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Ph.D. Thesis Dissertation

Optimization and Generalization of Lifting Schemes: Application to Lossless Image Compression

Author: Joel Solé Rojals

Advisor: Prof. Philippe Salembier

Image Processing Group
Department of Signal Theory and Communications
Universitat Politècnica de Catalunya

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Als meus pares,

Abstract

This Ph.D. thesis dissertation addresses multi-resolution image decomposition, a key issue in signal processing that in recent years has contributed to the emergence of the JPEG2000 image compression standard. JPEG2000 incorporates many interesting features, mainly due to the discrete wavelet transform stage and to the EBCOT entropy coder.

Wavelet analysis perform multi-resolution decompositions that decorrelate signal and separate information in useful frequency-bands, allowing flexible post-coding. In JPEG2000, decomposition is computed through the lifting scheme, the so-called second generation wavelets. This fact has focused the community interest on this tool. Many works have been recently proposed in which lifting is modified, improved, or included in a complete image coding algorithm.

The Ph.D. thesis dissertation follows this research line. Lifting is analyzed, proposals are made within the scheme, and their possibilities are explored. Image compression is the main objective and it is principally assessed by means of coding transformed signal with EBCOT and SPIHT coders. Starting from this context, the work diverges in two distinct paths, the linear and the nonlinear one.

The linear lifting filter construction is based on the idea of quadratic interpolation and the underlying linear restriction due to the wavelet transform coefficients. The result is a flexible framework that allows the creation of new transforms using different criteria and that may adapt to the image statistics.

The nonlinear part is founded on the adaptive lifting scheme, which is extensively analyzed and as a consequence, a generalization of the lifting is proposed. The discrete version of the generalized lifting is developed leading to filters that achieve good compression results, specially for biomedical and remote sensing images.

Resumen

Esta tesis aborda el problema de la descomposición multi-resolución, tema clave en procesado del señal que ha llevado estos últimos años a la creación del sobresaliente estándar JPEG2000 de compresión de imágenes. JPEG2000 incorpora una serie de funcionalidades muy interesantes debido básicamente a la transformada wavelet discreta y al codificador entrópico EBCOT.

La transformada wavelet realiza una descomposición multi-resolución que decorrela la señal separando la información en un conjunto de bandas frecuenciales útiles para la posterior codificación. En JPEG2000, la descomposición se calcula mediante el esquema lifting, también llamado wavelet de segunda generación. La integración del esquema lifting en el estándar ha centrado el interés de muchos investigadores en esta herramienta. Recientemente, han aparecido numerosos trabajos proponiendo modificaciones y mejoras del lifting, así como su inclusión en nuevos algoritmos de codificación de imágenes.

La tesis doctoral sigue esta línea de investigación. Se estudia el lifting, se hacen propuestas dentro del esquema y sus posibilidades se exploran. Se ha fijado la compresión de imágenes como el principal objetivo para la creación de nuevas transformadas wavelet, que se evalúan en su mayor parte mediante la codificación de la señal transformada con EBCOT o SPIHT. Dentro de este contexto, el trabajo diverge en dos caminos distintos, el lineal y el no lineal.

La construcción de filtros lifting lineales se basa en la idea de interpolación cuadrática y la restricción lineal subyacente de los coeficientes wavelet. El resultado es un marco de trabajo flexible que permite la creación de transformadas con distintos criterios y adaptables a la estadística de la imagen.

La parte no lineal tiene sus fundamentos en el esquema lifting adaptativo, del cuál se ofrece un extenso análisis y como consecuencia se propone una generalización del lifting. Su versión discreta se desarrolla consiguiendo filtros lifting que obtienen buenos resultados, sobretodo en imágenes biomédicas y de detección remota.

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Notation

Boldface upper-case letters denote matrices, boldface lower-case letters denote column vectors, upper-case italics denote sets, and lower-case italics denote scalars.

$\mathbb{R}$, $\mathbb{Z}$	The set of real and integer numbers, respectively.
$\mathbb{R}_+$, $\mathbb{R}_{++}$	The set of non-negative real and positive real numbers, respectively.
$\mathbb{Z}_n$	A set of n consecutive integer numbers.
$\mathbb{R}^{n\times m}$	The set of $n \times m$ matrices with real-valued entries.
$\mathbf{X}^T$	Transpose of the matrix $\mathbf{X}$.
$\mathbf{X}^{-1}$	Inverse of the matrix $\mathbf{X}$.
$[\mathbf{X}]_{i,j}$	$(i,j)^{\text{th}}$ component of the matrix $\mathbf{X}$.
$\mathbf{I}_n$	Identity matrix of dimensions $n \times n$ (the dimension is not explicitly indicated if it is clear from the context).
$\mathbf{e}_i$	Canonical vector with all the elements being zeros except the i^{th} one which is equal to one.
$\mathbf{a} \geq \mathbf{b}$	Elementwise relation $a_i \geq b_i$.
$\lvert x \rvert$	Absolute value of the scalar x.
$\lVert \mathbf{x} \rVert$	Euclidean norm of the vector $\mathbf{x}$: $\lVert \mathbf{x} \rVert = \sqrt{\mathbf{x}^T\mathbf{x}}$.
arg	Argument.
max, min	Maximum and minimum.
$(\cdot)^\star$	Optimal value.
$\bigcap$, $\bigcup$	Intersection and union.
$[a,b]$, (a,b)	Closed interval ($a \leq x \leq b$) and open interval ($a < x < b$), respectively.

$\Pr(\cdot)$	Probability.
$\mathbb{E}[\cdot]$	Mathematical expectation.
$\lvert\mathcal{A}\rvert$	Cardinality of the set $\mathcal{A}$, i.e., number of elements in $\mathcal{A}$.
$L^2(\mathbb{R})$	The space of square integrable functions.
$l^2(\mathbb{Z})$	The space of square summable sequences.
$*$	Linear convolution.
$\propto$	Equal up to a scaling factor (proportional).
$\triangleq$	Defined as.
$\simeq$	Approximately equal.
$\cong$	Equivalent to.
$\nabla_{\mathbf{x}} f$	Gradient of the function f with respect to $\mathbf{x}$.
$P(\cdot)$	Prediction lifting step.
$U(\cdot)$	Update lifting step.
$\lceil \cdot \rfloor$	Rounding to nearest integer.
$Q(\cdot)$	Quantization.
$\exp(\cdot)$	Exponential.
$\log(\cdot)$	Natural logarithm.
$\log_a(\cdot)$	Base-a logarithm.
$\delta[\cdot]$	Kronecker delta.

Acronyms

1-D, 2-D, 3-D	One-dimensional, two-dimensional, and three-dimensional, respectively.
AR, AR-m	Auto-regressive and auto-regressive model of m^{th} order, respectively.
bpp	bits per pixel.
DWT	Discrete Wavelet Transform.
EBCOT	Embedded Block Coding with Optimized Truncation.
EZW	Embedded Zero-tree Wavelet coding.
FB	Filter Bank.
FIR	Finite Impulse Response.
GL	Generalized Lifting.
GLS	Generalized Lifting Step.
IEEE	Institute of Electrical and Electronics Engineers.
IIR	Infinite Impulse Response.
ISO	International Organization for Standardization.
ITU-T	International Telecommunications Union (Standardization Sector).
JPEG	Joint Photographic Experts Group (image standard).
JPEG-LS	JPEG Lossless image standard.
KKT	Karush-Kuhn-Tucker.
LC	Local Characteristics.
LHS	Left-Hand Side.
LMS	Least Mean Square.
LS	Lifting Scheme.
LSB	Least Significant Bit.
LUT	Look-Up Table.

LWT	Lazy Wavelet Transform.
MRA	Multi-Resolution Analysis.
MSB	Most Significant Bit.
MSE	Mean Square Error.
PLS	Prediction Lifting Step.
pdf	probability density function.
PR	Perfect Reconstruction.
PSNR	Peak Signal to Noise Ratio.
ROF	Rank-Order Filter.
RHS	Right-Hand Side.
SNR	Signal to Noise Ratio.
SPIHT	Set Partitioning in Hierarchical Trees.
SST	Sea Surface Temperature.
s.t.	subject to.
ULS	Update Lifting Step.
w.r.t.	with respect to.