 1$. Notice that due to the moral hazard problem,

$$\eta = -\frac{\mathbb{E}\left[(1+\mu_\omega)q_{1\omega}\theta'_\omega s_{1\omega s}\right] + \frac{q}{q^r}\mathbb{E}\left[\theta'_\omega\right]s_s}{c''(e)k} < 0$$

which increases the cost of selling of both at $t = 0$ and $t = 1$. Regarding the financial claims sold, at least one of μ_ω is positive, and it is easy to check that among the potential cases only the following is possible: $\mu_b > 0$ and $\mu_g \geq 0$. Let's study under which conditions each of the cases occurs:

- **Only ex-post risk sharing** ($\mu_b > 0$ and $\mu_g > 0$) so that $d = min\{q_\omega \theta_\omega(e) s_{1\omega s}\}$.
  This case will occur when the following condition holds:

$$\mathbb{E}\left[\left(\frac{1-q_{1\omega}}{q_{1\omega}}\right)\theta_\omega(e)\right] + \mathbb{E}\left[\frac{\theta'_\omega}{q_{1\omega}\mathbb{E}\left[\theta'_\omega\right]}\right] \times \left(\frac{q}{q^r}-1\right)\mathbb{E}\left[\theta_\omega(e)\right] - \left(\frac{q^s}{q^r}-1\right)\theta_b(e)$$
$$< \left(\left(\frac{q}{q^r}-1\right)\mathbb{E}\left[\theta_\omega(e^m)\right] - \left(\frac{q^s}{q^r}-1\right)\theta_b(e^m)\right) \times \left(\frac{\theta_g(e)-\theta_b(e)}{\theta_g(e^m)-\theta_b(e^m)}\right) \tag{2.38}$$

  Notice that in equilibrium $e = e^m$ which implies that the latter condition will hold when $q_\omega \geq 1$. If this is the case then, $b^m_s = 0$.

- **Ex-post and ex-ante risk sharing**. There are two cases:
  * If $\frac{\pi_b + \left(\frac{q^s}{q^r}-1\right)\frac{\pi_b \theta'_b(0)}{\mathbb{E}[\theta'_\omega(0)]}}{\pi_b + \left(\frac{q^s}{q^r}-1\right)} < q_{1b} < 1$ ($\mu_b > 0$ and $\mu_g > 0$) then $s_{1gs} = 0$ and

$$\left(\frac{q}{q^r}-1\right)\mathbb{E}\left[\theta_\omega(e)\right] - \left(\frac{q_{1b}\theta'_b s_{1bs} + \frac{q}{q^r}\mathbb{E}\left[\theta'_\omega\right] s_s}{c''(e)k}\right)\mathbb{E}\left[\theta_\omega(e)\right] = 0$$
$$s_{1bs} = \left(\frac{\theta_g(e^m)-\theta_b(e^m)}{q_{1b}\theta_b(e)}\right) b^m_s$$

  * If $q_{1b} = \frac{\pi_b + \left(\frac{q^s}{q^r}-1\right)\frac{\pi_b \theta'_b(0)}{\mathbb{E}[\theta'_\omega(0)]}}{\pi_b + \left(\frac{q^s}{q^r}-1\right)}$ ($\mu_b > 0$ and $\mu_g > 0$) then $s_{1gs} = 0$ and $s_{1bs} \in \left(0, min\left\{\left(\frac{\theta_g(e)-\theta_b(e)}{q_{1b}\theta_b(e)}\right) b_s, Ak - s_s\right\}\right)$, then equation (2.33) - (2.35) and (2.36) - (2.5) must hold.

- **Only ex-ante risk sharing** ($\mu_b > 0$ and $\mu_g = 0$) so that $d = \theta_\omega(e^m) b^m_\omega$.
  This holds if $q_{1b} < \frac{\pi_b + \left(\frac{q^s}{q^r}-1\right)\frac{\pi_b \theta'_b(0)}{\mathbb{E}[\theta'_\omega(0)]}}{\pi_b + \left(\frac{q^s}{q^r}-1\right)} < 1 \implies s_{1bs} = 0$ and s_s is determined in equations (2.33) - (2.35).

Equilibrium characterization Focusing on an interior solutions ($\lambda_\omega = 0$), depending on the parameter values:

(a) $\frac{1}{F'(w^{li}_{1b})} < \frac{\pi_b + \left(\frac{q^s}{q^r}-1\right)\frac{\pi_b \theta'_b(0)}{\mathbb{E}[\theta'_\omega(0)]}}{\pi_b + \left(\frac{q^s}{q^r}-1\right)}$ only ex-ante risk sharing prevails, and the optimality conditions is the same as the one described in ex-ante risk-sharing section.

(b) $\frac{1}{F'(w_{1b}^{li})} > \frac{\pi_b + \left(\frac{q^s}{q^r}-1\right)\frac{\pi_b \theta_b'(0)}{\mathbb{E}[\theta_\omega'(0)]}}{\pi_b + \left(\frac{q^s}{q^r}-1\right)}$ then, the

$$q = q^r + \xi(e^m)\left(q^s - q^r\left[1 - \sigma(e^m)\pi_b\left(\frac{1-q_{1|b}}{q_{1|b}}\right)\right]\right)$$

where $\xi(e^m, e) \equiv \frac{\frac{\mathbb{E}[\theta_\omega']}{\mathbb{E}[\theta_\omega(e)]}}{(1-\sigma(e^m))\frac{\mathbb{E}[\theta_\omega']}{\kappa(e^m)\mathbb{E}[\theta_\omega(e)]} + \sigma(e^m)\frac{\pi_b \theta_\omega'}{q_{1|b}\theta_b(e)}}$ and $\sigma(e) \equiv \frac{\theta_g(e)-\theta_b}{\theta_b(e)}$. The amount of safe assets in the the economy, $d = \theta_g(e)s_{gs} = \theta_b(e)s_{bs} + q_{1|b}\theta_b(e)s_{1s|b}$, is implicitly determined in the following system of equations

Lemma 21 *When markets are exogenously incomplete and effort is not observable, bankers issue safe debt only when $q^s - q^r > 0$, and its level is determined in*

$$\begin{aligned}(1-\sigma(e^m)) \times &\left[\left(\frac{q^s}{q^r}-1\right) - \left(-\frac{\partial e}{\partial d}\right) \times \left(\left[\frac{\mathbb{E}[\theta_\omega']}{\kappa(e^m)\mathbb{E}[\theta_\omega(e)]}\right]\right) d\right] \\ + \sigma(e^m) \times &\left[\left(\frac{q^s}{q^r} - \pi_g - \pi_b\left(\frac{1}{q_{1|b}}\right)\right) - \left(-\frac{\partial e}{\partial d}\right) \times \left(\left[\frac{\pi_b \theta_b'}{q_{1|b}\theta_b(e)}\right]\right) d\right] = 0 \end{aligned} \tag{2.39}$$

where $\sigma(e^m) \equiv \frac{\theta_g(e^m)-\theta_b(e^m)}{\theta_g(e^m)}$ and e is implicitly determined in

$$\mathbb{E}[\theta_\omega'(e)]A - \left[(1-\sigma(e^m))\frac{\mathbb{E}[\theta_\omega']}{\kappa(e^m)\mathbb{E}[\theta_\omega(e)]} + \sigma(e^m)\frac{\pi_b\theta_b'(e)}{q_{1|b}\theta_b(e)}\right] d - c'(e) = 0 \tag{2.40}$$

so that $\frac{\partial e}{\partial d} = -\left(\frac{(1-\sigma(e^m))\frac{\mathbb{E}[\theta_\omega']}{\kappa(e^m)\mathbb{E}[\theta_\omega(e)]} + \sigma(e^m)\frac{\pi_b\theta_b'(e)}{q_{1|b}\theta_b(e)}}{c''(e) - \left[(1-\sigma(e^m))\frac{\mathbb{E}^2[\theta_\omega']}{\kappa(e^m)\mathbb{E}'^2[\theta_\omega(e)]} + \sigma(e^m)\frac{\pi_b\theta_b'^2(e)}{q_{1|b}\theta_b^2(e)}\right] d}\right) < 0$ *and q^r is implicitly determined in*

$$\frac{1}{q^r} = \frac{\mathbb{E}[\theta_\omega(e)]A - c(e) - \left(1 - \sigma(e)\pi_b\left(\frac{1-q_{1|b}}{q_{1|b}}\right)\right) d}{1 - q^s d}$$

Supply curve is upward sloping and it is in between the supply curves of the previous sections. The **demand curve** is also upward sloping. The equilibrium prices of safe and risky assets are given by

$$q^s = \frac{w}{d} \text{ and } q^r = \frac{1-w}{\mathbb{E}\left[\theta_\omega(e)\right] A - c(e) - \left(1 - \sigma(e)\pi_b \left(\frac{1-q_{1|b}}{q_{1|b}}\right)\right) d}$$

where d and e are implicitly determined in the aforementioned system of equations.

Social Planner

The state variables, on which the decisions and consumptions are contingent, are the realisation of the aggregate and idiosyncrasy risk, determining the publicly observable realisation of output. The social planner decides on the consumption of infinitely risk averse investors ($c^{ra}(z)$), risk neutral investors ($c(z)$), and late investors ($c^{li}(z)$), as well as the amount of risk neutrals' assets ($\alpha(z)$) and late investors' assets ($\alpha^{li}(z)$) that are transferred to infinitely risk averse investors, to maximize the expected utility of the three group of investors (with respective Pareto weights W^{ra}, W^{rn} and W^{li}):

$$U^{sp} = W^{ra}\left[\underset{\omega}{min}\left\{c^{ra}_{2z}\right\}\right] + W^{rn}\left[\mathbb{E}_e\left[c^{rn}_{2z}\right] - c(e)k^{rn}_0\right]$$

subject to the following constraints:

(i) The resource constraint at $t = 0$ and $t = 1$

$$k_0 \leq 1$$
$$k^{li}_{1\omega} + k^s_{1\omega} \leq w^{li}$$

(ii) Feasibility constraint

$$c^d_{2\omega} \leq k^s_{1\omega}$$
$$c^d_{2\omega} + \theta_\omega(e)c^b_{2\omega s}(1 - \theta_\omega(e))c^b_{2\omega f} + c^{li}_{2\omega} \leq \theta_\omega(e)Ak_0 + F(k^{li}_1) + k^s_{1\omega}$$

(ii) The participation constraint for each of the groups:

$$\begin{aligned}
&\underset{\omega}{min}\left\{c_{2\omega}^{d}\right\} \geq 0 \\
&\mathbb{E}_{e}\left[c_{2z}^{b}\right] - c(e) \geq \left(\mathbb{E}_{e^{max}}\left[A_{\iota}\right] - c(e^{max})\right)(1-w) \\
&c_{2\omega}^{li} \geq F(w^{li})
\end{aligned}$$

where $q_{1\omega}$ is the price of the safe capital in the intermediate date, such that $\frac{1}{q_{1\omega}} k_{\omega}^{s} \leq \theta_{\omega}(e) t_{\omega s}$. where e^{max} is implicitly determined in $\mathbb{E}\left[\theta'_{\omega}\right] A - c'(e^{max}) = 0$, and $t_{1\omega s}$ is a transfer to late investors

(iii) Incentive compatibility constraint:

$$\begin{aligned}
e^{*} &\equiv \underset{e' \in [0,\bar{e}]}{argmax} \mathbb{E}_{e'}\left[c_{2z}\right] - c(e')k \\
k_{1\omega}^{s*} &\equiv \underset{k_{1\omega}^{s'} \in \left[0, w^{li}\right]}{argmax}\, F(w^{li} - k_{1\omega}^{s'}) + \frac{1}{q_{1\omega}} k_{1\omega}^{s'}
\end{aligned}$$

(iv) Non-negativity constraint for c_{tz}^{i} for each group of agent i, date t and state z.

From the incentive compatibility constraint, $\mathbb{E}\left[\theta'_{\omega} \times (c_{\omega s} - c_{\omega f})\right] - c'(e) = 0$. Thus, to minimize the burden of the moral hazard problem, positive transfers (if any) to risk neutral agents will be to the success, i.e. $c_{2\omega f}^{b} = 0$. For the second incentive compatibility constraint, we have that

$$q_{1\omega} = \frac{1}{F'(w^{li} - k_{1\omega}^{s'})}$$

Lets focus in the interesting case in which $\underline{c_{2}^{d}} > 0$. Therefore, the first constraint is biding (since $F'(W^{li}) > 1$, so that $\mu_{\omega} > 0$, and $k_{1\omega}^{s} = q_{1\omega}\theta_{\omega}(e)t_{1\omega s} > 0$. Then,

$$\begin{aligned}
\frac{\partial q_{\omega}}{\partial t_{\omega}} &= -\frac{\frac{(-F''(.))}{F'^{2}(.)} q_{1\omega}\theta_{\omega}(e)}{1 + \frac{(-F''(.))}{F'^{2}(.)}\theta_{\omega}(e)t_{1\omega s}} = -\frac{\left(-\frac{\partial q_{1|b}}{\partial(\sigma(e)d)}\right) q_{1\omega}\theta_{\omega}(e)}{1 + \left(-\frac{\partial q_{1|b}}{\partial(\sigma(e)d)}\right)\theta_{\omega}(e)t_{1\omega s}} < 0 \\
\frac{\partial q_{\omega}}{\partial e} &= -\frac{\frac{(-F''(.))}{F'^{2}(.)} q_{1\omega}\theta'_{\omega}(e)s_{1\omega s}}{1 + \frac{(-F''(.))}{F'^{2}(.)}\theta_{\omega}(e)s_{1\omega s}} = -\frac{\left(-\frac{\partial q_{1|b}}{\partial(\sigma(e)d)}\right) q_{1\omega}\theta'_{\omega}(e)t_{1\omega s}}{1 + \left(-\frac{\partial q_{1|b}}{\partial(\sigma(e)d)}\right)\theta_{\omega}(e)t_{1\omega s}} < 0
\end{aligned}$$

For, $k_{1\omega}^{s} < w_{1}^{li}$, the Langrangian of risk neutral investors' problem at $t = 1$ is given by

$$
\begin{aligned}
\mathcal{L}_0 \quad =& W^d \underline{c_2^d} + W^b \mathbb{E}\left[\theta_\omega(e) \times (A - t_s - t_{1s\omega}) - c(e)\right] - \mathbb{E}\left[\lambda_\omega \left[t_s + t_{1\omega s} - A\right]\right] \\
& -\eta \left[\mathbb{E}\left[\theta'_\omega \times (A - t_s - t_{\omega s})\right] - c'(e)\right] - \mathbb{E}\left[\mu_\omega \left[\underline{c_2^d} - q_{1\omega}\theta_\omega(e) t_{\omega s} - \theta_\omega(e) t_s\right]\right]
\end{aligned}
$$

$$
\begin{aligned}
\lambda_\omega \geq 0 \text{ and } \lambda_\omega \left[t_{1\omega s} - A\right] = 0 \\
\eta \geq 0 \text{ and } \eta \left[\mathbb{E}\left[\theta'_\omega \times (A - t_{\omega s})\right] - c'(e)\right] = 0 \\
\mu_\omega \geq 0 \text{ and } \mu_\omega \left[\underline{c_2^d} - q_{1\omega}\theta_\omega(e) t_{\omega s} - \theta_\omega(e) t_s\right] = 0 \\
\lambda_\omega \geq 0 \text{ and } \lambda_\omega \left[t_s + t_{1\omega s} - A\right] = 0
\end{aligned}
$$

Unless, $q_{1\omega} \geq 1$ late sales will never be the unique option. Then, for certain Pareto-weights $W^d = \frac{q^s}{q^s + q^r}$, and after some algebra:

$$
\begin{aligned}
& \frac{q^s}{q^r} - \frac{\mathbb{E}\left[\theta_\omega\right]}{\theta_g} - \sigma(e) \frac{\pi_b (1 + \varepsilon_b)}{q_{1|b}} \\
& - \left(-\frac{de}{dd}\right) \times \left(\frac{\mathbb{E}\left[\theta'_\omega\right]}{\theta_g} + \sigma(e) \frac{\pi_b \theta'_b(e)}{q_{1|b}\theta_b} + \left(-\sigma'(e)\right) \pi_b \left(\frac{1 + \varepsilon_b}{q_{1|b}} - 1\right)\right) d = 0 \\
& \frac{de}{dd} = \frac{-\left(\frac{\mathbb{E}[\theta'_\omega]}{\theta_g} + \sigma(e) \frac{\pi_b \theta'_b(e)(1+\varepsilon_b)}{q_{1|b}\theta_b}\right)}{c''(e) - \left(\frac{\mathbb{E}[\theta'_\omega]\theta'_g}{\theta_g^2} + \frac{\pi_b \theta'_b(e)}{q_{1|b}\theta_b} \left(\sigma(e) \frac{\theta'_b}{\theta_b(e)} + \left(-\sigma'(e)\right)(1 + \varepsilon_b)\right)\right) d}
\end{aligned}
\tag{2.41}
$$

The different sources of inefficiencies are shown when compared to lemma 21.

Chapter 3

Safe Assets, Capital Flows, and Macroeconomic Outcomes

3.1 Introduction

There is a growing global need for safe assets. The diversity in the ability to produce safe stores of value around the world, coupled with mismatches between local supply and demand for this class of assets, has fuelled cross-border capital flows that search for safety. As a result, global imbalances in the US (Bernanke et al. (2011)), but also in other industrialised economies (Carvalho and Fidora (2015)), have loomed large. Many of the concerns regarding these imbalances derive from emerging markets' experiences, where capital flows are fickle as emphasised in the literature on sudden stops. However, the financial and economic fragilities associated with capital flows that search for safety are less understood.

In this paper, we contribute to the sparse literature on this topic by assessing the relationship between foreign safe assets' positions and the real economic activity (GDP growth), and further rationalise the preliminary findings through the lenses of a model. We use the international financial accounts data in Diebold and Richter (2021) to construct a cross-country dataset of safe assets and liabilities of different economic sectors in a sample of 38 OECD countries. Following Gorton et al. (2012), we define safe assets as debt or other debt-like contracts issued by governments (public supply) and financial institutions (private supply).

An analysis of the cyclical variation of safe asset positions reveals that changes in foreign sectors' net safe asset positions (demand side) are mapped almost one-to-one with changes in privately issued safe liabilities (supply side). The correlation between foreign sectors' demand and governments' debt (public supply) is negative but not significant. Public supply, instead, follows suit the business cycle, increasing during distress periods in accordance with their fiscal needs. Public supply by governments and private supply by financial intermediaries are negatively correlated, supporting the view that these two sources of supply are substitutes. While the real sector's holdings (households and non-financial corporations) have stayed relatively stable, risky lending to the real sector positively and strongly co-moves with the private supply of safe assets.

How do these developments relate to economic growth? We find that an increase in the net supply of safe assets by financial intermediaries (or net demand by the foreign sector) is associated with lower GDP growth over the following years. The strong correlation holds even when controlling for the credit to the real sector, among other controls. The real sectors' and governments' safe assets position do not exhibit a significant correlation with GDP growth. This preliminary evidence, while only suggestive, it raises important questions: Is there something intrinsic in the foreign demand for safety? alternatively, is private supply hindering economic growth? Would a public response ameliorate the burden? How is this pattern affected by the business cycle?

To answer these questions, we propose a two-country model that studies the determinants of safe assets demand and supply at the international level. Both economies are populated by households, whose endowment is risky (*iid* across households), and who demand safe assets as they derive a convenience service from holding them. There are two sources of supply catering to safety demand: (i) government bonds backed by future tax revenues, and (ii) privately issued safe assets backed by pools of private loans granted to households. The public supply is set to meet certain fiscal requirements (exogenous to the model), while the private supply is determined by a competitive financial sector. Notice that it is a resource economy, so credit is not needed to undertake profitable investment opportunities. Instead, financial intermediaries grant loans in their asset side to back the issuance of safe debt in their liability side. Ultimately, the private supply is jointly determined by the size and quality of the loans they grant.

The key friction in the model, that tights together supply of safe assets and economic activity, is a moral hazard problem: households can exert effort to enhance their income prospects, but given that this effort is not observable, only the fraction of the income they retain – *skin-in-the-game* – provides the incentives to exert effort. Therefore, when households borrow or they anticipate taxes, effort incentives jeopardize, which is translated into lower future output.

Capital flows arise to exploit any cross-country differences in their relative demand and supply for safety. [1] That is, the model predicts that the heterogeneity in the ability to produce safe stores of value across countries and the domestic demand relative to the global demand explain the direction and magnitude of capital flows. Countries with more developed financial systems can accommodate a higher demand for safety, which is consistent with the observation that the strong demand for safety has been targeted to a few industrialised economies. Still, the economic outlook or fiscal capacity are also important determinants.

The model predicts that an increase in the foreign demand for safety encourages the private sector to increase its supply. Financial intermediaries increase their lending, in turn increasing the collateral that serves to issue safe liabilities. Nonetheless, increasing the size of the loans reduces households' incentives to exert effort on detriment to output. So, the higher supply is met by more loans of lower quality; the foreign demand creates a bad type of credit boom, decoupled from productivity dynamics, which is followed by worse economic fundamentals.

Foreign demand harms real activity, yet the magnitude of the real cost depends on the strength of the global demand for safety and the country's ability to absorb it. Hence, there is nothing intrinsically different about foreign demand compared to domestic demand when comparing their respective effects on output; if the domestic demand soars and is met by domestic supply, the same mechanism would

[1] The only source of risk is idiosyncratic risk. Full diversification can be obtained within the country, thus, there is no need for further risk-sharing across borders. In addition, absent any aggregate risk, we also disregard international exchanges of risky assets.

be at play.

Then, is the private supply the problem? Would a public response be better? The model is not well suited to answer this question. The model suggests that both private and public supply hamper effort incentives. However, it is assumed that the financial sector is better equipped to cope with asymmetric information, ameliorating the moral hazard problem. Therefore, safe assets backed by private loans are less harmful to the real economy than safe assets backed by taxes. While this channel might play a role, it is at odds with our empirical findings, suggesting that this is not the complete story.

Regarding the business cycle, the private supply decreases when economic prospects deteriorate. There are two channels at play that reinforce each other. On one hand, the weakened expected output de facto reduces the skin-in-the-game of households and their incentives to exert effort. In turn, the equilibrium loan size decreases as the moral hazard problem becomes more pronounced. Therefore, both the size and quality of loans drop, and so does the private supply. This is a novel channel that explains the contraction of the private supply during downturns, which is not related to a loss in their safety status. In addition, financial intermediaries' supply decreases as a response to an increase in the public supply; public supply partially crowds out private supply and private investment, thus exacerbating the drop in the private supply.

3.1.1 Literature review

This paper relates to several strands in the literature. Some researchers (e.g., Caballero et al. (2008), Mendoza et al. (2009)) view these global imbalances as an efficient outcome of financial integration when countries have heterogeneous financial development. Similarly, in our paper difference in financial development, namely, the ability of financial intermediaries to deal with informational frictions in the credit market, is an important driver of capital flows, yet it is not the only one. As argued by He et al. (2019), the relative fiscal capacity and economic conditions also matter in our model, but they add the size of the debt as an attracting factor due to strategic complementarities.

This paper also relates to the growing literature that studies how countries have responded to the increase in the foreign demand for safety, and the fragilities associated with this type of capital flows. Caballero and Krishnamurthy (2009) argue that foreign demand for safe debt instruments left the US economy fragile by increasing the level of leverage of the domestic financial sector, amplifying the effects of bad shocks. On the other hand, Acharya and Schnabl (2009) contend that it was lax financial sector regulation in a world with global banking that contributed to the crisis right from its inception. In our paper, instead, the fragilities build as demand factors fuel a credit boom, decoupled from productivity, which worsen the quality of investment and the economic outlook.

The scarcity of safe assets has been a research focus, especially in the aftermath of the financial crisis (e.g., Caballero et al. (2017)). Caballero and Farhi (2018) claim that the global scarcity of safe assets, coupled with the zero lower bound for global interest rates, negatively affects global output, as the latter becomes the only adjustment mechanism to ameliorate the scarcity of safety (*safety trap*). In our model, while the source of fragility is different, safe asset scarcity exacerbates the cost of issuing safe assets in the economy.

On the conceptual side, this model builds on Azzimonti and Yared (2019), where they develop a theory of optimal government debt in which publicly-issued and privately- issued safe assets are substitutes. But both models differ in the friction.

The paper proceeds as follows. Section 3.2 describes the data sources, and presents the headline results. Section 3.3 presents the model to better understand the preliminary results. Section 3.4 concludes.

3.2 Data analysis

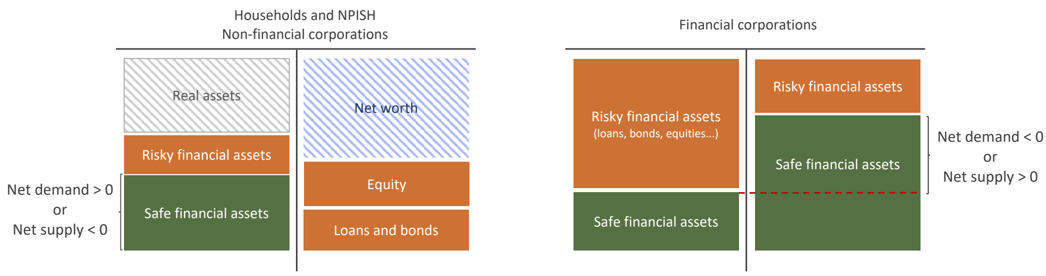
3.2.1 Database

This project uses the international financial accounts data in Diebold and Richter (2021) to construct a cross-country dataset of safe assets and liabilities of different institutional sectors. Following Gorton et al. (2012), we define safe assets as: Government bonds (central and local), and bonds issued by financial corporations. This leaves non-financial bonds, loans, shares, insurance and pension assets, derivatives and stock options, and other accounts receivable as risky assets. We construct safe assets and liabilities of the following five sectors: general government, central bank, financial institutions (both banks and non-banks), real sector (households, NPISH, and non-financial firms), and rest of the world.

We focus our analysis on the net safe asset supply defined by:

$$Net\ safe\ supply_{i,j,t} = Safe\ Liabilities_{i,j,t} - Safe\ Assets_{i,j,t}$$

where i, j, t are country and sector indexes. A positive net safe asset supply, captures the safe liabilities that are backed by risky assets. A negative net safe asset supply, captures the safe assets that are funded with risky liabilities. Therefore, we are abstracting from the safe assets (public and private) that are hold within the financial sector. The following section clearly illustrates this point.



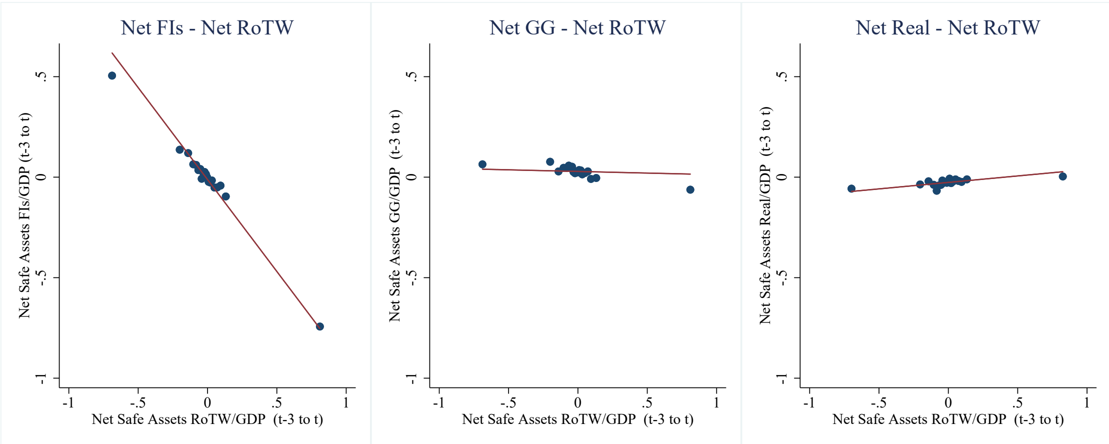
3.2.2 Preliminary evidence

Figure 3.6 in the appendix illustrates the evolution of net safe assets position of the different institutional sectors across countries. Despite the idiosyncrasies of each country, some interesting patters arise.

On the demand side (in the negative territory), the foreign sector shows significantly higher variation compared to the real sector, whose holdings remain relatively constant. Most importantly, the net positions of financial intermediaries and the foreign sector seem to mirror each other, i.e., financial sector supply strongly co-moves with foreign sector's demand. On the supply side (in the positive territory), the figure suggests a negative correlation between the private and public supply of safe assets. In most countries, private supply by financial intermediaries increases in the period preceding the global financial crisis and stops in the aftermath as the public supply by governments increases.

To further test this, the following graphs plot how changes in net safe assets positions interact across different sectors.

Figure 3.1: Correlations between changes in sectoral net positions

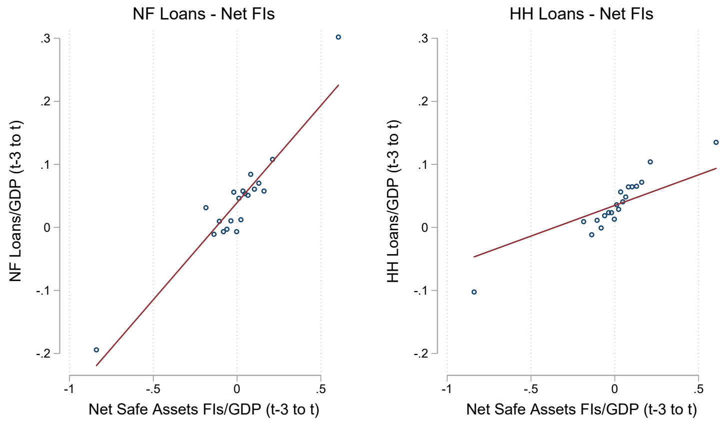


Notes: Observations are collapsed into 20 equal-sized bins according to the change in net safe asset position of the foreign sector with the last 3 years. Each point represents the group specific means of this variable and the net safe asset position of the stated sector. Fitted regression lines illustrate the correlation between the two variables.

Figure 3.1 suggests that net changes in the foreign sector's safe asset demand maps almost one-to-one with changes in the financial sector's supply. While the correlation with the public supply is also negative, it is significantly lower in magnitude. The real sector is positively but not significantly correlated with changes in net foreign position.

Financial intermediaries' net issuance of safe assets is backed by risky assets, e.g., corporate bonds, loans to the real sector, loans within the financial sector, etc. The Figure 3.2 illustrates how the issuances of private safe assets relates to risky lending from the financial to the real sector. An increase in the net safe asset supply of financial intermediaries is associated with more risky loans to households

Figure 3.2: Correlations between FI net positions and loans to the real sector



Notes: Observations are collapsed into 20 equal-sized bins according to the change in net safe asset position of the financial sector with the last 3 years. Each point represents the group specific means of this variable and changes in loans to the non-financial corporations (on the left-hand side) and changes in loans to households (on the right-hand side). Fitted regression lines illustrate the correlation between the these variables.

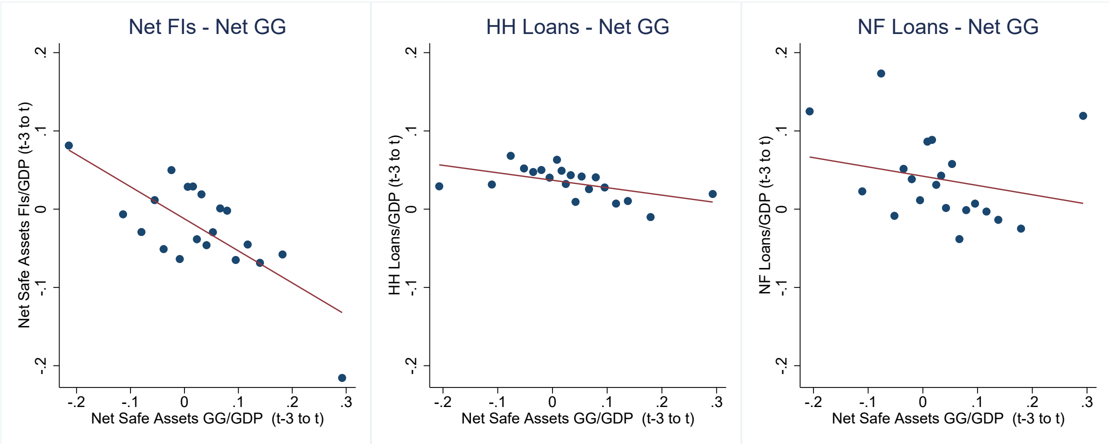
and non-financial corporations. That is, increases in the private supply of safe assets are associated with credit booms.

Figure 3.3 illustrates the correlation between changes in the net safe asset position of governments and financial intermediaries, along with the correlation of the former with risky lending by the financial sector to households and non-financial corporations. Figure 3.3 shows that the net safe asset supply by the government is negatively correlated with private supply and with private investment by reducing financial intermediaries' risky lending to the real sector. This supports the view that private and public supply are substitutes and that government debt might crowd out private investment.

Ultimately, the objective is to assess how these dynamics in safe asset position across sectors relate to macroeconomic outcomes. To do so, we propose the following regression in which we focus on the association between changes in net positions and subsequent GDP growth:

$$\Delta_3 ln(Y)_{it+3} = \alpha_i + \beta \Delta_3 N^j_{it} + \sum_{j=1}^{3} \beta^j_y \Delta y_{it-j} + \gamma X_{it} + u_{it+3}$$

Figure 3.3: Correlations between GG net positions and loans to the real sector



Notes:Observations are collapsed into 20 equal-sized bins according to the change in net safe asset position of the General Government with the last 3 years. Each point represents the group specific means of this variable and change in net safe asset position of the financial sector with the last 3 years (in the left-hand-side) changes in loans to the households (the middle one) and changes in loans to non-financial corporations (in the right-hand-side). Fitted regression lines illustrate the correlation between the these variables.

where t and i represent year and country indexes, $\Delta_3 ln(Y)_{it+3}$ is the cumulative 3-year GDP growth, N_{it}^j is cumulative 3-year change in the net safe asset position to GDP ratio of sector j, and X_{it} is a vector of controls including 3-year changes in household and non-financial credit, and foreign sector's cumulative 3-year change in the net risky asset position to GDP ratio. It also includes three lags of the dependent variables and country fixed effects. Time fixed effects are not included as the latter might capture demand factors that are common to the countries in the sample, which are of interest in the analysis.

Increasing the net supply of safe assets by financial intermediaries is associated with lower GDP growth over the following years. Increase in the net demand of the foreign sector is also associated with lower GDP growth over the following years. We have seen before that both sectors' net safe positions pick up similar variation. The significance of this result holds even when controlling for the credit boom that suits follows. Therefore, one channel in which credit booms might be harmful is when they are fuelled by safety demand and supply factors. In addition, by controlling for the changes in the foreign sector's risky asset position, we are trying to evaluate whether it is the overall capital flows or rather the safety fraction of it that is problematic. The latter exhibits a stronger correlation.

Figure 3.4: Correlations between changes in sectoral net safe asset positions and future GDP growth

	Dependent variable : Real GDP Growth$_{i,t+3}$			
	(1)	(2)	(3)	(4)
Δ_3*Net Safe RoTW*$_{i,t-1}$	0.09**			
	(0.04)			
Δ_3*Net Safe FI*$_{i,t-1}$		-0.11***		
		(0.03)		
Δ_3*Net Safe GG*$_{i,t-1}$			0.03	
			(0.04)	
Δ_3*Net Safe Real*$_{i,t-1}$				0.06
				(0.07)
Δ_3*Net Risky RoTW*$_{i,t-1}$	-0.01	0.00	-0.06**	-0.05*
	(0.02)	(0.02)	(0.03)	(0.03)
Δ_3*HH Loans*$_{i,t-1}$	-0.29*	-0.23	-0.36**	-0.50***
	(0.16)	(0.16)	(0.15)	(0.15)
Δ_3*NF Loans*$_{i,t-1}$	-0.01	0.01	0.00	-0.07
	(0.06)	(0.06)	(0.07)	(0.06)
R^2	0.301	0.320	0.303	0.363
Country fixed effects	✓	✓	✓	✓
GDP growth(3 lags)	✓	✓	✓	✓
Observations	658	658	654	637

Notes: The dependent variable is the change in log real GDP between t and $t+3$. Explanatory variables are changes in the ratio of the respective variable and GDP between $t-4$ and $t-1$. LDV includes three lags of GDP growth. Country fixed effects included. Standard errors in parentheses are dually clustered on country and year. $*$,$**$,$***$ indicates significance at the 0.1, 0.05, 0.01 level, respectively.

The results suggest that increases in the supply for safety by the government or decreases in the demand for safety by the real sector are both associated with increases in economic growth, but such correlation is not statistically significant. There are several points to mention here: first, public supply is positively correlated with contemporaneous GDP, and as already argued the supply might increase during economic downturn, thus, when GDP growth is expected to grow. Second, given that the real sector's demand is relatively stable, that lack of variation can also lead to a delusive non-significance. Yet, the sign of the coefficient is in line with our prior.

While only suggestive, this table gives some food for thought. Does it matter who is the issuer or the holder of those safe instruments? This is, is there something intrinsically different about foreign demand compare to domestic demand? Or, is it better if the public sector responds as opposed to the private sector responding? To answer these questions, we propose a theoretical models to start thinking about this answer.

3.3 Model

This is an endowment economy that has two dates ($t = 0, 1$), one perishable good that can be used either for consumption or investment, and one source of uncertainty, namely, a shock that is *iid* across the population affecting the level of their future endowment.

Households. There is a continuum of households of size one. Each household's utility is given by

$$c_0 + \mathbb{E}\left[c_{1s}\right] + v\left(\underset{s}{min}\left\{c_{1s}\right\}\right) \tag{3.1}$$

where c_{ts} represents the consumption of a household at date t and state s, and $v(.)$, where $v'(0) = \infty$, $v'(.) > 0, v''(.) < 0$ and $v'''(.) > 0$, captures the convenience service associated to safe cash flows. We model the demand for safe assets in reduced form, in a similar vein as Stein (2012) and Krishnamurthy and Vissing-Jorgensen (2012).

Each household has an endowment y_0 at date $t = 0$ and y_{1s} at date 1 and state s. Future stochastic income depends on an idiosyncratic shock that is *iid* across the population, and it is given by

$$y_{1s} = \begin{cases} y_{1h} & \text{with probability} \quad \pi_h(e) = \bar{\pi}(1+e) \\ 0 & \text{otherwise} \end{cases}$$

where the probability of obtaining a high output, $\pi_h(e)$, is a positive function of the effort households exert, $\pi_h'(e) = \bar{\pi} > 0$. Hereafter, $\mathbb{E}_e\left[.\right]$ represents the expectation operator for a level e of effort. While effort increases households' productivity, it also entails a non-pecuniary convex cost given by

$$c(e) = \frac{\gamma}{2}e^2 \tag{3.2}$$

A key friction in the model is that effort is not observable nor contractible by a third party. This informational friction creates a moral hazard problem whenever households sell a risky financial claim backed by their future income. The buyer of the claim can track the seller's other choices to infer their real incentives to exert effort, and/or can monitor the seller to (partially) resolve the incentive problem. Both options are extremely costly for individual households, consequently, it is not optimal for households to trade risky financial claims among themselves. In this aspect, financial intermediaries within the country, who hold a diversified portfolio, have a comparative advantage in the spirit of Diamond (1984). Hence, financial intermediaries will buy the risky financial claims sold by households within that economy. Households, however, knowing the aggregate variables can infer the collective average effort exerted in market. Considering this preliminary result, each household faces the following budget constraint

$$\begin{aligned} c_0 &= \tilde{y}_0 + \sum_s p_s^l (\hat{e})\, l_s - p^d d \\ c_{1s} &= \tilde{y}_{1s} - l_s + d \end{aligned} \tag{3.3}$$

The government levies taxes, such that $\tilde{y}_{ts} \equiv (1 - \tau_t)\, y_{ts}$ is disposable income at date t and state s, and τ_t is the marginal tax rate. At date 0, households decide the amount of state s limited-liability claims to sell, $0 \leq l_s \leq \tilde{y}_{1s} + d$, at a price $p_s^l(\hat{e})$, which is a function of the effort level expected by financial intermediaries, denoted as $\hat{e}$. In addition, households choose the amount of safe assets to buy, d, at a price p^d from the government or financial intermediaries. Even if they are produced differently, they are perfect substitutes for households.

Government. The government levies a lump sum tax/transfer in their respective countries $T_0 \equiv \tau_0 y_0$ and $T_1 \equiv \tau_1 \mathbb{E}_{\bar{\hat{e}}}[y_{1s}]$ uniformly across the population, where $\bar{\hat{e}}$ capture the average effort expected to exert by all households. The level of government debt, denoted as b is exogenous in the model, and the tax rate is such that the dynamic budget constraint is satisfied at dates 0 and 1:

$$\tau_0 y_0 + p^d b = 0 \quad \text{and} \quad \tau_1 \mathbb{E}_{\bar{\hat{e}}}[y_{1s}] - b = 0 \tag{3.4}$$

Due to the law of large number, government debt is risk-free, thus, the price of such debt is given by p^d.

Financial intermediation. There is a set of perfectly competitive financial intermediaries in the economy. These financial intermediaries, whose objective is to maximize expected profits, buy the risky financial claims sold by households, forming a diversified assets portfolio that costs $p_s^l(\bar{\hat{e}})$, and sell safe debt to households at a price p^d. Hence, the total safe debt they can issue is delimited by the cashflow of the claims bought, given by $\mathbb{E}_{\bar{\hat{e}}}\left[l_s\right]$. In addition, financial intermediaries have a monitoring technology that serves to partially restore the incentives, ameliorating the moral hazard problem. This technology, that is mechanical and involves no choice from financial intermediaries, will be later described.

Capital flows. Trade across *domestic* and *foreign* economies can arise to exploit potential sources of heterogeneity. Capital flows are exclusively driven by safety motives, and are determined in the international market for safe assets.

The following assumption delimits the value of certain parameters to focus on a realistic parameterization of the model.

Assumption 2 *Let's assume that* $\gamma < 0.5\bar{\pi}y_{1h}$ *and* $\phi > 0.5$.

The first condition ensures that the optimal effort is less than $\overline{e} = 0.5$, this is, effort can not increase more that 50% the probability of obtaining a high output. The second condition realistically delimits financial development.

Definition of Equilibrium. Given fiscal policy $\{b, \tau_0, \tau_1\}$, a competitive equilibrium corresponds to a set of prices $\left\{p^d, p^l(e)\right\}$ and a level of safe assets and risky financial claims $\{d^j, l_s^j\}$ in both economies $j \in \{domestic, foreign\}$ which satisfy the following conditions:

1. Households maximize (3.1) subject to (3.3) and the incentive compatibility constraint given prices
2. Governments satisfies their budget constraint (3.4)
3. Zero profit condition hold for financial intermediaries
$$p^l = p^d \pi_s(e) \tag{3.5}$$
4. The (international) safe asset market clear for prices p^d
5. Belief consistency: $e = \hat{e}$

3.3.1 Characterization of Equilibrium

In this section, we will characterize the equilibrium. First, we will analyse the partial equilibrium at the country level, to understand the supply and demand functions. Next, we will put the optimality conditions of both countries together, and study the equilibrium prices and capital flows, and how they vary under different institutional and economic conditions.

Partial equilibrium within the country

Let's first study the maximization problem of households. Effort is unobservable, and it is chosen after all financial transactions are materialized. Thus, the optimality condition for effort is given by [2]

$$\bar{\pi}\left[\tilde{y}_{1h} - \phi\left(l_h - l_l\right)\right] - \gamma e^* = 0 \tag{3.6}$$

The marginal benefit of effort is given by the differential in the high and low state cash flows retained. Households do not have incentives to exert effort for the fraction of the income they sell or pay in taxes, as they are not exposed to their corresponding increase in the expected. This is a standard moral hazard problem in which income retention serves as *skin-in-the-game* to spur effort. Financial intermediaries observe households' skin-in-the-game, infer the effort they exert, and price their financial claims accordingly. In addition, parameter ϕ captures financial intermediaries' monitoring technology that serves to ameliorate the strategic misconduct of households due to the unobersbability of effort.

Therefore, households take prices as given and choose $\langle d, \{l_s\}\rangle$ to maximize their utility in (3.1) subject to the budget constraint given by (3.3) and the incentive compatibility constraint that ensures $\hat{e} = e^*$ where e^* is determined in equation (3.6), in addition to the nonnegativity constraint for consumption.

From the first order conditions, the inverse demand function for safe assets is

[2] In particular, the optimal effort level is result to $e^* \equiv \underset{e' \in [0,\bar{e}]}{argmax} \mathbb{E}_{e'}\left[\tilde{y}_{1s} - \phi l_s + d\right] - c(e')$.

given by

$$p^d \;=\; 1 \;+\; \underbrace{\psi v'(d^*)}_{\text{convenience yield}} \tag{3.7}$$

Hence the marginal cost of safe assets (price) equates the expected payoff plus the convenience yield associated to the value households attach to holding safe assets. This convenience yield is downward sloping as a function of d, so is the demand function, $d^*(p^d)$.

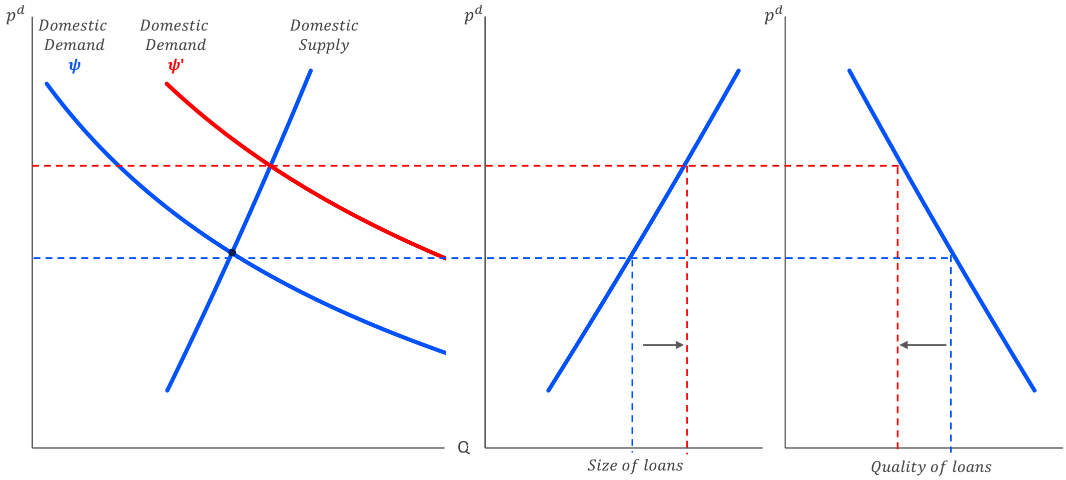
Regarding the financial claims sold, the model has a multiplicity of equilibrium; it has a clear prediction for the net financial position of households, but it is silent about the gross financial position. What matters for allocation and welfare is the effective safe cash flows, $d - l_l$, and the net transfer across state of nature, namely, $l_h - l_l > 0$. Thus, on the benefit of simplicity let's assume hereafter that $l_l = 0$. So, we can interpret the financial claims sold by households and bought by financial intermediaries as loans, with face value l_h and backed by the future income of households. The size of the loan demanded is determined in

$$\left(p^d - 1\right)\pi_h(e^*) - \left(\frac{\partial p_h^l(\hat{e})}{\partial \hat{e}}\right) \times \left(-\frac{\partial \hat{e}}{\partial l}\right) \times l_h^* = 0 \tag{3.8}$$

where effort is determined in (3.6) so that $\frac{\partial e}{\partial l} = -\frac{\bar{\pi}\phi}{\gamma} < 0$, and $p_h^l(e)$ is determined in the zero profit condition so that $\frac{\partial p_h^l(e)}{\partial e} = p^d\bar{\pi}$. Let's carefully interpret this equation, as it encapsulates the main trade off in the model. The first term represents the differential between the price of the loan and the value households attach to the underlying risky cash flows of the loan. Notice that in equilibrium, this difference is set by the convenience yield, as the loans are ultimately used as inputs in the safe asset production. However, when households increase their borrowing, they reduce as a byproduct their skin-in-the-game, signalling to the market a lower effort, which ultimately increases the cost of the loan (lower price which is equivalent to higher interest payment).

Financial intermediaries buy the claims households sold, i.e., lend to households. They bundle a large number of these loans together to get rid of the (idiosyncratic) risk. In turn, financial intermediaries issue safe assets backed by the diversified pool of loans: $\pi_h(\bar{\hat{e}})l$. Hence, the private supply of safe assets is determined by both the size and the quality of loans. Due to competition, however, financial intermediaries' net interest margin equals zero, so that at the prevailing equilibrium prices they are indifferent with the amount of safe assets they issue. Therefore, in equilibrium the loan size and quality are determined in households' optimality conditions already described. The following lemma summarizes the private supply schedule.

Lemma 22 *Under assumption (2), the private supply function is upward sloping,* $\frac{d(\pi_h(e)l)}{dp^d} > 0$*. However, as the supply of safe assets increases with* p^d*, the size of loans increases while their quality decreases, i.e.,* $\frac{dl_h}{dp^d} > 0$ *and* $\frac{de}{dp^d} < 0$*.*



Underneath the upward supply curve, there is a substitution between loan size and quality along the curve. In fact, due to the moral hazard problem, there is a trade-off between the two inputs to the private supply, i.e., when households increase their borrowing, *ceteris paribus*, they decrease their skin-in-the-game and effort, consequently, deteriorating their productivity and the quality of the loan. When p^d increases, this increase fully passes-through to the price of loans due to competition in the financial sector, reducing the cost of borrowing (a lower interest payment). This event encourage households to borrow more undermining their incentives to exert effort. Absent any other friction or externality, this result will unconditionally hold due to the optimality conditions of households.

There is an externality, however, that households do not internalize, and that it is related to the dual function of effort: the collective effort exerted by households does not only support the production of safe cash flows, but it also increase aggregate output by enhancing productivity. It is important to highlight the latter, as it affects the tax policy of governments. This is, a government whose target is to keep certain level of sobering debt for fiscal needs, might reduces the marginal tax rate when economic prospects improve, i.e., when aggregate output is higher.

$$\frac{de}{dl} = -\frac{\bar{\pi}\phi}{\gamma - \bar{\pi}(-\tau'(e))y_{1h}} \quad \text{where} \quad \tau'(e) = -\frac{\bar{\pi}}{\pi_h(e)}\tau(e) < 0 \quad \text{such that} \quad \left|\frac{de}{dl}\right| > \left|\frac{\partial e}{\partial l}\right|$$

For the parameters of interested, this externality does not change qualitatively the result, and it just has an effect on the magnitude. Given that our focus is on the positive analysis, we are going to abstract from the normative analysis for now.

After describing the demand and supply curves, the following proposition characterizes the partial equilibrium in the domestic economy.

Proposition 23 *When assumption (2) holds, for an equilibrium p^d, capital inflows are determined in*

$$\underbrace{d^f}_{\text{capital inflow}} = \underbrace{\tau_1(e^*)\mathbb{E}_{e^*}[y_{1s}]}_{\text{public dom. supply}} + \underbrace{\pi_h(e^*)l^*}_{\text{private dom. supply}} - \underbrace{d^*}_{\text{dom. demand}}$$

where d^* is determined in equation (3.7), loan size and effort are determined in equations (3.8) and (3.6), respectively, and the tax rate $\tau_1(e^*) = \frac{b}{\mathbb{E}_{e^*}[y_{1s}]}$ where b is exogenous. Capital inflows will be positive (negative) whenever the domestic demand is lower (higher) than the domestic supply. Consumption is given by

$$c_0 = y_0 + p^d d^F$$

$$c_{1h} = \underbrace{y_{1h} - \left(\frac{b}{\pi_h(e^*)} - l^*\right)}_{\text{retained risky income}} + \underbrace{(b + \pi_h(e^*)l^*) - d^f}_{\text{dom. holdings of safe assets}} > c_{1l} = \underbrace{(b + \pi_h(e^*)l^*) - d^f}_{\text{dom. holdings of safe assets}}$$

Both sectors, financial intermediaries and the government, can cater to the demand for safety. Public and private supply are perfect substitutes for households, but they are created differently. The former is backed by taxes, which do not discriminate across effort levels. Thus, income retention does not ameliorate the moral hazard problem created by taxes. The former is backed by a pool of diversified loans, endogenously determined in equilibrium, and given that the price of the loans depend on the expected effort level inferred from the skin-in-the-game, households optimally decide to retain risky income. In addition, due to the ability of financial intermediaries to partially solve the incentive problem derived from the informational friction, absent any other friction, the private supply of safe assets is more efficient than the public one.

Whenever, the domestic supply is higher (lower) than the domestic demand at the prevailing equilibrium price, there will be a capital inflow to (outflow from) the country. Whenever capital flows are positive, the country is going to be a net borrower, i.e., the country is going to bring forwards future resources for consumption today. Notice that there is not full risk-sharing, due to the fact that households optimally hold some skin-in-the-game to exert effort, hence, markets are endogenously incomplete (as described in Biais et al. (2021)). In addition, the foreign debt repayment make consumption tomorrow more volatile. We are further going to analyse next how the direction of the capital flows depend on various parameters of the model.

3.3.2 Capital flows

In this section we characterized the equilibrium price of safe assets, and the capital flows derived from differences in relative demand and supply for safe assets. A note on notation: parameters of the domestic (foreign) economy are denoted without (with) a hat, while the equilibrium value of endogenous variables are denoted with a * ($^+$).

The direction of the capital flows will depend on the relative demand and supply of safe assets across countries; the country with the relatively high (low) demand will experience a capital outflow (inflow), while the country with the relatively high (low) supply will experience a capital inflow (outflow). This happens because, whenever the equilibrium price within a country (close economy) is higher compared to the other country's, either because demand is relatively high or supply relatively low, this motivates an outflow of capital from the country whose safe assets are relatively expensive. The magnitude of the flow will be such that the price is equated in both countries, restoring equilibrium at the international level. In the following proposition summarizes how different parameters of the model affect the relative supply and demand, and makes a comparison across country.

Proposition 24 *The direction of capital flows depend on:*

- *Lower relative demand explains a positive capital inflow to the country:* $\psi < \hat{\psi} \implies l^* = l^+$ *and* $e^* = e^+ \implies \pi_h(e^*)l^* = \hat{\pi}_h(e^+)l^+$, *but* $d^* < d^+ \implies d^f > 0$

- *Higher financial development explains a positive capital inflow to the country:* $\phi < \hat{\phi} \implies d^* = d^+$, *but* $l^* > l^+$ *and* $e^* = e^+$ $\implies \pi_h(e^*)l^* > \hat{\pi}_h(e^+)l^+ \implies d^f > 0$

- *Bigger public debt explains a positive capital inflow to the country:* $b > \hat{b} \implies d^* = d^+$, *but* $l^* < l^+$ *and* $e^* < e^+$, *however,* $b + \pi_h(e^*)l^* > \hat{b} + \hat{\pi}_h(e^+)l^+ \implies d^f > 0$

- *Higher future output explains a positive capital inflow to the country:* $y_{1h} > \hat{y}_{1h} \rightarrow d^* = d^+$, *but* $l^* > l^+$ *and* $e^* > e^+ \implies \pi_h(e^*)l^* > \hat{\pi}_h(e^+)l^+$ $\implies d^f > 0$

In what follows, I will discuss the proposition, and further pursue a comparative statics mapping the subsequent discussion to the ongoing events.

Foreign demand

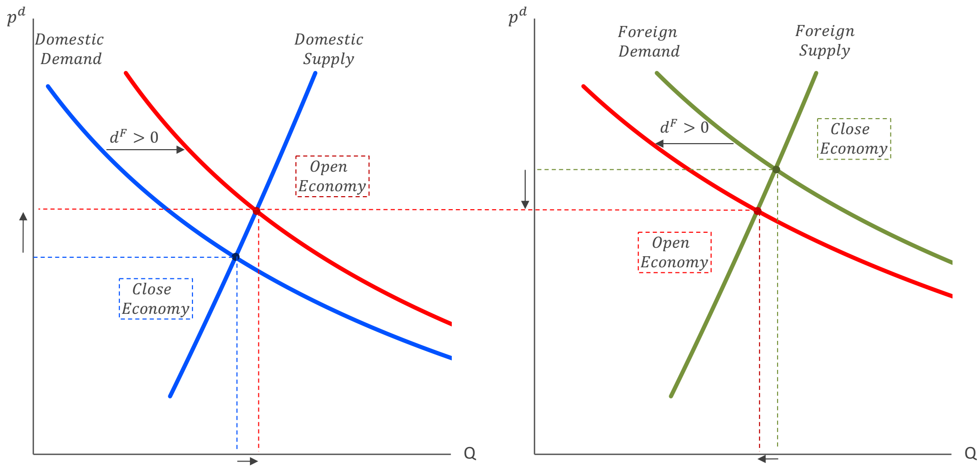
Higher $\hat{\psi}$ (demand shifter), shifts the foreign demand curve upwards, which implies that the foreign demand is relatively higher than the domestic. This causes a capital inflow to the domestic economy, balancing the demand across countries. Figure (3.5) clearly illustrates this point. However, given that both countries have the same supply schedule, not only the level of safe assets, but also the size-quality mix is the same in both countries.

In recent years there has been a structural increase in the global demand for safety (Bernanke et al. (2011)). A growing debate centres on the potential consequences for the country supplying safe assets to the rest of the world. The following proposition summarizes the prediction of the model.

Proposition 25 *When* $\hat{\psi} < \hat{\psi}'$, *then in* $d^+ < d^{+'}$, *so that* $d^f < d^{f'}$. *Since* $p^d < p^{d'}$ *then* $\pi_h(e^+)l^+ = \pi_h(e^*)l^* < \pi_h(e^{+'})l^{+'}\pi_h(e^{*'})l^{*'}$ *where* $l^+ = l^* < l^{+'} = l^{*'}$ *and* $e^+ = e^* < e^{+'} = e^{*'}$.

The model predicts that an increase in the foreign demand creates a credit

Figure 3.5: Relatively high foreign safety demand



Notes: This graph illustrates capital flows when foreign demand is relatively higher, i.e., $\hat{\psi} > \psi$. Domestic and foreign supply schedule are the same.

boom of lower quality which has pernicious effect on the real activity in both countries, i.e., aggregate output drops. However, the model further suggests that there is nothing intrinsically different about domestic and foreign demand; both countries respond equally to an increase in the global demand. Hence, it is the global demand level what matters in this aspect. Refer to the figure (3.7) in the appendix for a visual illustration of the comparative statics.

Financial development

A smaller ϕ means that financial intermediaries have a better technology to monitor households, which ameliorates the moral hazard problem inherent in the model. Hence, this parameter shifts the supply of safe assets upwards: for a given p^d the size of loans is higher while effort reminds the same (the positive direct effect of ϕ on effort and the negative indirect effect of higher loan size perfectly offset each other). Hence, $\phi < \hat{\phi}$ implies the domestic economy has relatively higher supply, attracting foreign capital.

A higher financial development could be the reason why most capital flows in search for safety are concentrated in a few countries. A growing literature claims that the observed large and persistent global capital flows can be the outcome of financial integration when countries have heterogeneous financial development. [3]

[3] For instance, Caballero et al. (2008), highlight the central role played by the heterogeneity

The following proposition summarizes the effects of an improvement in domestic financial development.

Proposition 26 *When $\phi > \phi'$, then in the domestic economy $\pi_h(e^*)l^* < \pi_h(e^{*'})l^{*'}$ where $l^* < l^{*'}$ and $e^* < e^{*'}$, and $d^f < d^{f'}$ which causes in the foreign economy $\pi_h(e^+)l^+ > \pi_h(e^{+'})l^{+'}$ where $l^+ > l^{+'}$ and $e^+ < e^{+'}$. Since $p^d > p^{d'}$, $d^* = d^+ < d^{*'} = d^{+'}$*

When ϕ declines, the domestic private supply increases, and that increase is met with higher loan sizes and higher effort, the latter enhancing economic prospects.There is a capital inflow to the domestic economy due to the relatively higher supply, which is match with better economic prospects. Refer to the graph (3.8) in the appendix for a visual illustration.

Putting both pieces together, if the higher demand boosts the incentives of financial institutions to innovate, ameliorating incentives problems in financial markets, the effects on economic activity would be ambiguous depending on which of the two channels dominates.

Public supply

Higher domestic public supply shifts upwards the domestic supply curve. However, the horizontal displacement is lower than the increase in the public supply. The reason is that public supply (partially) crowds out private supply, even when disregarding the general equilibrium effects: a higher government debt implies that taxes increase to meet the new debt obligations, which has a negative effect on effort as it reduces the skin-in-the-game hold by households. The lower effort, reduces the marginal benefit of borrowing, thus, reducing the size of the loan contracted – the first effect on effort dominates to the second effect. Yet, when assumption (2) hold, the net effect on total supply is positive, consequently, the country receives a capital inflow.

The empirical evidence suggest that the financial sector is strongly correlated with global demand for safety in the period preceding to the global financial crisis. To understand if it would have been better a public response, the following

in countries' ability to produce financial assets for global savers, while in Mendoza et al. (2009) "financial development" is captured by the extent to which financial contracts are enforceable.

proposition summarizes the effects of an increase in the public supply.

Proposition 27 *If assumption (2 holds), when* $b < b'$*, then in the domestic economy* $\pi_h(e^*)l^* > \pi_h(e^{*'})l^{*'}$ *where* $l^* > l^{*'}$ *and* $e^* > e^{*'}$*, and* $d^f < d^{f'}$ *which causes in the foreign economy* $\pi_h(e^+)l^+ > \pi_h(e^{+'})l^{+'}$ *where* $l^+ > l^{+'}$ *and* $e^+ < e^{+'}$*. Since* $p^d > p^{d'}$*,* $d^* = d^+ < d^{*'} = d^{+'}$*.*

The general equilibrium effects exacerbate the crowing out effect. The reduction in the private supply is led by a lower size and quality of loans. In line with the empirical evidence, the model predicts that public and private supply are substitutes (Krishnamurthy and Vissing-Jorgensen (2015), Kacperczyk et al. (2021)). While substitutes, the question is which one of the two is less harmful for the real activity. The model predicts that public supply deteriorates households' incentives to exert effort and exacerbates the impact of safe assets on the real economy. The private sector has a better technology to deal with the moral hazard problem, ameliorating the cost of risk-sharing on effort, thus, making more efficient the production of safe assets. Absent any other friction, any country is better off when safe assets are privatively produced.

The model assumes, however, that both sources of supply are equivalent to households. This seems at odds with the main premise in the literature where public supply is the quintessential source of supply. This could be explained by the fact that taxes are senior to any private claim, thus, government has a priority in collecting resources. The latter might be relevant when there is a shock that is not anticipated, which is not captured in this model with rational and perfect foresighted agents.

Future output

Higher future output shifts upwards the supply curve. Better economic prospects favour the production of safe assets, by increasing the marginal benefit of effort. Higher effort, in turn, increase the marginal benefit of borrowing, increasing the size of the loan which partially offset the initial increase in effort. Thus, the increase in the supply of safe assets is supported by both, higher loan size and quality. The relatively higher supply of safety attracts foreign demand to restore equilibrium in the safe asset market.

Notice, however, that in this setting, better economic prospects do not affect the demand. We abstract from the cyclical component of the safety demand, to focus on the structural determinants. The following proposition summarizes the effect of an increase in future output.

Proposition 28 *when* $y_{1h} < y'_{1h}$*, then in the domestic economy* $\pi_h(e^*)l^* < \pi_h(e^{*'})l^{*'}$ *where* $l^* < l^{*'}$ *and* $e^* < e^{*'}$*, and* $d^f < d^{f'}$ *which causes in the foreign economy* $\pi_h(e^+)l^+ > \pi_h(e^{+'})l^{+'}$ *where* $l^+ > l^{+'}$ *and* $e^+ < e^{+'}$*. Since* $p^d > p^{d'}$*,* $d^* = d^+ < d^{*'} = d^{+'}$*.*

When y_{1h} increases, the private supply increases, creating a credit boom of better quality. The higher households' incentives to exert effort, increase their productivity and has a positive effect on output. There is a capital inflow to the domestic economy, which is accompanied with better economic prospects. Therefore, not all credit booms are of bath quality; the driving force of the credit boom matters in this aspect.

Therefore, the private supply decreases when economic prospects deteriorate. There are two channels at play that reinforce each other. On one hand, the weakened expected output de facto reduce the skin-in-the-game of households and their incentives to exert effort. In turn, the equilibrium loan size decreases as the moral hazard problem becomes more pronounced. Therefore, both the size and quality of loans drop, and so does the private supply. This is a novel channel that explains the contraction of the private supply during downturns, which is not related to a loss in their safety status. In addition, financial intermediaries' supply decreases as a response to an increase in the public supply; public supply partially crowds out private supply and private investment, thus, exacerbating the drop in the private supply.

3.4 Conclusion

The model predicts that an increase in the global demand for safety, has negative effects on productivity. It rationalises that the higher demand puts upward pressure on the price for safety, encouraging the private sector to produce safe assets. However, the increase in the private supply is met by bigger loans of lower quality. The negative trade-off between loan size and quality arises due to a moral hazard problem; the higher the borrowing the lower the skin-in-the-game, reducing households' incentives to increase their productivity through effort. Hence, more levered households reduce their effort negatively affecting output. The model further predicts that economies with better financial systems will absorb a bigger fraction of the global demand, explaining why capital flows that search for safety are concentrated in a few countries with more developed financial sectors. The model suggests that public and private supply are substitutes. The questions is which one of the two is less harmful for the real activity. In this aspect, the private sector has a better technology to deal with the moral hazard problem inherent in the model. But in this respect, the model might lack important ingredients for this assessment.

Mathematical appendix

The optimality condition for effort is given by

$$\bar{\pi}\left[\tilde{y}_{1h} - \phi\left(l_h - l_l\right)\right] - \gamma e^* = 0 \tag{3.9}$$

Hence, the effort expected by the market is $\hat{e} = e^*\left(\{l_s\}\right)$, and households internalize the effect of l_s on $\hat{e}$. Then, the Lagrangian of the problem is given by

$$\begin{aligned} \mathcal{L} &= \tilde{y}_0 + p^d \mathbb{E}_{\hat{e}}\Big[l_s\Big] - p^d d + \mathbb{E}_e\Big[\tilde{y}_{1s} - l_s + d\Big] + \psi v\left(S\right) - c(e) \\ &- \lambda\Big[p^d d - p^d \mathbb{E}_{\hat{e}}\Big[l_s\Big] - \tilde{y}_0\Big] - \eta_h\Big[S - \tilde{y}_{1h} + l_h - d\Big] - \eta_l\Big[S + l_l - d\Big] \end{aligned}$$

The first-order conditions are given by

$$\begin{aligned} S &: & \psi v'(S) - \eta_h - \eta_l = 0 \\ d &: & -p^d + 1 - \lambda p^d + \eta_h + \eta_l = 0 \\ l_s &: & p^d \pi_s\left(\hat{e}\right) + p^d \bar{\pi}\left(l_h - l_l\right)\frac{\partial \hat{e}}{\partial l_s} - \pi_s + \lambda\left(p^d \pi_s\left(\hat{e}\right) + p^d \bar{\pi}\left(l_h - l_l\right)\frac{\partial \hat{e}}{\partial l_s}\right) - \eta_s = 0 \end{aligned}$$

where $\frac{\partial \hat{e}}{\partial l_h} = -\frac{\bar{\pi}\phi}{\gamma} < 0$ and $\frac{\partial \hat{e}}{\partial l_l} = \frac{\bar{\pi}\phi}{\gamma} > 0$. Notice that there is a multiplicity of equilibrium, and that all optimality conditions and constraints can be re-written in terms of $l^* \equiv l_h - l_l$ and $d^* \equiv d - l_l$. So lets find the optimum of the latter variables. In addition, in equilibrium $p^d > 1$ (due to $v'(0) = \infty$), so let's focus the partial equilibrium analysis in this range:

CASE I: If $1 < p^d < \bar{p}^d$ then $\lambda = 0$ so that $c_0 > 0$ and the demand for safe cash flows is given by

$$\psi v'(S^*) = p^d - 1 \quad \text{where} \quad S^* = \min_s\left\{\tilde{y}_{1s} - l_s + d\right\} \tag{3.10}$$

And the optimal l^*:

(I.a) If $\eta_h > 0, \eta_l = 0 \iff S^* = c_h < c_l \iff 0 < \tilde{y}_{1h} < l^*$, so that

$$l_l : \left(p^d - 1\right)\left(1 - \pi_h(e)\right) + \frac{p^d \bar{\pi}_h^2 \phi l^*}{\gamma} > 0 \implies \eta_l > 0 \quad \text{Contradiction!}$$

(I.b) If $\eta_h = 0, \eta_l > 0 \iff c_h > c_l = S^* \iff 0 \gtreqless l^* < \tilde{y}_{1h}$, so that

$$l_h : \left(p^d - 1\right) \pi_h(e) - \frac{p^d \bar{\pi}^2 \phi l^*}{\gamma} \;\left|\; \begin{matrix} > 0 \\ = 0 \end{matrix} \;\right|\; \begin{matrix} \text{if} \quad l^* \leq 0 \quad \Longrightarrow \eta_h > 0 \quad \text{Contradiction!} \\ \text{if} \quad l^* > 0 \quad \text{This is possible!} \end{matrix} \tag{3.11}$$

This case holds as long as

$$l_h : \left(p^d - 1\right) \pi_h(\underline{e}) - \frac{p^d \bar{\pi}^2 \phi \tilde{y}_{1h}}{\gamma} < 0 \tag{3.12}$$

where $\underline{e}$ is determined in equation (3.9) when $l^* = \tilde{y}_{1h}$. Otherwise, case (**I.c**) holds:

(I.c) If $\eta_h > 0, \eta_l > 0 \iff S^* = c_h = c_l \iff 0 < l^* = \tilde{y}_{1h}$.

Then, $c_h \geq c_l$, which implies that $0 < l^* \leq \tilde{y}_{1h} - \tilde{y}_{1l}$. When conditions (3.12) hold, the solution is interior, and l^* is determined in equation (3.11) (when it holds with equality), d^* is determined in equation (3.10), and effort is determined in equation (3.9). Otherwise, we have a corner solution in which there is full risk-sharing/diversification, so that $l^* = \tilde{y}_{1h}$ and effort equals zero.

CASE II: If $p^d \geq \bar{p}^d$ then $\lambda = \frac{1+\psi v'(S) - p^d}{p^d} > 0$ so that $c_0 = 0$ and

$$d^* = \frac{\tilde{y}_0 + p^d \pi_h(e) l^*}{p^d} \tag{3.13}$$

And the optimal l^*:

(II.a) If $\eta_h > 0, \eta_l = 0 \iff S^* = c_h < c_l \iff 0 < \tilde{y}_{1h} < l^*$, so that

$$l_l : \psi v'\left(c_h\right)\left(1 - \pi_h(e)\right) + \frac{\left(\psi v'\left(c_h\right)\right) \bar{\pi}_h^2 \phi l^*}{\gamma} > 0 \Longrightarrow \eta_l > 0 \quad \text{Contradiction!}$$

(II.b) If $\eta_h = 0, \eta_l > 0 \iff c_h > c_l = S^* \iff 0 \gtreqless l^* < \tilde{y}_{1h}$, so that

$$l_h : \psi v'\left(d^*\right) \pi_h(e) - \frac{\left(1 + \psi v'\left(d^*\right)\right) \bar{\pi}^2 \phi l^*}{\gamma} \;\left|\; \begin{matrix} > 0 \\ = 0 \end{matrix} \;\right|\; \begin{matrix} \text{if} \quad l^* \leq 0 \quad \Longrightarrow \eta_h > 0 \\ \text{if} \quad l^* > 0 \quad \text{This is possible!} \end{matrix} \tag{3.14}$$

This case holds as long as

$$l_h : \psi v' \left(\frac{\tilde{y}_0}{p^d} + \pi_h(\underline{e})\tilde{y}_{1h} \right) \pi_h(\underline{e}) - \frac{\left(1 + \psi v' \left(\frac{\tilde{y}_0}{p^d} + \pi_h(\underline{e})\tilde{y}_{1h} \right)\right) \bar{\pi}^2 \phi \tilde{y}_{1h}}{\gamma} < 0 \tag{3.15}$$

where $\underline{e}$ is determined in equation (3.9) when $l^* = \tilde{y}_{1h}$. Otherwise, case (**II.c**) holds:

(II.c) If $\eta_h > 0, \eta_l > 0 \iff S^* = c_h = c_l \iff 0 < l^* = \tilde{y}_{1h}$.

Then, $c_h \geq c_l$, which implies that $0 < l^* \leq \tilde{y}_{1h}$. When conditions (3.15) hold, the solution is interior, and l^* is determined in equation (3.14) (when it holds with equality), d^* is determined in equation (3.13), and effort is determined in equation (3.9). Otherwise, we have a corner solution in which there is perfect insurance so that $l^* = \tilde{y}_{1h}$ and effort equals zero. Hence, $\bar{p}^d$ is implicitly determined in $1 + \psi v' \left(\frac{\tilde{y}_0}{\bar{p}^d} + \pi_h(e) l^* \right) - \bar{p}^d = 0$. We will assume now on that the parameters are such that the solution is interior such that Case I.b holds. Then,

$$H^e(e^*, l^*; \mathbb{P}) \equiv \bar{\pi} \left[(1 - \tau(e)) y_{1h} - \phi l^* \right] - \gamma e^* = 0 \implies e^* (l; \mathbb{P})$$

$$H^l(l^*, p^d; \mathbb{P}) \equiv \left(p^d - 1\right) \pi_h(e^*) - \frac{p^d \bar{\pi}^2 \phi l^*}{\gamma} = 0 \implies l^* \left(p^d; \mathbb{P}\right)$$

$$p^d = 1 + \psi v'(d^*) \implies d^* \left(p^d; \mathbb{P}\right)$$

$$\tau(e) = \frac{b}{\pi_h(e) y_{1h}}$$

The **demand curve**, $d^*(p^d)$, is downward sloping since

$$\frac{\partial d^*}{\partial p^d} = \frac{-1}{(-\psi v''(d^*))} < 0$$

- Demand shifter $\psi \rightarrow$ if $\psi \uparrow \rightarrow d^*(.) \uparrow \forall p^d$

The **private supply** curve:

$$\frac{\partial e^*}{\partial l^*} = \frac{-\phi\bar{\pi}}{\gamma - \bar{\pi}\left(-\tau'(e)\right)y_{1h}} < 0 \text{ where } \tau'(e) = -\frac{\bar{\pi}\tau(e)}{\pi_h(e)} < 0$$

$$\text{since} \quad \gamma > \frac{\bar{\pi}^2\tau(e)y_{1h}}{\pi_h(e)}$$

$$\frac{\partial l^*}{\partial p^d} = \frac{\pi_h(e) - \frac{\bar{\pi}^2 l}{\gamma} \overset{FOC}{>} 0}{\frac{p^d\bar{\pi}^2\phi}{\gamma} + (p^d-1)\bar{\pi}\left(-\frac{\partial e}{\partial l}\right) > 0}$$

$$\frac{\partial \pi_h(e) l}{\partial p^d} = \left(\frac{\partial \pi_h(e) l}{\partial l} + \frac{\partial \pi_h(e) l}{\partial e}\frac{\partial e}{\partial l}\right)\frac{\partial l}{\partial p^d} = \left(\pi_h(e) - \frac{\phi\bar{\pi}^2 l}{\gamma + \bar{\pi}\tau'(e)y_{1h}}\right)\frac{dl}{dp^d} > 0$$

When assumption (2) holds, then $\gamma - \frac{\bar{\pi}^2}{\pi_h(e)}\tau(e)y_{1h} > \bar{\pi}y_{1h}\left(1 - \frac{\bar{\pi}}{\pi_h(e)}\tau(e)\right) > 0$ and the supply curve is upward sloping.

- Private supply shifter $\phi \rightarrow$ if $\phi \uparrow \rightarrow$ private supply $\downarrow$ ($l \downarrow$ and $e =$)

$$\frac{\partial l}{\partial \phi} = \frac{H^l_\phi + H^l_e\frac{\partial e}{\partial \phi}}{-\left(H^l_l + H^l_e\frac{\partial e}{\partial l}\right) > 0} = -\frac{l}{\phi}\left(\frac{\frac{p^d\bar{\pi}^2_h}{\gamma} + (p^d-1)\frac{\bar{\pi}^2}{\gamma - \bar{\pi}(-\tau'(e))y_{1h}}}{\frac{p^d\bar{\pi}^2}{\gamma} + (p^d-1)\frac{\bar{\pi}^2}{\gamma - \bar{\pi}(-\tau'(e))y_{1h}}}\right) = -\frac{l}{\phi} < 0$$

$$\frac{de}{d\phi} = -\frac{\bar{\pi}l + \bar{\pi}\phi\frac{dl}{d\phi}}{\gamma - \bar{\pi}\left(-\tau'(e)\right)y_{1h}} = 0$$

- If public supply $b \uparrow \rightarrow \tau(e) \uparrow \forall e \rightarrow$ private supply $\downarrow$ ($l \downarrow$ and $e \downarrow$)

$$\frac{\partial l}{\partial \tau} = \frac{H^l_e\frac{\partial e}{\partial \tau_1}}{-\left(H^l_l + H^l_e\frac{\partial e}{\partial l}\right)} = -\frac{y_{1h}}{\phi}\underbrace{\left(\frac{(p^d-1)\frac{\bar{\pi}^2}{\gamma - \bar{\pi}(-\tau'(e))y_{1h}}}{\frac{p^d\bar{\pi}^2}{\gamma} + (p^d-1)\frac{\bar{\pi}^2}{\gamma - \bar{\pi}(-\tau'(e))y_{1h}}}\right)}_{\equiv \eta < 1} < 0$$

$$\frac{de}{d\tau} = \frac{-\bar{\pi}y_{1h} - \bar{\pi}\phi\frac{dl}{d\tau}}{\gamma - \bar{\pi}\left(-\tau'(e)\right)y_{1h}} = \frac{-\bar{\pi}y_{1h}\left(1-\eta\right)}{\gamma - \bar{\pi}\left(-\tau'(e)\right)y_{1h}} < 0$$

$$\frac{d\pi_h(e)l}{d\tau} = -\pi_h(e)y_{1h}\underbrace{\left(\eta\frac{1}{\phi} + (1-\eta)\frac{\bar{\pi}}{\pi_h(e)}\frac{\bar{\pi}l}{\gamma - \bar{\pi}\left(-\tau'(e)\right)y_{1h}}\right)}_{\equiv \xi}$$

$$\frac{d\left(b + \pi_h(e)l\right)}{db} = 1 - \xi$$

After some algebra $\xi = \frac{2\bar{\pi}l}{\bar{\pi}\phi l + \frac{\pi_h(e)}{\bar{\pi}}(\gamma - \bar{\pi}(-\tau')y_{1h})}$. A sufficient condition for the latter to be lower than one is $l_h < 0.5(1-\tau)y_{1h}$ or that $e < 0.5$ and $\phi > 0.5$ both of them realistic assumptions.

- Economic conditions y_{1h}

$$\frac{\partial l}{\partial y_{1h}} = \frac{(p^d-1)\bar{\pi}\frac{\bar{\pi}(1-\tau)}{\gamma-\bar{\pi}(-\tau'(e))y_{1h}}}{\frac{p^d\bar{\pi}^2\phi}{\gamma}+(p^d-1)\frac{\bar{\pi}^2\phi}{\gamma-\bar{\pi}(-\tau'(e))y_{1h}}} = \frac{\bar{\pi}(1-\tau)}{\bar{\pi}\phi}\underbrace{\left(\frac{(p^d-1)\frac{\bar{\pi}^2\phi}{\gamma-\bar{\pi}(-\tau'(e))y_{1h}}}{\frac{p^d\bar{\pi}^2\phi}{\gamma}+(p^d-1)\frac{\bar{\pi}^2\phi}{\gamma-\bar{\pi}(-\tau'(e))y_{1h}}}\right)}_{\equiv\rho<1} > 0$$

$$\frac{de}{dy_{1h}} = \frac{\bar{\pi}(1-\tau)(1-\rho)}{\gamma-\bar{\pi}\left(-\tau'(e)\right)y_{1h}} > 0$$

$$\frac{d\pi_h(e)l}{dy_{1h}} > 0$$

Equilibrium: open economy Having the supply and demand functions for each economy, lets characterize the equilibrium price for safe assets and the equilibrium capital flows between both economies. Lets denote the parameter values for the foreign economy with a hat, and the optimums with a plus instead of a star.

There is a unique market where demand and supply from both countries determine the equilibrium price for safe assets. The market clearing conditions:

$$\underbrace{d^*}_{\text{dom. demand}} + \underbrace{d^f}_{\text{capital inflow}} = \underbrace{\tau(e^*)\mathbb{E}_{e^*}\left[y_{1s}\right]}_{\text{public dom. supply}} + \underbrace{\pi_h(e^*)l^*}_{\text{private dom. supply}}$$

$$\underbrace{d^+}_{\text{for. demand}} - \underbrace{d^f}_{\text{capital outflow}} = \underbrace{\hat{\tau}(e^+)\mathbb{E}_{e^+}\left[\hat{y}_{1s}\right]}_{\text{public for. supply}} + \underbrace{\hat{\pi}_h(e^+)l^+}_{\text{private for. supply}}$$

The direction of capital flows depend on the following parameter values:

- if $\psi < \hat{\psi} \implies l^* = l^+$ and $e^* = e^+ \implies \pi_h(e^*)l^* = \hat{\pi}_h(e^+)l^+$, but $d^* < d^+ \implies d^f > 0$
- if $\phi < \hat{\phi} \implies d^* = d^+$, but $l^* > l^+$ and $e^* = e^+ \implies \pi_h(e^*)l^* > \hat{\pi}_h(e^+)l^+ \implies d^f > 0$
- if $b > \hat{b} \implies d^* = d^+$, but $l^* < l^+$ and $e^* < e^+$, however, $b + \pi_h(e^*)l^* > \hat{b} + \hat{\pi}_h(e^+)l^+ \implies d^f > 0$
- if $y_{1h} > \hat{y}_{1h} \rightarrow d^* = d^+$, but $l^* > l^+$ and $e^* > e^+ \implies \pi_h(e^*)l^* > \hat{\pi}_h(e^+)l^+ \implies d^f > 0$

Comparative statics

Demand shock. When global demand increases, both economies move upwards along the supply curve. As already explained, the corresponding increase in the supply is met with bigger loan sizes of lower quality, the latter having a negative effect on output. Whether the shock is in domestic or foreign demand, only affects to capital flows, but the effects are the same as both countries respond equally to any demand shock. The figure 3.7 properly illustrates this point.

When $\hat{\psi} < \hat{\psi}'$, then in the domestic economy $d^+ < d^{+'}$, so that $d^f < d^{f'}$. Since $p^d < p^{d'}$ then $\pi_h(e^+)l^+ = \pi_h(e^*)l^* < \pi_h(e^{+'})l^{+'}\pi_h(e^{*'})l^{*'}$ where $l^+ = l^* < l^{+'} = l^{*'}$ and $e^+ = e^* < e^{+'} = e^{*'}$.

Shock to financial development. A decrease in ϕ constitutes an improvement in the financial development of a country, consequently, shifts in the supply in the aforementioned manner (higher loan size and same effort for each p^d). Hence, there is a capital inflow in the domestic economy as illustrated in figure (3.8). Graphically, it is clear that in the new equilibrium effort increases, yet, the effect on the loan size is not obvious. If we proof that when ϕ decreases the loan size is higher in the close economy (point (B)), then so will be in the open economy (point (C)). Then $\frac{dl}{d\phi}$ equals

$$-\frac{l}{\phi}\underbrace{\left(\frac{\frac{p^d\bar{\pi}^2}{\gamma}+\left((p^d-1)\bar{\pi}-\left(\pi_h(e)-\frac{\bar{\pi}^2\phi l}{\gamma}\right)\psi\left(-v''(d)\right)\bar{\pi}l\right)\frac{\bar{\pi}}{\gamma-\bar{\pi}(-\tau'(e))y_{1h}}}{\frac{p^d\bar{\pi}^2}{\gamma}+\left((p^d-1)\bar{\pi}-\left(\pi_h(e)-\frac{\bar{\pi}^2\phi l}{\gamma}\right)\psi\left(-v''(d)\right)\bar{\pi}l\right)\frac{\bar{\pi}}{\gamma-\bar{\pi}(-\tau'(e))y_{1h}}+\left(\pi_h(e)-\frac{\bar{\pi}^2\phi l}{\gamma}\right)\psi\left(-v''(d)\right)\frac{\pi_h(e)}{\phi}}\right)}_{<1}<0$$

assuming $v'(x) + v''(x)x > 0$ is a sufficient condition to ensure that the derivative reminds negative. Therefore, in the close economy, when ϕ increase (decreases) then, the size of the loan decreases (increases). So we can conclude that in point (C) the loan size is higher.

When $\hat{\psi} < \hat{\psi}'$, then in the domestic economy $d^+ < d^{+'}$, so that $d^f < d^{f'}$. Since $p^d < p^{d'}$ then $\pi_h(e^+)l^+ = \pi_h(e^*)l^* < \pi_h(e^{+'})l^{+'}\pi_h(e^{*'})l^{*'}$ where $l^+ = l^* < l^{+'} = l^{*'}$ and $e^+ = e^* < e^{+'} = e^{*'}$.

Public supply We have already proven that an increase in the public supply crowds out the private supply, however, the crowd out is partial. Hence, netting out both effect the supply increases, and the supply curve shifts in the in the aforementioned manner (lower loan size and lower effort for each p^d). Hence, there is a capital inflow in the domestic economy as illustrated in figure (3.9). Graphically, it is clear that in the new equilibrium loan size decrease, yet, the effect on effort is not obvious. If we proof that when b increase the effort is lower in the close economy (point (B)), then so will be in the open economy (point (C)). Then $\frac{dl}{d\tau_1}$ equals

$$-\frac{\bar{\pi} y_{1h}}{\bar{\pi}\phi} \underbrace{\left(\frac{\left(\pi_h(e) - \frac{\bar{\pi}^2 \phi l}{\gamma}\right) \psi\left(-v''(d)\right) \frac{\pi_h(e)}{\bar{\pi}} + \left((p^d - 1)\bar{\pi} - \left(\pi_h(e) - \frac{\bar{\pi}^2 \phi l}{\gamma}\right) \psi\left(-v''(d)\right) \bar{\pi} l\right) \frac{1}{\gamma - \bar{\pi}(-\tau'(e)) y_{1h}}}{\frac{p^d \bar{\pi}}{\gamma} + \left((p^d - 1)\bar{\pi} - \left(\pi_h(e) - \frac{\bar{\pi}^2 \phi l}{\gamma}\right) \psi\left(-v''(d)\right) \bar{\pi} l\right) \frac{1}{\gamma - \bar{\pi}(-\tau'(e)) y_{1h}} + \left(\pi_h(e) - \frac{\bar{\pi}^2 \phi l}{\gamma}\right) \psi\left(-v''(d)\right) \frac{\pi_h(e)}{\bar{\pi}\phi}} \right)}_{<1} < 0$$

assuming $v'(x) + v''(x)x > 0$ is a sufficient condition to ensure that the derivative reminds negative. Given that the second term is still lower than one, then effort is lower in point (B), and so is in point (C).

If assumption (2 holds), when $b < b'$, then in the domestic economy $\pi_h(e^*)l^* > \pi_h(e^{*'})l^{*'}$ where $l^* > l^{*'}$ and $e^* > e^{*'}$, and $d^f < d^{f'}$ which causes in the foreign economy $\pi_h(e^+)l^+ > \pi_h(e^{+'})l^{+'}$ where $l^+ > l^{+'}$ and $e^+ < e^{+'}$. Since $p^d > p^{d'}$, $d^* = d^+ < d^{*'} = d^{+'}$.

Future output An increase in future output shifts the supply curve in the in the aforementioned manner (higher loan size and higher effort for each p^d). Hence, there is a capital inflow in the domestic economy as illustrated in figure (3.10). Graphically, it is clear that in the new equilibrium effort increase, yet, the effect on loan size is not obvious. If we proof that when y_{1h} increase the effort is lower in the close economy (point (B)), then so will be in the open economy (point (C)). Then $\frac{dl}{dy_{1h}}$ equals

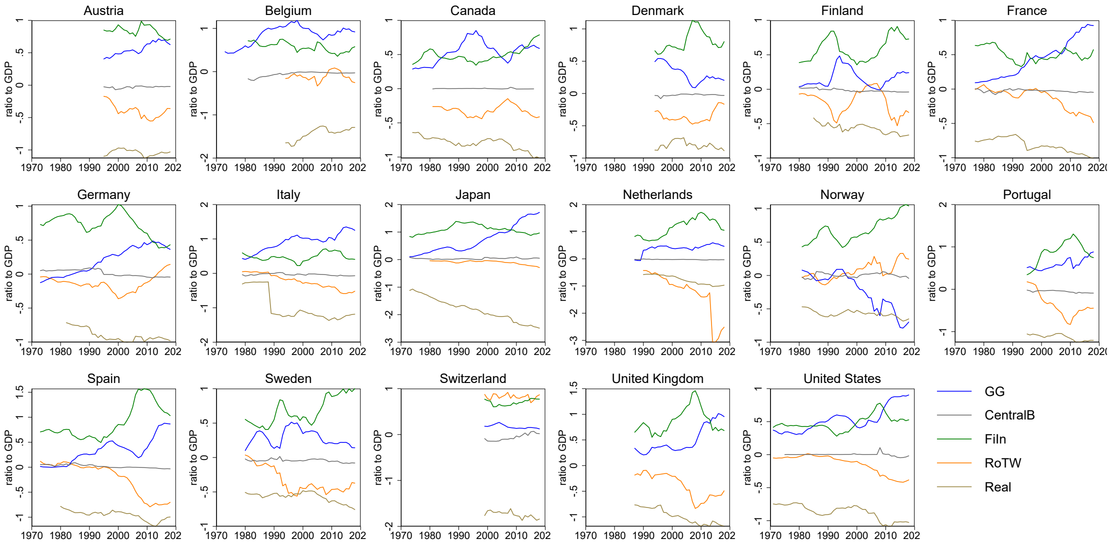
$$-\frac{\bar{\pi}(1 - \tau(e))}{\bar{\pi}\phi} \underbrace{\left(\frac{\left((p^d - 1)\bar{\pi} - \left(\pi_h(e) - \frac{\bar{\pi}^2 \phi l}{\gamma}\right) \psi\left(-v''(d)\right) \bar{\pi} l\right) \frac{1}{\gamma - \bar{\pi}(-\tau'(e)) y_{1h}}}{\frac{p^d \bar{\pi}}{\gamma} + \left((p^d - 1)\bar{\pi} - \left(\pi_h(e) - \frac{\bar{\pi}^2 \phi l}{\gamma}\right) \psi\left(-v''(d)\right) \bar{\pi} l\right) \frac{1}{\gamma - \bar{\pi}(-\tau'(e)) y_{1h}} + \left(\pi_h(e) - \frac{\bar{\pi}^2 \phi l}{\gamma}\right) \psi\left(-v''(d)\right) \frac{\pi_h(e)}{\bar{\pi}\phi}} \right)}_{<1} > 0$$

assuming $v'(d) + v''(d)\pi_h(e)l > 0$ is a sufficient condition to ensure that the derivative reminds negative. Thus, the loan size is higher in point (B), and so is in point (C).

When $y_{1h} < y'_{1h}$, then in the domestic economy $\pi_h(e^*)l^* < \pi_h(e^{*'})l^{*'}$ where $l^* < l^{*'}$ and $e^* < e^{*'}$, and $d^f < d^{f'}$ which causes in the foreign economy $\pi_h(e^+)l^+ > \pi_h(e^{+'})l^{+'}$ where $l^+ > l^{+'}$ and $e^+ < e^{+'}$. Since $p^d > p^{d'}$, $d^* = d^+ < d^{*'} = d^{+'}$.

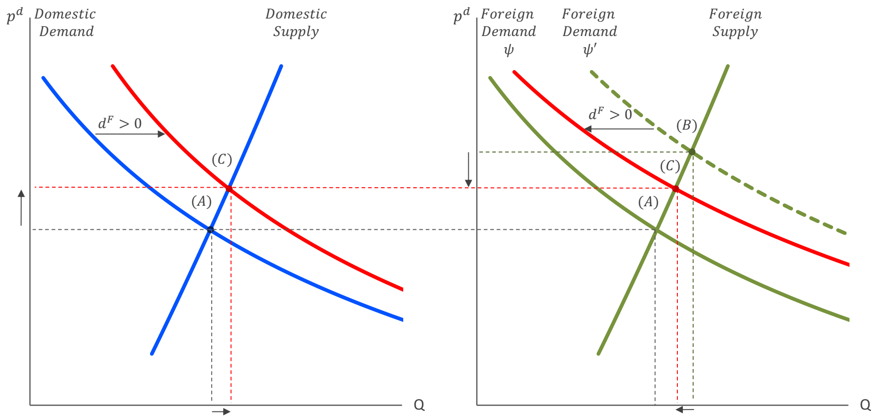
Figures

Figure 3.6: Net safe asset positions across sectors by country



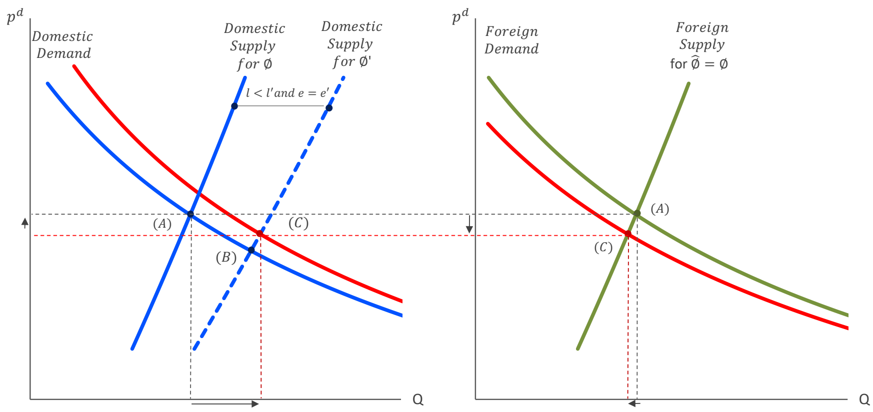
Notes: Net positions equal safe liabilities minus safe assets. The sectors are: general government, central banks, financial sector, foreign sector and the real sector (households and non-financial corporations).

Figure 3.7: Positive shock to foreign safety demand



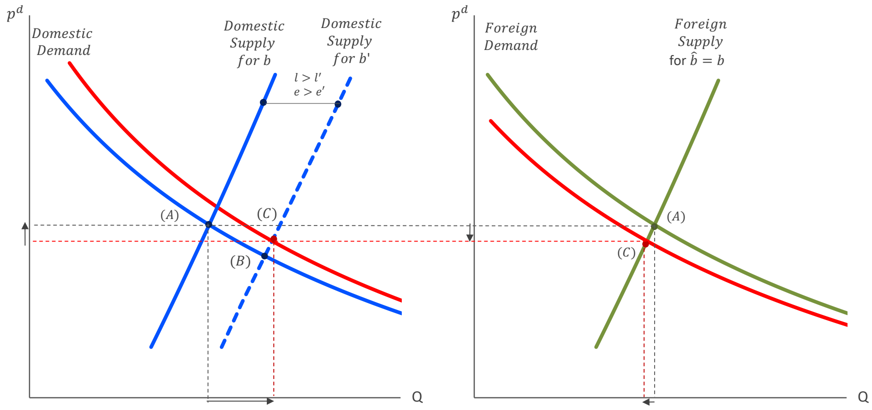
Note: This graph illustrates the comparative static of a positive shock to foreign demand, i.e., $\hat{\psi} < \hat{\psi}'$. The initial equilibrium is (A), where given that both countries are homogeneous the capital flow is zero. When the foreign economy face a positive shock in its demand the demand curve shifts, ceteris paribus, the economy jumps to (B). However, the price and capital flows adjust to restore equilibrium in the international setting at point (C).

Figure 3.8: Positive shock to domestic financial development



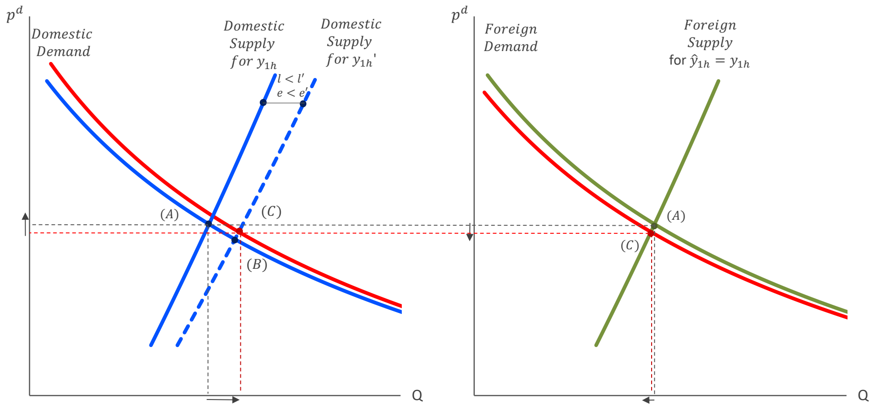
Note: This graph illustrates the comparative static of a positive shock to domestic financial development, i.e., $\phi > \phi'$. The initial equilibrium is (A), where given that both countries are homogeneous the capital flow is zero. When the domestic economy face a positive shock to the financial development, ceteris paribus, the economy jumps to (B). Notice that the for the same level of p^d, the new supply curve has the same effort level but higher loan size. Thus, from (A) to (B), taking into account how size-effort changes along the curve, we can conclude that effort is higher, and so is in (C) when equilibrium is restored after the capital flows and price have been adjusted. The ultimate loan size, compare to the initial level is ambiguous.

Figure 3.9: Positive shock to domestic public supply



Note: This graph illustrates the comparative static of a positive shock to domestic public supply, i.e., $b > b'$. The initial equilibrium is (A), where given that both countries are homogeneous the capital flow is zero. When the domestic economy face a positive shock to the public supply, ceteris paribus, the economy jumps to (B). Notice that for the same level of p^d, the new supply curve has the lower effort level and loan size. Thus, from (A) to (B), taking into account how size-effort changes along the curve, we can conclude that loan size is lower, and so is in (C) when equilibrium is restored after the capital flows and price have been adjusted. The ultimate effect on effort is ambiguous.

Figure 3.10: Positive shock to domestic future output



Note: This graph illustrates the comparative static of a positive shock to domestic future output, i.e., $y_{1h} > y'_{1h}$. The initial equilibrium is (A), where given that both countries are homogeneous the capital flow is zero. When the domestic economy face a positive shock to future output, ceteris paribus, the economy jumps to (B). Notice that the for the same level of p^d, the new supply curve has the higher effort level and higher loan size. Thus, from (A) to (B), taking into account how size-effort changes along the curve, we can conclude that effort is higher, and so is in (C) when equilibrium is restored after the capital flows and price have been adjusted. The ultimate loan size, compare to the initial level is ambiguous.

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