Chapter 1

Introduction

1.1 Motivation

Wavelet multi-resolution decomposition of images has shown its efficiency in many image processing areas and specifically in compression. Transformed coefficients are obtained by expanding a signal on a wavelet basis. The transformed signal is a different representation of the same underlying data. Such representation is efficient if a relevant part of the original information is found in a relative small number of coefficients. In this sense, wavelets are near optimal bases for a wide class of signals with some smoothness, which is the reason of its interest for compression.

A new family of wavelet-based image coders emerged from the original work of J. Shapiro in 1993 when a way to profit from the two dimensional compact wavelet representation of images was shown. The wavelet-based image encoders improved compression performance relative to the previously existing JPEG standard, as well as having other nice features such as a completely embedded bit-stream representation.

However, one of the initial assumptions on which the wavelet-based image coders relies is not precisely fulfilled. Images do not belong to a class of functions which are optimally represented with a wavelet transform. Real images are inherently non-stationary. Most of images are not smooth. Even a first approximation image model has to include different regions separated by edges. Indeed, regions are not usually flat or constant. Patterns and textures exist and they have not the smoothness property that would point to wavelets as their optimal bases. In wavelet-based coders it is observed that textures and contours need the major part of bit-rate. Coding these singularities is costly. Therefore, a fixed wavelet decomposition is unable to efficiently represent the complexity of a real image.

Wavelet family is broad. The choice of a wavelet basis is conditioned by the application at hand or the given objective. In coding, some wavelets are more adequate for smooth regions and others behave better near discontinuities. Hence, many researchers have proposed adaptive schemes that modify the underlying wavelet basis according to local signal characteristics.

Filter banks are the fundamental tool to create discrete wavelet transforms. They are formed by the analysis and synthesis low- and high-pass filters and the intermediate stages composed by down- and up-samplings. Several subsignals are the output of the filter bank analysis part. Each subsignal comes from a different filter channel.

Initially, the complexity and challenge of adaptivity was to assure the filter bank reversibility in order to recover the original data from the subsignals. The design of a whole filter bank with such property is a difficult task. However, several works attained this goal. Later, the lifting scheme proposed by W. Sweldens gave a suitable framework for developing perfect reconstruction time-varying and nonlinear wavelet filters mainly because lifting structure itself assures reversibility and so, freedom in the design stage is greatly increased.

Many contributions use or modify the lifting scheme. These works try in different ways to exploit the correlation existing among the decomposition channels. For instance, the signal local shape or the statistics in one channel may be considered to obtain a good prediction/interpolation of another channel. This is known as space-varying lifting.

Adaptive and nonlinear decompositions take into account the non-stationarity of images and in this way they achieve a sparser description of images than classical wavelets. Going one step further, adaptation may be improved if the information given by the same channel to be filtered is considered. There exist methods for searching the best basis for the entire signal or for a class of signals, but this requires book-keeping and so additional bit-rate to attain reversibility. H. Heijmans and G. Piella showed that a point-wise adaptation is possible by using a transform-invariant criterion. In consequence, the analysis filters can be recovered at the decoder side allowing the correct synthesis filter choice without any book-keeping.

Following the latter line of research, this Ph.D. thesis dissertation sets out a generalization of the lifting scheme in which additions and substraction in classical lifting steps are embedded to include any kind of operation. The generalization opens a door to new decompositions and design criteria. Lifting prediction and update steps are designed within the new framework, which is essentially devoted to nonlinear processing.

The interest of the proposed scheme is demonstrated for lossless applications. There are many applications in which original image data has to be exactly recovered. For example, in biomedical imaging several legal and regulatory issues work in favor of lossless compression. Similarly, exact coders are required in remote sensing imaging in order to ensure accurate values of physical ground parameters.

The discrete version of the generalized lifting is developed in order to construct prediction and update lifting steps. The scheme performance is mainly assessed by coding transformed coefficients by means of wavelet-based image coders, but also by drawing some relevant statistics from the coefficients. For instance, the mean energy or the entropy are informative about the goodness of a transform for compression purposes.

Three dimensional version of the proposed transforms and the implemented 3-D extension of a known image entropy coder permits the test of the new schemes for the coding of volumes or video sequences. The 3-D version is practical also for biomedical and remote sensing imaging. For example, magnetic resonance imaging and multi-spectral imaging are applications in which the gathered data is highly correlated in each of the three dimensions and so, the extended schemes may attain excellent compression ratios.

This Ph.D. thesis dissertation provides contributions to the study of point-wise adaptive and nonlinear decompositions within the lifting scheme, trying to fill in the room opened in these topics. Moreover, there is place in the linear setting for new ideas in space-varying, signal-dependant, and adaptive lifting. The linear framework is studied and contributions are made. The convergence of adaptive quadratic interpolation and the theory of convex optimization with the filter bank field leads to the improvement of existing linear lifting steps and the construction of new ones.

The concrete issues and aspects addressed in this Ph.D. thesis dissertation concerning these general objectives and their distribution within the text are detailed in the next section.

1.2 Thesis Organization

The Ph.D. thesis dissertation is divided in six chapters. This introductory chapter is followed by an overview on the discrete wavelet transform and surrounding field, which introduces the main concepts employed in the dissertation. The next three chapters are devoted to the presentation of the thesis contributions. Final chapter concludes this work.

Outline of the thesis dissertation:

Chapter 2 presents an overview of the discrete wavelet transform, filter banks §2.1, and the lifting scheme §2.2. The usefulness of wavelet transform in image compression is established. Space-varying and the adaptive lifting strategy are two variants of lifting employed in the dissertation. They are introduced in §2.2.5 and §2.2.6, respectively. A state-of-the-art review regarding the design and optimization in the lifting scheme is found in §2.3. Wavelet-based image entropy coders are described in §2.5.

Chapter 3 is dedicated to the optimization of linear lifting filters. An adaptive quadratic interpolation method is described in §3.2 which combined with the theory of convex optimization allows the construction of interpolative prediction steps. Then, the method is employed for the improvement of lifting steps and the construction of new ones in §3.3. Experiments and results are described in §3.4.

Chapter 4 starts with the description of the adaptive lifting scheme §4.1. Then, an extended analysis of this adaptive scheme is provided §4.2. Section 4.3 includes the proposal of two steps within this framework. The analysis also leads to the definition of the generalized lifting and its discrete version in §4.4.

Chapter 5 proposes the construction of concrete discrete generalized lifting steps. A geometrical approach to the design of a prediction step is described in §5.1.1. The rest of the section §5.1 is devoted to the optimization of generalized prediction steps, while section §5.2 optimizes generalized update steps. Experimental results including the 3-D version are also detailed.

Chapter 6 draws the main conclusions from the Ph.D. thesis dissertation and details possible future lines of research.

1.3 Research Contributions

The main contribution of this Ph.D. thesis dissertation is the analysis and development of adaptive wavelet decompositions within the lifting scheme and the creation of a linear framework for the development of new lifting steps. The details of the research contribution and publications in each chapter are as follows.

The main results in chapter 3 concern the design of linear lifting steps. A common formulation for quadratic interpolation and lifting design is presented. The connection permits the construction of a variety of new lifting steps, as well as a study of the optimality of known filters according to different criteria. Some of the results appear in the following conference papers:

- [Sol06a] J. Solé and P. Salembier, “A common formulation for interpolation, prediction, and update lifting design”, in *Proceedings of International Conference on Acoustics, Speech, and Signal Processing*, Vol. 2, pags. 13-16, May 2006.
- [Sol06b] J. Solé and P. Salembier, “Adaptive quadratic image interpolation methods”, *accepted to Research in AVR Barcelona*, July 2006.
- [Sol06c] J. Solé and P. Salembier, “Adaptive quadratic interpolation methods for lifting steps construction”, *accepted to the IEEE International Symposium on Signal Processing and Information Technology*, August 2006.

Chapter 4 characterizes adaptive lifting scheme from a new point of view, leading in a natural way to the proposal of two adaptive lifting constructions and the generalized lifting scheme and its discrete version. The generalized scheme description in chapter 4 and the part of chapter 5 regarding the geometrical approach to the discrete generalized prediction design §5.1.1 are presented in two papers:

- [Sol04a] J. Solé and P. Salembier, "Adaptive discrete generalized lifting for lossless compression", in *Proceedings of International Conference on Acoustics, Speech, and Signal Processing*, Vol. 3, pags. 57-60, May 2004.
- [Sol04b] J. Solé and P. Salembier, "Discrete generalized lifting for lossless image compression", in *Proceedings of Research in AVR*, pags. 337-340, February 2004.

Chapter 5 develops the discrete generalized scheme proposing the optimization of several prediction and update steps within the framework. An optimized generalized prediction and its space-varying version have been published in two conference papers:

- [Sol04c] J. Solé and P. Salembier, "Prediction design for discrete generalized lifting", in *Proceedings of Advanced Concepts for Intelligent Vision Systems*, pags. 319-324, September 2004.
- [Sol05] J. Solé and P. Salembier, "Adaptive generalized prediction for lifting schemes", in *Proceedings of International Conference on Acoustics, Speech, and Signal Processing*, Vol. 2, pags. 205-208, March 2005.