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Three Essays on Econometric Identification

Skander Garchi Casal

Universitat Autònoma de Barcelona

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A dissertation submitted in fulfillement of requierments for the degree of Doctor of Philosophy in the International Doctorate of Economic Analysis (IDEA), Departament d'Economia i d'història Econòmica of the Universitat Autònoma de Barcelona.

Advisor: Hugo Rodríguez Mendizábal

To Chafik and Hakim

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enriched my life and taught me valuable lessons on being inquisitive and becoming a better human being. To all my friends, whether we have shared laughter or tears, your presence has profoundly impacted my life, and I am grateful to count on all of you.

Preface

The field of econometrics offers a rich and diverse set of tools to analyze complex economic phenomena and understand the causal relationships between different variables. In this thesis, titled "Three Essays on Econometric Identification," I contribute to the exploration of various identification strategies in econometrics. Through a comprehensive analysis, I aim to shed light on important questions surrounding loan delinquency rates, credit demand, and output dynamics, as well as the identification of structural vector autoregression (SVAR) models with proxy variables.

The first chapter of this thesis focuses on loan delinquency rates and investigates the relative importance of supply and demand factors in explaining this phenomenon. Previous literature has primarily emphasized the role of supply factors, such as banks' risk-taking behavior influenced by monetary policy. However, empirical evidence on the significance of supply factors compared to demand factors has been scarce. To address this gap, I propose a panel reduced-form approach combined with a novel identification strategy. Surprisingly, the results reveal that demand factors play a more substantial role in driving loan delinquency rates than previously suggested. Furthermore, I extend the analysis by examining the relationship between corporate tax rates and credit risk, providing insights into the causal drivers of this connection.

The second chapter investigates the interplay between loan delinquency rates, credit demand, and output, which has long captivated economists and policymakers. Understanding the factors governing these dynamics is crucial for promoting financial stability and sustainable growth. However, the scarcity of granular loan-level data poses a challenge to studying this relationship comprehensively. To overcome this limitation, I leverage bank balance sheet data, offering valuable insights despite the absence of detailed individual loan information. Employing a structural microeconometric model, I generate counterfactual experiments to explore the impact of various loan delinquency drivers on credit demand and output. Through this methodology, I provide valuable insights for policymakers, such as the optimal response to a fiscal contraction.

The third and final chapter is based on collaborative work with Srečko Zimic and focuses on the identification of structural VAR models with proxy variables. Current methodologies in this area often rely on the exogeneity of instruments, which imposes limitations on the number of proxies that can be used. To address this issue, we propose a novel methodology inspired by the orthogonal Procrustes problem. This approach allows for the inclusion of an unrestricted number of instruments and enables the use of plausibly exogenous instruments in the identification process. By applying this methodology, we overcome the limitations associated with instrument endogeneity and demonstrate the benefits of using a complete set of instruments to identify structural shocks.

Each chapter in this thesis contributes to the econometric literature by addressing important questions and proposing innovative methodologies. While previous research has made significant contributions to these areas, my study builds upon the existing body of knowledge and provides nuanced analyses that account for the limitations of the data. Through empirical investigations and careful econometric modeling, I aim to deepen our understanding of loan delinquency rates, credit demand, output dynamics, and the identification of structural VAR models. By doing so, I hope to provide valuable insights for policymakers, economists, and researchers interested in these areas of study.

Finally, I hope that the findings presented in this thesis contribute to the ongoing discussions in the field of econometrics and inspire further research that advances our understanding of economic phenomena and their causal relationships.

Contents

Chapter 1

Are Supply Factors Essential to Explain Loan Delinquency?

Abstract

This chapter presents a collection of empirical findings regarding loan delinquency in the United States. It emphasizes the significance of credit demand in comprehending fluctuations in loan delinquency rates. Through estimation using US bank balance sheet data, the study reveals that credit demand factors play a more substantial role in explaining changes in loan delinquency rates compared to supply factors. Furthermore, the analysis documents a positive correlation between increases in corporate taxes and higher loan delinquency rates.

Keywords: *Panel Data, Credit Demand, Credit Supply*
JEL Classification: *E62, E63, G21, G33.*

1.1 Introduction

This paper aims to present empirical evidence regarding the factors influencing loan delinquency rates.[1] Two contrasting perspectives can be examined in this context. On one hand, loan delinquency can be viewed as a supply-driven occurrence, linking delinquency to the evolution of banks' credit standards. On the other hand, loan delinquency can be considered a demand-side phenomenon by focusing on the contributions of borrowers to loan delinquency.

In recent decades, the majority of discussions have primarily centered around the supply side, with recent publications often referencing the risk-taking channel

[1]Loan delinquency is defined to include loans that are 90 days past due.

of monetary policy. These publications argue that low-interest rate environments create incentives for banks to lend to riskier borrowers, as mentioned by Borio and Zhu (2012) and Jiménez, Ongena, Peydró, and Saurina (2014). Similarly, Ioannidou, Ongena, and Peydró (2015) assert that loan spreads for riskier firms are relatively lower during periods of monetary easing, suggesting that banks require inadequate compensation for taking on additional risk. Furthermore, these observations hold true across a wide range of countries, as demonstrated by (Maddaloni & Peydró, 2011), highlighting the significance of the relationship between interest rates and risk-taking.

It is important to note that these papers primarily focus on banks' incentives to engage in risk-taking (i.e., supply factors) and their impact on credit markets, as discussed by Delis, Hasan, and Mylonidis (2012). However, empirical evidence demonstrating the relative importance of supply factors versus demand factors in explaining credit risk is still lacking. This raises the following question: Are supply factors essential for explaining loan delinquency?

To address this question, this study proposes a panel reduced-form approach coupled with a novel and straightforward identification strategy for supply and demand factors. Intuitively, the identification is achieved by constructing dummy variables based on sign restrictions. Surprisingly, the results indicate that the majority of the variation in loan delinquency can be attributed to demand factors rather than supply factors. This implies that the role of supply factors may be less significant than suggested by existing literature. The analysis continues by examining corporate tax rates and documenting a positive relationship with credit risk. The causal drivers of this relationship are further explored in the second chapter.

1.2 Methodology

In the following analysis, I aim to examine the impact of supply and demand factors on changes in the loan delinquency rate. Specifically, I focus on the significance of supply factors in determining loan delinquency. To achieve this objective, I employ an identification strategy that allows me to disentangle the influence of both supply and demand forces. To establish my identification strategy, I utilize a straightforward theoretical framework that forms the foundation of my analysis. This framework establishes a relationship between shifts in supply or demand and certain variables of interest, namely interest rates and credit growth.

My identification strategy draws guidance from the existing theoretical banking literature, and I make two key assumptions regarding credit supply and demand.[2] Firstly, it is assumed that, on average, borrowers' desire for loan demand decreases as interest rates rise. Secondly, it is assumed that, on average, banks' incentives to lend increase as interest rates rise. By employing these straightforward assumptions, I am able to outline four distinct cases, which are illustrated in figure 1.1.

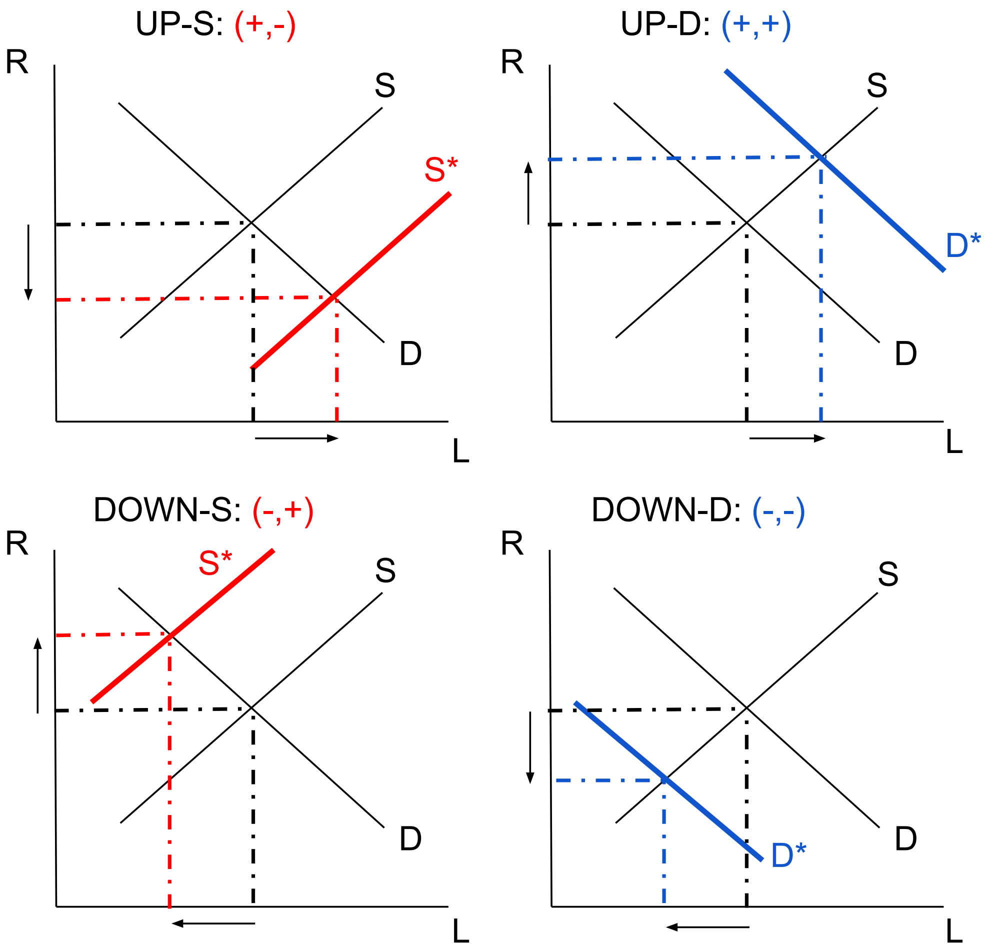
Figure 1.1 shows that an upward shift in credit supply (UP-S: (+,-)) leads to an increase in credit availability and a decrease in interest rates. Conversely, an increase in credit demand (UP-D: (+,+)) results in both higher credit availability and higher interest rates. Similarly, a reduction in credit supply (DOWN-S: (-,+)) diminishes credit availability while causing interest rates to rise. In contrast, a decrease in credit demand (DOWN-D: (-,-)) reduces both credit availability and interest rates. By employing this analytical framework and leveraging the insights from the theoretical banking literature, I can effectively examine the importance of supply factors in determining loan delinquency.

This straightforward theoretical framework facilitates the identification of credit supply and demand and their respective effects on loan delinquency. This identification is achieved by examining the co-movement of credit and interest rates within each of the four cases outlined in figure 1.1. To disentangle the effects of supply and demand, a simple strategy is employed, involving the use of the first differences in banks' interest rates ($\Delta r_{j,t}$) and annual loan growth ($\Delta log(L_{j,t})$). These two variables provide directional clues regarding the nature of the credit shift, allowing for the identification of the observed case among the four alternatives presented in the figure. Furthermore, the magnitude of the co-movement between $\Delta r_{j,t}$ and $\Delta log(L_{j,t})$ offers additional insights into the size of the credit shift.

This identification strategy is based on a simple framework that can be easily implemented within an econometric model. However, it is important to note a crucial assumption underlying this strategy. Specifically, it assumes that supply and demand do not move simultaneously, which may initially appear restrictive. In cases where credit demand/supply shifts occur simultaneously, they should be interpreted as movements in excess demand/supply. To illustrate this point, please refer to figure A.1 in the appendix. This figure depicts a scenario where supply and demand move simultaneously and in the same direction, with a larger change in supply than in demand. In this case, the identification strategy still provides a directional element. If my data were to align with the observation in figure A.1, it would be defined as a positive excess in credit supply, characterized

[2] These assumptions are derived from a simple Monti-Klein model. For further details, please refer to page 78 of Freixas and Rochet (2008).

Figure 1.1: Identification Strategy



This figure represents the supply and demand cross in the credit market. On the y-axis, I describe the interest rate R. Alternatively, on the x-axis, I represent the total credit. I describe four possible scenarios (UP-S: (+,-), UP-D: (+,+), DOWN-S: (-,+), DOWN-D: (-,-)) and their implication for credit and interest rates. Note that shifts in supply are represented in red and demand changes are in blue.

by an increase in credit and a reduction in interest rates.

Ultimately, although there are distinctions between supply/demand and excess supply/demand, these terminological differences are not crucial for the purpose of this section, as they do not hinder the implementation of the identification strategy. Given that the primary goal is to understand the importance of supply relative to demand, the nuances between the two terms are not of primary significance within the scope of this paper.

1.3 Results

The results of the identification process are presented based on bank balance sheet data collected from the Federal Deposit Insurance Corporation (FDIC), covering the period from 1992 to 2022. This dataset comprises comprehensive information on bank characteristics, including assets, liabilities, and geographical positions. Importantly, it encompasses the entire universe of banks in the United States and is accompanied by unique identifiers for each bank and county.

1.3.1 Supply versus Demand

I propose to start with a simple framework using a linear regression model found hereafter

$$\begin{aligned}\Delta\delta_{j,t} = \alpha_j + \alpha_t + x'_{j,t}\beta &+ \sum_{k=0}^{\bar{k}} |\Delta r_{j,t-k}\Delta log(L_{j,t-k})| d^{(+,-)}_{j,t-k}\gamma^{(+,-)}_{-k} \\ &+ \sum_{k=0}^{\bar{k}} |\Delta r_{j,t-k}\Delta log(L_{j,t-k})| d^{(-,+)}_{j,t-k}\gamma^{(-,+)}_{-k} \\ &+ \sum_{k=0}^{\bar{k}} |\Delta r_{j,t-k}\Delta log(L_{j,t-k})| d^{(+,+)}_{j,t-k}\gamma^{(+,+)}_{-k} \\ &+ \sum_{k=0}^{\bar{k}} |\Delta r_{j,t-k}\Delta log(L_{j,t-k})| d^{(-,-)}_{j,t-k}\gamma^{(-,-)}_{-k} + \epsilon_{j,t}. \end{aligned} \tag{1.1}$$

It can be written in a more compact form as follow

$$\Delta\delta_{j,t} = \alpha_j + \alpha_t + x'_{j,t}\beta + \sum_{c\in\mathcal{C}}\sum_{k=0}^{\bar{k}} |\Delta r_{j,t-k}\Delta log(L_{j,t-k})| d^{c}_{j,t-k}\gamma^{c}_{-k} + \epsilon_{j,t}$$
$$with\ \mathcal{C} = \{(+,-),(-,+),(+,+),(-,-)\}.$$

Regression 1.1 aims to explain the annual change in loan delinquency rates ($\Delta\delta_{j,t}$) within a specific bank j during period t. To account for the within-bank variation in loan delinquency rates, the regression includes bank fixed effects (α_j) and a set of control variables ($x_{j,t}$), such as indicators of bank market concentration, and other bank-level factors like the number of offices and employees. Additionally, time fixed effects (α_t) are incorporated to control for cyclical variations in the delinquency rate.

The second part of the regression is crucial for implementing my strategy. To identify the nature of the shift and select one of the four cases

$$c \in \mathcal{C} = \{(+,-),(-,+),(+,+),(-,-)\},$$

I define a dummy variable $d^c_{j,t-k}$. This dummy variable takes the value of 1 if case c is satisfied for bank j in period $t-k$. Moreover, to capture the magnitude of the shift, I create an interaction term by multiplying the change in interest rate ($\Delta r_{j,t}$) with the annual credit growth ($\Delta\log(L_{j,t})$) and taking the absolute value. These terms, with regression coefficients γ^c_{-k}, serve as regressors. The coefficient γ^c_{-k} represents the correlation between a supply/demand shift in period $t-k$ and the change in the loan delinquency rate in period t. Finally, the error term component is denoted as $\epsilon_{j,t}$.

I want to focus my attention to the dynamic relationship between the loan delinquency rate and credit supply/demand. Therefore, I include a certain number of lags $\bar{k}$ (20 quarters) and compute the running sum of the coefficients γ^c_{-k},

$$\sum_{k=\bar{k}}^{k_i} \gamma^c_{-k}, \;\; with \; k_i \in \{0,1,...,\bar{k}\} \; and \; c \in \mathcal{C},$$

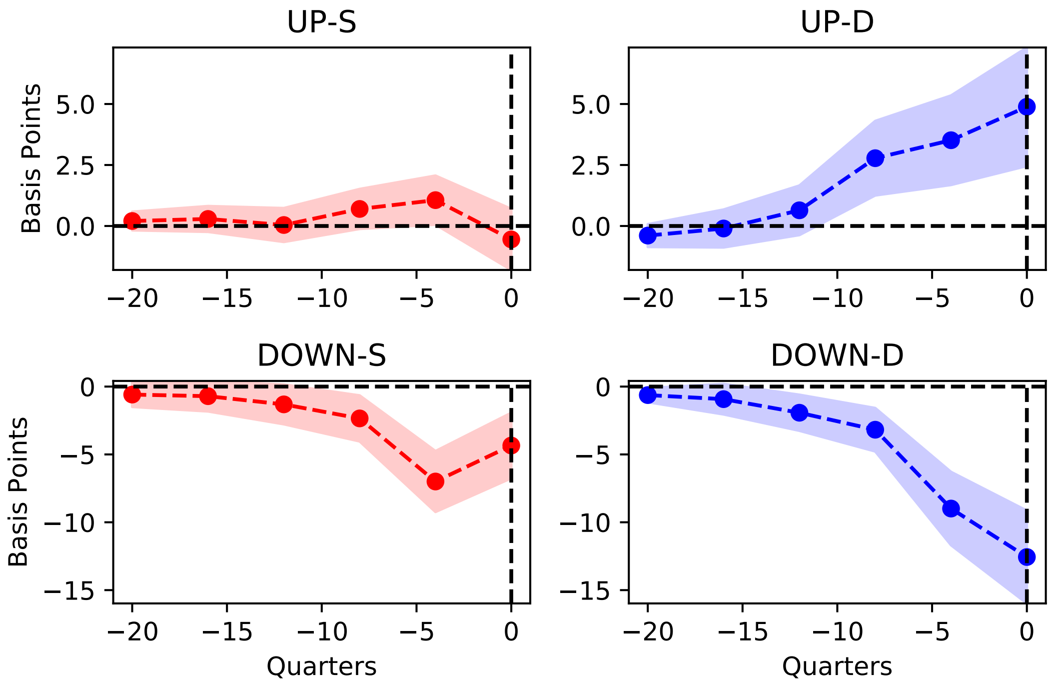
which captures the cumulative differential response of changes in delinquency rates to demand or supply factors. The running sum serves as the central focus of this section. To ensure the interpretability of my findings, I find it advantageous to convert the regressors and the dependent variable into basis points.

Figure 1.2 presents the results of regression 1.1, showcasing plots that provide insight into the decomposition of loan delinquency rates based on past supply and demand factors. For clarity, I will provide a detailed interpretation of one of the panels. I invite the reader to direct their attention to the top right panel (UP-D) in Figure 1.1, which coincides with an upward shift in demand. The x-axis of the panel represents quarters leading up to the current period t, while the y-axis represents changes in the loan delinquency rate expressed in basis points.

Analyzing the figure in quarter 0 reveals that all the marginal past marginal upward shifts in demand, occurring from 20 quarters before the current period, are associated, on average, with a 5.0 basis point increase in the current delinquency rate. Similarly, examining the figure in quarter -4 suggests that past upward shifts in demand, spanning from 20 quarters to 4 quarters before the current period, relate to an average increase of 3.75 basis points in the current delinquency rate. By subtracting 3.75 from 5.0, we find that a marginal upward shift in demand in the current year is, on average, associated with a 1.25 basis point increase in the loan delinquency rate.

Next, I proceed to compare the right and left panels with each other. Upon careful examination, I note that past upward shifts in demand exhibit a notable and positive association with loan delinquency rates, commencing from two years prior to the current period and extending onward. In contrast, the relationship between past supply increases and loan delinquency rates is found to be insignificant. Shifting my attention to the lower panels, I observe a negative and significant correlation between a reduction in past credit demand and the current loan delinquency rate, whereas downward shifts in supply demonstrate a considerably weaker relationship. These four empirical findings collectively emphasize the greater significance of demand over supply in elucidating variations in loan delinquency rates. Consequently, these results provide motivation for researchers to prioritize studies that focus on credit demand rather than supply as a means of explaining loan delinquency.

Figure 1.2: Decomposition of Loan Delinquency Rates in Supply and Demand Factors



This figure represents the results of regression 1.1. The panels in the figure show the sum of coefficients $\sum_{k=\bar{k}}^{k_i} \gamma_{-k}^{c}$ and the associated 95% confidence interval. The x-axis displays the quarters, where quarter 0 corresponds to the current quarter. The y-axis describes basis point changes in loan delinquency rates. The relationship is color-coded in blue/red to represent the decomposition of loan delinquency rates in demand/supply factors. The sample period spans from 1994 to 2021.

1.3.2 The Drivers of Loan Delinquency

After presenting the arguments that underscore the importance of studying credit demand, it is essential to focus on understanding the key drivers of loan delinquency. This subsection explicitly examines the interplay between interest rates, effective corporate tax rates, and their impact on loan delinquency. Similar to the previous subsection, the analysis decomposes these drivers into their respective supply and demand components.

Firstly, I begin by dissecting the relationship between interest rates and loan delinquency rates. The econometric model employed in this analysis closely resembles the one discussed in regression 1.1. The model's structure and formulation are further elaborated upon hereafter

$$\begin{aligned}\Delta\delta_{j,t} = \alpha_j + \alpha_t + x'_{j,t}\beta &+ \sum_{k=0}^{\bar{k}} |\Delta r_{j,t-k}| d^{(+,-)}_{j,t-k}\gamma^{(+,-)}_{-k} \\ &+ \sum_{k=0}^{\bar{k}} |\Delta r_{j,t-k}| d^{(-,+)}_{j,t-k}\gamma^{(-,+)}_{-k} \\ &+ \sum_{k=0}^{\bar{k}} |\Delta r_{j,t-k}| d^{(+,+)}_{j,t-k}\gamma^{(+,+)}_{-k} \\ &+ \sum_{k=0}^{\bar{k}} |\Delta r_{j,t-k}| d^{(-,-)}_{j,t-k}\gamma^{(-,-)}_{-k} + \epsilon_{j,t}. \end{aligned} \tag{1.2}$$

It can be written in a more compact form as follow

$$\begin{aligned}\Delta\delta_{j,t} = \alpha_j + \alpha_t + x'_{j,t}\beta + \sum_{c\in\mathcal{C}}\sum_{k=0}^{\bar{k}} |\Delta r_{j,t-k}| d^{c}_{j,t-k}\gamma^{c}_{-k} + \epsilon_{j,t} \\ with\ \mathcal{C} = \{(+,-),(-,+),(+,+),(-,-)\}.\end{aligned}$$

It is important to note that the only difference compared to regression 1.1 lies in the substitution of $|\Delta r_{j,t-k}\Delta\log(L_{j,t-k})|$ with a single variable, $|\Delta r_{j,t-k}|$. While the previous specification enabled the decomposition of delinquency rates into supply and demand factors, this regression offers additional insights by breaking down the impact of interest rate changes on delinquency rate changes.

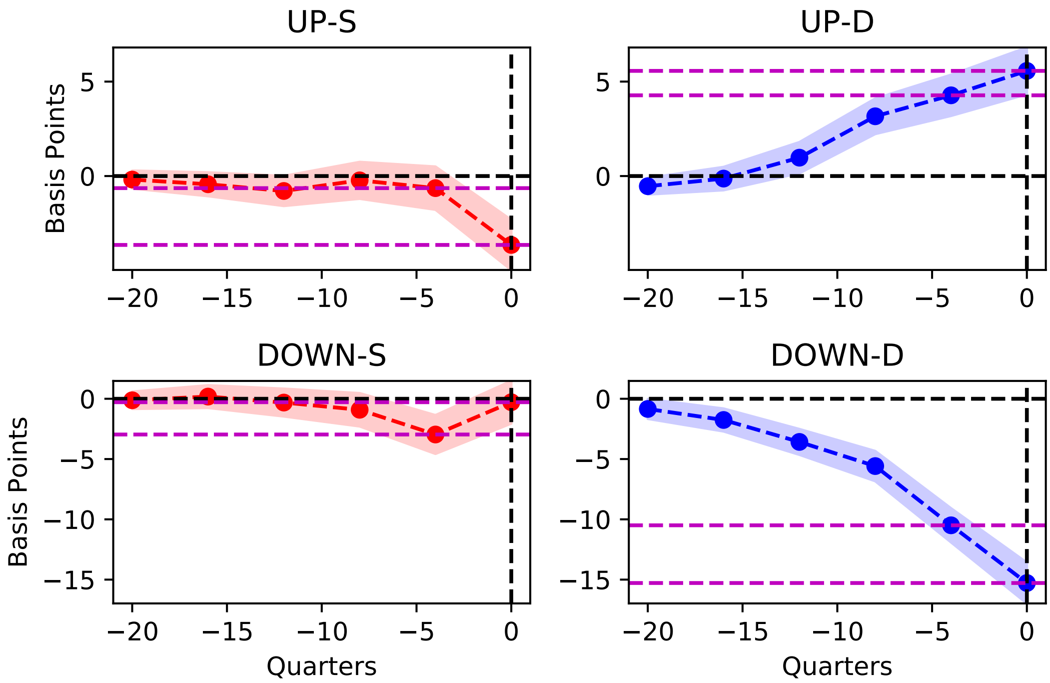
Figure 1.3 illustrates the outcomes of regression 1.2, displaying the decomposition of the relationship between loan delinquency rates and interest rates into supply and demand factors. Once again, I will provide a detailed interpretation of one of the panels for a comprehensive understanding. I encourage the reader to focus on the top right panel (UP-D) of the figure, which corresponds to an upward shift in demand, aligning with the top right scenario in figure 1.1. The x-axis of the panel represents quarters before the current period t, while the y-axis represents changes in the loan delinquency rate expressed in basis points.

Examining the figure at quarter 0 suggests that all past one basis point increases in interest rates (5 basis points in total), stemming from demand shifts occurring between 20 quarters ago and the current quarter, coincide with an average 5.4 basis point increase in the current delinquency rate. Similarly, analyzing the figure at quarter -4 indicates that all the past one basis point increases in interest rates, resulting from demand shifts spanning from 20 quarters ago to 4 quarters before the current period, are associated with an average 4.7 basis point increase in the current delinquency rate. Once again, it is worth noting that the contemporaneous relationship can be obtained by subtracting 4.7 from 5.4 (measured by the space between the purple dashed horizontal lines). Therefore, a one basis point increase in interest rates in the current year due to demand shifts is, on average, associated with a 0.7 basis point increase in the loan delinquency rate.

Subsequently, I proceed to compare the right and left panels. Initially, I observe a significant and positive relationship between interest rates and changes in loan delinquency rates on the demand side. However, the association becomes more ambiguous when considering the supply side. Firstly, although the contemporaneous relationship is significant, past changes in interest rates do not exhibit a significant correlation with changes in the loan delinquency rate. Secondly, the contemporaneous relationship deviates from that observed in the demand panels.

These findings provide additional evidence supporting the effectiveness of the identification strategy in decomposing supply and demand factors, as they align with theoretical and empirical insights documented in the literature. First, the risk-taking channel of monetary policy highlights that banks (on the supply side) tend to extend loans to riskier borrowers when interest rates decline, which corresponds to my empirical findings indicating a negative contemporaneous relationship between interest rates and loan delinquency rates. Second, the theoretical literature on credit default establishes a positive relationship between borrowers' risk-taking behavior and interest rates, which is also consistent with the results obtained in my study.

Figure 1.3: Decomposition of Interest Rate's Relationship to Loan Delinquency Rates in Supply and Demand Factors



This figure represents the results of regression 1.2. The panels in the figure show the sum of coefficients $\sum_{k=\bar{k}}^{k_i} \gamma_{-k}^{c}$ and the associated 95% confidence interval. The x-axis displays the quarters, where quarter 0 corresponds to the current quarter. The y-axis describes basis point changes in loan delinquency rates. The relationship is color-coded in blue/red to represent the decomposition of loan delinquency rates in demand/supply factors. The sample period spans from 1994 to 2021.

While acknowledging the significance of the connection between interest rates and loan delinquency, it is important to recognize the impact of fiscal policy on delinquency as well. Therefore, the following discussion will primarily concentrate on the correlation between loan delinquency and the effective corporate tax rate. The effective corporate tax rate is computed by dividing collected corporate taxes by the sum of collected corporate taxes and corporate profits. It is worth noting that this paper presents the initial documented analysis that explores the empirical relationship between corporate taxes and loan delinquency. By delving into this unexplored territory, I aim to shed light on the potential effects of corporate tax policies on loan delinquency.

Moving forward, I proceed with the decomposition of the relationship between effective corporate taxes and loan delinquency rates. The econometric model employed for this analysis is outlined below.

$$\begin{aligned}\Delta\delta_{j,t} = \alpha_j + p_t'\rho + x_{j,t}'\beta &+ \sum_{k=0}^{\bar{k}} \Delta T_{t-k} d_{j,t-k}^{(+,-)} \gamma_{-k}^{(+,-)} \\ &+ \sum_{k=0}^{\bar{k}} \Delta T_{t-k} d_{j,t-k}^{(-,+)} \gamma_{-k}^{(-,+)} \\ &+ \sum_{k=0}^{\bar{k}} \Delta T_{t-k} d_{j,t-k}^{(+,+)} \gamma_{-k}^{(+,+)} \\ &+ \sum_{k=0}^{\bar{k}} \Delta T_{t-k} d_{j,t-k}^{(-,-)} \gamma_{-k}^{(-,-)} + \epsilon_{j,t}. \end{aligned} \tag{1.3}$$

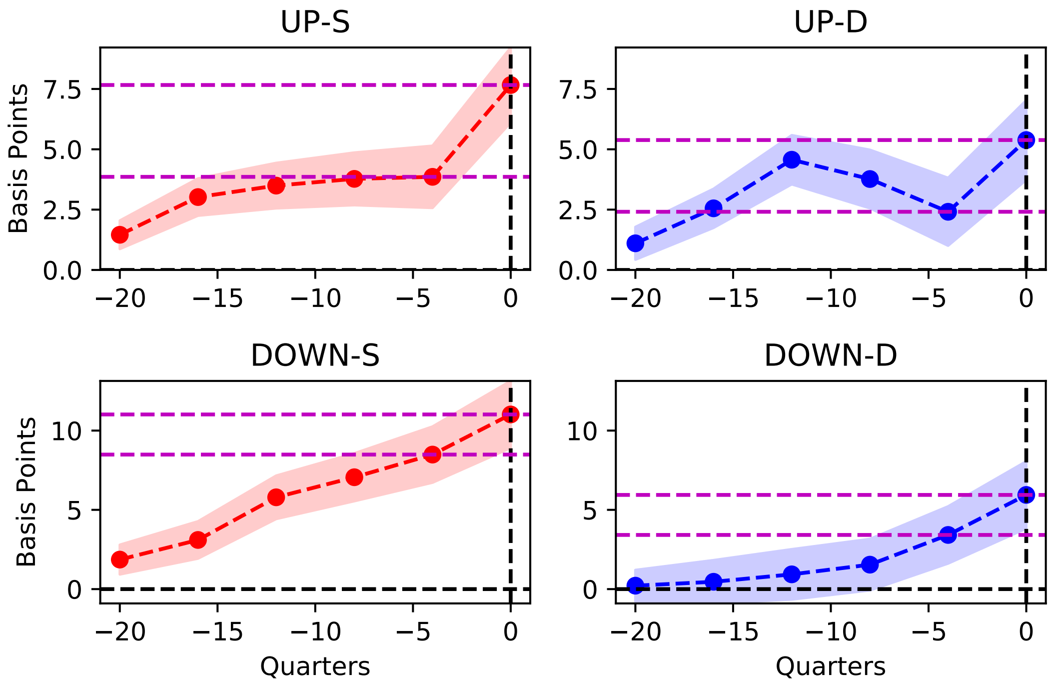
It can be written in a more compact form as follow.

$$\Delta\delta_{j,t} = \alpha_j + p_t'\rho + x_{j,t}'\beta + \sum_{c\in\mathcal{C}} \sum_{k=0}^{\bar{k}} \Delta T_{t-k} d_{j,t-k}^{c} \gamma_{-k}^{c} + \epsilon_{j,t}$$
$$with\ \mathcal{C} = \{(+,-),(-,+),(+,+),(-,-)\}.$$

It is important to note that there are several differences in comparison to the previous regression. Specifically, I replace the absolute value of interest rate changes, denoted as $|\Delta r_{j,t-k}|$, with the change in the effective corporate tax rate, represented as ΔT_{t-k}. Additionally, to address the issue of perfect collinearity with the regressor ΔT_{t-k}, I remove the time fixed effect α_t. However, to account for cyclical components unrelated to changes in the corporate tax rate, it remains necessary to include a set of time-dependent Chebyshev polynomials, denoted as $p_t'\rho$.

Figure 1.4 displays the outcomes of regression 1.3, elucidating the relationship between loan delinquency rates and corporate tax rates, further decomposed into supply and demand factors. On average, the results exhibit consistency across the four panels, indicating that an increase in the corporate tax rate aligns with an increase in the delinquency rate. Moreover, upon comparing the top panels with the bottom panels, it becomes apparent that when interest rates rise, increases in the corporate tax rate coincide with more substantial movements in the contemporaneous loan delinquency rate compared to when interest rates decline. On average, this amounts to a 3.65 basis point increase when rates rise, compared to a 2.5 basis point increase when they decline.

Figure 1.4: Decomposition of Corporate Taxes' Relationship to Loan Delinquency Rates in Supply and Demand Factors



This figure represents the results of regression 1.3. The panels in the figure show the sum of coefficients $\sum_{k=\bar{k}}^{k_i} \gamma_{-k}^{c}$ and the associated 95% confidence interval. The x-axis displays the quarters, where quarter 0 corresponds to the current quarter. The y-axis describes basis point changes in loan delinquency rates. The relationship is color-coded in blue/red to represent the decomposition of loan delinquency rates in demand/supply factors. The sample period spans from 1994 to 2021.

1.4 Conclusion

This paper delves into the dynamic relationship between loan delinquency rates and factors related to credit supply and demand. Through a rigorous econometric analysis, I examine various drivers, including interest rates, effective corporate tax rates, and their impact on loan delinquency.

My findings contribute to the existing literature by offering novel insights and shedding light on important aspects of credit dynamics. The results highlight the significance of demand relative to supply factors in explaining variations in loan delinquency rates. Specifically, past upward shifts in demand demonstrate a significant and positive relationship with loan delinquency rates, while the association with supply factors appears to be more ambiguous and less pronounced.

The empirical evidence supports the successful decomposition of supply and demand factors, consistent with theoretical expectations and previous research. The risk-taking channel of monetary policy aligns with our results, revealing that banks tend to extend loans to riskier borrowers when interest rates decline. This finding underscores the negative contemporaneous relationship between interest rates and loan delinquency rates. Additionally, the theoretical literature on credit default provides further support, as our results indicate a positive relationship between borrowers' risk-taking behavior and interest rates.

Furthermore, this study provides new evidence on the relationship between effective corporate tax rates and loan delinquency rates. My analysis demonstrates that an increase in the corporate tax rate coincides with an increase in the delinquency rate, emphasizing the importance of considering corporate tax dynamics in understanding credit dynamics.

Chapter 2

A Demand-Oriented Analysis of Loan Delinquency, Credit and Output

This study examines the impact of corporate loan delinquency drivers on credit demand and output. Using bank balance sheet data, a structural model of borrowers' risk-taking is estimated to shed light on this relationship. The findings reveal that increases in interest rates have a significant effect on enhancing the credit composition by reducing the profitability of unproductive borrowers. Specifically, a 1% rise in interest rates leads to an average productivity increase of over 1%. This improvement in the credit composition presents an opportunity for fiscal authorities to leverage high-interest rate environments by expanding corporate taxes. The model suggests that implementing such a policy could result in a substantial 37% enhancement in the output response relative to the benchmark.

Keywords: *Credit Demand, Loan Delinquency, Interest Rates, Corporate Taxes*
JEL Classification: *E62, E63, G21, G33.*

2.1 Introduction

In the field of economics, the intertwined relationship between loan delinquency rates, credit demand, and output has captivated many. This was especially true in the aftermath of the great financial crisis where credit risk ultimately materializes and affects output prospects for the years to come. Unraveling the causal relationship among these factors has long interested policymakers, as understanding their interplay holds the secret to financial stability and sustainable growth.

At the heart of this topic a central question needs to be asked: How do corporate loan delinquency drivers influence credit demand and output? Exploring this question is essential, as it sheds light on the mechanisms that shape borrowing, risk-taking, and ultimately, economic activity.

However, understanding the factors governing loan delinquency rates, credit demand, and output has proven to be difficult. One of the main challenges in investigating this relationship lies in the scarcity of granular data on loans, which hampers our understanding of borrower behavior. While detailed information at the individual loan level would provide valuable insights, such data is often unavailable or highly restricted due to privacy concerns and data limitations.

To overcome this challenge, my study turns to an alternative source of information: bank balance sheet data. Although these data aggregate information at the bank level, they still offer valuable insights into the dynamics of loan delinquency rates, credit demand, and output. By leveraging the available bank balance sheet data, I aim to shed light on the relationship between corporate loan delinquency drivers, credit demand, and output, despite the limitations imposed by the absence of granular loan-level data.

To answer the main question of this paper my methodology employs a structural microeconometric model, which allows me to generate counterfactual experiments and gain a deeper understanding of how borrowers take risks, despite the inherent limitations of the data. In a nutshell, I borrow an identification strategy from the literature of credit demand estimation (Ho and Ishii (2011), Dick (2007)), where information about borrowers' sensitivity to interest rates changes is inferred from bank level data.

By simulating counterfactual scenarios based on my structural model, I can explore the impact of various loan delinquency drivers on credit demand and output, even in the absence of granular loan-level data. This allows me to understand how to improve the output response to a fiscal contraction. In particular, I find that fiscal authorities should take advantage of high-interest rate regimes to increase the corporate tax rate as high-interest rate environments eliminate unproductive borrowers from the market and therefore allow to tax firms that are less sensitive to changes in the tax rate. Such insights are crucial for policymakers as they guide decision-making in the face of imperfect data.

While previous research has made significant contributions to the field, to the best of my knowledge, bank balance sheet data has never been used to understand borrowers' incentives to take credit risk and especially not to understand the impact of changes in corporate taxes rates on loan delinquency, credit demand, and output. My study acknowledges some challenges and strives to provide a nuanced analysis that accounts for the limitations of the data.

2.2 Literature Review

The objective of this paper is to explore the factors that drive loan delinquency. In pursuit of this objective, I have extensively reviewed various articles on risk-taking in the credit market. A significant focus of my research has been on the literature concerning the risk-taking channel of monetary policy. Notably, studies by Borio and Zhu (2012), and Jiménez et al. (2014) suggest that low interest rates incentivize lending to riskier borrowers. Additionally, Ioannidou et al. (2015) argue that during periods of monetary easing, loan spreads for riskier firms are relatively lower, indicating that banks require less compensation for the risks they undertake. These findings have been consistently observed across diverse countries, emphasizing the crucial relationship between interest rates and risk-taking, as demonstrated by (Maddaloni & Peydró, 2011).

As mentioned in the first chapter, it is worth noting that these papers primarily focus on banks' incentives to take risks (i.e., supply factors) and their effect on credit markets, as discussed by Delis et al. (2012). However, I have also complemented this existing literature, by introducing new empirical evidence that examines the influence of demand factors and interest rates on loan delinquency.

A significant portion of my paper investigates the causal relationship between interest rates, corporate taxes, and loan delinquency. To establish causality in economics, it is popular to employ structural econometric models. In Blundell (2017), such a model is considered to fully incorporate *"the structure of decision making"*. As opposed to conventional reduced-form analysis, I can specify borrowers' decision-making within the model by employing this approach, enabling robust causal inferences. Although using structural models presents an appealing avenue for studying causality, it necessitates bridging the gap between theoretical frameworks describing agents' decisions and empirical estimation strategies.

In constructing the theoretical foundation of my paper, I leverage the work of Martinez-Miera and Repullo (2010) and Martinez-Miera and Repullo (2017). While these papers primarily focus on banks' risk-taking, their framework provides essential insights into borrowers' decision-making, which is a fundamental aspect of my model. However, my research stands apart from previous articles by introducing dynamics, considering borrower productivity, and employing a precise estimation procedure, facilitating counterfactual experiments.

Alternatively, another approach to modeling borrowers could have been adopted by drawing from the literature on consumer default, as explored by Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007), Nakajima and Ríos-Rull (2019), and Dempsey and Ionescu (2021). These papers model borrowers' decision to default on their loans as a discrete choice. Although this approach offers realism, it incurs substantial computational costs, making it less practical. In contrast, my approach assumes that borrowers choose a risk level based on the probability of loan default. This approach proves more practical by reducing the computational burden.

On the empirical side, my research closely aligns with the literature on structural banking models, as demonstrated by Corbae, D'erasmo, et al. (2013), Corbae and D'Erasmo (2019), and Wang, Whited, Wu, and Xiao (2020). Although these studies primarily focus on the banking industry, they offer valuable identification strategies and insights into the primary dataset employed in my paper, which comprises balance sheet data at the bank level from the Federal Deposit Insurance Corporation (FDIC).

Furthermore, the estimation of demand curves is a critical aspect of my research, requiring reference to the credit demand estimation literature. This literature recognizes the simultaneity bias inherent in such estimations. To correct for this bias, I adopt the practice of employing a firm's cost of production as a relevant instrument for regression, as established by S. T. Berry (1994), S. Berry, Levinsohn, and Pakes (1995) and Nevo (2001). Similarly, credit and deposit demand estimation use bank-level costs as relevant instruments, which is at the center of my analysis, as in Dick (2007) and Ho and Ishii (2011). Borrowing these procedures from the literature allows me to estimate the parameters of my model accurately and quantify borrowers' risk-taking with bank-level data.

2.3 The Model

2.3.1 A Random Utility Model

The model begins at period t with a group of potential borrowers represented by the mass M_t. These borrowers are interested in obtaining funding for risky projects and can choose discretely from a range of options (j), summarized as follows:

$$j \in 0, 1, ..., j, ..., J, J + 1.$$

There are J commercial banks available for borrowing, denoted as options 1 to J. Alternatively, borrowers can opt for the outside options 0, which signifies not borrowing, or $J + 1$, which allows borrowing from the bond market. In addition to selecting their funding source, borrowers also choose the type of project they wish to undertake. In this study, project type aligns with loan types available. For example, a borrower obtaining a commercial and industrial (C&I) loan is involved in a C&I project. This simplification enables the representation of the extensive margin of credit demand. Borrowers have two project options:

$$\tau \in \{C\&I, nonres\},$$

representing commercial and industrial, and non-residential real estate projects, respectively. These loan types hold significant importance in the analysis conducted in this paper. To differentiate between project types, an additional index τ is introduced.

For a borrower choosing option j at period t and project type τ, the expected value is denoted as $V_{j,t,\tau}$. Detailed explanations regarding the computation of $V_{j,t,\tau}$ can be found in the subsequent subsection. Considering the values resulting from the discrete choices made by borrowers, the following random utility is expected:

$$U_{j,t,\tau} = \xi_j + x'_{j,t}\beta_q + V_{j,t,\tau}\beta_v + \Delta\xi_{j,t} + \epsilon_{j,t,\tau}.$$

Here, $\Delta\xi_{j,t}$ represents a credit demand shock specific to option j at time t. The term $x'_{j,t}\beta_q$ accounts for a set of explanatory variables that reflect the convenience of option j during period t. For instance, certain banks might have a stronger presence in particular geographical locations, making them more convenient for local borrowers. These convenience factors are typically captured by variables like the number of employees per branch or the total number of bank offices. These variables are consolidated in the vector $x_{j,t}$ and have a linear impact on the random utility through the coefficients in β_q. Further explanations on this topic can be found in Wang et al. (2020) and Dick (2007). The model incorporates a bank fixed effect ξ_j to account for variations in utility across different banks, and the utility experienced by borrowers is subject to random fluctuations represented by the extreme value distributed shock $\epsilon_{j,t,\tau}$.

Furthermore, agents can also choose to borrow from the bond market to undertake a riskless project for which they get the value $V_{J+1,t}$. Similarly, as before, the utility of borrowers depends on the value $V_{J+1,t}$, and an extreme value shock $\epsilon_{J+1,t}$, yielding the expression

$$U_{J+1,t} = V_{J+1,t} + \epsilon_{J+1,t}.$$

I assume that the value of not undertaking a project is κ such that the utility of the choice 0 is

$$U_0 = \kappa + \epsilon_{0,t}.$$

I adopt the standard assumption that $\epsilon \equiv (\epsilon_{0,t}, \epsilon_{j,t,\tau}, \epsilon_{J+1,t})$ follows a generalized extreme value distribution with cumulative distribution function given by

$$F(\epsilon) = exp(-exp(-\epsilon)).$$

It yields the following closed-form solution for $P_{j,t,\tau}$; the probability of selecting bank j, in period t and project type τ:

$$P_{j,t,\tau} = \frac{exp(\xi_j + x'_{j,t}\beta_q + V_{j,t,\tau}\beta_v + \Delta\xi_{j,t})}{exp(\kappa) + exp(V_{J+1,t}) + \sum_{k\in\mathcal{A}^\tau}\sum_{l\in\mathcal{A}^b} exp(\xi_l + x'_{l,t}\beta_q + V_{l,t,k}\beta_v + \Delta\xi_{l,t})}.$$

The formula gives that an increase in the value of option j in t increases the probability of choosing that option. Alternatively, an increase in the value of the outside option $J+1$ reduces the likelihood of selecting option j.

It is straightforward to write this joint probability as a product of its conditional and marginal distribution

$$P_{j,t,\tau} = \frac{exp(\xi_j + x'_{j,t}\beta_q + I_{j,t} + \Delta\xi_{j,t})}{exp(\kappa) + exp(V_{J+1,t}) + \sum_l exp(\xi_l + x'_{l,t}\beta_q + I_{l,t} + \Delta\xi_{l,t})} \frac{exp(V_{j,t,\tau}\beta_v)}{\sum_k exp(V_{j,t,k}\beta_v)},$$

where $I_{j,t} = log(\sum_k exp(V_{j,t,k}\beta_v))$. The first term of the expression corresponds to $P_{j,t}$ the probability of choosing bank j in period t, and the second term is $P_{\tau|j,t}$ is the probability of selecting project τ given that she has chosen bank j in period t. This decomposition of the joint probability allows identifying the credit composition, which is fully characterized by the conditional probability $P_{\tau|j,t}$. This constitutes an additional contribution to the literature, as explained in Corbae and D'Erasmo (2019), one aspect of the risk-taking channel that remains misunderstood involves compositional changes in credit.

2.3.2 The Project's Value

I follow the theoretical framework of Martinez-Miera and Repullo (2010) and assume that borrowers can choose the failure probability of their project ($\delta_{j,t,\tau}$). The cash flows obtained from the project depend on the failure probability. Indeed, borrowers can expect the following payoffs:

$$\begin{cases} \eta_{j,t,\tau}\alpha(\delta_{j,t,\tau}) & with\ \ 1-\delta_{j,t,\tau} \\ 0 & with\ \ \delta_{j,t,\tau}. \end{cases}$$

With probability $\delta_{j,t,\tau}$ the project fails, and the borrower does not collect cash flows. Alternatively, with probability $1-\delta_{j,t,\tau}$ the project is successful and yields $\eta_{j,t,\tau}\alpha(\delta_{j,t,\tau})$. The function $\alpha(.)$, is increasing and concave in the failure probability, which yields larger returns for larger risk levels.[1] The variable $\eta_{j,t,\tau}$ is known by the borrowers and represents the productivity of the match between bank j and the borrower in period t and for project τ.

To remain in the market of project type τ, borrowers need to pay every period a utility cost of I_τ and a proportional tax T_t on corporate profits Π. Moreover, borrowers need to pay a share μ_τ of their principal as well as the interest payment $r_{j,t,\tau}$. Finally, I follow the corporate finance literature (Strebulaev & Whited, 2012) in assuming that borrowers' discount factor depends on the interest rate charged by the bank such that the future is discounted at rate $\frac{1}{1+r_{j,t,\tau}}$.

In period t, a new borrower observes the quality of its match with all banks ($\eta_{j,t,\tau}$) as well as the extreme value distributed shock ($\epsilon_{j,t,\tau}$). After choosing a bank, they borrow one unit of funding, pay their utility cost I_τ, and start undertaking the project. In the subsequent periods, borrowers have to pay rates μ_τ and $r_{j,t,\tau}$ on the remaining share of their principal $(1-\mu_\tau)^k$, with k referring to the number of periods after loan issuance.

Hence, we can proceed computing the project's value ($V_{j,t,\tau}$)

$$V_{j,t,\tau} = \max_{\delta_{j,t,\tau}} \quad (1-\delta_{j,t,\tau})(A_{j,t,\tau} - C_{j,t,\tau} - T_t\Pi_{j,t,\tau}), \tag{2.1}$$

where $A_{j,t,\tau}$ is the discounted sum of cash flows received from the project such that

$$A_{j,t,\tau} = \eta_{j,t,\tau}\alpha(\delta_{j,t,\tau})\frac{1+r_{j,t,\tau}}{r_{j,t,\tau}}.$$

The discounted sum of borrowers' costs is represented by $C_{j,t,\tau}$ and reads as

$$C_{j,t,\tau} = I_\tau\frac{1+r_{j,t,\tau}}{r_{j,t,\tau}} + (1+r_{j,t,\tau})(1-\mu_\tau)^k.$$

Finally, discounted corporate profits are given by

$$\Pi_{j,t,\tau} = \eta_{j,t,\tau}\alpha(\delta_{j,t,\tau})\frac{1+r_{j,t,\tau}}{r_{j,t,\tau}} - (1+r_{j,t,\tau})(1-\mu_\tau)^k.$$

Notice, that I_τ is not included in the taxable corporate profits as it is a utility cost.

The failure probability $\delta_{j,t,\tau}$ is the result of value maximization as represented in equation 2.1. The solution is computed by solving for the following first-order

[1] $\alpha(\delta_{j,t,\tau}) = (a_\tau + \delta_{j,t,\tau})^{\gamma_\tau}$, where $a_\tau > 0$ and $\gamma_\tau \in [0,1]$.

condition with repect to $\delta_{j,t,\tau}$,

$$-R(\delta_{j,t,\tau}, r_{j,t,\tau}) + C(r_{j,t,\tau}, k) + T_t \pi(\delta_{j,t,\tau}, r_{j,t,\tau}, k) \\ + (1 - \delta_{j,t,\tau})(R'(\delta_{j,t,\tau}, r_{j,t,\tau}) - T_t \pi'(\delta_{j,t,\tau}, r_{j,t,\tau}, k)) = 0. \tag{2.2}$$

Unfortunately, the F.O.C doesn't admit a closed-form solution and therefore needs to be solved numerically.

Using equation 2.2 allows the computation of variable $\eta_{j,t,\tau}$. The variable can be considered a residual of the model, allowing to fit delinquency rates. Therefore, it is simple to compute $\eta_{j,t,\tau}$ by isolating it in equation 2.2 and plugging the values observed in the data as follows

$$\eta_{j,t,\tau} = \frac{r_{j,t,\tau} + \frac{I_\tau}{1-T_t}}{\alpha(\delta_{j,t,\tau}) - (1 - \delta_{j,t,\tau})\alpha'(\delta_{j,t,\tau})}.$$

Alternatively, borrowers from the bond market undertake a riskless project generating a constant cash flow ϕ. The maturity of the corporate bond is modeled through a geometrically decaying bond of parameter μ_b. Additionally, the bond issuer needs to pay a utility cost of I_b and make interest payments equal to the fed fund rate plus a credit spread ($f_t + s_t$). Therefore, the value of alternative $J + 1$ can be written as

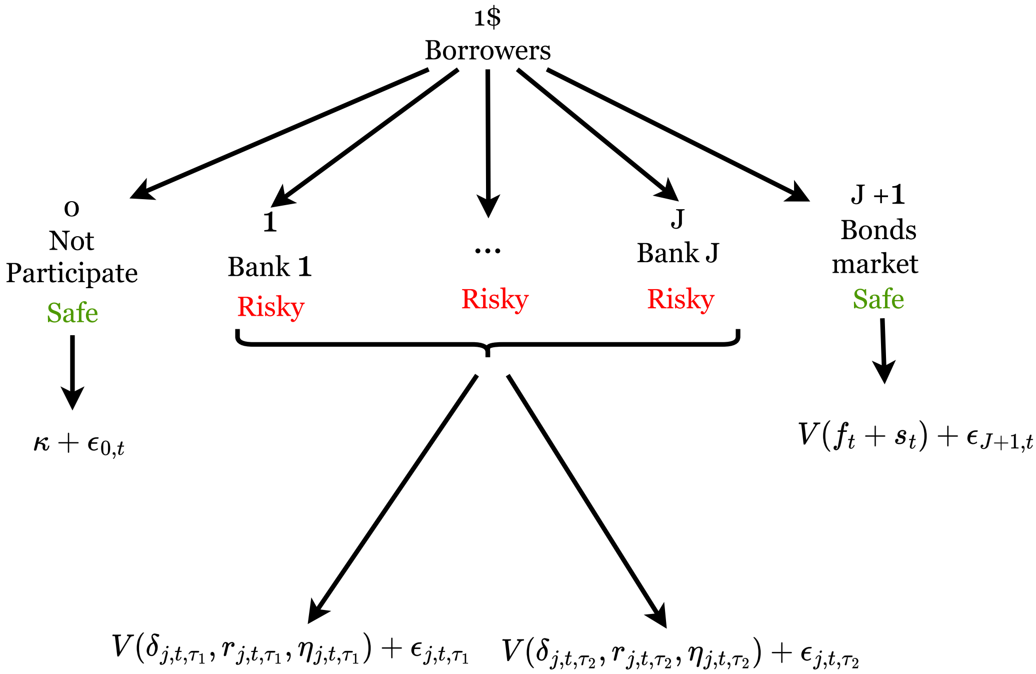
$$V_{J+1,t} = (1 + f_t + s_t)(1 - T_t)(\frac{\phi}{f_t + s_t} - 1) - I_b \frac{1 + f_t + s_t}{f_t + s_t}. \tag{2.3}$$

Here again, this equation is the result of the sum of discounted future profits from choosing alternative $J + 1$.

2.3.3 Trade-Offs and Dynamics

This subsection aims to provide the reader with a comprehensive understanding of the trade-offs and dynamics inherent to the model. Illustrated in Figure 2.1, the borrowing decision process for a borrower seeking a dollar revolves around selecting a funding source. The borrower has the option not to participate in the credit market (option 0), resulting in a value of κ (discounted home production). Alternatively, the borrower can seek funding from a commercial bank (options 1 to J), choosing between two projects (τ_1 or τ_2) and incurring an interest rate of $r_{j,t,\tau}$ on the loan. The borrower's choice is influenced by the value derived from borrowing at a bank for a specific project ($V(.)$). Additionally, the agent can choose to borrow from the bond market, which assumes a federal fund rate along with a credit spread for all borrowers. Incorporating the bond market and the option of non-participation is crucial for comprehending the impact of changes in the federal fund rate or taxes on credit participation [2].

Figure 2.1: Borrowers' Discrete Choices



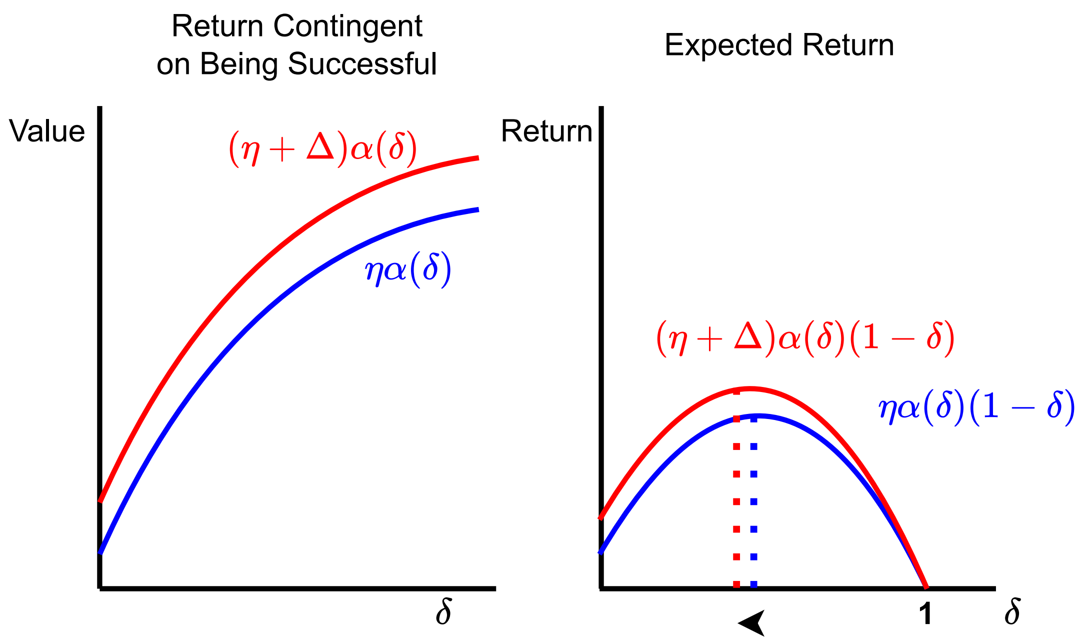
Borrowers are characterized by the productivity of their match with a bank ($\eta_{j,t,\tau}$), which represents their ability to undertake a profitable project. The left panel of Figure 2.2 illustrates how an increase in productivity influences a project's return contingent on success. The x-axis represents the project's risk level (the failure probability), while the y-axis represents the value of the return. The figure demonstrates that returns increase with the level of risk (δ). Moreover, returns grow as productivity (η) increases, as depicted by the shift from the blue to the red curve.

To analyze the impact of increasing productivity on the optimal level of risk, let us turn our attention to the right panel of Figure 2.2, which represents the project's expected return. The expected return incorporates the project's probability of success ($1 - \delta$), resulting in the curves displayed in the right panel. A rise in productivity leads to a decrease in the argmax of the curve, indicating a reduction in the level of risk (δ). This reduction in risk-taking illustrates a standard outcome in

[2]Modeling the bond market and the option of not participating is essential to understanding how changes in the federal fund rate or taxes affect credit participation.

this model[3]. Similarly, the model demonstrates that an increase in the loan's interest rate heightens delinquency through greater risk-taking, and the same effect is observed when taxes increase.

Figure 2.2: Borrowers' Project Return



So, what are the key trade-offs in this model? Firstly, an increase in productivity leads to a decrease in loan delinquency and an increase in credit market participation. This is because higher productivity results in an increase in the value of projects, thereby reducing borrowers' incentives to take risks. Secondly, an increase in interest rates or corporate taxes alters the credit composition by reducing the profitability of unproductive projects. Consequently, the average productivity of borrowers rises when costs are high. However, this increase in borrowers' costs also hampers credit market participation through the same mechanism. Therefore, the impact of changes in interest rates and taxes on output remains ambiguous, necessitating model estimation to accurately quantify the relationship.

2.3.4 Estimation

The estimation of the model relies on two procedures. First, I obtain some parameters externally through simple regressions. Then, I estimate the remaining parameters internally using the model to match moments from the data. This subsection focuses exclusively on describing the internal estimation procedure. Please

[3]For a formal proof, please refer to the subsection in the appendix.

refer to the dedicated section in the appendix for readers interested in the external estimation of parameter $\mu_{j,\tau}$.

The estimation relies on targeting a set of moments. The first moments correspond to orthogonality conditions described in this paragraph. From the model, I can obtain the ratio of the probability of choosing bank j and the likelihood of selecting the bond market $\left(\frac{P_{j,t}}{P_{J+1,t}}\right)$. Similarly, there exists a data counterpart using the ratio of bank j's market share and the bond market share $\left(\frac{S_{j,t}}{S_{J+1,t}}\right)$. Therefore, I can write

$$\log\left(\frac{S_{j,t}}{S_{J+1,t}}\right) = \log\left(\frac{P_{j,t}}{P_{J+1,t}}\right),$$

which yields the following equation

$$\log\left(\frac{S_{j,t}}{S_{J+1,t}}\right) = \xi_j + x'_{j,t}\beta_q + I_{j,t} - V_{J+1,t} + \Delta\xi_{j,t}.$$

The term $\Delta\xi_{j,t}$ represents a demand shock for bank j in period t. Additionally, remember that $I_{j,t}$ and $V_{J+1,t}$ depend on interest rates, which banks determine. Given that banks observe the demand for their products, interest rate changes are likely to depend on $\Delta\xi_{j,t}$ creating a simultaneity bias. To correct this bias, I follow the literature on credit demand estimation, providing a set of relevant instruments referred to as supply shifters. Supply shifters are variables generating a shift in supply that is uncorrelated with changes in demand. Typically, the cost of a bank is used as shifters as they affect the supply but don't affect the utility of borrowers, which ultimately keeps the demand unchanged. For my estimation, I use six instruments gathered in the vector $z_{j,t}$; provision for credit losses, salaries and employee benefits, premises and equipment expense, additional non-interest expense, total employees, and the number of offices. Therefore, the first set of moments contributes to matching the following orthogonality condition

$$\mathbb{E}[z'_{j,t}\Delta\xi_{j,t}] = 0,$$

yielding the first moment to be

$$m^1_{j,t} = z'_{j,t}\Delta\xi_{j,t}.$$

For the second group of moments, my objective is to match the contemporaneous correlation among interest rates, corporate tax rates, and delinquency rates, which I have documented in the appendix (see figures A.2, A.3, A.4, and A.5). Specifically, a marginal reduction in interest rates corresponds to a 10bp decrease in commercial and industrial delinquency rates and a 9.5bp reduction in non-residential real estate delinquency rates. Similarly, a marginal increase in the effective corporate tax rate coincides with a 7bp increase in commercial and industrial delinquency rates and a 4.2bp increase in residential real estate delinquency rates. I gather these four moments into the vector $m^2_{j,t}$.

The third set of moments, denoted as $m^3_{j,t}$, contributes to matching some aggregate moments. Specifically, I aim to ensure that, on average, the probability of borrowing from the bond market ($P_{J+1,t}$) is equal to the bond market share, and the likelihood of not participating in the credit market ($P_{0,t}$) is equal to the deposits market share. Finally, for the fourth set of moments ($m^4_{j,t}$), I aim to match the aggregate delinquency rates for both loan types to the observed data.

To incorporate all these conditions into the estimation process, I utilize the non-linear generalized method of moments (GMM). Let's define the vector of parameters as Θ, the vector of moments as $g_{j,t}(\Theta) \equiv (m^1_{j,t}, m^2_{j,t}, m^3_{j,t}, m^4_{j,t})'$, and W as a consistent estimate of the weight matrix $\mathbb{E}[g_{j,t}(\Theta)g_{j,t}(\Theta)']$. Then, the non-linear GMM estimator is given by

$$\hat{\Theta} = argmin_{\{\Theta\}} \quad g(\Theta)' W^{-1} g(\Theta),$$

where $g(\Theta) = \frac{1}{T \times N} \sum_{j,t} g_{j,t}(\Theta)$.

2.4 Results

The estimation procedure successfully yielded point estimates, as shown in Table B.1 in the appendix. The external estimation reveals average loan durations of 3.2 quarters for commercial and industrial loans, 5.4 quarters for non-residential real estate loans, 12.8 quarters for the bond market, and an average commitment duration of 13.4 quarters for non-participation.[4] The disparity in durations between external and internal options creates significant sensitivity of the model to changes in interest rates, as explored in the subsequent subsection. In essence, as interest rates rise, borrowers tend to delay borrowing from banks and instead commit to longer durations by either opting for the bond market or choosing not to participate at all.

To evaluate the fit of the internal estimation, Table B.2 assesses the deviation of certain moments from their target values. The table suggests that the model encounters challenges in matching the contemporaneous effects of interest rates and taxes on loan delinquency. However, it performs better in fitting the levels of

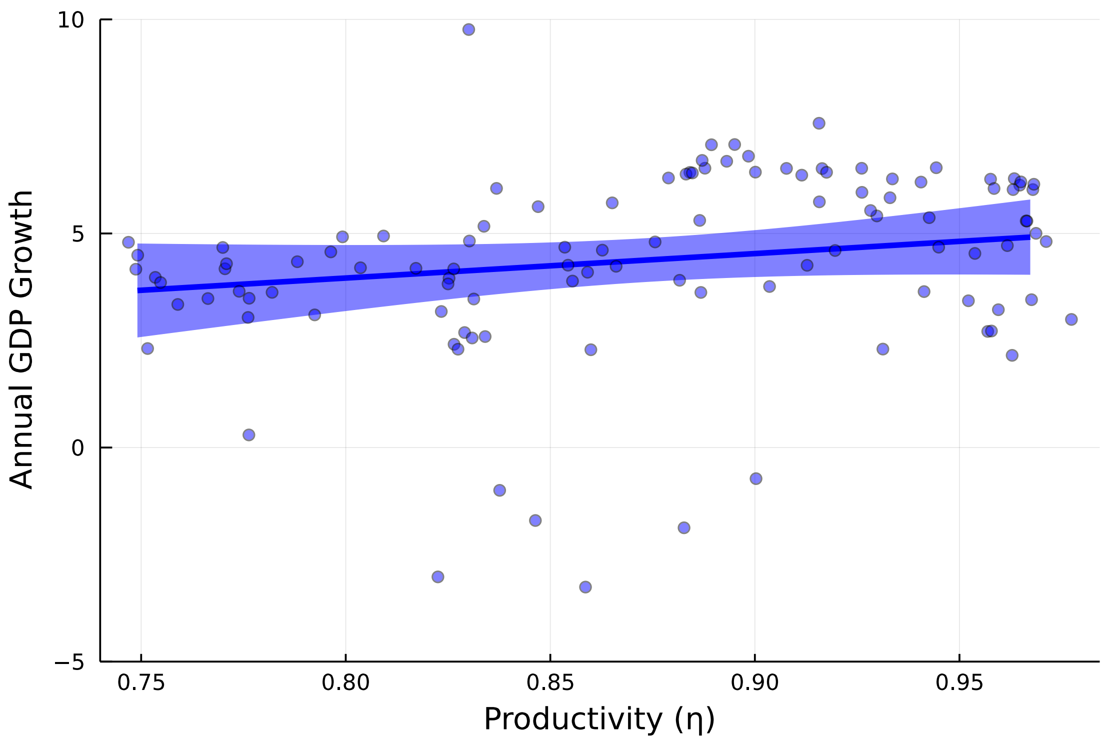
[4]These durations are computed as the inverse of the repayment share μ_τ.

delinquency rates and the aggregate proportion of outside options. Consequently, the subsequent subsection of the analysis focuses on permanent changes in policy rather than temporary ones.

The productivity parameter (η), which is crucial for the match between borrowers and banks, is estimated using the model. It determines the value of borrowing from a bank and influences borrowers' risk-taking behavior. To gain a deeper understanding of this filtered productivity, I present a plot of η against US annual GDP growth in Figure 2.3. As expected, there is a positive correlation between the two, indicating that, on average, US annual GDP growth serves as a reasonable proxy for the productivity of the match between borrowers and banks.

This section aims to provide policymakers with insights into the effects of policy instruments on macroeconomic variables. The analysis is divided into two parts. First, I examine how permanent changes in interest rates and corporate tax rates of varying magnitudes impact the economy. Subsequently, I focus on tax changes to explore the effects of permanent tax adjustments under two different interest rate regimes: low and high. This approach allows for a comprehensive understanding of the implications of policy decisions.

Figure 2.3: GDP Growth and Productivity



2.4.1 The Tale of Two Policies

What are the effects of changes in interest and corporate tax rates on delinquency rates, credit, and output? To answer this question, it is necessary to simulate the model under different interest rates and tax levels and compare the results to a benchmark. The outcomes of these simulations are presented in Figures B.1 to B.5, with two panels: one for changes in interest rates (Panel A) and one for changes in corporate tax rates (Panel B). The x-axis represents the average productivity value within a period, while the y-axis represents the variable of interest. Each figure displays the results for every period as scattered data points, along with a polynomial regression line and 95

Starting with Figure B.1, it is evident that both increases in taxes and interest rates have a positive impact on delinquency rates. However, changes in interest rates have a significantly larger effect on delinquency rates compared to changes in taxes. On average, increases in interest rates are four times more effective in driving up loan delinquency rates than changes in taxes. Moreover, the relationship between these factors appears to be highly non-linear. As shown in the figure, there is a kink around the productivity level of 0.82. When the average productivity of borrowers is sufficiently low (less than 0.82), even small increases in their costs result in substantial increases in the delinquency rate. However, when the average productivity is relatively high (greater than 0.82), increases in borrowers' costs have a less pronounced effect on the delinquency rate. Since borrowers' productivity is a pro-cyclical variable, these findings help explain the observed large fluctuations in delinquency rates during economic downturns, as unproductive borrowers are more prone to taking excessive risks to maintain the profitability of their projects.

Figure B.2 illustrates the changes in credit market participation, measured as the variation in the proportion of individuals engaged in borrowing. Overall, both an increase in interest rates and taxes lead to a reduction in credit market participation by diminishing the value of projects ($V_{j,t,\tau}$). Similarly to the previous analysis, there are notable differences in the magnitude of the effects of taxes and interest rates on participation. On average, an increase in interest rates reduces credit market participation four to eight times more than an increase in taxes. For instance, a 1% rise in the corporate tax rate is estimated to result in a maximum reduction of participation by 1.3%. In contrast, the same increase in interest rates can cause a maximum decline in participation of up to 10.4%.

Once again, the average productivity of borrowers and loan delinquency rates play a critical role in understanding credit market participation. Less productive borrowers, who take on greater risks for lower profits, tend to experience delinquency more frequently and are less inclined to undertake projects. Consequently, they end up abstaining from participating in the credit market. Additionally, non-linearities come into play, determined by the productivity threshold of 0.8. In general, productivity growth limits the impact of interest rate and tax increases on credit market participation by enhancing the attractiveness and value of projects. However, for productivity levels below 0.8, marginal increases in productivity seem to amplify the effects of interest rates and taxes on participation. This finding may appear counterintuitive but can be explained by the significant rise in delinquency rates among unproductive borrowers (with productivity levels below 0.82) observed in Figure B.1. These borrowers engage in substantial risk-taking, resulting in a significant share of them exiting the market, thereby improving the average productivity of the remaining borrowers. Consequently, credit market participation collapses for marginal increases in productivity, creating an amplification effect.

One unique feature of this model is its ability to analyze compositional changes in credit demand, specifically by examining variations in average borrowers' productivity, as depicted in Figure B.3. Panel A reveals that an increase in interest rates leads to a corresponding increase in the average productivity of borrowers. For instance, a 1% rise in interest rates results in a slightly over 1% boost in productivity, on average. The model demonstrates that interest rate increases predominantly impact unproductive borrowers, reducing their presence in the credit market. To illustrate this point, consider interest expenses as a linear cost that affects borrowers' profits. Unproductive borrowers are more susceptible to these expenses, as they are more likely to incur negative profits, potentially leading to insolvency. Consequently, interest rate increases have a positive effect on average productivity by excluding unproductive borrowers from the market.

Similarly, on average, increases in tax rates positively affect productivity. However, corporate taxes prove to be over ten times less effective in improving borrowers' productivity compared to interest expenses. Furthermore, for productivity levels below a certain threshold (less than 0.80), increases in taxes reduce average productivity. The significant disparity in the effects of interest expenses versus taxes on average productivity can be attributed to how taxes impact borrowers' profits. In the model, corporate taxes can be viewed as expropriating a share of borrowers' profits. As this share remains constant across borrowers, increases in the tax rate affect the value of both productive and unproductive borrowers proportionally. Consequently, taxes are less potent in generating compositional changes in credit demand.

These changes in productivity directly translate into output per capita, as demonstrated in Figure B.4. For instance, a 1% increase in interest rates leads to a slightly more than 1% increase in productivity and a slightly less than 1% increase in output per borrower. The two figures exhibit a close resemblance, highlighting the crucial link between production per capita and productivity. However, it is important to note that these changes in individual output do not necessarily translate into increases in aggregate output, as further explained in the subsequent analysis.

Up to now, I have explained various mechanisms operating in this economy. However, it is crucial to ascertain the impact on output. Figure B.4 illustrates the response of aggregate output. It is worth noting that although there is a positive effect on productivity, it does not translate into a positive impact on overall production. For example, on average, a 1% increase in interest rates leads to a 2% reduction in output. Nevertheless, the response is heavily contingent on productivity levels. Notably, there are significant disparities in output response beyond the average productivity level of 0.82, where the average response becomes slightly less than 1%. By comparison, a 1% increase in the corporate tax rate yields an average output reduction of 40 basis points (bps), which is five times less pronounced than the impact of interest rates.

The response of output exhibits predictable non-linearities. Specifically, increasing interest expenses have a more pronounced effect on production at lower productivity levels (ranging from 0.75 to 0.82) compared to higher levels. The rationale behind this observation lies in the fact that unproductive borrowers have stronger incentives to take risks and are more likely to default, as explained in Figure B.1. The resulting surge in delinquency rates amplifies the negative effect of rising expenses on output. Additionally, the observed decline in credit market participation further diminishes output. However, this reduction in participation predominantly stems from unproductive borrowers, as demonstrated in Panel A of Figure B.3, thereby mitigating the adverse impact of interest expenses on output beyond the productivity level of 0.82. Regrettably, this mitigating effect is less prominent in Panel B, implying that corporate tax increases do not substantially enhance the credit composition (improved average productivity levels) that could have supported output figures.

2.4.2 Evaluating Austerity Measures

In this subsection, I aim to examine the impact of corporate taxes on the economy, focusing specifically on a 100 basis point (bp) increase. Similar to previous analyses, I employ model simulations to generate counterfactual scenarios and compare them against a benchmark. The objective is to understand the most effective approach to implementing austerity measures through adjustments in the corporate tax rate. In essence, I seek to maximize government tax revenues while minimizing the distortions that may arise in output (negative effects on production). To gain insights into optimal tax policies, I investigate two distinct interest rate regimes in which taxes can be collected: a high-interest rate regime and a low-interest rate regime. In the high-interest rate regime, I define the benchmark as an economy with interest rates elevated by 100 bps (Δr of 1%), while the low-interest rate regime maintains the benchmark unchanged (Δr of 0%). This comparative analysis enables a comprehensive assessment of the effects of corporate tax adjustments under varying interest rate conditions.

To maintain consistency with the previous subsection, I will begin my analysis in a similar order. First, let's turn our attention to figure B.6, which illustrates the impact of a 100 basis point (bp) increase in the corporate tax rate on the delinquency rate. Overall, the findings exhibit similarities across both regimes, but there are notable differences between the two figures. Notably, for productivity levels below 0.78, the low-interest rate regime experiences more pronounced increases in the delinquency rate due to the expansion of the tax rate. Specifically, the low regime exhibits an average increase of 1.75 bp, while the high regime shows a slightly lower increase of 1.3 bp. Conversely, for productivity levels above 0.92, the high-interest rate regime results in a 0.7 bp increase in the delinquency rate, whereas the low regime sees a smaller increase of 0.5 bp. In summary, the high-interest rate regime curbs risk-taking among unproductive borrowers while encouraging it among productive ones. However, the effects of taxes on delinquency rates remain relatively modest and do not translate into significant changes in output.

Moving on, let's examine the response of credit market participation, presented in figure B.7. On average, in the high-interest rate regime, a 100 bp increase in the tax rate reduces participation by 30 bp. In contrast, the low-interest rate regime experiences a more substantial reduction, averaging 75 bp (more than double). This discrepancy can be attributed primarily to the disparity in credit composition between the two regimes. As discussed in the previous section, higher interest rates diminish the proportion of unproductive borrowers by diminishing the profitability of their projects. Consequently, the remaining borrowers become less sensitive to changes in their expenses, including changes in the tax rate. This accounts for the diminished impact of taxes on credit market participation in the high-interest rate regime.

Unsurprisingly, alterations in credit market participation correspond to changes in average productivity, as depicted in figure B.8. For instance, in the low-interest rate regime, a 100 bp increase in the tax rate yields an average increase in productivity growth of 4 bp. Conversely, the same experiment in the high-interest rate regime leads to a reduction in productivity growth by 2 bp. As mentioned earlier, taxes do not generate significant shifts in productivity growth. However, their impact on productivity, although negligible, indicates the presence of more productive borrowers in the high-interest rate regime. The negative response to tax increases can be attributed to the departure of some highly productive borrowers from the market due to growing expenses, while the overall representation of productive borrowers remains substantial.

Thus far, we have explored the key distinctions between the two regimes and examined the effects of increasing the corporate tax rate on delinquency, participation, and productivity. However, what is the ultimate impact on output? Figure B.9 provides the answer to this question. In the high-interest rate regime, corporate taxes result in a smaller reduction in output compared to the low-interest rate regime. Specifically, a 100 bp increase in the corporate tax rate leads to an average output reduction of 25 bp in the high regime, whereas the low regime experiences a larger reduction of 40 bp. Several factors contribute to this disparity between the regimes. Undoubtedly, the amplified decline in participation in the low regime compared to the high regime plays a crucial role in explaining the significant output reduction in the former. Additionally, the compositional changes in borrowers' productivity from the low to the high regime should not be overlooked, as they represent approximately a 1% increase in productivity (see figure B.3).

Taken together, the model allows us to assess the efficacy of austerity measures driven by corporate tax rate increases. The key finding is that in high-interest rate environments, austerity measures result in smaller reductions in production compared to low-interest rate regimes. The average output reduction improves by 37% relative to the benchmark when transitioning from a reduction of 40 bp to 25 bp. Consequently, the policy recommendation in this paper emphasizes implementing austerity measures during periods of high interest rates. The primary rationale behind this recommendation lies in the fact that interest rates act as a filter, facilitating the enhancement of borrowers' productivity. As taxes do not exert the same influence on credit composition, the implementation of austerity measures should be highly contingent on interest rate levels.

2.5 Conclusion

In conclusion, this paper has provided a comprehensive analysis of the impact of interest and corporate tax rate changes on various economic factors, including delinquency rates, credit market participation, productivity, and output. Through simulations and comparisons to a benchmark, we have gained valuable insights into the mechanisms at play in the economy.

The findings reveal that interest rate changes have a more substantial effect on delinquency rates compared to tax rate changes. Increases in interest rates are four times more effective in generating higher loan delinquency rates than tax rate increases. Additionally, the relationship between interest rates and delinquency rates exhibits non-linear behavior, with a kink observed around a certain productivity level. This suggests that lower productivity borrowers are more vulnerable to small increases in costs, leading to significant spikes in delinquency rates. These results shed light on the dynamics of delinquency rates, particularly during economic downturns when unproductive borrowers are more inclined to take disproportionate risks to maintain profitability.

Furthermore, the study reveals that both interest and tax rate changes impact credit market participation and average borrowers' productivity. Increases in interest rates lead to a significant reduction in credit market participation, with a greater effect observed in the low-interest rate regime. The composition of borrowers is crucial in explaining these changes, as higher interest rates make projects less profitable for unproductive borrowers, reducing their representation in the credit market. Tax rate increases also positively affect productivity, albeit to a lesser extent than interest rate changes. However, the impact of taxes on credit market participation and productivity is mitigated compared to interest rates, as taxes affect borrowers uniformly and do not induce significant compositional changes.

In terms of overall output, the study highlights that while increases in average productivity are positively associated with higher interest rates, they obviously do not translate into positive impacts on aggregate production. However, the response of output to interest rate and tax rate changes exhibits non-linear patterns with varying effects across different productivity levels. Particularly, unproductive borrowers' risk-taking behavior and the resulting delinquency rate amplify the negative impact of rising expenses on output. Moreover, changes in credit market participation play a significant role in shaping output outcomes.

A closer examination of the high-interest rate and low-interest rate regimes further underscores the importance of interest rates in determining the effectiveness of austerity measures. The study demonstrates that austerity measures, implemented through corporate tax rate increases, have a smaller adverse impact on output in high-interest rate environments compared to low-interest rate regimes. This is primarily attributed to interest rates acting as a filter that improves borrowers' productivity, while taxes do not generate the same effect on credit composition.

Based on these findings, the paper concludes by recommending the implementation of austerity measures during periods of high interest rates to minimize the adverse effects on production. The interplay between interest rates, credit composition, and borrower productivity highlights the need to consider the specific economic environment when designing and implementing policy measures aimed at maximizing tax revenue while minimizing distortions in output.

Chapter 3

A Multivariate Proxy Identification of SVARs With Plausibly Exogenous Instruments

[1] with ***Srečko Zimic***

Abstract

This paper proposes a novel methodology to identify structural VARs using multiple proxy instruments. The proposed methodology does not impose any restrictions on the total number of proxy variables used and allows for the incorporation of plausibly exogenous and/or weak instruments. In a nutshell, our method identifies the VAR by matching a targeted relationship between shocks and instruments. Monte Carlo experiments suggest, among other things, that using a complete set of instruments (meaning at least one instrument per shock) improves the identification for the entire system of shocks. Additionally, using instruments for individual shocks with lower cross-correlations seems to alleviate endogeneity problems. We apply our method to a small-scale VAR identifying demand, supply, and monetary drivers in US data.

Keywords: *External instruments; Proxy VAR; Plausibly exogenous; Structural vector autoregressive model*

JEL Classification: *C32; C36.*

[1]The title was recently changed to "The Power of Many: The Procrustes Approach to Proxy-SVAR Identification with Multiple Instruments".

3.1 Introduction

In Greek mythology, Polypemon used to invite tired travelers into his home, where he would generously offer his guests a place to stay for the night. To the detriment of those who fell for his hospitality, Polypemon had a twisted mind as he wanted the bed to perfectly fit the travelers who lay in it. If his guests were too tall, Polypemon would cut their limbs and if they were too small, he would stretch them, which earned him the pseudonym Procrustes translating as "the stretcher". Procrustes' story has been used as an analogy in many fields and in particular in the field of mathematics, where Procrustes analysis refers to the process of performing a shape-preserving Euclidean transformation to a set of shapes. In this paper, we focus on the Procrustes problem and how it can be used in the context of structural vector autoregression (SVAR).

Identifying SVARs with proxy variables has gained interest in the empirical macro literature. However, current methodologies rely primarily on the exogeneity of instruments to some selected shocks, constraining the number of proxies one can use in practice. With the increasing availability of usable instruments, the constraint on the number of proxies is ever more binding. In this paper, we propose a methodology that does not restrict the total number of proxy variables and allows for plausibly exogenous instruments to be used for the identification.

Historically, the identification of structural VARs with external instruments can be attributed to two seminal papers by Stock and Watson (2012) and Mertens and Ravn (2013). In their framework, the identification is performed through a two-step procedure where reduced form residuals are regressed on instrumental variables to obtain the identification. In more recent years, important developments have emerged in the literature. Notably, papers such as Caldara and Herbst (2019) and more recently Arias, Rubio-Ramírez, and Waggoner (2021) propose methods relying on augmenting the VAR with instruments, allowing for estimating and identifying shocks with instrumental variables in a unified one-step model. Ultimately, these methods suffer from an important shortcoming. The ex-ante imposition of instruments' perfect exogeneity with respect to other shocks is achieved through exclusion restrictions. The number of exclusion restrictions one can impose in a linear model is limited, which implies a constraint on the number of proxy variables econometricians can use to identify shocks of interest.

In contrast, our method allows the inclusion of an unspecified number of instruments. We borrow from an old idea, the orthogonal Procrustes problem, solved in Schönemann (1966) where he describes optimally choosing an orthogonal matrix. Similarly, we propose to write the identification of our proxy-VAR as a problem of optimal orthogonal matrix selection. When the number of instruments is lower than the number of exclusion restrictions one can impose, our method boils down to a standard ex-ante imposition of exogeneity as in Mertens and Ravn (2013) and Stock and Watson (2012). Alternatively, when more instruments are provided, our method infers the causal relationship between the instruments and the shocks of interest. Intuitively, the method finds the structural identification that is as close to satisfying exogeneity assumption as possible considering observed data and instruments.

The key advantage of our method is that when the number of instruments is large enough, the instruments' perfect exogeneity is not imposed ex-ante. Indeed, proxies are likely to suffer from instrument endogeneity. For instance, as explained in Jarociński and Karadi (2020), the use of high-frequency proxies can be challenging to identify monetary policy shocks as they can be polluted by the central bank's information shocks. Our framework is helpful in these cases as it allows the use of plausibly exogenous instruments. The flexibility of the method is especially valuable when researchers have access to a multitude of instruments but suspect instruments to be of poor quality.

Besides the microeconometric literature as in Conley, Hansen, and Rossi (2012), using plausibly exogenous instruments remains scarce. Still, we would like to emphasize recent work from Braun and Brüggemann (2022), which is most closely related to our paper. In their paper, they propose identifying structural VARs using exogenous and plausibly exogenous instruments. Their augmented VAR framework is based on Arias et al. (2021) but incorporates additional restrictions on the correlations and variance contributions between instruments and structural shocks. One of the contributions of their paper is to use ranking restrictions on the variance contribution of specific shocks to their instruments, which constitutes an important step forward as it allows for less arbitrary decision-making. However, our method differs from theirs in three important dimensions. First, our method achieves point identification, while their method produces a set of models consistent with the imposed constraints. Point identification generates sharper inference and allows a more straightforward use of classical methods to estimate the model in case of preference. Second and related, we show that when the number of instruments is lower than the number of possible exogeneity restrictions, our results will ex-post boil down to a standard approach of proxy identification, which is not the case with their approach. Last, in their applications, they do not explore the cases we find most interesting, using information from a large set of instruments to identify structural shocks primarily because of the binding restrictions on the total number of used proxies. In particular, we show the advantages of using a complete set of instruments to identify our shocks, which is not explored in their paper. We show that cross-correlations between instruments can be corrected by using at least one instrument per shock.

We highlight our method's performance through simulations. We assume a data generating process for a structural VAR and a set of instruments, after which we attempt to identify our shocks. Our instruments are plagued with different degrees of contamination, meaning that they are not only correlated to the shock of interest. We find that by imposing shocks' orthogonality with a complete set of instruments, our methods allow for contamination between instruments to be offset in many cases. Moreover, we see clear advantages in combining instruments for the same shock with lower cross-correlations, which allows to identify the common variation between them as originating from the shock itself. Finally, we describe how to implement our method with sign restrictions and show the benefits of using it to improve our identification strategy.

The method is applied to the US economy and identify a monthly VAR representing the dynamics between the fed fund rate, output gap, and inflation. We find that our methodology allows replicating dynamics consistent with economic theory by relying exclusively on plausibly exogenous instruments. In this application, we show the advantages of using our identification with a complete system of instruments. Finally, we show how using the two high-frequency instruments from Jarociński and Karadi (2020) can alleviate the problem of the central bank information shocks without relying on sign restrictions.

3.2 Methodology

3.2.1 An Augmented SVAR

We consider the SVAR model given by

$$y_t^T = \nu^T + \sum_{i=1}^{p} y_{t-i}^T A_i + \epsilon_t^T S, \quad \epsilon_t \sim (0, I_n),$$

where y_t is an $n \times 1$ vector of endogenous variables, ν is an $n \times 1$ vector of intercepts, and A_i, $i = 1, \ldots, p$ are $n \times n$ matrices of autoregressive coefficients. The model is assumed to be driven by a vector of $n \times 1$ normalized and orthogonal structural shocks ϵ_t. The matrix S is an $n \times n$ contemporaneous impact matrix, which determines the direct impact of ϵ_t on y_t.

We conveniently incorporate our instruments into our SVAR, yielding the following augmented SVAR

$$\underbrace{\begin{bmatrix} y_t^T & m_t^T \end{bmatrix}}_{\tilde{y}_t^T} = \underbrace{\begin{bmatrix} \nu^T & \nu_m^T \end{bmatrix}}_{\tilde{\nu}^T} + \sum_{i=1}^{p} \underbrace{\begin{bmatrix} y_{t-i}^T & m_{t-i}^T \end{bmatrix}}_{\tilde{y}_{t-i}^T} \underbrace{\begin{bmatrix} A_i & \Gamma_{y,i} \\ 0_{k\times n} & \Gamma_{m,i} \end{bmatrix}}_{\tilde{A}_i} + \underbrace{\begin{bmatrix} \epsilon_t^T & \eta_t^T \end{bmatrix}}_{\tilde{\epsilon}_t^T} \underbrace{\begin{bmatrix} S & \kappa_\epsilon \\ 0_{k\times n} & \kappa_\eta \end{bmatrix}}_{\tilde{S}}, \tag{3.1}$$

where m_t is a $k \times 1$ vector of instruments, and $\tilde{y}_t^T = [y_t^T \;\; m_t^T]$ an $\tilde{n} \times 1$ vector, where $\tilde{n} = n + k$. Hence, we allow for m_t^T to be affected by past values of the dependent variable through the $n \times k$ matrices of coefficients $\Gamma_{y,i}$. Additionally, we allow for persistent dynamics of the instruments through the $k \times k$ matrix of coefficients $\Gamma_{m,i}$. Moreover, we assume the contemporaneous relationship between the instruments and the structural shock to be driven by the $n \times k$ matrices of coefficients κ_ϵ, and correlations between instruments to be driven by a $k \times 1$ vector of measurement error η_t^T through the $k \times k$ matrix of coefficients κ_η. Furthermore, the instruments' past values and the measurement error are assumed not to affect the dependent variable y_t^T. Finally, the measurement error is assumed to be orthogonal to the structural shocks such as $\tilde{\epsilon}_t \sim (0, I_{\tilde{n}})$.

3.2.2 A Procrustean Identification

Without any additional assumption, the previous augmented SVAR is identified up to orthogonal transformations of the form $\bar{S} = \tilde{Q}\tilde{S}$ with $\tilde{Q}$ being an orthogonal matrix of the form $diag(Q_\epsilon, Q_\eta)$, where $Q_\epsilon^T Q_\epsilon = I_n$ and $Q_\eta^T Q_\eta = I_k$. The block structure of $\tilde{Q}$ allows us to impose the condition that the measurement error η_t is orthogonal to y_t, meaning that the lower left block of $\tilde{S}$ contains exclusively null values.

Let $\tilde{u}_t$ be our error term from equation 3.1, such that $\tilde{u}_t \equiv \begin{bmatrix} \epsilon_t^T & \eta_t^T \end{bmatrix} \begin{bmatrix} S & \kappa_\epsilon \\ 0_{k\times n} & \kappa_\eta \end{bmatrix}$, and $\tilde{u}_t \sim (0, \tilde{\Sigma})$. Then we can obtain the orthogonal reduced-form characterization of our augmented SVAR such that

$$\tilde{y}_t^T = \tilde{x}_t^T \tilde{A}_+ + \tilde{\epsilon}_t^T \tilde{Q} h(\tilde{\Sigma}) \tag{3.2}$$

where $h(\tilde{\Sigma})$ can be deconstructed in blocks as $\begin{bmatrix} h_{\epsilon,y} & h_{\epsilon,m} \\ 0_{k\times n} & h_{\eta,m} \end{bmatrix}$ and is an $\tilde{n} \times \tilde{n}$ matrix satifying the property $h(\tilde{\Sigma})^T h(\tilde{\Sigma}) = \tilde{\Sigma}$. We follow the literature by using $h(.)$ as the Cholesky decomposition of $\tilde{\Sigma}$, making $h(\tilde{\Sigma})$ an upper triangular matrix.

Furthermore, notice that

$$\tilde{Q}h(\tilde{\Sigma}) = \begin{bmatrix} Q_\epsilon & 0_{n\times k} \\ 0_{k\times n} & Q_\eta \end{bmatrix} \begin{bmatrix} h_{\epsilon,y} & h_{\epsilon,m} \\ 0_{k\times n} & h_{\eta,m} \end{bmatrix} = \begin{bmatrix} Q_\epsilon h_{\epsilon,y} & Q_\epsilon h_{\epsilon,m} \\ 0_{k\times n} & Q_\eta h_{\eta,m} \end{bmatrix},$$

yielding the following potential identification

$$\begin{bmatrix} S & \kappa_\epsilon \\ 0_{k\times n} & \kappa_\eta \end{bmatrix} = \begin{bmatrix} Q_\epsilon h_{\epsilon,y} & Q_\epsilon h_{\epsilon,m} \\ 0_{k\times n} & Q_\eta h_{\eta,m} \end{bmatrix}.$$

In this paper, we propose to find the Q_ϵ minimizing the distance between κ_ϵ and a targeted matrix $\bar{\kappa}_\epsilon$. Hence, we write the following minimization[2]

$$\begin{aligned} &min_{\{Q_\epsilon\}} ||Q_\epsilon h^*_{\epsilon,m} - \bar{\kappa}_\epsilon||_{\mathbf{F}} \\ &s.t. \\ &Q_\epsilon^T Q_\epsilon = I_n. \end{aligned}$$

It amounts to solving the Procrustes problem, which was initially introduced in Schönemann (1966) and further discussed in Gower and Dijksterhuis (2004). This problem offers a direct solution and can be expressed using a singular value decomposition. Ultimately, if $\bar{\kappa}_\epsilon h^{*^T}_{\epsilon,m}$ can be decomposed as follows

$$\bar{\kappa}_\epsilon h^{*^T}_{\epsilon,m} = U\Omega V^T,$$

then the solution is

$$Q^*_\epsilon = UV^T.$$

For completeness, we provide the proof in the appendix even though it can also be found in Schönemann (1966) and Gower and Dijksterhuis (2004).

The choice of $\bar{\kappa}_\epsilon$ remains the only input from the econometrician and is based on the assumption that our instruments are mostly correlated to their respective

[2] $h^*_{\epsilon,m}$ is a normalisation of the matrix $h_{\epsilon,m}$ such that $h^*_{\epsilon,m} = ||h_{\epsilon,m}||_R^{-\frac{1}{2}} h_{\epsilon,m}$, where $||X||_R^{-\frac{1}{2}} = \sum_i (e_i^T X^T X e_i)^{-\frac{1}{2}} O_i$, with e_i a vector having a 1 in entry i and zeros otherwise, and with O_i a matrix having a 1 in entry (i,i) and zeros otherwise.

shocks. So, we propose the following representation

$$\bar{\kappa}_\epsilon = \begin{bmatrix} 1^T_{|j(1)|} & 0^T_{|j(2)|} & \dots & 0^T_{|j(n)|} \\ 0^T_{|j(1)|} & 1^T_{|j(2)|} & \dots & 0^T_{|j(n)|} \\ \vdots & \ddots & \ddots & \vdots \\ 0^T_{|j(1)|} & 0^T_{|j(2)|} & \dots & 1^T_{|j(n)|} \end{bmatrix},$$

where $\bar{\kappa}_\epsilon$ is an $n \times k$ matrix, $j(i)$ is a function mapping shock i to its instruments $j(i)$, $1_{|j(i)|}$ is a vector of ones of length $|j(i)|$, and $0_{|j(i)|}$ is a vector of zeros of length $|j(i)|$.[3] The matrix $\bar{\kappa}_\epsilon$ shows the target relationship between the shocks and the instruments. The matrix imposes instruments for shock i only to be affected by shock i, meaning that our instruments are imposed to be relevant and exogenous. Notice, that in practice these targeted coefficients between instruments and shocks can't be obtained unless the instruments we use for the same shock are perfectly collinear to each other. However, by imposing this target coefficient structure we are able to come as close to exogeneity and relevance as it is possible. A crucial step here is to normalize the columns of matrix $h_{\epsilon,m}$ by its columns' implied variance yielding $h^*_{\epsilon,m}$. This step allows the implementation of the method without the incorporation of weights, which would lead us to use Wahba's problem described in Wahba (1965) instead of the Procrustes problem.

Finally, our choice of $\bar{\kappa}_\epsilon$ is motivated by its link to Mertens and Ravn (2013). In particular, we present the following proposition and refer to the proof in the appendix.

Proposition 1. *When using at most one instrument per shock and when $k \leq n$, the relationship between the shocks and the instruments implied by the Procrustean identification can be summarized as follows:*

$$Q^*_\epsilon h^*_{m,\epsilon} = \begin{bmatrix} \left(h^{*T}_{m,\epsilon} h^*_{m,\epsilon}\right)^{\frac{1}{2}} \\ 0_{n-k,k} \end{bmatrix}.$$

Corollary 1. *When $k = 1$, the Procrustean identification simplifies to the formulation presented in Mertens and Ravn (2013), which can be expressed as follows:*

$$Q^*_\epsilon h^*_{m,\epsilon} = \begin{bmatrix} 1 \\ 0_{n-1,1} \end{bmatrix}.$$

In this case, the Procrustean identification reduces to the form established by Mertens and Ravn (2013), where $n - 1$ exogeneity restrictions are imposed.

[3]Notice that the sum of all the total number of instruments per shock yields $\sum_i |j(i)| = k$.

3.3 Results

3.3.1 Simulations

In this subsection, we conduct Monte Carlo experiments assuming knowledge of the true data generating process (DGP) for both our dependent variables and instruments. Our main focus is to examine how the degree of instrument contamination, represented by the structure of κ_ϵ, affects the quality of identification. Unlike Mumtaz, Pinter, and Theodoridis (2018), we employ augmented SVAR simulations instead of a DSGE model to mitigate the impact of model misspecification on our results.[4]

To facilitate the discussion, we concentrate on a VAR model with three dependent variables. The number of instruments varies across cases, which can be inferred from the size of κ_ϵ provided in each instance. The VAR's DGP employs a single lag for simplicity, although similar results hold for cases with multiple lags. We generate simulated data using a stylized SVAR model with dimensions 3×3. This model effectively captures the conditional correlations among "monetary," "demand," and "supply" shocks. The instruments used in this simulation are linear combinations of structural shocks and measurement errors. In the appendix, Figure C.1 presents the main data generating process, and the corresponding impulse response functions (IRFs) can be found there.

$$
\begin{bmatrix} y_t \\ m_t \end{bmatrix}^T = \begin{bmatrix} y_{t-1} \\ m_{t-1} \end{bmatrix}^T \begin{bmatrix} 0.7 & -0.26 & 0 & & & \\ 0.03 & 0.7 & 0.18 & & 0_{3\times 3} & \\ 0.21 & 0.26 & 0.8 & & & \\ & & & 0.59 & -0.33 & -0.17 \\ & 0_{3\times 3} & & 0.35 & 0.85 & 0.05 \\ & & & -0.22 & -0.17 & 0.80 \end{bmatrix}
$$
$$
+ \begin{bmatrix} \epsilon_t \\ \eta_t \end{bmatrix}^T \begin{bmatrix} 0.5 & -0.25 & -0.25 & & & \\ 0.375 & 0.5 & 0.5 & & \kappa_\epsilon & \\ 0.25 & -0.5 & 0.5 & & & \\ & & & 0.1 & 0 & 0 \\ & 0_{3\times 3} & & 0 & 0.1 & 0 \\ & & & 0 & 0 & 0.1 \end{bmatrix}
$$

In our initial cases, we examine a favorable scenario where the instruments are both relevant and exogenous. We present first, case I.A. where only the first shock is identified using a single instrument. As illustrated below, the vector κ_ϵ has dimensions of 3×1, with zeros in positions 2 and 3, indicating that ϵ_2 and ϵ_3 have no impact on the instrument, rendering it exogenous. The non-zero value in position 1 confirms the instrument's relevance, as it is associated with the shock to

[4] For a detailed VAR specification, please refer to the appendix.

be identified (ϵ_1).

$$\kappa_\epsilon = \begin{bmatrix} 0.1 \\ 0 \\ 0 \end{bmatrix}.$$

By employing the Procrustean identification, we obtain $\hat{\kappa}_\epsilon$, our estimator representing the contemporaneous relationship between the shocks and the instruments:

$$\hat{\kappa}_\epsilon = \begin{bmatrix} 0.0988 \\ 0 \\ 0 \end{bmatrix}.$$

In this particular instance, the identification process was successful, as both the instrument's exogeneity and relevance were easily recovered. However, this outcome is expected since our method imposes exogeneity and relevance when the number of instruments is sufficiently low. Essentially, this case mirrors a straightforward instrumental variable (IV) procedure, as demonstrated in panel C.2a, which compares our method to Mertens and Ravn (2013). The panel's impulse response functions (IRFs) display our method (in blue), the IV procedure by Mertens and Ravn (2013) (in red), and the true IRF (in black). As evident, the Procrustean identification aligns precisely with the IV procedure in this specific scenario.

One significant advantage of our method is its flexibility regarding the number of instruments employed. Specifically, we have the capability to use one instrument per shock. We demonstrate this characteristic in Case I.B., where the data generating process (DGP) is defined by a diagonal matrix κ_ϵ, presented below. The diagonal structure signifies the presence of a single exogenous and relevant instrument for each shock.

$$\kappa_\epsilon = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}.$$

The identification process provides us with the following matrix $\hat{\kappa}_\epsilon$. Once again, the obtained results closely align with the data generating process (DGP), enabling us to generate impulse response functions (IRFs) displayed in panel C.2a that closely approximate the true IRFs. However, the exogeneity restrictions are slightly less binding than in the previous case. This can be attributed to the increased number of instruments, resulting in an over-identified system of equations.

$$\hat{\kappa}_\epsilon = \begin{bmatrix} 0.0986 & -0.0005 & -0.0003 \\ -0.0005 & 0.1045 & 0.0024 \\ -0.0002 & 0.0025 & 0.1015 \end{bmatrix}.$$

The initial challenge that our method encounters involves the introduction of contamination in the instruments. Therefore, we propose examining a second case in which only 58% of the variance in the instruments is attributed to the shock we aim to identify. In contrast, the previous case featured 100% of the variance being explained by that specific shock. Intuitively, our objective is to generate instruments of lower quality, meaning they deviate further from satisfying the exogeneity restriction.

Let us begin by considering the following case, depicted below, where the first and second shocks positively affect the instrument, while the third shock has a negative impact on it:

$$\kappa_\epsilon = \begin{bmatrix} 0.1 \\ 0.06 \\ -0.06 \end{bmatrix}.$$

As previously mentioned, the first shock explains most of the variation in the instrument but shocks two and three also have a sizeable impact.

The result from the identification yields biased results represented by the vector

$$\hat{\kappa}_\epsilon = \begin{bmatrix} 0.1303 \\ 0 \\ 0 \end{bmatrix}.$$

Again, our method is equivalent to imposing exogeneity restrictions leading to incorrect identification of κ_ϵ. As expected, these biases affect the IRFs significantly on panel C.3a where both methods perform poorly. [5]

While significant contamination in instruments can pose challenges to model identification, there are alternative approaches available to address this issue. Specifically, we demonstrate how incorporating additional instruments for a specific shock can enhance the identification procedure. To illustrate this, we propose the

[5] Similar conclusions can be drawn from cases with different signs such as in the case $\kappa_\epsilon = \begin{bmatrix} 0.1 \\ 0.06 \\ 0.06 \end{bmatrix}$, which gives the identification $\hat{\kappa}_\epsilon = \begin{bmatrix} 0.1336 \\ 0 \\ 0 \end{bmatrix}$.

following data generating process (DGP) that generates two instruments for shock ϵ_1. As shown below, both instruments exhibit a positive correlation with shock ϵ_1, but their correlations with shocks ϵ_2 and ϵ_3 differ. Specifically, the first instrument carries a positive coefficient of 0.06 for shock ϵ_2, while the second instrument bears a negative coefficient. Likewise, the first instrument has a negative coefficient of -0.06 for shock ϵ_3, whereas the second instrument has a positive coefficient of 0.06. Overall, this scenario reflects a relatively low cross-correlation between instruments, as the primary correlation stems from the variation of the shock we seek to identify (ϵ_1).

$$\kappa_\epsilon = \begin{bmatrix} 0.1 & 0.1 \\ 0.06 & -0.06 \\ -0.06 & 0.06 \end{bmatrix}.$$

The subsequent identification results demonstrate promising outcomes. Similar to Case II.A., the percentage of variance explained by the target shock on our instruments remains at 58%, yet the identification is largely successful. The impulse response functions (IRFs) presented in case C.3b confirm that the Procrustean identification closely aligns with the true IRFs, exhibiting minimal errors. In contrast, the IV procedure by Mertens and Ravn (2013) significantly fails to accurately identify the structural parameters. Moreover, the estimation of κ_ϵ exhibits substantial improvement, although there are incorrect signs for two entries.

$$\hat{\kappa}_\epsilon = \begin{bmatrix} 0.098 & 0.0985 \\ -0.0194 & 0.0195 \\ -0.0838 & 0.0843 \end{bmatrix}.$$

The inability to match rows 2 and 3 to the data generating process indicates, to a significant extent, the lack of identification for shocks 2 and 3. To further illustrate this point, let us consider the theoretical expression of $\hat{\kappa}\epsilon$, which can be represented as the product of $Q\epsilon^*$ and $h^*_{m,\epsilon}$. Notably, if we decompose Q^*_ϵ into blocks of rows and $h^*_{m,\epsilon}$ into blocks of columns, the following expression holds true:

$$\hat{\kappa}_\epsilon = Q^*_\epsilon h^*_{m,\epsilon} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} \begin{bmatrix} h_1 & h_2 \end{bmatrix} = \begin{bmatrix} q_1 h_1 & q_1 h_2 \\ q_2 h_1 & q_2 h_2 \\ q_3 h_1 & q_3 h_2 \end{bmatrix}.$$

Upon comparing the expression to our findings, it is evident that the terms associated with q_1 are precisely determined. However, the terms involving q_2 and q_3 lack precise determination. Consequently, only the first row of the rotation matrix Q_ϵ^* is identified, indicating that the first shock is successfully identified while the identification of the remaining shocks is uncertain. This observation highlights a notable advantage of Procrustean identification: the ability to effectively tackle instrument endogeneity by introducing multiple instruments with reduced cross-correlations for the same shock.

In the following instance, we examine the characteristics of Procrustean identification when utilizing a complete set of instruments, which implies that we have at least one instrument per shock. As indicated by κ_ϵ below, all instruments are subject to contamination, with 58% of their variance explained by the target shock. Specifically, the shock to be identified is assigned a coefficient of 0.1, while the other shocks are assigned a coefficient of 0.06.

$$\kappa_\epsilon = \begin{bmatrix} 0.1 & 0.06 & 0.06 \\ 0.06 & 0.1 & 0.06 \\ 0.06 & 0.06 & 0.1 \end{bmatrix}.$$

Once again, the Procrustean identification proves successful in accurately matching the true κ_ϵ. The reported results demonstrate precise alignment for all entries. Moreover, the impulse response functions (IRFs) in panel C.4 highlight how our method effectively reproduces the true IRFs, while the IV procedure fails to do so.

$$\hat{\kappa}_\epsilon = \begin{bmatrix} 0.1014 & 0.0619 & 0.0622 \\ 0.0607 & 0.1028 & 0.0631 \\ 0.0609 & 0.063 & 0.1029 \end{bmatrix}.$$

The example makes it evident that the identification of shocks within a system greatly enhances the overall performance of the identification process. This improvement stems from the elimination of contaminations that may arise in the instruments.

Finally, we examine the performance of all the combined combined cases, meaning that we identify the system of shocks with multiple instruments per shock. We employ the presented data generating process (DGP) denoted by κ_ϵ. This particular matrix enables the generation of two instruments for each shock, resulting in a complete set of instruments and ensuring multiple instruments are available for each shock. Additionally, the cross-correlation between the instruments remains relatively low.

$$\kappa_\epsilon = \begin{bmatrix} 0.1 & 0.1 & 0.06 & -0.06 & -0.06 & 0.06 \\ 0.06 & -0.06 & 0.1 & 0.1 & 0.06 & -0.06 \\ -0.06 & 0.06 & -0.06 & 0.06 & 0.1 & 0.1 \end{bmatrix}.$$

We find reassurance in observing that the identification remains successful regardless of the level of endogeneity in our instruments. As demonstrated below, the results obtained for $\hat{\kappa}_\epsilon$ align closely with the Data Generating Process (DGP), as evidenced by coefficients displaying similar signs and magnitudes. This consistency further strengthens our confidence in the accuracy of the identification process.

$$\hat{\kappa}_\epsilon = \begin{bmatrix} 0.1023 & 0.1019 & 0.0618 & -0.0599 & -0.0604 & 0.0606 \\ 0.0611 & -0.0595 & 0.1003 & 0.0992 & 0.0603 & -0.0609 \\ -0.0609 & 0.0597 & -0.0616 & 0.0608 & 0.0999 & 0.1016 \end{bmatrix}.$$

3.3.2 Applications

In our initial application, we concentrate on analyzing the US economy. Our dependent variables are sourced from the FRED database and cover the period from 1984 to 2023. As for our instruments, they are collected from various sources, primarily from the OECD database. We estimate a simple VAR model encompassing three dependent variables: the federal funds rate (FF Rate), the output gap, and the year-on-year inflation (yoy Pi). The output gap is derived using real GDP and a one-sided hpfilter, while the year-on-year inflation represents the annual growth rate of the CPI index over a one-month rolling window. To account for potential seasonality at the monthly frequency, we include twelve lags in the VAR model.

Application: Identifying Monetary Policy

In our initial application, we concentrate on the identification of monetary policy. Our baseline identification approach involves utilizing two high-frequency instruments obtained from Jarociński and Karadi (2020). Specifically, we employ the surprise in the three-month fed funds futures (ff4-hf) and the negative value of the surprise in the S&P 500 ((-)sp500-hf) as instruments. The authors of the aforementioned paper argue that these high-frequency proxies of monetary policy are not solely driven by monetary policy shocks. Instead, they suggest that surprises in fed funds futures and the S&P 500 can also arise from the Fed revealing information about the economy, commonly referred to as a central bank information shock.

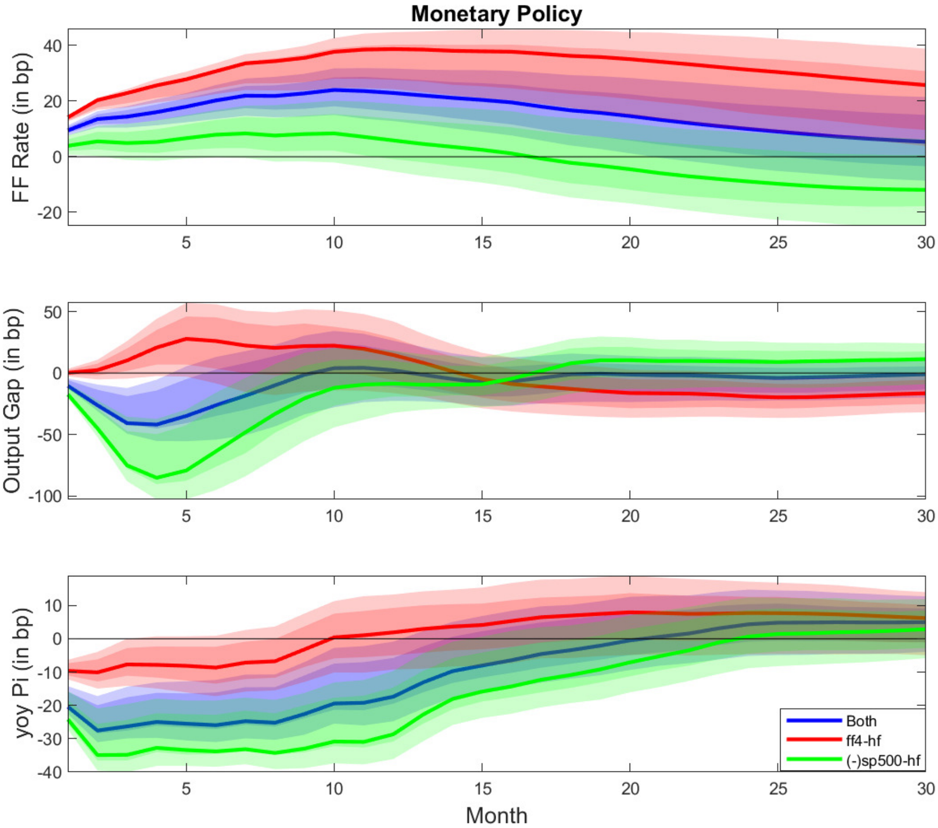
In their paper, Jarociński and Karadi (2020) elegantly address this issue by utilizing an augmented VAR with sign restrictions. However, the use of sign restrictions may be perceived as restrictive in some cases. Therefore, in this subsection, we demonstrate that similar results can be achieved using Procrustean identification, thereby avoiding reliance on sign restrictions. We proceed to identify the model using three different approaches presented below.

1. We identify the shock using exclusively the surprise in the three-month fed funds futures (ff4-hf).
2. We identify the shock using exclusively the surprise in the S&P 500 ((-)sp500-hf).
3. We identify the shock using both instruments together.

The results of the identification are presented in Figure 3.1. The red and green IRFs represent the identification using the surprise in the fed funds futures and the surprise in the S&P 500, respectively. Both identifications exhibit some degree of failure. The red IRF suggests that a contractionary monetary policy would have a positive impact on the output gap, which contradicts economic theory. On the other hand, the green IRF provides qualitatively sound predictions, but the quantitative effect of the monetary policy shock on the fed funds rate is insignificant.

In contrast, the blue IRF, obtained through a Procrustean identification that combines both instruments, yields predictions consistent with economic theory without relying on sign restrictions, as in Jarociński and Karadi (2020). This application closely resembles Case II.B., where instruments with relatively low cross-correlations effectively correct for instrument contamination. In this specific case, the combination of instruments successfully eliminates the bias introduced by the central bank information shock.

Figure 3.1: Procrustean Identification of Monetary Policy



Application: Identifying the System of Shocks

In our second application, our goal is to identify three shocks: monetary policy, aggregate demand, and aggregate supply. We propose to compare a joint identification approach using six instruments to an identification method that considers the instruments individually.

Similar to the previous application, we utilize the surprise in the fed funds futures and the surprise in the S&P 500 to identify monetary policy shocks. For demand shocks, we employ retail sales and the University of Michigan consumer sentiment index (UMCSENT) as instruments. In the case of aggregate supply shocks, we consider inventories and the index of real economic activity from the industrial commodity markets by Kilian (2009).

We believe that inventories play a crucial role in constraining goods production, making them excellent instruments for capturing aggregate supply shocks. To illustrate this point, let's consider an extreme scenario where a firm has extremely low inventories. In such a case, production becomes unresponsive to demand shocks as the limited stocks constrain production capacity. Consequently, even small changes in inventories can significantly impact quantities and prices, relatively independent of demand factors. A compelling real-world example of this phenomenon is the recent Covid crisis, where widespread shortages of production inputs resulted in substantial negative supply shocks. These shocks are reflected in the data through low stocks and inventories.

By considering these instruments collectively, we aim to uncover the interplay between monetary policy, aggregate demand, and aggregate supply. This approach allows us to gain a comprehensive understanding of the dynamics at play in the US economy. We propose to identify the shocks in two ways, as described below. There is a joint identification of shocks presented in blue and an individual identification presented in red.

1. Jointly
   - **Monetary Policy** - ff4-hf, (-)sp500-hf
   - **Aggregate Demand** - UMCSENT (University of Michigan: Consumer Sentiment), Retail Sales
   - **Aggregate Supply** - Kilian Index (index of real economic activity from Kilian (2009)), Inventories
2. Individually
   - **Monetary Policy** - ff4-hf
   - **Aggregate Demand** - UMCSENT (University of Michigan: Consumer Sentiment)
   - **Aggregate Supply** - Kilian Index (index of real economic activity from Kilian (2009))

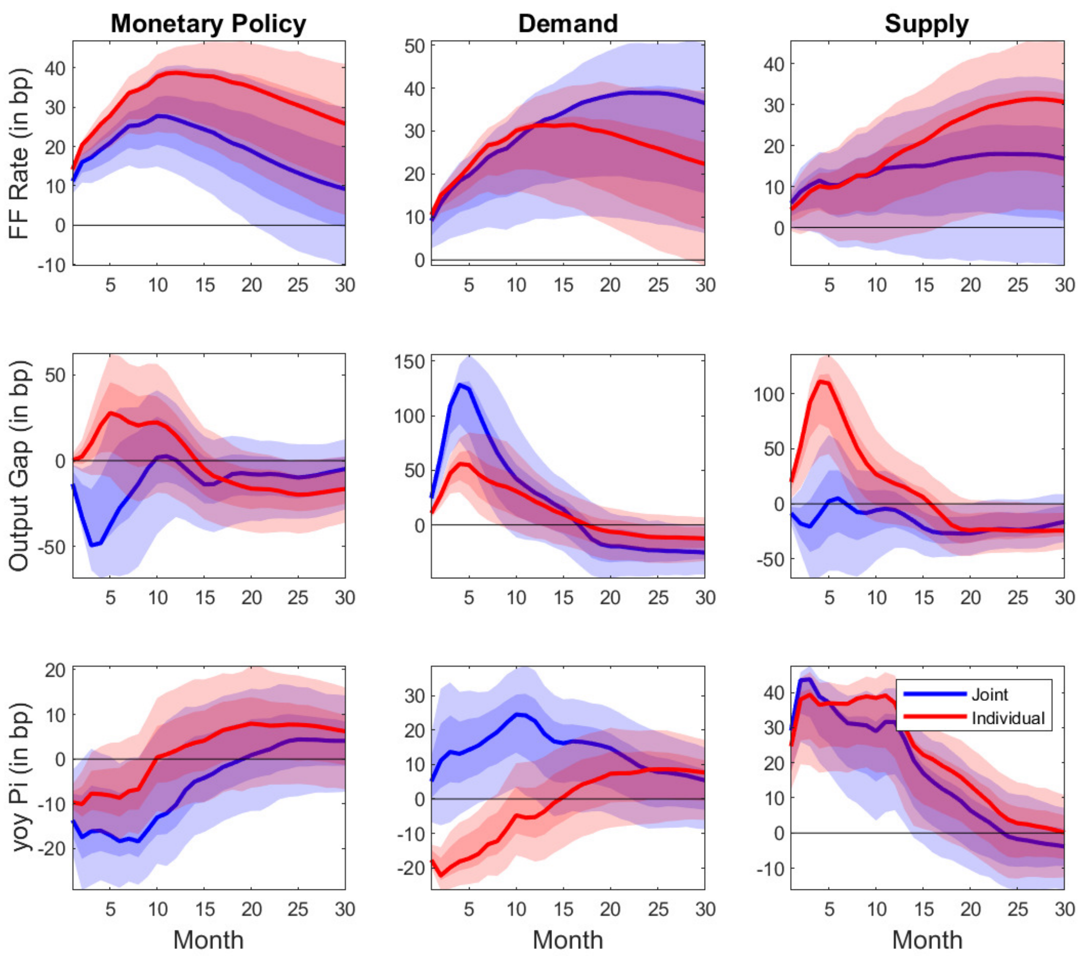
The results are presented in figure 3.2, revealing interesting insights into the identification of shocks. The blue lines in the figure, which correspond to the joint identification approach, demonstrate consistency with economic theory. On the other hand, the red lines, representing the individual identification approach, mostly yield inconsistent results.

For instance, when using the individual identification, a contractionary monetary policy shock is predicted to have a positive impact on the output gap, which contradicts economic expectations. However, the joint identification approach aligns with economic theory, correctly predicting a negative impact on the output gap. Similarly, in the case of demand shocks, the individual identification approach suggests a positive impact on the output gap and a negative effect on prices. Such relationships are typically associated with supply shocks rather than demand shocks.

However, the joint identification approach presents a more accurate depiction by predicting a positive effect of demand shocks on both output and prices. Furthermore, the individual identification strategy for supply shocks also deviates from economic theory. It indicates positive effects on both output and prices, which is inconsistent with our understanding of supply shocks. In contrast, the joint identification method captures the correct relationship by predicting a decrease in output accompanied by an increase in prices, which aligns with economic expectations.

This application serves as a direct illustration of the final case depicted in the simulations, where the combination of a larger number of instruments for different shocks allows for the correction of contamination present in individual shocks. By jointly identifying multiple shocks, we achieve more accurate and consistent results that are in line with economic theory.

Figure 3.2: Procrustean Identification of Multiple Shocks



The previous two applications provide compelling evidence for the effectiveness of our identification strategy. Our approach leverages a substantial number of instruments and expands the pool of usable instruments by accommodating the inclusion of endogenous instruments to identify shocks. Overall, these applications demonstrate the practical value of our identification approach. It empowers researchers and policymakers to effectively analyze complex systems and identify more accurately their underlying economic dynamics.

3.4 Conclusions

In this paper, we have presented a comprehensive identification strategy for structural VARs that effectively addresses the challenges associated with plausibly exogenous instruments. Inspired by the Procrustes orthogonal problem, our method incorporates a rich set of instruments to identify shocks and captures their relationship with dependent variables. Through a range of applications and simulations, we have demonstrated the efficacy of our strategy in accurately identifying and analyzing various shocks, including monetary policy, aggregate demand, and aggregate supply.

Our findings not only align with economic theory but also provide valuable insights into the contamination of instruments. By incorporating a large number of instruments and allowing for the inclusion of plausibly exogenous variables, our approach offers a robust and efficient tool for SVAR identification. The results of our simulations highlight the benefits of our identification strategy, enhancing our understanding the method's benefits.

We are optimistic that our approach will find practical policy applications. As we believe that our method holds significant advantages. Its simplicity, efficiency, and accuracy make it a valuable tool for analyzing the impact of different shocks in various economic settings. Additionally, our method addresses challenges associated with instrument endogeneity, expanding the set of usable instruments in a Proxy-VAR. We encourage researchers to explore and further develop our identification strategy, as it has the potential to advance the field and provide deeper insights into the causal relationships within economic systems.

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Appendix A

A.1 Estimating Interest Rates

The FDIC provides information about the interest income but does not report it per loan type. Nonetheless, it is still possible to obtain estimates of the interest rates. In what follows, I describe how the reader can get these estimates.

Denote $r_{j,t,\tau}$ and $S_{j,t,\tau}$ to be respectively the interest rate set by bank j in period t for loan type τ and the proportion of type τ loans held by bank j in period t. From the FDIC data, it is possible to compute $r_{j,t}$, which is the interest income over the total loan value. Notice that the interest income corresponds to the sum of interest incomes over loan types. Additionally, the interest income is the product of the interest rate and the total loan value. Thus, I can write the expression

$$L_{j,t}r_{j,t} = \sum_{\tau} L_{j,t,\tau}r_{j,t,\tau}.$$

Dividing by $L_{j,t}$ on both sides allows to write the following

$$r_{j,t} = \sum_{\tau} S_{j,t,\tau}r_{j,t,\tau},$$

where the interest rate $r_{j,t}$ is expressed as a weighted sum of interest rates across loan types.

Next, I assume a functional form for $r_{j,t,\tau}$ such that

$$r_{j,t,\tau} = \alpha_{\tau,c} + K_{j,t}\beta_{\tau,c} + \epsilon_{j,t},$$

where α_c is a county fixed effect, and $K_{j,t}$ gathers a set of explanatory variables.[1] Moreover, I allow the coefficients in $\beta_{\tau,c}$ to depend on the loan type and the county

[1] $K_{j,t}$ is a vector containing the Herfindahl index at the county level and the federal fund rate. Additionally, it has supply sifters to capture bank decision-making. These variables include bank salaries over the number of employees, additional non-interest expenses over total assets, premises and equipment expense over total assets, and provisions for credit losses over total assets.

to obtain a better fit. Finally, $\epsilon_{j,t}$ is an error component common to all loan types.

It follows that

$$r_{j,t} = \epsilon_{i,t} + \sum_{\tau} S_{j,t,\tau}\alpha_{\tau,c} + S_{j,t,\tau}K_{j,t}\beta_{\tau,c},$$

where this expression can be easily estimated with constrained OLS

$$\begin{aligned}
\hat{\beta}_{\tau,c} =& argmin_{\{\beta_{\tau,c}\}} \sum_{j,t}(r_{j,t} - \sum_{\tau} S_{j,t,\tau}\hat{r}_{j,t,\tau})^2 \\
& s.t \\
\hat{r}_{j,t,\tau} =& \alpha_{\tau,c} + K_{j,t}\beta_{\tau,c} \\
\hat{r}_{j,t,\tau} \geq& f_t \\
\hat{r}_{j,t,\tau} \leq& 0.8,
\end{aligned}$$

f_t is the federal fund rate value in period t.[2]

[2] I impose some conditions on my estimator. In particular, I set interest rates to be larger than the federal fund rate and lower than 80% annual interest rate. Finally, the regression obtains an R squared of 55%.

A.2 Figures

Figure A.1: Identification Strategy

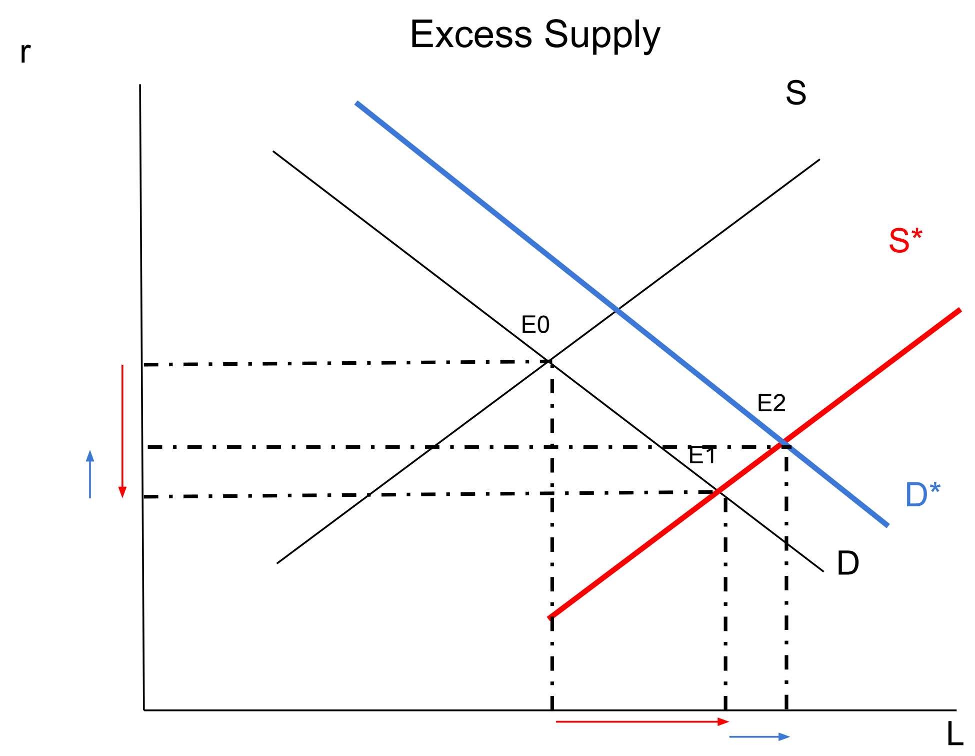
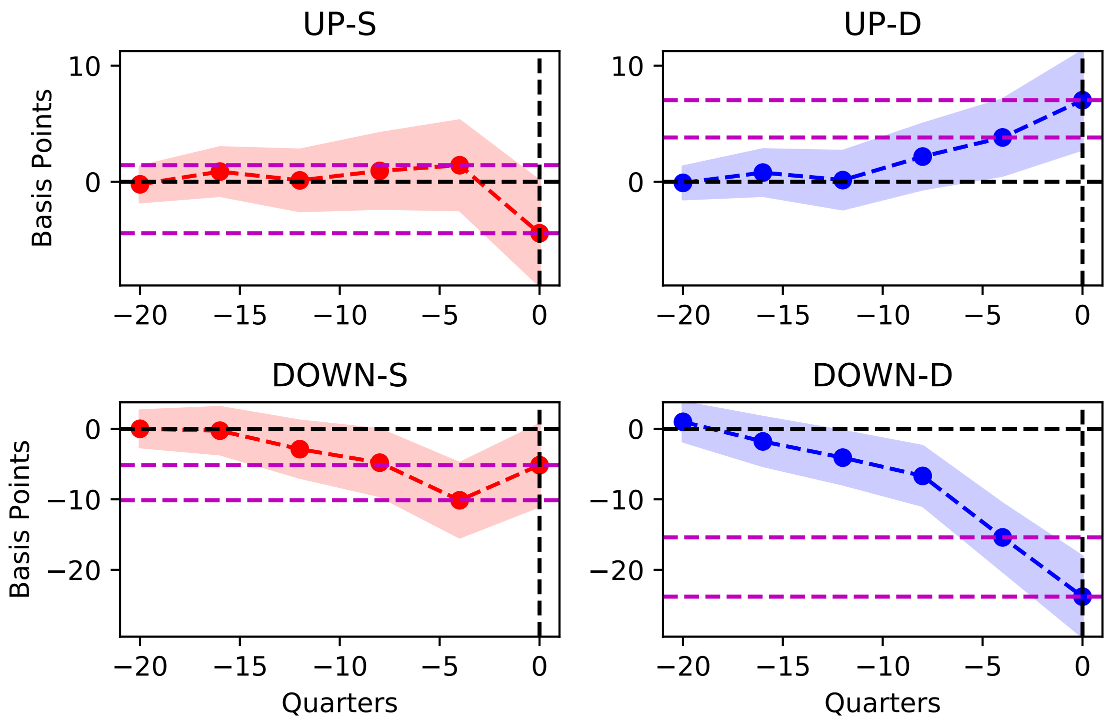
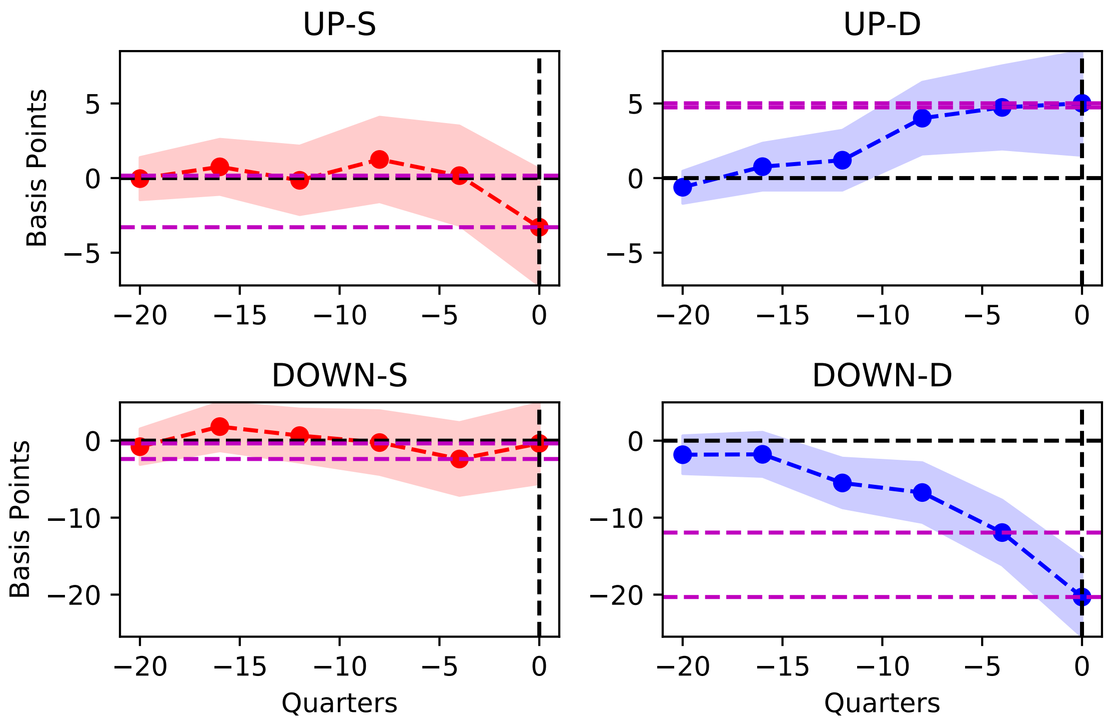


Figure A.2: Decomposition of Interest Rates' Relationship to Commercial and Industrial Loan Delinquency Rates in Supply and Demand Factors



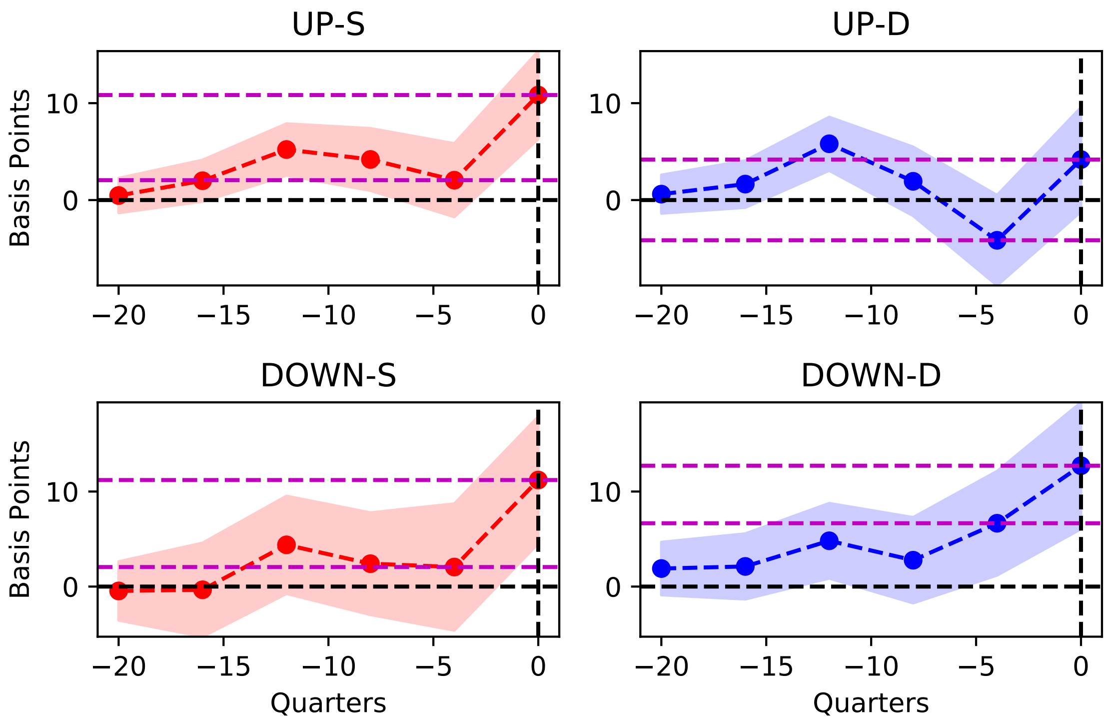
The panels in the figure show the sum of coefficients $\sum_{k=\bar{k}}^{k_i} \gamma_{-k}^{c}$ and the associated 95% confidence interval. The x-axis displays the quarters, where quarter 0 corresponds to the current quarter. The y-axis describes basis point changes in loan delinquency rates. The cumulative sum is scaled to the regressor's variance for interpretability purposes. The relationship is color-coded in blue/red to represent the decomposition of loan delinquency rates in demand/supply factors. The sample period spans from 1994 to 2021.

Figure A.3: Decomposition of Interest Rates' Relationship to Non Residential Real Estate Loan Delinquency Rates in Supply and Demand Factors



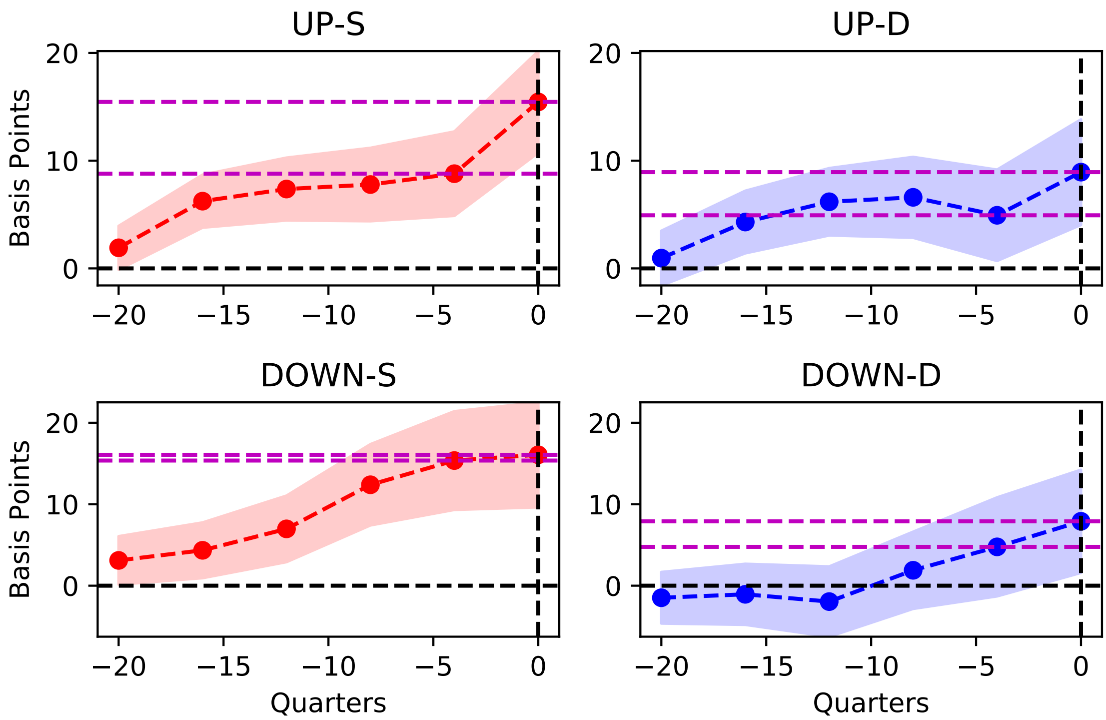
The panels in the figure show the sum of coefficients $\sum_{k=\bar{k}}^{k_i} \gamma_{-k}^{c}$ and the associated 95% confidence interval. The x-axis displays the quarters, where quarter 0 corresponds to the current quarter. The y-axis describes basis point changes in loan delinquency rates. The cumulative sum is scaled to the regressor's variance for interpretability purposes. The relationship is color-coded in blue/red to represent the decomposition of loan delinquency rates in demand/supply factors. The sample period spans from 1994 to 2021.

Figure A.4: Decomposition of Corporate Taxes' Relationship to Commercial and Industrial Loan Delinquency Rates in Supply and Demand Factors



The panels in the figure show the sum of coefficients $\sum_{k=\bar{k}}^{k_i} \gamma_{-k}^{c}$ and the associated 95% confidence interval. The x-axis displays the quarters, where quarter 0 corresponds to the current quarter. The y-axis describes basis point changes in loan delinquency rates. The cumulative sum is scaled to the regressor's variance for interpretability purposes. The relationship is color-coded in blue/red to represent the decomposition of loan delinquency rates in demand/supply factors. The sample period spans from 1994 to 2021.

Figure A.5: Decomposition of Corporate Taxes' Relationship to Non Residential Real Estate Loan Delinquency Rates in Supply and Demand Factors



The panels in the figure show the sum of coefficients $\sum_{k=\bar{k}}^{k_i} \gamma_{-k}^{c}$ and the associated 95% confidence interval. The x-axis displays the quarters, where quarter 0 corresponds to the current quarter. The y-axis describes basis point changes in loan delinquency rates. The cumulative sum is scaled to the regressor's variance for interpretability purposes. The relationship is color-coded in blue/red to represent the decomposition of loan delinquency rates in demand/supply factors. The sample period spans from 1994 to 2021.

Appendix B

B.1 Estimating the payment schedule (μ)

The estimation of the payment schedule (μ) is performed outside the main structural model. It relies on a constrained least squared estimation. To do so, assume the following law of motion for the total value of the loan stock $L_{j,t,\tau}$ of a bank j in period t for loan type τ,

$$L_{j,t,\tau} = L_{j,t-1,\tau}(1 - \delta_{j,t-1,\tau})(1 - \mu_{j,\tau}) + l_{j,t,\tau},$$

where $l_{j,t,\tau}$ is the value of newly issued loan. I propose the following regression

$$L_{j,t,\tau} = L_{j,t-1,\tau}(1 - \delta_{j,t-1,\tau})\beta_{j,\tau} + c^{(0)}_{j,\tau} + c^{(1)}_{j,\tau}t + \epsilon_{j,t,\tau},$$

where I assume $\beta_{j,\tau} \equiv 1 - \mu_{j,\tau}$ and $l_{j,t,\tau} \equiv c^{(0)}_{j,\tau} + c^{(1)}_{j,\tau}t + \epsilon_{j,t,\tau}$. Therefore, this means that $1 - \mu_{j,\tau}$ is the main parameter of our regression and $l_{j,t,\tau}$ depends on a constant term, a trend component and an error. Additionally, I want my regression to yield positives levels of loan issuance (i.e., $l_{j,t,\tau} \geq 0$). Therefore, I solve the following minimization problem to find my estimator

$$\begin{aligned}
\hat{\beta}_{\tau,c} =& argmin_{\{\beta_{\tau,c}\}} \sum_t \Big(L_{j,t,\tau} - L_{j,t-1,\tau}(1 - \delta_{j,t-1,\tau})\beta_{j,\tau} - c^{(0)}_{j,\tau} - c^{(1)}_{j,\tau}t\Big)^2 \\
s.t& \\
L_{j,t,\tau} \geq& L_{j,t-1,\tau}(1 - \delta_{j,t-1,\tau})\beta_{j,\tau}
\end{aligned}$$

B.2 Propositions

Proposition 1. The following relations hold:

- An increase in productivity $\eta_{j,t,\tau}$ reduces the delinquency rate $\delta_{j,t,\tau}$

- An increase in the interest rate $r_{j,t,\tau}$ increases the delinquency rate $\delta_{j,t,\tau}$
- An increase in the corporate tax rate T_t increases the delinquency rate $\delta_{j,t,\tau}$

Proof. The value of borrowing from bank j, in period t, for project τ is

$$V_{j,t,\tau} = \max_{\delta_{j,t,\tau}} \quad (1-\delta_{j,t,\tau})\left(\left(\eta_{j,t,\tau}\alpha(\delta_{j,t,\tau})\frac{1+r_{j,t,\tau}}{r_{j,t,\tau}} - (1+r_{j,t,\tau})(1-\mu_{j,\tau})^k\right)(1-T_t) - I_\tau\frac{1+r_{j,t,\tau}}{r_{j,t,\tau}}\right)$$

The First Order Condition yields the following equation

$$\alpha(\delta^*_{j,t,\tau}) - \alpha'(\delta^*_{j,t,\tau})(1-\delta^*_{j,t,\tau}) = \frac{(1-\mu_{j,\tau})^k r_{j,t,\tau} + \frac{I_\tau}{1-T_t}}{\eta_{j,t,\tau}},$$

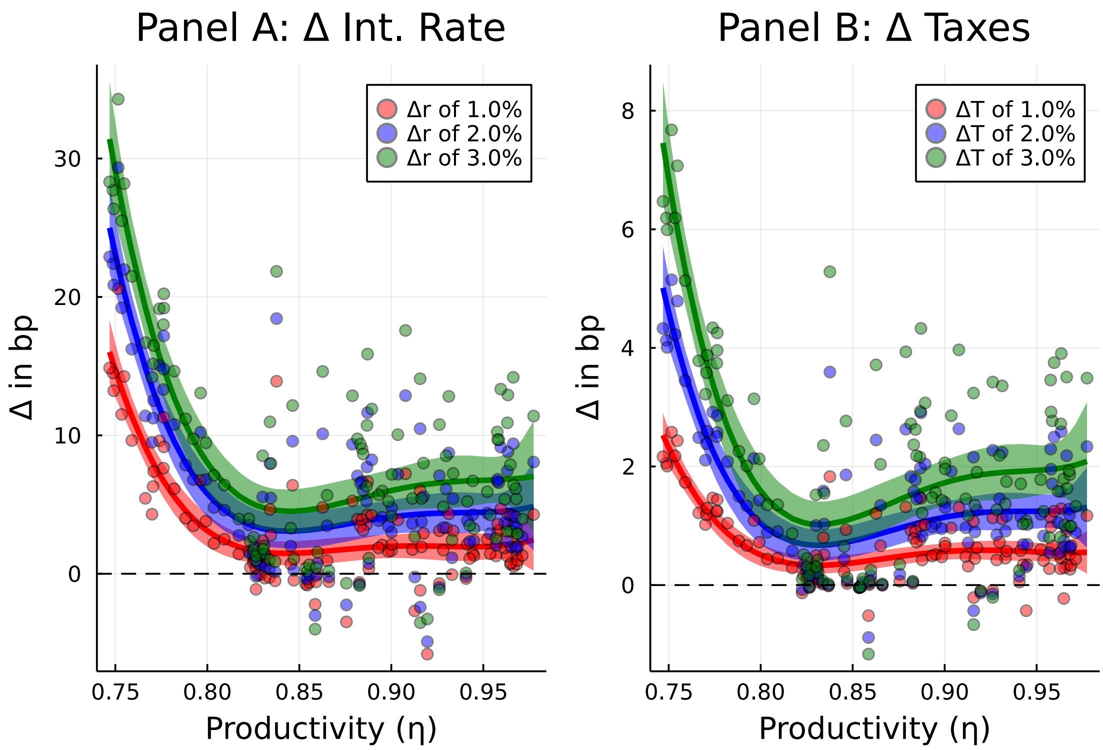
where $\delta^*_{j,t,\tau}$ is the equation's root.

Let $G(\delta^*_{j,t,\tau}) \equiv \alpha(\delta^*_{j,t,\tau}) - \alpha'(\delta^*_{j,t,\tau})(1-\delta^*_{j,t,\tau})$. It is straightforward to prove that $G(.)$ is a monotonically increasing function from which follows all the results in the proposition.

- An increase in $\eta_{j,t,\tau}$ reduces $G(\delta^*_{j,t,\tau})$ yielding that $\delta^*_{j,t,\tau}$ must decrease to satisfy the First Order Condition.
- An increase in $r_{j,t,\tau}$ increases $G(\delta^*_{j,t,\tau})$ yielding that $\delta^*_{j,t,\tau}$ must increase to satisfy the First Order Condition.
- An increase in T_t increases $G(\delta^*_{j,t,\tau})$ yielding that $\delta^*_{j,t,\tau}$ must increase to satisfy the First Order Condition.

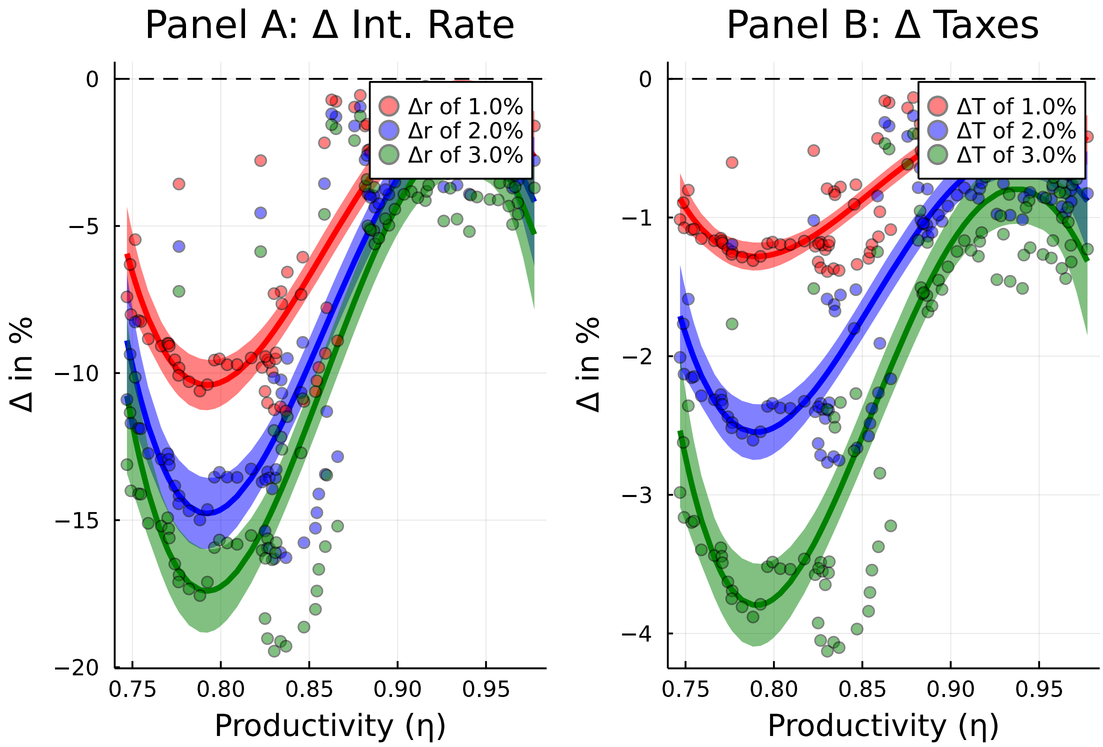
B.3 Figures and Tables

Figure B.1: Effect of Change in Interest Rate and Taxes on the Delinquency Rate



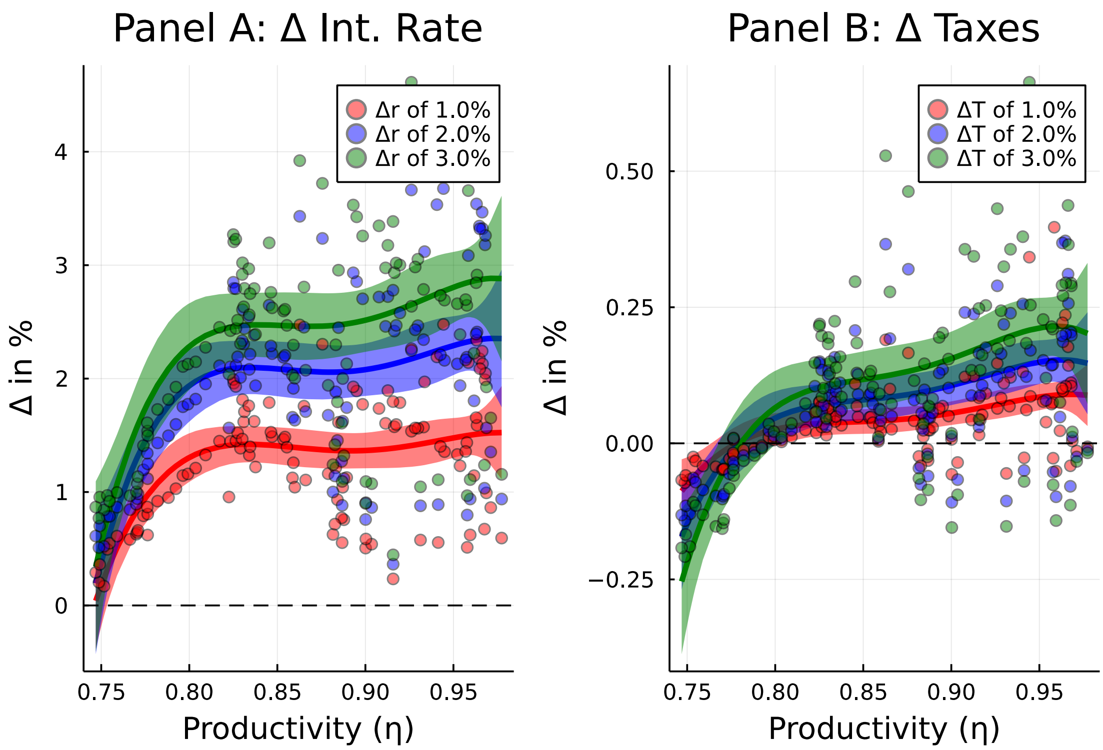
The panels in the figure represent two panels, one for changes in interest rates (panel A) and one for changes in corporate tax rates (panel B). On the x-axis, I represent the average productivity value within a period and plot it against the variable of interest. The figures scatter the result for every period and plot a polynomial regression with 95% confidence intervals.

Figure B.2: Effect of Change in Interest Rate and Taxes on the Credit Market Participation



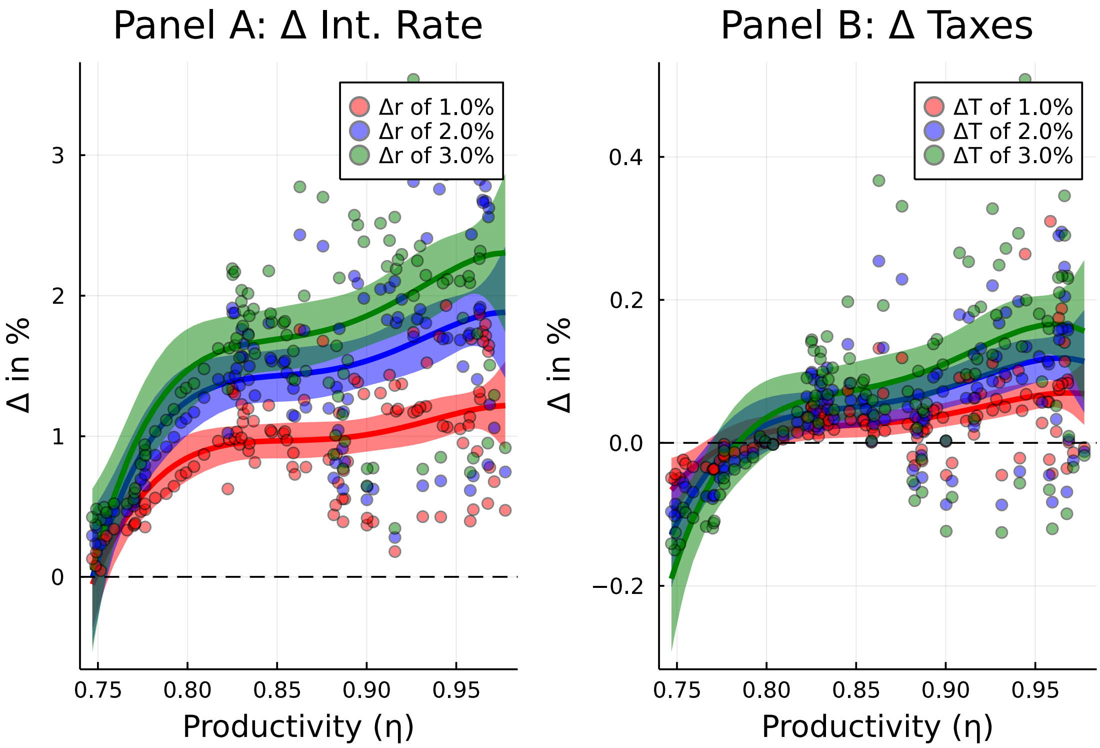
The panels in the figure represent two panels, one for changes in interest rates (panel A) and one for changes in corporate tax rates (panel B). On the x-axis, I represent the average productivity value within a period and plot it against the variable of interest. The figures scatter the result for every period and plot a polynomial regression with 95% confidence intervals.

Figure B.3: Effect of Change in Interest Rate and Taxes on Productivity



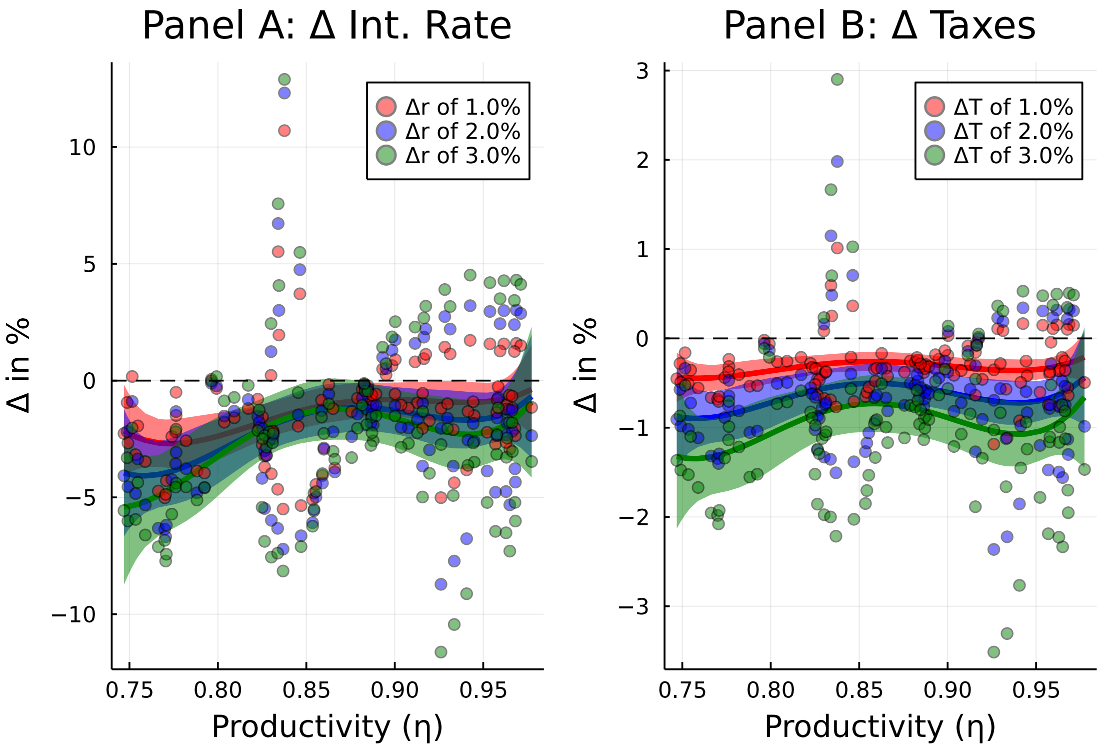
The panels in the figure represent two panels, one for changes in interest rates (panel A) and one for changes in corporate tax rates (panel B). On the x-axis, I represent the average productivity value within a period and plot it against the variable of interest. The figures scatter the result for every period and plot a polynomial regression with 95% confidence intervals.

Figure B.4: Effect of Change in Interest Rate and Taxes on Output per Capita



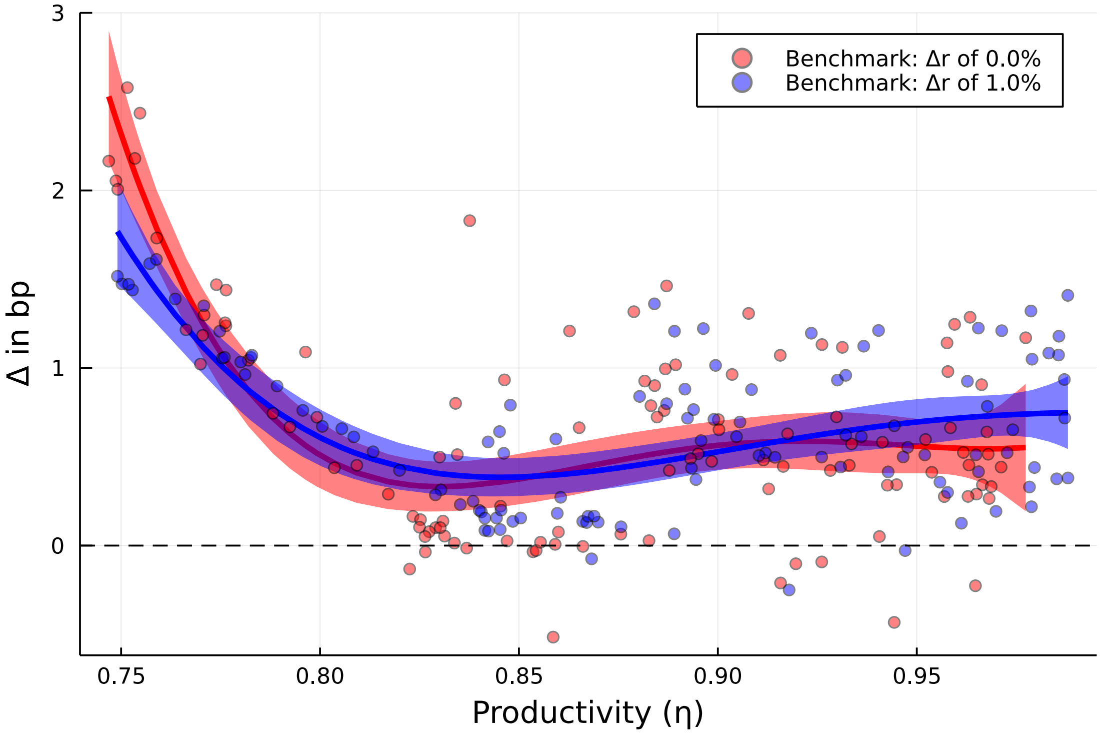
The panels in the figure represent two panels, one for changes in interest rates (panel A) and one for changes in corporate tax rates (panel B). On the x-axis, I represent the average productivity value within a period and plot it against the variable of interest. The figures scatter the result for every period and plot a polynomial regression with 95% confidence intervals.

Figure B.5: Effect of Change in Interest Rate and Taxes on Output



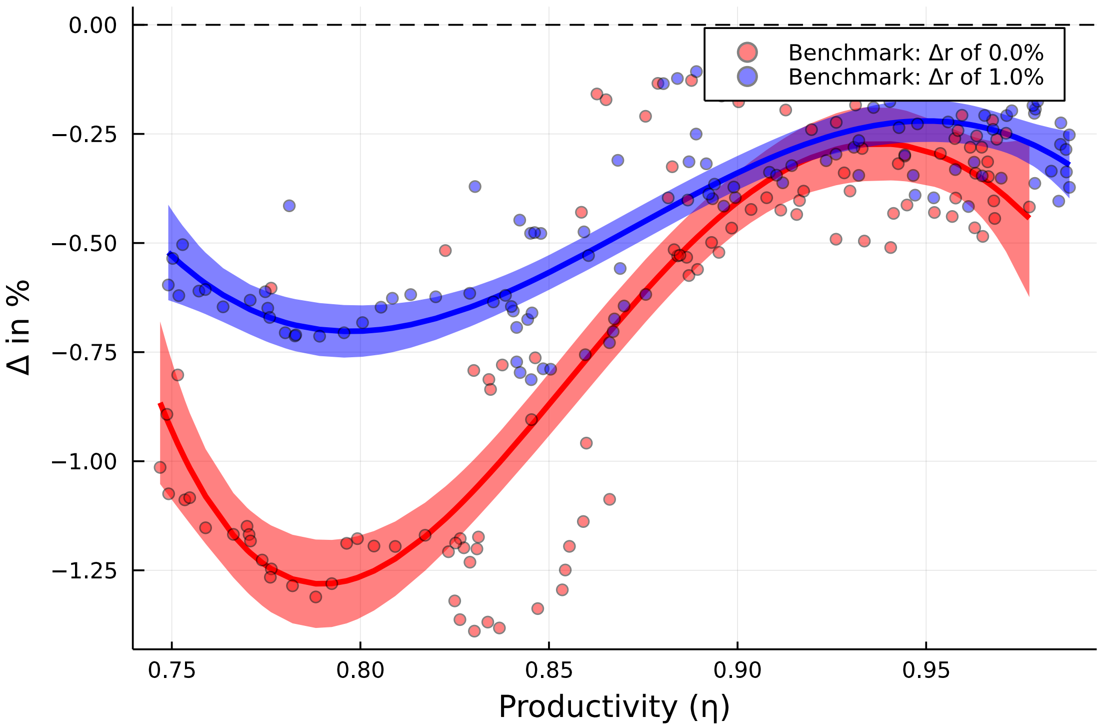
The panels in the figure represent two panels, one for changes in interest rates (panel A) and one for changes in corporate tax rates (panel B). On the x-axis, I represent the average productivity value within a period and plot it against the variable of interest. The figures scatter the result for every period and plot a polynomial regression with 95% confidence intervals.

Figure B.6: Effect of Change in Interest Rate and Taxes on the Delinquency Rate



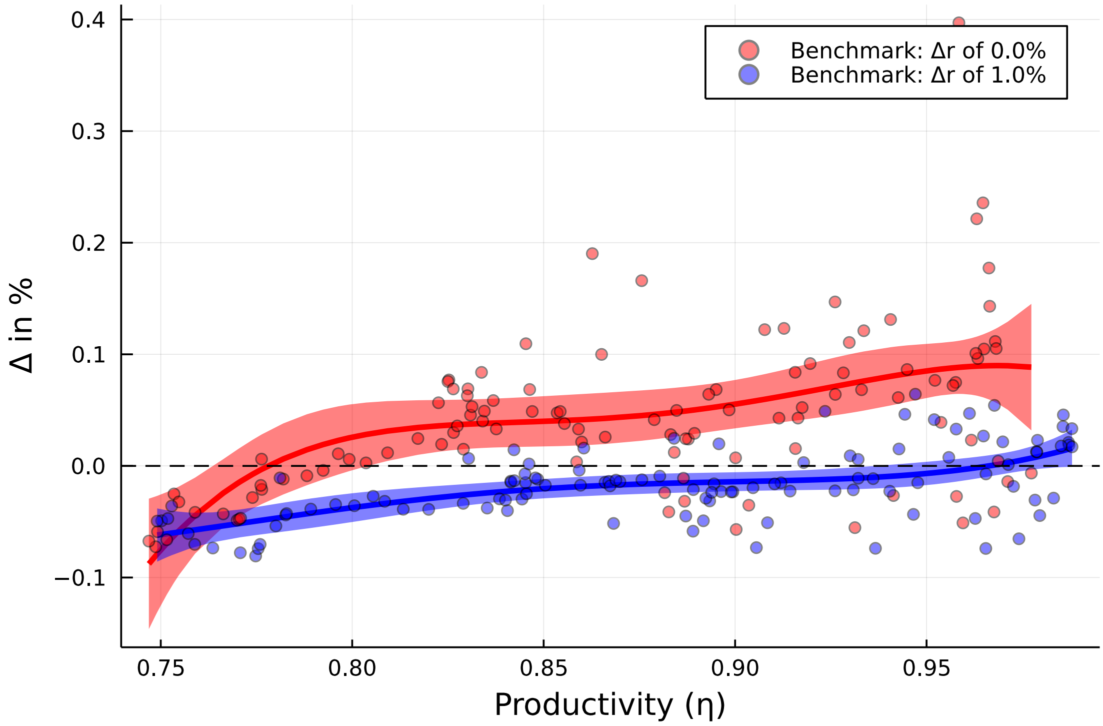
The figure represent two interest rate regimes. The high-interest rate regime defines the benchmark as an economy where interest rates are 100 bp higher (Δr of 1%), whereas the low-interest rate regime leaves the benchmark unaffected (Δr of 0%). On the x-axis, I represent the average productivity value within a period and plot it against the variable of interest. The figures scatter the result for every period and plot a polynomial regression with 95% confidence intervals.

Figure B.7: Effect of Change in Interest Rate and Taxes on the Credit Market Participation



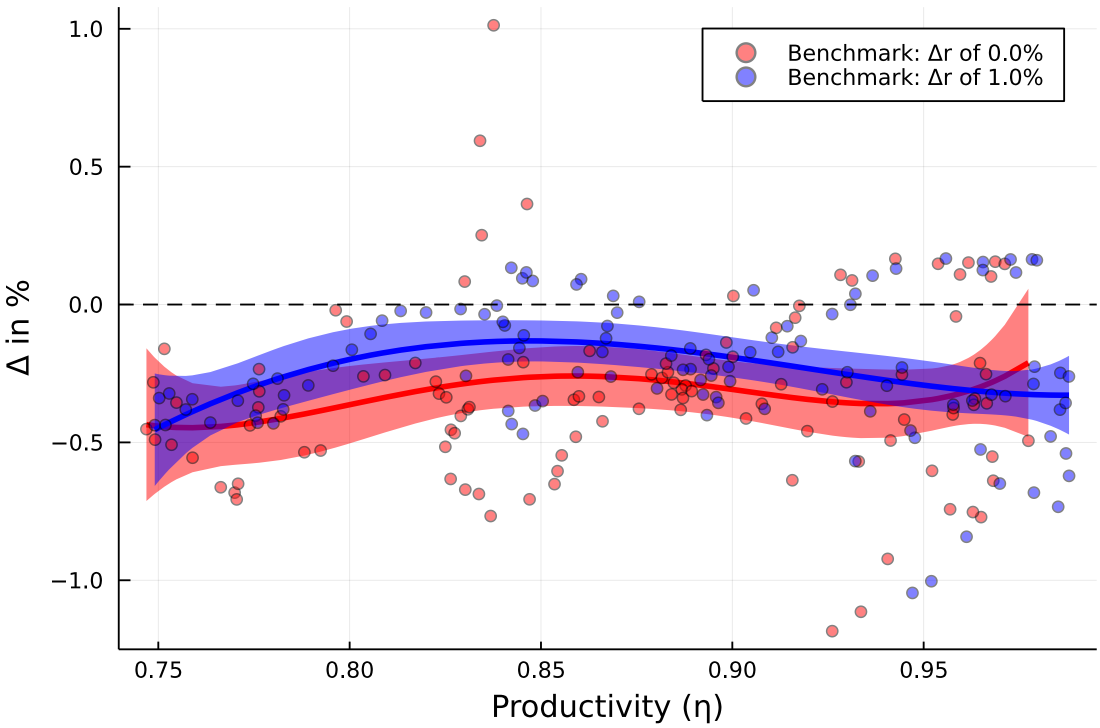
The figure represent two interest rate regimes. The high-interest rate regime defines the benchmark as an economy where interest rates are 100 bp higher (Δr of 1%), whereas the low-interest rate regime leaves the benchmark unaffected (Δr of 0%). On the x-axis, I represent the average productivity value within a period and plot it against the variable of interest. The figures scatter the result for every period and plot a polynomial regression with 95% confidence intervals.

Figure B.8: Effect of Change in Interest Rate and Taxes on Productivity



The figure represent two interest rate regimes. The high-interest rate regime defines the benchmark as an economy where interest rates are 100 bp higher (Δr of 1%), whereas the low-interest rate regime leaves the benchmark unaffected (Δr of 0%). On the x-axis, I represent the average productivity value within a period and plot it against the variable of interest. The figures scatter the result for every period and plot a polynomial regression with 95% confidence intervals.

Figure B.9: Effect of Change in Interest Rate and Taxes on Output



The figure represent two interest rate regimes. The high-interest rate regime defines the benchmark as an economy where interest rates are 100 bp higher (Δr of 1%), whereas the low-interest rate regime leaves the benchmark unaffected (Δr of 0%). On the x-axis, I represent the average productivity value within a period and plot it against the variable of interest. The figures scatter the result for every period and plot a polynomial regression with 95% confidence intervals.

Table B.1: Estimates

Symbol	*Description*	*Value*
	Panel A: External Parameters	
$\mu_{C\&I}$	*Payment Rate $C\&I$*	*0.3174*
		(0.1947)
μ_{NotRes}	*Payment Rate $NotRes$*	*0.1851*
		(0.16494)
μ_{J+1}	*Payment Rate Bond Market*	*0.0778*
μ_0	*Commitment to No Participation*	*0.0746*
	Panel B: Internal Parameters	
$x_{j,t}[1]$	*log(Number of Offices)*	*0.5646*
$x_{j,t}[2]$	*log(Number of Employees)*	*0.6272*
$I_{C\&I}$	*Utility Cost $C\&I$*	*0.1544*
I_{NotRes}	*Utility Cost $NotRes$*	*0.1118*
$\gamma_{C\&I}$	*Technology's Curvature $C\&I$*	*0.0986*
γ_{NotRes}	*Technology's Curvature $NotRes$*	*0.1204*
$a_{C\&I}$	*Risk Free Return $C\&I$*	*0.1712*
a_{NotRes}	*Risk Free Return $NotRes$*	*0.1902*
ϕ	*Bond Market Return*	*0.2075*
I_b	*Utility Cost Bond Market*	*0.1265*
κ	*Discounted Value of No Particpation*	*1.9457*

Table B.2: Estimates

Symbol	*Moment*	*Deviation (in %)*
	Internal Parameters	
$I_{C\&I}$	m^2 *(Contemp.)*	*49 %*
I_{NotRes}	m^2 *(Contemp.)*	*91 %*
$\gamma_{C\&I}$	m^4 *(Del. Rate)*	*17 %*
γ_{NotRes}	m^4 *(Del. Rate)*	*4 %*
$a_{C\&I}$	m^2 *(Contemp.)*	*607 %*
a_{NotRes}	m^2 *(Contemp.)*	*676 %*
ϕ	m^4 *(Agg. Prop.)*	*-6 %*
I_b	m^4 *(Agg. Prop.)*	-
κ	m^4 *(Agg. Prop.)*	*17 %*

Appendix C

C.1 Estimation

In this subsection, we describe how to estimate the model in 3.1 with classical methods, while imposing the lower left block of $\tilde{A}_i$ to be null. A simple way of imposing these constraints is through linear restrictions on the VAR's SUR representation, as explained in this subsection.

For convenience, rewrite 3.1 in compact form,

$$\tilde{y}_t^T = \tilde{x}_t^T \tilde{A}_+ + \tilde{\epsilon}_t^T \tilde{S},$$

where $\tilde{x}_t^T = [\tilde{y}_{t-1}^T, \ldots, \tilde{y}_{t-i}^T]$ and $\tilde{A}_+ = [\tilde{A}_1^T, \ldots, \tilde{A}_p^T]^T$.
Stacking observations yields

$$\tilde{Y} = \tilde{X}\tilde{A}_+ + \tilde{E}\tilde{S},$$

where $\tilde{Y} = [\tilde{y}_1, \ldots, \tilde{y}_T]^T$, $\tilde{X} = [\tilde{x}_1, \ldots, \tilde{x}_T]^T$, and $\tilde{E} = [\tilde{\epsilon}_1, \ldots, \tilde{\epsilon}_T]^T$.
Furthermore, the SUR representation writes

$$\underbrace{vec(\tilde{Y})}_{\tilde{y}} = (I_{\tilde{n}} \otimes \tilde{X}) \underbrace{vec(\tilde{A}_+)}_{\tilde{a}_+} + (I_{\tilde{n}} \otimes \tilde{E}) \underbrace{vec(\tilde{S})}_{\tilde{s}}. \tag{C.1}$$

Using C.1, it is now straightforward to impose linear restrictions on $\tilde{a}_+$, such that $R\tilde{a}_+ = d$, where R is an $n_r \times \tilde{n}^2 p$ matrix of linear restrictions and d is an $n_r \times 1$ vector. The least square estimator boils down to the simple following expression

$$\begin{bmatrix} \tilde{a}_+ \\ \lambda \end{bmatrix} = \begin{bmatrix} (I_{\tilde{n}} \otimes \tilde{X})^T (I_{\tilde{n}} \otimes \tilde{X}) & R^T \\ R & 0_{n_r \times n_r} \end{bmatrix}^{-1} \begin{bmatrix} (I_{\tilde{n}} \otimes \tilde{X})^T \tilde{y} \\ d \end{bmatrix},$$

where λ is a vector of Lagrange multipliers for the equality conditions.

C.2 Procrustes Problem

We add the proof for completeness. Please refer to .. for additional details.

$$min_{\{Q_\epsilon\}} ||Q_\epsilon h^*_{\epsilon,m} - \bar{\kappa}_\epsilon||_{\mathbf{F}}$$
$$s.t.$$
$$Q_\epsilon^T Q_\epsilon = I_n$$

where $||A||_{\mathbf{F}}$ corresponds to the Frobenius norm of A and is equivalent to $Tr(A^T A)$. Hence,

$$||Q_\epsilon h^*_{\epsilon,m} - \bar{\kappa}_\epsilon||_{\mathbf{F}} = Tr((Q_\epsilon h^*_{\epsilon,m} - \bar{\kappa}_\epsilon)^T (Q_\epsilon h^*_{\epsilon,m} - \bar{\kappa}_\epsilon))$$

$$= Tr(h^{*^T}_{\epsilon,m} Q_\epsilon^T Q_\epsilon h^*_{\epsilon,m} - h^{*^T}_{\epsilon,m} Q_\epsilon^T \bar{\kappa}_\epsilon - \bar{\kappa}_\epsilon^T Q_\epsilon h^*_{\epsilon,m} + \bar{\kappa}_\epsilon^T \bar{\kappa}_\epsilon)$$

The problem becomes equivalent to minimizing an auxiliary objective function

$$Tr(Q_\epsilon h^*_{\epsilon,m} \bar{\kappa}_\epsilon^T) = Tr(Q_\epsilon \bar{\kappa}_\epsilon h^{*^T}_{\epsilon,m}).$$

Using the singular value decomposition we have [1]

$$Tr(Q_\epsilon \bar{\kappa}_\epsilon h^{*^T}_{\epsilon,m}) = Tr(Q_\epsilon U \Omega V^T) = Tr(U^T Q_\epsilon V \Omega)$$

Since $U^T Q_\epsilon V$ is orthogonal the solution is reached for $U^T Q_\epsilon V = I$. Therefore, we obtain the following solution to the Procrustes problem

$$Q_\epsilon^* = UV^T.$$

[1]Notice that in the case with sign restrictions we have:
$Tr(Q_\epsilon \bar{S} h^{*^T}_{\epsilon,y}) + Tr(Q_\epsilon \bar{\kappa}_\epsilon h^{*^T}_{\epsilon,m}) = Tr(Q_\epsilon(\bar{S} h^{*^T}_{\epsilon,y} + \bar{\kappa}_\epsilon h^{*^T}_{\epsilon,m}))$.

C.3 Procrustes Implied Instrument Shock Relationship

Assume we have at most one instrument per shock and $k \leq n$.
What is $\bar{\kappa}_\epsilon h^{*^T}_{m,\epsilon}$?

$$\bar{\kappa}_\epsilon h^{*^T}_{m,\epsilon} = \begin{bmatrix} I_k \\ 0_{n-k,k} \end{bmatrix} h^{*^T}_{m,\epsilon} = \begin{bmatrix} h^{*^T}_{m,\epsilon} \\ 0_{n-k,n} \end{bmatrix}$$

The singular value decomposition yields:

$$\begin{bmatrix} h^{*^T}_{m,\epsilon} \\ 0_{n-k,k} \end{bmatrix} = U\Omega V^T$$

Lets partition our matrices

$$U\Omega V^T = \begin{bmatrix} U_L & U_R \end{bmatrix} \begin{bmatrix} \Omega_{TL} & 0_{k,n-k} \\ 0_{n-k,k} & 0_{n-k,n-k} \end{bmatrix} \begin{bmatrix} V_L^T \\ V_R^T \end{bmatrix}$$

So we have

$$\bar{\kappa}_\epsilon h^{*^T}_{m,\epsilon} = \begin{bmatrix} h^{*^T}_{m,\epsilon} \\ 0_{n-k,n} \end{bmatrix} = U_L \Omega_{TL} V_L^T$$

What is $Q^*_\epsilon h^*_{m,\epsilon}$?
Notice that

$$\bar{\kappa}_\epsilon^T \bar{\kappa}_\epsilon = I_k$$

Therefore,

$$\begin{aligned}
Q^*_\epsilon h^*_{m,\epsilon} =& U_L V_L^T h^*_{m,\epsilon} = U_L \Omega_{TL}^{-1} U_L^T U_L \Omega_{TL} V_L^T h^*_{m,\epsilon} = U_L \Omega_{TL}^{-1} U_L^T \bar{\kappa}_\epsilon h^{*^T}_{m,\epsilon} h^*_{m,\epsilon} \\
Q^*_\epsilon h^*_{m,\epsilon} =& U_L \Omega_{TL}^{-1} U_L^T \bar{\kappa}_\epsilon h^{*^T}_{m,\epsilon} h^*_{m,\epsilon} \bar{\kappa}_\epsilon^T \bar{\kappa}_\epsilon \\
Q^*_\epsilon h^*_{m,\epsilon} =& U_L \Omega_{TL}^{-1} U_L^T U_L \Omega_{TL}^2 U_L^T \bar{\kappa}_\epsilon \\
Q^*_\epsilon h^*_{m,\epsilon} =& U_L \Omega_{TL} U_L^T \bar{\kappa}_\epsilon
\end{aligned}$$

What is $U_L \Omega_{TL} U_L^T$?
Notice that

$$U_L \Omega_{TL}^2 U_L^T = \begin{bmatrix} h_{m,\epsilon}^{*T} \\ 0_{n-k,n} \end{bmatrix} \begin{bmatrix} h_{m,\epsilon}^{*} & 0_{n,n-k} \end{bmatrix} = \begin{bmatrix} h_{m,\epsilon}^{*T} h_{m,\epsilon}^{*} & 0_{k,n-k} \\ 0_{n-k,k} & 0_{n-k,n-k} \end{bmatrix}$$

$$U_L \Omega_{TL} U_L^T = \begin{bmatrix} \left(h_{m,\epsilon}^{*T} h_{m,\epsilon}^{*} \right)^{\frac{1}{2}} & 0_{k,n-k} \\ 0_{n-k,k} & 0_{n-k,n-k} \end{bmatrix}$$

Therefore,

$$Q_\epsilon^* h_{m,\epsilon}^* = \begin{bmatrix} \left(h_{m,\epsilon}^{*T} h_{m,\epsilon}^{*} \right)^{\frac{1}{2}} & 0_{k,n-k} \\ 0_{n-k,k} & 0_{n-k,n-k} \end{bmatrix} \bar{\kappa}_\epsilon = \begin{bmatrix} \left(h_{m,\epsilon}^{*T} h_{m,\epsilon}^{*} \right)^{\frac{1}{2}} & 0_{k,n-k} \\ 0_{n-k,k} & 0_{n-k,n-k} \end{bmatrix} \begin{bmatrix} I_k \\ 0_{n-k,k} \end{bmatrix}$$

Yielding

$$Q_\epsilon^* h_{m,\epsilon}^* = \begin{bmatrix} \left(h_{m,\epsilon}^{*T} h_{m,\epsilon}^{*} \right)^{\frac{1}{2}} \\ 0_{n-k,k} \end{bmatrix}$$

We get $h_{m,\epsilon}^{*T} h_{m,\epsilon}^{*}$ diagonal only if the eigen values are all equal we have $h_{m,\epsilon}^{*T} h_{m,\epsilon}^{*} = I_k$. Therefore, the normalization of $h_{m,\epsilon}^{*T} h_{m,\epsilon}^{*}$ is critical to find solutions that relate to Mertens and Ravn (2013). However, the case with one instrument clearly boils down to Mertens and Ravn (2013) as

$$Q_\epsilon^* h_{m,\epsilon}^* = \begin{bmatrix} (1)^{\frac{1}{2}} \\ 0_{n-1,1} \end{bmatrix} = \begin{bmatrix} 1 \\ 0_{n-1,1} \end{bmatrix}.$$

C.4 Modified Mertens and Ravn (2013)

We modify Mertens and Ravn (2013) to allow for multiple instruments per shock and present the estimator in this subsection. From the relevence and exogeneity of our instruments we have: $E(m_{j(i),t} \epsilon_{i,t}^T) = \underbrace{\Phi_i}_{|j(i)| \times 1}$ and $E(m_{j(i),t} \epsilon_{-i,t}^T) = \underbrace{0}_{|j(i)| \times N-1}$, where $-i \equiv \{k : k \neq i\}$. We have

$$\begin{bmatrix} u_{i,t}^T & u_{-i,t}^T \end{bmatrix} = \begin{bmatrix} \epsilon_{i,t}^T & \epsilon_{-i,t}^T \end{bmatrix} S.$$

Following Mertens and Ravn (2013), and without loss of generality, we can partition S such that $S = \begin{bmatrix} s_i \\ s_{-i} \end{bmatrix}$, yielding

$$\begin{bmatrix} u_{i,t}^T & u_{-i,t}^T \end{bmatrix} = \begin{bmatrix} \epsilon_{i,t}^T & \epsilon_{-i,t}^T \end{bmatrix} \begin{bmatrix} s_i \\ s_{-i} \end{bmatrix}.$$

After pre-multiplying by $m_{i,t}$ and taking expected values, we get

$$\begin{bmatrix} E(m_{i,t}u_{i,t}^T) & E(m_{i,t}u_{-i,t}^T) \end{bmatrix} = \begin{bmatrix} E(m_{i,t}\epsilon_{i,t}^T) & E(m_{i,t}\epsilon_{-i,t}^T) \end{bmatrix} \begin{bmatrix} s_i \\ s_{-i} \end{bmatrix}$$

$$\begin{bmatrix} \underbrace{\Sigma_{m_i,u_i^T}}_{|j(i)|\times 1} & \underbrace{\Sigma_{m_i,u_{-i}^T}}_{|j(i)|\times N-1} \end{bmatrix} = \begin{bmatrix} \Phi_i & 0 \end{bmatrix} \begin{bmatrix} s_i \\ s_{-i} \end{bmatrix}.$$

Therefore, we wright the following two equations

$$\Sigma_{m_i,u_i^T} = \Phi_i s_{i,i} \tag{C.2}$$

$$\Sigma_{m_i,u_{-i}^T} = \Phi_i s_{i,-i}, \tag{C.3}$$

where without loss of generality $s_i = \begin{bmatrix} s_{i,i} & s_{i,-i} \end{bmatrix}$. Equation (4) and (5) can be transformed as

$$(\Phi_i^T \Phi_i)^{-1}\Phi_i^T = s_{i,i}\Sigma_{m_i,u_i^T}^T(\Sigma_{m_i,u_i^T}\Sigma_{m_i,u_i^T}^T)^{-1}$$

$$(\Phi_i^T \Phi_i)^{-1}\Phi_i^T \Sigma_{m_i,u_{-i}^T} = s_{i,-i}.$$

Combining both equations yields the final identification

$$\frac{s_{i,-i}}{s_{i,i}} = \Sigma_{m_i,u_i^T}^T(\Sigma_{m_i,u_i^T}\Sigma_{m_i,u_i^T}^T)^{-1}\Sigma_{m_i,u_{-i}^T}.$$

C.4.1 Figures

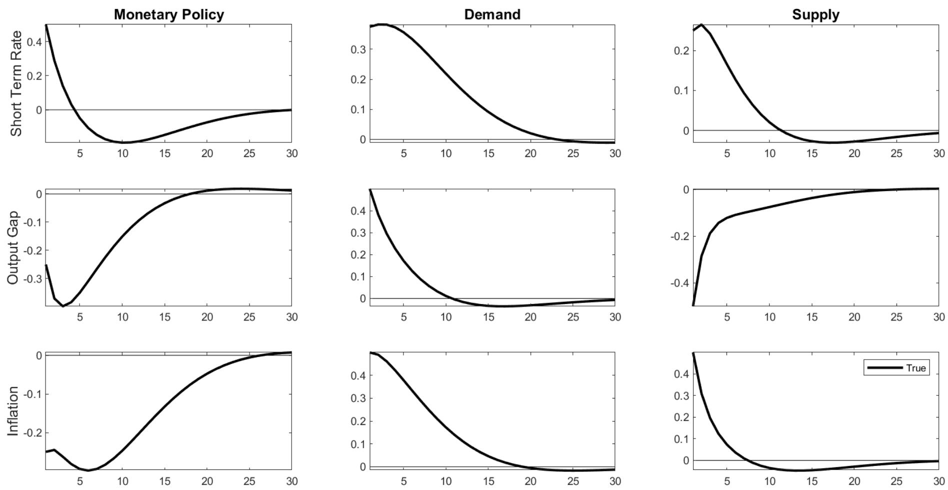
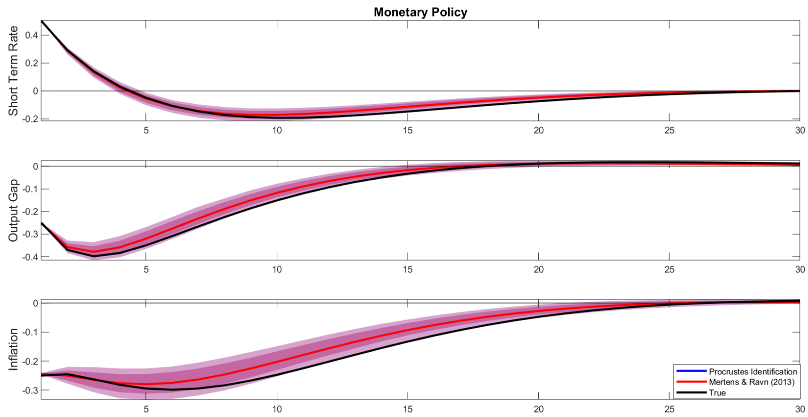
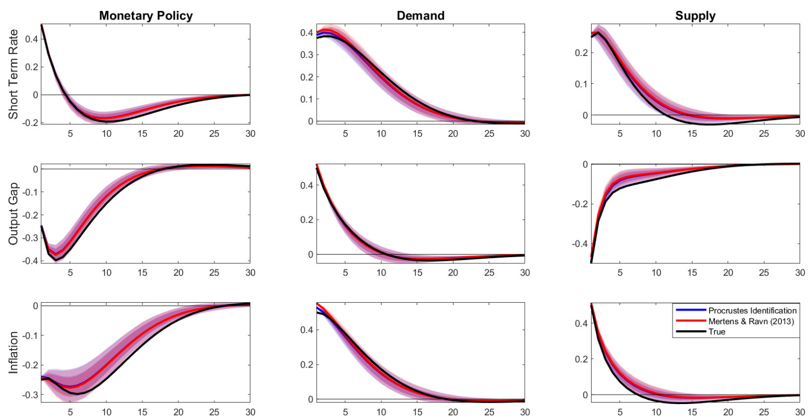


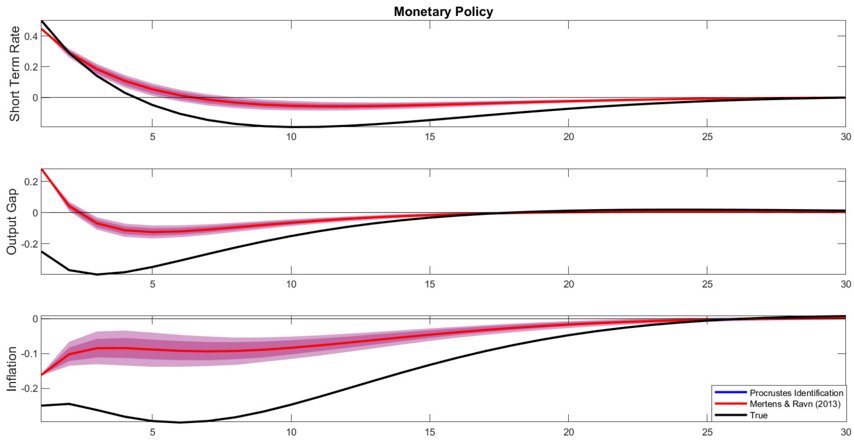
Figure C.1: IRF from Data Generating Process



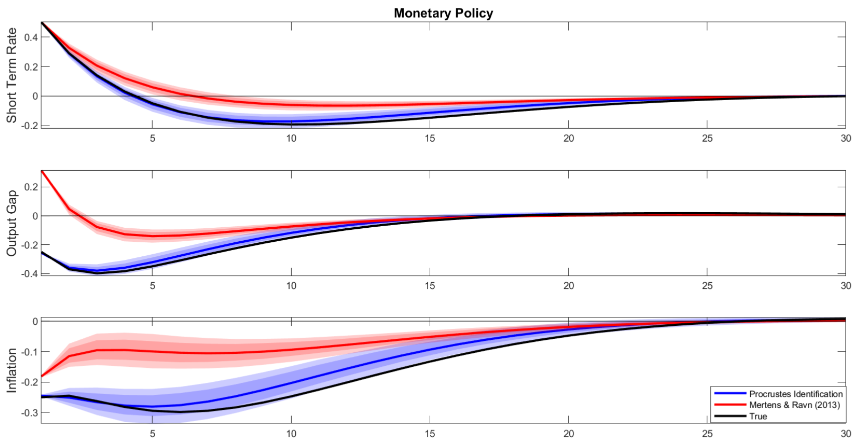
(a) Case I.A.



(b) Case I.B.



(a) Case II.A.



(b) Case II.B.

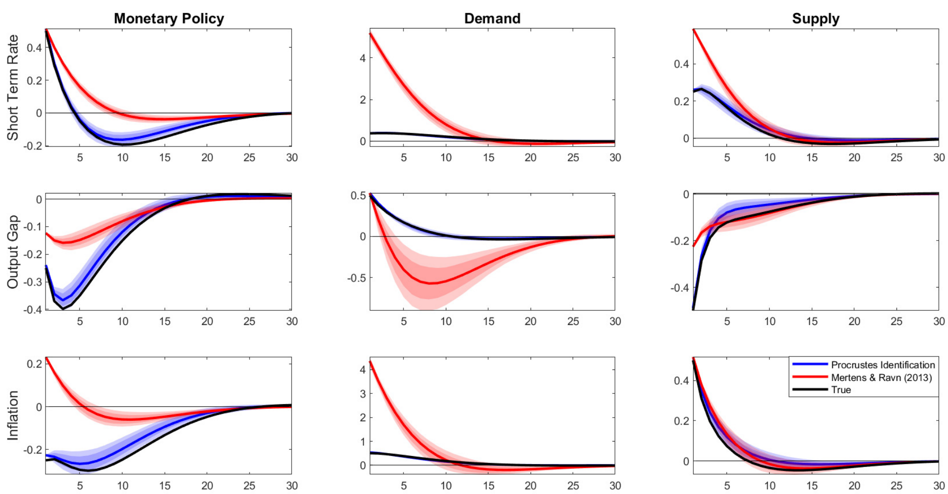


Figure C.4: Case III.

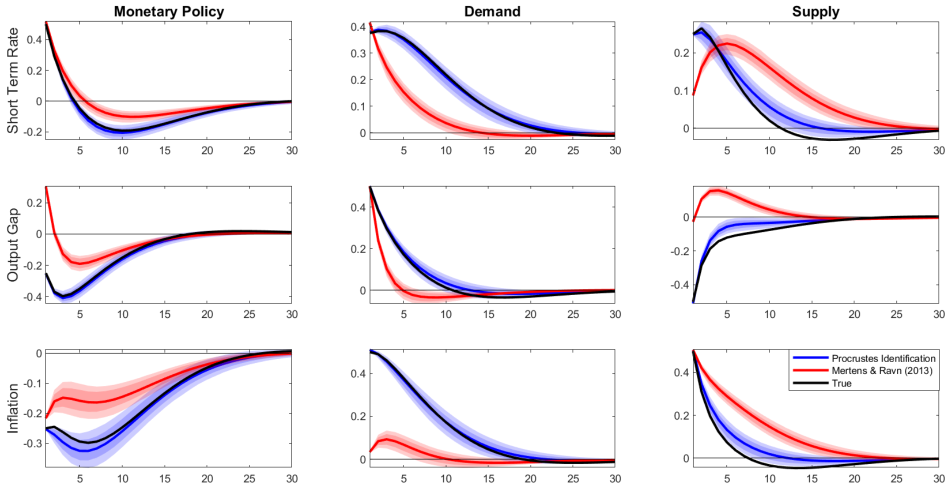


Figure C.5: Case IV.