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Spatial Structures in Analogical Reasoning

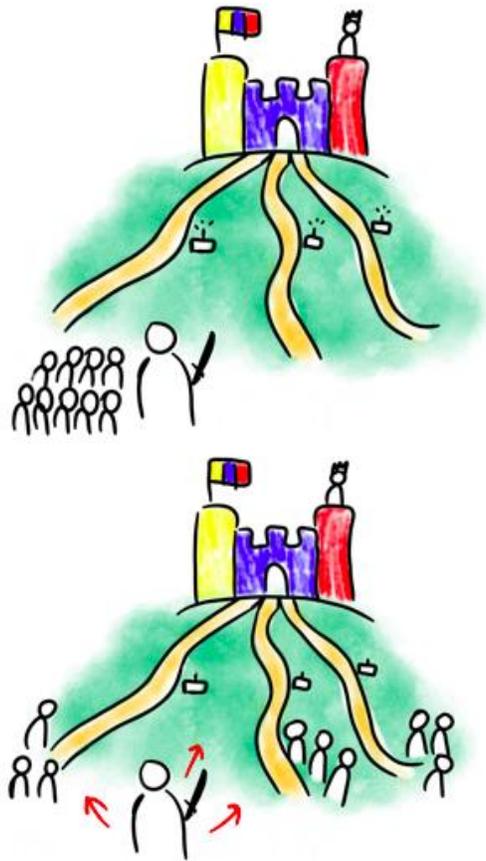
Amin Hashemi

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Spatial Structures in Analogical Reasoning



PhD Dissertation

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Spatial Structures in Analogical Reasoning

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June, 2023

To my parents,
my first humanities teachers

Acknowledgement

First of all, I would like to express my heartfelt gratitude to my supervisor, Elisabet Tubau, for her exceptional guidance and unwavering support throughout my doctoral journey. Her patience, expertise, and dedication have been invaluable in shaping my research and leading me from the starting point to the culmination of this thesis. Beyond her role as a supervisor, Elisabet has been an incredible mentor, teaching me the intricacies of scientific inquiry and fostering a deep understanding of the scientific process. I am truly fortunate to have had her as my guide, and her influence will resonate with me as I embark on future endeavors in the field of academia.

I would also like to extend my appreciation to Joan López-Moliner and Hans Supèr for their indispensable contributions to my research journey. Joan provided me with a remarkable opportunity to engage in research at the VISCA research group. During the months I spent working with him, I have been learning valuable research techniques for conducting scientific investigations and experimental implementations. Similarly, I would like to express my gratitude to Hans, with whom I had the privilege of conducting research for several months. Working with him, I had the opportunity to acquire specialized knowledge and skills related to the eye tracking approach.

Many thanks to Cristina de la Malla and Matthias Kiel for their insightful comments and suggestions to this research endeavor, Jordi Espada for his generous assistance with some experimental implementations, and Alba Bautista Conejo for her kind contribution of the first image used in this thesis.

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Abstract

Analogical reasoning is fundamental to human thinking; it plays an important role in a variety of problem-solving contexts. Analogical problem-solving involves recognizing relational similarities between two situations and mapping the structure of the familiar situation (known as the source) to the unfamiliar situation (known as the target) to generate inferences about the target. A wide range of research has been done on different factors affecting analogical reasoning. However, the role of spatial structures of the source and target in analogical transfer has not received enough attention. This research mainly investigates the role of the global spatial structure in analogical reasoning in both visual and verbal tasks.

To this end, we designed and implemented some visual analogies to separate the global path (the encoding and mapping of relations between the global-spatial shapes of the images) from the object path (the encoding and mapping of relations between the individual objects of the images). Overall, the results demonstrated that the use of either of these paths depended on the saliency of the global shape (global spatial structure) and certain individual differences. Specifically, some participants tended to encode global relations to a larger extent than object relations, and this measure seemed to be a good indicator of both abstract visual reasoning abilities and analogical problem-solving skills.

The effect of spatial relations in solving a verbal story-like problem was also investigated. As shown in previous studies, the results showed that people could transfer the underlying principles of visual diagrams to solve a target problem described verbally. Moreover, the results demonstrated, for the first time as far as we know, that explicit spatial properties in either the source, the target, or both had significant effects on the rate of successful analogical transfer. They also showed that drawing a spatial representation of the verbal source facilitated analogical transfer compared to writing a summary.

Taken together, the results of the experiments in both the visual and verbal domains emphasize the importance of spatial structure in analogical problem-solving. This may lead us to think about analogical reasoning not as a strict local-to-global process (in which individual local correspondences between two systems are generated and then integrated into a global correspondence), but rather as a more flexible process. Sometimes, people can map a source to a target problem simply based on their spatial structures, regardless of the individual elements of both systems. This research also has implications for the development of strategies that may facilitate abstraction and analogical transfer.

Resum

El raonament analògic és fonamental per al pensament humà; té un paper important en diversos contextos de resolució de problemes. La resolució analògica de problemes implica reconèixer les similituds relacionals entre dues situacions per tal de fer un mapeig de l'estructura de la situació familiar (coneguda com a font) sobre la situació desconeguda (coneguda com a objectiu), possibilitant la generació d'inferències sobre l'objectiu. S'ha fet una àmplia gamma d'investigacions sobre diferents factors que afecten el raonament analògic. Tanmateix, el paper de les estructures espacials del problema font en la transferència analògica no ha rebut prou atenció. Aquesta investigació investiga principalment el paper de l'estructura espacial global en el raonament analògic tant en tasques visuals com verbals.

Amb aquesta finalitat, vàrem dissenyar i implementar algunes analogies visuals per separar la via global (la codificació i mapeig de relacions entre les formes global-espacials de les imatges) de la via d'objecte (la codificació i mapeig de les relacions entre els objectes individuals de les imatges). En general, els resultats van demostrar que l'ús de qualsevol d'aquestes vies depenia de la rellevància de la forma global (estructura espacial global) i de certes diferències individuals. Concretament, alguns participants tendien a codificar relacions globals més que relacions d'objectes, sent aquesta mesura un bon indicador tant de la capacitat de raonament visual abstracte com de les habilitats de resolució de problemes per analogia.

També es va investigar l'efecte de les relacions espacials en la resolució d'un problema presentat en una narració. Com es va mostrar en estudis anteriors, els resultats van demostrar que les persones podien transferir els principis subjacents dels diagrames visuals per resoldre un problema objectiu descrit verbalment. A més, els resultats van demostrar, fins on sabem per primera vegada, que les propietats espacials explícites al problema font, a l'objectiu o a ambdós poden tenir efectes significatius en la taxa de transferència analògica exitosa. També van mostrar que dibuixar una representació espacial del problema font facilita la transferència analògica en comparació amb escriure un resum.

En conjunt, els resultats dels experiments tant en el domini visual com en el verbal emfatitzen la importància de l'estructura espacial en la resolució analògica de problemes. Això ens pot portar a pensar en el raonament analògic no com un procés que evoluciona estrictament d'aspectes locals a globals (en el qual es generen correspondències locals individuals entre dos sistemes i després s'integren en una correspondència global), sinó com un procés més flexible. De vegades, les persones poden mapejar un problema font sobre un problema objectiu simplement basant-se en les seves estructures espacials, independentment dels elements individuals d'ambdós sistemes. Aquesta investigació també té implicacions per al desenvolupament d'estratègies que puguin facilitar l'abstracció i la transferència analògica.

Chapter 1

General Introduction

General Introduction

Analogy is a kind of similarity in which shared relations can be found across different elements (Gentner & Maravilla, 2018). It is at the core of human cognition (Gentner & Kokinov, 2001; Goswami, 1992; Hofstadter, 2001) and it is considered a major contributor –arguably the major contributor- to our remarkable cognitive capacity (Gentner, 2003; Kurtz et al, 1999; Penn et al, 2008). Reasoning by analogy is ubiquitous in human cognition; people often understand a new situation by drawing an analogy to a familiar situation and transfer knowledge from the familiar to the unfamiliar (Gentner & Smith, 2013; Holyoak, 2012).

Analogical processes are central in human thought. Even in everyday life, people draw on experiential analogies to form mental models of phenomena in the world (Gentner & Smith, 2013). They draw analogies to form mental models of both physical phenomena (Kaiser & Proffitt, 1985; McCloskey, 1983) and social phenomena (Andersen & Chen, 2002; Mussweiler & Rüter, 2003).

In educational settings, a familiar domain is used to help students learning an unfamiliar new domain (Gentner & Smith, 2013; Goswami, 1992; Hummel & Holyoak, 1997). Gentner & Smith (2013) suggested that analogy could foster knowledge via inference projection –transferring information from a familiar situation to an unfamiliar one, generalization –formation of new relational abstractions, difference detection –noticing differences in situations which share alignable structures, and re-representation.

Analogy has also an important heuristic role in scientific discoveries (Bartha, 2013; Goswami, 1992). Archimedes' discovery of a method to distinguish gold from metal, Kepler's theory of celestial mechanics, and Maxwell's discovery of electromagnetic fields are among classic novelties in science led by analogies (Bartha, 2013; Gentner & Markman, 1997; Goswami, 1992; Nersessian, 2009). As Bartha (2013) pointed, Priestley (1769) wrote:

“Analogy is our best guide in all philosophical investigations; and all discoveries, which were not made by mere accident, have been made by the help of it”.

In experimental settings, analogical problem solving tasks have been designed and implemented in visual as well as verbal formats. The abstract form of an analogy is represented as four terms A:B::C:D in which the relation between A and B is similar to the relation between C and D. For example, in the hand: glove:: head: hat analogy, the relation between the elements of

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the first pair (hand and glove) is similar to the relation in the second pair (head and hat). In both pairs, the first element “covers” the second element.

There are other forms of verbal analogies which are more complex and necessitate more cognitive resources than just evaluating connections between a few elements (e.g., Reed, Ernst, & Banerji, 1974; Hayes, 1977; Gick and Holyoak, 1980 and 1983). More complex analogical reasoning is often assessed by presenting a short story describing a problem and its solution (the source of the analogy) followed by an analogous problem that participants must solve (the target). To solve it, participants are expected to map the source onto the target and generate the analogically similar solution for the target problem. As developed below, the analogical transfer from the source to the target can be a difficult task and different ways to improve performance have been investigated.

Apart from verbal domains, different studies have explored people’s cognitive capacities in solving visual analogies. There is considerable evidence that the same kinds of analogical processes operate across these domains (Gentner & Smith, 2013; Lovett & Forbus, 2017). Furthermore, studies found that verbal analogies, geometric analogies, and matrix problems engage a common brain region, the left rostrolateral prefrontal cortex, which may be associated with relational reasoning (Hobeika et al., 2016). Accordingly, visual analogies can measure analogical problem solving abilities in general.

Whereas verbal analogies usually require previous semantic knowledge, relations in visual analogies often can be inferred from the presented images (for example, when the relations refer to spatial location, shape, color, or number of elements). Hence, in addition to a general perception capacity, just a general capacity for relational reasoning would be sufficient for solving such visual analogies. In this way, visual analogies have been vastly used to explore analogical problem solving abilities (e.g. Gray & Holyoak, 2017; Kubricht et al., 2015; Stevenson et al., 2013; Vendetti et al., 2014).

1.1.The role of spatial structures in analogical reasoning

As outlined above, reasoning by analogy involves two main steps: (i) recognizing a common relational system between the source and the target, and (ii) generating further inferences guided by mapping the source onto the target (Bartha, 2013; Gentner, 1983; Gentner & Maravilla,

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2018; Gentner & Markman, 1997; Holyoak & Thagard, 1989; Hummel & Holyoak, 1997; Kokinov & French, 2003). Recognizing similarities between the source and the target can occur at multiple levels of abstraction. At a low level, one can find surface similarities between individual objects of the two systems; that is, some objects in the source share similar attributes to the objects of the target. At a higher level, one can recognize similarities in relations between objects rather than individual objects themselves. Such similarities are called relational similarities which are crucial for analogical reasoning (Holyoak, 2012, Gentner & Smith, 2013; Goswami, 1992). In this way, while in analogy only relational structure is shared between two systems, in similarity, both relational structure and object attributes are shared (Gentner & Maravilla, 2018).

After recognizing that two systems are similar based on their relational structure, the source is being mapped to the target to generate inference about the problem at hand (the target). According to the structure-mapping theory, proposed by Gentner (1983), reasoners map the structure of the source to the target through establishing a structural alignment between two representations based on their common relational structure (Gentner & Maravilla, 2018). In this process, information that is being transferred from the source to the target would lead to generate inferences about the target.

In line with the structure-mapping theory, Sagi et al. (2012) showed that images with alignable structures were easier for reasoners to compare and identify similarities than those without alignable structures. Moreover, derived from the structure-mapping theory, Matlen et al. (2020) demonstrated that alignment between the spatial structures of two visuals enhanced comparison. Although these studies explored how aligned spatial structures of the images would have a role in structural comparison, results could have implications on the importance of spatial structure in structure-mapping, and, more broadly, in analogical reasoning in visual domains (Lovett & Forbus, 2017). Specifically, recognizing relational similarities and differences between images and also mapping one onto the other may depend partly on the spatial structure of the images and their alignability. Nevertheless, as far as we know, there has been no systematic research exploring the role of spatial structure in visual analogical reasoning.

Apart from that, although these studies were done in the visual domain, the results would have implications on structural alignment, and, hence, analogical reasoning even in verbal domains. The mechanisms and strategies supporting effective visual analogical problem solving

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are similar to those supporting verbal analogical problem solving (Gentner & Smith, 2013; Lovett & Forbus, 2017). Given that in verbal domains reasoners build mental representations for the source and target, and also due to the fact that mental representations resemble, in part, percepts (Tversky, 2005), it is expected that the spatial structure of the built mental representations would have effects on analogical reasoning also in verbal domains.

1.2.Spatial structure in visual analogical problem solving

Commonly, a visual analogical problem refers to a set of images with a missing one and reasoners must select the missing element from several response options. Figure 1 shows an example of a simple visual analogy made by geometric shapes. It is considered that solving a visual analogical problem involves (i) perception, (ii) relation identification through comparison, and (iii) mapping the inferred relations onto the proposed response options to complete the missing element by selecting the appropriate option (Lovett & Forbus, 2012; Chen et al, 2016; Sternberg, 1977).

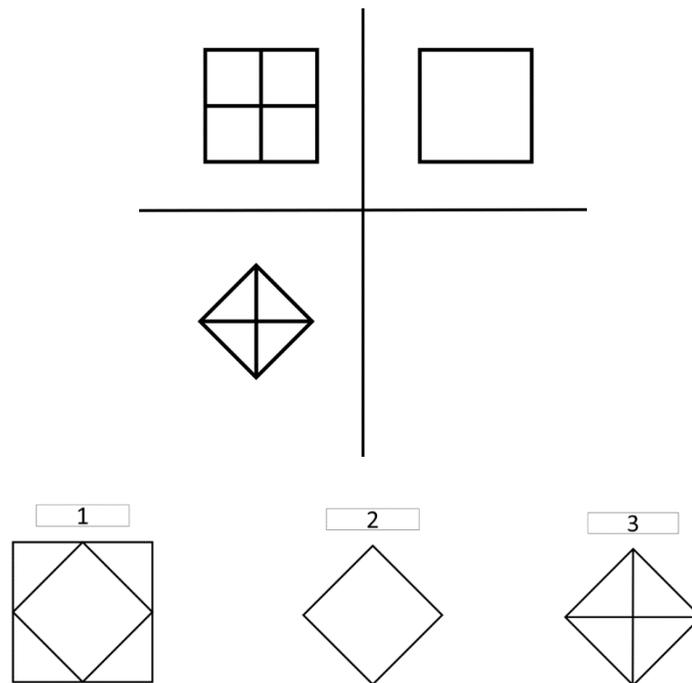


Figure 1. An example of a simple visual analogy with three response options (the second option is correct)

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Perception of the images can be characterized as hierarchical in the sense that each image can be represented at multiple levels in a spatial hierarchy (Martin, 1979; Navon, 1977; Palmer, 1977). Particularly, Navon (1977) designed and tested sets of stimuli to investigate the global vs local precedence in perception. He used small letters (at the low level of the spatial hierarchy) together making a large letter (at the high level) –see Figure 2. He showed the precedence of global vs local features in visual perception.

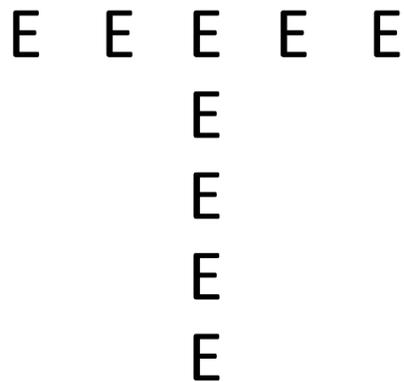


Figure 2. An example of two-level perceptual stimuli designed and tested by Navon (1977).

Likewise, the images of a visual analogy can be perceived in a spatial hierarchy. Given that there might be separate qualitative and quantitative components at each hierarchical level, comparison, relation identification, and mapping (analogical reasoning) may also be performed at different levels (Forbus, 2017; Love et al., 1999).

Although analogical reasoning at different hierarchical levels may lead to different answers for the missing image, the visual analogies used in previous studies did not allow for this possibility. In fact, some of the visual analogies employed contain only a few sparse elements that do not form any global organization, while in others the end-result of reasoning at different hierarchical levels are identical (Arendasy & Sommer, 2005; Christie & Gentner, 2010; Chen et al., 2016; Meo et al., 2007; Perez-Sales et al., 2013; Raven, Raven, & Court, 1993; Richardson, 1991; Richardson & Webster, 1996; Siegler & Svetina, 2002; Stevenson et al., 2013). However, as developed in chapter 3, some designs can dissociate global spatial-based from object-based analogical reasoning.

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For example, Figure 3a, presents an example of a visual analogy consisting of compound images with a global shape (a vertical line) composed of smaller objects (fruit shapes). One can consider the global shapes and infer that, for the missing image, the elements are becoming smaller from top to bottom. On the other hand, one can consider the objects and infer that, from left to right, the carrot becomes the biggest, the watermelon the second biggest, and so on. As a result, solving the visual analogy represented in Figure 3a by considering global spatial relations would imply choosing the response shown in Figure 3b, while solving it by considering object-based relations would lead to selecting the response shown in Figure 3c.

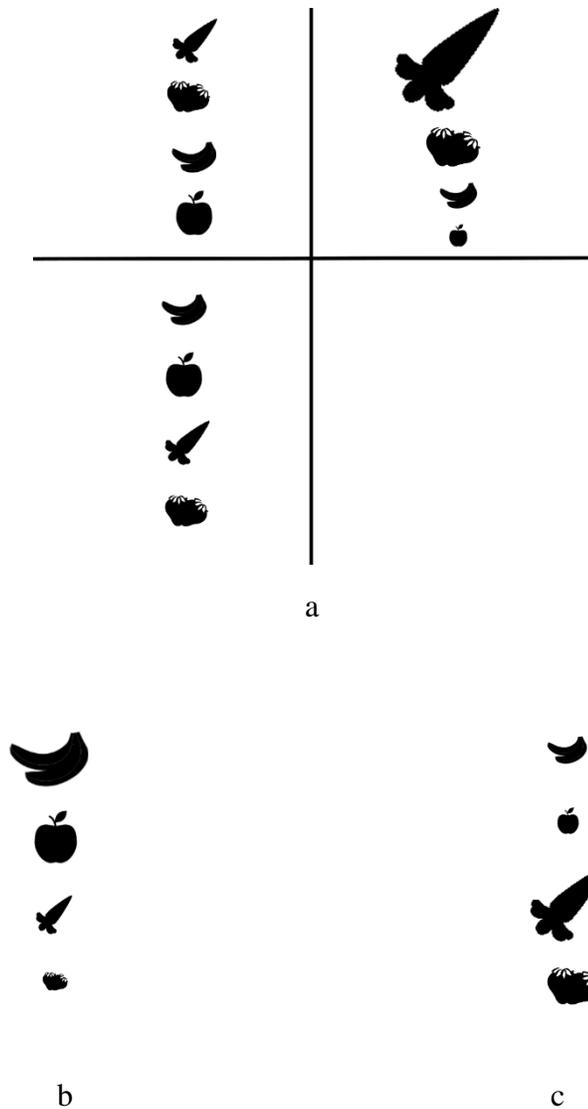


Figure 3: a) A visual analogy that separates the results of the global and object paths. b) The result of the global path. c) The result of the object path.

Considering such visual analogies, where the end-result of encoding and mapping global relations is separated from that of encoding and mapping object-based relations, both global features (i.e. the saliency of the global shapes of the images of a visual analogy) and object features (i.e. the familiarity of the individual objects of a visual analogy and the color condition) can have effects on noticing global/object relations. Previous studies showed that more salient global shapes would facilitate the perception of the global *versus* local features compared to less salient global shapes (Kimchi, 2015; LaGasse, 1993; Martin, 1979; Poirel et al., 2006). However, the effect of the global shapes in visual analogical problem solving has not been explored.

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Furthermore, previous studies demonstrated that object saliency may play a significant role in the identification and mapping of relations between objects (Meo et al., 2007; Perez-Sales et al., 2012; Roberts et al., 2000; Richardson, 1991). For example, Perez-Sales et al. (2012) showed that colorful objects facilitated the identification and mapping of relations between objects compared to black and white objects. Moreover, Meo et al. (2007) showed that familiar objects promoted the identification of relations between objects to a larger extent than unfamiliar objects. However, all visual problems used in those studies had few objects and no clear global organization. In the presence of global organization, object features might have different effects.

1.3. Spatial structure in story-like analogical problem solving

It has been suggested that the mechanisms and strategies supporting effective visual analogical problem solving are similar to those supporting verbal analogical problem solving (Lovett & Forbus, 2017). So, the distinction between global spatial and non-spatial relations, introduced in visual analogies, could be applied to verbal analogies as well, particularly to story-like analogies introduced above (Gray & Holyoak, 2017; Markman & Gentner, 1996).

As mentioned before, complex analogical problem solving is presented by a short story describing a problem and its solution –source- followed by an analogous story-like problem –target. In this way, the story-like analogical problem solving process is considered to involve four main steps: (i) constructing a mental representation of the source and target problems, (ii) noticing analogous relations between them, (iii) mapping the correspondences between the key elements of the source and target, and (iv) generate the parallel solution based on the source for the target (Chen, 1995; Gentner, 1989; Holyoak, 1988). People may spontaneously retrieve the source and transfer it onto the target problem to analogically solve it (Gick & Holyoak, 1980). If people are not successful in analogical transfer spontaneously, they may do so after a hint suggesting to use the source to solve the target problem (ibid). The hint is particularly needed in the case of analogies across different semantic domains, where participants tend to have difficulties in the spontaneous analogical transfer (e.g., Holyoak & Koh, 1987).

The source and the target of a story-like analogy commonly used in previous research have spatial as well as non-spatial properties (such as categorical or other semantic properties of the objects). For example, regarding the problems used in chapter 5 (see Appendix D and E), the fortress story as well as the radiation problem refer to specific meaningful objects (fortress,

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general, ray, tumor, etc.) and specific relations between these objects (the general vows to capture the fortress, the rays are a type of force able to destroy the tumor, etc.). Some relations may lead reasoners to the solution (i.e., the dispersion-convergence solution) but some others may distract from it, leading to irrelevant inferences. Critically, scenarios related to the dispersion-convergence solution generally describe some spatial relations between certain objects. For example, in the fortress story, it is mentioned that the fortress is in the middle of a country. Also, many roads radiate outward from the fortress like *spokes on a wheel*. Therefore, a verbal story as the source of an analogy may describe spatial properties leading to spatial relations in addition to non-spatial properties leading to non-spatial relations.

Analogous relations –whether spatial or non-spatial- described in the source and the target can lead to map the source onto the target and finally analogically solve the target problem. However, the extent to which spatial relations are critical for analogical transfer across different semantic domains is still an open question.

Previous studies showed spatial relations represented via visual diagrams could communicate the conception of the dispersion-convergence solution in the case of the radiation problem as the target (Beveridge & Parkins, 1987; Gick & Holyoak, 1983; Pedone et al., 2001). Particularly, Beveridge & Parkins (1987) and Pedone et al. (2001) showed that, by improving some perceptual properties, abstract visual diagrams could be regarded as the source of an analogy even when spontaneous transfer was considered. In this way, visual diagrams that intended to represent movement (e.g., sequences of arrows; Pedone et al., 2001) or gradual transformation of intensity (Beveridge & Parkins, 1987) seemed to communicate the dispersion-convergence solution more effectively.

However, the stimuli used in those studies did not have a problematic situation followed by a solution, the structure used for the verbal sources in story-like analogies. In fact, the stimuli seemed to communicate only the dispersion-convergence solution. Due to the lack of the problematic state and its corresponding key elements, the process of solving the radiation problem with such visual diagrams lacks a critical stage in analogical problem solving: the structural mapping between the problems of the source and of the target. More critically, most of the visual representations previously used as the source for the radiation problem show some surface similarities with the common visual representations of tumor (circles) and rays (arrows).

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Accordingly, the high rate of analogical transfer for the visual representations reported in previous studies might be partly due to surface similarities leading to similarity-based mappings (Gattis, 2004; Gentner & Markman, 1997; Gentner & Maravilla, 2018) rather than structural mappings (Gentner, 1983; Gentner & Maravilla, 2018; Holyoak & Koh, 1987; Gattis, 2004; Markman & Gentner, 1993). These critics may cast doubt on the nature of problem solving in cases of a visual source and a verbal target in previous studies.

Apart from that, while there are studies exploring the role of spatial relations in analogical transfer from a visual source to a verbal target, there is no research investigating the effect of spatial relations on the process of mapping from a verbal source to a verbal target. As far as we know, there are no studies which explore the effect of explicitly mentioned spatial relations on analogical reasoning. Gick and Holyoak (1980) proposed that finding common spatial relationships would facilitate solving the radiation problem and suggested an idea for future studies. However, none of the future studies worked on the effect spatial properties in the verbal-verbal format, while many studies were dedicated to do research on the visual-verbal format of analogical reasoning.

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Chapter 2

Research Goals and Outline

2.1. Research goals and hypotheses

Research Goals and Outline

The first goal of the present research was to answer the question: How people would solve a visual analogy if reasoning at the different hierarchical levels lead to different answers? To this end, and inspired by the high and low hierarchical levels entailed in image perception (i.e., the global shape and its constituent elements) suggested by Navon (1977), we considered two levels at which visual reasoning could be established. Accordingly, two paths of comparison, relation identification, and mapping were defined: (i) the global path, in which reasoners would identify and map relations between the global shapes of the images –at a higher hierarchical level, and (ii) the object path, in which reasoners would identify and map relations between objects –at the lower level. In this way, some visual analogies were designed in which the relations between the objects led to one response whereas the relations between the global shapes led to a different one –see Figure 1.

Specifically, to identify some of the conditions that may foster the encoding and mapping of either global or object relations, two variables were studied: (i) object saliency: according to the previous literature (Meo et al., 2007; Perez-Sales et al., 2012; Roberts et al., 2000; Richardson, 1991) more salient objects were expected to facilitate the encoding and mapping of relations between objects, compared to less salient objects, and (ii) global shape saliency: in line with previous studies (Love et al., 1999), more salient global shapes were expected to facilitate the encoding and mapping of global relations, compared to less salient global shapes.

Another objective of this thesis referred to individual differences. Based on the observation that there are individual differences in global *versus* local processing tendencies in perception (Navon, 1977; de-Wit & Wagemans, 2015), individual differences in the tendency to encode and map global relations *versus* object relations were also expected in solving visual analogies. It was also expected that a stronger tendency to map global vs object relations would be associated with better performance in visual analogical problem solving. Furthermore, previous studies have shown a positive association between the ability to solve visual analogies and the ability to solve story-like analogies (Gray & Holyoak, 2017). In this way, it was expected that people with a higher tendency to encode and map global relations would be also better at solving story-like analogies.

A third goal of this research was dedicated to investigating the role of spatial relations in story-like analogical problem solving. In order to investigate the role of spatial relations on this type of analogies, we aimed to somehow separate spatial relations from non-spatial ones. To do

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so, like what were done in previous studies (Beveridge & Parkins, 1987; Gick & Holyoak, 1993; Pedone et al, 2001), we designed visual diagrammatic representations made by geometric shapes as the source of an analogy. Visual diagrams are semantically poor; so, it is difficult to communicate non-spatial relations by visual diagrams. On the other hand, they are spatially rich; so, most of the found relations in such visuals are spatial. The results of previous studies showed that people can spontaneously transfer good visuals as the source to a verbal target (Beveridge & Parkins, 1987; Pedone et al, 2001). However, there are some critical points in the visuals casting doubt on the analogical transfer that people can make from visual diagrams to a verbal target.

Related with the third objective, we also aimed to investigate the role of the spatial relations described in either the source or the target, which has not been studied before, as far as we know. To do so, although the spatial and non-spatial relations are so entangled that it seems difficult to separate them (Gattis, 2004), we manipulated the number of relevant spatial properties explicitly mentioned. In other words, we added or removed some explicitly mentioned spatial properties in order to investigate their effects on analogical transfer. It was expected that explicit spatial cues in either the source or the target would facilitate the analogical transfer, compared to less explicit spatial cues. Finally, we also manipulated the task requested to the participants to enhance comprehension of the source: schematic drawing *versus* writing a summary. Given that drawing implies developing an explicit spatial representation, it was also expected that this task would facilitate analogical transfer, compared to writing.

2.2. Structure of the thesis

To address these goals, in Chapter 3 we introduce and explain the designed stimuli for investigating the conditions affecting the reasoning path (the global vs the object path). After this introduction, the method, the results, and the discussion of four experiments using these stimuli are reported. To investigate the different reasoning paths, we measured accuracy, response time and eye movements of the participants solving several analogies. In addition to the study of the variables introduced above (saliency of the global shape, color, and familiarity of the objects), we investigated the effect of the presentation of the source and the target (simultaneous or sequential). Furthermore, individual differences in the tendency to encode and map global/object relations are reported. Specifically, we investigate and discuss the associations between this tendency and visual as well as story-like analogical problem solving abilities.

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In Chapter 4, we develop the role of spatial relations in story-like analogies by elaborating previous studies and introducing our stimuli. The effect of dynamic *versus* static diagrams and also the effect of explicit *versus* implicit spatial features in narratives are investigated and discussed. Specifically, we assess the previous designed visuals, and then introduce and explain our stimuli to investigate the extent to which they facilitate recognizing, encoding, and mapping spatial relations represented in a set of visual diagrams onto a verbal target. Moreover, the effect of dynamic visual representations compared to static ones is studied because previous studies showed that dynamic visuals are more effective in learning and reasoning (Pedone et al, 2001; Tversky, 2005). As mentioned above, we also investigated the effect of spatial properties in the verbal-verbal format of analogical problem solving in two ways. First, we controlled the spatial properties of the source regarding the fact that narratives including explicit spatial properties lead to build mental representations with different characteristics from mental representations built for narratives without explicit spatial properties (Johnson Laird, 1981). Second, we explored how spatial properties described in the verbal target enhanced analogical transfer when the source and the target of an analogy share a similar spatial structure which can be inferred from the narratives.

Finally, Chapter 5 presents a general discussion about the findings of our experiments, their limitations, and their implications for future studies. As developed there, we argue that that the global precedence observed in image perception also seems to apply in the context of visual analogical reasoning. More generally, spatial properties of an image play important roles not only in perception but also in more complex tasks such as analogical problem solving. In this way, individuals differ in the tendency to encode and map global spatial relations instead of relations between objects; this tendency might be associated with different individuals' capacities. Analogously, in story-like analogies, spatial relations can facilitate analogical transfer from the source to the target. Moreover, dynamic visuals facilitate the process of analogical reasoning compared to static ones. Regarding the fact that spatial relations might be encoded between some images globally and independently of their constituent objects, this research can be regarded as the starting point for thinking about reasoning in ways other than the local-to-global way –i.e. generating local correspondences and coalescing them into a global correspondence (Love et al, 1999).

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Chapter 3

Global Relations vs Object Relations in Visual Analogies

3.1. Introduction

Global Relations vs Object Relations in Visual Analogies

We reason by analogy in verbal as well as visual contexts. There is considerable evidence that the same kinds of analogical processes operate across these domains (Gentner & Smith, 2013; Kokinov & French, 2003; Markman & Gentner, 1993). However, while comprehending verbal analogies usually require previous knowledge, relations in visual analogies which are construed with geometric shapes can be inferred from the presented images without requiring semantic knowledge. Therefore, solving a visual analogical problem involves (i) perception, (ii) relation identification through comparison, and (iii) mapping the inferred relations onto the proposed response options¹ to complete the missing image (Chen et al, 2016; Lovett & Forbus, 2012; Stevenson & Hickendorff, 2018).

Perception of complex images of a visual analogy can proceed at different levels of abstraction in a spatial hierarchy (Navon, 1977; Palmer, 1977). Thus, the global shape of an image is located at the highest level and details of the constituent elements at the lowest. Given that there might be separate qualitative and quantitative components at each hierarchical level, comparison, relation identification, and mapping may be performed at different levels as well (Forbus & Lovett, 2021; Lovett & Forbus, 2017).

As mentioned in Chapter 1, although it is possible that comparison, relation identification, and mapping at different hierarchical levels may lead to different answers for the missing image, the visual analogies used in previous studies did not allow for this possibility. In fact, some of the visual analogies employed contain only a few sparse elements that do not form any global organization, while in others the end-products of reasoning at different hierarchical levels are identical (Chen et al., 2016; Christie & Gentner, 2010; Meo et al., 2007; Perez-Sales et al., 2013; Raven et al., 1993; Richardson, 1991; Richardson & Webster, 1996; Siegler & Svetina, 2002; Stevenson et al., 2013). With such visual analogies, there is no way to investigate in what hierarchical level reasoners compare images and identify relations between them to arrive at the result. The aim of the present research was to design visual analogical problems that would enable us to separate different analogical reasoning paths.

The global path vs the object path. Inspired by the distinction between global and local levels entailed in image perception (i.e., the global shape and its constituent elements; Navon,

¹ Most of visual analogical problem-solving tasks include a several options as proposed responses

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1977), we also considered two levels at which visual analogical reasoning could be established. Accordingly, two paths of comparison, relation identification, and mapping were defined:

- (i) The global path would lead to infer relations between global shapes. To solve a visual analogy through the global path, the global shapes of the images in a visual analogy are compared to identify global commonalities and differences. Then, the inferred global relations are mapped onto the missing element, regardless of the individual objects.
- (ii) The object path would lead to infer relations between individual objects. To solve a visual analogy through this path, the objects constituting the images in a visual analogy are compared to identify object commonalities and differences. Then, the inferred object relations are mapped onto the missing image. Hence, it is the objects and their features rather than the global shapes that matter.

Visual analogies with compound images. To investigate these different problem-solving paths, it was necessary to design visual analogies consisting of images related by rules at two levels of abstraction corresponding to each path. Based on research on hierarchical levels of perception, we designed visual analogies containing compound images, i.e., a global shape composed of an arrangement of smaller objects (see Figure 4a). Unlike the visual analogies used previously, the current ones yielded different end-product of reasoning through either the global path or the object path.

Consider Figure 4a, an example of a visual analogy consisting of compound images with a global shape (a vertical line) composed of smaller objects (fruit shapes). According to the global path, one can consider the global shapes and infer that, for the missing image, the elements are becoming smaller from top to bottom. On the other hand, according to the object path, one can consider the objects and infer that, from left to right, the carrot becomes the biggest, the strawberry the second biggest, and so on. As a result, solving the visual analogy represented in Figure 4a through the global path would imply choosing the response shown in Figure 4b, while solving it through the object path would lead to selecting the response shown in Figure 4c.

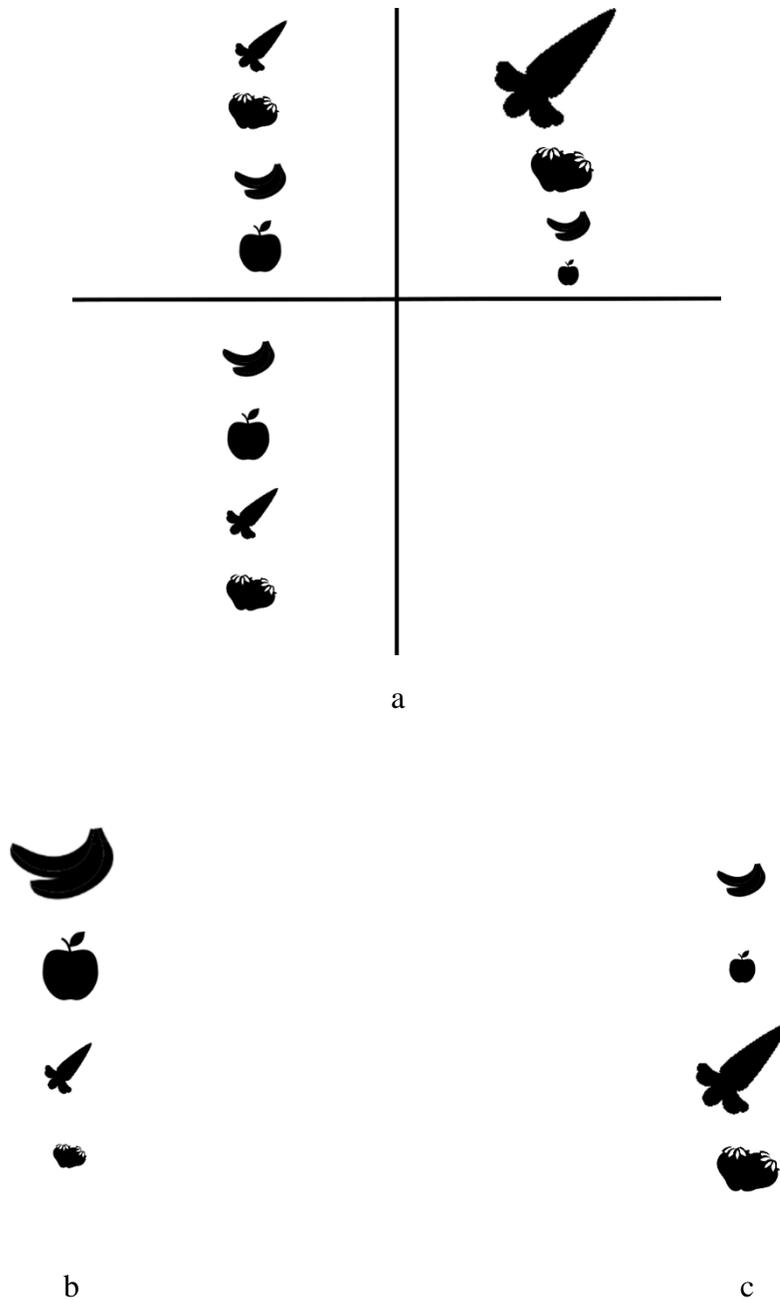


Figure 4: a) A visual analogy that separates the results of the global and object paths. b) The result of the global path. c) The result of the object path.

3.2. Objectives and hypotheses

Global shape saliency and the mapping of global/object relations. One of the main objectives of this research was to determine when reasoners encode and map global relations and when they encode and map object relations. Previous evidence shows that, in general, higher

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hierarchical levels are perceived first when the global shape of the stimulus is a good Gestalt form, no matter what the local elements are (Navon, 1977; Poirel et al., 2008). Therefore, if the global shapes of the images in a visual analogy are good forms, the global shapes will be perceived first, regardless of the constituent objects. This will lead to global comparisons before any object-by-object comparisons (Love et al., 1999). Accordingly, we hypothesized that when the global shapes of the images in a visual analogy are good forms that can be aligned, global relations would be identified and mapped more often than object relations.

Previous studies have also suggested that if the global shape of a stimulus is less salient, the global precedence effect might be reduced or even reversed (Kimchi, 2015; LaGasse, 1993; Martin, 1979; Poirel et al., 2006). Accordingly, we expected that when the spatial arrangement of the objects in the images forms a less salient global shape, the likelihood of identifying and mapping object relations would be higher, compared to a visual analogy with more salient global shapes. In Experiment 1 and 2, we investigated the effect of the saliency of the global shapes on encoding and mapping global/object relations.

Object features and the mapping of global/object relations. In the case of visual analogies with less salient global shapes, we also expected that object features would play a more important role in determining the type of mapping. Previous studies have suggested that object saliency may play a significant role in the identification and mapping of relations between objects (Meo et al., 2007; Perez-Sales et al., 2012; Roberts et al., 2000; Richardson, 1991). For example, Perez-Sales et al (2012) showed that colorful objects facilitated the identification and mapping of relations between objects compared to black and white objects. In this way, we expected that problems with colored objects would be solved by mapping object relations to a larger extent than problems with black and white objects. In Experiment 1, we aimed to study the effect of color on encoding and mapping global-object relations.

Moreover, in the context of visual problems with few objects and no clear global organization, Meo et al. (2007) showed that familiar objects promoted the identification of relations between objects to a larger extent than unfamiliar objects. Accordingly, we expected that, in the case of problems with less salient global shapes, problems containing familiar objects would be solved by mapping object relations to a larger extent than problems containing less objects. In

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Experiment 2, we aimed to study the effect of familiarity on encoding and mapping global-object relations.

The global path/object paths for complex visual problems. Apart from the tendency to use the global/object paths, we aimed to investigate the ability to employ the global/object paths in problem solving. Due to the fact that solving visual analogies with simple rules are too easy for most of adults to measure problem solving abilities, more complex rules between the images of a visual analogy were used for this purpose. In this way, in Experiment 3 we designed and tested some visual analogies which had either only relations between the spatial global shapes of the images or only relations between the individual objects. Problems with only global spatial relations had to be solved using the global path while problems with only object-based relations had to be solved by using the object path. We expected that problems with only global spatial relations would be easier to solve than those containing only object relations.

Individual differences. We also investigated individual differences in mapping global relations *versus* object relations. Based on observed individual differences in global processing *versus* local processing tendencies in perception (Navon, 1977; de-Wit & Wagemans, 2015), we expected to find differences in the tendency to identify and map global relations *versus* object relations in visual analogies. Specifically, we expected that a higher tendency to map global rather than object relations would facilitate analogical problem-solving. In studies 2 and 4 we tested this hypothesis, using problems of Advanced Raven Progressive Matrices (ARPM; J. Raven et al., 1993). In Experiment 4, we aimed to explore possible associations between the tendency to notice global/object relations and analogical transfer abilities in the context of problems described in narratives.

Problem solving strategies. Finally, to more directly investigate individual differences in using different strategies to solve visual problems, in Experiment 5 we used an eye-tracking approach. As previous studies suggested, the eye tracking approach is useful to go through the steps required by analogical problem solving (Bethell-Fox et al., 1984; Chen et al., 2016; Glady et al., 2014, 2016; Gordon and Moser, 2007; Sternberg & Rifkin, 1979; Thibaut et al., 2011; Thibaut and French, 2016; Vendetti et al., 2016). In this way, the goal of Experiment 5 was to explore possible associations between different strategies of analogical problem solving and the tendency to use the global/object paths using the eye tracking approach (see Experiment 5).

3.3. Experiment 1

In this experiment, we aimed to investigate the effects of saliency of the global shapes and color of the individual objects on encoding and mapping global/object relations. As introduced above, we expected that problems with more salient global shapes would be solved by a global relation response option more often than problems with less salient global shapes. Also, problems with colorful elements were expected to be solved by an object relation response option more often than problems with black-and-white elements.

Method

Participants. One hundred and thirty undergraduates participated in this experiment. The ethics committee of the local university approved the research procedure. Participants were randomly divided into two groups (see procedure).

Materials. We created an online questionnaire that included 6 multiple choice visual analogies implemented by fruit shapes (see Appendix A). They were presented as 2*2 matrices with an empty matrix (as in Figure 4a) with 3 response options: one matched global relation/s (Figure 4b), one matched object relation/s (Figure 4c), and one was an irrelevant choice (a copy of the bottom left image). For each problem, the object relation choice and the global relation choice were considered relevant because they were the result of mapping relations between images. The rules used to design these problems were addition, subtraction, or change in size. These rules were extracted from Set II and III of the of Raven's Standard Progressive Matrices (RSPM J. Raven et al., 1993).

Two main independent variables were studied in this experiment: the saliency of the global shapes, which varied within-participant in two levels (more and less global shape saliency; see an example in Figure 4), and the color of the objects, which varied between-participants in two conditions (colorful and black and white). For both more and less global shape salient problems, the spatial position of the objects remained the same in all matrix images (i.e., from right to left and from top to bottom). That is, all the global shapes of the images could be aligned. The problems with salient global shapes were created based on simple global forms such as circles and vertical and horizontal lines, generated by the spatial arrangement of the individual objects (see Figure 5). In contrast, the spatial arrangements of the objects constituting less salient global shapes did not

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form a good global shape (see Figure 6). To avoid using similar patterns throughout the task, different kinds of global shapes were used.

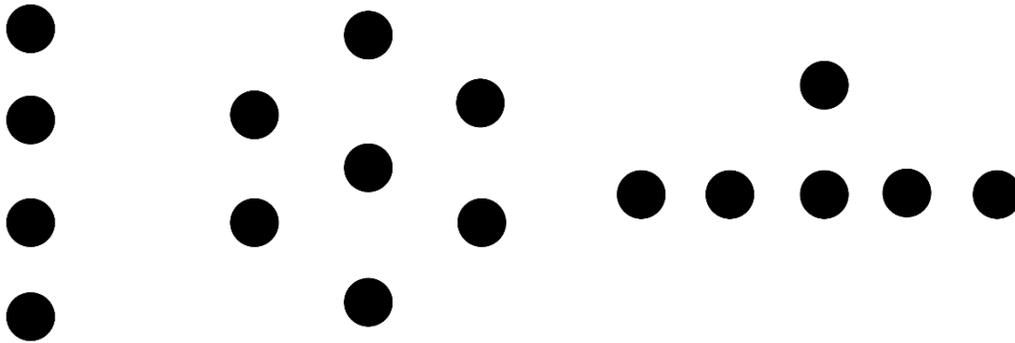


Figure 5: Good forms used as salient global shapes of the questionnaire.

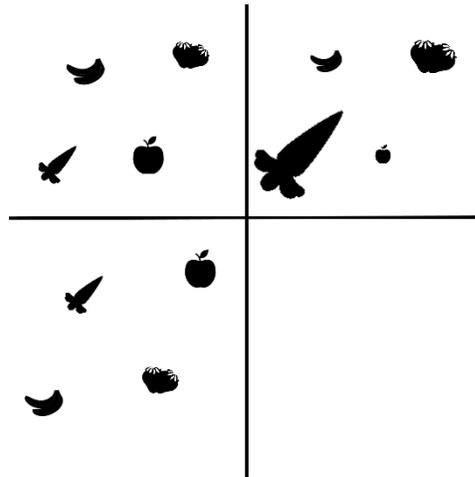


Figure 6: A visual analogy with the same objects as in Figure 4a but containing less salient global shapes.

To disentangle the effect of global shape saliency from other potentially confounding effects, all problems with salient global shapes had a counterpart with the same rule but with less salient global shapes. So, in each group, there were three problems with salient global shapes and three with less salient global shapes.

Regarding color, the problems presented to one group of participants were black and white, while the ones presented to the other group were colorful. Except color, there were no other

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differences between the problems of both groups. Colors were selected based on two considerations: (i) fruits were colored according to their natural colors, and (ii) fruits used in one problem had different colors to avoid repetition of colors.

Procedure. The questionnaire was administered via Qualtrics. According to the instruction page, participants were asked to fill the missing image of each visual analogy with the response option they thought fit the best. Following the instruction, the problems were shown one per page. Subjects had to select one of the response options before moving on to the next problem. Problems presented randomly. No time limit was set for completing the questionnaire.

Data Analysis. The selected response options were extracted from Qualtrics and imported into a Python environment for analysis. We used mixed-effect ANOVAs to analyze main effects and potential interactions between independent variables on problem-solving outcomes.

Results and discussion

Responses were analyzed at two levels. First, regarding relevant/irrelevant responses, we considered both global relations choice and object relations choice as relevant because they were the result of encoding and mapping relations between images. So, responses were labeled as relevant or irrelevant. We run a mixed-effect ANOVA to see possible effects of Saliency of the global shape (as the within-participant factor) and Color (as the between-participant one) on encoding and mapping relevant relations. Results showed that global shape Saliency had a significant effect on mapping relevant relations ($F(1,130)= 33, p< 0.001, \eta^2 = 0.2$). However, neither Color, nor its interaction with global shape Saliency showed a significant effect. Table 1 shows the rate of relevant answers for different conditions. Overall, results showed that, as expected, most of our designed visual analogies were answered by a relevant choice.

Table 1. Percentages of relevant answers in the different conditions for Experiment 1

	BW elements	Colorful elements
Less salient global shape	84	89
More salient global shape	94	96

Global Relations vs Object Relations in Visual Analogies

To compare global relation *versus* object relation answers, only problems with relevant responses (answered by either a global relation or an object relation choice) were considered. In this way, answers were labeled as global or object. We run another mixed-effect ANOVA with the same factors. In line with our hypothesis, the results demonstrated a significant effect of the saliency of global shapes in encoding and mapping global relations *versus* object relations ($F(1,130)= 131, p< 0.001, \eta^2 = 0.5$). Against our expectation, however, color did not have any effect. Table 2 shows the rate of global relations answers, among the problems solved by relevant answers.

Table 2. Percentages of global relations vs object relations answers in the different conditions for Experiment 1

	BW elements	Colorful elements
Less salient global shapes	46	53
More salient global shapes	81	80

Overall, the results confirmed that global shape saliency had a significant main effect on selecting the global/object relation choice. This means that, when the spatial arrangement of the objects in a visual analogy forms a good global shape, there is a higher likelihood of mapping global relations than in a visual analogy with less salient global shapes. However, color did not facilitate the encoding and mapping of object relations. We thought that color might be effective when used for unfamiliar shapes. Experiment 2 aimed to test this hypothesis.

3.4. Experiment 2

This experiment had three main goals: (i) it aimed to replicate the results of Experiment 1 about the significant effect of the saliency of global shapes on encoding and mapping global/object relations, (ii) it also aimed to investigate the effect of color, using unfamiliar individual elements, and (iii) finally, it aimed to study possible associations between the tendency to notice global/object relations and abstract visual reasoning skills. As mentioned in the introduction, we expected that unfamiliar elements, compared to familiar elements, would facilitate encoding and mapping global relations. We also expected that people with a higher tendency to encode and map global instead of object relations would be better visual problem solvers. To further investigate

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into these issues, in Experiment 2, we gathered response time in addition to the selected response option for each question. We expected that problems with salient global shapes would be solved faster. Moreover, in line with the results of global *versus* local tasks in image perception (Navon, 1977), it was anticipated that solutions based on the global path would be faster than those based on the object path.

Method

Participants. Eighty undergraduates participated in this Experiment. The ethics committee of the local university approved the research procedure. Participants were randomly divided into three groups (see procedure).

Materials. The questionnaire included two blocks. The first block consisted of 6 multiple choice visual analogies with different conditions corresponding to different groups. For one group, the problems were identical to the black-and-white visual analogies we had implemented in Experiment 1. For another group, visual analogies were the same as the visual analogies of the first group but with black-and-white unfamiliar elements (see Figure 7). For the last group, visual analogies were the same as the visual analogies of the black-and-white unfamiliar group but with colorful elements. To make unfamiliar elements colorful, elements used in one problem had different colors to avoid repetition of colors.

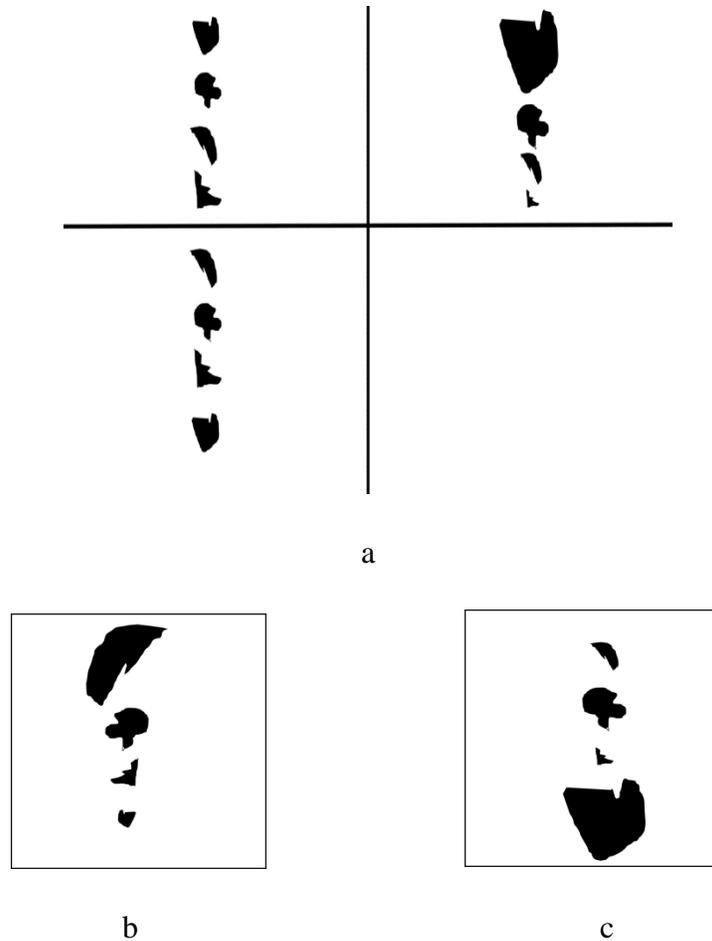


Figure 7: a) A visual analogy with the same objects as in Figure 4a but containing unfamiliar shapes, b) the global response option, and c) the object response option

For the unfamiliar shapes, we designed some shapes without any direct reference to familiar objects. To test whether the designed shapes would be unfamiliar for others too, we asked participants of the study a question about the familiarity of some of the shapes used in the visual analogy block, after participants finished the block. More than 97% of the answers confirmed that the designed unfamiliar shapes were unfamiliar for our participants.

The second block was mainly designed to measure abstract visual reasoning. To this end, we used the 12-item short version of ARPM designed by Arthur and Day (1994). The second block was the same for all groups.

Procedure. The questionnaire was administered via Qualtrics. Following the instruction page, the problems were shown one per page. Subjects had to select one of the response options

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before moving on to the next problem. The problems of the first block were presented in a random order. No time limit was set for completing the questionnaire. Therefore, all participants answered to one question about the familiarity of one of the designed unfamiliar shapes. They had to give yes or no. Finally, all participants did the second block in the original order. It was mentioned that to solve each problem of the second block, they had at maximum 2 minutes.

The datasets generated and analyzed during the current experiment are available from the corresponding author on request.

Data Analysis. The selected response options and response times were extracted from Qualtrics and imported into a Python environment for analysis. Like Experiment 1, we used mixed-effect ANOVAs to analyze main effects and potential interactions between independent variables on problem-solving outcomes. To study differences between averages, we employed t-test. Also, to investigate relationships between independent factors, Pearson's correlation was used.

To explore associations between the performances of participants in the global/object visual analogies block and the 12-item short version of ARPM, we considered participants who correctly solved more than one out of 12 problems of the ARPM. We suspected that participants who did not get this minimum score either did the questionnaire without sufficient concentration or they did not understand the basics of visual problem solving. In this way, the data of 5 participants were not extracted for analysis. Overall, the data of the performance of 75 participants (25 for each condition) was extracted and analyzed.

Results and discussion

First, regarding the relevant/irrelevant responses, we found no differences between conditions. In fact, the rate of relevant answers was very high for all conditions (0.95 for the black-and-white familiar condition, 0.98 for the black-and-white unfamiliar condition, and 0.95 for the colorful unfamiliar condition). We suspected that the higher rates compared to Experiment 1 was due to the elimination of participants who presumably did not have enough deliberation for the ARPM block. For Experiment 1, we considered all participants, and it was possible that some of them did the task haphazardly, and, consequently, selected irrelevant responses more often. The saliency of global shapes was not significantly effective in noticing relevant responses, but the direction was in line with what we had found in Experiment 1: problems with more salient global

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shapes were more likely to be answered by a relevant choice than problems with less salient global shapes (0.98 vs 0.96).

Second, to compare responses based on global *versus* object relations, only problems with relevant responses were considered. In this way, answers were codified as 1 when the response was global and 0 for object. With this measure, we run a mixed-effect ANOVA with the Saliency of global shapes as the within-participant factor and Color condition as the between-participant factor. The black-and-white unfamiliar block and the colorful unfamiliar block were considered for this purpose. In line with Experiment 1, the results replicated the significant effect of the saliency of global shapes in encoding and mapping global relations *versus* object relations ($F(1,48)= 22.3, p < 0.001, \eta^2 = 0.32$) and the non-significant effect of color. That is, color affected the type of response neither for familiar nor for unfamiliar elements. Table 3 shows the rate of global relations answers, among the problems solved by relevant answers.

Furthermore, to study the possible effect of the familiarity of the individual elements on encoding and mapping global/object relations, we run another mixed-effect ANOVA with the saliency of global shapes as the within-participant factor and familiarity condition as the between-participant factor. To this end, the block-and-white unfamiliar block and the black-and-white familiar block were considered. The results replicated the significant effect of the saliency of global shapes on encoding and mapping global relations *versus* object relations ($F(1,48)= 21.5, p < 0.001, \eta^2 = 0.31$). However, against our expectation, the familiarity of objects had no significant effect. Table 3 shows the rate of global relations answers, among the problems solved by relevant answers. Surprisingly, familiar problems were solved more by global relations answers than unfamiliar problems, particularly for problems with a more salient global shape, although this difference was not significant.

Table 3. Percentages of global relations *versus* object relations answers in the different conditions for Experiment 2

	Familiar BW	unfamiliar BW	unfamiliar colorful
Less salient global shape	54	53	44
More salient global shape	82	71	75

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Regarding response time, the results of a mixed effect ANOVA considering the unfamiliar blocks (black-and-white and colorful) showed that the saliency of global shapes had a significant effect but with a low effect size on response time ($F(1,48)= 4.6, p = 0.03, \eta^2 = 0.09$). This suggests that problems with salient global shapes tended to be solved faster than problems with less salient global shapes. Comparing the black-and-white blocks, results of another mixed-effect ANOVA again showed the saliency of global shapes had a significant effect with a low effect size on response time ($F(1,48)= 5.3, p = 0.02, \eta^2 = 0.1$) while familiarity did not have any considerable effects—see Table 4.

Finally, we compared the averages of response time for problems solved by either a global choice or an object choice. As expected, results of t-test showed that problems solved through the global path were answered faster than those solved through the object path (medians were 15.6 vs 17.7 respectively, $t = 2, p = 0.04, \text{Cohen's } d = 0.19$).

Table 4. Medians of response time for a visual analogy in the different conditions for Experiment 2

	Familiar BW	unfamiliar BW	unfamiliar colorful
Less salient global shape	24	19	16
More salient global shape	21	18	13

Individual differences. The average score for the second block (the 12-item short version of ARPM) was 6.4 out of 12. To explore possible associations between the tendency to encode and map global relations (the participants’ global to object ratio) and abstract visual reasoning abilities (the score in the ARPM short test), we calculated the Pearson’s correlation between these measure for each condition. For the black-and-white unfamiliar condition, the results showed that the tendency to notice global relations was marginally correlated with abstract reasoning accuracy ($r = 0.38, p = 0.06$). The correlation showed a similar trend, but it was not significant in the case of black-and-white familiar problems ($r = 0.13$). Overall, the tendency to notice global relations in black and white images had a marginal positive correlation with abstract visual reasoning abilities ($r = 0.24, p = 0.09$). However, for colorful problems, there was a significant negative correlation between these measures ($r = -0.45, p = 0.02$). To explain this unexpected finding, we suspected that color often creates another kind of global property: color distribution (Castelhano &

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Henderson, 2008). In other words, color can be regarded as a global feature rather than an object one. Accordingly, some of those more skilled participants who selected object-based options were indeed responding according to a global color-based rule.

3.5. Experiment 3

In Experiment 1 and 2 the tendency to notice global/object relations were investigated. There, we found that only the saliency of global shapes had effects on encoding and mapping global/object relations. Furthermore, regarding individual differences, Experiment 2 demonstrated that the tendency to use the global/object paths had associations with visual problem solving abilities. However, because the designed visual analogies contained easy rules (addition, subtraction, and changes in size), mostly individuals could encode and map them unproblematically. Therefore, Experiment 2 has almost nothing to do with the ability to use the global/object paths but with the tendency to use the global/object paths.

One of the main objectives of this research was to explore human abilities to use the global/object paths, and their possible associations with general problem solving abilities. To do so, we aimed to design visual analogies with more complex rules. Also, to measure the abilities instead of the tendencies, we designed visual analogies with only one correct response option instead of visuals with two relevant responses corresponding to encoding and mapping global/object relations. In this way, we extracted the rules of problems in SET IV and V of RSPM (see Method).

To study the ability to the global/object paths, we needed visual analogies in which problem solving was possible only through one of the paths. In other words, we needed two corresponding groups of problems: (i) in one group, only global relations existed and reasoners had to encode and map global relations, and (ii) in the other group, only object relations existed and reasoners should identify object relations.

To do so, for the visual analogies of the first group, we used only one object (lines) with different spatial configuration reflecting spatial rules². Therefore, we eliminated the possibility of identifying any object relations for such visual problems. To solve them, reasoners should encode

² To implement some of the SRPM problems which had different rules for different sets of objects, we used lines and dots. However, visual relations between the images were still based on spatial configuration.

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and map global spatial relations between the images, the global path. Figure 8a shows as example of our designed visual analogies for the global path group. This was an implementation of a SRPM problem (E5). To solve it, one should compare the global shapes of the images and identify that the right image is the subtraction of the middle image from the left image. She or he cannot infer the rule by considering individual elements as all individual elements are lines.

On the other hand, for the visual analogies of the second group, we used different objects which did not form any meaningful spatial relations (see Method). In this way, we removed any global spatial relations for such visual problems. To solve them, reasoners should encode and map object relations between the images, the object path. Figure 8b shows as example of our designed visual analogies for the object path group (corresponding to Figure 8a). To solve it, reasoners have to notice relations between individual objects of the images in order to infer that the right image is the subtraction of the middle image from the left image. It is not possible to infer the rule by considering global spatial relations as the global shapes of the images are not alignable and the spatial positions (both absolute and relational spatial positions) of the individual objects are changing throughout the images.

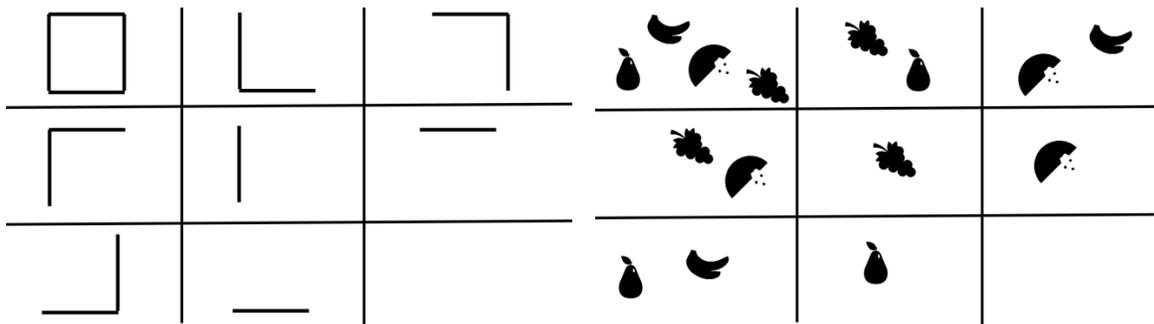


Figure 8: a) A visual analogy designed based on E5 from SRPM which contains only spatial relations, and b) a visual analogy designed based on E5 from SRPM which contains only object relations

There were two goals for Experiment 3: (i) to explore the difficulty of the global path/object path visual problems, and (ii) to investigate possible associations between the ability to use the global/object paths and visual problem solving abilities. To address these aims, we created an online questionnaire and distributed through Qualtrics.

Method

Participants. Sixty four undergraduates participated in this experiment. The ethics committee of the local university approved the research procedure. Participants were randomly divided into two groups (see procedure).

Materials. The questionnaire included two blocks. The first block was designed to measure problem solving skills. To this end, we used the 12-item short version of Advanced Raven Progressive Matrices (ARPM; J. Raven et al., 1993) designed by Arthur and Day (1994). The first block was the same for all groups.

The second block consisted of 7 multiple choice visual analogies with different conditions corresponding to different groups (see Appendix B and C). The problems were designed according to 7 problems of Set IV and V of SRPM (D7, D9, E5, E6, E7, E9, and E10). For one group, visual analogies were designed according to the global path –Figure 8a. There, only lines were used. To implement a problem based on its corresponding SRPM problem, each line in a problem of the global path group represents one involving element in its corresponding problem. For problems which have different rules for different groups of elements (D7 and E9), lines and dots were used to communicate two groups of elements. In these problems, rules applied to lines are different from rules applied to dots. However, like for the other problems, the rules could be inferred according to the spatial configuration of the images not specific elements (lines and dots).

For the other group, visual analogies were designed according to the object path –see Figure 8b. Problem of this group were designed using fruit shapes. To implement a problem based on its corresponding SRPM problem, each fruit in a problem of the object path group represents one involving element in its corresponding problem. For problems which have different rules for different groups of elements (D7 and E9), fruits shapes and geometric shapes were used to communicate two groups of elements. In these problems, rules applied to fruits shapes are different from rules applied to geometric shapes. The spatial positions of the fruits shapes and geometric shapes, however, were not separated in order not to induce any spatial categories. Spatial positions of the objects are changing throughout the images without communicating any possible spatial relations. So, the only way for inferring the rules is to consider relations between individual objects. Each problem in this group is corresponding to a problem in the global path group. In

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order to control possible effects of other factors in problem solving, corresponding problems have the same rule and the same number of elements.

Procedure. The questionnaire was administered via Qualtrics. Following the instruction page, the problems of the first block were shown one per page in the original order. It was mentioned that to solve each problem, they had at maximum 2 minutes. Subjects had to select one of the response options before moving on to the next problem. Thereafter, problems of the second block was shown in the order the reference problems appeared in SRPM. Half of participants solved the problems of the global path group and the other half solved the problems of the object path group.

The datasets generated and analyzed during the current experiment are available from the corresponding author on request.

Data Analysis. The selected response options and response times (only for the second block) were extracted from Qualtrics and imported into a Python environment for analysis. Regarding elimination rules, like what we did in Experiment 2, we considered participants who correctly solved more than one out of 12 problems of the second block. In this way, the data of 4 participants were not extracted for analysis. Overall, the data of the performance of 60 participants (30 for each condition) was extracted and analyzed.

Results and discussion

Regarding the first block, the average of the 12-item short version of ARPM was 6.8 out of 12. For the second block, the average score for the condition with only spatial rules was 5.83 out of 7 while the average for the condition with only object rules was 5.06 out of 7. Results of one-way ANOVA showed a significant difference between the conditions ($p = 0.05$) meaning that problems with only spatial relations were easier to solve than problems with the same rules but embedded in individual objects.

We also analyzed response times. To do so, we considered only response time for correctly solved problems. Results showed no significant differences in response time between problems containing only spatial rules and those containing only object rules (21.4 and 22.4 seconds respectively).

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Finally, we explored possible associations between two blocks. Results showed significant positive correlations between the accuracy of solving problems with only spatial rules and the accuracy of solving the short version of ARPM ($r = 0.46$, $p = 0.01$), and the accuracy of solving problems with only object rules and the accuracy taken from the short version of ARPM ($r = 0.7$, $p < 0.001$). These findings showed people with higher skills in rule induction are better in encoding and mapping both spatial and object rules.

3.6. Experiment 4

Findings of the first experiments suggest that people tend to select the global choice when solving visual analogies with two possible correct responses (i.e., based on either global or object relations), particularly when the global shape is salient. However, as in typical visual analogies, all the images (first and second rows, and the response options) were simultaneously presented in our previous studies.

This contrasts with some verbal analogical problems (i.e. story-like analogies), where the target (current problem) is separated from the source (previous story containing an analogous solution). It means that reasoners have to retrieve the mental representation that they built during confronting the source and map it onto the target to infer an analogous solution for the target. In this way, to solve story-like analogies, building a mental representation of the source and retrieving it after a while is necessary for analogical reasoning. A common finding with this type of sequential presentation (e.g., Gick & Holyoak, 1980; Holyoak and Koh, 1987) is that people rely more on object-based similarities between the source and the target (superficial similarities), rather than on more global properties (i.e., common structure or abstract causal relationships).

Here, we aimed to explore to what extent the type of presentation of visual analogies (simultaneous or sequential) would modulate the tendency to select global/object choices. Based on the reliance on object similarities in story-like analogies commented on above and regarding the fact the mechanisms and strategies that support effective analogizing are also those that support visual problem solving. (Lovett & Forbus, 2017), we expected to observe more object-based choices in sequential presentations. Furthermore, we aimed to investigate whether the saliency of the global shape would also affect the encoding of global/object relations in the case of sequential presentation of visual analogies.

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To this end, we used the same black-and-white visual analogies of Experiment 1 but, instead of a simultaneous presentation of the first row (the source of the visual analogy) and the second row (the target), we separated them. Between the source and the target there was a time gap. So, unlike the previous visual analogies, people had to build a mental representation of the source (the images of the first row) and memorize it. Then, when the target appeared (the images of the second row accompanied by some response options to fill the missing image), people had to retrieve the mental representation of the source to map the relations encoded in it onto the target (see Figure 9).

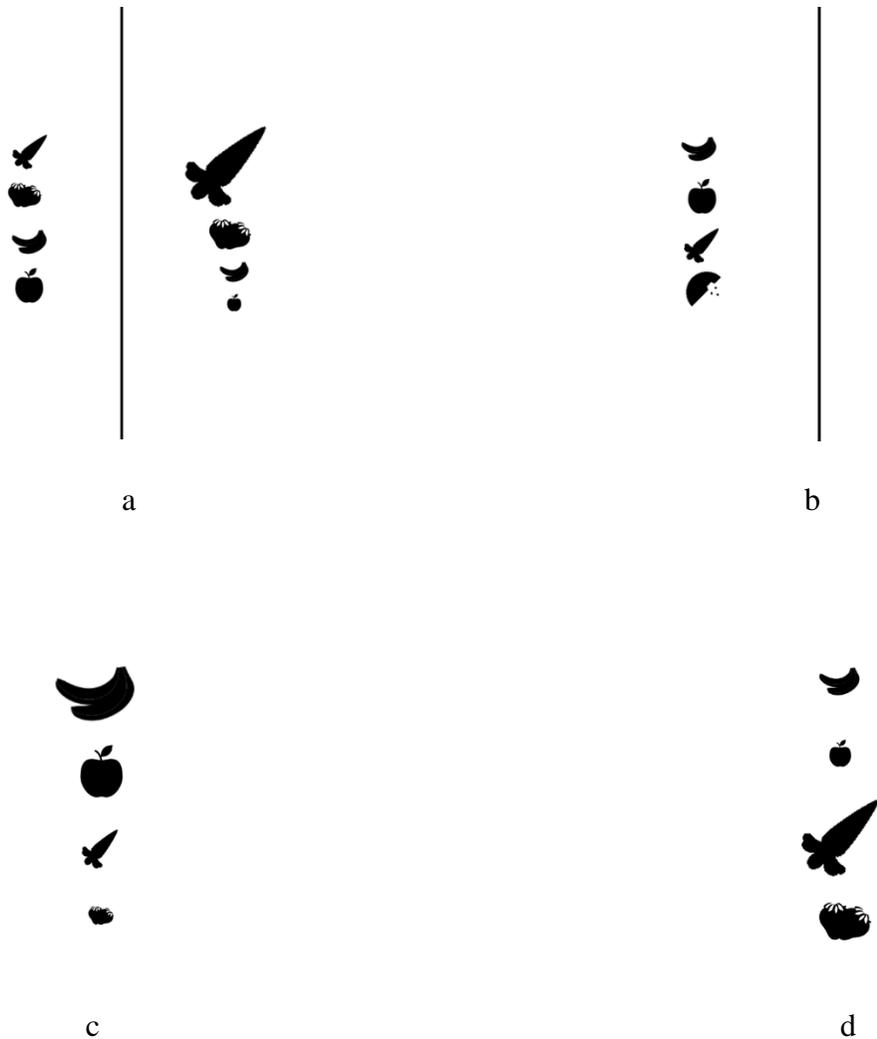


Figure 9: a) the first set of images (source) of a visual analogy in the sequential presentation, b) the second set (target), c) the global relation response option, and d) the object relation response option.

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Accordingly, a mental representation of the global shape or spatial properties of the source (i.e., the size of the fruit icons decreases from top to bottom in Figure 9) would lead to a global choice. That is, when the second row and the response options will be presented, the reasoner would tend to select the response option with analogous global relations (Figure 9c). On the other hand, a mental representation of the individual objects (i.e., the identity of the fruit that becomes the largest or the smallest in Figure 9) would lead to an object-based response. In this case, when the second row will appear, the reasoner would tend to select the option that shares analogous correspondences, based on the identity of the fruits (see Figure 9d).

Another objective of this experiment was to investigate the extent to which the tendency to build global mental representations in sequential visual analogies would facilitate analogical problem solving in the story-like format. To do so, we employed the famous fortress-story/radiation problem set which was firstly used by Gick and Holyoak (1980) and used by Gray and Holyoak (2019) in an individual differences study.

Method

Participants. Thirty-one undergraduates participated in this experiment. The ethics committee of the local university approved the research procedure.

Procedure. Following the instruction page, the problems were shown one by one. For each problem, participants firstly saw the first row –see Figure 9a. Then, they waited for six seconds. During this time, they saw “please wait” on the screen. After six seconds, the second row (an image on the left part and missing image on the right part) appeared along with the response options. Participants had to select one of the options before moving on to the next problem. The problems were presented randomly. No time limit was set for completing the questionnaire. After the visual analogy task, participants were asked to read the fortress story carefully. Thereafter, participants solved two multiple-choice comprehension questions about the story they had read (one was about the problem and the other was about the solution). Then, they performed an unrelated task for around 5 minutes after which they were asked to solve the radiation problem by writing their answers. Then, they were asked to rethink the radiation problem by considering the task the story they had first read.

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The datasets generated and analyzed during the current experiment are available from the corresponding author on request.

Results and Discussion

Regarding the verbal analogy task, solutions to the radiation problem were scored as to whether they proposed the critical idea of having weaker rays converging at the tumor (1 for successful analogical transfer and 0 for unsuccessful). The scores given by two raters had a reliable internal consistency (the inter-rater reliability was 0.96).

Like Experiment 1 and 2, responses were analyzed at two levels. First, regarding relevant/irrelevant responses, results of ANOVA showed that the saliency of global shapes had a significant effect on and mapping of relevant relations ($F(1,30)= 5$, $p = 0.03$, $\eta^2 = 0.14$). This is in line with the results of Experiment 1 and 2 showing that problems with more salient global shapes were more likely to be solved by a relevant response option than problems with less salient global shapes (0.95 vs 0.88 respectively). However, as expected, most of problems were solved by relevant response options.

Second, to compare global relation *versus* object relation answers, only problems with relevant responses (answered by either a global relation or an object relation choice) were considered. As expected from the results of Experiment 1 and 2, results of the ANOVA demonstrated the significant effect of the saliency of global shapes in encoding and mapping global relations *versus* object relations ($F(1,30)= 16.2$, $p < 0.001$, $\eta^2 = 0.35$). It means that the likelihood of encoding and mapping global relations was significantly higher in problems with more rather than less salient global shapes (0.73 vs 0.45 respectively).

We compared these results with the results of the black and white version of the first block of Experiment 1 in which there were the same visual analogies but without a separation between the first row (source) and the second row (target). Specifically, we calculated the global to object rate, which was defined in Experiment 2, for both groups. Results showed that although the global to object rate was lower for sequential than for simultaneous visual analogies (0.64 vs 0.72 respectively), the difference was not significant ($t = 1.6$, $p = 0.11$). However, further analysis revealed that the global to object rate in problems with salient global shapes was significantly lower for the sequential rather than simultaneous presentation (0.75 vs 0.87 respectively; $t = 2.1$,

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$p = 0.04$, Cohen's $d = 0.4$), while there was no difference regarding the global to object rate in problems with less salient global shapes (0.51 vs 0.51). This finding suggests that the tendency to notice global rather than object relations in problem with salient global shapes decreases when reasoners must build and remember a mental representation of the relations.

Regarding the analogical problem-solving task, solutions to the radiation problem were scored as to whether they proposed the critical idea of having weaker rays converging at the tumor. The scores given by the two raters had a reliable internal consistency (the inter-rater reliability was 0.94). Before the hint, only one out of 31 participants solved the radiation problem (3%). However, after the hint, 61% of them solved the radiation problem using analogical transfer from the fortress story. Both findings are in line with previous literature suggesting that although analogical transfer from a distant source is difficult, after a hint recommending reasoners to use the source, many reasoners can map the source onto the target and solve the target by generating an analogous solution.

Finally, results showed a significant positive correlation between the global to object rate and analogical problem-solving score after the hint (Spearman $r = 0.45$, $p = 0.01$), meaning that people with a higher tendency to notice global *versus* object relations have higher skills in analogical transfer between distant domains. Further analysis showed that while the global to object rate in problems with salient global shapes had a very strong correlation with the analogical problem-solving score ($r = 0.57$, $p < 0.001$), there was no significant correlation between the global to object rate in problems with less salient global shapes and the problem-solving score ($r = 0.2$, $p = 0.28$). This finding emphasizes on the significance of noticing and memorizing global *versus* object relations in sequential visual analogies with salient global shapes.

3.7. Experiment 5

In the previous studies, the idea of the global/object design was introduced, explained, and investigated. The results showed that there were individual differences based on the tendency to encode and map global/object relations in solving visual analogies. However, such individual differences were only reported for the end-product of visual reasoning –i.e. the selected choice. In this experiment, we aimed to employ the eye tracking approach to study possible associations between different strategies and the two paths of reasoning: the global vs the object path.

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Most models of analogical reasoning are based on behavioral findings (e.g., Day and Gentner, 2007; Markman and Gentner, 1993; Richland et al., 2006; Sternberg, 1977). That is, researchers have mostly gathered the selected choice and sometimes response time during visual problem solving. One potential limitation of behavioral data is that a behavioral response can provide a measurement for the outcome, but it does not capture the strategy used to arrive at that response (Vendetti et al, 2017). Eye tracking approach, however, allows researchers to identify more closely different strategies. This approach employs eye movements to study how participants inspect the visual displays in order to infer the critical relations.

Previous studies using eye movement analyses have found that more skillful reasoners prefer some specific strategies to other strategies for analogical problem solving (Glady et al., 2014; Gordon and Moser, 2007; Thibaut et al., 2011, Thibaut and French, 2016; Vendetti et al., 2016). Particularly, Vendetti et al. (2016) introduced three strategies of solving multiple-choice visual analogies with the 2*2 format (A:B::C:?). Theoretically, they have roots on analogical reasoning models.

- (i) Project-first strategy: analogies are solved by first generating a rule that relates the A and B terms, then mapping the A and C terms, and finally applying a similar rule filling the missing piece (Sternberg, 1977; Hummel and Holyoak, 1997; Doumas et al., 2008). Although this model was initially introduced for analogies with semantic relations, it can be applied to visual analogies as well. For example, to solve the simple visual analogy shown in Figure 1 through the project-first strategy, one firstly considers the top left and top right images (analogous to A and B in the A:B::C:D format), then identifies the relation between the two (the subtraction rule), and finally map the relation onto the second pair (the bottom left and bottom right images) to fill the missing image.
- (ii) Alignment-first strategy: This strategy begins with aligning the A and C terms, rather than the A and B terms. Once this alignment is made, the next step is to align the B item with the target (Gentner and Forbus, 2011; Gentner, 1983, 2010; Falkenhainer et al., 1989). As an example, to solve the analogy in Figure 1, one firstly tries to align the top left image with the bottom left (both consist of a geometric shape with two

- intersecting lines). Once the alignment is complete, she aims to find the proper choice which completes the alignment between the top right image and the bottom right.
- (iii) Limited strategy: The third strategy is based on the idea that the A, B, and C terms in an analogy problem are related in multiple ways (Chalmers et al., 1992; Thibaut et al., 2011; Glady et al., 2012). This strategy was considered as a less efficient strategy (Thibaut et al., 2011). To solve the visual analogy in Figure 1, one can move back and forth between the matrix area and the response space comparing the images with different response options in order to find an option which seems the best match for the bottom left image without possibly encoding relations between A, B, and C.

As Vendetti et al (2016) indicated, the project-first strategy seemed to be a more optimal strategy than the alignment-first strategy; the rate of using the project-first strategy was correlated with the accuracy of problem solving. Additionally, previous studies showed that the project-first strategy (primarily focusing on AB relations rather than AC relations) is optimal for solving analogy problems (Hummel and Holyoak, 1997; Doumas et al., 2008).

In this Experiment, we aimed to use the eye tracking approach to study problem-solving strategies in solving global/object visual analogies. Particularly, the association between the project first/ alignment first/ limited strategies and the global/object tendency was investigated. Based on the results of Experiment 2 indicating that the global path had a significant positive correlation with visual problem solving skills, we expected that the global path would be associated with the project-first strategy as the most optimal strategy for visual problem solving.

Method

Participants. Twenty-three undergraduates participated in this experiment. The ethics committee of University of Barcelona approved the research procedure.

Material. The questionnaire included 6 multiple-choice figural analogies, Problems were presented as 2*2 matrices with an empty matrix with 3 optional choices: One matched an object relation, another one matched a global relation, and the third one was a choice which was not the result of any true mapping –a copy of the bottom-left matrix. The problems were identical to the black-and-white problems used in Experiment 1.

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Eye tracking Apparatus. Stimuli were presented using the Psychopy software synchronizing stimulus presentation with an eye tracker. Participants were seated comfortably in front of a monitor (15.6-inch monitor, 1600×900 pixel resolution) which contained the Tobii Pro Fusion eye tracker. Distance was calibrated individually so that each participant focused on the middle of the screen, within a range of 50–80 cm. The calibration process was done before starting the main tasks using Tobii Software –a software designed for conducting experimental research. The Tobii Pro Fusion built-in cameras capture data with a temporal resolution of 250 Hz and average spatial resolution of 0.3° of visual angle. The cameras can automatically compensate for small head movements (within a 40×25 cm area at 65 cm distance); thus, participants' heads were not restrained. The cameras recorded eye gaze position of the left and right eyes. For each data point, the eye tracker gave a corresponding validity score indicating to what extent the data was valid. It could be 0 as non-valid or 1 as valid.

Procedure. After the instruction page, two easy visual analogies (with the true/false format) were presented to make participants familiar with the implementation of visual analogies. Responses were recorded for these problems. Thereafter, the visual analogy questionnaire was presented. The problems were shown one by one (one in each page). Participants could see each problem and the proposed choices simultaneously on the screen. They had to select one of the choices and click the next button (at the right end of the screen) before moving to the next problem; each problem had at most 60 seconds to respond.

Measures. For each question, the selected response, the response time, and relative gaze positions on the screen were recorded.

Data analysis. For the behavioral analysis, the data was extracted from Psychopy and imported to Python environment for analysis. To test the statistical significance of differences, we used t-test. Also, to investigate relationships between independent factors, Pearson's correlation was used.

For the eye movement analysis, we defined two levels of AOI. First, at the highest level, two AOIs were defined as the matrix area, where the images of the problem were presented, and the response area, where the response options were presented. Second, at a lower level, we explored eye movements between and within the different matrices. To do so, as shown in Figure

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4, 10 areas of interests (AOIs) were defined as: (i) A: the area of matrix A (zone A), (ii) B: the area of matrix B (zone B), (iii) C: the area of matrix C (zone C), (iv) G: the global relations choice, (v) O: the object relations choice, and (vi) I: the irrelevant choice (see Figure 10). We defined a fixation on an AOI when a participant fixated her or his eyes on the AOI more than 75 milliseconds.

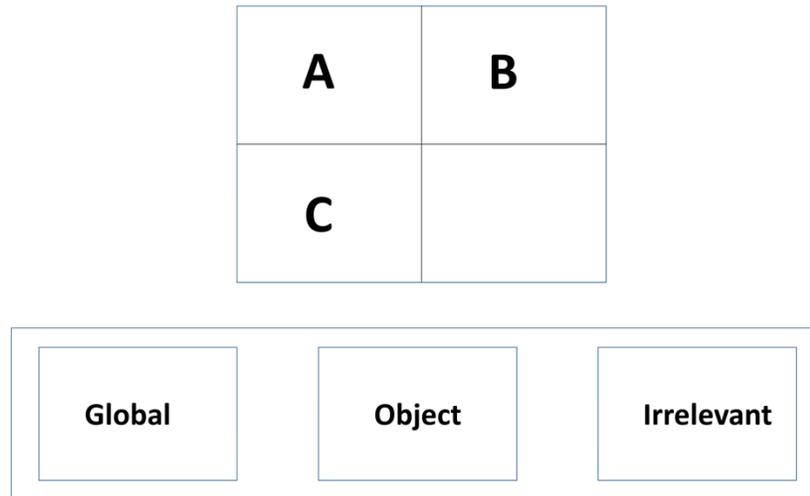


Figure 10: The implementation of areas for zone analysis (the order of the choices varied across the problems).

Specifically, to explore the problem-solving strategies, fixation transitions were considered. Based on the definitions of the project-first strategy, the alignment-first strategy, and the limited strategy, the considered transitions were: (i) AB transitions –transitions from A to B or from B to A, (ii) AC transitions –transitions from A to C or from C to A, (iii) BR transitions – transitions from B to R (the response area) or from R to B, (iv) CR transitions –transitions from C to R or from R to C, and (v) MR transitions: the transitions from any of the images on the matrix area to the response area or from the response area to the matrix area. Considering these transitions, for each solved problem, three tendencies were calculated:

- 1- The project-first strategy tendency: Because the project-first strategy involves back and forth movements within the first pair (A and B) to encode relations and then applying the relations to the second pair (C and the missing matrix), the project-first tendency was defined as the percentage of AB transitions to all transitions + the percentage of CR transitions to all transitions.

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- 2- The alignment-first strategy tendency: Because this strategy involves back and forth movements within the first element of the pairs (A and C) to make an alignment and then applying the same alignment to the second element of the pairs (B and the missing matrix), the alignment-first tendency was defined as the percentage of AC transitions to all transitions + the percentage of BR transitions to all transitions.
- 3- The limited strategy tendency: because the limited strategy involves back and forth between the matrix area and the response area, the limited tendency was defined as the percentage of AR, BR, and CR to all transitions.

Elimination rules. We eliminated eye movements of problems with an average of validity scores lower than 0.8. Eye movement analyses was done in Python environment after the data was transferred from Psychopy.

Results and discussion

Behavioral analysis. 12% of all problems were not answered or answered by irrelevant choices, 18% of them were answered by object choices, and the other 70% were answered by global choices. As expected, global shape saliency had a significant effect on choosing global choices ($t = 14, p < 0.001$) meaning that problems with a salient global shape were more likely to be answered by a global choice than problems with a less salient global shape (84% vs 55% respectively). Regarding response time (RT) analysis, in line with Experiment 2, statistics showed that global relations were encoded and mapped faster than object relations (means of solution times for problems solved with global *versus* object choices were 14.8 sec. vs 20.3 sec., respectively; $t=7, p = 0.009$).

Eye movement analysis. All problems solved by the 23 participants were taken into account because their average validity score was higher than 0.8. In total, the average validity was 0.98. To explore different strategies leading to encode and map either global relations or object relations, only problems solved by relevant choices (either a global choice or an object choice) were considered. Overall, 121 problems from 23 participants were analyzed.

At the highest level, we considered two AOIs: the matrix area and the response area. In total, 1610 fixations were extracted (859 for the matrix area and 751 for the response area). Regarding the number of fixations per question, results showed that problems solved by a global

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choice had significantly fewer fixations than problems solved by an object choice (mean numbers of fixations per question were 15.5 vs 21.7; $F=7$, $p=0.008$). This was in line with the behavioral finding indicating that problems answered by a global choice were solved faster than those answered by an object choice.

Regarding images as AOIs, in total, 3928 fixations were extracted (around 32.5 fixations in average for each question). As expected, the average number of fixations was significantly smaller for problems solved by global choices than for problems solved by object choices (29.8 vs 42.6; $t=8.2$, $p < 0.004$). Table 5 shows the distribution of the number of fixations among images, the proportion of time spent on each image, and the average of fixation time on each image. Furthermore, Figure 11 shows the proportion of time spent on each AOI for both types of responses. There were no significant differences between the global vs object response options regarding the proportion of time spent in the AOIs of the matrix area.

Table 5. The distribution of zone fixations

	Porportion of number of fixations	Proportion of time spent	Average of fixation duration
A	0.21	0.19	382 ms
B	0.22	0.19	365 ms
C	0.21	0.21	436 ms
G	0.16	0.20	553 ms
O	0.12	0.13	485 ms
I	0.07	0.07	427 ms

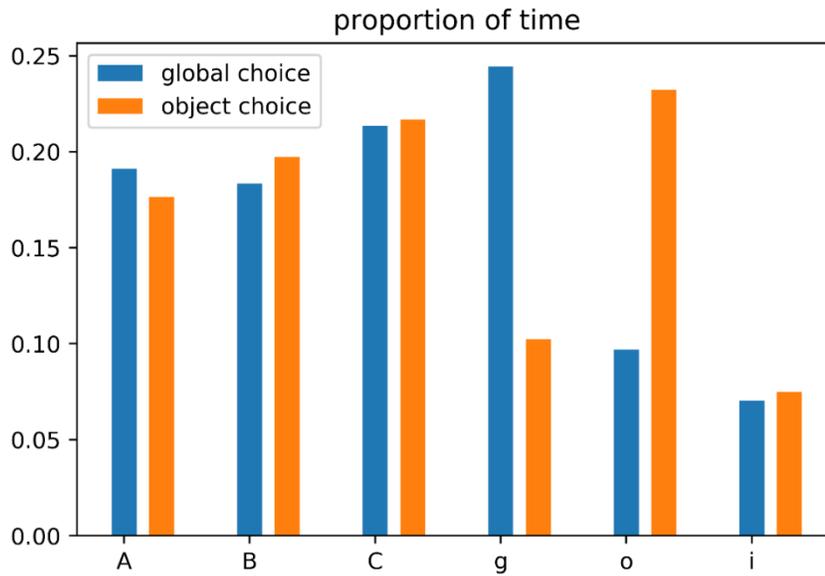


Figure 11: Proportion of time spent in each AOI for problems solved by a global/object response option

Transitions between images. Overall, 2480 valid transitions were extracted. Table 3 shows the proportions of each type of transition. Regarding all problems, the average of the project-first tendency was 38% and the average of the alignment-first tendency was 30%. Percentages of certain transitions varied according to the type of solution (global vs object choice). Specifically, as shown in Figure 12, solving a problem by a global relation choice included more A to B, B to A, C to R and R to C transitions which are the constituent transitions of the project-first tendency. On the other hand, solving a problem by an object relations choice involved more A to C, C to A, B to R and R to B transitions which are the constituent transitions of the alignment-first strategy. In this way, problems solved by global choices had a higher project-first tendency, compared to problems solved by object choices (54% vs 40%, $t = 10.1$, $p = 0.002$). Moreover, problems solved by global choices had a lower alignment-first tendency compared to problems solved by object choices (27% vs 38%, $t = 10.4$, $p = 0.002$). Regarding the limited strategy (any transitions from A, B, or C to the response area or vice versa), no differences were reported between the two groups. Results confirmed our expectation suggesting that the global path is associated with the most optimal strategy (the project-first strategy) while the object path is not.

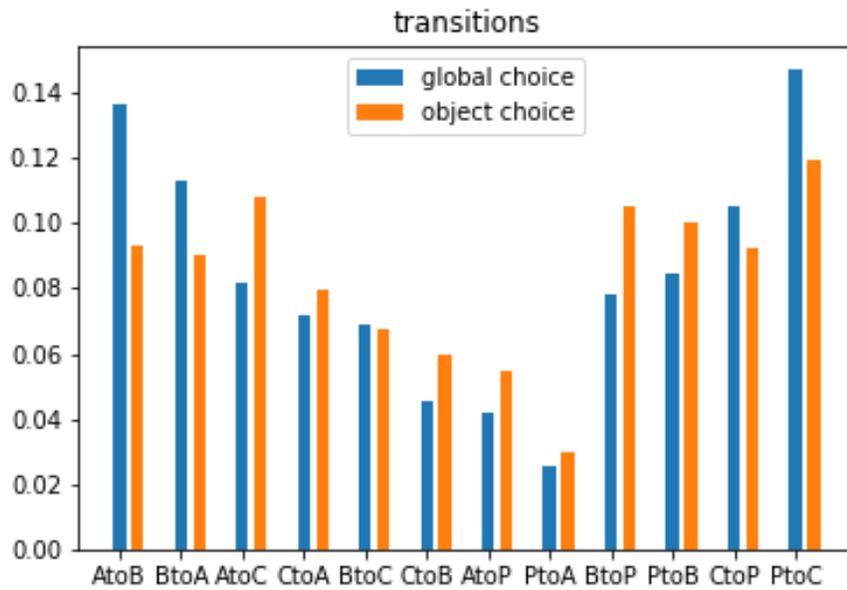


Figure 12. Proportion of transitions for problems solved by a global/object response option

Individual differences. Coherent with the association between RT and global to object rate, there was a significant negative correlation between the average of the number of fixations and the global to object rate ($r = -0.47$, $p = 0.02$) meaning that people with a higher tendency to map global relations had less eye fixations in visual problem solving. Figure 13 shows this association in the eye-movement patterns of two participants with low and high global to object rate. Furthermore, the project-first tendency was positively correlated with the global to object rate ($r = 0.58$, $p = 0.03$), while the alignment-first tendency was negatively correlated with the global to object rate ($r = -0.57$, $p = 0.04$). These findings were in line with our expectation confirming that a higher tendency to encode and map global relations is positively associated with the using the most optimal strategy (the project-first strategy). In other words, the results indicate that people with a higher tendency to use the global path are more skilled visual problem solvers. This is consistent with the results shown in Experiment 2 and 4: the global to object rate was correlated with both visual and verbal problem solving abilities.

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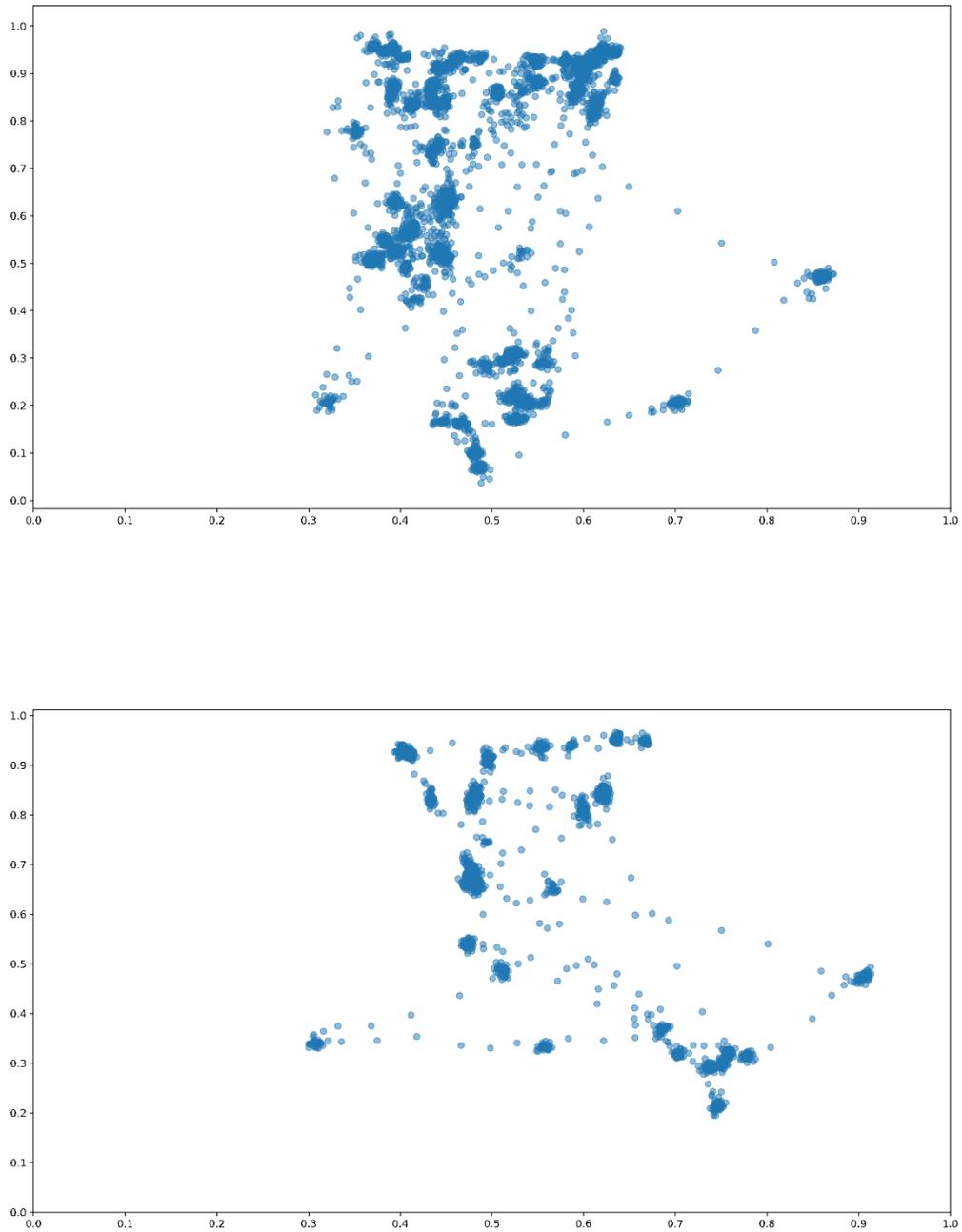


Figure 13: Top) the scatterplot of the eye movements of a participant with low global to object rate solving a problem, and bottom) the scatterplot of the eye movements of a participant with high global to object rate solving the same problem.

3.8. Discussion

One of the main objectives of this research was to investigate when reasoners identify and map object relations and when they identify and map global relations. Specifically, we hypothesized that problems with good global forms would promote the mapping of global relations. In the case of images with less salient global shapes, it was also expected that more salient objects would promote the mapping of relations between objects, compared to problems with less salient objects. The results of studies 1 and 2 consistently supported the first hypothesis: good global forms facilitated global relation answers. However, contrary to our expectation, neither object familiarity, nor color promoted noticing object relations.

Furthermore, the tendency to map global rather than object relations correlated with performance in visual and verbal analogical problem-solving tasks. Studies 2 and 4 showed a significant positive correlation between the global to object ratio (in the case of black-and-white problems) and different cognitive capacities (visual as well as verbal problem solving).

Below the tendency to use the global/object paths, Experiment 3 showed that problems solved by the spatial path alone were easier than those solved by the object path alone (83% vs 72% respectively). Moreover, performance in both types of problems was also correlated with visual problem-solving skills. Finally, Experiment 5 demonstrated that people with a higher tendency to notice global rather than object relation tended to use the most optimal strategy in visual problem solving. These points are further discussed in the following sections.

Global path vs object path. We designed visual analogies to investigate two distinct visual analogical reasoning paths. One choice in each problem was the end-product of the global path, whereas a different one was the end-product of the object path. We predicted that selection of one or the other would be the result of comparison, difference identification, and mapping through the corresponding path. Our results confirmed that visual reasoning could be performed at different hierarchical levels (Forbus & Lovett, 2021; Lovett & Forbus, 2017), highlighting the importance of the global path. Specifically, the findings that most of the problems were solved through the global path and that this path produced faster responses, regardless of the familiarity or color of the objects, suggest that global correspondences could be generated directly, bypassing the local correspondences (see also Love et al., 1999).

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While problems with more salient global shape were mostly solved through the global path, both paths were equally used in the case of problems with less salient global shape. It is also worth noting that, compared with participants with higher abstract reasoning skills, participants with lower reasoning skills tended to select the object choice rather than the global choice (see also below). Accordingly, although the object path produced slower responses, it was not necessarily more difficult than the global path.

Element saliency. Previous studies have suggested that familiar objects increase the likelihood of identifying relations between objects (Roberts et al., 2000). Accordingly, in visual analogies with less salient global shapes, we expected that familiar objects would increase the likelihood of identifying object relations compared to abstract objects. Against our expectation, however, the results ruled out the effect of object familiarity on mapping object relations. Furthermore, colored objects had no effect on the likelihood of encoding and mapping object relations. One of the differences between the problems that we designed and the problems used in previous studies was that our problems contained more objects that formed a global organization. Hence, we suspect that, in accordance with gradual global precedence in visual perception (Martin, 1979), global features become more important than local ones in visual problem-solving as the number of elements increases. Therefore, while object familiarity might affect the encoding of object relations in the case of problems with few elements, it will be less effective in the case of problems with more elements. Further research is needed to investigate the interactive effect between the number of elements and object saliency on visual problem-solving.

Meanwhile, regarding color, previous studies have shown that color distribution can be seen as a global property (Castelhano & Henderson, 2008). In other words, color distribution can be perceived without object identification (Greene & Oliva, 2009). Therefore, color-based relations can be regarded as global relations –relations between global features– rather than object relations, which involve object identification (also see Individual differences in mapping global relations).

Sequential visual analogies. To study possible associations between the tendency to map global/object relations and verbal analogical problem solving, we designed and tested visual analogies in which the source (the first set of images) and the target (the second set of images) were presented sequentially. One of the goals was to investigate whether memorizing and

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retrieving global instead of object relations would be associated with a higher capacity to use the underlying structure of the fortress story to solve the radiation problem. To this end, Experiment 3 tested, as far as we know for the first time, a sequential presentation of the global/object visual analogies. Results confirmed our expectation showing that better analogical problem solvers tend to notice, memorize, and retrieve global relation to a larger extent than less skilled participants. Furthermore, compared to the simultaneous presentation, in the sequential presentation, participants tended to map object relations more often. This finding suggests that memorizing and retrieving global/object relations can be regarded as another factor affecting visual problem-solving processes. Further research can study different aspects of the sequential vs simultaneous presentations of visual analogies.

Individual differences in mapping global relations. Our results showed that people who tended to map global relations more often than object relations were more able to solve visual analogies than people with the opposite tendency. These results are in line with the observation that experts take a more holistic approach compared to novices in specific problem-solving domains (Schiano et al., 1989).

Unexpectedly, for colored problems, the results obtained in Experiment 2 indicated that more skilled visual reasoners showed a higher tendency to encode and map object relations (based on color) instead of global relations (based on the global shape). One possible explanation is that, as mentioned before, color can be regarded as a global feature of an image (Castelhano & Henderson, 2008; Greene & Oliva, 2009). Accordingly, object relations in colored problems could be considered as global as the spatial arrangement of the individual objects. If this were the case, the global-to-object rate in colored problems should not be considered in a similar way to the global-to-object rate in black and white problems. Overall, the results of this research suggest that a higher tendency to notice global instead of object relations indicates higher skills in encoding visual rules. For colorful problems, better problem solvers might encode color-based relations to a larger extent than global shape-based relations.

Apart from that, Experiment 3 showed that the global to object rate in the sequential presentation of visual analogies was positively correlated with analogical transfer in the context of problems described in a narrative. Considering that the global to object rate is also positively

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associated with visual problem-solving skills, this finding is in line with what Gray and Holyoak (2019) found: visual and verbal analogical problem-solving skills are correlated with each other.

Finally, Experiment 5, in line with studies 2 and 4 showed a positive correlation between the global to object ratio and using the optimal strategy in visual problem solving. Previous studies suggested that the project-first strategy, primarily focusing on AB relations rather than AC relations in visual analogy pairs, seemed to be a more optimal strategy than the alignment-first strategy, focusing on AC relations rather than AB relations (Vendetti et al, 2016; Hummel and Holyoak, 1997; Dumas et al., 2008). Together with the results of Experiment 5, people with a higher tendency to use the global rather than the object path were better in problem solving. They were better not only regarding the end-results of problem solving, but also regarding the strategy they used for problem solving.

Holistic vs piecemeal strategies. During our analysis in Experiment 5, we realized that some participants had longer fixations on the images of the matrices area. In fact, while the fixation duration did not differentiate the global and object paths of reasoning, there were differences among participants. Such differences were comparable to the differences in fixation durations reported by Khooshabeh et al (2013). There, researchers investigated two different strategies (holistic and piecemeal) in mental rotations in a classic mental rotation study. The holistic strategy was previously proposed by Shepard and Metzler (1974) as analogue mental rotation process akin to physical rotations. In other words, a holistic strategy involves rotating the mental image as a whole. However, Khooshabeh et al (2013) observed that people could also employ a piecemeal strategy which involves decomposing the mental image into pieces mentally rotating one piece into congruence with the comparison figure, and then applying the same rotation to the other parts of the figure to see if they match (Khooshabeh et al, 2013).

Although such strategies have been studied in mental rotations, they can be relevant in visual analogies –especially visual analogies in which the images contain spatial hierarchical levels. To compare two images, one can use either the holistic or the piecemeal strategy. One can holistically perceive one image as a whole, build a mental representation of it, and then compare the mental representation to the other image. This is similar to the holistic strategy in mental rotation tasks. On the other hand, one can break the images into some parts, then compare one part

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of one image to the corresponding part of the other. This is similar to the piecemeal strategy in mental rotation tasks.

Accordingly, there might be two different strategies for encoding and mapping global relations: the holistic and the piecemeal strategies. For example, to solve the visual analogy in Figure 4a, one can holistically perceive the top left image (a vertical line at the global level), build a mental representation of it, and then compare it with the top right image. On the other hand, one can compare the two images in a piecemeal way. That is, through the global path, one can compare the top element in the left image with the top element in the right, the second top with the second top and so forth. Further research is needed to investigate the holistic/piecemeal strategies in solving visual analogies, especially visual analogies with the global/object design

In our research, we did not explore fixations on individual objects of the images of a visual analogy. However, we suspect that longer fixation durations on the images of a visual analogy might be interpreted as perception and comparison of whole images (the holistic strategy) while shorter fixation durations as perception and comparison of parts of images (the piecemeal strategy). Our findings suggested no associations between the global/object paths of reasoning and the holistic/piecemeal strategies. However, differences in the average of fixation durations among participants might be related with different use the holistic and piecemeal strategies. As an example, we have two participants with the global to object rate equal to 1; the average of fixation durations in the matrices area for one of them was 555 ms, while for the other it was 296 ms³

³ We found no associations between the project-first/alignment-first strategies and fixation durations.

Chapter 4

Spatial Structures in Story-like Analogical Problem

Solving

4.1. Introduction

Analogical problem solving refers to the use of a known problem (the analogous *source*) as a guide for developing a solution for a novel problem (the *target*; Beveridge & Perkins, 1987; Gick & Holyoak, 1980 and 1983; Holyoak & Koh, 1987; Kubritch et al., 2017; Pedone et al., 2001). Gick and Holyoak (1980) performed the first systematic set of experiments to investigate analogical reasoning using the story-like format. The target they introduced was the radiation problem designed by Dunker (1945; see the Appendix E). Based on this problem, they designed the fortress story, which has frequently been used as a structurally analogous source (see below and the Appendix D).

In the story-like format of analogical problem solving, the source is a story describing a problematic situation and its solution. Therefore, there are two states for the source: a problem state (in the fortress story, the impossibility of sending the entire army along the same path to destroy the fortress) and a solution state (the dispersion of the army along different paths converging at the fortress). The target, however, consists of only a problematic state in which the structural elements (goal, operator, and main constraints) are analogous to those of the problem in the source (in the radiation problem, the goal of destroying the tumor located in the stomach, the ray that can be reduced in intensity, and the impossibility of sending the high intensity ray along the same path are analogous to the corresponding structural elements of the fortress problem). In this way, the process of analogical reasoning involves four steps: (i) constructing a mental representation of the source and target problems, (ii) noticing analogous relations between them, (iii) mapping the correspondence between the key elements, and (iv) generating an analogous solution for the target based on the source (Chen, 1995; Gentner, 1989; Holyoak, 1984).

Importantly, mapping can occur at multiple levels of abstraction (Gick & Holyoak, 1980). At a low level, the detection of similar surface features among the compared objects would lead to similarity-based or attribute mapping (Gattis, 2004; Gentner & Markman, 1997; Gentner & Maravilla, 2018). At a higher level, the focus on the relations between objects, rather than on individual objects, would lead to structural mapping (Gentner, 1983; Gentner & Maravilla, 2018; Holyoak & Koh, 1987; Gattis, 2004; Markman & Gentner, 1993). Indeed, structural mapping is critical for successful analogical transfer from semantically distant sources. Nevertheless, in general, novices have difficulty in the spontaneous transfer of structural analogies without surface

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similarities (e.g., novices cannot spontaneously retrieve the fortress story for solving the radiation problem; Gick & Holyoak, 1980). But, after a hint indicating the relevance of the potential analogue, most of them tend to succeed in the required structural mapping (Gick & Holyoak, 1980 and 1983; Holyoak and Koh, 1987). This suggests that the main difficulty of analogical problem solving resides in the spontaneous retrieval of a memory representation referring to different objects from those objects in the target problem, but with analogous structural relationships.

Supporting this proposal, Holyoak and Koh (1987) showed that surface similarity facilitated spontaneous analogical transfer (the transfer before the hint). Hence, surface similarities seem to be useful in the initial stages of learning (Novick, 1988), or during the spontaneous retrieval of analogous episodes in more naturalistic tasks (Trench & Minervino, 2015). However, they can also lead to the misleading retrieval of non-analogous, structurally dissimilar sources (de la Fuente et al., 1989; Ross, 1989). Furthermore, the reliance on structural rather than surface similarity promotes farther transfer, increasing the chance of solving a larger set of analogous problems. In this regard, most of the studies about analogical problem solving have focused on ways to enhance the spontaneous use of structural analogies without surface similarities (e.g., Gick & Holyoak, 1980 and 1983; Kubritch et al., 2017).

Visuospatial representations in structural mapping

Gick and Holyoak (1980) analyzed the verbal reports of participants while solving the radiation problem, after having read the fortress story. They realized that many of the subjects employed visuospatial mental images representing the essential aspects of the dispersion-convergence solution. The researchers proposed that a mental spatial representation of the fortress story might elicit the critical spatial relations needed to transfer the solution to the radiation problem. Based on this suggestion, Gick and Holyoak (1983) hypothesized that a visual diagram of the dispersion-convergence solution would facilitate its use for solving the radiation problem. To this end, they designed a visual source containing an image with two parts. The left part was designed to convey the initial problematic state (a spatial representation of the ray as a thick arrow) while the right part was designed to convey the dispersion-convergence solution (multiple converging thinner arrows; see Figure 14). Regarding spontaneous transfer, the results showed that such abstract visual representation was not useful. However, after a hint recommending its use, most reasoners could adapt the dispersion-convergence solution to the radiation problem.

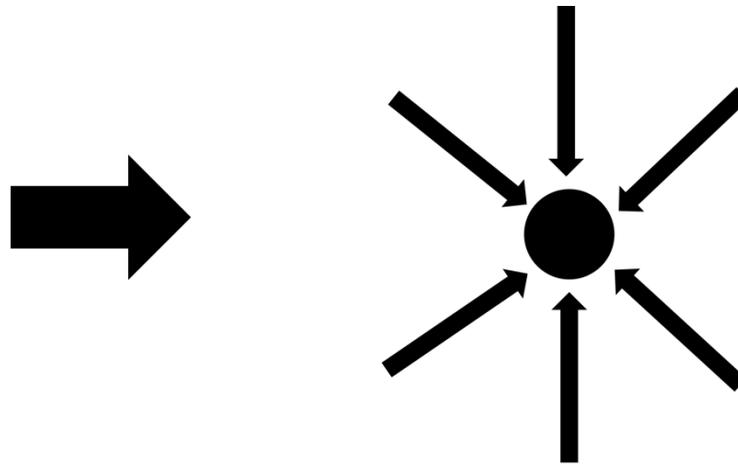


Figure 14. Visual diagrams similar to those used by Gick & Holyoak (1980) as the source of the radiation problem

Following this research, Beveridge & Parkins (1987) and Pedone et al. (2001) showed that, by improving some perceptual properties, abstract visual diagrams could be regarded as the source of an analogy even when spontaneous transfer was considered. Specifically, visual diagrams that intended to represent movement (e.g., sequences of arrows; Pedone et al., 2001) or gradual transformation of intensity (Beveridge & Parkins, 1987) seemed to communicate the dispersion-convergence solution more effectively. Pedone et al. (2001), and more recently Kubricht et al. (2017), also showed that the most effective diagrams for spontaneous analogical transfer were those that included the temporal dimension in an animated video.

However, it is not known whether the lower effectiveness of the static visual diagrams, compared to the dynamic ones, was due to the limitation in communicating the temporal dimension of the dispersion-convergence solution, or to the fact that they did not clearly represent a problem and its solution (see below). Furthermore, the benefit of the animations was previously observed in conditions where (i) the represented elements in the animation had some surface similarity with the common visual representation of rays and tumors (Pedone et al., 2001), or (ii) a verbal description of the dynamic diagram was also presented (Kubricht et al., 2017). Therefore, further research is necessary to better understand the elements of the visual displays that are more relevant for analogical problem solving.

4.2. Objectives and hypotheses

Based on the limitations commented on above, we aimed to investigate the effectiveness of visual diagrams that differed from previously used diagrams in the following ways. First, previously used visual diagrams generally lack a clear problematic state. For example, in the diagrams designed by Gick and Holyoak (1983), the right side of the image aims to represent the solution while the left side aims to represent the problem (see Figure 14). However, the latter does not show the problematic situation in which the thick element has to overcome some obstacles to arrive at the center. Beveridge and Parkins (1987) used more detailed visual representations, but they still failed to show the key elements of the radiation problem (i.e., obstacles to arriving at the center). Due to the lack of the problematic state and its corresponding key elements, the process of solving the radiation problem with such visual diagrams lacks a critical stage in analogical problem solving: the structural mapping between the problems of the source and of the target. We therefore designed, in Experiment 6, new visual analogues including the problem and solution states in a more explicit way than in previous studies.

Second, most of the visual representations previously used as the source for the radiation problem show some surface similarities with the common visual representations of tumor (circles) and rays (arrows). In previous literature, there are some visuals in which more dissimilar elements are used; however, they are presented with verbal descriptions (Kubricht et al., 2017). Accordingly, the high rate of analogical transfer for the visual representations reported in previous studies might be partly due to surface similarities and/or the presence of verbal descriptions. In this regard, in Experiment 7 we investigated the extent to which surface similarity in the visual animations might have a significant effect on the likelihood of analogical transfer, by comparing dynamic diagrams with and without surface similarities with the radiation problem, all without verbal descriptions.

Finally, to further investigate the role of spatial mental representations in structural mapping, Experiment 8, 9, and 10 were dedicated to explore the role of spatial properties in analogical transfer from a verbal source to a verbal target. In Experiment 8 we manipulated the number of spatial cues included in the verbal description of the fortress story. We hypothesized that a verbal source with fewer spatial cues than the standard fortress story would diminish the chance of analogical transfer, even after the hint. In Experiment 9, it was also investigated the role

of different types of summaries of the source. Specifically, we expected that a summary in the form of a visual schema would invite people to build a spatial mental representation of a scenario. Accordingly, the rate of analogical transfer would increase if a visual schema of the source were requested, compared with the request of a written summary. Finally, in Experiment 10, we manipulated the spatial properties of the target. We expected that a target with clearer spatial properties would facilitate analogical transfer, compared with the standard target, where the spatial properties are less explicit.

4.3. Experiment 6

Considering the issues discussed above, we aimed to design a set of new abstract visual representations as the source of the radiation problem with the following differences from previous representations: (i) the visual diagrams provided a more explicit representation of the problem/solution states, and (ii) there were no surface similarities between the elements of the visual diagrams and the common visual representations of the elements of the target. In particular, in line with Kubricht et al. (2017), we aimed to design visual representations of the fortress story as the source of the radiation problem, but representing the problem state more explicitly, using more abstract shapes, and without verbal descriptions (see Method).

We specifically wondered if the null effect of the length of the sequence of static images reported by Pedone et al. (2001) would be replicated in these new designs. Pedone et al. (2001) showed that a longer sequence of images (containing intermediate images between the initial and the final image) was no more effective than a shorter one (containing only the initial and final image; see Figure 14). Indeed, neither of them helped participants in the spontaneous analogical transfer, and both showed a similar rate of analogical transfer after the hint. However, we aimed to explore whether this effect would be significant in the context of a clearer representation of the initial problematic state. Given that a critical step in analogical problem solving is the understanding of both the problem structure and the process leading to the solution, a more extended representation of these elements might facilitate analogical transfer.

We also wondered whether the benefit of visual animations found in Pedone et al. (2001) and Kubricht et al. (2017), compared to static images, would be replicated with the new design. If the new static images were sufficient for communicating the dispersion-convergence solution, no difference in the likelihood of analogical transfer between static and dynamic visual displays

should be expected. However, if dynamic representations were more effective than static ones, this would confirm the proposal that movement (or apparent movement) is a critical aspect for communicating the causal relationship between the dispersion-convergence procedure and the goal of the problem (Pedone et al., 2001). As in Kubricht et al. (2017), we also assessed the extent to which participants understood the dispersion-convergence solution. Given that verbal descriptions were eliminated from our presentations, this measure would assess more clearly the comprehension of the visual displays. If animated displays communicate better the dispersion-convergence solution than static images, comprehension scores would be higher for the former than for the latter. Moreover, we expected to replicate the correlation between the quality of comprehension and successful analogical transfer.

Method

Participants. One hundred and sixty-two undergraduates participated in this experiment. The ethics committee of the University of Barcelona approved the research procedure. Participants were randomly divided into four groups, each of which was assigned to one condition (see procedure).

Materials. The target used in all the conditions was the radiation problem. For one condition (long sequence condition), the source was an abstract visual representation of the fortress story including the representation of the initial problematic state, some intermediate states, and the final state (the solution). In the design, all the key elements of the story (the fortress, the mines, and the group(s) of soldiers) were demonstrated with colorful geometric shapes (see Figure 15a). To avoid surface similarities with the elements of the radiation problem, a square was used for the fortress rather than a circle, because tumors are usually demonstrated with circles. Also, we did not employ arrows as demonstrations of groups of soldiers. Instead, these were represented as circles (see Figures 15 and 16).

Regarding the long sequence condition, the problem state showed a big circle being eliminated when approaching the border from different angles in two sets of images. The intermediate state showed in two sets of images that smaller circles could pass the border from different angles. One set of images showed that some (but not all) small circles could reduce the size of the central square (the summation effect). The final set of images showed that the small circles crossing the border through all possible paths caused the elimination of the square. To focus

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attention on the images, after each set, participants had to answer a visual question with two optional choices (see Figure 15). Each response was followed by a feedback page pointing to the correct choice. After viewing the sequence of images and completing the multiple-choice questions, participants were asked to write down what they understood from the visual images they had seen.

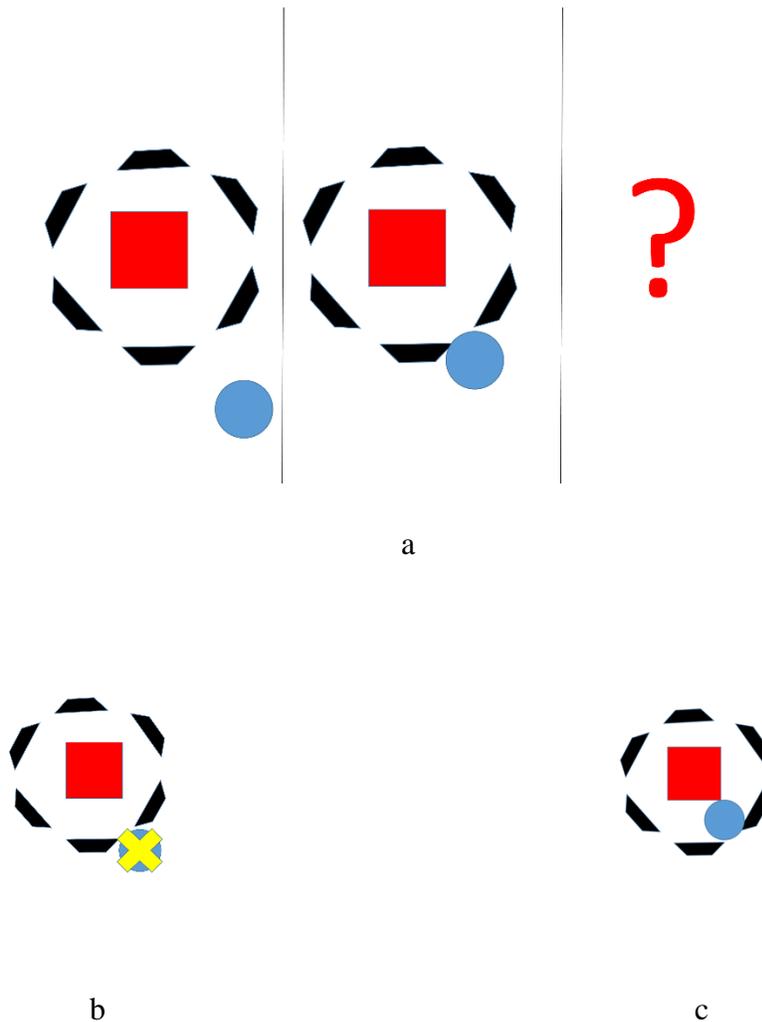


Figure 15: a) An example of a question for the long sequence of static images condition, b) the first choice (the correct one), and c) the second choice

For another condition (short sequence condition), there were only two sets of images simultaneously presented on the screen; one presented the problem of the fortress story and the other presented the solution. Participants were asked to observe them carefully (see Figure 16). Thereafter, they were requested to write a description of what they had just seen.

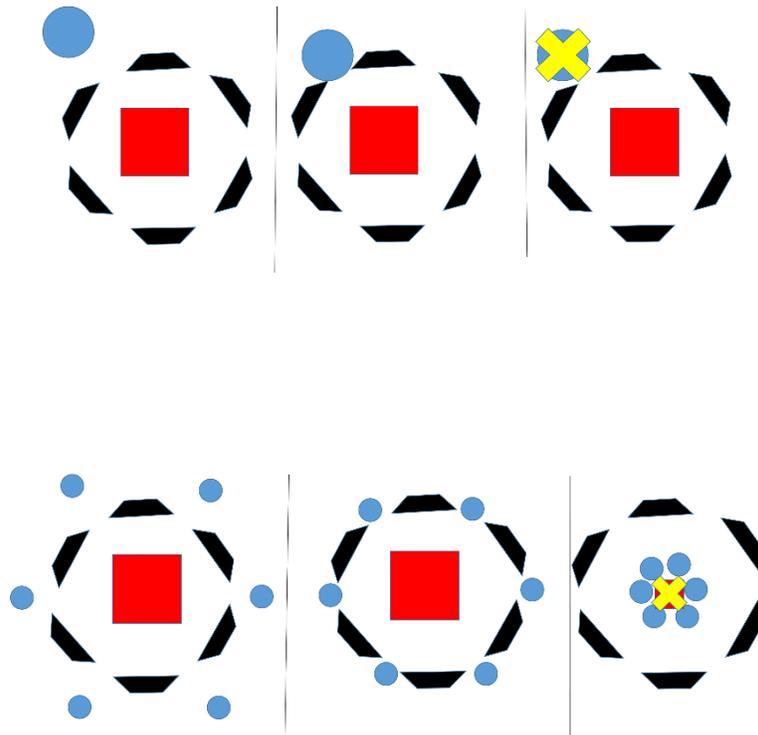


Figure 16: Top) the problematic state shown in the short sequence of static images condition, and bottom) the solution state (the dispersion-convergence solution)

For the last condition (video condition), the source included dynamic images as a short video. The video contained the six images of the long sequence condition in a dynamic format. In this way, circles were moving either toward the border, or, if they could pass the border, through the border toward the center. Also, the changes in the size of the central square were shown by gradually decreasing or eliminating the square (the video can be watched using the link: <https://osf.io/pjgtr>). Participants were allowed to watch the video multiple times. Thereafter, participants were asked to write down what they understood from the video.

Procedure. The task was implemented as an online questionnaire via Qualtrics. Participants were randomly divided into four groups. One group attempted to solve the radiation problem without any source (control condition). The other groups corresponded to the three conditions introduced above: (i) the long sequence of static visual images, (ii) the short sequence

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of static visual images, and (iii) the video. For these groups, after completing the initial task, participants performed an unrelated task for around 5 minutes. Thereafter, they were asked to solve the radiation problem by writing down their answers. On the next page, they were asked to rethink the radiation problem by considering the task they had first completed.

Results and Discussion

Solutions to the radiation problem were scored as to whether they proposed the critical idea of having weaker rays converging at the tumor (1 for successful analogical transfer and 0 for unsuccessful). Moreover, the description of the visual source each participant gave was scored as 1 if it pointed to the problem of big circles and its solution. Otherwise, it was scored as 0. To test the significance of differences between different conditions, Chi-square goodness of fit tests were used. For exploring possible correlations, Spearman's correlation was used. The scores given by the two raters had a reliable internal consistency (the inter-rater reliability was 0.96).

Table 6 shows the overall results of the conditions. Compared to the control group, the results showed that visual sources with static images were not effective for spontaneous transfer. The percentage of successful transfer before the hint was higher in the video condition ($\chi^2 = 11$; $df = 1$; $p < 0.001$), but this percentage was also very low (see Table 6).

After the hint, all conditions showed moderate effectiveness compared to the control group ($\chi^2 = 41$; $df = 1$; $p < 0.001$ for the long sequence of images, $\chi^2 = 46$; $df = 1$; $p < 0.001$, for the short sequence of images, $\chi^2 = 154$; $df = 1$; $p < 0.001$ for the video). Noticeably, after the hint, the video facilitated analogical transfer to a larger extent than the static images ($\chi^2 = 5.9$; $df = 1$; $p = 0.01$). However, as reported in Pedone et al. (2001), the difference between long and short sequence conditions was insignificant, both before and after the hint.

Table 6. Percentages of successful analogical transfer before and after the hint and the number of participants (N) in different groups.

Condition	Before hint	After hint	Total	N
Control	3	-	3	34
Longer sequence of images	5	15	20	40
Shorter sequence of images	7	13	20	45
Video	11	24	35	43

We also analyzed the scores participants achieved for the comprehension question. The results showed that the average comprehension score was higher for the video (0.86) than for the static images (0.2 for the larger sequences of images and 0.24 for the shorter sequence). Therefore, dynamic visuals were more effective at transferring the dispersion-convergence solution than the static ones, supporting previous findings using animations with surface similarities for the radiation problem (Pedone et al., 2001) or with verbal descriptions (Kubricht et al., 2017).

Finally, we calculated the correlations between understanding the visuals and analogical transfer after the hint. The results showed a significant positive correlation between the two scores ($r = 0.49$, $p < 0.001$), confirming that understanding the underlying structure of the visuals is critical for analogical transfer. However, the correlation was higher for the static than for the dynamic display. Specifically, while 74% of the participants who understood the static images could appropriately map the source onto the target after the hint, this percentage was 44% for the video. This suggests that the few participants who understood the static images had the required skills for analogical transfer. In contrast, the video was understandable even for participants without such skills.

4.4. Experiment 7

The results of Experiment 6 were in line with previous studies suggesting that dynamic images would be more effective than static images as the source of a verbal target (Kubritch et al., 2017; Pedone et al., 2001). Nevertheless, the effectiveness of the dynamic images was still very low, even after the hint. In Experiment 7, we aimed to investigate the role of surface similarity in

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abstract visual animations. If the higher effectiveness of dynamic images found in Pedone et al. (2001) was due to surface similarity, a video with elements more similar to the common visual representation of the elements of the radiation problem (i.e., rays as arrows and tumor as circle) would facilitate analogical transfer, compared with a video with dissimilar elements.

Method

Participants. Sixty undergraduates, who had not participated in the previous experiment, participated in this experiment. The ethics committee of the University of Barcelona approved the research procedure. Participants were divided into two groups, each of which was assigned to one condition (see procedure).

Materials. There were two videos corresponding to the two conditions. The first video was the one used in Experiment 6 but without the intermediate states. That is, it only showed the animation of the problem and solution states (the video can be watched using the link: <https://osf.io/juvkh>). The second video aimed to abstractly represent the radiation problem and its dispersion-convergence solution. Like the fortress video, it consisted of only two animations (the initial problematic state and the solution). To visually represent the radiation problem, we used geometric shapes that shared superficial similarities with the common visual representation of the elements of the radiation problem: blue arrows representing rays, a red circle representing the tumor, and a circle surrounding the red circle as the healthy tissues surrounding the tumor (the video can be watched using the link: <https://osf.io/bhjrj>).

Procedure. The task was also implemented as an online questionnaire via Qualtrics. Participants were divided into two groups, each of which was assigned to one of the conditions mentioned above. We asked participants to watch the videos carefully. Then, they were asked to briefly write about the video. Thereafter, participants performed an unrelated task for around 5 minutes, after which they were asked to solve the radiation problem by writing their answer. Then, they were asked to rethink the radiation problem by considering the task they had first completed.

Results and Discussion

As in Experiment 6, solutions to the radiation problem were scored as to whether they proposed the critical idea of having weaker rays converging at the tumor. Moreover, the description of the visual source that each participant gave was scored as 1 if it pointed to the problem of big

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circles and its solution (in the case of the video representing the fortress story) or the problem of big arrows and its solution (in the case of the video representing the radiation problem and its solution). Otherwise, it was scored as 0. As in Experiment 6, Chi-square goodness of fit tests and Spearman’s correlation were used. The scores given by the two raters had a reliable internal consistency (the inter-rater reliability was 0.9).

The two groups did not differ in terms of successful spontaneous transfer (see Table 7). However, after the hint, the performance of participants who viewed the video of the radiation story condition was significantly better than that of participants who viewed the video of the fortress story condition ($\chi^2 = 4$; $df = 1$; $p = 0.04$; see Table 2). This finding confirmed our hypothesis about the role of perceptual similarity in analogical transfer: perceptual similarities with the common visual representations of the elements of the radiation problem facilitated analogical transfer.

Table 7. Percentages of successful analogical transfer before and after the hint and the number of participants (N) in different groups.

Condition	Before hint	After hint	Total	N
Video of the fortress story	10	30	40	30
Video of the radiation story	17	43	60	30

Regarding the descriptions that participants gave of the visual displays, the average score was significantly higher for the video of the fortress story than for the video of the radiation story (0.83 vs 0.66; $\chi^2 = 3.8$; $df = 1$; $p = 0.05$). We suspect that circles (which were used in the video of the fortress story) are better than arrows (which were used in the video of the radiation story) in representing changes in size. Additionally, the border in the video of the fortress story (a set of rectangles defining a circular border; see Figure 16) could represent constraints in a clearer way than the border in the video of the radiation story (a round circle surrounding the smaller circle at the center).

However, as mentioned above, the rate of analogical transfer was higher for the video of the radiation story. Although only 66% of the participants seemed to understand the dispersion-

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convergence solution after viewing this video, 70% of them were able to successfully perform analogical transfer. In contrast, only 48% of the participants who showed correct understanding of the video of the fortress story were able to successfully perform analogical transfer. In other words, the rate of converting a true understanding of the source to a true mapping was significantly higher for the video of the radiation problem, compared to the video of the fortress story ($\chi^2 = 4$; $df = 1$; $p = 0.04$). Therefore, consistent with our hypothesis, perceptual similarities between the source and the target had a significant effect on the rate of analogical transfer.

Another finding that emphasizes the role of perceptual similarity in mapping was that 40% of the participants who showed poor understanding of the video of the radiation problem were able to solve the radiation problem after the hint. This unexpected finding suggests that the hint, which encouraged the participant to use the video, led some of those participants to perform backward mapping of the radiation problem onto the video based on perceptual similarities (i.e., by seeing the arrows similar to the rays). Using the verbal narrative, they were able to understand the video and solve the radiation problem. This did not happen for participants who produced a poor description of the fortress video. In fact, among the few participants who showed poor understanding of the video of the fortress story, none of them were able to successfully solve the radiation problem even after the hint.

Regarding spontaneous analogical transfer, the results showed that, among the participants who demonstrated poor understanding of the video, none could solve the radiation problem before the hint. That is, understanding the source was necessary, although insufficient, for spontaneous transfer. Among those who demonstrated a good understanding of the video, the rate of spontaneous transfer was marginally higher for the radiation video than for the fortress video (0.25 vs 0.12; $\chi^2 = 3.2$; $df = 1$; $p = 0.07$). This finding confirms the previously reported benefit of superficial similarity for spontaneous analogical transfer (Chen, 1995; Holyoak & Koh, 1987), although this benefit was weaker for our visual dynamic displays than for the verbal stories previously used.

4.5. Experiment 8

The results of studies 1 and 2 confirmed previous findings: relevant spatial relations, even in a semantically poor context (i.e., a set of abstract visual diagrams), could lead reasoners to solve a verbal problem that was structurally similar to the source. However, as Gattis (2004) mentioned,

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previous studies used visual sources to demonstrate the importance of spatial relations in analogical reasoning without addressing how visual diagrams and verbal stories are related. Therefore, the importance of spatial relations in mapping between a verbal source and a verbal target remains elusive. Instead of generalizing the effect from visual to verbal domains, one possible approach is to go beyond visual sources and investigate the effect of spatial relations on mapping a verbal source to a verbal target, which, as far as we know, has not been studied before.

Verbal stories provided as the source of an analogy are conceptually rich. Like other narratives, they refer to specific meaningful objects (fortress, general, ray, tumor, etc.) and specific relations between these objects (the general vows to capture the fortress, the rays are a type of force able to destroy the tumor, etc.). Some relations may lead reasoners to the solution (i.e., the dispersion-convergence solution) but some others may distract from it, leading to irrelevant inferences. Critically, scenarios related to the dispersion-convergence solution generally describe some spatial relations between certain objects. For example, in the fortress story, it is mentioned that the fortress is in the middle of a country. Also, many roads radiate outward from the fortress like *spokes on a wheel*. Therefore, a verbal story as the source of an analogy may describe spatial properties leading to spatial relations in addition to non-spatial properties leading to non-spatial relations.

Although, in many narratives, the spatial and non-spatial relations are so entangled that it seems difficult to separate them (Gattis, 2004), we can control the number of relevant spatial properties explicitly mentioned. In fact, as Mani & Johnson-Laird (1982) proposed, adding spatial properties to a narrative makes it more determinate in that more objects have determined spatial positions. Conversely, removing some spatial properties from a narrative makes it more indeterminate. Given that indeterminate spatial relations do not lead to a single integrated and unambiguous mental representation, people tend to remember non-spatial rather than spatial relations to a larger extent than in spatially determined scenarios (ibid).

In line with these previous findings, we hypothesized that spatial representations of the fortress story would be more ambiguous if critical spatial cues were removed from the verbal description. This might have consequences for memory and hence for retrieval and transfer. That is, without explicit spatial cues, mental representations would focus more on verbatim information rather than on global, spatial properties. Therefore, based on the proposal that finding common

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spatial relationships would facilitate solving the radiation problem (Gick and Holyoak, 1980), the explicitness of the spatial cues in the fortress story would affect such transfer. To test this hypothesis, we compared the standard fortress story (the same as the one used in Gick & Holyoak, 1980, including explicit spatial cues) with another version in which the spatial cues were removed. That is, in the second version, the relational spatial positions of the critical elements were not explicitly specified (see method). We expected that the version without explicit spatial cues would be less effective than the standard version for solving the radiation problem.

Method

Participants. Sixty-two undergraduates, who had not participated in the previous studies, participated in this experiment. The ethics committee of the University of Barcelona approved the research procedure. Participants were divided into two groups, each of which was assigned to one condition (see procedure).

Materials. There were two versions of the fortress story corresponding to the two conditions of this experiment. The first version was the Spanish translation of the version used first by Gick and Holyak (1980; see the Appendix D). After the story, we added two multiple-choice comprehension questions: one was about the initial problem (what happens if all the soldiers try to go along the same path to get to the fortress?) and the other one was about the solution (what happens if all the soldiers in small groups try to go along different paths from different places around the fortress to reach the fortress at the same time?).

The second version was like the original fortress story with a few differences. Specifically, we eliminated two sentences that contained explicit spatial properties: “The fortress was situated in the middle of the country” and “Many roads radiated outward from the fortress like spokes on a wheel”. Gick and Holyoak (1980) mentioned the latter as the important element that participants used to map the story to the radiation problem. Note that the eliminated sentences are not vital for understanding the dispersion-convergence solution. Without these sentences, the problem and the solution states of the story are sufficiently clear. We replaced the eliminated parts with irrelevant but not interfering information (see the Appendix D). As in the first version, two comprehension questions followed the story. The first comprehension question was exactly the same as the question used in the first version. However, in the second question, the spatial properties of the

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solution were not mentioned (what happens if all the soldiers in small groups try to go along different paths to reach the fortress at the same time?).

Procedure. The task was also implemented as an online questionnaire via Qualtrics. Participants were asked to read the story carefully. Thereafter, participants solved the two multiple-choice comprehension questions about the story they had read. Then, they performed an unrelated task for around 5 minutes after which they were asked to solve the radiation problem by writing their answers. Then, they were asked to rethink the radiation problem by considering the task they had first completed.

Results and Discussion

As in previous studies, solutions to the radiation problem were scored as to whether they proposed the critical idea of having weaker rays converging at the tumor. We also analyzed the response options participants chose for the two comprehension questions. Regarding statistics, Chi-square goodness of fit tests were used. The scores given by the two raters had a reliable internal consistency (the inter-rater reliability was 0.94).

First, regarding the comprehension questions, the average rate of correct answers was 0.88 for the questions about the fortress story with explicit spatial properties and 0.8 for the questions about the fortress story without explicit spatial properties. This means that participants were able to understand the critical points of both stories. As expected, removing the sentences explicitly mentioning spatial properties did not affect people's understanding of the fortress story.

Regarding the solutions, Table 8 shows the overall results of the two conditions. Before the hint, there were no significant rates of spontaneous analogical transfer in any of the conditions. After the hint, however, the rate of successful analogical transfer was significantly higher for the condition including the fortress story with explicit spatial features than for the condition including the fortress story without explicit spatial features ($\chi^2 = 7$; $df = 1$; $p = 0.006$). This finding suggests that a scenario that promotes building a spatial mental representation by means of explicit spatial cues facilitates analogical structural mapping, compared with a similar scenario but without explicit spatial properties.

Table 8. Percentages of successful analogical transfer before and after the hint and the number of participants (N) in different groups.

Condition	Before hint	After hint	Total	N
Fortress story with explicit spatial features	0	55	55	31
Fortress story without explicit spatial features	0	32	32	31

4.6. Experiment 9

The results of the Experiment 8 showed that the explicit spatial properties had a significant effect on analogical transfer after the hint. There, participants answered two multiple choice questions about the fortress story, and after an unrelated task, they tried to solve the radiation problem before and after a hint. Although most of the participants correctly understood the problem and the solution described in the fortress story, participants who read the version without the explicit spatial properties had more difficulties for analogical transfer. Therefore, beyond the comprehension of the causal relations between the means and the goal, which were common in both versions of the fortress story, the possibility to represent the global spatial structure of the scenario seems to facilitate the comparison between the source and the target and corresponding analogical transfer of the solution.

One way of compensating the lack of spatial properties could be to invite people to create a visual schema of the fortress story which, necessarily, must refer to spatial relations. Catrambone et al. (2006) showed that drawing did not facilitate analogical transfer compared to writing a summary. In that study, the standard version of the fortress story (the version which includes spatial properties) was used. Accordingly, we expected that drawing would not be different from writing in the case of the fortress story with spatial properties. However, for the fortress story without spatial properties, we expected that drawing would invite people to build a spatial mental representation of the story, and, therefore, compensate the lack of explicit spatial properties in the narrative. In other words, for the fortress story without explicit spatial properties, the rate of

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analogical transfer was expected to be significantly higher for participants drawing a visual schema, compared with those writing a summary. Experiment 9 aimed at testing this hypothesis.

Another goal of Experiment 9 was to explore if putting active comprehension questions after the source would have effects on analogical transfer. In line with previous literature (Kubritsch et al., 2017), we expected that active comprehension questions after the source (such as asking reasoners to write a summary of the source) would facilitate analogical transfer compared to passive ones (like the multiple choice questions used in Experiment 8).

Finally, we expected to replicate the results of Experiment 8 regarding a significant difference between the rates of analogical transfer with the fortress story with and without spatial properties if participants were asked to write a summary. However, as drawing might compensate the lack of spatial properties, it was expected that the fortress story with and without spatial properties would not be significantly different from each other if participants were asked to draw a visual schema.

Method

Participants. One-hundred twenty-four undergraduates, who had not participated in the previous studies, participated in this experiment. The ethics committee of the University of Barcelona approved the research procedure. Participants were evenly divided into four groups, each of which was assigned to one condition (see procedure).

Materials. There were two versions of the fortress story corresponding to the two conditions of this experiment. The first version was the Spanish translations of the version used firstly by Gick and Holyak (1980) and then other researchers; the version includes spatial properties. The second version was the version of the fortress story without spatial properties used in Experiment 8. Also, there were two kinds of active comprehension questions for this experiment. For the verbal comprehension questions, people were asked to write briefs summaries of the problem and the solution of the fortress story. For the visual comprehension questions, people were asked to schematically draw the problem and the solution of the fortress story. Participants could draw on paper or using computer programs. They had to upload their drawings; in the case of drawing on paper, they took a photo and uploaded it. So, there were four conditions in this experiment: (i) the fortress story without spatial properties and visual comprehension

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questions, (ii) the fortress story with spatial properties and visual comprehension questions, (iii) the fortress story without spatial properties and verbal comprehension questions, and (iv) the fortress story with spatial properties and verbal comprehension questions

Procedure. The task was also implemented as an online questionnaire via Qualtrics. Participants were asked to read the story carefully. Thereafter, participants were asked to solve the comprehension questions. Then, they performed an unrelated task for around 5 minutes after which they were asked to solve the radiation problem by writing their answers. Then, they were asked to rethink the radiation problem by considering the task they had first completed.

Results and discussion

Table 9 shows the overall results of Experiment 9. First, for the writing conditions, the results were in line with Experiment 8; the rate of analogical transfer was significantly higher for participants who read the fortress story with rather than without spatial properties ($\chi^2 = 6$; $df = 1$; $p = 0.01$). This finding suggests that that a scenario with explicit spatial properties facilitated the use of structural relations more effectively, compared with the same scenario but without explicit spatial properties. However, for the drawing conditions, both scenarios showed a similar effect on the rate of analogical transfer. This confirms our hypothesis regarding the possibility that drawing could compensate the lack of spatial properties in the fortress story. That is, drawing might have forced people to build a spatial mental representation of the story, even if the story did not have explicit spatial relations. Figure 17 shows two examples of drawings done by our participants. As seen in Figure 17 (the top image), the global spatial properties of the standard version of the fortress story might led participants to draw the problem and the situation based on the properties. Furthermore, even drawing the fortress story without spatial properties might led people to think about the spatial structure of the story (see the bottom image of Figure 17)

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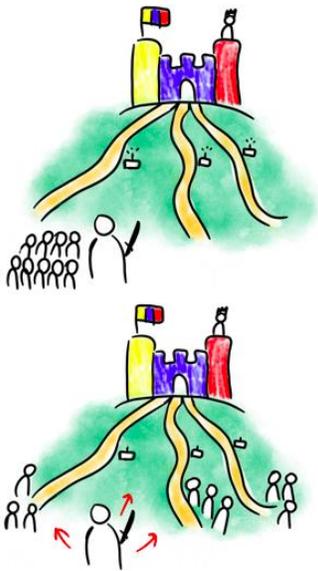
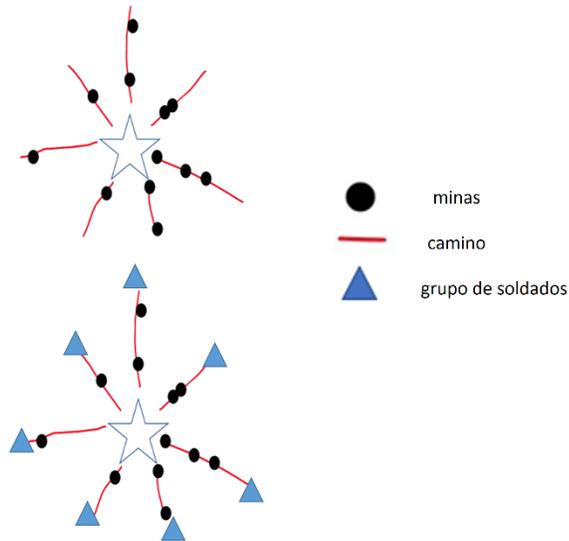


Figure 17: Top) a drawing done by an anonymous participant of the problem and the solution of the fortress story with spatial properties, and bottom) a drawing by an anonymous participant for the fortress story without spatial properties.

Table 9. Percentages of successful analogical transfer before the hint (spontaneous) and after the hint and the number of participants (N) for different groups.

Summary	Fortress story	Before hint	After hint	Total	N
Drawing	No spatial properties	9	55	64	31
	Spatial properties	3	58	61	31
Writing	No spatial properties	6	23	29	31
	Spatial properties	9	43	52	31

Moreover, for the fortress story with explicit spatial properties, in line with Catrambone et al (2006), drawing led to a higher rate of analogical transfer than writing, but the difference was not significant ($\chi^2 = 1.4$; $df = 1$; $p = 0.23$). However, regarding the fortress story without spatial properties, the rate of analogical transfer was significantly higher for drawing rather than writing ($\chi^2 = 16$; $df = 1$; $p < 0.001$). This finding also confirms the hypothesis that drawing compensates the lack of spatial properties, promoting the inference of spatial relations and, hence, facilitating the analogical transfer, even if the spatial properties were not explicit in the text.

Finally, to investigate the effect of active *versus* passive verbal comprehension questions, we compared the results of this experiment with those of Experiment 8. No significant differences were found indicating that against our expectations, writing a summary (as an active comprehension task) did not facilitate analogical transfer compared to solving multiple-choice questions (as a passive comprehension task).

4.7. Experiment 10

Results of studies 3 and 4 showed that describing explicit spatial properties in the source had significant effects on analogical transfer. In Experiment 10, we aimed to extend this finding by manipulating the spatial properties of the target. Specifically, given that the radiation problem used in the previous studies has no explicit spatial properties⁴, the goal of Experiment 10 was to

⁴ Although there is no verbally explicit spatial properties, People can mentally visualize the tumor inside the stomach which is inside the body

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study the effect of adding more explicit spatial properties in the description of the problem on analogical transfer.

Spatial properties can be at different levels of the spatial hierarchy. They can be more local, determining the spatial position of individual elements, or more global, determining the global spatial structure of the situation. In this experiment, we manipulated spatial properties at the level of the global structure since we expected that such spatial properties could more easily lead to notice structural similarities between the two situations. Accordingly, we hypothesized that adding global spatial properties to the target would facilitate analogical transfer.

Method

Participants. Fifty-eight undergraduates, who had not participated in the previous studies, participated in this experiment. The ethics committee of the University of Barcelona approved the research procedure. Participants were divided into two groups, each of which was assigned to one condition (see procedure).

Materials. The source was the Spanish translation of the version used first by Gick and Holyoak (1980) for the two conditions of this experiment. After the source, participants were requested to write a summary of the story they had just read. For the target, there were two different versions of the radiation problem corresponding to the two conditions of this experiment. The first version was the one used in the previous studies. In the second version, as pointed out above, we minimally manipulated the global spatial properties of the radiation problem. In fact, we only replaced the sentence “a patient who has a malignant tumor in his stomach” by “a patient who has a malignant tumor inside of the body surrounded by healthy tissues”. Although it is implicitly mentioned in the radiation problem that the tumor is surrounded by healthy tissues, in the second version, we made this relation more explicit. Moreover, to communicate a more global picture of the situation, we replaced “stomach” by “body” (see Appendix E).

Procedure. The task was also implemented as an online questionnaire via Qualtrics. Participants were asked to read the story carefully. Thereafter, participants were asked to write a summary of the story they had read. Then, they performed an unrelated task for around 5 minutes after which they were asked to solve the radiation problem by writing their answers. Then, they were asked to rethink the radiation problem by considering the task they had first completed.

Results and discussion

As in previous studies, solutions to the radiation problem were scored as to whether they proposed the critical idea of having weaker rays converging at the tumor. Moreover, the summary of the story that each participant gave was scored as 1 if it pointed to the problem and its solution (the dispersion-convergence solution). Otherwise, it was scored as 0. Regarding statistics, Chi-square goodness of fit tests were used. The scores given by the two raters had a reliable internal consistency (the inter-rater reliability was 0.96).

First, regarding the comprehension question, the average score of the summary of the fortress story was 0.83 meaning that most of our participants correctly understood the problem and the dispersion-convergence solution. In fact, in each condition, 25 out of 29 participants got 1 for the comprehension question.

Regarding the solutions, Table 10 shows the overall results of the two conditions. Surprisingly, the rate of successful spontaneous analogical transfer was significantly higher for the condition including the radiation problem with global spatial properties than for the condition including the radiation problem without explicit spatial features ($\chi^2 = 4.7$; $df = 1$; $p = 0.03$). This finding suggests that a description of the problem that promotes building a global spatial mental representation by means of explicit spatial cues facilitates spontaneous analogical transfer, compared with a similar problem but without global spatial properties. Regarding people who could solve the problem only after the hint, however, there was no significant difference between the two conditions. Overall, the results indicated global spatial properties of the target facilitate structural mapping in an analogical problem solving task even before any hint.

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Table 10. Percentages of successful analogical transfer before and after the hint and the number of participants (N) in different groups.

Condition	Before hint	After hint	Total	N
Radiation problem with global spatial features	24	52	76	29
Radiation problem without global spatial features	7	45	52	29

4.8. Discussion

This research was carried out to investigate the role of spatial relations in analogical problem solving, using the radiation problem as the target. Unlike the visual designs used in previous studies (Beveridge & Perkins, 1987; Gick & Holyoak, 1983; Pedone et al., 2001), our designs did not share any surface similarity with the common visual representation of the elements of the radiation problem (i.e., rays as arrows and tumor as circle). Also, to make the process of analogical problem solving similar to the process for the verbal-verbal format, our visuals represented a problematic state and a solution state in a more explicit way.

With these new designs, Experiment 6 replicated the null effect of the length of the sequence of static images on the likelihood of analogical transfer (Pedone et al., 2001). It also replicated the observed benefit of dynamic (compared to static) visual displays for solving the radiation problem (Kubricht et al., 2017; Pedone et al., 2001). Regarding spontaneous transfer, the visual animation only seemed to benefit a few participants, compared to a condition where no visual display was presented. The results of Experiment 7 demonstrated that, as reported for verbal sources (Holyoak & Koh, 1987), surface similarities between the visual animation and the radiation problem also modulate the likelihood of analogical transfer. Experiment 8, 9, and 10 explored, as far as we know for the first time, the role of spatial relations in structural mapping in the verbal-verbal format of analogical problem solving, using the fortress story as a source. The results of Experiment 8 and 9 showed that the removal of the spatial properties of the verbal story diminished the likelihood of solving the radiation problem, also emphasizing the importance of spatial relations for structural mapping. Finally, Experiment 10 demonstrated that global spatial properties of the target facilitated analogical transfer even before the hint. These points are discussed further in the following sections.

Dynamic visual diagrams. Consistent with previous studies (Kubritch et al., 2017; Pedone et al., 2001), Experiment 6 demonstrated that dynamic visuals are more effective than static ones in communicating the dispersion-convergence solution. In fact, while less than 25% of participants interpreted the static visuals in a way we had in mind, more than 80% of them interpreted the dynamic visuals properly. This finding is also consistent with what previous studies have suggested in other domains: animations are more effective than corresponding static graphics, particularly for the representation of temporal changes (Baek & Layne, 1988; Betrancourt & Tversky, 2000; Rieber & Kini, 1991; Tversky et al., 2002).

Accordingly, the higher rate of analogical transfer for the dynamic visuals could be attributed to the fact that the video was understood more accurately than the static diagrams. This confirms previous suggestions (e.g., Pedone et al., 2001) regarding the efficiency of visual animations in communicating causal relationships, such as the ones underlying the dispersion-convergence solution.

Yet, the current study also showed that, in contrast to the static images, accurate comprehension of the video did not imply accurate analogical transfer. As Kubritch et al. (2017) noted, animations may have decreased the impact of intelligence on comprehension. However, difficulties in analogical transfer were still presented by some of the participants who understood the video. In contrast, understanding the dispersion-convergence solution from a set of static images was more difficult, suggesting that higher cognitive skills were required. Such skills may have contributed to both the comprehension of the static diagrams and its analogical transfer. It is also possible that comprehension of the static diagrams occurred at a more optimal level of abstraction (Gick & Holyoak, 1980; 1983) than the dynamic displays, facilitating analogical transfer. For its part, the video may have promoted a more vivid representation of the objects, which might be coherent with the observed effect of surface similarities on analogical transfer. Nevertheless, the differences in the required skills and in the knowledge acquired from either static or dynamic displays require further research with larger samples and other measures of comprehension.

In contrast to the findings of Pedone et al. (2001), even with dynamic visuals and surface similarity, only a few participants showed spontaneous transfer. There are two possible explanations for this discrepancy. First, since analogical problem solving abilities depend on

individual factors (Gray & Holyoak, 2019; Kubritcht et al., 2017), differences in the rate of spontaneous transfer (as well as differences in the rate of analogical transfer after the hint)⁵ might be due to differences in the individuals between the two groups.

Second, differing from previous designs, in our designs there was an explicit representation of the problematic state. In Pedone et al. (2001), there was initially one object that was then divided into smaller pieces. In our designs, however, the problematic state included the three main elements and an abstract visual scenario relating the elements (i.e., the inability of the thick arrow to pass through the border to arrive at the central element). In this sense, our designs were more complex, which might have made mapping more difficult.

Surface similarities. In line with previous studies (Chen, 1995; Holyoak & Koh, 1987; Trench & Minervino, 2015), our results showed that perceptual similarities between the elements of the abstract visual source and the target had a significant effect on analogical transfer in the visual-verbal format. This suggests that superficial similarities enhance the alignment of critical elements even in the case of abstract visual sources. Of note, although most of the participants could understand visual analogues without superficial similarity, analogical mapping remained a barrier for most of them.

In our research, perceptual similarities had no effect on spontaneous transfer. However, considering only participants who understood the visuals properly, perceptual similarity facilitated spontaneous transfer. This might extend what previous studies have suggested into the visual-verbal format of analogical reasoning (Gentner & Maravilla, 2018; Holyoak & Koh, 1987): the spontaneous retrieval of a potential analogy (between a verbal source and a verbal target) initially occurs through surface similarities, as similarity-based mappings. Nevertheless, it is worth noting that superficial similarity in the visual-verbal format was more abstract than in the verbal-verbal format (for example, in Holyoak and Koh, 1987, the word “ray” appeared in both the source and the target, whereas in the video participants needed to realize that arrows could represent rays). This could explain why only a few participants benefited from superficial similarity in the visual-verbal format.

⁵ As Pedone et al. (2001) showed, 90% of participants were able to solve the radiation problem using a visual source either before or after the hint. In the case of our participants, the rate was 60% for the most effective set of visuals.

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As expected, people who understood the visuals in a way other than we had in mind (the problem of the big force and the dispersion-converging forces as the solution) were not able to spontaneously solve the radiation problem based on the convergence solution. However, some of them were able to solve it after the hint. It is possible that, after the hint, people started to map the verbal problem to the visual diagrams (backward mapping; Gick & Holyoak, 1980) and interpreted the visuals in a different way from the way they had interpreted them previously. Interestingly, this kind of re-interpretation only occurred among people watching the video with surface similarities. In fact, among people who did not understand the video without any surface similarity with the target, none solved the target even after the hint. This finding suggests that re-interpretation through backward mapping was triggered by surface similarities between some elements of the source (i.e., arrows) and the target (i.e., rays).

The importance of global spatial properties. Gick and Holyoak (1980) proposed that finding common spatial relationships would facilitate solving the radiation problem and suggested an idea for future studies. However, most of the following studies investigated the importance of spatial relations by means of designing and implementing visual diagrams as the source of the radiation problem. In this research, we explored the extent to which the inclusion of explicit references to spatial relations in the text affect analogical transfer. Experiment 8, 9, and 10 consistently demonstrated that spatial properties, either in the source or in the target, facilitated analogical transfer. These findings confirm the importance of spatial properties, at least in the case of the fortress story-radiations problem which are spatial in nature (the analogous *source*) as a guide for developing a solution for a novel problem (the *target*; Beveridge & Perkins, 1987; Gick & Holyoak, 1980), in analogical transfer.

Importantly, spatial properties added and removed in studies 8, 9, and 10 point to the global spatial structure of scenario. For example, the sentences of the Fortress story (“The fortress was situated in the middle of the country” and “Many roads radiated outward from the fortress like spokes on a wheel”), controlled in studies 8 and 9, describe the global spatial structure of the scenario instead of local positions of some individual elements. Similarly, the sentence included in Experiment 10, “a patient who has a malignant tumor inside of the body enclosed by healthy tissues” seems to invite people to think about the global spatial structure of the scenario instead of, or in addition to, considering individual elements.

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Spatial structures. Inspired by the results of Studies 6 and 7, we ran Experiment 8 to explore the role of explicit spatial properties in the verbal-verbal format of analogical reasoning. To do so, we made another version of the fortress story by removing the explicit spatial cues. The results of Experiment 8 showed that this removal did not affect the comprehension of the dispersion-convergence solution, the rate of correct responses to the comprehension questions being similarly high (about 0.8). The two conditions also showed similar inefficacy for spontaneous analogical transfer, confirming the difficulty in the spontaneous retrieval of distant analogues (Gick & Holyoak, 1980 and 1983). However, after the hint, the source with explicit spatial properties was significantly more effective than that without them for analogical transfer.

Therefore, while the spatial cues (central element surrounded by other elements) seem to be irrelevant for understanding the dispersion-convergence solution, certain non-spatial cues (those referring to groups of soldiers moving towards and arriving at the fortress at the same time), common in both versions of the fortress story, may be critical for comprehension. However, after the hint, both spatial and non-spatial cues may play a role in analogically solving the radiation problem. But then, how could explicit spatial properties make structural mapping easier if they seem irrelevant for understanding the required solution?

As suggested by Mani & Johnson-Laird (1981), having explicit spatial properties in a narrative facilitates representing the global spatial structure of the situation whereas lacking them leads to verbatim details being remembered more effectively. Therefore, the fortress story with the spatial properties may have encouraged the reasoner to consider and remember the global structure of the story (i.e., a center, a circular border, and some forces outside of the border trying to get inside) while the fortress story without spatial properties may have encouraged the reasoner to consider and remember the details of the story (i.e., the general, the fortress, the mines, and the soldiers).

Based on the above proposal, the spatial properties of the fortress story may have encouraged comparison of the global structures of the source and the target: both describe a central element (tumor and fortress, respectively) surrounded by other elements (healthy tissues and radial paths, respectively). Accordingly, after the hint, reasoners might have mapped the spatial structure of the source onto the target by aligning different parts of the spatial structure to different elements of the radiation problem (tumor as the center, skin as the border, rays as outsiders) to generate the

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dispersion-convergence solution. This spatial mapping is similar to the mapping from an abstract visual source onto a verbal target; it mainly involves spatial relations.

However, the mental representation of the fortress story without spatial properties may have consisted mainly of non-spatial instead of spatial relations. In this case, mapping is more likely to have been based on some analogous correspondences between conceptual or causal relations, such as between “the soldiers (destroy) the fortress”, and “the rays (destroy) the tumor”. Given that the fortress story with spatial properties provided reasoners with both spatial and conceptual relations, spatial cues may have enhanced structural mapping by allowing reasoners to employ spatial mapping apart from, or in addition to, conceptual mapping.

Similarities in spatial structures. Analogical reasoning in non-experimental situations usually occurs spontaneously, without any hint connecting a source to the problem at hand. However, in line with previous studies, the results of Experiment 8, 9, and 10 showed that spontaneous analogical transfer (without any hint) from a verbal source to a verbal target was difficult for non-expert people (Gick & Holyoak, 1980). This confirms that the main obstacle for spontaneous analogical transfer is the retrieval of a source which is only structurally –not superficially- similar to the target problem. However, in Experiment 10, by adding global spatial properties to the target, we finally got a significant increase in the rate of spontaneous analogical transfer.

To give an account for this finding, we think that if both the source and the target contain spatial properties, reasoners can spontaneously retrieve the mental global spatial representation of the source and map it onto the target. In other words, people can retrieve a global spatial image (of the source) which is somehow similar to the global mental image they are building (based on the target). This suggests that, apart from similarities between the individual objects of the source and those of the target (Holyoak & Koh, 1987; Gentner & Maravilla, 2018), similarities of the mental global image of the source and that of the target may facilitate spontaneous retrieval and mapping.

Why the mental images of the source and target had an effect on spontaneous retrieval only if both the source and target contained explicit global spatial properties? It might be the case that explicit spatial properties lead people to consider the global structure of a narrative instead of verbatim details. Therefore, explicit spatial properties of the source and target may make the global images of the source and target clearer. The findings of Experiment 10 suggest that a possible way

to facilitate spontaneous analogical transfer between two analogous problems is to highlight the common spatial structure, which seems to facilitate the processes of retrieval and mapping.

Drawing as a compensation for lack of spatial properties. Previous studies showed that drawing might be helpful for comprehension and inferring abstract relations (e.g., Butcher, 2006; Fiorella and Kuhlmann, 2019; Fiorella and Mayer, 2015; Fiorella and Zhang, 2018; Van Meter and Firetto 2013; Van Meter and Garner 2005). However, Catrambone et al (2006) demonstrated that it was not significantly more effective than writing for analogical problem solving. Given that drawing may invite people to build and represent a spatial mental image of the previously confronted situation, we run Experiment 9 to see if drawing would be equally effective in situations with and without spatial properties. The results confirmed what Catrambone et al (2006) found in the presence of spatial properties. However, in the case of the fortress story without spatial properties, drawing had a significant positive effect on analogical transfer.

This finding indicates that schematic drawing can compensate the lack of spatial properties because it invites people to infer the spatial relations that are not explicit in the text. The spatial structure inferred from the fortress story without spatial properties might be different from the one mentioned in the original text: “The fortress was situated in the middle of the country” and “Many roads radiated outward from the fortress like spokes on a wheel”. However, building and representing certain spatial structure in the process of drawing seems enough for analogical transfer after the hint for the majority of our participants, even in the case of the scenario without explicit spatial cues. Participants who wrote about the story had a significantly lower rate of success in analogical transfer because writing might not necessarily invite people to build a mental spatial structure of the story. Accordingly, the results of our studies demonstrated the importance of mental spatial structure in analogical reasoning. In this way, drawing can play a role in leading people to build a spatial structure if the source does not provide it.

Chapter 5

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The main objective of this research was to study the role of spatial relations in analogical reasoning –both in visual and verbal domains. To this end, we designed and performed ten studies. Here, a summary of the main findings, together with a discussion of their theoretical implications and limitations which may direct future research are presented.

5.1. Summary of main findings

In Chapter 3, we were inspired by the global/local distinction explored in visual perception, as introduced and studied by Navon (1977). We proposed two paths for visual analogical reasoning: the global path and the object path. The global path involves considering the overall shapes of the images and identifying and mapping relations between them (global relations). On the other hand, the object path involves considering the individual objects within the images and identifying and mapping relations between them (object relations). To investigate the extent to which participants employ the global or object path in solving visual analogies, we designed problems that separated the end-results of these two paths.

In Experiments 1 and 2, we examined how the saliency of the global shape (good vs. no good global forms) and the saliency of individual objects (familiar vs. unfamiliar objects and colorful vs. black-and-white objects) influenced the participants' choices regarding global/object relations. The results showed that the saliency of the global shapes increased the likelihood of encoding and mapping global relations, while the saliency of the individual objects did not have the same effect on object-based relations. Additionally, the tendency to encode and map global relations in black-and-white problems positively correlated with visual problem-solving skills (measured using a shortened version of the ARPM).

In Experiment 3, we found that problems containing global spatial relations were easier to solve compared to those containing object-based relations. Experiment 4 revealed a weaker inclination to rely on the global path when the source (first row of visual analogies) was separated from the target (second row, including the missing image), compared to the simultaneous presentation used in previous studies. Moreover, the tendency to encode and map global relations correlated positively with analogical problem-solving abilities. In Experiment 5, we employed eye-tracking techniques to study the process of visual reasoning. The results showed that the tendency to encode and map global relations was positively correlated with the use of the "project-first"

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strategy, which is considered the most optimal strategy in visual analogical problem solving (Vendetti et al., 2017).

Moving on to Chapter 4, we investigated the significance of spatial relations in solving a verbal target, specifically the radiation problem (Duncker, 1945). Our aim was to study analogical transfer from an abstract visual source to a verbal target, which had been previously examined in studies by Beveridge & Perkins (1987), Gick & Holyoak (1983), and Pedone et al. (2001).

To address the limitations of previous research (as discussed in Chapter 4), we designed new sets of abstract visual diagrams. The results were consistent with previous findings, demonstrating that dynamic visuals were more effective than static ones (Experiment 6). Additionally, Experiment 7 indicated that surface similarities facilitated analogical problem solving when abstract visual representations were used as the source for a verbal target. In Experiment 8, we explored whether spatial relations played a role in analogical transfer from a verbal narrative to a verbal problem. The results showed that both versions of the problem (with and without explicit spatial cues) could effectively communicate the problem and its solution. However, explicit spatial properties facilitated analogical transfer after the participants received a hint. Furthermore, Experiment 9 revealed that drawing, as opposed to writing, about a source story facilitated analogical reasoning only when the source lacked explicit spatial properties. Lastly, Experiment 10 found that the rate of spontaneous analogical transfer was higher for a version of the radiation problem that contained explicit global spatial properties.

These points will be further discussed in the subsequent sections.

5.2. The global/object paths in visual analogies

In contrast to commonly designed and used visual analogies, the visual analogies in our experiments differentiate between response options that reflect the end-results of encoding and mapping global relations (the global path) and those that reflect the end-results of encoding and mapping object relations (the object path). Other problem sets, such as SRPM, do not separate the global and object paths and can be solved through either of these two distinct paths.

Consider Figure 18 as an example of a visual analogy from SRPM. At the highest level of the spatial hierarchy, one can identify the global shapes of the top left image, which consists of two parts: the surrounding part and the surrounded part. By comparing the global shapes of the top

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left and top right images, one can infer that the surrounded part is being subtracted, leaving the surrounding element for the missing image. This represents the global path of visual analogical reasoning, where the individual objects do not hold significance.

On the other hand, at a lower level, one can identify the individual objects involved in the top row, namely the diamond and the two intersecting lines. By comparing these objects, reasoners can infer that from left to right, the intersecting lines are being subtracted, resulting in the square remaining for the missing image. This reflects the object path, where reasoners consider individual objects instead of the global shapes they might form.

In summary, the visual analogies in our experiments differentiate between the global and object paths, while other problem sets, like SRPM, may allow for either of these paths to be employed in solving the analogies.

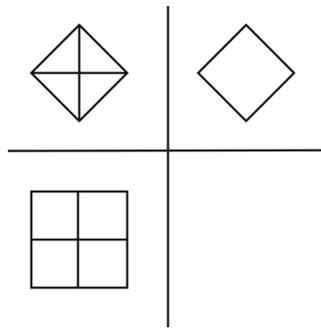


Figure 18: an example of SPM with the subtraction rule

Previous studies in visual problem solving have primarily focused on the object path, especially in the context of RPM. Consequently, the visual problems designed and implemented based on RPM often contain sparse and separated elements without forming a spatially integrated global organization. This approach has led to visual problem solving being predominantly carried out through the object path, as global spatial relations are lacking in these problems. As a result, the possibility of visual problem solving through the global path has been overlooked, despite the findings of this research indicating that reasoners tend to solve visual analogies based on global relations.

This research can be seen as a starting point for considering the importance of the global path in visual problem solving. Future studies could further investigate the significance of the global/object path in visual reasoning and explore the effects of global and object features on

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problem solving. By delving deeper into these aspects, we can gain a better understanding of how individuals utilize different paths and features in visual problem solving, leading to more comprehensive insights into visual analogical reasoning.

5.3. Spatial mental representations

The process of analogical reasoning initiates with constructing a mental representation of the source and target problems. Then, reasoners notice analogous relations between them, map the source onto the target, and finally generate an analogous solution for the target based on the source (Chen, 1995; Gentner, 1989; Holyoak, 1984). In solving the radiation problem using the fortress story, people firstly build a mental representation of the fortress story during reading. Experiment 8 showed that global spatial properties of the source facilitated analogical transfer. In line with Mani and Johnson-Laird (1981), we think that such properties could invite people to build a mental representation which is more based on global spatial relations than verbatim details such as relations between individual elements of the story. Furthermore, Experiment 10 showed that global spatial properties of the target facilitated analogical transfer. This finding may also indicate that narratives which lead to build global spatial mental representations rather than semantic detailed representations facilitate analogical transfer.

Experiment 4 (Chapter 3) also showed that people with a higher tendency to memorize the global spatial relations of the images were better problem solvers than people who tended to build a mental representation based on the individual objects. This finding is in line with the findings from Experiment 8, 9 and 10 (Chapter 4), suggesting that building global spatial mental representations is a crucial step in analogical transfer –at least in the case of the fortress story-radiation problem.

One can claim that the added spatial properties would make analogical transfer easier due to the amount of relevant relations between the source and the target. To respond to this, we run Experiment 9 in which we asked some of participants to draw the problem and the solution of the fortress story, based on the version without explicit spatial properties. For some others, we asked them to write about the story. Results showed that the effect of the explicit spatial properties on analogical transfer remained significant in the case of writing, but it was not significant in the case of drawing. This finding indicates that the spatial properties do not facilitate analogical transfer due to an increment of the relevant relations between the source and target. Rather, they might

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facilitate analogical transfer because they lead people to construct a global spatial mental representation of the source instead of a detailed one based on object properties.

5.4. Spatial-based *versus* object-based mapping

One of the main objectives of this research was to examine the role of spatial relations in analogical reasoning. By spatial relations, we referred to analogous relations between two situations characterized by two aspects: (i) the relations are independent of the non-spatial properties of the objects involved, and (ii) the relations are spatial and can be encoded at different levels of the spatial hierarchy, ranging from the global shape to more local levels. The results indicated that individuals could map spatial relations from the source to the target in both visual and verbal domains.

In visual domains, the findings demonstrated that reasoners could map the spatial global structures of the images from one set to another. This suggests that individuals are capable of mapping visual representations based on spatial relations, known as spatial mapping. In verbal domains, as evidenced by Experiments 6 and 7, people could map a set of abstract visual diagrams onto a verbal problem that also described specific spatial relations. It is unlikely that individuals map the visuals onto the verbal problem by considering relations between abstract elements of the visuals (such as circles and squares) and individual elements of the radiation problem (such as tumors and rays). Instead, we propose that people map the source onto the target based on spatial relations, which involves spatial mapping.

In addition to spatial mapping, we also considered another type of mapping that is based on relations between individual objects. In this case, two situations may have analogous object-based relations if the relations are defined based on non-spatial properties of the individual objects. In visual domains, people could map object-based relations from one set of images to another, disregarding spatial relations. Chapter 3 demonstrated that individuals can perceive relations between individual objects and overlook global spatial relations, particularly when the global shape is not salient.

Similarly, in verbal domains, Studies 8 and 9 revealed that some individuals could map a verbal scenario without explicit spatial properties onto a verbal problem. Although, as depicted in the drawings, reasoners inferred some spatial relations even when they were not explicitly stated,

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it is also possible to map non-spatial relations between the elements of a verbal source (e.g., the causal relation between soldiers and the destruction of a fortress) onto a verbal target (e.g., the causal relation between rays and the destruction of a tumor) and analogically solve the target. Therefore, similar to visual domains, individuals can primarily map the source onto the target based on non-spatial relations between the individual elements of the source and target in verbal analogical reasoning.

To illustrate the distinction between the two types of mapping, consider Figure 19a, a visual analogy composed of geometric shapes. From left to right, only spatial relations can be identified and mapped, while from top to bottom, the circles decrease in size. This example demonstrates situations where only spatial relations are involved. On the other hand, Figure 19b presents another visual analogy composed of fruit shapes, where there is no alignable spatial organization between the images. Therefore, the problem can only be solved based on relations between the individual elements (from the left image to the right image, a pear is being subtracted). This exemplifies situations where only object relations can be identified and mapped.

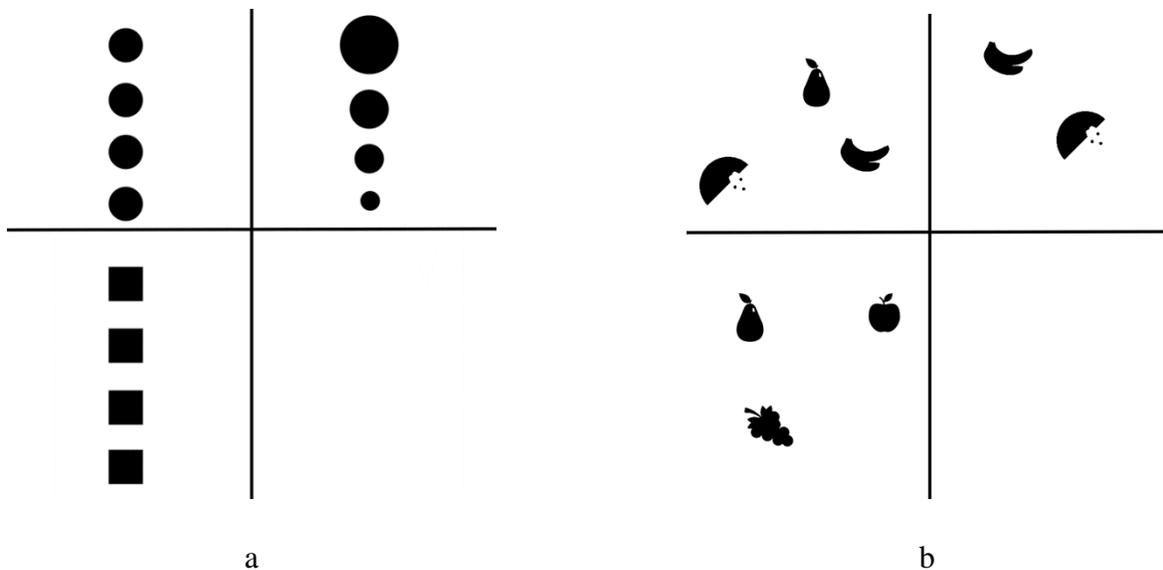


Figure 19: a) an example a visual analogy which can be solved only by the global path, and b) an example of visual analogy which can be solved only by the object path.

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In previous literature, there was no distinction between spatial mappings and object-based mappings. So, specifically in verbal domains, researchers designed situations in which both kinds of mappings were possible. This research can be considered as a starting point for separating spatial mappings from object-based mappings. Far broader visual and verbal problems can be designed to explore the factors that affect spatial mapping/object-based mappings.

5.5. Local vs global correspondences

As pointed out by Love et al. (1999), most models of analogical reasoning assume a strict local-to-global processing model, where local correspondences between the individual elements of the source and target are generated first, followed by the integration of these local correspondences into a global correspondence. This approach is commonly used for object-based mapping, where people identify similar relations between the individual elements of the source and target before mapping them.

However, for spatial mapping, this local-to-global model is not always applicable. Chapter 3 of this research demonstrated that many visual analogies could be solved by encoding and mapping global relations. In this case, individuals ignore the individual elements of the images and instead compare and map the global shapes of the images. This type of mapping is purely global, as it involves mapping the global spatial structure of the source onto the target without generating local correspondences. Thus, visual analogies can be solved using both the local-to-global approach and the global approach, in line with the non-strict sequential order of processes observed in visual perception.

Based on the results of this research, we know that the global way of reasoning occurs in visual domains. However, there is no clear evidence for the global way of reasoning in verbal domains. Considering the similarities between the mechanisms of visual reasoning and verbal reasoning, as well as the importance of the global spatial structure of the source and target in verbal analogical reasoning (as demonstrated in Experiments 8, 9, and 10), we believe that generating a global correspondence between the global structures of the source and target directly, without generating local correspondences, is possible in verbal analogical reasoning as well.

The global way of analogical reasoning may extend to a wide variety of situations. For example, in the well-known analogy between the solar system and the atomic model, which was

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explained as an example in Gentner (1983), beyond the relational similarities between the two systems, there is a similarity between the spatial structures of the solar system and the atomic model, regardless of the properties of the individual elements in the two systems. This spatial similarity could be the critical factor leading to the structural mapping between the two, in line with the spatial alignment principle (Matlen et al., 2020). Spatial alignment of the global structures facilitates noticing the similarities between the spatial structures of the two systems, and the mapping of the solar system onto the atomic model. However, if reasoning is considered a strictly local-to-global process, such a global structural correspondence between the two systems, regardless of any local correspondences, may not be possible.

5.6. The global/object tendency as a new measure of individual differences

In Experiment 2, a new measure called the global to object ratio was introduced, which provides valuable information about individual differences in visual analogical reasoning. This ratio indicates the tendency of individuals to encode and map global relations *versus* object relations. In the case of black-and-white problems, there was a positive correlation between this ratio and certain problem-solving skills, such as visual problem-solving skills in Study 2 and verbal analogical problem-solving skills in Experiment 4. Therefore, the global/object tendency in visual analogies can be considered a novel indicator of individual differences.

The key distinction between this measure and visual problem-solving scores obtained from different visual problem sets lies in their focus. The global to object ratio captures the inclination to encode and map two types of relations at different levels of the spatial hierarchy, while the scores in standard reasoning tasks (visual matrices or analogical problem solving) solely reflect a general relational reasoning ability (e.g., Gray & Holyoak, 2019). To perceive global relations, one needs to consider the overall shapes of the images beyond the potential relations between individual objects. On the other hand, to notice object relations, one can disregard global spatial features and concentrate on the possible relationships between individual objects.

5.7. Limitations and further directions

There are some limitations to this research that might guide further research. First, regarding visual analogies with the global/object format, we believe that the number of elements can influence the encoding and mapping of global/object relations, although this variable was not

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controlled for. Given the viewpoint that cognition is closely connected with perception (Barsalou, 1993; Finke, 1985; Gilbert, 1991), differences in perceptual processes may lead to variations in other cognitive processes, such as reasoning (Love et al., 1999). Therefore, we suggest that having more individual objects in a visual analogy facilitates problem-solving through the global path. Further research is needed to explore this hypothesis.

Similarly, the type and number of relations between elements may affect the tendency to notice global/object relations, although this aspect was not explored in this research. Our experiments utilized a few simple rules (addition, subtraction, and change in size) to investigate the global/object tendency. We did not examine potential differences between rules, as our control variables focused on global *versus* object features. Hence, an open question remains regarding the extent to which different rules have varying effects on employing the global/object paths. Additionally, each visual analogy we designed and implemented contained only one type of rule. Future studies could investigate whether different types of rules in a visual analogy influence the use of global/object paths.

Furthermore, the global-to-object ratio was introduced and examined in this research. However, regarding the potential associations between the ratio and other cognitive measures, we specifically studied the correlations with visual problem-solving skills (using a short version of ARPM) and verbal analogical problem solving (using the fortress story-radiation problem task). Future studies could explore the extent to which the global-to-object ratio correlates with other measures. For instance, previous studies have reported that individuals with a global processing style tend to discover structural relations to a greater extent than those with a more local processing style (Li et al., 2018). Investigating the possible association between the tendency to map global relations in visual problem-solving and the tendency to attend to global features when perceiving visual stimuli would be valuable in future research.

Regarding verbal domains, we only employed one target (the radiation problem), which, like any other verbal narrative, has its unique characteristics. Specifically, as mentioned in previous studies (Gick & Holyoak, 1980 and 1983), the dispersion-convergence solution of the radiation problem is spatial in nature. Future studies could incorporate different verbal problems with varying degrees of spatial properties to explore the effectiveness of dynamic *versus* static abstract visual representations as sources for structurally similar verbal problems. Additionally,

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investigating whether manipulating the spatial properties of a verbal source has a significant effect on analogical transfer to a verbal target requires further studies using different verbal scenarios.

Furthermore, the exploration of verbal analogical problem-solving using only one set of source-target did not provide us with the opportunity to investigate factors that influence structural mapping, specifically spatial mapping. We suggest that future studies should explore two factors that can influence spatial mapping.

Firstly, similar to visual domains, we believe that the number of elements in a verbal scenario may impact spatial *versus* object-based mapping. Just as the number of elements in a scene affects the precedence of global *versus* local features in perception (Martin, 1979), the number of elements in a verbal narrative can also have an effect on facilitating spatial or object-based mappings. Specifically, in scenes with few elements, local features might take precedence over global features, whereas in scenes with many elements, global features may be prioritized. Therefore, we hypothesize that having more elements involved in a verbal narrative would facilitate spatial mappings rather than object-based mappings. The fortress story exemplifies this spatial aspect, with its many elements, including multiple roads leading to the fortress and numerous small groups organized to attack it.

Secondly, in a given situation, some relations between elements are spatial, while others are non-spatial. For instance, in the fortress story, the relation between the general and the fortress (to-destroy) is non-spatial, whereas the relation between groups of soldiers and the fortress involves spatial movement (moving-toward through different paths). We propose that the extent to which analogical transfer relies on spatial mapping may depend on the number of relevant spatial relations in a situation. Even if the explicit spatial properties of the fortress story are removed, the story still implicitly contains spatial relations due to the movement of soldiers toward the fortress.

As a more specific consequence of the second factor, while we replicated previous studies that confirm the superiority of dynamic visuals over static visuals in analogical transfer due to the facilitating role of movements in understanding causal relationships, doubts still remain about why animations are more effective. Specifically, dynamic visuals representing the dispersion-convergence solution incorporate temporal dimensions that cannot be fully represented by static graphics. It remains uncertain whether the positive effect of dynamic visuals is solely due to their

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ability to represent temporal relations or if other factors, such as the capacity of animations to better motivate participants (Tversky et al., 2002) or demonstrate non-temporal relations more effectively, also contribute. To address this issue, further research using different verbal problems with and without temporal relations is needed.

Lastly, since our experiments only collected behavioral data, we did not have access to the mental representations individuals constructed while reading the source and target, or when contemplating the target problem. We recommend that future studies explore how spatial mental representations are constructed and utilized during reading and reasoning, using other experimental methods such as fMRI. In recent years, numerous studies have been dedicated to investigating how humans build and utilize cognitive maps in cognitive tasks such as problem-solving and navigation (Vigano et al., 2023; Bellmund et al., 2018; Bottini and Doeller, 2020). Exploring the construction and utilization of a mental global spatial structure during analogical problem-solving, particularly in narratives, would be an intriguing avenue of research.

Conclusions

This research aimed to explore the role of global structures in analogical reasoning. As far as we know, there has been no systematic research investigating how global structures, particularly those related to spatial relations, affect analogical reasoning in both visual and verbal domains. To this end, we designed and implemented visual analogies that separated the end-result of the global path (by encoding and mapping global spatial relations) and the object path (by encoding and mapping object-based relations). The results of the experiments consistently showed that the likelihood of reasoning through the global path was higher in the case of salient global shapes. However, object saliency, whether based on color or familiarity, had no significant effect on the likelihood of reasoning through either of these paths. The reported findings also showed that individuals with a higher tendency to employ the global path performed better in both visual and verbal analogical problem-solving tasks. This suggests that a higher tendency to encode and map global rather than object relations promotes the use of the most optimal problem-solving strategy.

The relevance of encoding the global structure for rule induction and analogical transfer was also observed in verbal presentations of the target problem. Consistent with previous studies, the results showed that visual analogues representing the spatial structure of the solution could be useful for analogical transfer. Moreover, the present findings demonstrated, as far as we know for the first time, that the amount of verbally explicit spatial properties in the verbal source had a significant effect on the rate of successful analogical transfer. This effect was stronger if both the source and the target contained explicit global spatial properties, increasing the chance of spontaneous analogical transfer. Finally, the findings of this research suggest that a global spatial mental representation of a narrative facilitates analogical transfer to a larger extent than a mental representation that is mainly based on verbatim details of individual objects. It is possible that spatial properties highlighted either in the text or in a picture may invite reasoners to induce more abstract structures, facilitating the recognition of relational commonalities between different situations.

Overall, the present findings have some theoretical implications, suggesting that analogical reasoning can occur not only through a strict local-to-global fashion but also in a global fashion, generating a global correspondence regardless of local correspondences between individual elements. The way analogical reasoning proceeds would depend on different factors of the

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situation and the reasoner's abilities and propensities. This flexibility is similar to how we, as humans, perceive a situation at different levels of the spatial hierarchy. Moreover, the spatial characteristics of the global way of reasoning can highlight the importance of our ability to build and manipulate spatial mental models from abstract concepts, which is the subject of various state-of-the-art studies.

The results of this research could also contribute to improving the design of teaching materials and useful learning strategies. For example, adding explicit global spatial properties or training learners to schematically draw may help reasoners induce more abstract knowledge than simply writing summaries. The reported findings also suggest new ways to assess individual differences in reasoning skills, such as the global-to-object ratio. Much more can be done in future research by expanding visual analogies with the global/object format or controlling the amount of verbally explicit spatial structures in a variety of verbal narratives.

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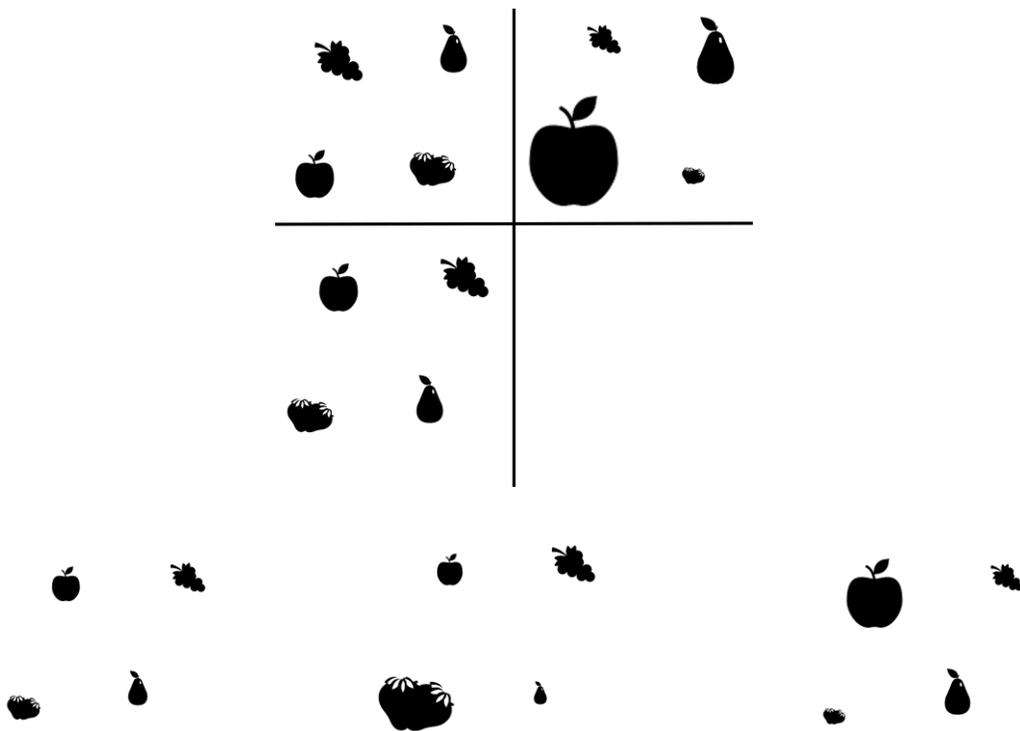
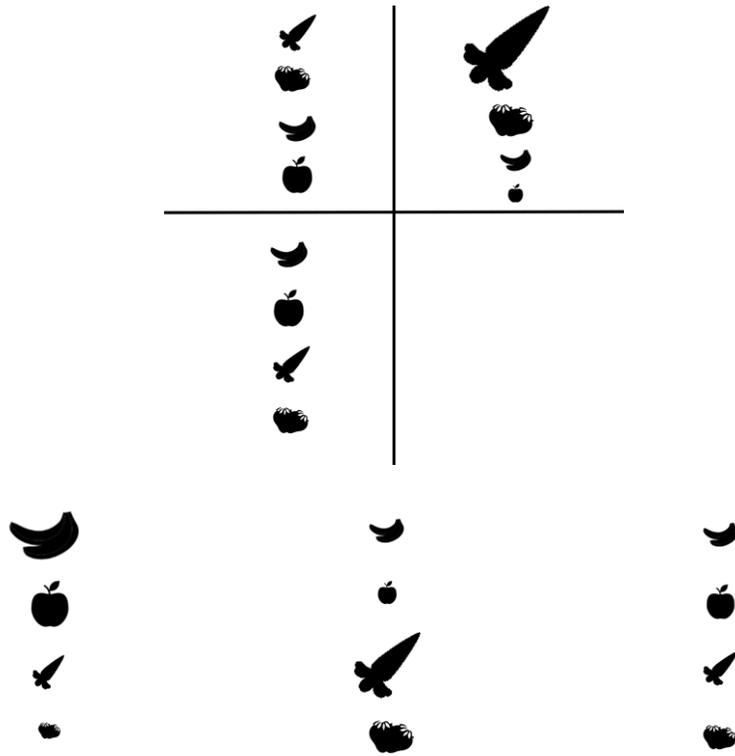
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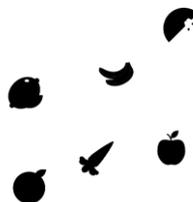
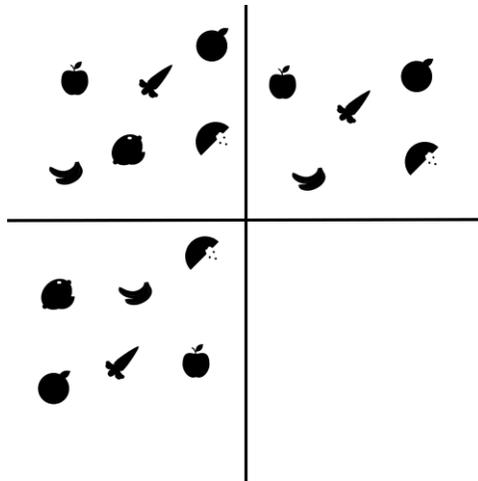
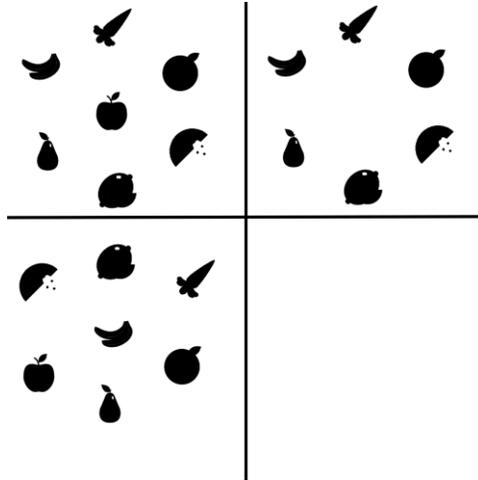
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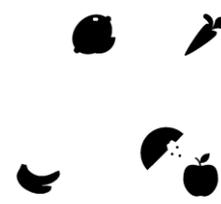
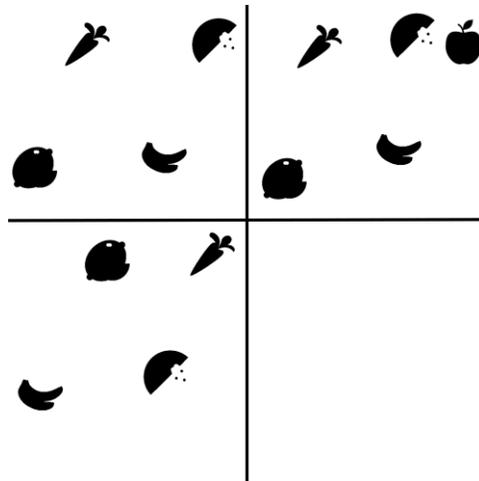
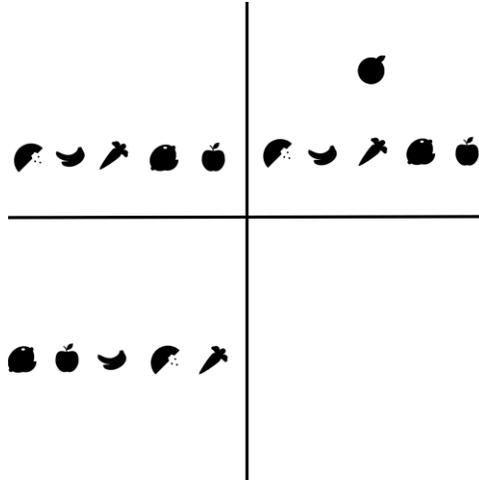
Appendix A: Black-and-white visual analogies with the global/object format



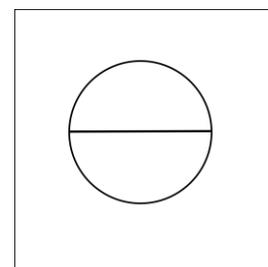
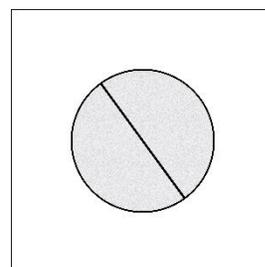
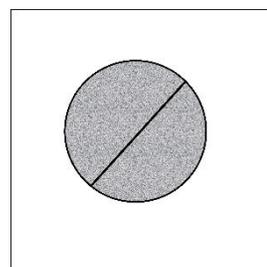
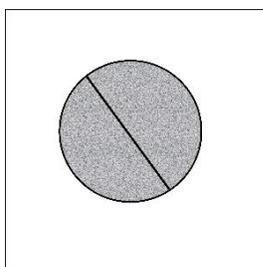
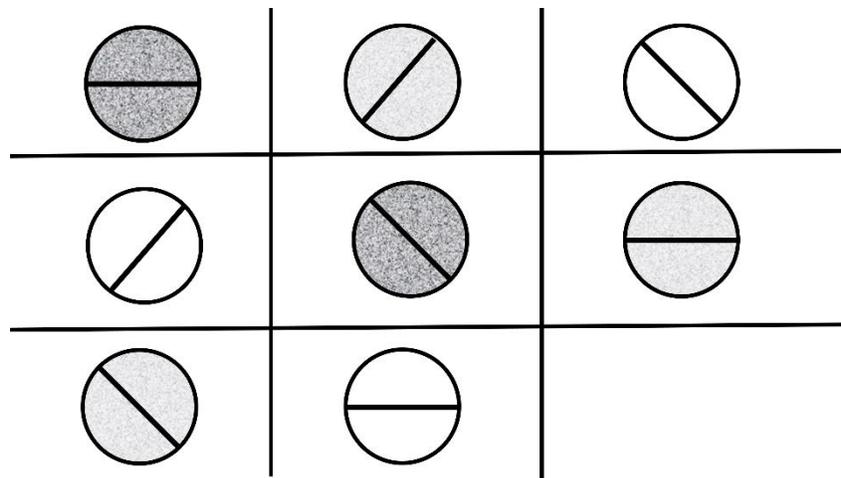
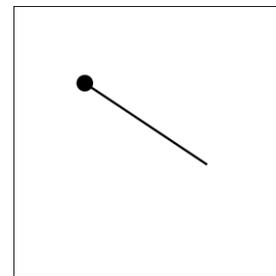
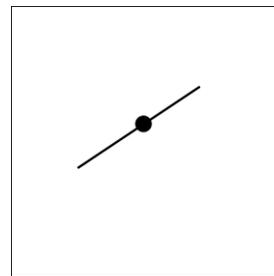
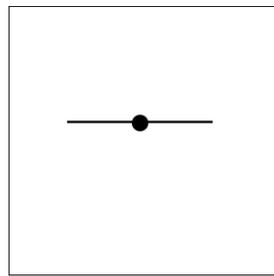
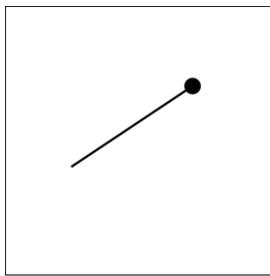
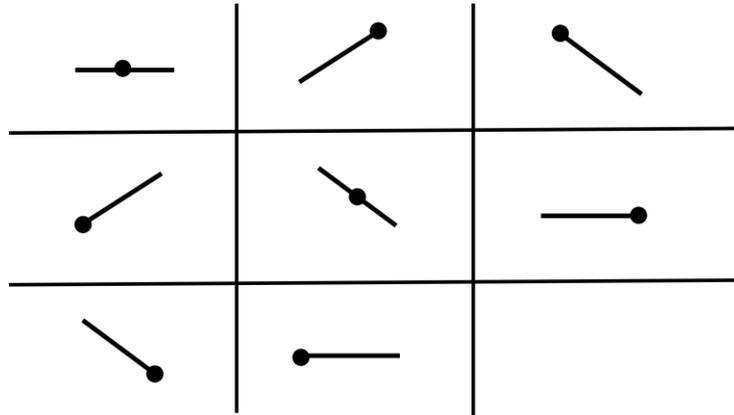
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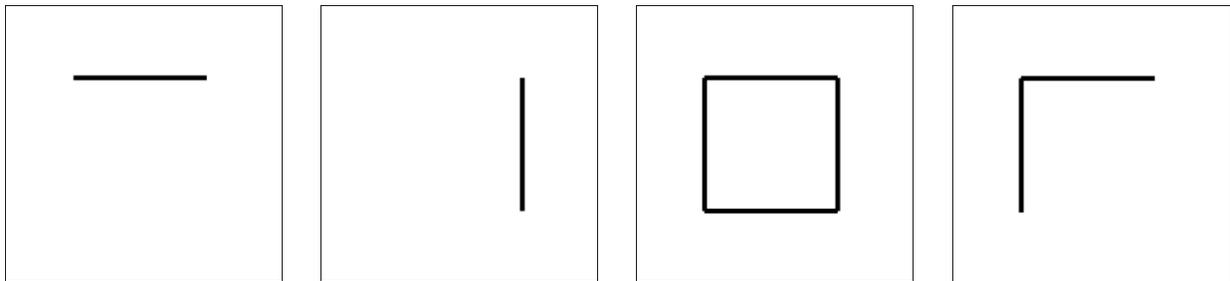
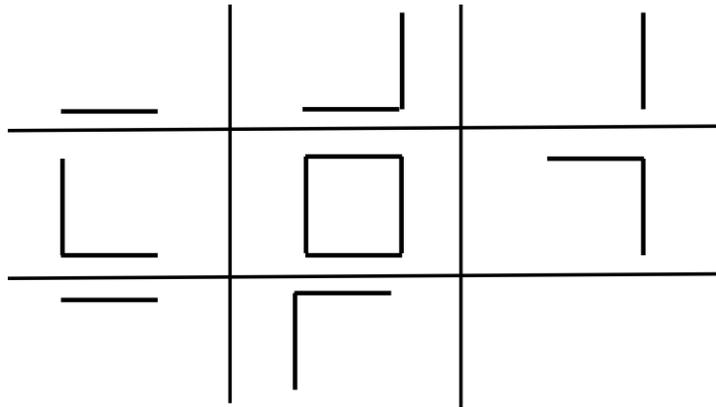
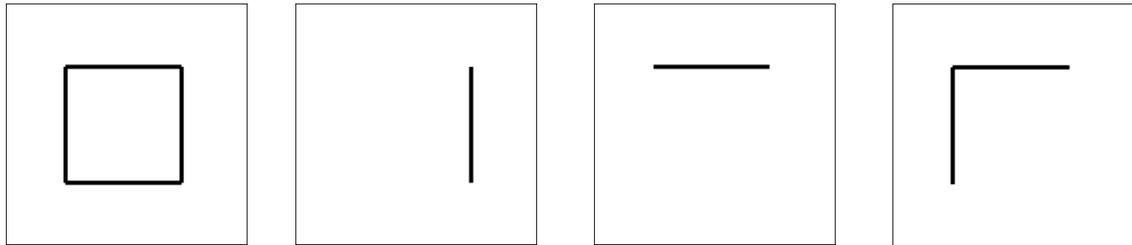
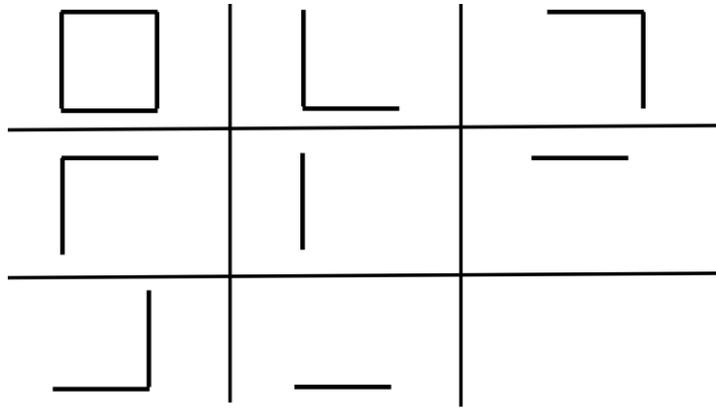
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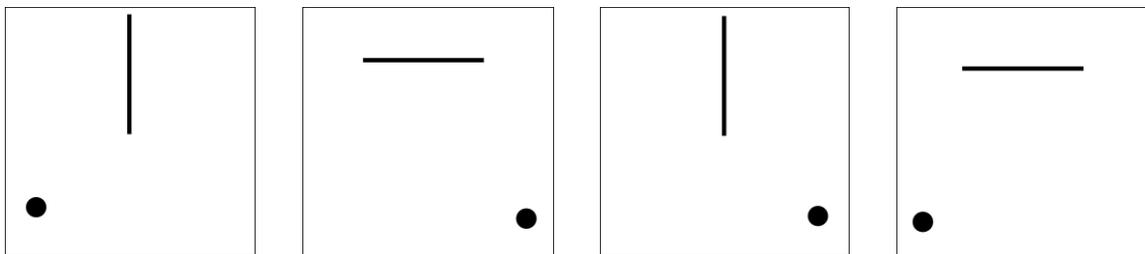
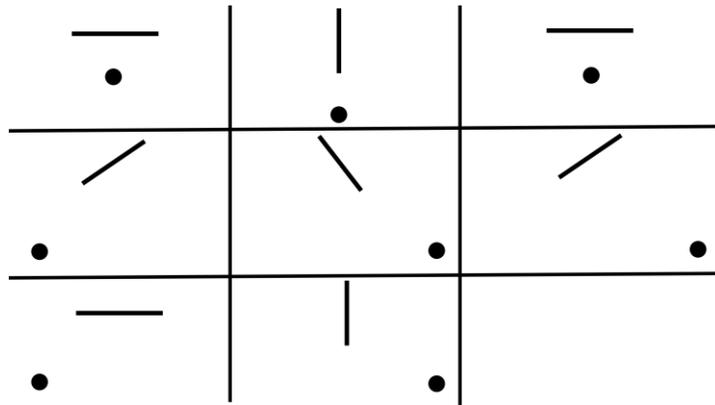
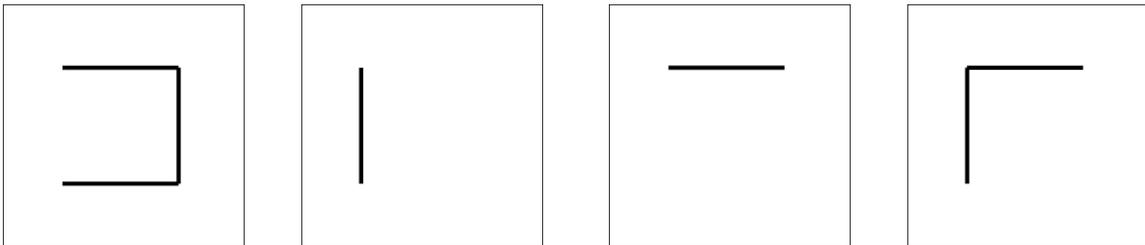
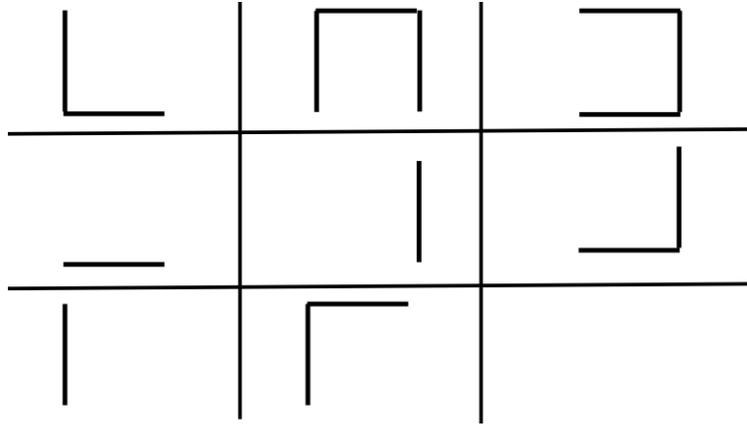
Appendix B: Visual analogies with only spatial relations



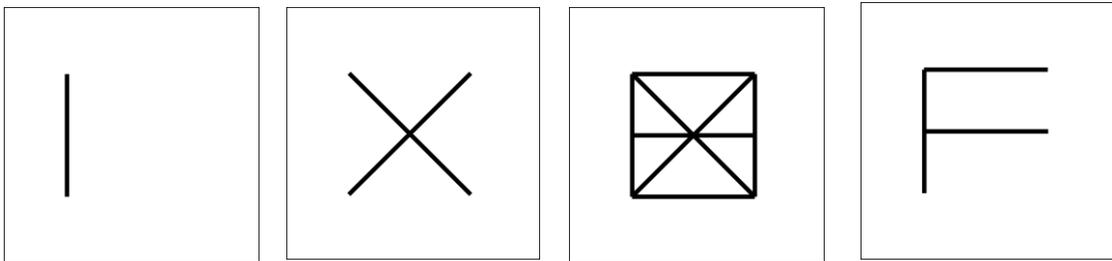
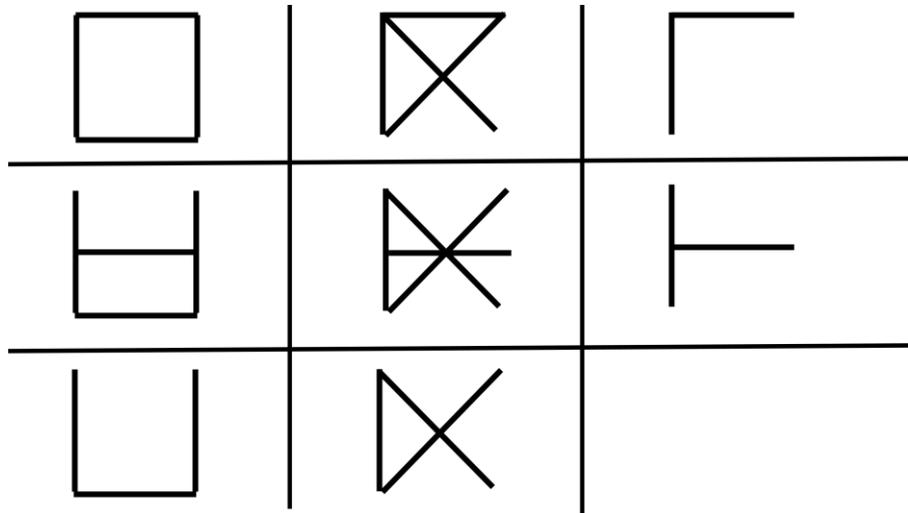
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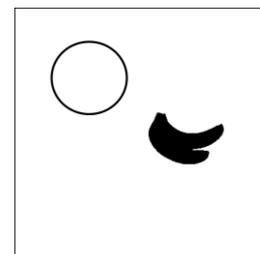
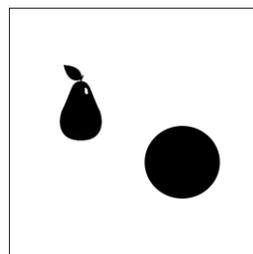
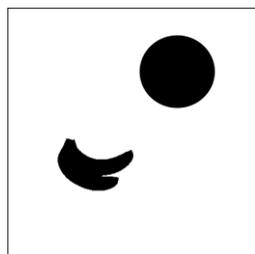
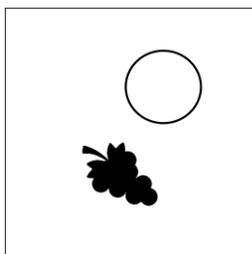
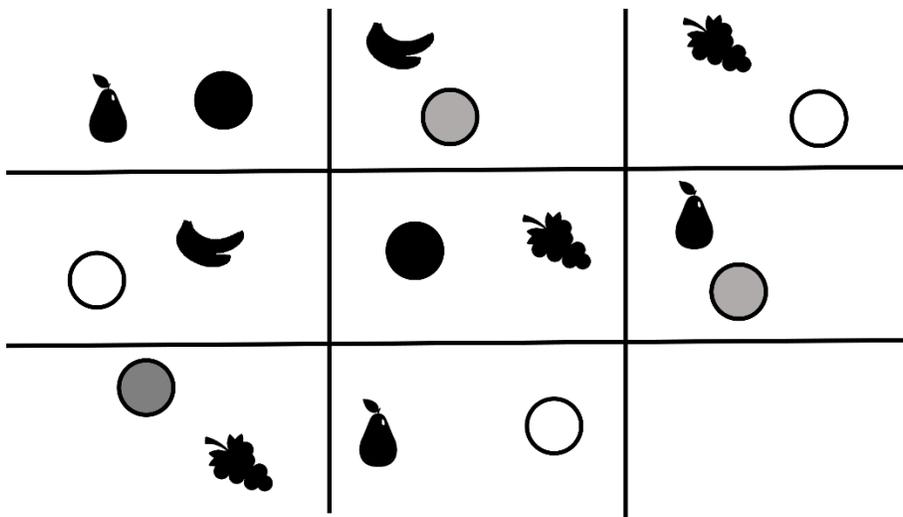
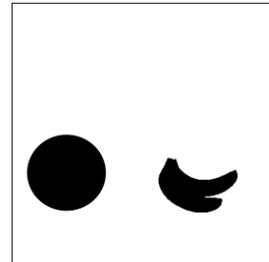
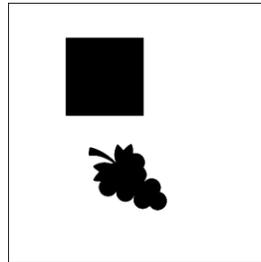
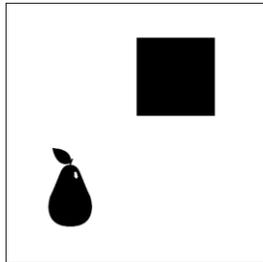
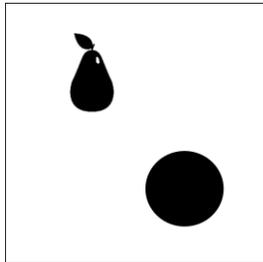
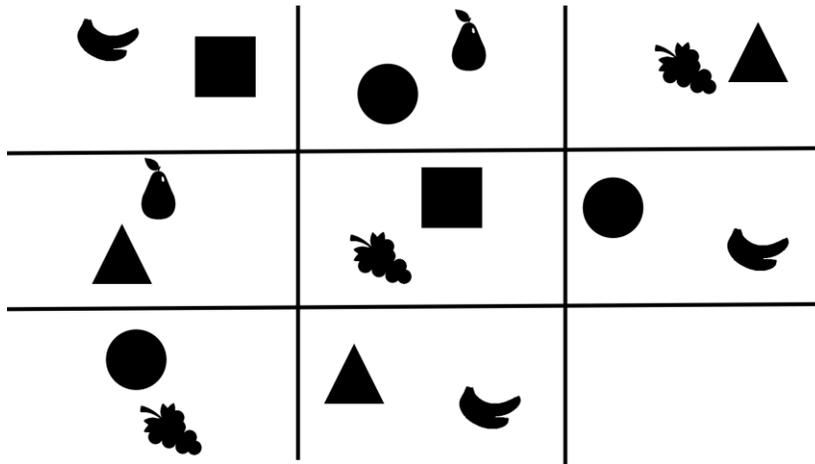
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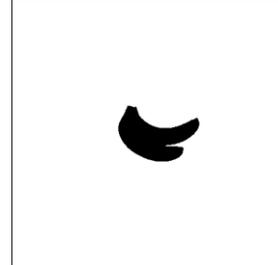
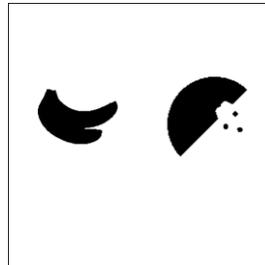
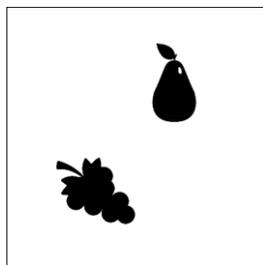
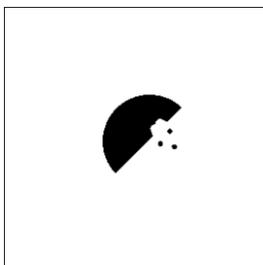
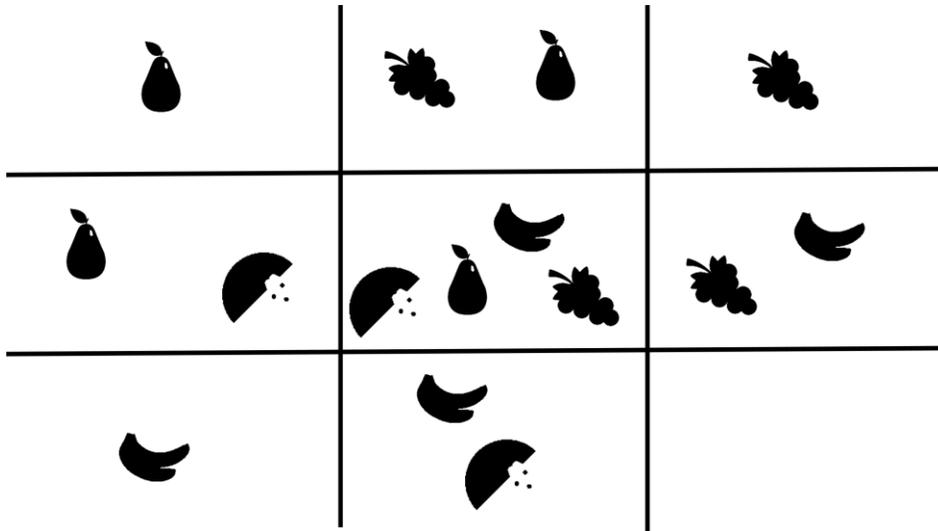
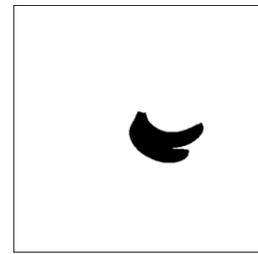
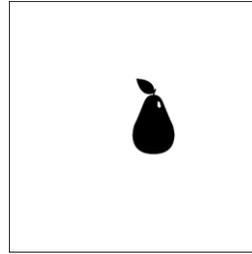
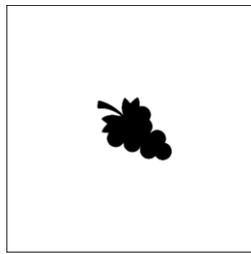
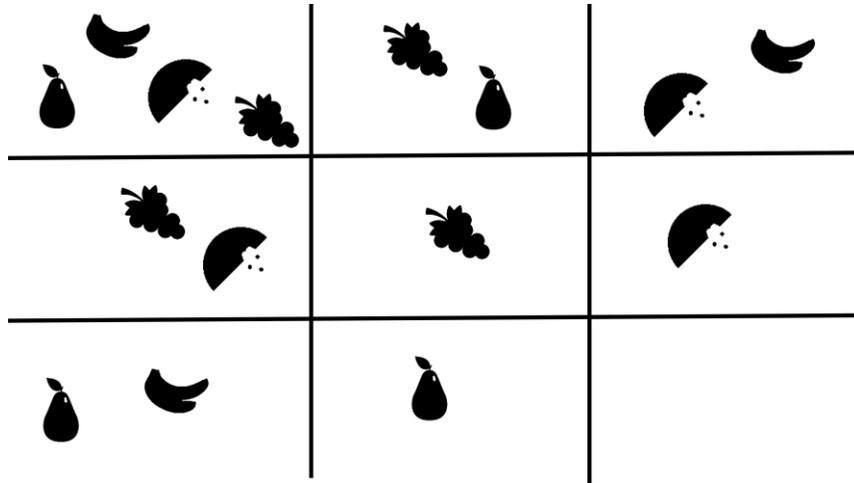
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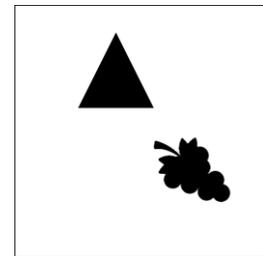
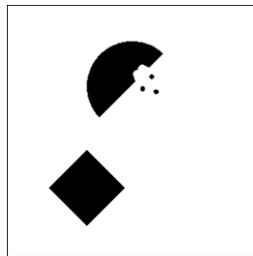
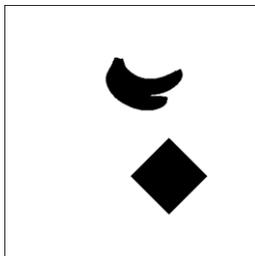
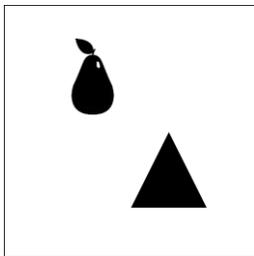
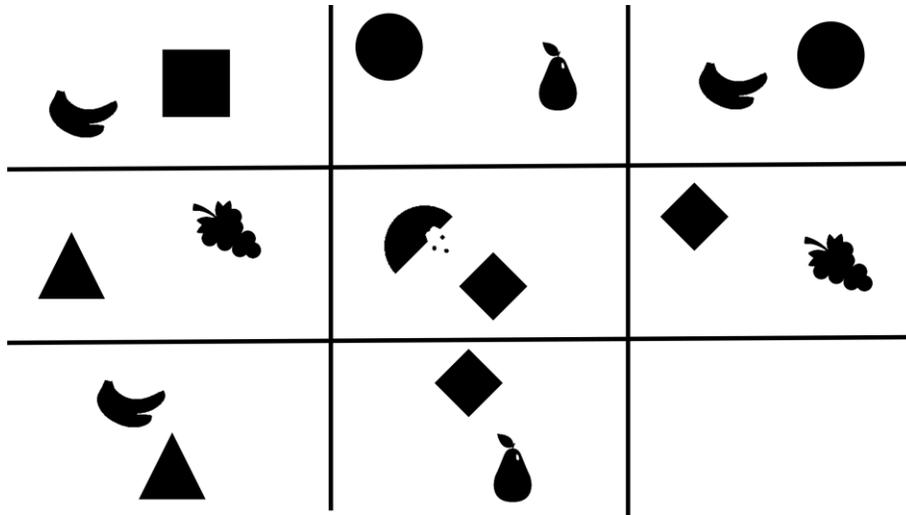
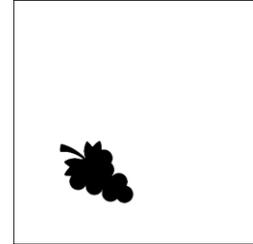
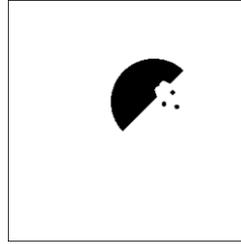
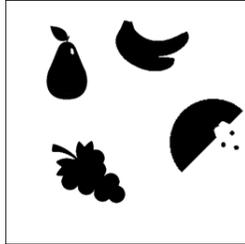
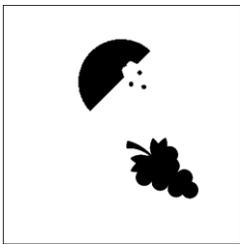
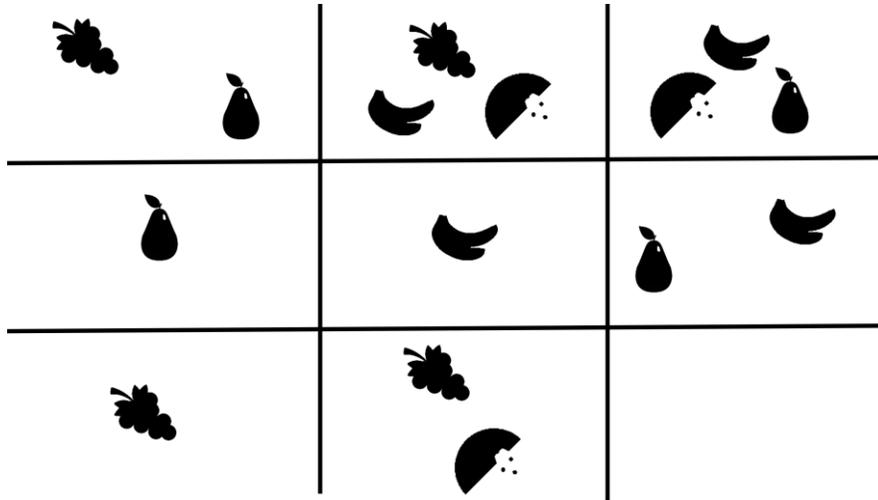
Appendix C: Visual analogies with only object relations



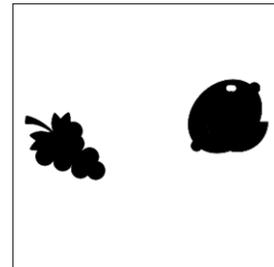
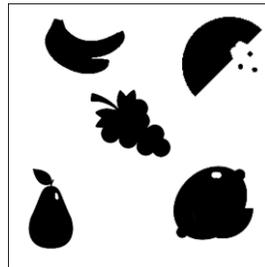
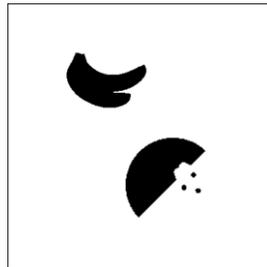
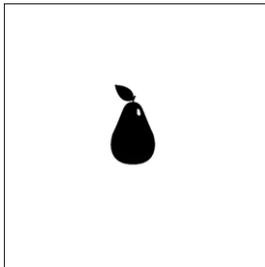
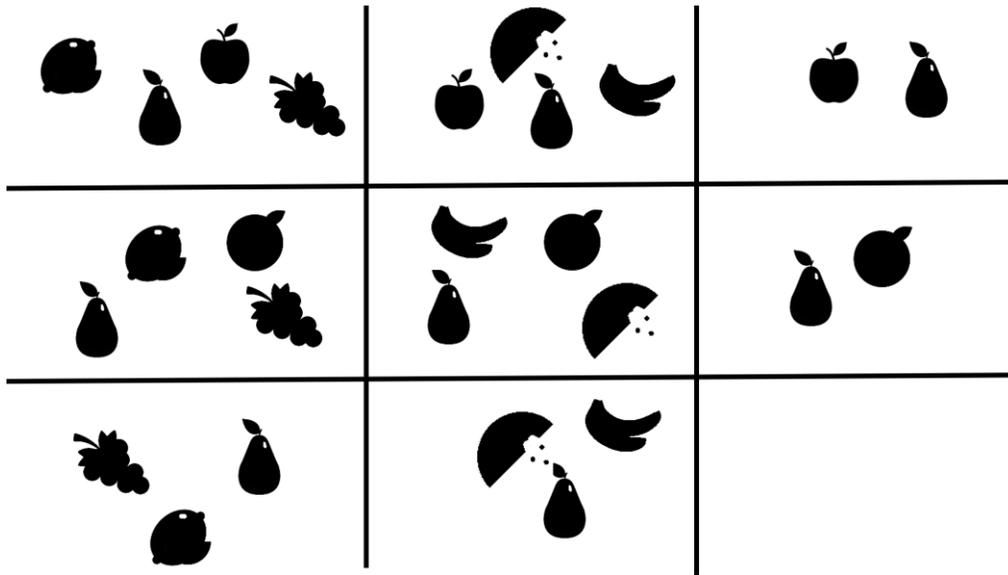
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Appendix D: The fortress story

The standard version of the story

A small country was controlled by a dictator. The dictator ruled the country from a fortress located in the middle of the territory. Many roads radiated from the fortress to the towns around like the spokes of a wheel. A general, leading a large army, rebelled and vowed to capture the fortress. However, the general was informed that the dictator had planted mines on each of the roads in such a way that the passage of the entire army through one of them would cause the destruction of the road, soldiers and surroundings. The general also knew that the mines would not go off with small groups of soldiers. Therefore, he divided his army into small groups so that each marched on a different path in a synchronized manner. In this way, the entire army arrived at the fortress at the same time and the general was able to capture the fortress.

The story without explicit spatial properties

A small country was controlled by a dictator. The dictator ruled the country from a fortress painted in the colors of the country's flag. There were many paths from the different towns to the fortress. A general, leading a large army, rebelled and vowed to capture the fortress. However, the general was informed that the dictator had planted mines on each of the roads in such a way that the passage of the entire army through one of them would cause the destruction of the road, soldiers and surroundings. The general also knew that the mines would not go off with small groups of soldiers. Therefore, he divided his army into small groups so that each marched on a different path in a synchronized manner. In this way, the entire army arrived at the fortress at the same time and the general was able to capture the fortress.

Appendix E: The Radiation Problem

The standard version of the radiation problem

Suppose you are a doctor faced with a patient who has a malignant tumor in his stomach. It is impossible to operate on the patient, but unless the tumor is destroyed the patient will die. There is a kind of ray that can be used to destroy the tumor. If the rays reach it all at once at a sufficiently high intensity, the tumor will be destroyed. Unfortunately, at this intensity the healthy tissue that the rays pass through on the way to the tumor will also be destroyed. At lower intensities the rays are harmless to healthy tissue, but they will not affect the tumor either. What type of procedure might be used to destroy the tumor with the rays, and at the same time avoid destroying the healthy tissue?

The radiation problem with explicit spatial properties

Suppose you are a doctor faced with a patient who has a malignant tumor in his body surrounded by healthy tissues. It is impossible to operate on the patient, but unless the tumor is destroyed the patient will die. There is a kind of ray that can be used to destroy the tumor. If the rays reach it all at once at a sufficiently high intensity, the tumor will be destroyed. Unfortunately, at this intensity the healthy tissue that the rays pass through on the way to the tumor will also be destroyed. At lower intensities the rays are harmless to healthy tissue, but they will not affect the tumor either. What type of procedure might be used to destroy the tumor with the rays, and at the same time avoid destroying the healthy tissue?