

APPENDIX 2

OVERLAP FACTOR TABLES FOR UNIFORM ILLUMINATION

1.1 SINGULAR POINTS

When the separation of the lidar and telescope axes lies between these extremes

$$|r_T(R) - W(R)| < d(R) < r_T(R) + W(R) \quad (1)$$

(for notation see Chap.3, Sect.1.3 and Fig.2b) the *overlap factor* (ovf) takes some value between zero and unity. Note that all the variables are functions of the range R . Replacing the inequality signs by equality signs yields the ranges where the ovf is zero or unity. There are four possible roots to eq.(1). By reference to Fig.4 of Chap.3, the first couple of roots ($R_{min}^{(1)}, R_{min}^{(2)}$) represents the distances where the ovf takes zero and unity values, respectively, in the rising edge of $\xi(R)$ (i.e in the ranges where the laser beam enters the field-of-view of the telescope). The second couple of roots ($R_{max}^{(2)}, R_{max}^{(1)}$) represents the distances where the ovf value is unity and zero, respectively, in the falling edge of $\xi(R)$ (i.e in the ranges where the laser beam leaves the field-of-view of the telescope). The superindexes ⁽¹⁾ and ⁽²⁾ indicate zero and unity values of the ovf function, respectively.

The four roots can be retrieved by solving the following independent set of equations, which are equivalent to the formulation of eq.(1) using equality signs:

$$[(\phi + \delta)^2 - \theta^2]R^2 + [2(r_o - d_o)(\phi + \delta)]R + [(d_o - r_o)^2 - w_o^2] = 0 \Rightarrow R_{min}^{(1)}, R_{min}^{(2)} \quad (2)$$

$$[(\phi - \delta)^2 - \theta^2]R^2 + [2(r_o + d_o)(\phi - \delta)]R + [(d_o + r_o)^2 - w_o^2] = 0 \Rightarrow R_{max}^{(1)}, R_{max}^{(2)} \quad (3)$$

For each tilting angle δ , there are up to four roots associated. At the end on this appendix, extensive tables are listed arranged by increasing $R_{min}^{(1)}$. The angle δ has been computed according to

$$d = r_T + W \Rightarrow \delta = \frac{d_o - (r_T + W)}{R_{min}^{(1)}} \quad (4)$$

1.2 ASYMPTOTIC VALUES

In the far-field, the following approximations hold for the laser beam radius, the distance between the telescope and the laser axes in the target plane and the radius of the field-of-view circle in the target plane

$$W(R) \approx \theta R ; d(R) \approx |\delta| R ; r_T(R) = \phi R \tag{5}$$

As for the angles ψ_W and ψ_r of the overlap area (see Chap.3, eq.19 and Fig.2b) we have

$$\psi_W = \cos^{-1} \left[\frac{\delta^2 + \theta^2 - \phi^2}{2\theta|\delta|} \right] \quad \psi_r = \cos^{-1} \left[\frac{\delta^2 + \phi^2 - \theta^2}{2\phi|\delta|} \right] \tag{6}$$

Substituting eqs.(5) and (6) into eq.(17) of Chap.3 yields the asymptotic value of the ovf

$$\xi(\infty) = \frac{\theta^2 \psi_W + \phi^2 \psi_r - \phi |\delta| \sin \psi_r}{\pi \theta^2} \quad |\delta| < \theta + \phi \tag{7}$$

But for the case when the asymptotic value of $\xi(R)$ is neither zero nor unity, eq.(7) can be made redundant and the asymptotic behaviour of $\xi(R)$ can be known by the number of positive roots of eqs.(2) and (3). This behaviour can be better understood by reference to Fig.1.

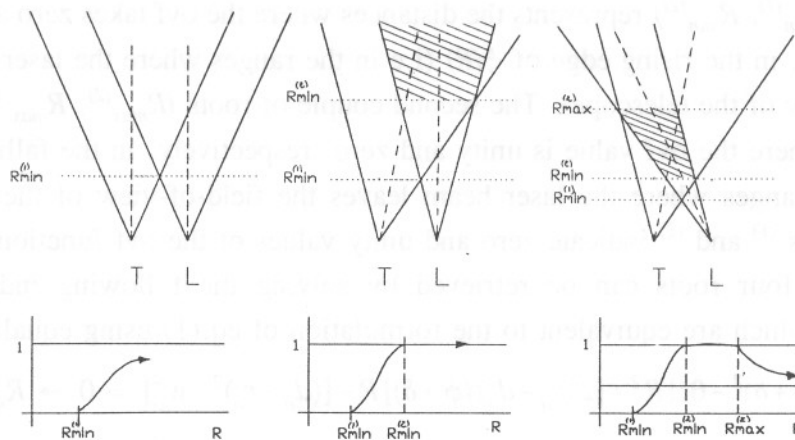


Fig.1 Graphic interpretation of the positive roots of eqs. (2) and (3).

1.3 TABLES

Next, ovf tables for different values of the laser divergence angle, field-of-view and laser-telescope separation are listed.

Laser half divergence angle [mrad] : 0.1
 Receiver-optics half opening angle [mrad] : 0.2
 Laser-Telescope separation axes [mm] : 170

Laser-Telescope inclination angle [mrad]	Rmin1 [m]	Rmin2 [m]	Rmax1 [m]	Rmax2 [m]
5.7950	10.0000	13.3592	46.2878	50.2576
2.7901	20.0000	26.8736	98.8153	109.9822
1.7853	30.0000	40.6969	158.4912	183.4941
1.2807	40.0000	54.9804	226.8862	277.0855
0.9764	50.0000	69.8740	306.2652	400.9918
0.7723	60.0000	85.5276	399.7734	573.5007
0.6256	70.0000	102.0941	511.8374	831.0269
0.5149	80.0000	119.7	648.9	1258.1
0.4283	90.0000	138.6	820.6	2106.4
0.3586	100.000	158.9	1042.3	4611.0
0.30	110.000	180	1340	221760
0.25	120.000	200	1760	-5760
0.21	130.000	230	2400	-3080
0.146	150.000	291	5807	-1757
0.038	200.000	514	-4371	-1029
-0.028	250.000	975	-2116	-822
-0.072	300.000	2514	-1571	-724
-0.104	350.000	-17486	-1325	-666
-0.128	400.000	-2486	-1186	-629
-0.147	450.000	-1486	-1095	-602
-0.162	500.000	-1122	-1032	-583
-0.1847	600.000	-819.1	-950.2	-555.2
-0.2010	700.000	-685.8	-898.8	-537.1
-0.2133	800.000	-610.8	-863.7	-524.2
-0.2228	900.000	-562.7	-838.8	-514.6
-0.2305	1000.00	-529.2	-818.8	-507.1

Laser half divergence angle [mrad] : 0.1
 Receiver-optics half opening angle [mrad] : 0.2
 Laser-Telescope separation axes [mm] : 190

Laser-Telescope inclination angle [mrad]	Rmin1 [m]	Rmin2 [m]	Rmax1 [m]	Rmax2 [m]
7.7950	10.0000	12.5176	36.7801	39.5991
3.7901	20.0000	25.1400	77.2578	84.4231
2.4520	30.0000	37.9702	121.7779	136.2810
1.7807	40.0000	51.1073	170.9303	197.4603
1.3764	50.0000	64.6461	225.5438	271.0772
1.1056	60.0000	78.6775	286.6905	361.6490
0.9113	70.0000	93.2895	355.7362	476.0712
0.7649	80.0000	108.5689	434.4329	625.4693
0.6505	90.0000	124.6031	525.0648	829.0677
0.559	100.000	141	631	1123
0.483	110.000	159	755	1586
0.420	120.000	178	905	2422
0.366	130.000	198	1088	4386
0.320	140.000	219	1317	14526
0.280	150.000	242	1611	-14365
0.138	200.000	383	7591	-1791
0.052	250.000	596	-6081	-1169
-0.005	300.000	957	-2753	-948
-0.047	350.000	1699	-1976	-834
-0.078	400.000	4111	-1630	-765
-0.102	450.000	-36889	-1434	-719
-0.122	500.000	-4089	-1308	-686

Laser half divergence angle [mrad] : 0.1
 Receiver-optics half opening angle [mrad] : 0.2
 Laser-Telescope separation axes [mm] : 200

Laser-Telescope inclination angle [mrad]	Rmin1 [m]	Rmin2 [m]	Rmax1 [m]	Rmax2 [m]
8.7950	10.0000	12.2373	33.6763	36.1411
4.2901	20.0000	24.5646	70.3584	76.4251
2.7853	30.0000	37.0697	110.2814	122.1456
2.0307	40.0000	49.8368	153.8453	174.8715
1.5764	50.0000	62.9456	201.6104	236.6248
1.2723	60.0000	76.4713	254.2898	310.1632
1.0542	70.0000	90.4855	312.7675	399.4113
0.8899	80.0000	105.0572	378.1388	510.1875
0.7616	90.0000	120.2547	451.7753	651.5289
0.6586	100.000	136.1466	535.4200	838.2940
0.346	150.000	229	1215	6456
0.188	200.000	352	3400	-2682
0.092	250.000	525	-38944	-1443
0.028	300.000	787	-4164	-1101
-0.018	350.000	1229	-2538	-941
-0.053	400.000	2136	-1961	-848
0.080	450.000	5060	-1666	-787
-0.102	500.000	-50490	-1487	-745

Laser half divergence angle [mrad] : 0.1
 Receiver-optics half opening angle [mrad] : 0.2
 Laser-Telescope separation axes [mm] : 180

Laser-Telescope inclination angle [mrad]	Rmin1 [m]	Rmin2 [m]	Rmax1 [m]	Rmax2 [m]	Asymptote
6.7950	10.0000	12.8781	40.8186	44.1136	0
3.2901	20.0000	25.8817	86.3366	95.0774	0
2.1187	30.0000	39.1341	137.0913	155.5746	0
1.5307	40.0000	52.7556	194.0073	229.2004	0
1.1764	50.0000	66.8627	258.3930	321.2263	0
0.9390	60.0000	81.5693	331.9871	439.9626	0
0.7685	70.0000	96.9881	417.0939	599.4487	0
0.6399	80.0000	113.2330	516.8181	825.494	0
0.5394	90.0000	130.4	635.4	1171.3	0
0.459	100.000	148.7	779.1	1767.5	0
0.392	110.000	168	957	3041	0
0.336	120.000	189	1182	7674	0
0.289	130.000	211	1478	-25976	0.0172
0.213	150.000	261.4	2472.8	-3222.1	0.3670
0.088	200.000	431.2	-23723.8	-1320.2	1.0000
0.012	250.000	718.6	-3194.4	-971.4	1.0000
-0.039	300.000	1312.2	-2019.9	-824.8	1.0000
-0.075	350.000	3262.3	-1597.9	-744.	1.0000
-0.103	400.000	-25987.	-1380.6	-692.9	0.9955
-0.125	450.000	-3237.6	-1248.1	-657.6	0.9078
-0.142	500.000	-1899.4	-1158.9	-631.7	0.8103
-0.168	600.000	-1169.4	-1046.4	-596.5	0.6482
-0.1867	700.000	-916.1	-978.3	-573.5	0.5295
-0.2008	800.000	-787.5	-932.7	-557.4	0.4418
-0.2117	900.000	-709.8	-900.0	-545.4	0.3754
-0.2205	1000.00	-657.6	-875.4	-536.2	0.3239

Laser half divergence angle [mrad] : 0.05

Receiver-optics half opening angle [mrad] : 0.2

Laser-Telescope separation axes [mm] : 180

Laser-Telescope inclination angle [mrad]	Rmin1 [m]	Rmin2 [m]	Rmax1 [m]	Rmax2 [m]
6.7988	10.0000	12.8624	40.8855	43.9839
3.2975	20.0000	25.7562	86.8755	93.9622
2.1296	30.0000	38.7128	138.7993	151.6106
1.5450	40.0000	51.7633	197.7165	219.2013
1.1938	50.0000	64.9383	265.0301	299.8680
0.9593	60.0000	78.2681	342.6236	398.0821
0.7915	70.0000	91.7826	433.0493	520.5054
0.6654	80.0000	105.5111	539.8192	677.5753
0.5670	90.0000	119.4825	667.8756	886.6656
0.4882	100.000	133.7	824.4	1179.0
0.4235	110.000	148.3	1020.1	1617.2
0.3695	120.000	163.1	1272.2	2347.0
0.3236	130.000	178.4	1609.0	3801.9
0.2842	140.000	194.0	2082.3	8181.4
0.1293	200.000	297	-13523	-2316
0.0560	250.000	400	-2981	1440
0.0066	300.000	523	-1956	-1147
0.0290	350.000	673	-1568	-1000
-0.0559	400.000	862	-1363	-912
-0.0769	450.000	1107	-1237	-853
-0.0939	500.000	1437	-1152	-811
-0.1194	600.000	2624	-1043	-755
-0.1377	700.000	6525	-977	-719
-0.1515	800.000	-51975	-932	-694

Laser half divergence angle [mrad] : 0.2

Receiver-optics half opening angle [mrad] : 0.2

Laser-Telescope separation axes [mm] : 180

Laser-Telescope inclination angle [mrad]	Rmin1 [m]	Rmin2 [m]	Rmax1 [m]	Rmax2 [m]
6.8	10.000	12.9	40.6	44.6
3.3	20.000	26.4	85.0	98.7
2.1	30.000	40.8	133.9	167.7
1.5	40.000	56.6	188.3	260.2
1.1	50.000	74.3	249.7	391.3
0.9	60.000	94.5	319.8	592.9
0.7	70.000	117.7	400.8	943.4
0.6	80.000	144.8	495.4	1706.6
0.5	90.000	176.9	607.7	4660.0
0.3764	100.00	216	743	-11860
0.1225	150.00	656	2285	-1008
-0.0062	200.00	-12994	-45490	-688