Chapter 6

Conclusions and Future Work

This Ph.D. dissertation has considered lifting scheme design for image compression applications. Two design framework have been proposed and analyzed, namely the interpolative and projection-based linear lifting setting and the adaptive and generalized nonlinear setting.

6.1 Conclusions

Chapter 3 is devoted to explain an interpolation framework and its modification for the development of new linear lifting steps, including a variety of space-varying steps. An optimality analysis and results are derived from the proposed designs. The first part of this section reviews chapter 3 and draws the main conclusions.

A quadratic interpolation method is presented. The algorithm is able to interpolate by any factor and to model properties of the data acquisition system. The quadratic model may be determined from the local image data, from an image model, or from a combination of both. The proposed formulation of the quadratic interpolation as a convex optimization problem provides the flexibility to introduce knowledge in different ways and to derive the corresponding optimal solutions. Some of the additional considered information is the number of bits representing the pixels, the weighting factors of the local image patches, the signal energy, or the possible smoothness of the original image. This additional knowledge is incorporated by modifying the objective function or by adding linear equality and inequality constraints. Closed-form solutions are found by means of the KKT conditions. Problems with inequality linear constraints or with l^1-norm objective functions have no closed-form solutions. In these cases, the optimal interpolation is obtained by iterative optimization strategies efficiently implemented in many software packages.

The interpolation methods are not the best in literature when they face up to a direct interpolation. Their performance is evaluated with the PSNR and it tends to be slightly better than the bi-cubic interpolation. Nevertheless, if the image is low-pass filtered before the down-sampling, then the proposed methods outperform the bi-cubic. The provided experiment averages four pixels and then down-samples. The subsequent interpolation is 1.5-2 dB better w.r.t. the bi-cubic. Furthermore, the methods are useful for the lifting step construction by the addition of a set of linear equality constraints reflecting in the formulation the inner products due to DWT coefficients.

This latter variation allows the construction of novel first and second PLS and a ULS. The optimization criteria are the detail signal energy for the PLS and the gradient of the updated sample w.r.t. its non-updated neighbors for the ULS. Note however that any of the precedent interpolation solutions may be applied to obtain new lifting steps. Two other optimization criteria are considered for the ULS. The objective functions are the approximation signal gradient and the coarser level detail signal energy. Therefore, first prediction, updates, and second prediction steps are designed within a common framework.

The formulation has also turned out to be an adequate tool for the optimality analysis of existing wavelet transforms according to the selected criteria and image models. The image model seems to be a sensitive choice. The experiments are performed using auto-regressive models, but others may be considered. Once the model is selected, the parameters for a given image class are estimated, the auto-correlation matrix is derived, and the optimal lifting filter is determined. The study of the first PLS confronts the LS resulting from the AR-1 image model with the LeGall 5/3 prediction. This model is widely used by the image processing community, but when applied to derive an optimal first PLS with 2 samples the result is not the common sample mean, which reports the best image coding results. This leads to also adopt the AR-2 model for the subsequent experiments. However, the approach consistence is confirmed by the better coding of AR-1 processes with the optimal prediction and updates compared to their LeGall 5/3 counterparts. On the other hand, the first PLS derived with the AR-2 model reduces to the two-sample mean for the parameter values that appear in practice.

The study of the second PLS optimality helps to recognize the type of images that should be coded with the 5/11-a filter, with the 5/11-b, with any of the intermediate optimal PLS, or even when the second PLS should be avoided. The choice depends on the estimated first-order auto-regressive parameter.

For some image classes, like the mammography or the SST, the optimal ULS with the auto-regressive model of first and second order are found to be far from the usual LeGall 5/3 case. In this case, the difference implies an improvement that is supported by the optimized ULS application to image compression within the JPEG2000 environment. The bit-stream size diminishes 8% for the synthetic and 4% for the SST images, but increases for the mammography. This last case is treated separately and overcome using an ULS specific for the foreground and another one for the background. Several experiments and a variety of modifications are provided:

lifting filters on a quincunx grid, space-varying steps, and local adaptive steps in a line-wise basis. These experiments illustrate the richness of the approach.

Chapters 4 and 5 develop the nonlinear lifting scheme framework. The initial adaptive lifting analysis reveals some important clues for the contributions made in the nonlinear lifting. LS considered as a mapping between real spaces leads to the construction of the two adaptive ULS introduced in 4.3 and the generalized lifting scheme formulation. The discrete GL scheme is explored providing prediction and update step designs. Competitive coding results are obtained. The rest of the section extends this nonlinear lifting summary and provides the main conclusions.

The adaptive lifting scheme is analyzed in §4.2 from a new point of view: as a mapping between real spaces. The analysis is useful to gain an insight in such kind of transforms and unveils some hints for the further nonlinear lifting development. The new interpretation leads to two novel adaptive ULS designs with a median-based decision function and a variance-based decision function. Results in terms of the weighted entropy for both adaptive ULS are given showing their potential in lossy and lossless image compression.

The generalized lifting scheme is proposed. It defines the lifting step as a mapping between spaces. The continuous version is introduced and some of its relevant properties are stated. Among them, the capacity to recover the decomposition basis from the transform coefficients is explained in more detail. However, the quantization arises as a problem for such scheme and in consequence, the discrete version is formulated to overcome it.

Several generalized discrete designs are given. First, the geometrical prediction is introduced. The original three-rule design shows the GL scheme potential and flexibility. Good image compression results are obtained with the SPIHT coder. A 3-D version of this coder is developed and employed for the coding of 3-D images. The geometrical prediction outperforms the LeGall 5/3 wavelet for these images with the 3-D SPIHT.

Then, the image pdf is employed to optimize the generalized prediction. Propositions 5.1 and 5.2 are demonstrated. They show that the optimized prediction attains the minimal detail signal energy as well as the minimal entropy at the same time. In the experimental setting, the optimized prediction is derived for the natural, mammography, and SST image classes. Optimization based on the natural image class pdf is found to be similar to the LeGall 5/3 prediction and so, coding gain is small. However, the gain is considerable for the last two classes, that is, when the pdf is remarkably different from the natural image class pdf.

The optimized prediction main drawback is the LUT storage in the coder and decoder side. This fact impels the creation of the adaptive optimized generalized prediction. Despite the adaptation algorithm is simple, it offers good convergence properties as seen in the experiments in §5.1.3.2. This amounts to compression results only slightly worse than the non-adaptive version. Additionally, the adaptive prediction is applicable to images that do not belong to a class of images with a common pdf. For example, LeGall 5/3 wavelet is clearly outperformed by the adaptive generalized scheme for the synthetic images.

The possibilities of generalized ULS construction are explored. The need to preserve the signal properties for a multi-resolution processing is established. Then, the problem is divided into the update-first and update-last structures and a design for each structure is proposed. The minimal entropy mapping is used for the creation of a generalized update-last step. The design requires an accurate label selection. A gradient minimizing design is proposed for the update-first design. Results are obtained for both designs: they are acceptable, but in general they are worse than the LeGall 5/3 results for the test set. However, the used entropy coders are specifically devoted to linear wavelet coefficients. Better results are expected from the use of an entropy coder that takes into account the characteristics of the coefficients arising from the nonlinear ULS schemes. Once this is said, the conclusion is that the generalized update step development remains a widely open problem. Some hints have been found but clearly a large amount of work is still required.

6.2 Future Work

There exist several lines for future research that can be taken as an extension of the work carried out in this dissertation. Concerning a more theoretical part, there are several points which could be focused in a following of this Ph.D. dissertation:

- Chapter 3 explains an optimality analysis of existing wavelets based on the stated design criteria. The converse would be interesting, that is, the study of the new transforms considering some usual informative mathematical parameters of the wavelet transform: number of vanishing moments, coding gain, Riesz bounds, regularity (Hölder and Sobolev), angle between the analysis and synthesis spaces, etc. This study would give an additional comparison basis for the proposed linear schemes. Similarly, an analysis of the nonlinear decompositions properties is possible. The study may include reversibility in lossy compression, stability, synchronization, artifacts, and frequency-domain characteristics (if appropriate) of the approximation signal. Following this line, the resulting bitstream functionalities is a relevant issue. For instance, the proposed nonlinear schemes attain resolution embedded bitstreams, but SNR and quality scalability is not obtained. It would be interesting to profit from the previous analysis to construct generalized transforms with SNR embedded and scalable bitstream (for lossy compression purposes).

- The linear framework deserves more attention. For instance, the linear transforms may be evaluated in lossy compression. Perhaps, the objective functions may be changed in order to design specific transforms for lossy compression. The formulation accepts many other design modifications: a possible approach is to design different ULS for the even and odd samples, which makes sense because the subsequent function of these samples is also different. Another working topic within the linear framework is the search and use of other image models than the auto-regressive one.

- Generalized lifting may be improved in several ways. Usually, the underlying GLS transform support is smaller than the 2-D LeGall 5/3 wavelet support. This fact leads to the extension of the proposal by enlarging the generalized lifting support, possibly improving the current results. A proportional increase of computation and memory requirements should be avoided if possible. Eventually, 2-D transforms may be developed, both non-separable and 1-D direction-selective. Finally, an important effort should be focused on the update step development. One possibility is to identify appropriate update steps for the predictions proposed in §5.1.2 and §5.1.3. A second way is to study variations on the gradient minimization generalized ULS, which seems promising.
- The discrete generalized version is a fruitful approach that has demonstrated its richness. However, it may restrict the generalized lifting scope since it is a concrete case of the continuous version. The continuous generalized lifting requires a further study which may lead to appropriate mappings for lossy compression applications.

There are some practical implementation issues that could help the verification of the potential of this work and that would extend its range of application:

- The entropy coders used in this work are devoted to specific linear wavelet transforms. Better results are expected for the GL scheme by using an entropy coder that takes into account the characteristics of the nonlinear coefficients. The development of a simple JPEG-LS-like entropy coder for the nonlinear transforms would be interesting.
- The study of the nonlinear schemes interest for video coding would be relevant if the theoretical development of a lossy scheme is successful. Currently, the search for appropriate transforms in video coding is a hot topic.