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Essays in Labor Markets and Expectations

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Essays in Labor Markets and Expectations

Dissertation submitted in fulfillment of the requirements for the degree of

Doctor of Philosophy

by

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International Doctorate in Economic Analysis

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To my grandmother, Marilina, who always believed in me and encouraged me to follow my dreams.

Acknowledgements

Foremost, I would like to express my deepest gratitude to my supervisor, Albert Marcet, whose continuous support, inspirational suggestions, and guidance were invaluable throughout my PhD journey. I would like to thank him for all the work he has done helping to develop my research with his insightful and honest advice and for his belief in my potential. Under his mentorship, I have not only learned the intricacies of research but also gained insights on how to approach complex problems in the simplest way. The topic of this thesis and my enduring interest in it were sparked by his course on Expectations and Optimal Policy, which ignited my curiosity about deviations from Rational Expectations.

I am deeply grateful to Timothy Kehoe for his unwavering support throughout my PhD journey since meeting him during my master's at UAB. I would like to thank him for all the work and suggestions he has done helping to became a good researcher and for believing in my capability. Our numerous discussions on economics have been incredibly enlightening. I am also thankful for his invitation to contribute to an issue of Cuadernos Económicos de I.C.E., and for welcoming me at the University of Minnesota to further my research. Also, special thanks to Juan Pablo Nicolini, who welcomed me at the Federal Reserve Bank of Minneapolis and provided valuable comments and support during the job market.

My appreciation extends to the faculty members of the International Doctorate in Economic Analysis and Universitat Autònoma de Barcelona, from whom I have learned immensely. In particular, I would like to thank Luis Rojas, Jordi Caballé, Ines Macho-Stadler, Javier Blanco, Joan Jull, Hugo Rodríguez, and Francesc Obiols for their insightful comments and suggestions. I am also thankful for the administrative staff of IDEA, especially Angels and Merce, for their kindness and assistance over these years. I am grateful to BSE for the opportunity to teach in the Master's program in Macroeconomic Policy and Financial Markets. Additionally, special thanks to Jaime Alonso, who first believed in me, encouraged me to pursue my PhD, and instilled in me a passion for research.

A heartfelt thanks to my co-author and partner, Clemente, for his constant support and companionship during these years. He has been the cornerstone of my PhD journey, from whom I have learned immensely. His patience, belief in my abilities, and shared joy in our work have been significant sources of encouragement. I am looking forward for our future projects together.

I extend my sincere thanks to all the master's and PhD students with whom I have interacted at IDEA, specially Konstantin, Joan, Las Marias and Mridula. I have been fortunate to be surrounded by exceptional colleagues, some of whom have become friends. Special mention to "Dark" Maria - Barcelona will be so different without our brunch, coffees and gossip. Thanks also to the wonderful group of graduate students at the University of Minnesota, and to Alvaro and Clara, who made my visiting there all the more enjoyable. Thanks also to Lidia Cruces for always being available to help me. My thanks as well to Adriana, Mart, Paloma, and Manuel for creating a safe and empathetic space when I needed it most, and to Anna, also my co-author, for fostering such a supportive environment.

Last, but certainly not least, I am indebted to my family, without whom none of this would have been possible. Special thanks to my parents, grandparents and my aunt Maria for their unwavering support and for providing me with the tools to pursue my goals. To my friends, Tamara, Elena, Belén, Ana and Lola, thank you for being there whenever I needed you.

This research has been greatly beneficial from founding provided by the European Research Council (ERC) under grant Horizon2020 grant agreement no. 788547 (APMPAL-HET) - Asset Prices and Macro Policy When Agents Learn and are Heterogeneous - and 804989 (DYMOLAMO) - Dynamic Modeling of Labor Market Mobility and Human Capital Accumulation.

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Introduction

This thesis is a compendium of three papers about labor markets and expectations. The search and matching model (DMP) has become the standard equilibrium unemployment theory, and hence, an essential tool for evaluating a range of labor market policies, both existing and prospective. Therefore, it is crucial for the selected model to accurately reflect empirical facts.

The first chapter, entitled *"The Role of Wage Expectations in the Labor Market"*, develops a search and matching model applied to the business cycle in which agents have subjective wage expectations, deviating from rational expectations. This model is motivated by the empirical finding that survey wage expectations from professional forecasters and workers covary differently with labor productivity than than rational wage expectations do. I formally reject the hypothesis that this is compatible with rational expectations. Furthermore, I show that this model can replicate several stylized facts of the US labor market, such as the high fluctuations in labor market variables and the near-zero correlation between productivity and labor market tightness post-1990, which are challenging to rationalize within the framework of rational expectations. Additionally, this approach adeptly aligns with wage expectation survey data from professional forecasters. The incorporation of this novel approach to modeling expectations carries substantial policy implications. In light of this model, certain countercyclical unemployment insurance policy rules may lead to instability in the belief system, making them undesirable.

The second chapter, entitled *"The Effect of Non-Technological News Shocks on Unemployment Fluctuations: The Case of Europe"*, co-authored with Clemente Pinilla, identifies a novel source of unemployment fluctuations, non-technological news shocks, from firm and household survey data across 22 European countries. By extending the identification scheme of Beaudry and Portier (2006), our study introduces a two-step procedure. Initially, the sequential scheme serves as a signal to ascertain if selected forward-looking variables contain news shock information. Subsequently, a simultaneous identification framework is employed to distinguish between fundamental and various types of news shocks. We show that non-technological news shocks play a significant role in explaining the variance in unemployment in the medium to long run. Moreover, neither technological news nor technological shocks are the main driver of unemployment throughout the business cycle. A search and matching model that incorporates deviations from rational expectations—through the use of adaptive learning for forecasting labor market tightness—effectively reproduces the response of unemployment to non-technological news shocks. This model accounts for a significant portion of the unemployment variance decomposition observed in our empirical analysis, whereas the rational expectations version of the model fails to generate similar outcomes. This analysis not only contributes a new perspective on unemployment fluctuations but also enriches the theoretical discussions on how expectations and external information affect labor markets.

The third chapter, entitled *"Labor Market Dynamics and Imperfect Market Knowledge: A comparative study"*, conducts an analysis that compares the model proposed in the first chapter with different approaches that incorporate departures from rational expectations into a DMP model applied to the business cycle, proposed in the literature. This chapter evaluates the efficacy of these models in replicating labor market dynamics. Specifically, I juxtapose my model with the one advanced by Di Pace et al. (2021), wherein agents formulate wage expectations using an autoregressive model, and with the framework proposed by Menzio (2022), who endows workers with stubborn beliefs about the productivity process. The analysis shows that while all the compared models capture noticeable fluctuations in labor market variables, both the Di Pace et al. (2021) and Menzio (2022) models fail to notably reduce the correlation between productivity and labor market tightness or to accurately replicate the fluctuations observed in real wages. Whereas the approach in Di Pace et al. (2021) yields heightened relative wage fluctuations compared to empirical data, the model of Menzio (2022) produces the opposite effect. Furthermore, my model not only fits the empirical data more closely, but also demonstrates a superior capacity to align with the co-movements between survey wage expectations and labor productivity forecast by professional forecasters. Therefore, this chapter highlights the distinctive ability of my model to provide a more accurate and complete description of labor market dynamics.

1

The Role of Wage Expectations in the Labor Market

Marta García Rodríguez

Abstract

The standard search and matching model does not reproduce some key aspects of the US labor market, in particular, the high volatility in vacancies and unemployment and the null contemporaneous correlation between the vacancy-unemployment ratio and labor productivity from 1990 to 2020. Additionally, I document that survey wage expectations and rational wage expectations covary differently with labor productivity. I formally reject the hypothesis that this is compatible with rational expectations. This paper develops a search and matching model applied to the business cycle with internally rational agents. Even though agents hold subjective expectations about wages, they behave rationally given these expectations. The inclusion of learning significantly improves the model's fit with US data compared to its rational expectations counterpart. During expansionary periods, agents underestimate future wages, amplifying the effect of productivity shocks on the labor market. In light of this model, certain countercyclical unemployment insurance policy rules may lead to instability in the belief system, making them undesirable.

Keywords: Internal Rationality, Wage Expectations, Labor Market, Subjective Expectations, Belief Shock.

JEL Classification: E24; E32; D83; J64

This work was produced as part of my PhD thesis at Universitat Autónoma de Barcelona and BSE. First version: July 2021. This version: August 2023. I thank Albert Marcet for his guidance. Besides, I have benefited from comments from participants in seminars at Universitat Autónoma de Barcelona, SAEe2022, University of Minnesota, 1st PhD Conference on Expectations, SAET2023, XXVI Workshop on Dynamic Macroeconomics and EEA-ESEM-2023. This research has been greatly beneficial from founding provided by the European Research Council (ERC) under grant Horizon2020 grant agreement no. 788547 (APMPAL-HET) - Asset Prices and Macro Policy When Agents Learn and are Heterogeneous - and 804989 (DYMOLAMO) - Dynamic Modeling of Labor Market Mobility and Human Capital Accumulation.

1.1 Introduction

The Search and Matching Model (DMP) has become the standard equilibrium unemployment theory. However, several studies question the model's ability to accurately represent labor market fluctuations in the United States.[1] In particular, the standard DMP struggles to replicate observed fluctuations in the labor market and the propagation of productivity shocks. Targeting the ratio of standard deviations between labor market variables and productivity has been the focus of much research. However, the near zero correlation between productivity and labor market tightness post-1990 has been largely neglected in the literature. In this paper, I show that a DMP model is able to reproduce these observations if one allows for small deviations from rational expectations (RE).

I study how to introduce internal rationality (IR) in a DMP model.[2] I relax the standard assumption that agents have perfect knowledge about the wage function obtained from the standard Nash bargaining process. Agents have limited foresight and can not perfectly predict the outcome of wage bargaining, instead workers and firms have subjective beliefs about wages, and they maximize their objective functions subject to their constraints. I call such agents ”internal rational” because they know all internal aspects of their problem and maximize their respective objective functions given their knowledge about the wage process. I consider systems of beliefs implying only a small deviation from rational expectations (RE), and that match some aspects of survey wage expectations. The model has a self-referential mechanism: shifts in beliefs about future returns to labor affect current wages, and agents use realized wages to update their beliefs. This generates an additional source of dynamics that helps to match the data. Framing the model under IR provides a microfoundation to previous adaptive learning papers on unemployment.[3]

Moreover, I present a formal econometric test of the null hypothesis that survey evidence is consistent with RE, and demonstrate that the hypothesis of rational wage expectations is rejected by the survey data. This adds another puzzle for the standard version of DMP. The datasets used for this analysis are sourced from the European Commission's professional forecasters and the New York Federal Reserve's panel data on workers' expectations. A notable aspect of this test is its capacity to offer insights into the reasons behind the failure of the RE hypothesis: the failure arises because survey expectations and rational expectations covary differently with the labor productivity. This

[1] See Shimer (2005), Hall (2005), Fujita and Ramey (2003), Costain and Reiter (2008).
[2] See Adam and Marcet (2011).
[3] See Schaefer and Singleton (2018) and Di Pace et al. (2021).

finding is used to discipline expectations in the model of IR.

To quantitatively evaluate the learning and the RE models, I consider how well they match labor market moments. I use formal structural estimation based on simulated moments (MSM), adapting the results of Duffie and Singleton (1990) to estimate some parameters. Subsequently, I conduct a formal test to determine whether the model statistics significantly deviate from their empirical counterparts. The learning model offers a more accurate representation of U.S. data compared to RE. A key finding is the model's capacity to yield a low contemporaneous correlation between labor market tightness and productivity, coupled with elevated relative volatilities in the labor market. For instance, it produces relative volatilities of unemployment and the vacancy-unemployment ratio that are 7.7 and 10.85 times higher than those generated under rational expectations, respectively. Most models under RE require wages to exhibit minimal responsiveness to productivity variations to achieve such volatility. This results in a wage volatility that is less than that of productivity, a scenario inconsistent with empirical data. In my approach, wages are not rigid, they are influenced by both productivity fluctuations and agents' expectations, enabling the model to exhibit a wage volatility that slightly surpasses that of productivity. Additionally, the model generates the positive correlation between wage forecast error and productivity found in surveys, a relationship that is non-existent under rational expectations.

The reduction in the correlation between labor market tightness and labor productivity stems from the additional source of variability introduced by learning, which affects job creation conditions. In a RE framework, labor market tightness solely depends on current productivity, yielding a correlation nearly equal to one. However, with IR, labor market tightness is influenced not only by productivity but also by the time-varying coefficients determining wage expectations, thereby reducing the aforementioned correlation.

Furthermore, learning introduces an endogenous amplification of productivity shocks in the labor market due to the slower adaptation of wage expectations, a result of the constant gain learning algorithm. Hiring decisions are contingent upon firms' projections of future profits per hire, requiring an estimation of the future marginal product and wages over an indefinite horizon. For instance, after a positive productivity shock, IR firms expect lower future wages compared to RE firms. Under RE, firms know perfectly how wages correlate with productivity, whereas in the IR model, they do not know exactly how changes in productivity translate into changes in wages instead, they learn about this relationship. The productivity shock generates a negative impact on the forecast error, updating the expectation downwards. In subsequent periods, agents revise their beliefs in response to changes in market opportunities. This causes wage expectations to be lower compared to RE for a while. It will take some periods to adjust its expectations upward, and in the meanwhile, firms will post more jobs, so that for a while the response of unemployment is contrary to the needed adjustment.

The quantitative model is next used to assess the welfare implications of the current US unemployment insurance (UI). The UI programs, in United States, become more generous during economic downturns. This issue has gained renewed attention given the recent recession. I find that in an economy where agents learn about wages, the welfare costs are significantly higher compared to a RE model, and also, the policy introduces relatively more uncertainty in the economy. Additionally, I found that such policy may destabilize the macroeconomic system when agent learn, specially if the UI is linked to unemployment. Policymakers should steer clear of rules that induce to instability.

The rest of the paper is organized as follows. Section 2 reviews the literature. Section 3 tests the RE assumption with data from professional forecasters and consumer. Section 4 describes the model. Section 5 presents the calibration of the model and summarizes the main results. Section 6 studies welfare properties of some labor market policies. Section 7 performs some robustness exercises. Lastly, section 8 concludes.

1.2 Related Literature

This model aligns with efforts to solve the Shimer puzzle in the search and matching model literature. Two solutions stand out in the literature. (I) Change in wage formation, in wage formation, as suggested by Shimer (2005), Hall (2005), Gertler and Trigari (2009), where wages don't fully adjust to productivity shifts, spurring job creation. (II) Calibration changes, as proposed by Hagedorn and Manovskii (2008), enhance firm bargaining power and unemployment benefits, inducing endogenous wage rigidities. Yet, these methods face critiques, and there is no consensus in the literature about how to solve the puzzle.[4] Although these models generate volatility in the labor market, they fall short in explaining the near-zero correlation between labor market metrics and productivity, and the

[4]These approaches were criticized by Pissarides (2009), Haefke et al. (2013), Mortensen and Nagypal (2007) and Costain and Reiter (2008). There are more solutions to generate volatility in the labor market; see Costain and Reiter (2008), Silva and Toledo (2009), Reiter (2007), Menzio (2005) among others.

slightly higher wage volatility relative to productivity. Departing slightly from Rational Expectations (RE), I introduce more rigid expectations rather than rigid wages. To the best of my knowledge, this is the first paper to propose a model that is able to generate high volatility in the labor market, a subdued correlation between vacancy-unemployment ratio and labor productivity, flexible wages, and a rationale for wage expectation surveys.

Some recent papers study DMP departing from full information rational expectations (FIRE). For example Morales-Jiménez (2022) and Menzio (2022). In this papers, workers misperceive the true process for productivity. Moreover, workers are assumed to know the mapping from productivity to wages. In these models, agents still require immense knowledge of market behavior. Alternatively, I endow agents with uncertainty regarding how wages are linked to productivity. This fact is tested using wage expectation surveys. My model adeptly addresses the observed correlation between the forecast error of wages and productivity documented in surveys. This is achieved by showcasing a significantly reduced covariance between wage expectations and productivity, as compared to what is implied by rational expectations. In contrast, Morales-Jiménez (2022) and Menzio (2022) do not consider surveys of workers to test the productivity hypothesis.

This paper extends the adaptive learning literature, with applications outlined in Evans and Honkapohja (2012), Bullard and Mitra (2002) and Eusepi and Preston (2011). Recently, the introduction of a standard adaptive learning approach in the search and matching model has been studied. Schaefer and Singleton (2018) find that when agents make one-step-ahead forecast of labor market tightness, the learning model struggles to capture labor market volatility. Conversely, Di Pace et al. (2021) find that when agents use a misspecified model for wage expectations, while it amplifies labor market dynamics, it overstates wage fluctuations and does not appreciably adjust the correlation between labor market variables and productivity. Di Pace et al. (2021) is the paper most akin to mine. The main difference lies in the way agents form wage expectations. In my paper, agents use productivity directly to form wage expectations, a fact I test with survey data, while in their paper they form wage expectations using an autoregressive model, implying that agents have an inaccurate model to form such expectations. This paper builds on the adaptive learning literature, but maintains the rationality of the agents. Importantly, it is also specific about beliefs system that the agents have in the economy.[5] Both these modelling features are the hallmark of the Internal Rationality framework developed by Adam and Marcet (2011). This approach has not been applied to the search and matching model before and can provide a micro-foundation for adaptive learning models.

A large literature studies the optimality of UI policies including Fredriksson and Holmlund (2001), Coles and Masters (2006), Lehmann and Van der Linden (2007), Landais et al. (2010), Mitman and Rabinovich (2015); among others.[6] I quantify the policy bias in the cost-benefit calculation of unemployment policies that depends on the state of the economy in job creation when using a RE model instead of a learning model. Results show that the cost/benefit of unemployment benefits on job creation is significantly underestimated in rational expectations models.

A vibrant literature has recently developed studying the behavior of expectation surveys. Some papers show that there is a significant discrepancy between the expectations implicit in the macroeconomic model under RE and the expectations coming from the survey data; see Conlon et al. (2018), Greenwood and Shleifer (2014), Adam et al. (2017), Malmendier and Nagel (2016), Coibion and Gorodnichenko (2012),Coibion and Gorodnichenko (2015); among others. I applied the statistical test proposed by Adam et al. (2017) to see whether the data support the rational expectation assumption regarding the formation of future wages. I show that neither workers, firms, nor professional forecasters form wage expectations following rational expectations. Moreover, the test provides clues about why the RE hypothesis fails, which I used as a guide for modeling expectations.

1.3 Wages and Wage Forecast

Wage expectations play an important role in the labor market decisions. In the search and matching framework, they affect the match surplus and therefore, current wages and also, the hiring decisions made by firms. In the standard DMP model, workers and firms bargain about the wage and the equilibrium wage equation is known by them. More precisely, all agents are assumed to know the mapping from observed productivity shocks to equilibrium wages. This "complete information" as-

[5] The adaptive learning literature does not specify what agents' views are on the evolution of macro-variables. They only equip them with a recursion, which tracks some moments of the variable. If beliefs are not fully specified in the model, then why, exactly, agents must form expectations according to a given recursion and how this relates to rational behaviour is unclear.

[6] Optimal benefit levels strikes a balance between insurance and incentives, providing insurance against unemployment risk and providing firms with incentives for vacancy creation. I do not address the mention tradeoff, but only highlight the importance of the RE assumption in predicting effects of UI on job creation.

sumption is commonly made, although rarely proven, because expectations are very rarely observed.[7]

This section shows that forecast of wages are inconsistent with the notion that agents hold rational wage expectations. I present a formal econometric test following Adam et al. (2017) showing that expectations and RE covary differently with the labor productivity. To run the test, Section 3.1 employs survey data from professional forecasts provided by the European Commission, while Section 3.2 uses data from the Survey of Consumer Expectations (SCE) Labor Market Survey by the New York Fed, which contains workers' wage expectations. In both datasets, the observed covariance between wage expectations and productivity is significantly lower than the one implied by rational expectations.

1.3.1 Professional Forecasters

This section conducts a test for rational expectations using survey data that comprises the average forecast of annual wage growth in the United States, reported by the European Commission for the period of 1999 to 2020.[8]

Let $E_t^{\mathcal{S}}$ denote the agent's subjective expectation operator based on information up to time t, which can differ from the rational expectation operator E_t. Let $\hat{w}_{t+2}$ denote the two-period ahead realized annual growth of wages, and let s_k be a measure of agent's subjective beliefs regarding future growth of wages that are possibly subject to measurement error, μ_t, obtained from survey data. Therefore, $s_{t+2} = E_t^{\mathcal{S}}(\hat{w}_{t+2}) + \mu_t$ represents an estimate of the agents' subjective beliefs about annual wage growth two semesters ahead. Given the forecast horizon of professional forecasters, t stands for 2 semesters.

$$\hat{w}_{t+2} = c^R + b^R \hat{y}_t + \nu_t, \tag{1.1}$$

$$s_{t+2} = c^E + b^E \hat{y}_t + \epsilon_t, \tag{1.2}$$

where $\hat{y}$ represents annual productivity growth. Under the null hypothesis of RE ($H_0 : E_t = E_t^{\mathcal{S}}$), if $\hat{y}_t$ is in the informational set of agents for time period t, the prediction error must be orthogonal to

[7]Conlon et al. (2018), using the Survey of Consumer Expectations (SCE) Labor Market Survey from the New York FED, found a significant correlation between labor force's revisions of wage offer forecasts and their forecast errors. This finding supports the existence of information rigidities in forming expectations about future wage offers.

[8]The forecast is reported twice a year in Autumn and Spring. They just report the average forecast. Link reports: `https://ec.europa.eu/economy_finance/publications/european_economy/forecasts/index_en.htm`

Indep. variable	b^R	b^E	P-value $H_0 : b^R = b^E$	P-value $H_0 : b^R \leq b^E$
$\hat{y}_t$	0.75**	0.15*	0.0487	0.025
	(2.35)	(1.9)		
$\hat{y}_{t-1}$	0.78***	0.12	0.0234	0.0202
	(2.71)	(1.18)		

Table 1.1: RE test

Note: ***,**,* denote sig. at 1%, 5% and 10% levels, respectively. t statistics in parentheses. The Table presents the results of the test $b^R = b^E$. The third row shows the results of the test where I include the independent variable with a lag of half a year. The p-values for the test are constructed using bootstrapping, 1000 bootstrap samples. The number of observations is 42.

$\hat{y}_t$. $\hat{b}^R$ and $\hat{b}^E$ must be estimates of the same regression coefficient because $b^R = b^E$. If coefficients across equations are different I reject RE. [9]

Table (1.1) shows the result of the test. Column 4 shows the p-values.[10] Additionally, column 5 shows the p-values for the one-sided test. As a robustness exercise, in the third row, I report the results when the test is performed with annual productivity growth lagged. The results provide evidence against the notion that survey expectations of wages are compatible with RE. This rejection arises because survey expectations and rational expectations covary differently with the labor productivity. Therefore, the forecast error of wages is correlated with productivity growth.

An intriguing observation emerges from the data: during recessions, professional forecasters tend to overestimate wage growth, whereas during expansionary periods, they underestimate it. For instance, amidst the Great Recession, forecasters predicted an average annual wage growth of 0.99%. In contrast, the actual average annual growth for that period experienced a decline of 3.4%. Between Q1-2011 and Q3-2016, a period of economic expansion, the pattern reversed. Forecasts anticipated a growth of 0.76%, yet the actual realization was an impressive 2.51%. Such disparities in wage growth predictions could potentially account for the pronounced fluctuations observed in the labor market.

[9] Coibion and Gorodnichenko (2012, 2015) bring evidence in favor of information rigidity in expectation formation described by a significant correlation between forecast revisions and forecast error.

$$s_{t+2/t} - s_{t+2/t-1} = c + b(\hat{w}_{t+2} - s_{t+2/t}) + \epsilon_t$$

Using my data, b=-0.08, the non-significance can be due to the fact that the measurement error of the survey data makes the explanatory variables correlated with the residual and gives a bias b.

[10] The p-values are constructed using a small sample correction procedure. To construct the p-values for the test I rely on Monte-Carlo simulations rather than on asymptotic results. Please refer to Section 2 and Appendix A.3 of Adam et al. (2017) for additional details of the test.

b^1	b^2	Is $\hat{y}_t$ included?	R^2
-	0.094***	No	0.09
	(2.57)		
0.204**	0.149***	Yes	0.16
(1.96)	(1.97)		

Table 1.2: RE test

Note: ***,**,* denote sig. at 1%, 5% and 10% levels, respectively. t statistics in parentheses. The Table presents the results of regression 1.3. The first row shows the results of the regression when I do not include productivity growth as independent variable. The p-values for the test are constructed using bootstrapping, 1000 bootstrap samples. The number of observations is 42.

For example, during an expansionary period, a firm that anticipates lower future wages might be inclined to post more job vacancies.

Di Pace et al. (2021) posits that agents rely on an autoregressive models to shape wage expectations.[11] This implies that agents do not use directly productivity to form wage expectations. To test that assumption, I run the following regression:

$$\hat{s}_{t+2} = c + b^1\hat{y}_t + b^2\hat{w}_t + \varepsilon_t. \tag{1.3}$$

Table 1.2 reveals that productivity remains a significant factor in forecasting wage growth, even after accounting for the realized wage growth.

1.3.2 Consumer Expectations

The data on consumer expectations is sourced from the Survey of Consumer Expectations (SCE) by the Federal Reserve Bank of New York. For the Rational Expectation test, I utilize two datasets: (1) the SCE, which includes detailed demographic information of participants, and (2) the SCE Labor Market Survey.[12] The latter dataset comprises two primary sections: (I) the "Experiences" section, capturing labor market outcomes such as recent wage offers, search behavior, and job satisfaction, and (II) the "Expectations" section, recording expectations regarding wage offers, job transitions, and retirement.

[11] Di Pace et al. (2021) employ the same survey data up to 2018Q3. However, rather than employing regression (1) and (2) to test Rational Expectations (RE), they examine the correlation between the forecast error, $\hat{w}_{t+2} - s_{t+2}$, and GDP growth. While this is a valid approach, as supported by Coibion and Gorodnichenko (2012, 2015), it does not allow for an exploration into the potential association between GDP growth and the forecast of wage growth.

[12] Details can be found at https://www.newyorkfed.org/microeconomics/databank.html.

The panel data enables me to explore how individual expectations align with realizations over the subsequent 4-month period, offering insights into the accuracy and formation of expectations in the labor market. Each interview date is denoted by the subscript t. Respondents are surveyed quarterly for up to a year, and each respondent is identified by the subscript i. To calculate forecast errors and conduct a statistical test, respondents must participate in at least two consecutive surveys. I focus on data from November 2014 onwards, a time when the survey began including questions about current and anticipated job offers.[13]

The distinction of the rational expectations test conducted in this section, compared to the one proposed by Adam et al. (2017)), lies in the nature of the forecast: agents are predicting their own wage offers rather than aggregate economic variables.

Let $E_t^{S,i}$ denote agent i's subjective expectation operator based on information up to time t, which can differ from the rational expectation operator E_t^i. Let w_{t+1}^i denote the realized wage offer that the agent receives four months ahead, and let s_k^i be a measure of agent i's subjective beliefs regarding future wage offers that are possibly subject to measurement error, ν_t^i obtained from survey data. Therefore, $s_{t+1}^i = E_t^{S,i}(w_{t+1}^i) + \nu_t^i$ represents an estimate of agent i's subjective beliefs about his/her wage offer four months ahead. Given the expectation horizon in the Labor Market Survey, t stands for four months.

$$w_{t+1}^i = a + \delta \hat{y}_t + \sum_{n=1}^{N} \alpha_n X_{i,n} + \epsilon_{i,t}, \tag{1.4}$$

$$s_{t+1}^i = a^e + \delta^e \hat{y}_t + \sum_{n=1}^{N} \alpha_n^e X_{i,n} + \mu_{i,t}. \tag{1.5}$$

Where $\hat{y}$ represents quarterly productivity growth and N is the number of control variables such as income, age, race, numeracy, gender, location, education, type of industry, search effort, and employment status (dummies and categorical variables).

Table (1.3) shows the results of the test under different specifications of the independent variable. The last column presents the p-values of the test.

[13] Section A.2 of the Appendix provides a detailed overview of the survey data and clarifies the assumptions I adopted.

Indep. variable	δ	δ^e	p-value $H_0 : \delta = \delta^e$
y_t	3.24*	0.208	0.017
	(1.729)	(1.768)	
y_{t-2}	3.62**	0.55	0.011
	(1.694)	(0.532)	
y_{t-4}	2.85**	-0.47	0.009
	(1.467)	(0.195)	

Table 1.3: Rational Expectation Test, $H_0 : \delta = \delta^e$

Note: Robust standard errors in parenthesis.***,**,* denote sig. at 1%, 5% and 10% levels, respectively. The number of observations are 740. The regression in the second row uses productivity growth with two lags (six-month lagged) as an independent variable instead of contemporaneous productivity growth. The regression of the third row uses productivity growth with 4 lags (one year lagged) as independent variable of contemporaneous productivity growth.

The results presented in Table (1.3) demonstrate that the null hypothesis is rejected in all cases, implying that rational expectations do not find empirical support. Aggregate labor productivity growth correlates with the forecast error of wage offers at a significance level of less than 0.020. Therefore, the forecast error is not orthogonal to productivity growth.

A similar result is evident among professional forecasters. The correlation between actual wages and productivity surpasses that between wage expectations and productivity. Additionally, it appears that, on average, the labor force does not fully incorporate the impact of productivity when forming their beliefs about future wages.

This empirical observation does not necessarily mean that workers are irrational. One plausible explanation could be that each worker is privy to the time series data of their own wages, but lacks access to the comprehensive panel data that includes wage offers for a broad spectrum of workers. As a result, when a worker conducts a regression of their personal wage offers against aggregate productivity, the derived coefficient may lack statistical significance due to the limited sample size. Consequently, workers might discount aggregate labor productivity as a non-informative factor in predicting their future wages.

The next section spells out the microfundations of a DMP model under IR where wage expectations are formed using a adaptative approach. Therefore, forecasting errors are not supposed to be necessarily orthogonal to the variables agents observe when making the predictions.

1.4 The Model

I propose a model featuring labor market search and matching friction as in Mortensen and Pissarides (1994) applied to the business cycle. Under the standard setting of RE, agents understand how productivity maps to wages. Instead, I assume the lack of common knowledge of general equilibrium wage mapping and equip agents with a fully specified system of beliefs. Agents form their expectations about the future path of wages based on their respective perceived law of motion (PLM) and update their beliefs as new information becomes available. Given their expectations, agents take optimal decisions. Two shocks can hit the economy: a productivity shock and a shock that affects the agents' beliefs about their expected wages. At the start of a period, shocks occur. Agents forecast future wages, influencing employment surplus of workers, firms' hiring surplus and vacancy decisions. Should a match occur, wages are then bargained over. The period concludes with certain jobs destroyed exogenously.

1.4.1 The Labor Market

Following the standard literature, this economy is characterized by frictions in the labor market. There is a time-consuming and costly process of matching workers and job vacancies, which is capture by a standard constant returns to scale matching function *m(u,v)* where u denotes the unemployment rate and v is the vacancy rate. I refer to $\theta_t = \frac{v_t}{u_t}$ as the market tightness at time *t*. Hence, the rate at which unemployed workers find a jobs, $f(\theta)$, and vacancies are filled $q(\theta)$ depend of the vacancy-unemployment ratio, where $f(\theta) = \theta q(\theta)$ and $f(\theta)' > 0$, $q(\theta)' < 0$. The unemployment rate increases when jobs are destroyed at a exogenous rate, λ, and decreases when workers find jobs. Thus,employment evolves according

$$n_{t+1} = (1-\lambda)n_t + q(\theta_t)v_t. \tag{1.6}$$

The labor productivity takes the form of stationary AR(1) in logs:

$$\ln(y_t) = (1-\rho)\ln(\overline{y}) + \rho\ln(y_{t-1}) + \epsilon_t,\ 0 < \rho < 1. \tag{1.7}$$

Where $\epsilon_t \sim N(0, \sigma^2)$ and ρ measures the persistence.

1.4.2 Worker's Problem

There is a continuum of identical, risk neutral workers with total measure one and an infinite horizon. These workers can either be employed or unemployed in each period.

An employed worker earns a wage w_t at t, and faces a probability λ of losing his job in the subsequent period. Conversely, an unemployed worker receives unemployment benefits b and has a probability $f(\theta_t)$ of finding a job in the next period. The wage process, w_t, and the tightness of the labor market, θ_t, are given by individual workers. Individual workers have nothing to choose, whether they are employed or not is determined exogenously. The primary calculation where their expectations will play a role is the net surplus of the match that is used to bargain the wage with the firm if the match is realized. This surplus is the difference between the value of being employed and unemployed.

Deriving the standard surplus of the worker hides many assumptions that I wish to bring out in this section. The worker surplus depends on expectations and expectations are determined with a probability measure $\mathcal{P}^w$. The definition of $\mathcal{P}^w$ depends on exactly how much workers are assumed to know about the equilibrium process for n, θ and w and about the properties of these variables. So, I start with a general definition of $\mathcal{P}^w$ that is consistent with the above setup and that encompasses a number of standard equilibrium concepts that are found in the literature. This will be useful, first, to unveil some assumptions in the adaptive learning literature that are often not explicitly stated and it will allow me to extend those equilibrium concepts. Then, I obtain step by step some familiar derivations in the literature and explain how each derivation depends on an increasing amount of assumptions. This provides a clear comparison of the IR equilibrium studied in the paper with RE and with some adaptive learning versions of the model.

A generic worker problem under Internal Rationality

Consider first the case where I do not make any assumption about the relation between workers' beliefs and actual equilibrium. The next subsection will cover the case of RE as well as the case of Bayesian/RE.

If workers are rational, at the very least, the state space for the measure $\mathcal{P}^w$ has to contain the payoff relevant variables for individual workers that are beyond the agent's control, therefore $\mathcal{P}^w$

puts probabilities on sequences $\{(w,\theta,n)^t\}_{t=0}^{\infty}$.[14] $(w,\theta,n)^t$ is the usual notation describing sequences up to t, and it is understood that $E_t^{\mathcal{P}^w}$ responds to the usual definition meaning "conditional expectation given $(w,\theta,n)^t$".

Following, I state the first assumption on beliefs

Assumption 1. The belief system $\mathcal{P}^w$ is Markov up to a state vector m. More precisely,

$$\begin{aligned} Prob^{\mathcal{P}^w}(w_t,\theta_t,n_t \mid (w,\theta,n)^{t-1}) &= \mu(m_{t-1}), \\ m_t &= g(m_{t-1},y_t,w_t,\theta_t,n_t). \end{aligned} \tag{1.8}$$

For some given functions μ, g conformable to their arguments and for a vector m_t that contains θ_t, w_t, n_t. In standard IR models, m will also contain variables that in the workers' mind summarize the best forecast of future wages, as is the case in the main sections of this paper.

Now, I can formulate the value functions for the worker.

The present value of working for an agent is as follows:

$$\mathcal{W}(m_t) = w_t + \beta E_t^{\mathcal{P}^w}\left[(1-\lambda)\mathcal{W}(m_{t+1}) + \lambda\mathcal{U}(m_{t+1})\right]. \tag{1.9}$$

On the other hand, workers can be unemployed. The present value of unemployment is given by:

$$\mathcal{U}(m_t) = b + \beta E_t^{sw}[f(\theta_t)\mathcal{W}(m_{t+1}) + (1-f(\theta_t))\mathcal{U}(m_{t+1})]. \tag{1.10}$$

Where $\mathcal{W}$ and $\mathcal{U}$ are time-invariant functions.

It may seem that this is enough to arrive at a standard equation for worker's surplus, $\mathcal{W} - \mathcal{U}$. But since I have not given any market knowledge to agents, they still do not necessarily know the equilibrium process of θ unless I make the following additional assumption.

[14] From the point of view of probability theory I should also state that the probabilities $\mathcal{P}^w$ are defined on the sets of a sigma algebra of the mentioned sequence space, but since, it is obvious how to set this up and it does not have an impact on any application to search models we will not mention sigma algebras anywhere else in the paper.

Assumption 2. Individual workers have a model that forecast correctly the true evolution of θ. Formally, $Prob^{\mathcal{P}^w}(\theta_t = \theta | m_t = m) = Prob^{\mathcal{P}}(\theta_t = \theta | m_t = m)\ \forall(\theta, m)$.

Only under all these assumptions I get the workers' share of the total surplus is:

$$\mathcal{W}(m_t) - \mathcal{U}(m_t) = w_t - b + \beta\left(1 - \lambda - f(\theta_t)\right) E_t^{\mathcal{P}^w}\left(\mathcal{W}(m_{t+1}) - \mathcal{U}(m_{t+1})\right). \tag{1.11}$$

This equation would be satisfied when agents learn about wages, as long as Assumptions 1-2 hold. Learning problem remains hidden in the belief structure $\mathcal{P}^w$. In section 1.4.4, I provide an explicit system of beliefs $\mathcal{P}^w$.

The individual problem under RE

Assume now that agents are endowed with the knowledge that wages are a function of the productivity, y_t, that is I include in (1.8) an equation giving w_t as an exact function of y_t. This is summarize in assumption 3.

Assumption 3. The system of equations (1.8) includes

$$w_t = \mu_w(y_t). \tag{1.12}$$

In addition, assume that agents know the law of motion of productivity, i.e. they know equation (1.7). In this paper, I focus on the RE equilibrium that takes the form of the fundamental or minimum state variable solution (MSV).[15] With these additional assumptions then, indeed, we have that $m_t = (y_t)$. In this case, market wages carry only redundant information. This allows to exclude wages from the state space without loss of generality.

Additionally, I have to assume the following.

Assumption 4. Agents' beliefs are correct, that is, in equilibrium $\mathbf{w}_t = \mu_w(y_t)$.

Then workers have RE.

[15] While there may be RE equilibria contradicting this assumption, with added lags in wage determination, Campbell (1994) shows that the RE solution has w_t as an ARMA(2,1) process. Adhering to McCallum (1983), I select the minimal state variable set that's indispensable for a solution.

1.4.3 Firms Problem

Consider an economy populated by a mass of infinity firms. Firms' revenues are $y_t n_t$, where n_t and y_t are exogenous to the firms. The productivity, y_t follows a AR(1) process (1.7). Firms pay a total of $w_t n_t$ at t, the wage process is taken as given by firms. Each period firms choose the number of vacancies v to post at a constant ongoing cost c. Their period-t profits, Π_t, is $y_t n_t - w_t n_t - cv_t$.

A key feature of equilibria will be the firms' expected discounted profits from period t onwards, given by

$$\Pi_t \equiv E_t^{\mathcal{P}^f}\left(\sum_{j=0}^{\infty}\beta^j[y_{t+j}n_{t+j} - w_{t+j}n_{t+j} - cv_{t+j})]\right), \tag{1.13}$$

where $\mathcal{P}^f$ is the firm' probability measure about relevant future variables.

To derive the standard job creation condition, it is common in the literature to appeal to dynamic programming to write Π_t in a forward recursive form. In the next subsection, I set out the necessary assumptions to derive this equation. The definition of $\mathcal{P}^f$ depends on exactly how much firms are assumed to know about the equilibrium process for w, y, θ, n, and about the properties of these variables. Therefore, as in the workers problem, I start with a general definition of $\mathcal{P}^f$ and derive step by step some familiar derivations and explain how each derivation depends on a large amount of assumptions.

A generic firm problem under Internal Rationality

This subsection will cover the case of RE as well as the case of Bayesian/RE for the firms' problem. I do not make any assumption about the process for equilibrium variables nor about the relation between firms' beliefs and actual equilibrium.

The state space for the measure $\mathcal{P}^f$ has to contain all payoff-relevant variables for individual firms. Hence, $\mathcal{P}^f$ puts probabilities on sequences $\{(w,\theta,y)^t\}_{t=0}^{\infty}$. $(w,\theta,y)^t$ is the sequences up to t. $E_t^{\mathcal{P}^f}$ in (1.13) represents the "conditional expectation given $(w,\theta,y)^t$".

Since Π_t is still a function of the whole sequence $(w,\theta,y)^t$, to obtain a recursive formulation, I need to add *assumption* 1 of the workers' problem, that set that the belief system is a Markov up to a state vector, together with the transversality condition, $E_t^{\mathcal{P}^f}\beta^j\Pi_{t+j} \to 0$ as $j \to \infty$ almost surely in

$\mathcal{P}^f$.

Additionally, to set the problem and derive the job creation condition, I have to add *assumption* 2 from the worker' problem, that firms forecast correctly the labor market tightness, and the following *assumption* 5.[16]

Assumption 5. Individual firms know the law of motion of n.

Taking into account previous assumptions, firms make contingent plans for vacancy posting subject to the evolution of employment (1.6). Now, I can state the maximization problem of the firms:

$$\Pi(m_t) = \max_{v_t \geq 0} \quad y_t n_t - w_t n_t - c v_t + E_t^{\mathcal{P}f} \; \Pi(m_{t+1}) \tag{1.14}$$

subject to

$$n_{t+1} = (1-\lambda) n_t + q(\theta_t) v_t. \tag{1.15}$$

Below, I will specify the probability measure through some perceived law of motion describing the firm's view about the evolution of (w_t, y_t) over time, together with a prior distribution about the parameters governing this law of motion. Optimal behavior will then entail learning about these parameters, in the sense that agents update their posterior beliefs about the unknown parameters in the line of new wage, and productivity observations. For the moment, this learning problem remains hidden in the belief structure $\mathcal{P}^f$.

Optimality Conditions. The firm's optimal plan is characterized by the first order condition, together with the envelop condition with respect to n_t.

$$E_t^{\mathcal{P}^f} \mathcal{J}_{t+1} = \frac{c}{\beta q(\theta_t)}, \tag{1.16}$$

$$\mathcal{J}_t = y_t - w_t + \beta(1-\lambda) E_t^{\mathcal{P}^f} \mathcal{J}_{t+1}. \tag{1.17}$$

[16] In Garcia-Rodriguez and Pinilla-Torremocha (2021), we relax assumption 2 in the DMP model.

where $\mathcal{J}_t = \frac{\partial \Pi(m_t)}{\partial n_t}$ represents the marginal value of having an additional worker employed at the firm. Therefore, equation (1.17) gives the surplus of the firm coming from a match. Combining (1.16) and (1.17) and iterated forward, I come up with the job creation condition

$$\frac{c}{q(\theta_t)} = E_t^{\mathcal{P}^f} \sum_{j=1}^{\infty} [\beta(1-\lambda)]^j \left[\frac{y_{t+j} - w_{t+j}}{1-\lambda}\right]. \tag{1.18}$$

This equation would be satisfied when agents learn about wages, as long as all previous assumptions hold. Therefore, in this case I have that the usual job creation condition, but I still need a generic m in (1.16) and (1.17).

The individual firm problem under RE

Analogous to section 1.4.2 of the worker's problem, assume now that firms are endowed with some knowledge of how wages are formed, i.e., assumption 4 holds. Also, firms know the law of motion of y and firms' beliefs are correct, that is, in equilibrium $\mathbf{w}_t = \mu_f(y_t)$.

Then, firms have RE.

1.4.4 Agents' Belief System

Once one departs from rational expectations, beliefs become part of the microfoundations of the model. Previous sections left open how $\mathcal{P}^w$ and $\mathcal{P}^f$ incorporate wage beliefs. In this section, I introduce a fully specified probability measure $\mathcal{P}$ and derive the optimal belief updating equation it implies. For simplicity, I assume that this part of beliefs is common to $\mathcal{P}^w$ and $\mathcal{P}^f$.[17] Nevertheless, agents may not know that this is true prior to wage bargaining. It is important to understand how agents view the wage process to specify an internally consistent rational agent model. The belief system of internally rational agents requires that they do not make obvious mistakes while learning.

Agents have the following perceived law of motion (PLM) which they use to make forecast of wages:

$$\begin{aligned} w_t &= d_t^c + d_t^y y_{t-1} + \epsilon_t, \\ D_t &= D_{t-1} + \nu_t. \end{aligned} \tag{1.19}$$

[17]In the section 7.3, I build a version that allow workers and firms have a different belief system for wages.

Where $D_t = [d_t^c \ d_t^y]$. Shocks $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$ and $\nu_t \sim \mathcal{N}(0_{2,1}, \sigma_\nu^2 I_2)$ are independent of each other. This PLM considers a fundamental or minimal state variable solution with unobserved coefficients.

Consider the case where agents' prior beliefs are centered at the REE with the prior variance $\sigma_{D,0}^2$:

$$D_0 \sim \mathcal{N}(D^{REE}, \sigma_{D,0}^2 I_2). \tag{1.20}$$

Where $\sigma_{D,0}^2$ is set to the steady-state Kalman filter variance. Note that the agents' beliefs encompass the REE of the model. In particular, when agents believe $\sigma_\nu^2 = 0$ and assign probability 1 to $D_0 = D^{REE}$, I have that $D_t = D^{REE}$ for all $t \geq 0$ and wages are given by RE equilibrium wages in all periods. Alternatively, if (1.20) is combined with a belief that σ_ν^2 is small, even though the resulting dynamics of the economy are not going to be precisely given by REE, it will be close to REE.

Agents' posterior beliefs at any time t are given by

$$D_t \sim \mathcal{N}(\hat{D}_t, \sigma_{D,t}^2 I_2). \tag{1.21}$$

Given that agents are rational, they update $\hat{D}_t$ according to the recursive least squares (RLS) algorithm:

$$\begin{aligned} \hat{D}_t &= \hat{D}_{t-1} + \gamma R_t^{-1} z_{t-1}[\mathbf{w}_{t-1} - \hat{D}_{t-1}^{'} z_{t-1}] + \epsilon_t^\beta, \\ R_t &= R_{t-1} + \gamma(z_{t-1} z_{t-1}^{'} - R_{t-1}). \end{aligned} \tag{1.22}$$

Where $\hat{D}_t = [\hat{d}_t^c \ \hat{d}_t^y]'$ represent the estimated coefficients, R_t denotes the moment matrix for $z_{t-1} = [1 \ y_{t-1}]$ and $\mathbf{w}_t$ denotes the realized previous wage. $\epsilon_t^\beta \sim \mathcal{N}(0_{2,1}, \sigma^{\beta 2} I_2)$ is a shock to wage beliefs and γ denotes the steady state Kalman gain $\in$ (0,1) that determines the rate at which older observations are discounted.[18] Strictly speaking, given the above information structure the Kalman filter requires $\sigma^{\beta 2} = 0$. This shock to beliefs can be interpreted as additional information about ν_t available to agents or as a departure from fully rational belief formation.

These beliefs constitute a small deviation from RE beliefs in the limiting case with vanishing in-

[18] The variable w_t is *not* introduced with a delay in the estimation of $\hat{D}$, is a standard assumption in the learning literature. This approach conveniently avoids the simultaneous determination of forecasts and endogenous variables. As proved by Marcet and Sargent (1989a), this does not alter the asymptotic results obtained in the following as compared to the algorithm allowing for simultaneity.

novation to the random walk process. Agents' prior uncertainty then vanishes, and the optimal gain goes to zero. As a result, one recovers the RE equilibrium value for wages.

1.4.5 Wage Bargaining

Wages are negotiated according to a Nash bargaining process. Each agent calculates its respective surplus from its problem, taking into account its system of beliefs of wages, before going to the bargaining process. The wage w_t maximizes the joint surplus of a match between workers and firms,

$$\max_{w_t} \quad [\mathcal{W}(m_t) - \mathcal{U}(m_t)]^{\alpha} \, \mathcal{J}_t^{1-\alpha} \tag{1.23}$$

where α represents the bargaining power of the worker. The first order condition of this problem gives the standard sharing rule that characterizes the optimal split of the aggregate surplus,

$$(1-\alpha)(\mathcal{W}(m_t) - \mathcal{U}(m_t)) = \alpha(\mathcal{J}_t). \tag{1.24}$$

Assuming that agents know that (1.24) holds in expectations, the equilibrium wage mapping $\mathbf{w}_t$ is given by

$$\mathbf{w}_t = \alpha(y_t + c\theta_t) + (1-\alpha)b. \tag{1.25}$$

Since agents do not hold rational wage expectations, I need to distinguish between the stochastic process for equilibrium wages $\mathbf{w}_t$ and agents' perceived wage process w_t. The wage equation is the weighted average of the marginal product of employment, the cost of replacing the worker, and the opportunity cost of working, b. Labor market tightness is a function of expectations; therefore, expectations play an important role in determining wages in equilibrium.

1.4.6 Equilibrium Dynamics under Learning

Under internal rationality, the solution of the model is summarized by (1.18), (1.25) and (1.22). It follows from (1.19) and (1.7) that beliefs about wages k periods ahead are given by

$$E_t^{\mathcal{P}}(w_{t+k}) = \hat{d}_t^c + \hat{d}_t^y((1-\rho^{k-1}) + \rho^{k-1}y_t). \tag{1.26}$$

Inserting equation (1.26) into equation (1.18) and, then the resulting one into (1.25), one can write the actual law of motion (ALM) of wages as follows:

$$\mathbf{w}_t = T_c(\hat{d}_t^c \;\; \hat{d}_t^y) + T_y(\hat{d}_t^y)y_{t-1} + T_\epsilon(\hat{d}_t^y)\epsilon_t, \tag{1.27}$$

where T_c, T_y and T_ϵ are functions of the estimated coefficients of the PLM.[19] T_c, T_y and T_ϵ represent the coefficients of the the equilibrium wage equation and therefore, implicitly defines the mapping from the PLM to the ALM. The interpretation of the ALM is that describes the stochastic process followed by wages if forecasts are made under the fixed rule given by the PLM. To formulate the T-mapping, $T(\hat{D}) = (T_c, T_y)$, I following the method of Marcet and Sargent (1989b) and Evans and Honkapohja (2012). This function maps the agents' perceptions about wage coefficients ($\hat{D}$) to their realized values ($T(\hat{D})$). The T-mapping is not know to agents.

The fixed point of this mapping is the REE of the model.

Definition: A rational expectations equilibrium is a matrix $D = [d^c, d^y]$ that satisfies $D = T(D)$. Thus a rational expectations equilibrium is a fixed point of the mapping T. Let me denote such equilibrium by $d^{c,RE}$ and $d^{y,RE}$.

T-mapping determines the evolution of beliefs in the transition to long-run equilibrium. The fact that agents learn about D_t introduces a different dynamic behavior. In particular, if firms believe that wages are going to be high tomorrow, this expectation will be transmitted to the actual realized wage through (1.27), and wages respond to this belief. This is a key feature of self-referential learning models that are absent in Bayesian learning models. Wage expectations affect realized wages, and agents use wages to update their expectations and so on.

Intuitively, the reason learning matters is the following. The higher the wage expectation, the lower the number of vacancies that firms open up, because their expected profits are lower. This makes the labor market tighter, which in turn reduces the probability of finding a job. When firms and workers negotiate wages, -through the bargaining process- in the presence of lower expected profits and a lower probability of finding a job, wages tend to fall. Figures (1.1a) and (1.1b) shows the $T_y(\hat{d}_t^c \hat{d}_t^{y,RE})$ and $T_y(\hat{d}_t^y)$, respectively, represented by the dashed line, which are linear decreasing functions. Values

[19] For exact formula for Φ^c, Φ^y and Φ^ϵ and the derivations see Appendix C.

of the coefficient $\hat{d}_t^c$ and $\hat{d}_t^y$, on the right hand side of the fixed point, which is the intersection between the 45 degree line and the T-mapping, indicate that agents expect wages above their realization and vice versa. The negative slopes of the dashed lines reflects the negative relationship between wage expectations and wages present in the model.

Because the agent's equation of wages can differ from the truth, his beliefs evolve over time. To understand the dynamic behavior of $\hat{D}$, it helps to analyze whether the learning rule induces instability in the state evolution. Using the theorems of Sargent and Williams (2005), if g is small enough, to analyze local stability, I need to check the following condition, known as E-stability condition.[20] Accordingly, the stability of the systems (1.22) is governed by the following ordinary differential equations (o.d.e.):

$$\begin{bmatrix} \dot{\hat{d}}^c \\ \dot{\hat{d}}^y \end{bmatrix} = \begin{bmatrix} T_c(\hat{d}_t^c \ \hat{d}_t^c) - \hat{d}^c \\ T_y(\hat{d}_t^y) - \hat{d}^y \end{bmatrix}. \tag{1.28}$$

For local stability, I need all eigenvalues of Ω are less than 0 in real part:

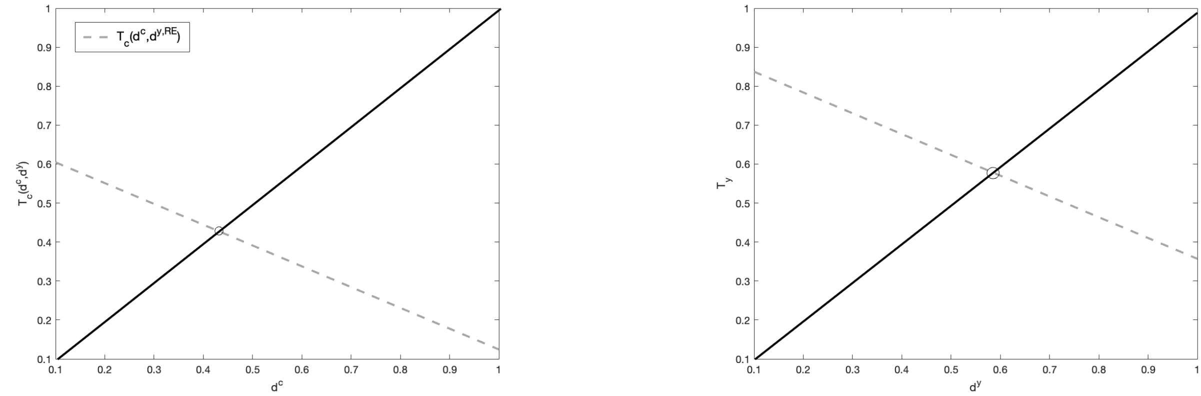
$$\Omega = \left. \frac{\partial [T(D) - D]}{\partial D} \right|_{D=D^{RE}} < 0. \tag{1.29}$$

The eigenvalues are real and negative, because the derivative of the T-mapping with respect to D is negative as one can see in figures (1.1a) and (1.1b), so that the condition for local stability of the learning mechanisms is satisfied. Therefore, one may expect constant gain models fluctuate around the REE, and least squared learning would converge.

1.4.7 Is the structure of Internal Rationality logically inconsistent? An heterogeneous approach

In the context of Internal Rationality (IR), the equilibrium wage function is solely dependent on productivity. This dependency induces a singularity in the objective density across wages and productivity. Given this scenario, a pertinent question arises: would the awareness of this singularity enable internally rational agents to accurately discern the equilibrium wage function through deductive reasoning? In other words, if agents are aware that productivity is the sole source of fundamental disturbances, does internal rationality inherently imply external rationality?

[20] If g is small enough, the local stability conditions are the same than assuming decreasing gain, $g = \frac{1}{t-1}$.



(a) Operator T-mapping for the constant coefficient

(b) Operator T-mapping for the productivity coefficient

Note: The dashed line represents the operator, T-mapping for each coefficient. The thick black line represents the 45-degree line. The ellipse represents the RE point—which is the fixed point of the T-mapping. In the first subplot, I assume that the coefficient of productivity is at its RE point. T-mapping is obtained under the calibration of the learning model specified in section 1.5.1.

The answer to this question turns out to be 'no'. In this section, I consider some sources of heterogeneity to highlight that an individual agent would not be able to infer the wage function from observations and her own behavior.

I consider a model in which firms are heterogeneous in some parameter values. Consider the previous RBC search and matching model with firms heterogeneous in the cost of opening a vacancy, c^j and their discount factor $\beta^{F,j}$, but they face the same productivity y_t that follows an AR(1) process. The values of the pair $(c^j, \beta^{F,j})$ are drawn from exogenously specified, possibly time-varying distribution. When solving their optimal problem, agents know their own values of $(c^j, \beta^{F,j})$. Therefore, the job creation condition for a firm endowed with $(c^j, \beta^{F,j})$ is as follows:

$$\frac{c^j}{q(\theta_t)} = E_t^{sf} \sum_{z=1}^{\infty} \left[\beta^{F,j}(1-\lambda)\right]^z \left[\frac{y_{t+z} - w_{t+z}}{1-\lambda}\right]. \tag{1.30}$$

Due to the fact that workers are homogeneous and there is no heterogeneity in productivity, there is no dispersion in wages. The equilibrium can be characterized by a degenerate distribution of wages arising from a bilateral bargaining problem between each firm and the average worker. The wage equation is represented by

$$\mathbf{w}_t = \alpha y_t + (1-\alpha)\left(b + \beta^W m\left(\int_j \frac{v_t^j}{u_t} dj\right) E_t^w \frac{\partial W_{t+1}}{\partial n_{t+1}}\right) \tag{1.31}$$

Wages are a function of aggregate vacancies. Equivalent, the previous equation can be written as follows

$$\mathbf{w}_t = \int_j \overline{\Phi}^{c,j} dj + \int_j \overline{\Phi}^{y,j} y_{t-1} dj + \int_j \overline{\Phi}^{\epsilon,j} \epsilon_t dj. \tag{1.32}$$

Assume that firms know that workers know the process of productivity and how to map productivity in their future surplus and the distribution of idiosyncratic parameters across firms. In, this case the firm can perfectly map productivity into the wage.

Instead, firms can know the process of productivity, know that workers form expectations in the right way, and still are not enough to know perfectly how productivity maps to the aggregate level of wages. In addition, I have to assume that firms know the distribution of the vacancy cost and discount factor across firms at each point in time.[21] From this example, I can conclude that it is logically consistent to assume that agents are rational and do not have perfect knowledge of the mapping between productivity onto wages. All I need to assume is that firms do not know the distribution of other firms' vacancy costs and utilities when they have to make the decision of posting vacancies, how the average worker forms its expectations at each point in time, or both.

In fact, section 1.7.1 extends the model to allow for discrepancies in the way workers and firms form expectations about wages. In that model, I arrive at an equilibrium wage equation that is different from the one obtained in the main paper.

1.5 Quantitative Analysis

In macroeconomics, search and matching models are essential tools for evaluating a range of labor market policies, both existing and prospective. Therefore, it's crucial for the selected model to accurately reflect observed moments in the data. However, the textbook search and matching model is not able to explain the observed fluctuations of unemployment and vacancies in the US

[21] In appendix C, you can see the structural form of the parameters Φ. $\overline{\Phi}$ are the parameters Φ evaluated at the RE point.

economy in response to productivity shocks of plausible magnitude. Additionally, the model demonstrates a lack of propagation, evidenced by an almost 1 contemporaneous correlation between the vacancy-unemployment ratio and productivity, a stark contrast to the near-zero correlation observed in empirical data. The model proposed by Di Pace et al. (2021) does not account for the latter fact.

This section evaluates the quantitative performance of the search and matching model with subjective wage beliefs. I formally estimate and test the model using a mixed strategy calibration that includes the Method of Simulated Moments (MSM). Testing helps me to focus on the ability of the model to explain the specific moments of the data described in Table 1.6.

1.5.1 Estimation of the Model

This section describes the calibration/estimation of the model parameters. The parameterization strategy is threefold. The model has 11 parameters: a subset is selected from the literature, another subset is picked from the US data, and the rest is estimated following the Method of Simulated Moments (MSM). [22]

Specifically, the vector $\hat{Z} = [\beta, \lambda, \alpha, \overline{y}, \nu]$ is obtained directly from the literature. I normalize time to one-quarter. Following the literature, I assume that the matching function is Cobb-Douglas. Without loss of generality, the steady state of productivity is normalized to 1. The value of the discount factor β is set to generate an annual real interest rate of approximately 5%. The value of the separation rate is set following Shimer (2005), who suggests a quarterly separation rate of 0.10. Hence, on average, jobs last for approximately 2.5 years.

Parameter	Description	Value	Source
β	discount factor	0.99	r=0.05
λ	separation rate	0.10	Shimer (2005)
α	bargaining power worker	0.50	Hosios rule: $\alpha = 1 - \nu$
ν	elasticity of matching function	0.50	standard
$\overline{y}$	steady state productivity	1.00	Normalization
σ_ϵ	st. dev. of productivity shocks	0.0058	Data
ρ	persistency of productivity	0.73	Data

Table 1.4: Calibrated quarterly parameters from literature and data

[22] This constitutes another difference with respect to the paper of Di Pace et al. (2021), they do not use MSM to estimate some parameters of the model.

I set the value of the elasticity of the matching function at 0.5 in line with the literature. This value lies within the plausible interval of [0.5 0.7] as surveyed by Petrongolo and Pissarides (2001). Following Hosios (1990), I set the bargaining power of the worker to 0.5. Using US data, I set the standard deviation and persistence of the productivity process to match the empirical behavior of labor productivity from 1990 to 2020. I find a quarterly autocorrelation and standard deviation of 0.7518 and 0.0058, respectively.

Defining $Z = [c, A, g, \sigma^{\beta}, b]$ as the vector of parameters to be estimated using an extension of the Simulated Method of Moments. These parameters are estimated to match the first 11 moments reported in table 1.6, are standardly used in the search and matching literature to summarize the main features of the labor market.[23] The MSM estimator is given by

$$\min_{Z} \quad (\hat{\mathcal{S}} - \tilde{\mathcal{S}}(Z))' \hat{\Sigma}_{\mathcal{S}}^{-1} (\hat{\mathcal{S}} - \tilde{\mathcal{S}}(Z)) \ . \tag{1.33}$$

where $\tilde{\mathcal{S}}(Z)$ is the vector of empirical moments to be matched, $\hat{\mathcal{S}}$ is the model moments counterpart and $\hat{\Sigma}_{\mathcal{S}}$ is the weighting matrix, which determines the relative importance of each statistic deviation from its target. I use a diagonal weighting matrix whose diagonal is composed of the inverse of the estimated variances of the data statistics.[24] Model-implied statistics are generated through a Montecarlo experiment with 10000 realizations. I formally test the hypothesis that any individual model statistics differ from its empirical counterpart.

The calibrated gain is inside the values found in the literature, which range from 0.002-0.05. Additionally, when compared against wage forecasts from the European Commission, the estimated gain is 0.086.[25] Meanwhile, the standard deviation of the belief shock, introduced in a variant of the learning model, is notably conservative. It is markedly smaller than the empirical standard deviation, which I estimated using the survey of the European Commission, standing at 0.01. These values estimated by survey data can be interpreted as upper bounds.

[23] I include functions of moments, instead of pure moments. I target 13 functions of moments. See appendix D for more details.

[24] In practice the estimated variances of the data moments, $\hat{\mathcal{S}}$ is used. The variances are obtained using a Newey-West estimator and the delta method as in Adam et al. (2016).

[25] Learning in the model is about wage level, therefore I have transformed the annual wage growth forecasts into de-trended levels to estimate the gain, ensuring the forecast generated by the European Commission remains parallel to forecasts implied by the model's learning mechanism. I estimate the gain parameter using a nonlinear least squares to minimize the distance between expectations implied by a constant gain algorithm and the survey expectations.

Parameter	Description	Values Learning	Values RE
c	cost of open a vacancy	0.45	1.30
A	efficiency matching technology	0.97	1.10
g	constant gain	0.009	0.00
σ^{β}	Std. wage belief shocks	0.0009	0.00
b	unemployment benefits	0.75	0.80

Table 1.5: Estimated quarterly parameters from SMM

1.5.2 Statistical Properties

In this section, the estimation results are reported. Table 1.6 contains statistics from the US labor market data and those implied by the model under rational expectations and learning dynamics.[26] The sample length of one simulation is T=120 quarters. I simulate the model 10,000 times and report the mean values of the statistics of interest as deviations from the steady state, facilitating comparison to earlier studies.[27] The first horizontal panel of Table 1.6 presents moments targeted in the estimation. The statistics considered are the relative standard deviation of each labor market variable with respect to the standard deviation of labor productivity, the correlation between each labor variable and labor market tightness, the latter's autocorrelation and wages, and the Beveridge curve represented by the correlation between unemployment and vacancies. The second horizontal panel displays a set of non-targeted moments, encompassing additional autocorrelations and correlations, coefficients from regressions (1.1) and (1.2) in Section 1.3.1 used to test Rational Expectations with forecast data from Professional Forecasters at the European Commission, and the correlation and standard deviation of their forecast errors. The second column in Table 1.6 reports the labor market moments from the data. The third and fourth columns present the moments and t-statistics of the learning model, respectively, while the fifth and sixth columns provide those of the RE model.

The simplest version of the DMP model with learning performs remarkably well quantitatively. The model statistics pass almost all the t-tests. It can generate a low contemporaneous correlation between labor market tightness and productivity, together with the high relative volatilities in the labor market, solving the two puzzles, the propagation and the amplification puzzle. This is achieved without generating rigid wages. This represents a significant success, being problematic for the standard

[26]The sources for the data can be found in Appendix E.

[27]The initial values of the employment, n_t, unemployment, u_t, productivity, y_t and wages w_t needed to initialize the algorithm are set to the steady state values. The initial value R is given by $R_0 = T^{-1} z_T' z_T$ where T is 155 quarters that represent a pre-sample period before 1990-Q1. The initial D_0 are set to the RE values, that is $d^{c,RE} = 0.4359$ $d^{y,RE} = 0.5869$ under the learning calibration.

Moment's Symbol	Data	Learning Model	t-stat	RE model Re-est	t-stat
Targeted moments					
$\sigma_{\tilde{u}/\sigma_{\tilde{y}}}$	11.952	8.591	1.622	0.767	5.400
$\sigma_{\tilde{v}/\sigma_{\tilde{y}}}$	13.221	16.162	-1.587	1.773	6.176
$\sigma_{\tilde{\theta}/\sigma_{\tilde{y}}}$	24.713	22.105	0.664	2.426	5.673
$\sigma_{\tilde{w}/\sigma_{\tilde{y}}}$	1.737	1.972	-1.022	0.741	4.328
$\rho(\tilde{y}_t, \tilde{\theta}_t)$	-0.040	0.145	-0.429	0.991	-2.400
$\rho(\tilde{v}_t, \tilde{\theta}_t)$	0.984	0.948	3.956	0.981	0.261
$\rho(\tilde{u}_t, \tilde{\theta}_t)$	-0.980	-0.968	-1.102	-0.894	-7.969
$\rho(\tilde{w}_t, \tilde{\theta}_t)$	0.780	0.949	-0.480	0.991	-0.600
$\rho(\tilde{\theta}_{t-1}, \tilde{\theta}_t)$	0.941	0.786	2.348	0.618	4.454
$\rho(\tilde{w}_{t-1}, \tilde{w}_t)$	0.826	0.785	1.302	0.703	3.840
$\rho(\tilde{u}_t, \tilde{v}_t)$	-0.927	-0.844	-4.134	-0.791	-6.771
Non Targeted moments					
$\rho(\tilde{u}_{t-1}, \tilde{u}_t)$	0.939	0.948	-0.50	0.867	3.7379
$\rho(\tilde{y}_t, \tilde{v}_t)$	0.034	0.159	-2.00	0.947	-14.99
$\rho(\tilde{y}_t, \tilde{u}_t)$	0.119	-0.1253	3.95	-0.945	17.187
$\rho(\tilde{w}_t, \tilde{u}_t)$	-0.796	-0.913	2.07	-0.945	2.63
$\rho(\tilde{w}_t, \tilde{v}_t)$	0.7389	0.87	-2.03	0.99	-3.88
b^E	0.15	0.39	-2.14	0.62	-4.20
b^R	0.75	0.85	0.35	0.74	0.035
$\rho(FE_t, FE_{t-1})$	0.68	0.75	-	0.00	-
$\sigma(FE)$	0.03	0.05	-	0.00	-

Table 1.6: Labor Market Statistics

Note: Data moments are computed over the period 1990Q1: 2020Q1. Moments have been computed as averages over 1000 simulations. b^R is the coefficientes of regression (1.1) and b^E is the coefficientes of regression (1.2) running in Section 1.3.1. Survey data: European Commision from 1990-2020. t-ratios are defined as $\sqrt{T}$(data moment-model moment)/(estimated standard deviation of the model moment).

real business cycle model. Job creation is driven by the difference between the expected productivity and the expected cost of labor in new matches. In my model, the learning mechanism makes wage expectations less responsive to changes in productivity, and this generates the amplification. Additionally, the equilibrium wage is now a function not only of productivity but also of expectations. This additional dynamics generated by learning about D as described in section 1.4.6, lead to wages fluctuating more extensively than productivity.

Furthermore, the model gives an explanation for the fact that the labor market tightness is not strongly correlated with productivity. This phenomenon occurs because the model introduces a novel source of fluctuations stemming from wage learning. The labor market tightness is not only a function of productivity but also of the estimated coefficients of wage expectations. Moreover, it effectively reduces the correlation between labor productivity and other labor market variables, such as unemployment and vacancies, an outcome that the REE version of the model fails to achieve.

Additionally, the model provides a more accurate interpretation of some facts observed in forecasts made by professional forecasters at the European Commission. It closely matches the coefficient b^E from regression (1.2), consistent with survey data findings, despite this coefficient not being a direct target of the model. Furthermore, the learning mechanism generates a correlation across forecast errors and a standard deviation greater than zero, lending support to the necessity of deviating from rational expectations to explain these facts fully. The model is not without its limitations. Large t-ratios of certain moments highlight areas of improvement, though it's important to consider the model's simplicity compared to others within the DMP literature.

I posit a scenario where agents are learning about two coefficients influencing wages and introduce a belief shock. There might be speculations on the specific coefficient driving these results or debates on whether the outcomes are attributed to learning or merely the shock in expectations. In section 1.7 dedicated to robustness, I demonstrate that the predominant factor is indeed the learning about the coefficient that goes with productivity. This re-calibrated model with just learning about d_t^y, achieves a relative standard deviation of the vacancy-unemployment ratio and unemployment at 19.9 and 5.22, respectively, and reduces the correlation between the vacancy-unemployment ratio and productivity to 0.39.

The key to understanding where the volatility in the learning model of d_t^y comes from, lies in the job creation equation (1.18). This equation is a function of the discounted presented value of profits, the difference of the infinite sums of expected revenues (Θ_y) and expected labor costs (Θ_w). That different can be written as:

$$\Theta_y - \Theta_w = \underbrace{C}_{R1} + \underbrace{\frac{\rho - \hat{d}_t^y}{1-\beta(1-\lambda)\rho} y_t}_{R2} + \underbrace{\left(\frac{1}{1-\beta(1-\lambda)\rho} - \frac{1}{1-\beta(1-\lambda)}\right)\hat{d}_t^y}_{R3}, \tag{1.34}$$

where $C = \frac{1-d^{c,RE}}{1-\beta(1-\lambda)} - \frac{\rho}{1-\beta(1-\lambda)\rho}$ is a constant.[28] Under RE the volatility in the discounted presented value of profits just come from R2. R3 is constant, so the volatility of that term is zero under RE, $var(\Theta_y - \Theta_w)^{RE} = (\frac{\rho - d^{y,RE}}{1-\beta(1-\lambda)\rho})^2 var(y_t)$. The solutions that tries to solve the volatility puzzle keeping RE, try to make the coefficient $d^{y,RE}$ smaller to increase $\frac{\rho-\hat{d}_t^y}{1-\beta(1-\lambda)\rho} y_t$. My mechanism does not operate in that way. With a small standard deviation of $\hat{d}_t^y$, R3 can add a significant volatility to the discounted presented value of profits, due to the large value of the difference that multiplies $\hat{d}_t^y$. Moreover, the small deviations of RE coming from the learning of $\hat{d}_t^y$ not just make R3 volatile, but also generate a higher volatility in R2. For example, under the proposed calibration R2 is 3 times more volatile than in the RE model.

The Effects of a Productivity Shock. To develop more the intuition on the role of wage expectations in labor market fluctuations, consider the model's impulse response functions of the labor market tightness, unemployment and the estimated coefficient of wages expectations, d_t^y to a positive standard deviation productivity shock. The impulse response functions of the labor market tightness and unemployment are expressed in percentage deviations from steady state.

Figure 1.2 reports the median impulse response functions of the labor market tightness and unemployment. It illustrates a pronounced impact of a productivity shock on the labor market under a learning framework compared to Rational Expectations (RE). Figure 1.3 shows the dynamics following for one of estimated coefficients that determined wage expectations, d_t^y after a positive productivity shock. Under learning, the productivity shock leads to revisions in wage beliefs that commence the period after the disturbance. Subsequent dynamics are largely driven by revisions to wage beliefs. Particularly, after the shock, firms think that the wages will be lower compared to the RE economy. For a number of periods, the coefficient undergoes a downward adjustment because

[28] See Appendix B for the details.

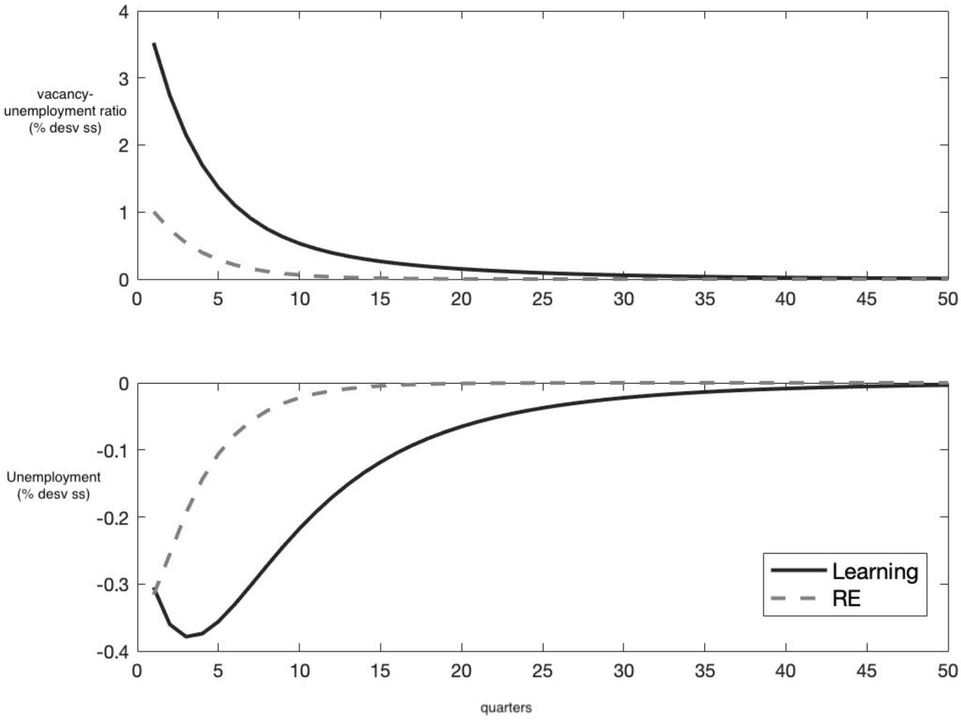


Figure 1.2: Impulses Responses to a positive Productivity Shock

Note: Impulse response functions of labor market tightness and unemployment following a one standard deviation positive productivity shock. The dashed line represents the learning model (calibrated under the RE model), while the solid line represents the RE model. The horizontal axis displays the number of quarters after the shock.

the forecasting error is negative, i.e., $w_{t-1} < d^{c,RE} + \hat{d}_t^y y_{t-1}$, until some given period. However, after a certain point, economic recovery ensues. Firms start the process of upwardly adjusting their expectations, $w_{t-1} > d^{c,RE} + \hat{d}_t^y y_{t-1}$ until they align with the RE benchmark. During this adjustment phase, firms' expectations temporarily deviate from the rational expectations framework as they adapt to integrating productivity changes into wage-setting. This adaptive period, where firms anticipate comparatively lower wages than under RE, prompts them to post more vacancies, leading to a decrease in unemployment. Additionally, it takes time to converge to the steady state, so the positive effect of the productivity shock in the labor market is more persistent.

1.6 Labor Market Policies and Welfare

Introduce learning as in surveys has more reliable and robust macroeconomic implications. However, arguably, an even more important value of the new models lies in their usefulness for analyzing policy.

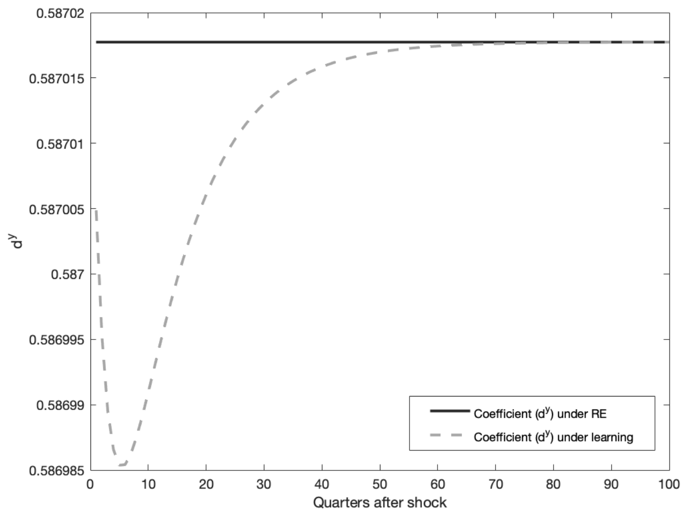


Figure 1.3: Impulses Responses to a positive Productivity Shock

Note: Impulse response functions of coefficient $\hat{d}_t^y$ following a one standard deviation positive productivity shock. The dashed line represents the estimated time-varying coefficient under the learning model (using the RE model calibration), and the solid line represents the constant coefficient $\hat{d}^{y,RE}$ under the RE model. The horizontal axis displays the number of quarters following the shock.

Given that the learning model reproduces the dynamics of the U.S. labor market data remarkably well, the next step is to analyze the effects of some labor market policies using such model. The goal is to provide policymakers with a more accurate understanding of the potential costs and benefits of these policies and highlight the importance of considering the impact of expectations on labor market outcomes. Learning could amplify the effects of a given policy. If policy makers do not take this effect into account, they may obtain biased estimates and perhaps incur a large cost to the economy after implementation. In the following section, I evaluate the differences in welfare and the standard deviation of unemployment when policymakers use the RE model versus the learning model to assess certain labor market policies. The standard deviation of unemployment can be considered a measure of uncertainty in the economy.

To quantify the welfare effects, I use the compensating variation method. This method calculates the number of consumption units that I should give to the representative individual of the economy, uniformly period after period, so that he or she would be indifferent between the economy subject to the base policy and the economy subject to the reform. The welfare measure in these comparisons,

λ, is defined from

$$E_0[\sum_{t=0}^{\infty} \beta^t (1+\lambda) c_t] = E_0[\sum_{t=0}^{\infty} \beta^t c_t^R], \tag{1.35}$$

where c_t is the aggregate consumption under the benchmark case and c_t^R is the aggregate consumption under a particular experiment. If $\lambda > 0$ there is a welfare gain; otherwise, there is a welfare loss.

1.6.1 Asymmetric Countercyclical UI Policy

Although unemployment insurance (UI) in principle remains constant regardless of labor market conditions, the United States adjusts its generosity during economic downturns. For instance, during the 2007-2011 labor market downturn, the weekly benefit amount increased by $25. More recently, during the pandemic, interventions such as the FPUC, which offered a weekly supplement in addition to full social security benefits, were implemented. Papers that try to look at the impact of these policies on the economy, for instance Schwartz (2013), they assume that agents have RE. It is important to analyze whether the fact that agents learn about wages amplifies the effects of such a policy.

I consider two rule-based systems that link the level of UI benefits to either GDP, denote by z, or unemployment, and vary the elasticity of the response to changes in these variables. The gdp-based rule is the following:

$$b_t = b - \phi \tilde{z}_{t-1} 1_{\tilde{z}_{t-1}<0}, \tag{1.36}$$

where b is the calibrated benchmark UI of each respective model, and ϕ represents the elasticity of UI with respect to gdp, that is, the percentage increase in UI for each percent drop in gdp with respect to their steady-state value, $\tilde{y}$. The UI is financed using taxes proportional to wages. Alternatively, the rule can be linked to unemployment rather than gdp as follows:

$$b_t = b + \phi \tilde{u}_t 1_{\tilde{u}_t<0}. \tag{1.37}$$

Notice that now, in the IR economy with the introduction of the time-varying UI, if agents internalize such policy in their expectations, the PLM of wages becomes:

$$w_t = d_t^c + d_t^y y_{t-1} + d_t^b b_t + \epsilon_t,$$
$$D_t = D_{t-1} + \nu_t. \tag{1.38}$$

Where $D_t = [d_t^c \;\; d_t^y \;\; d_t^b]$. Shocks $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$ and $\nu_t \sim \mathcal{N}(0_{3,1}, \sigma_\nu^2 I_3)$ are independent of each other. Such PLM incorporates additional learning on how unemployment benefits are mapped to wages. The RE economy in this context is the fixed point of the T-mapping implied by the above PLM.

Tables 1.7 and 1.8 showcase how λ values and unemployment standard deviation change when contrasting two policy rules against an economy where UI remains stable throughout business cycles. In both cases, the models predict decreased welfare and increased unemployment volatility. These results arise from the more generous UI benefits, which lead to elevated wages and reduced anticipated profits, thereby diminishing firms' motivation to create new job openings. It is noteworthy, however, that the outcomes diverge based on the chosen model and rule.

	Learning		RE	
ϕ	$\lambda(\%)$	$\text{std}(\tilde{u}_t)$	$\lambda(\%)$	$\text{std}(\tilde{u}_t)$
0	0	0.083	0	0.007
0.5	-0.1	0.09	-0.47	0.017
1	-0.44	0.117	-0.83	0.026
1.25	-2.78	0.20	-1.00	0.032

Table 1.7: GDP-based rule

Note: Values of λ, compared with the benchmark economy where UI is constant over the business cycle and the standard deviation of unemployment in an economy that undergoes UI reform, utilizing the GDP-based rule. The calibration used for each model is in table 3.3.1 and 3.3.2.

The Learning model appears to be more sensitive to changes in ϕ than the RE model, especially when $\phi > 1$. This implies that economies with agents that learn about wages may experience more pronounced welfare reductions when UI benefits become more reactive to changes in GDP. In specific terms, with ϕ at 1.25, the welfare reduction in the Learning model hits -2.78%, whereas the RE model shows a milder reduction of -1.00%. This stark contrast underlines the learning model's heightened

	Learning		RE	
ϕ	$\lambda(\%)$	std($\tilde{u}_t$)	$\lambda(\%)$	std($\tilde{u}_t$)
0	0	0.083	0	0.007
0.03	-1.31	0.09	-0.04	0.0079
0.05	-2.27	0.10	-0.1	0.0085
0.08	-3.80	0.14	-0.54	0.014

Table 1.8: Unemployment-based rule

Note: Values of λ, compared with the benchmark economy where UI is constant over the business cycle and the standard deviation of unemployment in an economy that undergoes UI reform, utilizing the Unemployment-based rule. The calibration used for each model is in table 3.3.1 and 3.3.2.

sensitivity and indicates a potential policy consideration.

Additionally, the standard deviation of unemployment escalates with ϕ in both models, suggesting that tying UI to GDP fluctuations introduces more uncertainty into the economy. Again, the learning model's reactions are considerably more accentuated, drawing attention to the intensified outcomes when agents are learning wage expectations.

When UI benefits are tethered to unemployment rates, the potential detriment to overall welfare is evident, and becomes concerning if the linkage is too responsive. To illustrate, the Learning model exhibits a welfare reduction from -1.31% at $\phi = 0.03$ to -3.80% at $\phi = 0.08$. In comparison, the RE model remains relatively stable against changes in ϕ. Unemployment's standard deviation generally amplifies with rising ϕ. The pronounced volatility observed in the learning model underscores the importance of acknowledging expectation formation methods among agents. This variability between models serves as an essential insight for policymakers: comprehending real-world expectation formulation is pivotal in understanding policy repercussions.

When expectations enter explicitly in model, certain policy rules may be associated with instability of the REE. Making UI responds to GDP and unemployment might on the other hand introduce additional volatility in the economy which might destabilize the system. Policimakers should only advocate policy rules which induce E-stable REE. Figures 1.4 and 1.5 show the values of ϕ under the two rules that induce to instability. While small reactions to GDP might not always lead to instability, policymakers should exercise caution when tying UI to unemployment, given its inherent

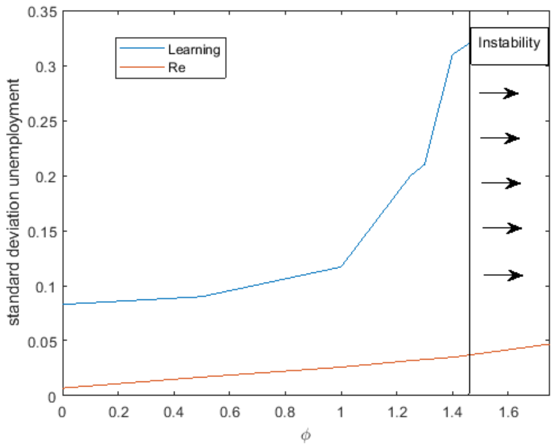


Figure 1.4: GDP-based rule

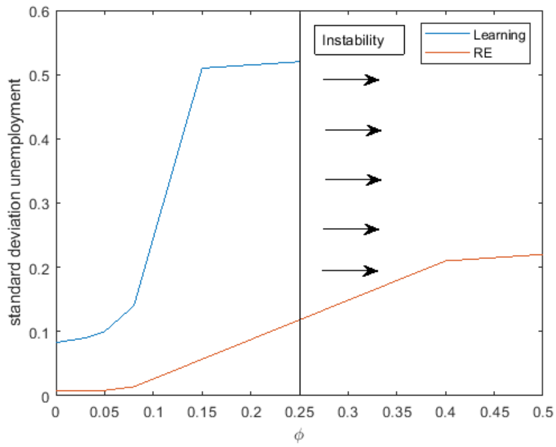


Figure 1.5: Unemployment-based rule

Note: The figure present the stability and instability (regions with arrows) for different values of ϕ. Y axis show the standard deviation of the unemployment associated at each value of ϕ. The stability of the system is given by (1.29).

volatility.

1.6.2 Symmetric Procyclical UI Policy

In an economy where agents learn, the existing US unemployment insurance system results in notable welfare costs due to its impact on job creation.[29] Can pro-cyclical unemployment benefits smooth cyclical fluctuations in unemployment and deliver substantial welfare gains? The current section seeks to answer that.

Unemployment fluctuations are driven by expectations. Policymakers might adjust unemployment benefits in response to economic indicators like unemployment or GDP to influence firms' expectations.

I explore two policy rules: a linear response to lagged GDP (rule I in table 1.9) and a linear response to unemployment (rule II in table 1.9). To grasp their implications, I study three scenarios: (I) an economy with rational expectations, (II) one where agents learn about wages and internalize the UI policy, and (III) where they learn about wages but disregard the UI policy

[29]Note that in this simplified model, workers are risk neutral, so I do not take into account the positive effects of unemployment benefit as insurance for workers against the risk of unemployment. The optimal policy of this model without risk adverse workers is zero IU.

I. GDP linear rule	$\mathrm{b}_t = b + \phi \tilde{y}_{t-1}$
II. Unemployment linear rule	$\mathrm{b}_t = b - \phi \tilde{u}_t$

Table 1.9: Unemployment Benefit Policy rules

Note: $\tilde{y}_{t-1}$ and $\tilde{u}_t$ represent deviations from the steady state. b is the unemployment benefits estimated from section 4.1.

The prior analysis assumed agents internalize the UI policy's response to unemployment or GDP during recessions. Thus, when forecasting wages, they'd factor in government adjustments to UI levels. However, it's plausible agents might overlook the government's business cycle reactions due to unclear policy communication or skepticism about government commitment. In this section, additionally, I explore the impact of such an information gap, assuming agents disregard this rule in their PLM, even as the government adjusts based on GDP or unemployment deviations.[30]

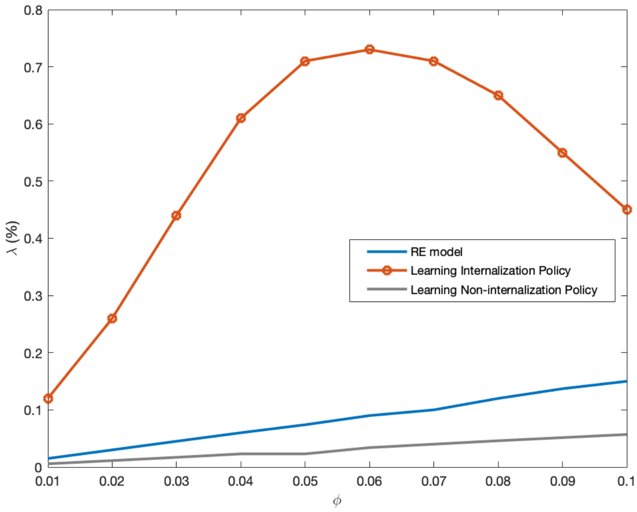


Figure 1.6: Welfare Implications of UI GDP linear rule

Note: The figure shows the welfare gains λ as defined in equation (31) for different combinations of coefficients for GDP using rule I defined in table (1.9). Welfare gains were computed as averages over 1000 simulations, each including 120 time periods.

Figure 1.6 depicts the welfare benefits of UI rules based on GDP from table 1.9 for varying GDP response coefficients. When agents internalize the policy during wage learning, substantial welfare improvements result. The policy impacts both wages and wage expectations. In expansions, subsidies

[30] In this case, the agents' PLM do not incorporates unemployment benefits, as in the benchmark model: $w_t = d_t^{f,c} + d_t^{f,y} y_{t-1} + \epsilon_t^f$.

decrease to boost job creation, while in recessions, the UI rises. Yet without such internalization, welfare gains shrink, mirroring gains under rational expectations. The welfare gain's relationship to GDP-rule has an inverted U-shape, indicating excessive GDP reactions can diminish welfare as UI rises in expansions.

Figure 1.7 displays welfare benefits from UI rules based on unemployment, as detailed in Table 1.9, across various unemployment response coefficients. While linear and symmetrical UI responses to unemployment could potentially add volatility, figure 1.7 indicates stability for smaller reactions. Within this stability zone, left of the dotted line, findings align with those of the GDP-based rule. When agents internalize and learn this policy, the potential welfare gains substantially outpace those under rational expectations or non-internalization scenarios.

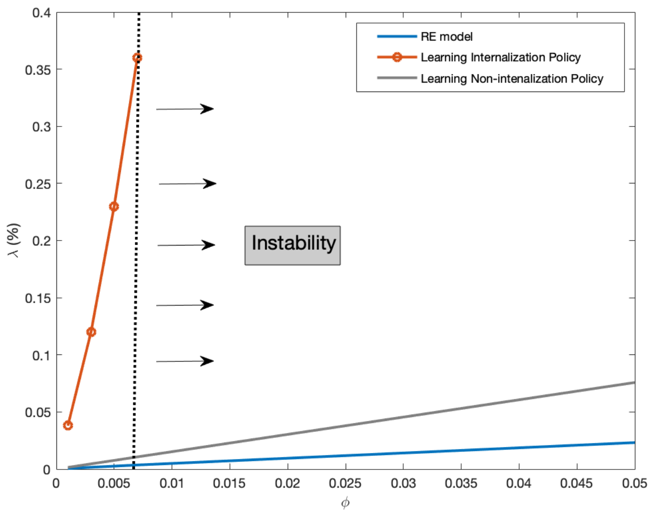


Figure 1.7: Welfare Implications of UI Unemployment linear rule

Note: The figure shows the welfare gains λ as defined in equation (31) for different combinations of coefficients for unemployment, using rule II defined in table (1.9). Welfare gains have been computed as averages over 1000 simulations, each one including 120 periods.

1.7 Robustness

In this section, I examine the performance of the learning mechanism under alternative assumptions and extensions. All tables I refer to in Section F.1 in the Appendix.

1.7.1 Asymmetric Perceived Law of Motions

In this section, I extend the standard search and matching frictions to account for differences in how both workers and firms anticipate future wages. The survey of Consumer Expectations indicates that workers on average do not internalize the effect of aggregate labor productivity on the formation of such beliefs.

To incorporate this fact, I have adjusted the system of beliefs of the worker. Particularly, I have introduced a new Assumption 2, which simplifies the model.[31]

Assumption 2. Individual workers perceived that the labor market tightness θ is constant over time, therefore $E_t^{\mathcal{P}^w}(\theta_{t+k}) = \overline{\theta}$.

Consequently, based on this assumption, workers consistently perceive the job-finding probability as a constant value, $\overline{f}$ over time.[32] When these assumptions are considered, the resultant workers' share of total surplus can be expressed as follows:

$$\mathcal{W}(m_t) - \mathcal{U}(m_t) = \sum_{j=1}^{\infty} \beta^{j-1}(1 - \lambda - \overline{f})^{j-1} E_t^{sw}[w_{t+j-1} - b]. \tag{1.39}$$

Firms adopt a perceived law of motion for wages that aligns with the minimal state variable, the one used in the main paper.

$$\begin{aligned} w_t &= d_t^{f,c} + d_t^{f,y} y_{t-1} + \epsilon_t^f, \\ D_t^f &= D_{t-1}^f + \nu_t. \end{aligned} \tag{1.40}$$

Where $D_t^f = [d_t^f \;\; d_t^y]$. Shocks $\epsilon_t^f \sim \mathcal{N}(0, \sigma_\epsilon^2)$ and $\nu_t \sim \mathcal{N}(0_{2,1}, \sigma_\nu^2 I_2)$ are independent of each other.

In contrast, workers do not use productivity to form wage expectations in line with the documented fact. Therefore, I assume workers believe that wages follow an unobserved component model of the

[31] This assumption is made to find a closed form solution if equilibrium wages when firms and workers form wage expectations differently.

[32] This assumption is in line with the empirical fact documented by Balleer et al. (2021). Using the Survey of Consumer Expectations, they find that workers do not update their expected labor market transition probabilities. Therefore, they find no empirical evidence of learning about labor market transition probabilities over the life cycle.

following form:

$$w_t = d_t^{w,c} + \epsilon_t^w,$$
$$d_t^{w,c} = d_{t-1}^{w,c} + u_t. \tag{1.41}$$

Shocks $\epsilon_t^w \sim \mathcal{N}(0, \sigma_\epsilon^2)$ and $u_t \sim \mathcal{N}(0, \sigma_u^2)$ are independent of each other. The previous setup defines a filtering problem in which agents need to decompose observed wages into its persistent and transitory elements.

The estimation takes place using the recursive least squares (RLS) algorithm. Agents estimate equations (1.41) and (1.40) and update their coefficient estimates for every period as new data become available. Workers' and firms' beliefs evolve according to the following schemes, respectively:

$$\hat{d}_t^{w,c} = \hat{d}_{t-1}^{w,c} + \gamma[\mathbf{w}_{t-1} - \hat{d}_{t-1}^{w,c}] + \epsilon_t^\beta \tag{1.42}$$

$$\hat{D}_t^f = \hat{D}_{t-1}^f + \gamma R_t^{-1} z_{t-1}[\mathbf{w}_{t-1} - \hat{D}_{t-1}^{f'} z_{t-1}] + \epsilon_t^\beta$$
$$R_t = R_{t-1} + \gamma(z_{t-1} z_{t-1}^{'} - R_{t-1}) \tag{1.43}$$

Where $D_t^f = [d_t^{f,c} \;\; d_t^{f,y}]'$ and $z_{t-1} = [1 \;\; y_{t-1}]$. ϵ_t^β is a shock to wage beliefs (sentiment shock), $\mathbf{w}_{t-1}$ denotes the realized previous wage, and γ denotes the constant gain $\in$ (0,1).

The actual law of motion of wages stemming from prior assumptions and the bargaining process is the following:

$$\mathbf{w}_t = [T_c(\hat{d}_t^{f,c} \;\; \hat{d}_t^{w,c}) \;\; T_y(\hat{d}_t^{f,y})][1 \;\; y_{t-1}]' + C_\epsilon \epsilon_t, \tag{1.44}$$

where $T_c(\hat{d}_t^{f,c} \;\; \hat{d}_t^{w,c})$ and $T_y(\hat{d}_t^{f,y})$ represents the T-mapping. I follow the method of Marcet and Sargent (1989a) and Evans and Honkapohja (2012) to formulate the function T-mapping that maps the agents' expectations - D=$[\hat{d}_t^{f,c} \;\; \hat{d}_t^{f,y} \;\; \hat{d}_t^{w,c}]$' - to their realized values. The T-mapping obtained in this section is different from that described in the main article.[33]

When the forecast model of some agents is misspecified, the natural limit of adaptive learning dynam-

[33] For T-mapping details, see Appendix D.

ics is called restricted perception rational expectation equilibrium (RP-REE). To this end, I apply the theory of Marcet and Sargent (1989a). In this version of the model, the worker makes decisions using a misspecified model to form wage expectations. In other words, the worker does not use one relevant state variable for forecasting.

Formally, there exists an n x 1 state vector z_t. Let z_{it} be any n_i x 1 vector $z_{it} = e_i z_t$, where $1 \geq n_i \geq n$ and e_i are the selector matrices for $i = w, f$. There are two types of agents, firms and workers, types f and w, which use $z_{ft} = z_t$ and $z_{wt} = e_w z_t$, respectively. In my environment, the state and the noise of the model at t are specified as

$$z_t = \begin{bmatrix} 1 \\ y_t \end{bmatrix}, \epsilon_t. \tag{1.45}$$

Firm behaves competitively, it forecasts w_t using $z_{ft} = [1 \;\; y_t]'$. On the other hand, worker behaves competitive as well. However, to forecast w_t, he uses a subset of z_t such that $z_{wt} = 1$. Under the described settings, the operator that determines the REE of my model is related to, but distinct from the described T-mapping. The restricted perception of one of the agents alters the relevant operator.

If the ALM of w_t is (1.44), then the linear least-squares projection of w_t on z_{t-1} for each agent is given by

$$E(w_{it}|z_{it-1}) = S_i(D) z_{it-1}, \tag{1.46}$$

where

$$S_i(D) = T(D)[M_{z_i}(D)^{-1} M_{z_i.z}(D)]', \;\; for \;\; i = f, w. \tag{1.47}$$

Where $M_{z_i}(D) = E z_{it} z_{it}'$ and $M_{z_i.z}(D) = E z_{it} z_t'$, $i = f, w$. Notice that for the firm, $S_f(D) = T(D)$. The operator $S_i(D)$ maps the perceptions D=[$\hat{d}_t^{f,c}$ $\hat{d}_t^{f,y}$ $\hat{d}_t^{w,c}$]'coefficients ($T(D)$, $S_w(D)$). The S-mapping determines the evolution of beliefs in transition to the Restricted Perception long-run equilibrium (RP-REE).

I now advance the following definition.

Definition: A Restricted Perception Rational Expectation Equilibrium (RP-REE) is a vector D=$[\hat{d}_t^{f,c} \;\; \hat{d}_t^{f,y} \;\; \hat{d}_t^{w,c}]'$ that satisfies D=S(D).

Thus the rational expectations equilibrium or the long-run equilibrium of this economy is a fixed point of the mapping S.[34] Let me denote such equilibrium D^{RPREE}. Notice that this concept of a rational expectations equilibrium is relative to the fixed information sets z_{wt} and z_{ft} specified by the model builder.

Agent's equation of wages can differ from the truth, his beliefs evolve over time. In this case, the stability of systems (1.42) and (1.43) is governed by the following ordinary differential equations (o.d.e.):

$$\begin{bmatrix} \dot{\hat{d}}^{f,c} \\ \dot{\hat{d}}^{f,y} \\ \dot{\hat{d}}^{w,c} \end{bmatrix} = \begin{bmatrix} T_c(\hat{d}_t^{f,c} \;\; \hat{d}_t^{w,c}) - \hat{d}^{f,c} \\ T_y(\hat{d}_t^{f,y}) - \hat{d}^{f,y} \\ S_w(\hat{d}_t^{f,c} \;\; \hat{d}_t^{w,c} \;\; \hat{d}_t^{f,y}) - \hat{d}^{w,c} \end{bmatrix}. \tag{1.48}$$

Figure 1.8 describes the phase diagram of this economy. The intersection between the 3 planes is the RP-REE.[35]

Table 1.E.3 shows the statistics coming from the simulation of the models. The calibration of the model is summarized in Tables (1.E.4) and (1.E.5). The learning model with a lower gain than the learning model in the main paper, is able to match very well the moments in the data. It is able to generate fluctuations in the labor market, account for the low correlation between the vacancy-unemployment ratio and productivity, together with flexible wages. Using t-statistics derived from asymptotic theory, I cannot reject the hypothesis that any of the individual model moments differ from the moments in the data in the estimated earning with asymmetric PLMs. Therefore, it performs slightly better than the learning model present in the main paper. Each time a productivity shock hits the economy, the worker becomes pessimistic (optimistic), which affects the realized wage

[34] For exact formula for S, and the derivations see Appendix D.

[35] For local stability, I need all eigenvalues of Ω are less than 0 in real part:

$$\Omega = \left.\frac{\partial[S(D) - D]}{\partial D}\right|_{D=D_f} < 0. \tag{1.49}$$

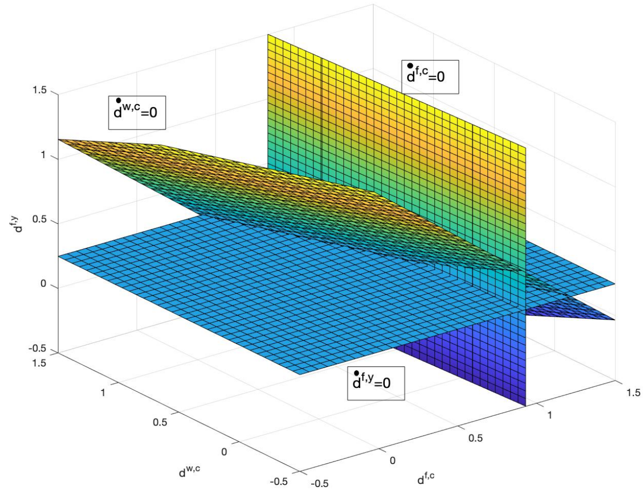


Figure 1.8: Phase Diagram

and generates a mistake in the enterprise's wage forecasts. This mechanism endogenously causes agents to deviate from rational expectations and propagate productivity shocks.

Additionally, to explores whether the model has the potential to quantitatively address the fact that I found in the survey of consumers. In each iteration, I carry out a similar test to the one carried out in the empirical part using the theoretical wage expectations of workers.[36] The productivity coefficient is not statistically significant on wage expectations 88% of the time at 5% significant level. Moreover, I reject the null hypothesis that coefficients are equal across regressions of the realize wages and wage' expectations 98% of the time.

1.7.2 Alternative Calibration

I assess the robustness of the expectation channel proposed in the paper to introducing changes in the calibration of some parameter values. Results are collected in the Table 1.E.1.

First, I examine the performance of the baseline model under RE with the solution proposed by

[36]Regressions run with the model to check RE:

$$\tilde{w}_{t+1} = a^1 + b^1 \tilde{w}_{t-1} + \delta_t^1 \tilde{y}_t + \epsilon_t \tag{1.50}$$

$$E_t \tilde{w}_{t+1} = a^2 + b^2 \tilde{w}_{t-1} + \delta_t^2 \tilde{y}_t + \nu_t \tag{1.51}$$

Hagedorn and Manovskii (2008) in order to compare both solutions under the same framework. According to them, the standard DMP model is unable to match the data because of an erroneous parametrization of two parameters: the instantaneous utility of being unemployment and workers' bargaining power. With a higher calibrated value for unemployment benefits closed to the steady-state of wages ($b = 0.955$), and a low bargaining power of the worker, close to zero ($\alpha = 0.05$), the model generates endogenous wage rigidities.[37] This can be seen in the significant drop in the RE value of the coefficient that goes with productivity in the linear wage equation, $d^{y,RE}$. Under the learning model 1, that coefficient moves around 0.58, while under the calibration of Hagedorn and Manovskii (2008) it is reduced to 0.11. The rigid wages increase second moment of the vacancy-unemployment ratio. However, the RE model, under Hagedorn and Manovskii (2008) calibration, fails in generating a relative standard deviation between wages and productivity higher than 1. Also, is no able to decrease significantly the correlation between the vacancy-unemployment ratio and productivity.

Second, I simulate the learning model 1 under the calibration of Shimer (2005), keeping the gain parameter equal to their estimated value reported in Table 1.5, third column. This exercise is running in order to check that the amplification is coming from the learning process instead of an alternative calibration of other parameters such us a higher bargaining power of the worker, $\alpha = 0.72$ or lower unemployment benefits, $b = 0.4$. Column 5 of Table 1.E.1 shows that the model still delivers the amplification of the labor market tightness, and a lower correlation between vacancy-unemployment ratio and productivity compared to the RE version of the model.

1.7.3 Learning about d_t^y

In the main paper, I introduce a framework where agents learn about two coefficients that impact wages and incorporate a belief shock. This might spark speculation about which specific coefficient yields these results or raise questions regarding whether the outcomes stem from the learning process or merely from the shock to expectations.

In this subsection, I evaluate the performance of the learning model when agents learn about how productivity correlates with wages, represented in the model by d_t^y. In this case, d_t^c is fixed at the RE value, and I exclude belief shocks during the model's simulation.

[37]The calibration of the remaining parameters follows table 3.3.1 and the fifth column of table 3.3.2.

Table 1.E.2 displays the moments derived from this learning model along with the respective t-statistics. The model that learns about d_t^y demonstrates impressive quantitative performance. The model statistics pass many of the t-tests. It can generate a low contemporaneous correlation between labor market tightness and productivity, together with the high relative volatilities in the labor market, solving the two puzzles, the propagation and the amplification puzzle. This represents a significant success. For instance, the model can generate unemployment's relative volatility that is 7.7 times greater than that produced under rational expectations.

1.7.4 Learning about the Constant

In the previous subsection, I argue that the primary factor behind labor market fluctuations in the learning model is the learning process about the coefficient governing the relationship between wages and productivity in the wage linear equation, denoted as productivity in the linear equation of wages, d^y.

Subsequently, I investigate the performance of the learning model when agents are provided with the rational expectations (RE) coefficient $d_t^y = d^{y,RE}\ \forall t$, with their learning focused solely on the constant term of the wage linear equation, d^c. I also make an assumption that the model environment is devoid of belief shocks to singularly emphasize the impact of learning about the constant. The findings, as presented in Column 7 of Table 1.E.1, reveal that the revised model yields a relative standard deviation of labor market tightness, which is 2.3 times higher than what's observed with the RE model. Nevertheless, it is almost 5 times lower compared to learning about d_t^y. Notably, the model effectively reduces the contemporaneous correlation between labor market tightness and labor productivity.

1.7.5 Information Assumption

In the baseline model, I assume that agents do not observe period wages at the time they make their forecasts. This is a standard assumption in the learning literature to avoid the simultaneous determination of forecast and endogenous variables. I will move away from that assumption, and I will assume that the forecast of wages, the decision of vacancies and realized wages are determined simultaneously. Consequently, agents beliefs evolve according to the following scheme:

$$\hat{D}_t = \hat{D}_{t-1} + \gamma R_t^{-1} z_{t-1}[w_t - \hat{D}'_{t-1} z_t],$$
$$R_t = R_{t-1} + \gamma(z_{t-1} z'_{t-1} - R_{t-1}). \quad (1.52)$$

Where $\hat{D}_t^f = [\hat{d}_t^c \;\; \hat{d}_t^y]'$ represent the estimated coefficients and $z_{t-1} = [1 \;\; y_t]$. Note that in this case, $(w_t - \hat{D}'_{t-1} z_t)$ is the most recent forecast error.

As you can see in Table 1.E.1, the re-calibrated model still delivers the amplification of the labor market tightness, and a lower correlation between vacancy-unemployment ratio and productivity compared to the RE version of the model. However, this is achieved at the cost of making wages too volatile.

1.7.6 Sentiment Shocks under RE

In this article, I claim that the key model that would solve the puzzle in the labor market is the combination of learning with expectation shocks. A question that may arise is whether the learning mechanism is necessary or whether the same results can be achieved by maintaining rational expectations and adding a sentiment shock.

To address this question, I do two exercise: (1) I simulate the RE model under the same calibrated parameter values than in the main paper and I introduce sentiment shock with the same standard deviation than in the learning model with sentiment shocks. (2) I simulate the RE model and I estimate the standard deviation of the expectation shock to match 2 moments, the amplification and the propagation.

Table 1.E.7 reports the results coming from the two exercises. Three things are observed: (I) the model under rational expectations with the same shock as in the learning version generates a volatility almost four times lower than the previous model and a higher correlation. (II) The introduction of this shock in RE generates negative autocorrelations in the labor market variables. (III) To generate the same relative volatility between labor market tightness and productivity as in learning, the volatility of the shock needs to be approximately multiplied by a factor of four. But the higher the volatility of the shock, the higher the negative autocorrelation of the unemployment vacancy ratio.

It seems that this shock does not operate in an economy where agents do not make small mistakes, as it leads to negative autocorrelations in the labor market.

1.8 Conclusions

A simple search and matching model applied to the business cycle is able to quantitatively replicate a number of important labor market facts in US, provided that one slightly relaxes the assumption that agents perfectly know how wages are formed in the market. I assume that agents are internally rational, in the sense that they formulate their doubts about market outcomes using a consistent set of subjective beliefs about wages and behave optimally given this set of beliefs. The system of beliefs is internally consistent in the sense that it specifies a proper joint distribution of wages and fundamentals at all dates. Moreover, the perceived distribution of wage behavior, although different from the true distribution, is nevertheless close to it and the discrepancies are hard to detect.

In such a setting, optimal behavior implies that agents learn about equilibrium wage process from past wage behavior. This gives rise to a self-referential model of learning about wages. I document that the relation between wage expectations and wages is negative in this model. Higher wage expectations will lead to larger drops in wages.

There are some facts in the labor market that appear puzzling from the RE viewpoint, and many papers question the quantitative consistency of the search and matching models. Sticky wages has gained attention among the literature to solve the puzzling behavior. However as Pissarides (2009) and Haefke et al. (2013) have point out, this mechanisms in matching models is difficult to justify on empirical ground. The learning model performs remarkably well, despite its simplicity. It generates fluctuations in the labor market variables, and it is not subject to the previous critics, in the sense that, the learning approach does not generate rigid wages. Moreover, RE is not supported by survey data in the formation of wage expectations. My result suggest that learning about wage behavior may be a crucial ingredient in understanding labor market volatility.

The finding that large labor market fluctuations can result from optimizing agents with subjective beliefs is also relevant from a policy perspective. As I show in the last part of the paper, If policy makers rely on RE models instead of IR ones, they can get bias estimates for effects of policies related to unemployment benefits. Also, it will be interesting include risk averse agents in the model, and take into account such channel, to get a non-zero optimal policy for unemployment benefits. Therefore, computing the optimal policy of unemployment benefits under internal rationality, and see if such optima policy is time dependent with respect to business cycle, appears to be an interesting avenue for further research.

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Appendix

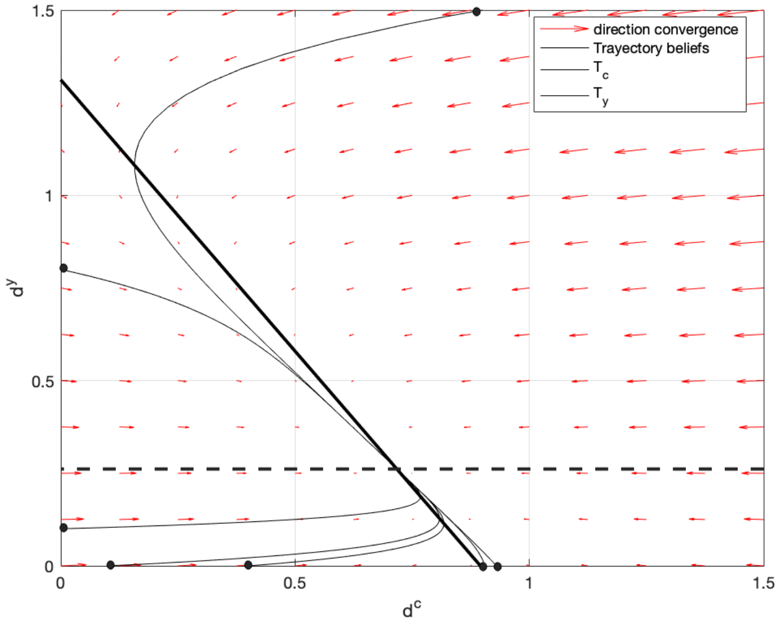


Figure 1..9: Phase Diagram

Note: The figure plots the trajectories for beliefs about the output gap and its actual realisations given different initial conditions. The thick black line represents the combination of beliefs $hatd^c$ and $hatd^y$ such that $\dot{\hat{d}}^c = 0$. The dashed line the represents the values of $hatd^y$ such that $\dot{\hat{d}}^y = 0$.

Appendix 1.A Survey Data

1.A.1 Business Leaders Survey

Survey design

In this section, I analyze the expectations of firms about future wages. For that, I use the Business Leaders Survey that is a monthly survey conducted by the Federal Reserve Bank of New York that asks service firms in its district - which includes New York State, upstate New Jersey, and Fairfield County, Connecticut - about recent and expected trends in key business indicators. Service sector participants respond to a questionnaire and report on a range of indicators, both in terms of recent and expected changes.

The survey is sent on the first business day of each month to the same pool of about 150 business executives, usually the president or CEO, in the region's service sector. In a typical month, about 100 responses are received by around the tenth of the month when the survey closes.

Questions and description of the data

I am interested in the questions regarding wages paid to the company's average worker. I am focus in two variables: wpcdina - current wages - this variable tells me how wages have changed in the last 3 months on average, and wpfdina - future wages - lets me know how companies expect wages to change in the next 6 months. Both variables are express in diffusion indexes -the difference between the percentage of firms that report an increase of wages minus the percentage of firms that report a decrease.

Before running the test, I have to deal with a problem of different horizons between the diffusion index of wage realization and the one for wage expectation. To solve this problem and keep things simple, assume that firms have in mind that wages are generated by the following process:

$$w_t = w_t^p + \epsilon_t, \tag{1.53}$$

$$w_t^p = w_{t-1}^p + \nu_t. \tag{1.54}$$

Where w_t^p is a persistent component and ϵ_t is a transitory component. Persistent component depends on the past persistent component and on a shock ν_t. Both shocks are independently and normally distributed with zero mean. Under this assumption, $E_t(w_{t+2}) = E_t(w_{t+1})$.

RE test

In this part, I construct a test to verify whether the assumption of RE holds in the data. I test if firms are rational on average when they form expectations about wages. Moreover, this test allow me to know how the firms form their expectations and which variables are important for the determination of them. Particularly, I want to test if aggregate productivity in New York State, New Jersey, and Connecticut is used by the firms on average, when they form their expectations.

To be rational, expectations have to efficiently use the available information. Forecasting errors, in RE models, have to be orthogonal to all information that was available and relevant to the agents at the moment of making forecasts. The realizations and expectations for each agent should identically incorporate the information contained in his/her past realizations.

b^R	b^E	p-value $H_0 : b^R = b^E$	p-value $H_0 : b^R \leq b^E$
0,0098 (0,0882)	0,16 (1,7378)	0,039	0,00427

Table 1.A.1: RE test firms

Note: t statistics HAC covariance estimator in parentheses. This table presents the results of the test (1.56). Monthly data from January 2007 to March 2022.

Let E_t^P denote firms' subjective expectations operator based on information up to time t, which can differ from the rational expectations operator E_t. Let $\tilde{w}_{t+1}$ denote the realized wages that the firms paid 3 months ahead, and let s_k be a measure of firms' subjective beliefs regarding future wages paid to their company's average worker that are possibly subject to measurement error, obtained, from survey data. Therefore, $s_{t+1} = E_t^P(\tilde{w}_{t+1})$ represents an estimate of firms' subjective beliefs about their wage paid 3 months ahead. Given the expectations horizon in the Business Leaders Survey, t+1 stands for 3 months -quarterly measure-.

$$w_{t+1} = a^R + \delta_t^R Y_{t-1} + \epsilon_t, \tag{1.55}$$

$$s_{t+1} = a^E + \delta_t^E Y_{t-1} + \nu_t. \tag{1.56}$$

Where Y represents the quarterly labor productivity growth in New York State, New Jersey, and Connecticut.

Under the null hypothesis of the information structure of RE ($H_0 : E_t = E_t^P$), $\hat{\delta}_R = \hat{\delta}_E$ must be estimates of the same regression coefficient, because $d_E = d_R$ under RE. Under the null hypothesis, the coefficients should equal. If coefficients across equations are different, we reject RE. Under RE, if Y_{t-1} is in the informational set of the agents for the time period t, the prediction error must be orthogonal to Y_{t-1}.

Table 10 shows the result of the test. Column 3 shows the p-values for the test. Additionally, column 4 shows the p-values for the one-sided test. The results provide evidence against the notion that firm survey expectations of wages are compatible with RE. The null hypothesis is rejected for the considered period, implying that rational expectations with respect to real wages do not provide empirical support in that period. The forecast error of wages is correlated with the productivity. It can be seen that firms on average underestimate the effect of productivity in the formation of wage expectations.

1.A.2 Survey of Consumer Expectations

Description of the Data

The survey data on expectations comes from the Survey of Consumer Expectations conducted by the Federal Reserve Bank of New York. To conduct the Rational Expectation test, I use two data sets: (1) the Survey of Consumer Expectations which report information on many demographic variables of the participants, and (2) the Survey of Consumer Expectations (SCE) Labor Market Survey.[38] The participants in the Labor Market Survey are members who participate in an SCE monthly survey in the prior three months. Since respondents are on the SCE panel for a maximum of 12 months, they end up participating in one, two or three labor market surveys during their tenure in the panel.

The SCE Labor Market Survey has two main sets of questions: (I) an ”Experiences” category that takes data on labor market outcomes, such us wage offers received in the past 4 months, search behavior, reservation wages, job satisfaction and (II) ”Expectations” category, which takes data on expectations related to job offer wage expectations, expected job transitions, and retirement.

The panel data enables me to explore how each individual's expectations relate to realizations in the next 4-months period, which allows me to assess the accuracy of expectations and how individuals form their expectations in the labor market. The data from the Labor Market Survey covers the waves from March 2014 to March 2020. The date on which each interview was conducted is represented by the subscript t. Individuals are surveyed every four months for up to one year, and I will identify each individual with the subscript i. In the sample, $26,01\%$ took one labor market survey, $33,29\%$ took two surveys, and $40,71\%$ took three surveys. To compute the forecast error for each agent and carry out a statistical test, agents must participate in at least two consecutive surveys. Therefore, I focus on the last two groups.

It is important to clarify the data assumptions that I made. I turn the variables of salary offers

[38] See https://www.newyorkfed.org/microeconomics/databank.html for details.

and expectations into real terms, the base period of March 2014, using the consumer price index (CPI) for All Urban Consumers from the Federal Reserve Bank of St. Louis. [39] I transform the annual earnings of offers and expectations into hourly earnings, taking into account whether the contract is part-time or full-time. If people work full-time, we divide earnings by 2080 (52 weeks times 40 hours), and if people work part-time, we divide earnings by 1040 (52 weeks, 20 hours). With respect to beliefs, if anyone has received only part-time offers, we assume that her/his beliefs are about part-time work; otherwise, we assume that her/his beliefs are about full-time work. I drop respondents whose revision in beliefs between surveys or the gap between the realizations and the previous period's expectation is greater (lower) than quartile 99 (quartile 1). Finally, I focus on the data from November 2014 onward, when questions about current job offers and expectations about future offers were added to the survey.

Main Questions

Question NL2 give me the realized wage offer of each agent, w_t^i, where t denotes four-month period. It asks participants the annual salary of the three best offers they received in the last 4 months, and whether they were full-time or part-time offers. More precisely, the question is the following:

What was the annual salary of this job offer? An was it for a full-time or part-time job?/ Thinking about 3 best job offers that you received in the last 4 months. What was their annual salary? And were they for a full-time or a part-time job? Note the best offer is the offer you would be most likely to accept.

Each agent can report at most 3 offers; therefore, I calculate and average offer for each agent.

On the other hand, question OO2a, allow me to know the expected wage offer of each agent, $E_t^i(w_{t+1}^i)$, where $t+1$ represents the next four-month period. It asks respondents reporting a non-zero percent chance of receiving a job offer about the average salary of the offers they may receive within the coming four months. Particularly, the question is the following:

Think about the job offer that you may receive within the coming four months. Roughly speaking,

[39]Due to the fact that the survey is four-monthly, I transformed the quarterly data into four-monthly data using interpolation methods. Source: U.S. Bureau of Labor Statistics, Consumer Price Index for All Urban Consumers: All Items in U.S. City Average [CPIAUCSL], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/CPIAUCSL, June 14, 2021.

what do you think the average annual salary for these offers will be for the first year?

Appendix 1.B Data Source

The time series are presented as seasonally adjusted quarterly series. The period covered is from 1990-Q1 to 2020-Q1, a total of 277 quarters.

Unemployment: U.S. Bureau of Labor Statistics, Unemployment Level [UNEMPLOY], retrieved from FRED, Federal Reserve Bank of St. Louis;
`https://fred.stlouisfed.org/series/UNEMPLOY`, October 20, 2022.

Vacancies: To get the vacancy level, I combine two sources. Barnichon, Regis. 2010. "Building a composite Help-Wanted Index", retrieved from
`https://sites.google.com/site/regisbarnichon/data` and U.S. Bureau of Labor Statistics, Job Openings [JTS10000000JOL], retrieved from
`https://data.bls.gov/timeseries/JTS10000000JOL`, October 20, 2022.

Labor productivity: U.S. Bureau of Labor Statistics, Nonfarm Business Sector: Output per Job for All Employed Persons [PRS85006163], retrieved from FRED, Federal Reserve Bank of St. Louis;
`https://fred.stlouisfed.org/series/PRS85006163`, October 20, 2022.

Wages: U.S. Bureau of Economic Analysis, Gross domestic income: Compensation of employees, paid: Wages and salaries [A4102C1Q027SBEA], retrieved from FRED, Federal Reserve Bank of St. Louis;
`https://fred.stlouisfed.org/series/A4102C1Q027SBEA`, October 12, 2022.

Consumer Price Index: U.S. Bureau of Labor Statistics, Consumer Price Index for All Urban Consumers: All Items in U.S. City Average [CPIAUCSL], retrieved from FRED, Federal Reserve Bank of St. Louis;
`https://fred.stlouisfed.org/series/CPIAUCSL`, October 12, 2022.

Appendix 1.C Theoretical Model

1.C.1 Equilibrium equations

To determine the labor market tightness of the economy, I have to start with the job creation condition:

$$\frac{c}{q(\theta_t)} = E_t^{sf} \sum_{j=1}^{\infty} [\beta(1-\lambda)]^j [\frac{y_{t+j} - w_{t+j}}{1-\lambda}] \tag{1.57}$$

Plugging the expectations of productivity and wages; $E_t y_{t+j} = (1-\rho^j) + \rho^j y_t$ and $E_t w_{t+j} = \hat{d}_t^c + \hat{d}_t^y((1-\rho^{j-1}) + \rho^{j-1} y_t)$, the labor market tightness can be writte as follows:

$$\theta_t = (\frac{A\beta}{c})^{\frac{1}{1-\nu}} [\Theta_y - \Theta_w]^{\frac{1}{1-\nu}}, \tag{1.58}$$

where Θ_y represents the present discount revenues and Θ_w the present discount labor costs. These two can be writte as

$$\begin{aligned} \Theta_y &= \frac{1}{1-\beta(1-\lambda)} + \frac{\rho}{1-\beta(1-\lambda)\rho}(y_t - 1) \\ \Theta_w &= \frac{\hat{d}_t^c + \hat{d}_t^y}{1-\beta(1-\lambda)} + \frac{\hat{d}_t^y}{1-\beta(1-\lambda)\rho}(y_t - 1). \end{aligned} \tag{1.59}$$

Therefore, vacancies are determine by

$$v_t = u_t(\frac{A\beta}{c})^{\frac{1}{1-\nu}} [\Theta_y - \Theta_w]^{\frac{1}{1-\nu}}. \tag{1.60}$$

1.C.2 Wages

Wages are negotiated according to a Nash bargaining process. The wage maximizes the joint surplus of a match between workers and firms. The maximization problem is the following:

$$\max_{w_t} \quad [\mathcal{W}(m_t) - \mathcal{U}(m_t)]^{\alpha} \mathcal{J}_t^{1-\alpha} \tag{1.61}$$

where α is the workers' bargaining power. The first order condition is as follows:

$$\alpha\left(\mathcal{W}-\mathcal{U}\right)^{\alpha-1}\left(\mathcal{J}_t\right)^{1-\alpha}+\left(\mathcal{W}-\mathcal{U}\right)^{\alpha}\left(1-\alpha\right)\left(\mathcal{J}_t\right)^{-\alpha}\left(-1\right)=0,$$
$$\alpha\left(\mathcal{W}-\mathcal{U}\right)^{\alpha-1}\left(\mathcal{J}_t\right)^{1-\alpha}=\left(\mathcal{W}-\mathcal{U}\right)^{\alpha}\left(1-\alpha\right)\left(\mathcal{J}_t\right)^{-\alpha},$$
$$\alpha\left(\mathcal{J}_t\right)=\left(1-\alpha\right)\left(\mathcal{W}-\mathcal{U}\right). \tag{1.62}$$

Therefore, the following equalities are satisfied:

$$\mathcal{W}-\mathcal{U}=\alpha S_t, \tag{1.63}$$

$$\mathcal{J}_t=(1-\alpha)S_t. \tag{1.64}$$

Where S_t is the total surplus of the match.

$$S_t=(\mathcal{W}-\mathcal{U})+\mathcal{J}_t. \tag{1.65}$$

Plugging the surpluses into (1.62), I come up with:

$$\alpha\left[y_t-w_t+\beta\left((1-\lambda)E_t^{\mathcal{P}^f}(\mathcal{J}_{t+1})\right)\right]=$$
$$(1-\alpha)\left[w_t-b+\beta\left((1-\lambda-f(\theta_t))E_t^{\mathcal{P}^w}(\mathcal{W}(m_{t+1}))-\mathcal{U}(m_{t+1})\right)\right]. \tag{1.66}$$

Assuming that agents belief that (62) and (63) hold in expectations,

$$\alpha\left[y_t-w_t+\beta E_t^{\mathcal{P}^f}\left[(1-\lambda)((1-\alpha)S_{t+1})\right]\right]=$$
$$(1-\alpha)\left[w_t-b+\beta E_t^{\mathcal{P}^w}\left[(1-\lambda-f(\theta_t))(\alpha S_{t+1})\right]\right] \tag{1.67}$$

Doing some algebra, I come up with

$$(1-\alpha)w_t-(1-\alpha)b+(1-\alpha)\beta E_t^{\mathcal{P}^w}\left[(1-\lambda-f(\theta_t))(\alpha S_{t+1})\right]=$$
$$\alpha y_t-\alpha w_t+\alpha\beta E_t^{\mathcal{P}^f}\left[(1-\lambda)((1-\alpha)S_{t+1})\right] \tag{1.68}$$

Let's assume that $E_t^{\mathcal{P}^f}=E_t^{\mathcal{P}^w}=E_t^{\mathcal{P}}$,

$$w_t=\alpha y_t+(1-\alpha)b+\beta f(\theta_t)\alpha E_t^{\mathcal{P}}(S_{t+1})(1-\alpha).$$

Finally, if both agents know that the FOC of firms hold, $(1-\alpha)\beta E_t^{\mathcal{P}}(S_{t+1}) = \frac{c\theta_t}{f(\theta_t)}$, I come up with the following expression:

$$\begin{aligned} w_t &= \alpha y_t + (1-\alpha)b + f(\theta_t)\alpha \frac{c\theta_t}{f(\theta_t)} \\ &= \alpha(y_t + c\theta_t) + (1-\alpha)b. \end{aligned} \tag{1.69}$$

1.C.3 T-mapping

First all, I linearize the job creation condition applying a first-order Taylor polynomial of this equation at the steady state $\theta = \overline{\theta}$, $w = \overline{w}$ and $y = \overline{y} = 1$. The job creation condition is represented by the following equation:

$$\frac{c}{\beta q(\theta_t)} = E_t^{\mathcal{P}f} \sum_{j=1}^{\infty} [\beta(1-\lambda)]^{j-1} \left[y_{t+j} - w_{t+j}\right] \tag{1.70}$$

I take the first-order Taylor polynomial of each component of the previous equation:

$$\frac{c}{\beta q(\theta_t)} = \frac{c}{\beta q(\overline{\theta})} - \frac{c}{\beta q(\overline{\theta})^2} \frac{\partial q(\overline{\theta})}{\partial \overline{\theta}} (\theta_t - \overline{\theta}) \tag{1.71}$$

$$E_t^{\mathcal{P}f} \left(\sum_{j=1}^{\infty} [\beta(1-\lambda)]^{j-1} y_{t+j} \right) = \frac{1}{1-\beta(1-\lambda)} + E_t^{\mathcal{P}f} \left(\sum_{j=1}^{\infty} [\beta(1-\lambda)]^{j-1} (y_{t+j} - 1) \right) \tag{1.72}$$

$$E_t^{\mathcal{P}f} \sum_{j=1}^{\infty} [\beta(1-\lambda)]^{j-1} w_{t+j} = \frac{\overline{w}}{1-\beta(1-\lambda)} + E_t^{\mathcal{P}f} \sum_{j=1}^{\infty} [\beta(1-\lambda)]^{j-1} (w_{t+j} - \overline{w}) \tag{1.73}$$

Therefore, I can write equation (1.70) as

$$\frac{c}{\beta q(\overline{\theta})} - \frac{c}{\beta q(\overline{\theta})^2} \frac{\partial q(\overline{\theta})}{\partial \overline{\theta}} (\theta_t - \overline{\theta}) = E_t^{\mathcal{P}f} \sum_{j=1}^{\infty} [\beta(1-\lambda)]^{j-1} [y_{t+j} - w_{t+j}], \tag{1.74}$$

$$\theta_t = \overline{\theta} + \phi E_t^{sf} \sum_{j=1}^{\infty} [\beta(1-\lambda)]^{j-1} [y_{t+j} - w_{t+j}]. \tag{1.75}$$

where $\phi = \frac{\beta q(\overline{\theta})^2}{c(q'(\overline{\theta}))}$. I plug the previous equation into the wage equation, I come up with

$$w_t = \alpha \left(y_t + c \left(\overline{\theta} + \phi E_t^{\mathcal{P}^f} \sum_{j=1}^{\infty} [\beta(1-\lambda)]^{j-1} [y_{t+j} - w_{t+j}] \right) \right) + (1-\alpha)b. \tag{1.76}$$

Taking into account the expectation; $E_t^{\mathcal{P}} y_{t+j} = (1-\rho^j) + \rho^j y_t$ and $E_t^{\mathcal{P}} w_{t+j} = \hat{d}_t^c + \hat{d}_t^y((1-\rho^{j-1}) + \rho^{j-1} y_t)$, I come up with:

$$w_t = \Phi^c + \Phi^y y_{t-1} + \Phi^\epsilon \epsilon_t, \tag{1.77}$$

where

$$\Phi^c = \alpha \left[c\overline{\theta} + \frac{c\phi}{1-\beta(1-\lambda)} \left[1 - (\hat{d}_t^c + \hat{d}_t^y) \right] + (1-\rho) - \rho \left[\frac{\rho - \hat{d}_t^y}{1-\beta(1-\lambda)\rho} \right] \right] + (1-\alpha)b,$$

$$\Phi^y = \rho \left[\alpha + \phi\alpha c \left[\frac{\rho - \hat{d}_t^y}{1-\beta(1-\lambda)} \right] \right],$$

$$\Phi^\epsilon = \left[\alpha + \phi\alpha c \left[\frac{\rho - \hat{d}_t^y}{1-\beta(1-\lambda)} \right] \right]. \tag{1.78}$$

1.C.4 Method of Moments

$$\min_{\theta} \quad (\hat{\mathcal{S}}_i - \tilde{\mathcal{S}}_i(\theta))' \hat{\Sigma}_{\mathcal{S}}^{-1} (\hat{\mathcal{S}}_i - \tilde{\mathcal{S}}_i(\theta)) \ . \tag{1.79}$$

Where $\hat{\Sigma}_{\mathcal{S}}$ is the varianza of the moments.

The Statistics and Moment Functions

This section gives explicit expressions for the statistics function S(.) and the moment functions *h(.)*.

The undrerlying sample moments needed to construct statistics of interes are:

$$\hat{M}_N = \frac{1}{N} \sum_{t=1}^{N} h(y_t), \tag{1.80}$$

where $h(.)$ is defined as
$$\begin{bmatrix} \tilde{v}_t \\ \tilde{u}_t \\ \tilde{\theta}_t \\ \tilde{y}_t \\ \tilde{w} \\ \tilde{v}_t^2 \\ \tilde{u}_t^2 \\ \tilde{\theta}_t^2 \\ \tilde{y}_t^2 \\ \tilde{w}_t^2 \\ \tilde{v}_t\tilde{\theta}_t \\ \tilde{u}_t\tilde{\theta}_t \\ \tilde{y}_t\tilde{\theta}_t \\ \tilde{w}_t\tilde{\theta}_t \\ \tilde{v}_t\tilde{u}_t \\ \tilde{\theta}_t\tilde{\theta}_{t-1} \\ \tilde{w}_t\tilde{w}_{t-1} \end{bmatrix}$$

The 13 statistics I consider can be expresed as functions of the moments as follows:

$$\mathcal{S}(M) = \begin{bmatrix} \sigma_{\tilde{v}} \\ \sigma_{\tilde{u}} \\ \sigma_{\tilde{\theta}} \\ \sigma_{\tilde{y}} \\ \sigma_{\tilde{w}} \\ \rho(\tilde{v}_t, \tilde{\theta}_t) \\ \rho(\tilde{u}_t, \tilde{\theta}_t) \\ \rho(\tilde{y}_t, \tilde{\theta}_t) \\ \rho(\tilde{y}_t, \tilde{\theta}_t) \\ \rho(\tilde{w}_t, \tilde{\theta}_t) \\ \rho(\tilde{u}_t, \tilde{v}_t \\ \rho(\tilde{\theta}_{t-1}, \tilde{\theta}_t) \\ \rho(\tilde{w}_{t-1}, \tilde{w}_t) \end{bmatrix} = \begin{bmatrix} \sqrt{M_6 - M_1^2} \\ \sqrt{M_7 - M_2^2} \\ \sqrt{M_8 - M_3^2} \\ \sqrt{M_9 - M_4^2} \\ \sqrt{M_{10} - M_5^2} \\ \frac{M_{11} - M_1 M_3}{\sqrt{(M_6 - M_1^2)(M_8 - M_3^2)}} \\ \frac{M_{12} - M_2 M_3}{\sqrt{(M_7 - M_2^2)(M_8 - M_3^2)}} \\ \frac{M_{13} - M_4 M_3}{\sqrt{(M_9 - M_4^2)(M_8 - M_3^2)}} \\ \frac{M_{14} - M_5 M_3}{\sqrt{(M_{10} - M_5^2)(M_8 - M_3^2)}} \\ \frac{M_{15} - M_1 M_2}{\sqrt{(M_2 - M_1^2)(M_7 - M_2^2)}} \\ \sqrt{M_8 - M_3^2} \\ \sqrt{M_{10} - M_5^2} \\ \frac{M_{17} - M_3^2}{S_{11}^2} \\ \frac{M_{18} - M_5^2}{S_{12}^2} \end{bmatrix}, \tag{1.81}$$

where M_i drenotes the ith element of M.

I compute the t-statistics for a particular statistic i as follows:

$$\sqrt{N}\frac{\mathcal{S}_i - \mathcal{S}_i^M}{\hat{\sum}_{\mathcal{S}}}, \tag{1.82}$$

where $\mathcal{S}_i$ are the i statistic of the data and $\mathcal{S}_i^M$ is the i statistic coming from the model. $\hat{\sum}_{\mathcal{S}}$ is the variance for the sample statistics $\mathcal{S}$:

$$\hat{\sum}_{\mathcal{S}} = \frac{\partial S}{\partial M'}\hat{\mathcal{S}}_w\frac{\partial S'}{\partial M} \tag{1.83}$$

I can test if the ability of the model to explain individual moments using t-statistics based on formal asymptotic distribution:

$$\sqrt{N}\frac{\mathcal{S} - \mathcal{S}^M}{\hat{\sum}_{\mathcal{S}}} \to N(0,1). \tag{1.84}$$

Appendix 1.D Asymmetric Perceived Law of Motions

1.D.1 Bargainig

Wages are negotiated according to a Nash bargaining process. The wage maximizes the joint surplus of a match between workers and firms. The FOC of the problem is:

$$\alpha\left(\mathcal{J}_t\right) = (1-\alpha)\left(\mathcal{W}(m_t) - \mathcal{U}(m_t)\right)$$

$$\alpha(\tfrac{1}{1-\beta(1-\lambda)\rho}y_t + \tfrac{\beta(1-\lambda)}{1-\beta(1-\lambda)} - \tfrac{\beta(1-\lambda)\rho}{1-\beta(1-\lambda)\rho} - w_t - \tfrac{\beta(1-\lambda)}{1-\beta(1-\lambda)}\hat{d}_t^{f,c} - \hat{d}_t^{f,y}[\tfrac{\beta(1-\lambda)}{1-\beta(1-\lambda)} + \tfrac{\beta(1-\lambda)}{1-\beta(1-\lambda)\rho}(y_t-1)] = (1-\alpha)(w_t + \tfrac{1}{1-\beta(1-\lambda-\bar{f})}[\beta(1-\lambda-\bar{m})\hat{d}_t^{w,c} - b]),$$

$$\mathrm{w}_t = \alpha[\tfrac{\beta(1-\lambda)}{1-\beta(1-\lambda)}(1-\hat{d}_t^{f,c}) + \tfrac{1-\rho}{1-\beta(1-\lambda)\rho} - \tfrac{\beta(1-\lambda)}{1-\beta(1-\lambda)\rho}] + (1-\alpha)[\tfrac{b}{1-\beta(1-\lambda-\bar{f})} - \tfrac{\beta(1-\lambda-\bar{m})}{1-\beta(1-\lambda-\bar{f})}\hat{d}_t^{w,c}] + ...$$

$$...+\alpha\rho[\tfrac{1}{1-\beta(1-\lambda)\rho} - \tfrac{\beta(1-\lambda)\hat{d}_t^{f,y}}{1-\beta(1-\lambda)\rho}]y_{t-1} + \alpha[\tfrac{1}{1-\beta(1-\lambda)\rho} - \tfrac{\beta(1-\lambda)\hat{d}_t^{f,y}}{1-\beta(1-\lambda)\rho}]\epsilon_t.$$

Therefore,

$$T_c = \alpha[\frac{\beta(1-\lambda)}{1-\beta(1-\lambda)}(1-\hat{d}_t^{f,c}) + \frac{1-\rho}{1-\beta(1-\lambda)\rho} - \frac{\beta(1-\lambda)}{1-\beta(1-\lambda)\rho}] + ...$$
$$... + (1-\alpha)[\frac{b}{1-\beta(1-\lambda-\bar{f})} - \frac{\beta(1-\lambda-\bar{f})}{1-\beta(1-\lambda-\bar{f})}\hat{d}_t^{w,c}], \tag{1.85}$$

$$T_y = \alpha\rho[\frac{1}{1-\beta(1-\lambda)\rho} - \frac{\beta(1-\lambda)\hat{d}_t^{f,y}}{1-\beta(1-\lambda)\rho}], \tag{1.86}$$

$$C_\epsilon = \alpha[\frac{1}{1-\beta(1-\lambda)\rho} - \frac{\beta(1-\lambda)\hat{d}_t^{f,y}}{1-\beta(1-\lambda)\rho}]. \tag{1.87}$$

1.D.2 S-mapping

In this section, I derive the mapping S. Firstly, I will derive the operator S for the worker, the agent that have a misspeficied model in mind to form the expectations of wages. Following Marcet and Sargent (1989a) the formula is the following:

$$S_w(D) = T(D)[M_{z_w}(D)^{-1}M_{z_w.z}(D)]', \tag{1.88}$$

where $T(D) = [T_c \;\; T_y]$, $M_{z_w}(D) = Ez_w z_w' = 1$ and $M_{z_w.z}(D) = Ez_w z' = E[1 \;\; y]'$. Assuming that the expectation of y is set to 1, $S_w(D) = T_c + T_y\bar{y}$.

On the other hand, given the fact that the firm has the right model in mind to form expectations, $T_f(D) = T(D)$. Therefore,

$$S(D) = \begin{bmatrix} T(D) \\ S_w(D)] \end{bmatrix} = \begin{bmatrix} T_c \\ T_y \\ T_c + T_y \frac{\bar{y}}{1-\rho} \end{bmatrix}.$$

The operator S governs the dynamics of $D = [d_t^{f,c} \;\; d_t^{w,c} \;\; d_t^{f,y}]$.

Appendix 1.E Tables of Summary Statistics of Robustness

	Data	Learning[1] model	RE model[2] Hagedorn and Manovskii	Learning[3] Shimer	Learning[4] Simultaneous (Re-est)	Learning[5] Constant (Re-est)
$\sigma_{\tilde{\theta}/\sigma_{\tilde{y}}}$	24.713	19.891	15.63	16.02	19.31	4.00
		(1.228)	(2.31)	(2.21)	(1.37)	(5.27)
$\sigma_{\tilde{w}/\sigma_{\tilde{y}}}$	1.737	1.314	0.10	1.38	6.43	0.73
		(1.834)	(7.11)	(1.54)	(-20.42)	(4.37)
$\rho(\tilde{y}_t, \tilde{\theta}_t)$	-0.040	0.389	0.998	0.21	0.11	0.65
		(-0.998)	(-2.42)	(-0.57)	(-0.35)	(-1.60)

Table 1.E.1: Summary Statistics: Alternative calibration, learning constant coefficient and information assumption

Note: **1**. Learning model about d_t^y with productivity shocks, $d_t^c = d^{c,RE}$. Calibration follows tables 3.3.1 and 3.3.2. **2**. Calibration follows tables 3.3.1 and 3.3.2 of the RE model except for parameters $b = 0.955$ and $\alpha = 0.05$. **3**. Calibration follows tables 3.3.1 and 3.3.2 of the learning model, expect for parameters $b = 0.4$ and $\alpha = 0.72$. **4**. Calibration follows tables 3.3.1 and the estimated coefficients coming from SMM are: $c = 0.9$, $A = 0.6$, $g = 0.051$ and $b = 0.09$. **5**. Calibration follows tables 3.3.1 and the estimated coefficients coming from SMM are: $c = 0.8105$, $A = 0.5737$, $g = 0.02$ and $b = 0.75$.The moments of the data are calculated for the period 1990Q1: 2020Q1. The moments are calculated as averages of 1,000 simulations. The t-statistics are defined as (data moment-model moment)/E.S. of the data moment.

Moment's	Data	Learning Model[1]	t-stat	RRE model	t-stat
$\sigma_{\tilde{u}/\sigma_{\tilde{y}}}$	11.952	5.220	3.250	0.767	5.400
$\sigma_{\tilde{v}/\sigma_{\tilde{y}}}$	13.221	16.240	-1.628	1.773	6.176
$\sigma_{\tilde{\theta}/\sigma_{\tilde{y}}}$	24.713	19.891	1.228	2.426	5.673
$\sigma_{\tilde{w}/\sigma_{\tilde{y}}}$	1.737	1.314	1.838	0.741	4.328
$\rho(\tilde{y}_t, \tilde{\theta}_t)$	-0.040	0.389	-0.998	0.991	-2.400
$\rho(\tilde{v}_t, \tilde{\theta}_t)$	0.984	0.964	2.163	0.981	0.261
$\rho(\tilde{u}_t, \tilde{\theta}_t)$	-0.980	-0.966	-1.283	-0.894	-7.969
$\rho(\tilde{w}_t, \tilde{\theta}_t)$	0.780	0.863	-0.234	0.991	-0.600
$\rho(\tilde{\theta}_{t-1}, \tilde{\theta}_t)$	0.941	0.797	1.935	0.618	4.454
$\rho(\tilde{w}_{t-1}, \tilde{w}_t)$	0.826	0.763	1.987	0.703	3.840
$\rho(\tilde{u}_t, \tilde{v}_t)$	-0.927	-0.875	-2.612	-0.791	-6.991
$b^R - b^E$	0.60	0.24	-	0.12	-

Table 1.E.2: Labor Market Statistics

Note: 1. Learning model of d_t^y with productivity shocks, $d_t^c = d^{c,RE}$. Calibration follows tables 3.3.1 and the estimated coefficients coming from SMM are: $c = 0.6$, $A = 1$, $g = 0.05$ and $b = 0.7$.Data moments are computed over the period 1990Q1: 2020Q1. Moments have been computed as averages over 1000 simulations. t-ratios are defined as (data moment-model moment)/ S.E of data moment. $b^R - b^E$ represents the difference in the coefficients coming from regressions 3.A and 1.5

	Data	Learning Model	t-stat	RP-RRE model	t-stat
$\sigma_{\tilde{u}/\sigma_{\tilde{y}}}$	11.952	8.814	1.515	0.310	5.621
$\sigma_{\tilde{v}/\sigma_{\tilde{y}}}$	13.221	13.918	-0.376	0.762	6.721
$\sigma_{\tilde{\theta}/\sigma_{\tilde{y}}}$	24.713	22.360	0.599	1.026	6.030
$\sigma_{\tilde{w}/\sigma_{\tilde{y}}}$	1.737	1.682	0.238	1.059	2.944
$\rho(\tilde{y}_t, \tilde{\theta}_t)$	-0.040	0.046	-0.199	0.993	-2.404
$\rho(\tilde{v}_t, \tilde{\theta}_t)$	0.984	0.989	-0.594	0.983	0.072
$\rho(\tilde{u}_t, \tilde{\theta}_t)$	-0.980	-0.970	-0.897	-0.892	-8.161
$\rho(\tilde{w}_t, \tilde{\theta}_t)$	0.780	0.705	0.216	0.993	-0.605
$\rho(\tilde{\theta}_{t-1}, \tilde{\theta}_t)$	0.941	0.907	-1.747	0.999	-2.653
$\rho(\tilde{w}_{t-1}, \tilde{w}_t)$	0.826	0.840	-0.417	1.000	-5.415
$\rho(\tilde{u}_t, \tilde{v}_t)$	-0.927	-0.923	-0.200	-0.793	-6.651

Table 1.E.3: Labor Market Statistics. Asymmetric learning

The calibration for the two models is described in tables 1.E.4 and 1.E.5. The moments of the data are calculated for the period 1990Q1: 2020Q1. The moments are calculated as averages of 1,000 simulations. The t-statistics are defined as (data moment-model moment)/E.S. of the data moment.

Variable	description	value	source
β	discount factor	0,99	Kyndland & Prescott (1982): r=0,04.
λ	separation rate	0,1	Shimer (2005).
1-α	bargaining power firm	0,5	Standard
ν	elasticity of matching function	0,5	Hosios rule (1990): $\alpha = 1 - \nu$.
b	unemployment benefit	0,4	Shimer (2005).
$\tilde{y}$	steady state productivity	1	Normalization.
σ_ϵ	st. dev. of productivity shocks	0,0058	Calibrated
ρ	persistency of productivity	0,7318	Calibrated

Table 1.E.4: Calibrated quarterly parameters. Asymmetric Learning

Variable	Description	Values (Learning)	Values (RRPE)
c	cost of open a vacancy	0,24	0,195
A	efficiency matching technology	0,63	0,543
g	constant gain	0,011	-
σ^β	Std. wage belief shocks	0,928*10^{-3}	-

Table 1.E.5: Estimated quarterly parameters from SMM. Asymmetric Learning

	Data	Learning Model[1]	t-stat	Learning Model[2] Productivity (Re-est)	t-stat
$\sigma_{\tilde{u}}/\sigma_{\tilde{y}}$	11.952	7.461	2.168	8.621	1.608
$\sigma_{\tilde{v}}/\sigma_{\tilde{y}}$	13.221	16.850	-1.958	15.183	-1.058
$\sigma_{\tilde{\theta}}/\sigma_{\tilde{y}}$	24.713	23.401	0.334	21.569	0.801
$\sigma_{\tilde{w}}/\sigma_{\tilde{y}}$	1.737	1.912	-0.765	1.958	-0.963
$\rho(\tilde{y}_t, \tilde{\theta}_t)$	-0.040	0.097	-0.319	0.152	-0.447
$\rho(\tilde{v}_t, \tilde{\theta}_t)$	0.984	0.984	-0.018	0.950	3.719
$\rho(\tilde{u}_t, \tilde{\theta}_t)$	-0.980	-0.907	-6.743	-0.980	0.051
$\rho(\tilde{w}_t, \tilde{\theta}_t)$	0.780	0.947	-0.475	0.954	-0.495
$\rho(\tilde{\theta}_{t-1}, \tilde{\theta}_t)$	0.941	0.825	-1.217	0.830	1.713
$\rho(\tilde{w}_{t-1}, \tilde{w}_t)$	0.826	0.831	-0.141	0.808	0.572
$\rho(\tilde{u}_t, \tilde{v}_t)$	-0.927	-0.817	-5.460	-0.873	-2.720

Table 1.E.6: Learning about wages and productivity

Note: 1. Learning model of d_t^y and d_t^c with productivity and sentiment shocks. 2. Learning model of d_t^y, d_t^c, a_t^c and a_t^y with productivity and sentiment shocks. Calibration of learning model 1 follows table 3.3.1 and the forth column of table 3.3.2. Calibration of learning model 2 follows table 3.3.1 and the estimated parameters coming from SMM are: $c = 0.4$, $A = 0.5$, $b = 0.7$, $g = 0.02$ and $\sigma^\beta = 0.0016$. The moments of the data are calculated for the period 1990Q1: 2020Q1. The moments are calculated as averages of 1,000 simulations. The t-statistics are defined as (data moment-model moment)/E.S. of the data moment.

	RE* No sent. shocks	RE (1) sent. shocks	RE (2) sent. shocks
$\sigma_{\tilde{\theta}/\sigma_{\tilde{y}}}$	1.75	6.60	24.16
$\rho(\tilde{y}_t, \tilde{\theta}_t)$	1.00	0.31	0.09
$\rho(\tilde{\theta}_{t-1}, \tilde{\theta}_t)$	0,70	-0.27	-0.35

Table 1.E.7: Belief Shocks in the RE model

Note: *Calibration follows tables 3.3.1 and 3.3.2 of the RE model. The moments are calculated as averages of 1,000 simulations.

2

The effect of non-technological news shocks on unemployment fluctuations: The case of Europe

Marta García-Rodríguez & Clemente Pinilla-Torremocha

Abstract

This paper identifies a 'new source' of unemployment fluctuations, non-technological news shocks from firm and household survey data. We extend the identification scheme of Beaudry and Portier (2006) into a two-step procedure. Initially, the approach signals whether the selected data potentially contain news shock information. Subsequently, a simultaneous identification scheme is proposed to jointly identify fundamental and news shocks. For a panel of 22 European countries, we find that non-technological news shocks explain a significant proportion of unemployment's variance in the medium/long run. We show that a search and matching model applied to the business cycle with news shocks generates a realistic response of unemployment to such shock if one allows for small deviations from rational expectations. In the proposed theory, news shocks influence firms' perceptions of labor market tightness, which subsequently affects job creation and unemployment.

Keywords: Subjective Expectations, labour markets, search and matching frictions

JEL Classification: E24; E32; D83; J64

Marta García Rodríguez is a PhD candidate at UAB-BSE, Clemente Pinilla Torremocha is a PhD candidate at University of Alicante and a trainee at the ECB. We thank participants at the 22nd annual SAET conference, University of Minnesota Macroeconomic Workshop, Alicante Macroeconomic PhD Workshop, the UAB MacroClub for helpful comments. This research has been greatly beneficial from founding provided by the European Research Council (ERC) under the project number 804989 (DYMOLAMO).

2.1 Introduction

Much recent research emphasizes the role of expectations in macroeconomics particularly, the impact of anticipated changes in future technology, referred to as technological news shock, on economic activity (see Beaudry and Portier (2006), Barsky and Sims (2011), Barsky et al. (2015), among others). However, the effect of news shocks on the labor market, along with the specific nature of these shocks, whether technological or non-technological news, has received comparatively less attention. This paper fills this gap.

This paper identifies and studies the effect of news shocks on the European labor market. Specifically, we analyze data from 22 European countries spanning from 2000 to 2021. To identify news shocks, we use two data sources: survey data from firms and households, and the stock market index. The surveys are conducted monthly across economic agents —manufacturing, construction, retail trade, services, and consumers— capturing expectations regarding future production, employment, and the overall state of the economy. These variables allow us to extract news contained in their responses. Through the lens of a Mixed-Frequency Panel FAVAR model, we extract a country specific factor that combines all survey data for each country, summarizing overall expectations for each economy. To empirically disentangle these news shocks from other sources, we propose an augmented version of the news identification scheme of Beaudry and Portier (2006). Finally, we enrich a baseline real business cycle (RBC) model with search and matching frictions on the labor market. This theoretical framework aids in reconciling the empirical findings and testing our identification scheme on model-generated data.

Identification of news shocks is not an easy task as the news literature has proved. This is due to, standard identification techniques depend on the implicit assumption that economic shocks are retrievable from data; hence, assuming that data contain enough information to correctly estimate the shocks a researcher has in mind. Furthermore, the sequential identification scheme developed by Beaudry and Portier (2006) (BP) encounters limitations in jointly identifying exogenous variations from fundamental components and news shocks. Identified shocks tend to be correlated to each other preventing to know, for example, the contribution share of different shocks. To overcome these challenges, our study introduces an innovative empirical strategy encompassing two pivotal steps. Initially, we employ the BP sequential scheme, not as an identification if not as a signaling scheme to ascertain two critical aspects: firstly, whether the forward-looking variables —selected by the econometrician— potentially embody news information; and secondly, it informs about the nature of the news —specifically, whether news are related to the technological or non-technological side of the economy. Following this, we propose a simultaneous scheme to jointly identify uncorrelated shocks — technology, non-technology, technological news and non-technological news— whose restrictions are informed by the insights gained from the signaling scheme.[1]

Finally, we propose a theoretical mechanism that can rationalize the empirical responses observed in the data regarding unemployment and its relationship with news shocks in a search and matching RBC model. In this environment, we explore a learning dynamic theory where agents have an incomplete model of the economy, since a model under RE can not account for the empirical effects of news shocks on unemployment dynamics. In this framework, agents have subjective beliefs about how tight is the labor market. Firms make one-step-ahead forecast of the labor market tightness to decide optimally the vacancy posting, using adaptive learning. Their estimated forecast is affected by news shocks. This belief structure has the property that beliefs affect the true data generating process of the economy which in turn affects belief formation. In other words, these news shocks generate waves of optimism or pessimism unrelated to productivity. This expectational channel is proposed on the empirical grounds that a positive non-technological news granger-causes an increase in job vacancies.

We document the following findings. First, relying on the signaling scheme -that includes productivity, unemployment, and survey data- we find that a shock, that has no contemporaneous effects on productivity and unemployment, contains predictive information about long-term unemployment behavior. Notably, this shock exhibits a strong negative correlation (-0.95) with the non-technological long-term driver of unemployment. Second, upon expanding the system to include stock prices to identify technological new shocks, we observe that a shock, which has no contemporaneous effect on productivity, is significantly correlated (0.77) with the long-term behavior of productivity. This finding aligns with the observations reported by BP. Third, the simultaneous short and long-run identification scheme demonstrates that non-technology news shocks significantly affect unemployment, particularly in the medium to long run, accounting around 65% of its variance. This result

[1] Our identifying approach to news shocks is related to the family of "max-share" restriction approaches, but with much fewer restrictions. We do not take a stand on whether the contribution of our news shock should maximize the forecast-error variance of the targeted variable (in our case, unemployment) at a long but finite horizon. The "max-share" approach was first introduced by Faust (1998) and Uhlig et al. (2004), and adapted by Barsky and Sims (2011); Kurmann and Otrok (2013); Francis et al. (2014); and Angeletos et al. (2020) among others.

is robust to the identification of technological news shocks. Non-technology shocks emerge as the primary driver of unemployment fluctuations at the business cycle frequency, contributing over 70% of its short run variance. Neither technological news nor technological shocks are the main driver of unemployment throughout the business cycle, corroborating the findings of Angeletos et al. (2020). Finally, we ascertain that a search and matching model under adaptive learning can effectively reproduce the response of unemployment to non-technological news shocks. These shocks account for a significant portion of the unemployment variance decomposition observed in our empirical analysis, whereas the rational expectations version of the model fails to generate similar outcomes.

Related literature. Our paper bridges the empirical and theoretical literature on ”news driven business cycle hypothesis”. This literature states that business cycles can emerge without contemporaneous changes in fundamentals. In fact, since the seminal contribution of Beaudry and Portier (2006), the news economic literature, has primarily focused on identifying the effect of news about future productivity on the business cycle. However, we investigate the effect of non-technology news on the economy; in particular, labor market variables.

The news view in the empirical side. The original contributions of Beaudry and Portier (2006) suggest that business cycles might be, to a very significant extent, driven by expectations. In particular, they rely on forward-looking variables such as stock prices to identify technological news shocks. They argue that stock prices reflect news about future changes in technology, as they are clearly forward looking and free to jump in response to revised expectations. Subsequent works by Barsky and Sims (2011), Forni et al. (2014), and Barsky et al. (2015) have challenged these conclusions by using alternative identification strategies. They find that technology news shocks exist, and are relatively important in macroeconomic variables specially in the medium and long term, around 10% to 50%. In this paper, we proposed an augmented version of the news identification scheme of Beaudry and Portier (2006); in order to jointly identify exogenous variation from the non-technology and news shocks affecting unemployment fluctuations. In fact, our proposed version is related to the family of restrictions called ’max-shared’, which was popularized by Barsky and Sims (2011). However, our proposed scheme does not add strong assumption restrictions, like in the former paper. In fact, it does not take a stand on whether the contribution of news shock should maximize the forecast-error variance of the targeted variable (in our case, unemployment) at a long but finite horizon. Moreover, our proposed scheme can be extended to also include technology news shocks, in conjunction with non-technology news shocks.[2] The closest empirical paper to ours, in the sense of news affecting the unemployment rate, is Gambetti et al. (2023). They use data scraping techniques to extract unemployment news from major US newspapers, and analyze how these news affect the US unemployment rate. Instead, we infer news shocks exploiting the information contained in European survey data about firms and consumers.

This work also contributes to the ongoing research that investigates whether news shocks must be taken into account in the theoretical models. A wide range of theoretical papers have attempted to quantify the contribution of news - mainly related to productivity - to aggregate fluctuations. A subset list of theoretical contributions includes Jaimovich and Rebelo (2009), Den Haan and Kaltenbrunner (2009) and Theodoridis and Zanetti (2016). These papers are able to generate booms introducing news about tomorrow's technology through several channels. Jaimovich and Rebelo (2009) introduce variable capacity utilization and adjustment costs in investment process. This mechanism encourages faster capital depreciation in the present, resulting from an increased utilization rate that favors increased production today and thereby leading to a boom. Den Haan and Kaltenbrunner (2009) use a standard matching model augmented with endogenous labor force participation, investment in new projects increases early in response to anticipated productivity shocks. Together with the increase in investment in new projects, vacancies increase. In this model, unemployment is entirely driven by productivity news; however, this assumption is at odds with the nature of shocks that we identify empirically. Finally, Theodoridis and Zanetti (2016) its the closest paper related to the introduction of the nature of shocks using a search and matching model. However, they find that non-technological news - in their particular analysis these are shocks that affect the destruction rate and the efficiency of the matching function - and technology news shocks are not able to significantly explain unemployment fluctuations in their model. Hence, dynamics in the unemployment rate are governed by surprises in the destruction rate and the efficiency of the matching function. Our theoretical model mainly differs from theirs in the way that we introduce news that affect the expectation of the labor market tightness. This expectation affects the job creation and, hence, the unemployment rate. In summery, our theoretical model is able to reproduce the empirical variance decomposition of the unemployment rate.

The search and matching model, which has become the accepted theory of equilibrium unemploy-

[2]See Section 5, where we include stock prices to jointly retrieved technology shocks, technology news shocks, non-technology shocks and non-technology news shocks.

ment, has been found to be inconsistent with key observations of the business cycle. A seminal paper by Shimer (2005) demonstrates that the standard search and matching model, driven by productivity shocks, is unable to replicate the cyclical behavior of key indicators, such as unemployment in the United States and other developed countries. This discrepancy is referred to as the ”unemployment volatility puzzle” in academic literature. Our research suggests that one of the reasons for this puzzle is the lack of incorporation of news shocks in the model, as they have been found to explain a significant portion of the variance in unemployment. Finally, an extensive theoretical and empirical summery of the news literature - focused on technology news shocks- is provided by Beaudry and Portier (2014).

Structure. The remaining parts of this paper are organized as follows. Section 2 describes the data, its sources and transformations. Section 3 presents the econometric model and the signalling scheme of BP. Section 4 presents the augmented identification scheme and the main empirical findings. Section 5 presents the theoretical model. Section 6 provides a battery of extensions and robustness tests; and Section 7 provides some concluding comments.

2.2 Data: Sources, and Transformation

Sources. Data are gathered for the following countries: Austria, Belgium, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Italy, Ireland, Lithuania, Luxembourg, Netherlands, Poland, Portugal, Slovak Republic, Slovenia, Spain, Sweden, and the United Kingdom. The frequency of the data is monthly (M) and quarterly (Q). It spans the period 2000m1-2021m6. Data are collected from two main institutions: (i) the OECD Main Economic Indicators Database- gross domestic product per person employed (Q) and unemployment rate (M); (ii) European Commission - The Business and Consumer Survey. The business and consumer data are qualitative surveys reported as aggregated diffusion time series[3]. In particular, we use several business surveys related to expected production and employment. Concerning the households, we use surveys related to their expected financial and future economic situation in their country (in general terms and the number of unemployed people).[4]

[3]The goal of this survey data is to provide overall perceptions and expectations (anticipations) of the short-term developments of the economic cycle. These surveys are conducted under the principle of harmonization to produce comparable data; for example, high frequency, timeliness, and continuous harmonization are among their main qualities.

[4]For more details, see Appendix B.

Transformations. Productivity is transformed into the Napierian logarithm. The unemployment is transformed as follows:

$$v = \ln\left(\frac{100 \times \text{Variable}}{100 - \text{Variable}}\right).$$

This transformation is necessary because it allows us to argue that v is defined as a non-stationary series unbounded above and below.[5] To construct the confidence indicator for each economic agent, we follow the methodology of the European Commission. Confidence indicators are the arithmetic average of the answers to the questions that we consider. All variables are already available in the seasonally adjusted form. Finally, data enter standardized - $\mathcal{N}(0,1)$ - in the mixed-frequencies Panel FAVAR since it helps us to reconstruct better, from quarterly to monthly, the labor productivity of each economy.

2.3 Econometric Methodology

In this section, we introduce the empirical model and its restrictions that help us to distinguish between the fundamental view and the news view. First, we explain the mixed-frequency Panel FAVAR model and describe the estimation procedure.[6] Second, we explain the (signalling) scheme of Beaudry and Portier (2006) (BP).

2.3.1 Mixed Frequency (MF) Panel FAVAR model

MF-Panel FAVARs have the same structure as VAR models in the sense that all variables—observable and unobservable—are assumed to be endogenous and interdependent. However, a cross-sectional dimension is added to the representation. Formally, a panel FAVAR model comprises C units, which in our case are countries. As for a standard VAR, each country includes N endogenous variables and p lags, defined over T periods. We consider an unbalanced panel with mixing frequencies—at a monthly and quarterly frequency. If one were to consider a VAR at a monthly-quarterly frequency, then the vector of dependent variables has two missing observations in every quarter.

[5] These transformations are appropriate for us since most unemployment rates are I(1) processes using a standard unit-root test. See Farmer (2015) and Nicolau (2002) for more details.

[6] Our econometric approach takes general approach in the following aspect: Our VAR does not implicitly impose any cointegration relation on the different countries of analysis. In this sense, we estimate the model in log-levels, as suggested by Sims et al. (1990), instead of imposing an ad hoc number of co-integrating relationships in a VEC model as in Beaudry and Portier (2006). This approach allows us to (i) analyze countries where there exists a cointegration relationship between the variables, as well as countries where there is no such relationship, and (ii) avoid the problem of not having a unique solution that is emphasized by Kurmann and Mertens (2014). The augmented BP scheme and results are robust if variables enter in the model in stationary terms.

Suppose a general panel FAVAR model where observable and non-observable variables are known a priori. In addition, also suppose that there is a single frequency. This model can be written, as a panel VAR, in the following way:

$$\begin{pmatrix} y_{1,t} \\ y_{2,t} \\ \vdots \\ y_{C,t} \end{pmatrix} = \begin{pmatrix} A_1^1 & 0 & ... & 0 \\ 0 & A_2^1 & ... & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & ... & A_C^1 \end{pmatrix} \begin{pmatrix} y_{1,t-1} \\ y_{2,t-1} \\ \vdots \\ y_{C,t-1} \end{pmatrix} + ... \tag{2.1}$$

$$+ \begin{pmatrix} A_1^p & 0 & ... & 0 \\ 0 & A_2^p & ... & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & ... & A_C^p \end{pmatrix} \begin{pmatrix} y_{1,t-p} \\ y_{2,t-p} \\ \vdots \\ y_{C,t-p} \end{pmatrix} + \begin{pmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \vdots \\ \epsilon_{C,t} \end{pmatrix},$$

and

$$\begin{pmatrix} \Sigma_1 & 0 & ... & 0 \\ 0 & \Sigma_2 & ... & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & ... & \Sigma_C \end{pmatrix},$$

where $y_{i,t}$ is a vector that contains three variables, labor productivity, the unemployment rate, and the unobservable factor for country i, $\epsilon_{i,t}$ is a vector of random disturbances following $N(0, \Sigma_i)$, and A_i^p represent the matrix of coefficients for country i and endogenous variables with p lags. We assume that $\mathcal{A}_i$, where $\mathcal{A}_i = \{A_i^1, ..., A_i^p\}$, are related across i units according to the specification:

$$\mathcal{A}_i = \bar{a} + a_i,\ a_i \sim N(0, \Omega)$$

where $\bar{a}$ and Ω represent a common mean and variance. This specification simply means that the C units of the model are characterized by heterogeneous VAR coefficients (i.e., $\mathcal{A}_i \neq \mathcal{A}_j$ if $i \neq j$), but that these coefficients are random processes sharing a common mean. Therefore, the parameters of interest, $\bar{a}$, are the (cross-sectional) average coefficients of the group.[7]

Mixed-Frequencies FAVAR. The observed state-space FAVAR model of each country is

$$\underbrace{\begin{bmatrix} LP_{i,t} \\ UN_{i,t} \\ HH_{i,t} \\ IND_{i,t} \\ SER_{i,t} \\ BUL_{i,t} \\ RE_{i,t} \end{bmatrix}}_{y_{i,t}} = \underbrace{\begin{bmatrix} 1/3 & 0 & 0 & 1/3 & 0 & 0 & 1/3 & \dots & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & h_{i,1} & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & h_{i,2} & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & h_{i,3} & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & h_{i,4} & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & h_{i,5} & 0 & 0 & 0 & 0 & \dots & 0 \end{bmatrix}}_{H_i} \underbrace{\begin{bmatrix} \hat{LP}_{i,t} \\ UN_{i,t} \\ \hat{F}_{i,t} \\ \hat{LP}_{i,t-1} \\ UN_{i,t-1} \\ \hat{F}_{i,t-1} \\ \hat{LP}_{i,t-2} \\ UN_{i,t-2} \\ \hat{F}_{i,t-2} \\ \dots \\ \hat{LP}_{i,t-p} \\ UN_{i,t-p} \\ \hat{F}_{i,t-p} \end{bmatrix}}_{\hat{y}_{i,t}} + \begin{bmatrix} 0 \\ 0 \\ v^3_{i,t} \\ v^4_{i,t} \\ v^5_{i,t} \\ v^6_{i,t} \\ v^7_{i,t} \end{bmatrix}.$$

This equation states two important relations: 1. When a quarterly observation for labor productivity ($LP_{i,t}$) is available, it is computed as an average of the unobserved monthly data on ($\hat{LP}_{i,t}$). 2. The five survey series—consumer, industry, service, construction, and retail confidence indicators—are mapped to a single latent variable that we called factor ($\hat{F}_{i,t}$), plus a specific error term. This mapping is done through the restricted matrix H_i that depends on the free parameters $h_{i,1}, \cdots, h_{i,5}$.

[7]Different European frameworks have also been studied under the lens of PANEL VARs; for example: (i) price differential in monetary unions, Canova and Ciccarelli (2004), (ii) responses to monetary policy shocks in different regions of the same monetary union, Jarociński (2010), and (iii) how the structure of housing finance affects the monetary transmission mechanism Calza et al. (2013).

On the other hand, when an observation for $(LP_{i,t})$ is unavailable, the state-space model changes to

$$\underbrace{\begin{bmatrix} LP_{i,t} \\ UN_{i,t} \\ HH_{i,t} \\ IND_{i,t} \\ SER_{i,t} \\ BUL_{i,t} \\ RE_{i,t} \end{bmatrix}}_{y_{i,t}} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & h_{i,1} & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & h_{i,2} & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & h_{i,3} & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & h_{i,4} & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & h_{i,5} & 0 & 0 & 0 & 0 & \dots & 0 \end{bmatrix}}_{H_i} \underbrace{\begin{bmatrix} \hat{LP}_{i,t} \\ UN_{i,t} \\ \hat{F}_{i,t} \\ \hat{LP}_{i,t-1} \\ UN_{i,t-1} \\ \hat{F}_{i,t-1} \\ \hat{LP}_{i,t-2} \\ UN_{i,t-2} \\ \hat{F}_{i,t-2} \\ \dots \\ \hat{LP}_{i,t-p} \\ UN_{i,t-p} \\ \hat{F}_{i,t-p} \end{bmatrix}}_{\hat{y}_{i,t}} + \begin{bmatrix} v^1_{i,t} \\ 0 \\ v^3_{i,t} \\ v^4_{i,t} \\ v^5_{i,t} \\ v^6_{i,t} \\ v^7_{i,t} \end{bmatrix},$$

where $var(v^1_{i,t})$ is set to a large number. Notice that when observations of $(LP_{i,t})$ are missing, the first row of H_i is zero. Since we rely on the Kalman filter, this assumption effectively means that missing observations on $(LP_{i,t})$ are ignored when calculating the updated estimate of $(LP_{i,t})$. Therefore, the observation equation for this model changes over time depending on whether observations on $(LP_{i,t})$ are missing.

Hence, the unobserved state-space model of each country is a simple VAR with three variables (i.e., monthly labor productivity, unemployment rate, and the surveys-factor)

$$\hat{y}_{i,t} = A_{i,1}\hat{y}_{i,t-1} + A_{i,t-2}\hat{y}_{i,2} + \dots + A_{i,p}\hat{y}_{i,t-p} + \epsilon_{i,t}. \tag{2.2}$$

Therefore, the observed and unobserved equations represent the joint transition equations in the state-space model. The initial conditions $y_{i,0:-p+1} = (y'_{i,0}, \cdots, y_{i,-p+1})$ are assumed to be distributed according to $y_{i,0:-p+1} \sim N(0, V(\mathcal{A}_i, \Sigma_i))$, where $V(\mathcal{A}_i, \Sigma_i)$ represents the unconditional variance of $y_{i,0:-p+1}$.

The priors for the VAR coefficients $\mathcal{A}_i = \left(A_{i,1}, \ldots, A_{i,p}\right)$ and the covariance matrix Σ_i have a standard form, namely,

$$p\left(\text{vec}(\mathcal{A}_i) \mid \Sigma_i\right) = \mathcal{N}\left(\text{vec}(\underline{\mathcal{A}_i}), \Sigma_i \otimes \underline{\Sigma_i}\right) I(\text{vec}(\mathcal{A}_i)),$$
$$p\left(\Sigma_i\right) = I\mathcal{W}\left(n+2, (n+2)\,\underline{\Sigma_i}\right),$$

where $p\left(\Sigma\right) = I\mathcal{W}\left(n+2, (n+2)\,\underline{\Sigma}\right)$ denotes the inverse Wishart distribution with mode $\underline{\Sigma}$ and $n+2$ degrees of freedom, and $I(\text{vec}(\mathcal{A}_i))$ is an indicator function that is equal to 0 if the VAR is explosive—some of the eigenvalues of $\mathcal{A}_i(L)$ are greater than 1—and to 1 otherwise.[8]

The same priors are shared for all countries. Hence, this prior structure exploits the structure of coefficients, given that the C units of the model are sharing a common mean. The prior for the VAR parameters, $\text{vec}(\underline{\mathcal{A}})$, is a standard Minnesota prior with the hyperparameter for the overall tightness equal to the commonly used value of 0.2 (see Giannone, Lenza, and Primiceri (2015)). The prior for the VAR parameters $\text{vec}(\mathcal{A})$ are centered around zero, except for the "own-lag" parameter that is centered at 1 - this implies that the individual variables exhibit random walk behavior. The prior for the covariances Σ_i of the innovations, $\underline{\Sigma}$, is a relatively uninformative inverse Wishart distribution with just enough degrees of freedom (n+2) to have a well-defined prior mean, which is set to be a diagonal matrix. The prior for H_i is given by $p(h) = N(1, 0.5^2)$, the product of independent Gaussian distributions for each element $h_{i,1,\cdots,5}$ of the matrix H_i.[9] Turning to the initial conditions, all the country-specific $Y_{0:-p+1}$ have mean zero and standard deviations equal to one.

The state-space model is efficiently estimated with Bayesian methods using Kalman Filter, in conjunction with modern simulation smoothing techniques (Carter and Kohn (1994); Durbin and Koopman (2002)) that easily help us to accommodate missing observations and draw the latent states. All

[8]The enforcement of the stationarity constraint on the model coefficients becomes relevant to avoid that the updated covariance matrix in the Kalman Filter algorithm becomes singular and hence precluding the computation of its inverse.

[9]Elements in the matrix H_i are updated using Metropolis-Hastings algorithm. This algorithm involves a scaling matrix that the researcher selects to obtain the appropriate acceptance ratio of proposals. The general recommendation is to accommodate an acceptance rate between 20% and 40%.

results are based on 10,000 simulations, of which we discard the first 9,000 as burn-in draws.[10]

2.3.2 Signalling Scheme of BP

This section explains the scheme of Beaudry and Portier (2006), that we interpret as a signalling scheme. Two orthogonalization schemes are used, imposing sequentially, not simultaneously, either impact or long-run (at eight year horizon) restrictions on the reduced-form moving average representation of the data. The disturbance of the news shock is obtained by imposing impact restrictions (i.e., short-run) on the reduced-form residuals of equation 2.2,

$$\underbrace{\begin{pmatrix} \epsilon_t^{Prod} \\ \epsilon_t^{Un} \\ \epsilon_t^{F} \end{pmatrix}}_{\text{Reduced-form residuals}} = \underbrace{\begin{bmatrix} s_0^{11} & 0 & 0 \\ s_0^{21} & s_0^{22} & 0 \\ s_0^{31} & s_0^{32} & s_0^{33} \end{bmatrix}}_{\text{P}} \underbrace{\begin{pmatrix} {w_{1t}}^{\text{Shock 1}} \\ w_{2t}^{\text{Shock 2}} \\ w_{3t}^{\text{News Shock}} \end{pmatrix}}_{\text{Structural Disturbances}} . \tag{2.3}$$

To be specific, let the mapping between reduced-form and structural disturbances be $\epsilon_t = Pw_t$, where $w_t \sim N(0, I_n)$ is a $n \times 1$ vector of structural disturbances with unit variance. In particular, P is the restriction implemented using Cholesky factorization on Σ; hence, P is a lower-triangular matrix, with at least n(n-1)/2 additional restrictions. Our interpretation of a news shock is an advanced information or signal that agents receive about the future before these are realized, that affect their expectations.[11] Our news shock affects the factor contemporaneously and with a lag to productivity and unemployment. Moreover, the news shock is orthogonal to the other two innovations, shocks 1 and 2, which affect productivity and the unemployment rate contemporaneously. We leave the two first shocks without giving a formal interpretation.

On the other hand, by imposing long-run restrictions, at eight year horizon, on the reduced-form residuals of equation 2.2, we obtain the structural disturbances that have persistent effects on the system's variables. To be specific, let the mapping between reduced-form and structural disturbances

[10]To decreased the complexity and uncertainty of the model, given that the model needs to deal with missing observations, and draw the latent states, some shortcuts are taken. First, a Mixed-Frequency Favar model is estimated for each country using the same priors and initial conditions. Attempts to perform the estimation in stacked-form have been done, but the updated covariance matrix in the Kalman Filter algorithm becomes singular and hence precluding the computation of its inverse. Second, the posterior distributions of the reduced-form coefficients for each unit are averaged out across the entire cross-section of C units. This yields the posterior distributions of the (cross-sectional) average coefficients of the group. This estimation approach yields consistent estimates, initially proposed by Pesaran and Smith (1995). Intuitively, this estimation approach is equivalent as including a hyperprior on Ω, with a high value, allowing single country coefficients to differ between them, see Section 2.2 in Jarociński (2010).

[11]The errors are orthogonal $var(w_t) = var(P^{-1}\epsilon_t) = (P^{-1})\Sigma(P^{-1})' = P^{-1}\Sigma(P^{-1})' = P^{-1}(PP')(P^{-1})' = I_{(N)}$.

be $\epsilon_t = \tilde{P} w_t$, where $\tilde{w}_t \sim N(0, I_n)$ is a $n \times 1$ vector of structural disturbances with unit variance. In particular, $\tilde{P}$ has the following structure $C(1)^{-1}S$, where $C(1)$ represents the point estimate of the cumulated impulse responses in reduced form on the eight-year horizon, and S is the restriction implemented using Cholesky factorization on $C(1)\Omega C(1)'$; hence, S is a lower-triangular matrix, with at least n(n-1)/2 additional restrictions.[12] We are interested in the first two disturbances affecting productivity and unemployment in the long-run since we want to identify technological and non-technological shocks (i.e. fundamental shocks).

$$\underbrace{\begin{pmatrix} \epsilon_t^{Prod} \\ \epsilon_t^{Un} \\ \epsilon_t^{F} \end{pmatrix}}_{\text{Reduced-form residuals}} = \underbrace{C(1)^{-1} \begin{bmatrix} s_0^{11} & 0 & 0 \\ s_0^{21} & s_0^{22} & 0 \\ s_0^{31} & s_0^{32} & s_0^{33} \end{bmatrix}}_{\tilde{P}} \underbrace{\begin{pmatrix} \tilde{w}_{1t}^{\text{Technological Shock}} \\ \tilde{w}_{2t}^{\text{Non-Technological Shock}} \\ \tilde{w}_{3t}^{\text{Residual 3 Shock}} \end{pmatrix}}_{\text{Structural Disturbances}}. \tag{2.4}$$

The first shock drives the long-run behavior of all the variables in the system. In this sense, it affects the long-run dynamics of the three variables. The second one can influence the long-run movements of the unemployment rate and the factor, but it does not alter the long-run dynamics of productivity. Finally, the last structural shock cannot affect the dynamics of the first two variables in the long-run. The next step is to evaluate the potential connection between different structural economic shocks.

2.4 Augmented Identification Scheme and Empirical Results

This section documents our main empirical results obtained from the estimated model. First, we show that non-technology shocks, identified with long-run restrictions, and news shocks, identified using short-run restrictions, are highly correlated. It potentially implies (i) that our factor contains information in the form of news, and (ii) that these news shocks are related to the non-technological part of the economy given that they anticipate a significant part of non-technology shocks. Second, the signalling scheme of BP lacks the potential to assess the explanatory power of each structural shock (i.e. we cannot compare the contribution of news and non-technological shocks since they are identified in different systems). Hence, we propose a simultaneous short-long run identification that builds up on the signalling scheme. This new scheme enables us to jointly identify technological, non-technological and news shocks. In the next section, Section 2.5, we expand our focus and include stock prices in the system of variables to properly capture technological news; following the central idea of Beaudry and Portier (2006) .

[12]The errors are orthogonal $var(\tilde{w}_t) = var(\tilde{P}^{-1}\epsilon_t) = (\tilde{P}^{-1})\Sigma(\tilde{P}^{-1})' = S^{-1}C(1)\Sigma C(1)'(S^{-1})' = S^{-1}(SS')(S^{-1})' = I_{(n)}$.

2.4.1 Preliminar Results

We begin by estimating a MF Panel FAVAR for LP, UN, HH, IND, SER, BUL, and RE with 5 lags and recover three orthogonalized shocks series corresponding to the w_t and $\tilde{w}_t$ explained in Section 3. That is, the structural shocks w_t are recovered by imposing impact restriction, and the structural shocks $\tilde{w}_t$ are recovered by imposing long-run restrictions on a 100 periods horizon. Figure 2.4.1 shows the correlation between the news shock, w_{3t}, and the fundamental shocks (technological shock, $\tilde{w}_{1t}$, and non-technological shock, $\tilde{w}_{2t}$).

The striking observation is that long-run shocks appear to correlate with news shocks, particularly those related to the non-technological side of the economy (-0.95). More specifically, the dynamics associated with the w_{3t} shock - which by construction is an innovation in the estimated factor which is contemporaneously orthogonal to productivity and unemployment - seem to recover similar information to $\tilde{w}_{2t}$ - which by construction has long-lasting effects on unemployment. In other words, the estimated factor contains additional information that is translated into decreases in the unemployment rate. On the other hand, they also show a very modest correlation with changes in productivity, meaning that hardly any information is reflected in the factor before actually translating into productivity increases.

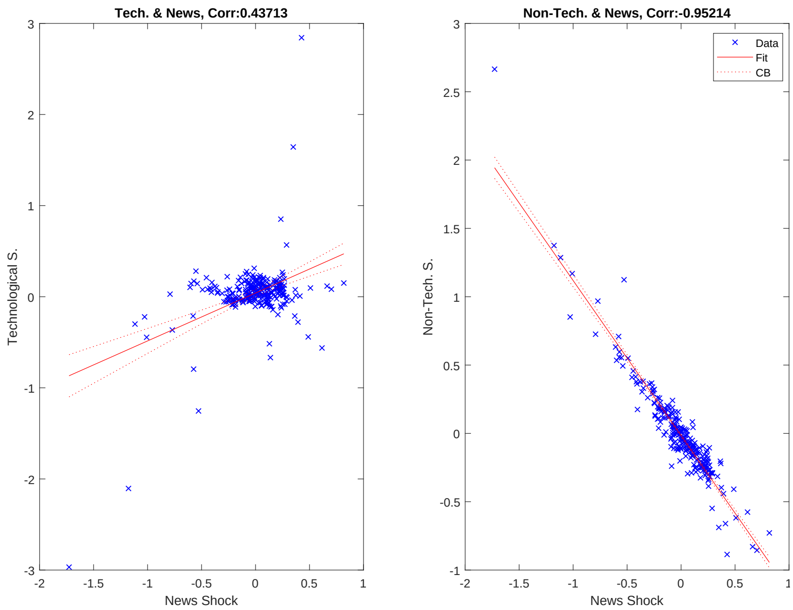


Figure 2.4.1: Plot of w_{3t} against $\tilde{w}_{1t}$ - left - and on the right w_{3t} against $\tilde{w}_{2t}$ in the MF Panel FAVAR with 5 lags

The interesting question then becomes, what potential explanatory power do news shocks have compared to the other fundamental shocks? Notice that under the BP scheme, we can only retrieve the different shocks of interest, but we cannot explore the relative importance of each. So in the next subsection, we carry out a simultaneous short-long run identification that enables us to jointly identify technological, non-technological and news shocks. This new structural identification will be consistent with the findings using the BP scheme.

2.4.2 Simultaneous Identification - Short and Long-run restrictions

The proposed identification scheme imposes, at the same time, short and long-run restrictions to help us to identify, under the same framework, (i) technological, (ii) non-technological, and (iii) news shocks. Equation 2.5 denotes the restricted elements of P (i.e. impact matrix) and L (i.e. long-run matrix). The implemented restrictions imply the following properties for the relationship between the variables in the system.

Assumption 1. Productivity can only be explained in the long-run by technological shocks. Moreover, technological shocks have contemporaneous and long-run effects on all the variables in the system.

Assumption 2. Non-technological shocks have contemporaneous effects on all the variables in the system, but they have no long-run effects on productivity.

Assumption 3. Non-technological news shocks do not immediately impact unemployment. However, they can have a contemporaneous effect on productivity. Furthermore, they only have the potential to explain the long-term dynamics of unemployment, as demonstrated in Figure 2.4.1.

$$P = \begin{pmatrix} * & * & * \\ * & * & 0 \\ * & * & * \end{pmatrix}, \quad L = \begin{pmatrix} * & 0 & 0 \\ * & * & * \\ * & * & * \end{pmatrix}. \tag{2.5}$$

Assumptions 1 and 2 follow common assumptions regarding productivity as a driving force of economic fluctuations. In particular, 1 is quite a natural representation - also reflected by a broad range of theoretical models - given that it resembles the standard long-run identification assumption, see Gali (1999); Galí (2004). In order to identify the same non-technological shocks, 2 represents the same restrictions as in equation 2.4. Finally, 3 combines (i) the standard properties of news shocks -

that they do not have a contemporaneous impact on the variable of interest (in this case unemployment) - and (ii) the potential ability to explain long-run unemployment fluctuations based on our previous correlation results.

2.4.3 How does unemployment respond to structural shocks of technology, labor market, and news?

Figure 2.4.2 reports unemployment's cumulated impulse response functions (CIRFs), i.e., the total change in unemployment, to positive innovations in the MF Panel FAVAR. Structural shocks are displayed on the columns. The horizontal axis measures time in months from impact to 100 months after innovations have occurred. The vertical axis represents the responses. All CIRFs are displayed with 90% probability density intervals.

The unemployment rate responds negatively and significantly to news innovations and positively and significantly to non-technological innovations. Moreover, the effect of news shocks on unemployment is larger than the effect of non-technological shocks in the medium and long term. On the other hand, although the median response of unemployment to a technology shock is negative, it does not become significant at any horizon. This insignificant response to the technological shock to unemployment may reflect heterogeneous responses across European countries, as reflected in the confidence bands. While in some European countries, improvements in technology could lead to a permanently lower unemployment rate, in others, it could destroy the job skills of some types of workers and lead to long-term unemployment. Figure 2.4.3 plots the share of variance of the unem-

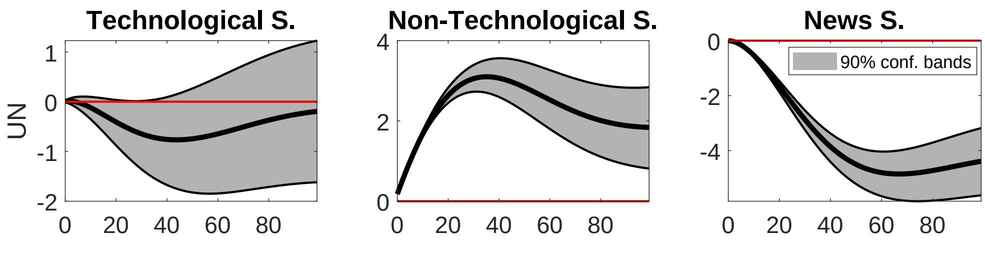


Figure 2.4.2: Response functions of unemployment to positive innovations from the MF PANEL FAVAR

Note: Posterior distributions of cumulative impulse response functions to a estimated shock of one standard deviation using short and long-run restrictions, as in Equation 2.5. Median (solid line) and 90% probability density intervals (shaded area) based on 10,000 draws.

ployment rate attributable to each shock in the system. In this sense, we can quantify the relative importance of the structural shocks under consideration - technological, non-technological and news shocks. This exercise is done at different frequencies from impact to 100 months ahead. In this subsection, we only focus our attention on the unemployment rate. In the short-term, fluctuations in

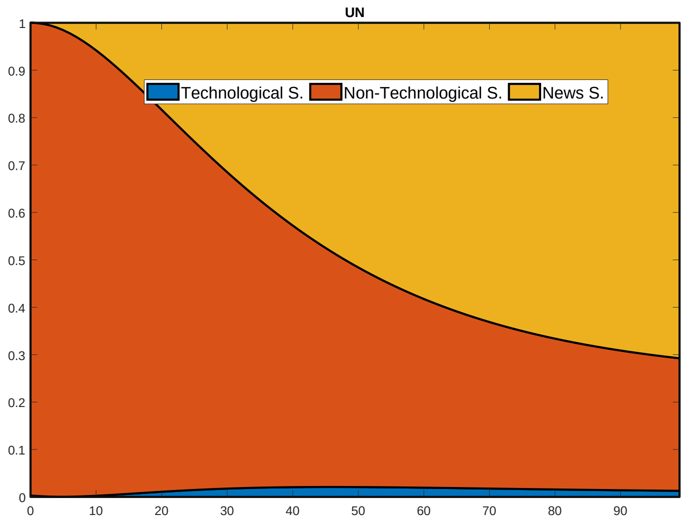


Figure 2.4.3: Variance decomposition at different frequencies

Note: The colored areas represent the point-wise median cumulative contributions of each identified shock to the forecast error variance contributions of the unemployment rate at horizons j = 0, 1, . . . , 100 using joint short and long-run restrictions as in equation 2.5.

unemployment can be attributed entirely to non-technological shocks. However, as time progresses, the impact of news shocks becomes increasingly significant. Eventually, they become the primary driver of the unemployment's variance in the medium and long-term. Specifically, they are found to account for approximately 70% of the variance in the long-term. [13]

2.5 Stock Prices and Technological News Shocks

Since surveys do not capture technological news, we increase our focus and look at the relevance of technological news to the labor market; following the central idea in Beaudry and Portier (2006) of including stock prices in the system of variables. Based on this idea, financial variables, especially stock prices, are likely to reflect news about future technological growth. Two recent papers, Beaudry and Portier (2014) and Barsky et al. (2015), revisit how to identify news shocks that can influence fluctuations of future TFP. These two papers conclude that the inclusion of stock prices, in a small VAR, may drastically change the effects of news shocks in the system of variables. Hence, we follow

[13] The complete results of the Figures 2.4.2 and 2.4.3 are reported in the Appendix, depicted in Figures C.2.C.1 and C.2.C.2.

the recommendations of these papers and include stock prices (SP) as our second variable in the system.

Following the previous procedure, we first apply the signaling scheme explained in Section 3.2 for the four variable case as follows:

Short-run restrictions

$$\underbrace{\begin{pmatrix} \epsilon_t^{Prod} \\ \epsilon_t^{SP} \\ \epsilon_t^{Un} \\ \epsilon_t^{F} \end{pmatrix}}_{\text{Reduced-form residuals}} = \underbrace{\begin{bmatrix} s_0^{11} & 0 & 0 & 0 \\ s_0^{21} & s_0^{22} & 0 & 0 \\ s_0^{31} & s_0^{32} & s_0^{33} & 0 \\ s_0^{41} & s_0^{42} & s_0^{43} & s_0^{44} \end{bmatrix}}_{\text{P}} \underbrace{\begin{pmatrix} w_{1t} \\ w_{2t} \\ w_{3t} \\ w_{4t} \end{pmatrix}}_{\text{Structural Disturbances}} .$$

Long-run restrictions

$$\underbrace{\begin{pmatrix} \epsilon_t^{Prod} \\ \epsilon_t^{SP} \\ \epsilon_t^{Un} \\ \epsilon_t^{F} \end{pmatrix}}_{\text{Reduced-form residuals}} = \underbrace{C(1)^{-1} \begin{bmatrix} s_0^{11} & 0 & 0 & 0 \\ s_0^{21} & s_0^{22} & 0 & 0 \\ s_0^{31} & s_0^{32} & s_0^{33} & 0 \\ s_0^{41} & s_0^{42} & s_0^{43} & s_0^{44} \end{bmatrix}}_{\tilde{P}} \underbrace{\begin{pmatrix} \tilde{w}_{1t} \\ \tilde{w}_{2t} \\ \tilde{w}_{3t} \\ \tilde{w}_{4t} \end{pmatrix}}_{\text{Structural Disturbances}} .$$

Upon incorporating stock market data into our analysis, we find that when implementing the previous scheme, w_{2t}, a shock that affects stock prices contemporaneously and with a lag to productivity, contains predictive information about $\tilde{w}_{1t}$, a shock that can only drive the long-run behavior of productivity. This is evidenced by the relatively high correlation (0.765) between w_{2t} and $\tilde{w}_{1t}$, as shown in Figure 2.5.1. Moreover, the correlation between w_{4t} and $\tilde{w}_{3t}$ is still significantly high (-0.78).

This finding prompts us to impose joint restrictions, as outlined in Eq. 2.6, at both the short-term (P) and long-term (L) in order to properly identify shocks related to technology, technological news shocks, non-technological shocks, and news shocks related to the non-technological side of the economy.

Assumption 1. Productivity can only be explained in the long-run by technological and news shocks

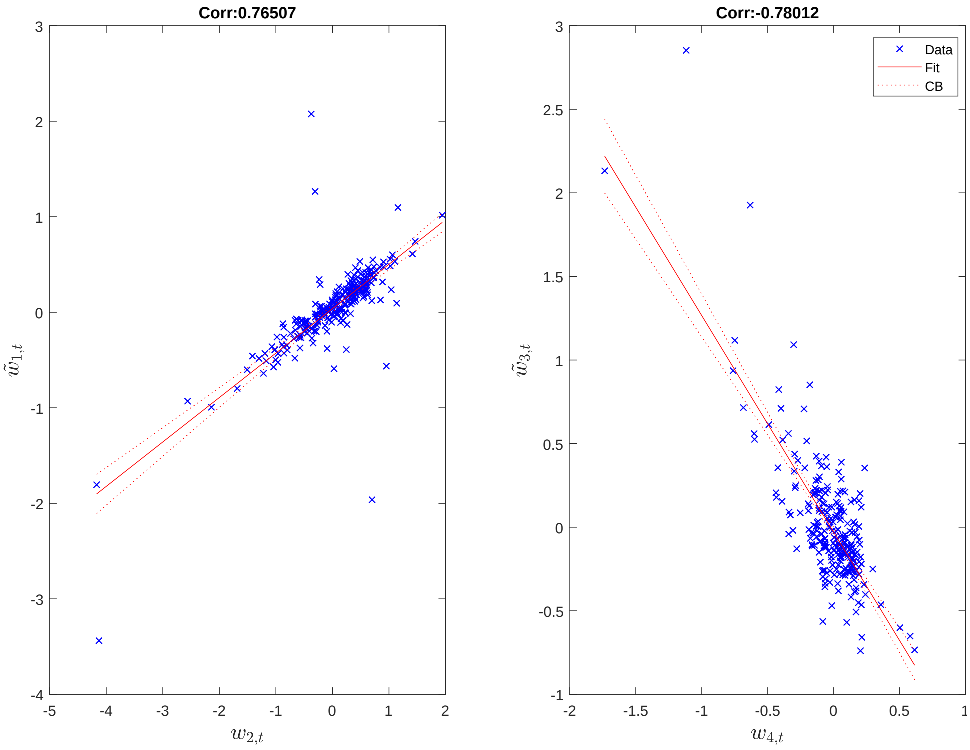


Figure 2.5.1: Plot of w_{2t} against $\tilde{w}_{1t}$ - left - and on the right w_{4t} against $\tilde{w_3}_t$ in the MF Panel VAR with with stock prices.

related to productivity. Moreover, technological innovations have contemporaneous and long-run effects on all the variables in the system. On impact, productivity responds with a lag to only news technological shocks.

Assumption 2. Technological news shocks do not have contemporaneous effects on productivity, but they can affect unemployment and their related news contemporaneously. Moreover, SP can affected by all shocks in the long-run. Technological news shocks only have the potential to explain the long-run dynamics of productivity - as it is suggested by Figure 2.5.1.

Assumption 3. Non-technological shocks have contemporaneous effects on all the variables in the system, but they have no long-run effects on productivity. On impact, unemployment is not affected by non-technological news shocks.

Assumption 4. Non-technological news shocks do not have contemporaneous effects on the unemployment rate, but they can affect productivity and their related news contemporaneously. Moreover, they have the potential to explain the long-run dynamics of unemployment and the factor - as it is suggested by Figure 2.5.1.

$$P = \begin{pmatrix} * & 0 & * & * \\ * & * & * & * \\ * & * & * & 0 \\ * & * & * & * \end{pmatrix}, \quad L = \begin{pmatrix} * & * & 0 & 0 \\ * & * & * & * \\ * & 0 & * & * \\ * & 0 & * & * \end{pmatrix} \tag{2.6}$$

Figure 2.5.2 shows the CIRFs of productivity and unemployment using the restrictions of Eq.2.6. We can see that positive technological news innovations have a permanent positive effect on productivity - first raw, second column -while they lead to increase unemployment in the short and medium term. This may be because positive productivity news lead to a reallocation of resources - labor for capital - within firms. This result is in line with Manuelli (2000) who argues that an anticipated improvement in technology is likely to lead to a long-lived (but not permanent) increase in the unemployment rate. In the long term, its effect on the labor market is not significant, meaning that the initial loss of employment is recovered. Due to the inclusion of the stock market and identification of a technology news shock, a positive productivity shock decreases unemployment significantly. This implies that, in Figure 2.4.2, the response of unemployment to productivity shocks is masking the effects of productivity and news productivity shocks.[14] The effect of the remaining shocks on unemployment is unchanged, reinforcing the previous results.

Regarding the variance decomposition, see Figure 2.5.3, technological news shocks explain about 20% of the variance of productivity in the long run. This result is in line with Barsky and Sims (2011). With respect to the unemployment drivers, non-technological news shocks continue to explain 60% of its variance in the long run. This result is not affected by the introduction of technological new shocks. Moreover, productivity news shocks would come to explain 20% of the variance of unemployment in the short and medium term, but in the long term, this percentage would be reduced to 5%. In addition, productivity shocks explain around 10% in the long-run, whereas previously in Figure

[14] This effect of technological shocks on the unemployment rate is in line with the results found by Gali (1999) and Christiano et al. (2004), where technological shocks decrease the amount of hours worked.

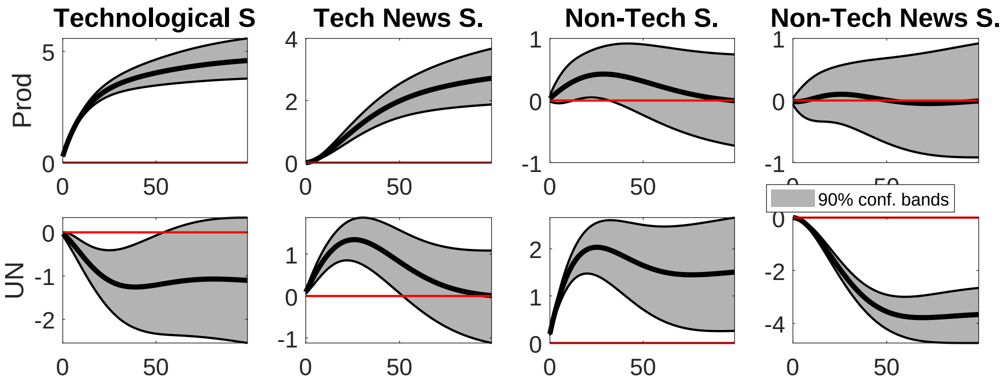


Figure 2.5.2: Response functions of unemployment to positive innovations from the MF PANEL FAVAR

Note: Posterior distributions of cumulative impulse response functions to a estimated shock of one standard deviation using short and long-run restrictions, as in Equation 2.6. Median (solid line) and 90% probability density intervals (shaded area) based on 10,000 draws.

2.4.3 explain near zero. [15]

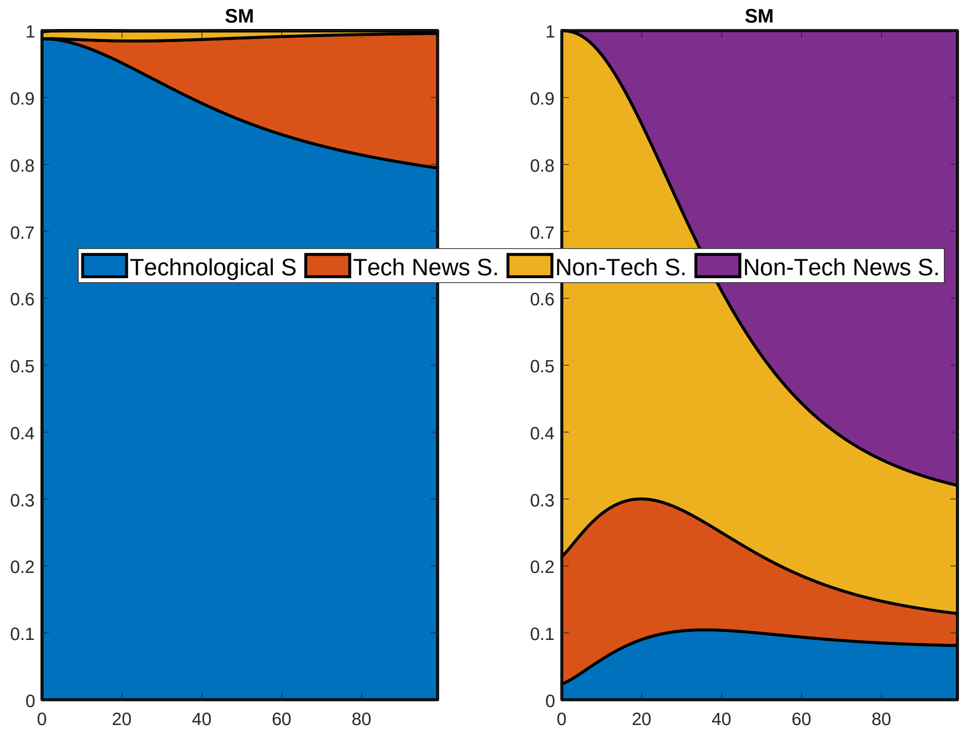


Figure 2.5.3: Variance decomposition at different frequencies

Note: The colored areas represent the point-wise median cumulative contributions of each identified shock to the forecast error variance contributions of the unemployment rate at horizons j = 0, 1, . . . , 100 using joint short and long-run restrictions as in equation 2.6.

[15]The complete results of the Figures 2.5.2 and 2.5.3 are reported in the Appendix, depicted in Figures C.2.C.3 and C.2.C.4.

2.6 Theoretical Model

We propose an expectation channel - based on empirical evidence - in a model featuring labor market search and matching friction applied to the business cycle under adaptive learning. In this economy, there is a continuum of workers who search for jobs if they are unemployed and work for firms if they are employed. There is also a continuum of firms that post vacancies and employ workers with a lag. They produce output using labor as the only input for production. The proposed expectations channel works through the recursive least squares (RLS) algorithm that agents use to forecast labor market tightness.[16]

2.6.1 Expectation channel; building a bridge between empiric and theory

The expectation channel proposed in the theoretical model is grounded on empirical evidence. This channel influences the job creation condition, where waves of optimism or pessimism, unrelated to productivity, affect vacancies and unemployment rates. To investigate this, we test the following hypothesis: Suppose our non-technological news are not related to the labor market. In that case, these news should not be followed by an increase in job vacancies, implying that our identified news shocks should not granger-causal job vacancies.[17]

Table 2.6.1 presents granger causality tests between the estimated news shocks and the job vacancies ratio (JV) - for two transformations of JV - with five lags.[18] On the left, the hypothesis that the news shocks do not Granger cause JV is rejected with a P-value less than 0.01, indicating strong evidence that information in the estimated news shocks helps forecast the job vacancy rate one quarter later. Conversely, the right side of the table shows that the hypothesis JV does not help predict the estimated news shocks. It has a P-value of 51% without transforming JV and 22% if JV is expressed in quarterly differences. Therefore, we observe a robust unilateral causal relationship from news shocks to European job vacancies. Based on this result, the following model establishes an expectation channel that operates through the job creation condition

[16] For simplicity, we do not include stock prices in the model, so we only introduce non-technology news shocks into the model.

[17] We obtain the job vacancy rate - defined as the number of job vacancies * 100 / (number of occupied posts + number of job vacancies) - from Eurostat. It is obtained as an aggregated measure for the EU 27 - individual weights are not available to calculate the joint job vacancy ratio for our 22 studied countries - from 2006.Q1 to 2021.Q2.

[18] The estimated news shocks are transformed to quarterly frequency by averaging every three observations. The Granger causality tests are run from 2006.Q1-2021.Q2 - when JV has no transformation - and 2006.Q2-2021.Q2 - when JV is expressed in quarterly differences.

Dependent variable: JV			Dependent variable: News		
Transformation JV	F-test	P-Value	Transformation JV	F-test	P-Value
None	14.94	2.7849e-04	None	0.44	0.51
Diff	10	1.8884e-04	Diff	1.51	0.22

Table 2.6.1: Granger causality tests

2.6.2 The Labor Market

Frictions in this labor market are characterized by a Cobb-Dougla matching function, $M_t = Av_t^{1-\nu}u_t^{\nu}$, where $A > 0$ and $0 < \nu < 1$, which describes the number of successful matches between unemployed workers, u_t and vacancies, v_t, reflecting increasing and concave dependencies on its inputs. The labor market tightness, defined as $\theta = \frac{v}{u}$, influences the likelihood of filling vacancies, $q(\theta_t) = A\theta_t^{-\nu}$, and the matching probability for unemployed workers, $m(\theta_t) = A\theta_t^{1-\nu}$.

The law of motion for the fraction of workers who are employed at the beginning of period $t+1$ is given by

$$n_{t+1} = (1-\lambda_t)n_t + m(\theta_t)u_t, \tag{2.7}$$

where λ follows an iid process

$$\lambda_t = \lambda + \epsilon_t^{\lambda}, \tag{2.8}$$

and $\epsilon_t^{\lambda} \sim N(0, \sigma_{\lambda}^2)$ denotes a shock to the destruction rate that directly affects unemployment and λ is the mean of the separation rate.

Finally, labor productivity is modeled as a stationary AR(1) process in logs

$$\ln(y_t) = (1-\rho)\ln(\overline{y}) + \rho\ln(y_{t-1}) + \epsilon_t, \;\; 0 < \rho < 1. \tag{2.9}$$

Where $\epsilon_t \sim N(0, \sigma^2)$ and ρ measures its persistence.

2.6.3 The Household

We consider an economy with a representative household of size one, where all workers are identical and risk-neutral, and there is perfect consumption insurance among the members. The expectation operator $E_t^{\mathcal{P}^w}$ is determined using a subjective probability measure $\mathcal{P}^w$. The household's decision-making can be represented by the following Bellman equation:

$$W(n_t, y_t) = w_t n_t + b(1 - n_t) + \beta E_t^{\mathcal{P}^w} W(n_{t+1}, y_{t+1}), \tag{2.10}$$

subject to the law of motion for employment given by Equation 2.7. $W(n_t, y_t)$ represents its current value. The household takes as given wages, w_t, and labor market tightness, θ_t. The period utility value from non-employment is represented by b, and β is the discount factor. The surplus from an additional member of the household being employed is captured by

$$\frac{\partial W(n_t, y_t)}{\partial n_t} = w_t - b + \beta(1 - \lambda_t - \theta_t q_t(\theta_t)) \frac{\partial E_t^{\mathcal{P}^w} W(n_{t+1}, y_{t+1})}{\partial n_{t+1}}. \tag{2.11}$$

This equation reflects the net employment value plus the expected continuation value.

2.6.4 The Firm

A representative firm with a linear production function aims to maximize its profits by choosing the number of vacancies, v, to post in each period at a constant ongoing cost, c. The firm's profit maximization problem is subject to the evolution of employment and taking as given wages, w_t and the labor market tightness, θ_t. The problem is formalized as:

$$\Pi(n_t, y_t) = \Pi_t = \max_{v_t \geq 0} \quad y_t n_t - w_t n_t - c v_t + \beta E_t^{\mathcal{P}^f} \Pi(n_{t+1}, y_{t+1}) \ , \tag{2.12}$$

subject to

$$n_{t+1} = (1 - \lambda_t) n_t + q(\theta_t) v_t, \tag{2.13}$$

where $\Pi(n_t, y_t)$ denotes the current value function. The problem defined above is standard; the only difference from the RE setting is that the expectation operator, $E_t^{\mathcal{P}^f}$, is determined using a subjective probability measure $\mathcal{P}^f$. The first order condition is given by

$$\frac{\partial E_t^{\mathcal{P}^f}\Pi(n_{t+1},y_{t+1})}{\partial n_{t+1}} = \frac{c}{\beta q(\theta_t)}, \tag{2.14}$$

indicating the marginal cost of posting a vacancy equals the discounted marginal gain from an additional employee. The net profit from hiring an additional worker, considering the expected continuation value, is:

$$\frac{\partial \Pi(n_t,y_t)}{\partial n_t} = y_t - w_t + (1-\lambda_t)\frac{c}{\beta q(\theta_t)}. \tag{2.15}$$

This represents the direct profit impact of an additional employed worker plus the adjusted expected future benefits.

2.6.5 Wage Negociation

Wages in this model are negotiated through a Nash bargaining process, where the wage w_t maximizes the joint surplus of a match between workers and firms. The objective function for wage determination is given by:

$$\max_{w_t}\left\{\left[\frac{\partial W(n_t,y_t)}{\partial n_t}\right]^{\alpha}\left[\frac{\partial \Pi(n_t,y_t)}{\partial n_t}\right]^{1-\alpha}\right\}, \tag{2.16}$$

where α is the bargaining power of the worker. The first-order condition for this maximization problem is:

$$(1-\alpha)\frac{\partial W(n_t,y_t)}{\partial n_t} = \alpha\frac{\partial \Pi(n_t,y_t)}{\partial n_t}, \tag{2.17}$$

which defines the standard sharing rule for splitting the aggregate surplus between workers and firms. Assuming that (2.17) holds in expectations, and using the FOC of the firm, (2.14), the wage is given by

$$w_t = \alpha(y_t + c\theta_t) + (1-\alpha)b. \tag{2.18}$$

2.6.6 Beliefs

Rational Expectation Equilibrium

Initially, we derive the REE where the expectations of workers and firms align and are measure with a objective probability measure. We denoted this expectations by E_t. The equilibrium is characterized by the labor market tightness at which the representative firm is indifferent to opening additional vacancy. This is captured by the free entry condition. By iterating forward the labor market tightness equation (2.15), and utilizing the firm's first-order condition (2.14), the wage equation (2.18), and the expected constant separation rate ($E_t\lambda_{t+1} = \lambda$), we come up with:

$$\frac{c}{\beta q(\theta_t)} = (1-\alpha)(E_t y_{t+1} - b) + \frac{(1-\lambda)c}{q(\theta_{t+1})} - \alpha c E_t \theta_{t+1}. \tag{2.19}$$

Deviations from this condition prompt immediate adjustments in θ_t through changes in firm vacancy decisions. The labor market tightness today is affected by expectations of the value of a filled vacancy in the next period.

Considering the productivity process (2.9), agents solve the system of equations given by (2.19) and (2.9) and linearizing around steady state values for $\overline{\theta}$ and $\overline{y} = 1$. This yields:

$$\theta_t = \hat{\phi}_0 + \hat{\phi}_1 y_{t-1} + \hat{\phi}_2 E_t \theta_{t+1} + \hat{\phi}_1 \rho^{-1} \epsilon_t. \tag{2.20}$$

In this paper, we focus on the RE equilibrium that takes the form of the fundamental or minimum state variable solution (MSV).[19] This solution can be guessed to be of the form

$$\theta_t = \overline{A} + \overline{B} y_{t-1} + \overline{C} \epsilon_t, \tag{2.21}$$

where $\overline{A}, \overline{B}$, and $\overline{C}$ are time-invariant coefficients known to the agents, ensuring that no systematic errors occur in their expectations.[20]

Agents' Model of Learning

We now relax the assumption of rational expectations by modeling agents as econometricians. We equipped agents with a perceived law of motion (PLM) that takes the form of MSV solution with with unobserved coefficients:

$$\theta_t = A_t + B_t y_{t-1} + \nu_t. \tag{2.22}$$

Agents estimate equation (2.22), estimating and updating their coefficients every period as new data become available. For that, they use a recursive least squares algorithm. Letting $\hat{x}_t' = (\hat{A}_t, \hat{B}_t)$ and

[19]2.20 can be written in ARMA(1,1) form. As Evans and Honkapohja (1986) point out, a complete listing of ARMA solutions brings into relief the problem of multiple equilibria. One selection rule has been proposed by McCallum (1983). His first principle is to choose a minimal set of state variables. One from which it is impossible to delete (i.e., set a coefficient of value zero) any single variable, or group of variables, while continuing to obtain a solution.

[20]Further details are provided in Appendix B showing how 2.21 can be obtained from 2.20.

$z_t' = (1, y_t)$, the algorithm can be written in recursive terms as:

$$R_t = R_{t-1} + g(z_{t-1}z_{t-1}' - R_{t-1}), \tag{2.23}$$
$$\hat{x}_t = \hat{x}_{t-1} + gR_t^{-1}z_{t-1}(\theta_{t-1} - z_{t-1}'\hat{x}_{t-1}) + \epsilon_{t-1}^{\beta}.$$

Where $\hat{x}_t$ denotes the current period's coefficient estimate, g $\in$ (0,1) denotes the constant gain, determining the rate at which older observations are discounted, and ϵ_t^{β} is a shock to labor market tightness beliefs. In this case, ϵ_t^{β} represents the new information about the transitory component of the labor market received at the end of the previous period (news shock).

From (2.22) it follows that agents' one-period forecasts of labor market tightness in a given period are given by

$$E_t^{\mathcal{P}}\theta_{t+1} = \hat{A}_t + \hat{B}_t y_t. \tag{2.24}$$

In any period t, agents inherit belief parameters determined by period $t-1$ data. It is assumed that although agents forecast θ_{t+1} by using y_{t-1}, the variable y_t is not in the information set for the estimation of A_t and B_t at the moment of making the forecast. Plugging (2.24) into (2.20) gives the actual law of motion (ALM) for labour market tightness

$$\theta_1 = \hat{\phi}_0 + \hat{\phi}_2\hat{A}_t + \hat{\phi}_2(1-\rho)\hat{B}_t + (\hat{\phi}_2\rho\hat{B}_t + \hat{\phi}_1)y_{t-1} + (\rho^{-1}\hat{\phi}_1 + \hat{\phi}_2\hat{B}_t)\epsilon_t. \tag{2.25}$$

Follow the method of Marcet and Sargent (1989) and Evans and Honkapohja (2012), we use the ALM (2.25) and the PLM (2.22) to formulate the function $T(\hat{A}_t, \hat{B}_t)$ that maps the agents' expectations about parameters A,B into their realised values

$$T(\hat{A}_t, \hat{B}_t) = [\hat{\phi}_0 + \hat{\phi}_2\hat{A}_t + \hat{\phi}_2(1-\rho)\hat{B}_t,\ \hat{\phi}_2\rho\hat{B}_t + \hat{\phi}_1]. \tag{2.26}$$

The fixed point in this mapping is a REE for the model mencioned in the subsection 6.6.1. The T-mapping determines the evolution of beliefs in transition to the long-run equilibrium.

Data Generating Process. Plugging (2.26) into (2.7) and (2.24), and solving delivers the actual data generating process

$$\mu_t = (\epsilon_t, \epsilon_t^\lambda, \epsilon_t^\beta) \sim N(0, \sigma_\mu^2 I_3), \sigma_\mu^2 = [\sigma^2, \sigma_\lambda^2, \sigma_\beta^2] \tag{2.27}$$

$$y_t = (1-\rho) + \rho y_{t-1} + \epsilon_t, \tag{2.28}$$

$$\lambda_t = \lambda + \epsilon_t^\lambda, \tag{2.29}$$

$$R_t = R_{t-1} + g(z_{t-1}z_{t-1}' - R_{t-1}), \tag{2.30}$$

$$\hat{x}_t = \hat{x}_{t-1} + gR_t^{-1} z_{t-1}(z_{t-1}'[T(\hat{A}_t, \hat{B}_t) - \hat{x}_{t-1} + V(\hat{B}_t)\epsilon_t] + \epsilon_t^\beta, \tag{2.31}$$

$$u_{t+1} = u_t + (1-u_t)\lambda_t - \mu(z_t' T(\hat{x}_t) + V(\hat{x}_t)\epsilon_t)^{1-\alpha} u_t \tag{2.32}$$

2.6.7 Calibration

This section describes the calibration of the model parameters, which total 12. The parameterization approach adopted is two-pronged: it involves selecting a subset of parameters from the existing literature and estimating the remaining parameters through a process of matching impulse responses of unemployment.

Specifically, the parameter vector $\theta_1 = [\beta, \alpha, \nu, \rho]$ is directly obtained from the literature. We normalize the time period to one month. The steady state of productivity is normalized to 1 without loss of generality. The discount factor β is set to 0.96, implying an annual real interest rate of approximately 5%. Direct evidence on workers' bargaining power is scarce; however, according to Petrongolo and Pissarides (2001), acceptable values fall within the interval [0.5, 0.7]. Mortensen and Nagypal (2007) suggests a value of 0.5, which aligns with conventional thinking in the literature. Following Hosios (1990), we set the parameter in the Nash bargaining problem such that $\alpha = 1 - \nu$. The value of the persistence of the productivity, ρ, is computed as an average of the 22 European countries.

The remaining parameters, collected in the vector $\Theta = [c, \lambda, A, g, b, \sigma, \sigma^\lambda, \sigma^\beta]$, are calibrated to match the model's unemployment responses after the three shocks with those observed empirically in our FAVAR. This calibration focuses on matching unemployment dynamics over horizons up to 60 months.

Let $\hat{\gamma}$ denote the vector collecting the IRFs of unemployment to the three structural shocks. The objective function targeted for optimization is defined as:

$$\mathcal{L}(\Theta) = (\hat{\gamma} - \gamma(\Theta))'\Omega^{-1}(\hat{\gamma} - \gamma(\Theta)), \tag{2.33}$$

where $\gamma(\Theta)$ represents the vector of IRFs generated by the model, and Ω is a weighting matrix. Specifically, Ω is a diagonal matrix where each diagonal element represents the variance of the corresponding IRF, with zeros elsewhere.

Variable	Description	Value	Source
α	bargaining power	0.50	Standard
β	discount factor	0.96	Annual real interest rate 0.05
ρ	persistence productivity	0.88	Empirical monthly productivity
ν	elasticity matching function	0.5	Hosios rule: $\alpha = 1 - \nu$

Table 2.6.2: Calibrated parameters from literature and data

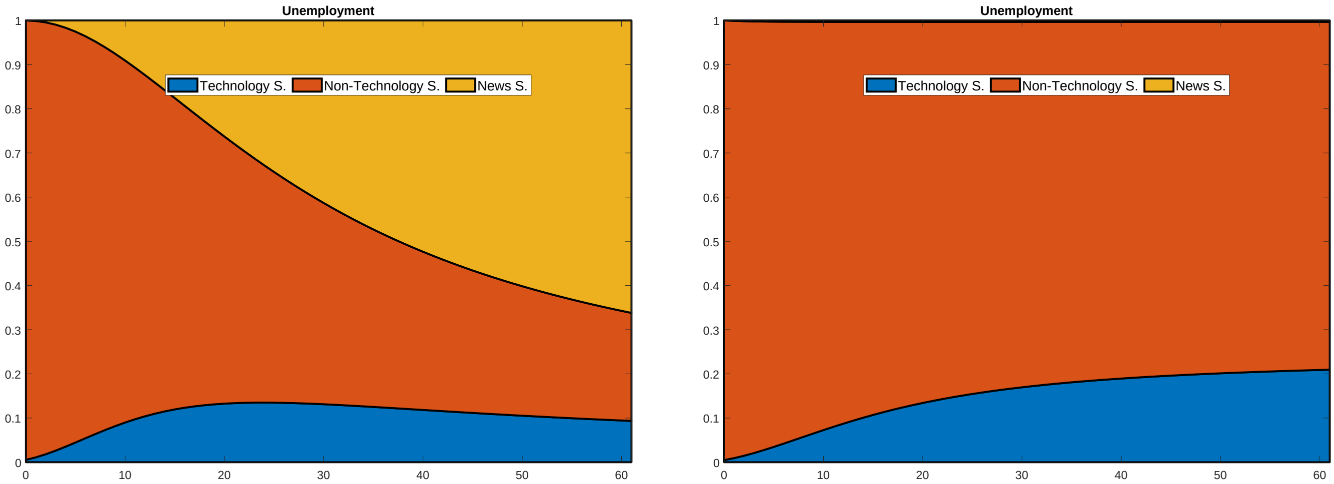
Variable	Description	Adap. Learning Estimates	RE Estimates wrt news shocks
c	cost of open a vacancy	0.29	0.32
μ	efficiency matching technology	0.11	0.11
g	constant gain	0.08	0.00
b	unemployment benefits	0.46	0.45
λ	separation rate	0.11	0.10
σ	Std. productivity shocks	0.0044	0.0049
σ^{λ}	Std. destruction rate shocks	0.0036	0.0040
σ^{β}	Std. news shocks	0.0031	0.0055

Table 2.6.3: Estimated monthly parameters from matching IRFs of unemployment

2.6.8 Theoretical Results

Figure 2.6.1 illustrates the share of unemployment variance attributed to each perturbation in the adaptive learning (AL) and rational expectations (RE) models. The AL model demonstrates an excellent fit, with short-term fluctuations in unemployment predominantly due to the destruction rate (i.e., non-technological shocks). Over time, the significance of news shocks grows, eventually accounting for more than 50% of the long-term variance, aligning with the empirical model (see Figure 2.4.3). In contrast, in the RE model, the contribution of news to the variance of unemployment is not different from zero at any time horizon. This result is in line with the findings of Theodoridis and Zanetti (2016). However, this fact is at odds with our empirical results.

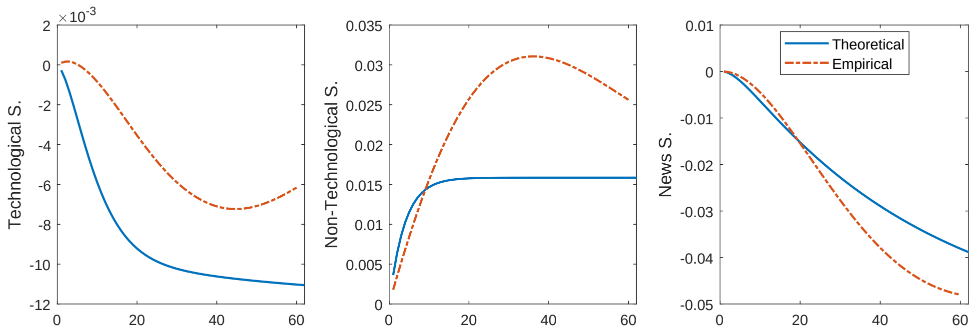
Figure 2.6.2 reports unemployment's cumulative impulse response functions to positive innovations



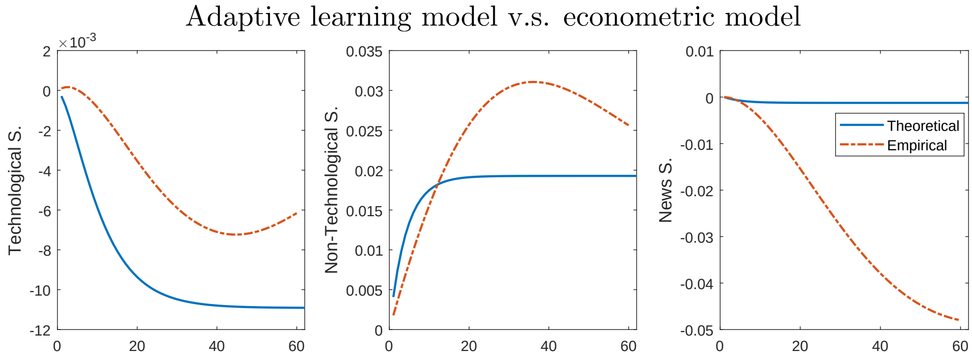
Adaptative Expectations Model Rational Expectations Model

Figure 2.6.1: Variance decomposition at different frequencies

Note: The colored areas represent the point-wise median cumulative contributions of each shock to the forecast error variance contributions of the unemployment rate at horizons j = 0, 1, . . . , 60.



Adaptive learning model v.s. econometric model



RE model v.s. econometric model

Figure 2.6.2: Cumulative response functions of unemployment to positive innovations from the theoretical and econometric model

Note: The solid blue line corresponds to CIRFs to the theoretical model, and dashed orange line to the median of the MF PANEL FAVAR.

in the AL and RE model, respectively. Model and empirical responses are displayed as blue lines and red dotted lines, respectively. Structural shocks are displayed on the rows. The horizontal axis measures time in months from impact up to 60 months post-innovation, while the vertical axis depicts the responses. Consistent with our empirical findings, the unemployment rate responds negatively to news innovations and positively to non-technological innovations in the AL model. The AL model successfully captures the persistent effect of news shocks on unemployment. In contrast, the RE model fails to generate a decrease in unemployment following a positive news shock, despite the news shock's standard deviation is close to two times larger than in the AL model. This indicates that the persistent effect generated in the AL model is not driven by a large standard deviation of such shock. Table 2.6.3 shows the standard deviations of each shock. The standard deviation of the news shock is lower compared to the other shocks. This fact differs from the ones used in the news literature, where the standard deviation is much larger than the fundamental ones, usually by a factor of two or three.

There are two principal takeaways from our estimation exercise. First, a parsimoniously specified labor search and matching model under adaptive learning successfully replicates the empirical dynamics of unemployment in response to news shocks. Therefore, these shocks have a cavity in a theoretical framework. Second, our results underscore the importance of accounting for non-technological news shocks, as they play a crucial role in driving labor market dynamics, both empirically and theoretically.

2.7 Robustness Tests

In this section, we perform several extensions to the baseline specification and check the robustness of our results to a battery of sensitivity checks. We include all the figures related to this section in Appendix D. The MF Panel FAVAR model presented in the previous section is estimated using five lags, long-run restrictions are imposed on an 8.3-year horizon (i.e. 100 months horizon), and using (i) labor productivity, (ii) unemployment rate and (iii) estimated factor from surveys of households and firms - manufacturing, services, retail and construction. Hence, we can include additional variables in the model and analyze the behavior of unemployment to these new shocks in the system. In addition, we follow Schorfheide and Song (2021) to handle the extreme observations from the COVID-19 outbreak. Moreover, we generate data from a random generating process to test whether the BP signalling strategy used in this paper induces a high correlation between the estimated structural shocks. Finally, we check whether our proposed augmented identification scheme of Beaudry and Portier (2006) is able to disentangle the different shocks when the data generating process is our theoretical model. We check the robustness of our results to changes in all of these specifications.

2.7.1 Alternative Long-Run Horizons, and Lag Specifications

In this subsection, we present the robustness of our baseline results to alternative long-run horizons and lag specifications. Figures D 2.D.5, 2.D.6, 2.D.7 and 2.D.8 present the correlation between w_{3t} against $\tilde{w}_{1t}$ and w_{3t} against $\tilde{w}_{2t}$ using the BP signalling scheme scheme and the IRFs under the joint identification scheme of equation 2.5.

Changing the horizon - to 50 and 150 periods - at which long-run restrictions are imposed does not affect the results presented in the previous section. The same is true if we use different lag specifications, 7 and 9 lags - see Figures D 2.D.1, 2.D.2, 2.D.3 and 2.D.4.

2.7.2 More Variables

In this subsection, we present the robustness of our baseline results to the inclusion of investment (INV) in the model. With the inclusion of INV, we can analyse the behavior of pigouvian cycles.

Including Investment. Pigouvian cycles require total investment to increase in response to an anticipated shock. In this sense, investment is included in the system as the last variable. The new joint restrictions at the short-long run are denoted in Equation 2.34, which maintain the same core assumptions as in equation 2.5. The only difference is that the extra shock has no long-term effects on productivity, unemployment and the factor.

Figure D 2.D.9 represents the response functions taking into account investment in the system. The fourth row and third column show that investment increases after a positive non-technological news shock, confirming the existence of Pigou cycles.

$$P = \begin{pmatrix} * & * & * & * \\ * & * & 0 & * \\ * & * & * & * \\ * & * & * & * \end{pmatrix}, \quad L = \begin{pmatrix} * & 0 & 0 & 0 \\ * & * & * & 0 \\ * & * & * & 0 \\ * & * & * & * \end{pmatrix} \tag{2.34}$$

2.7.3 Handling extreme observations from the COVID-19 outbreak a la Schorfheide and Song (2021)

In line with Schorfheide and Song (2021), we exclude Covid crisis observations - March, April, and May - from the estimation sample, given that it is another way of modeling outliers. Figure D 2.D.10 and 2.D.11 show that excluding a few months of extreme observations does not change our baseline results. In fact, it provides evidence in favor of that this method is a promising way of handling VAR estimation going forward, as an alternative of a sophisticated modeling of outliers.

2.7.4 Spurious correlation between the estimated structural shocks

In this subsection, we present noisy simulated data to check whether the BP signalling strategy used in this paper induces a high correlation between the estimated structural shocks. If this spurious correlation were to happen with simulated data, we could not impose the long-run behavior of news labor market shocks on unemployment in equation 2.5.

Figure D 2.D.12 plot the correlation between w_3 against $\tilde{w}_1$ and w_3 against $\tilde{w}_2$ for three different simulated data groups. The correlation figures plot an obvious point cloud in each simulated group, pointing this identification system does not generate spurious correlations between w_3 against $\tilde{w}_1$ and w_3 against $\tilde{w}_2$.

2.7.5 Simulating data from the theoretical model

In this subsection, we present two robustness exercises using the theoretical model as our data generating process. First, we create artificial series of productivity, unemployment rate and the expectation of the labor market tightness using the three shocks in our model. Then, we run a standard VAR model and apply the augmented version of BP (i.e., signalling scheme and a joint identification scheme - equation 2.5) to our estimation. Figures 2.D.13 and 2.D.14 show that our proposed augmented version of BP is able to fully identify the three shocks of interest. In our second robustness, we create a non-fundamentalness problem. This means that we create the same artificial series for productivity, unemployment rate and the expectation of the labor market tightness, but now only using the technological and non-technological shocks in our model. Figures 2.D.15 and 2.D.16 show two important things. First, the signalling scheme of BP correctly picks that the forward looking variable (the expectation of the labor market tightness) does not contain news shocks. Second, we proceed by ignoring the result of signalling scheme and impose the joint identification scheme of equation 2.5. It can be seen that in Figure 2.D.16 that the joint identification scheme does not identify any news shock. These robustness results make us confident with respect to the use of our proposed augmented version of BP.

2.8 Conclusion

This paper presents novel empirical evidence on the relationship between non-technological news shocks and unemployment fluctuations in Europe. Employing a mixed-frequency Panel FAVAR, using surveys from various economic agents, and formulating a simultaneous identification scheme (long and short run restrictions), we find significant insights. First, using the BP scheme, we find that surveys of economic agents' expectations contain non-technological news. Second, applying the proposed simultaneous identification to jointly identify technological, non- technological and news shocks, the non-technological news shocks are important to explain unemployment, especially in the medium/long run, accounting around 65% of its variance. This result is robust to the identification of technological news shocks including the stock market index. Moreover, we find that non-technology shocks emerge as the primary driver of unemployment fluctuations at the business cycle frequency, contributing over 70% of its short run variance. Neither technological news nor technological shocks are the main driver of unemployment throughout the business cycle.

The last fact runs counter to what is predicted by standard labor market models in which productivity shock drives fluctuations. We propose a theoretical mechanism that can rationalize the empirical findings. We apply adaptive learning to a dynamic and stochastic search and matching model where news shocks hit the economy. Through the job creation condition, waves of optimism or pessimism, unrelated to economic productivity developments, affect vacancies and unemployment due to imperfect knowledge of labor market tightness. In other words, in this model, agents recognize changes in labor market opportunities in advance of their effect on unemployment, and the recognition itself leads to a boom in vacancies, which precedes a reduction in unemployment. The introduction of these shocks together with adaptive learning generates the persistent effect of these shocks on unemployment that we observe in our empirical approach. In contrast, in the rational expectation version of the model, these shocks do not play a major role in long-run of unemployment.

Future research could explore the heterogeneous effects of news shocks on various labor market outcomes, such as wages, labor force participation, and job search behaviors. This analysis could further examine differences across different groups of workers, such as by age and education. Investigating the impact of labor market policies and institutional differences across countries could also shed light on the robustness and generalizability of our findings to other contexts.

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Appendix

Appendix 2.A A Gibbs Sampler for PANEL VARs

First, let me use the notation $z_{i,j:k}$ to denote the sequence $\{z_{i,j}, \ldots, z_{i,k}\}$ for a generic variable of a country $z_{i,t}$. The mixed-frequency Panel FAVAR, specified by the observed and unobserved equations in Section 3, is estimated using a Gibbs sampler, which involves the following blocks:

1. The first block involves draws from the joint distribution $y_{i,-p+1:T}, H_i \mid \text{vec}(\mathcal{A}_i), \Sigma_i, W_i, y_{i,1:T}$, which is given by the product of the marginal posterior of $H_i \mid \text{vec}(\mathcal{A}_i), \Sigma_i, W_i, y_{i,1:T}$ times the distribution of the initial observations $y_{i,-p+1:T} \mid H_i, \text{vec}(\mathcal{A}_i), \Sigma_i, W_i, y_{i,1:T}$. The marginal posterior of $H_i \mid \text{vec}(\mathcal{A}_i), \Sigma_i, W_i, y_{i,1:T}$ is given by:

$$p\left(H_i \mid \text{vec}(\mathcal{A}_i), \Sigma_i, W_i, y_{i,1:T}\right) \propto \mathcal{L}\left(y_{i,1:T} \mid H_i, \text{vec}(\mathcal{A}_i), \Sigma_i, W_i, y_{i,1:T}\right) p(H_i)$$

where $\mathcal{L}\left(y_{1:T} \mid H_i, \text{vec}(\mathcal{A}_i), \Sigma_i, W_i\right)$ is the likelihood obtained by using the Kalman Filter in the state-space model specified in the observed equation. Since $p\left(H_i \mid \text{vec}(\mathcal{A}_c), \Sigma_i, W_i, y_{i,1:T}\right)$ does not feature a known form, this step involves a Metropolis-Hastings algorithm. Then, we use Carter and Kohn (1994) and Durbin and Koopman (2002)'s simulation smoother to obtain draws for the estimated factors $y_{i,-p+1:T}$, for given H_i and $\text{vec}(\mathcal{A}_i), \Sigma_i, W_i, y_{i,1:T}$.

2. The second block involves the estimation of Equation 2.2, given $y_{i,-p+1:T}$. The posterior distribution of $\text{vec}(\mathcal{A}_i), \Sigma_i$ is given by:

$$p\left(\Sigma_i \mid y_{0:T}\right) = IW\left(\underline{\Sigma_i} + \hat{S}_{i,v}, (n+2) + T\right)$$
$$p\left(\text{vec}(\mathcal{A}_i) \mid \Sigma_i, y_{i,0:T}\right) = N\left(\text{vec}(\hat{\mathcal{A}}_i), \Sigma_i \otimes \left(X_i X_i' + \underline{\Sigma_i}^{-1}\right)^{-1}\right)$$

where $X_i = \left(y'_{i,-p+1}, \ldots, y'_{i,T-(p+1)}\right)'$, $\hat{S}_{i,v} = v_i v_i' + (\hat{\mathcal{A}}_i - \underline{\mathcal{A}_i})'\underline{\Sigma_i}^{-1}(\hat{\mathcal{A}}_i - \underline{\mathcal{A}_i})$, and
$\hat{\mathcal{A}}_i = \left(X_i X_i' + \underline{\Sigma_i}^{-1}\right)^{-1} \left(X' y_{i,1:T} + \underline{\Sigma_i}^{-1} \text{vec}(\underline{\mathcal{A}_i})\right)$,
and $v_i = y_i - \hat{\mathcal{A}}_i{}' X_i$ are the VAR residuals.

Appendix 2.B European Commission - The Business and Consumer Survey

To calculate the aggregate confidence indicator of each economic agent, we follow the procedure in the Joint Harmonised EU Programme of Business and Consumer Surveys of the European Commission.

Industrial confidence indicator.

The industrial confidence indicator is the arithmetic average of the balances (in percentage points) of the answers to the questions on production expectations, order books, employment expectations and stocks of finished products (the last with inverted sign).

Do you consider your current overall order books to be...?

- + more than sufficient (above normal)
- = sufficient (normal for the season)
- − not sufficient (below normal)

Do you consider your current stock of finished products to be...?

- + too large (above normal)
- = adequate (normal for the season)
- − too small (below normal)

How do you expect your production to develop over the next 3 months? It will...

- + increase
- = remain unchanged
- − decrease

How do you expect your firm's total employment to change over the next 3 months? It will...

- + increase
- = remain unchanged

- − decrease

Services confidence indicator.

The services confidence indicator is the arithmetic average of the balances (in percentage points) of the answers to the questions on business climate and on recent and expected evolution of demand and employment.

How has your business situation developed over the past 3 months? It has...

- + improved
- = remain unchanged
- − deteriorated

How has demand (turnover) for your company's services changed over the past 3 months? It has...

- + increase
- = remain unchanged
- − decrease

How do you expect the demand (turnover) for your company's services to change over the next 3 months? It will...

- + increase
- = remain unchanged
- − decrease

How do you expect your firm's total employment to change over the next 3 months? It will...

- + increase
- = remain unchanged
- − decrease

Retail trade confidence indicator.

The retail trade confidence indicator is the arithmetic average of the balances (in percentage points) of the answers to the questions on the present and future business situation, expected employment and on stocks (the last with inverted sign).

How has (have) your business activity (sales) developed over the past 3 months?

- + improved
- = remain unchanged
- − deteriorated

Do you consider the volume of stock currently hold to be...?

- + too large (above normal)
- = adequate (normal for the season)
- − too small (below normal)

How do you expect your business activity (sales) to change over the next 3 months? It (They) will...

- + improved
- = remain unchanged
- − deteriorated

How do you expect your firm's total employment to change over the next 3 months? It will...

- + increase
- = remain unchanged
- − decrease

Construction confidence indicator.
The construction confidence indicator is the arithmetic average of the balances (in percentage points) of the answers to the questions on order book and employment expectations.

Do you consider your current overall order books to be...?

- + more than sufficient (above normal)

- = sufficient (normal for the season)
- − not sufficient (below normal)

How do you expect your firm's total employment to change over the next 3 months? It will...

- + increase
- = remain unchanged
- − decrease

Consumer confidence indicator.

The consumer confidence indicator is the arithmetic average of the balances (in percentage points) of the answers to the questions on the past and expected financial situation of households, the expected general economic situation, the intentions to make major purchases over the next 12 months and expected unemployment (the last with inverted sign).

How has the financial situation of your household changed over the last 12 months? It has...

- ++ got a lot better
- + got a little better
- = stayed the same
- − got a little worse
- −− got a lot worse
- N don't know

How do you expect the financial position of your household to change over the next 12 months? It will...

- ++ got a lot better
- + got a little better
- = stayed the same
- − got a little worse

- −− got a lot worse
- N don't know

How do you expect the general economic situation in this country to develop over the next 12 months? It will...

- ++ got a lot better
- + got a little better
- = stayed the same
- − got a little worse
- −− got a lot worse
- N don't know

Compared to the past 12 months, do you expect to spend more or less money on major purchases (furniture, electrical/electronic devices, etc.) over the next 12 months? I will spend...

- ++ much more
- + a little more
- = about the same
- − a little less
- −− much less
- N don't know

How do you expect the number of people unemployed in this country to change over the next 12 months? The number will...

- ++ increase sharply
- + increase slightly
- = remain the same
- − fall slightly
- −− fall sharply
- N don't know

Appendix 2.C Main Results - Complete Figures

In this section, we present the complete figures from the augmented identification scheme of the baseline model MF Panel Favar using (i) labor productivity, (ii) unemployment rate and (iii) estimated factor from surveys of households and firms - manufacturing, services, retail and construction.

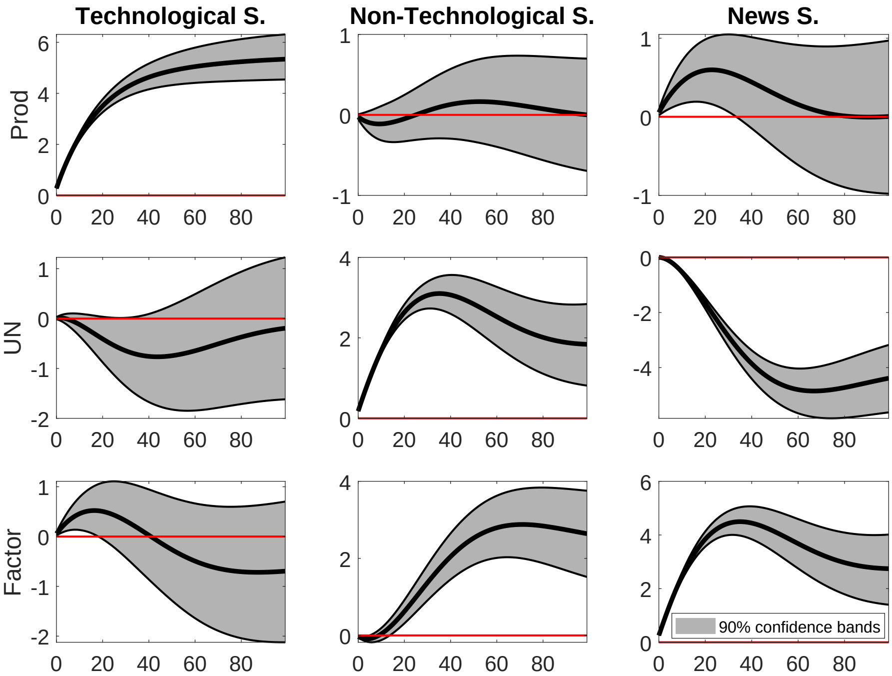


Figure 2.C.1: Impulse response functions to a permanent positive shock, as in Equation 2.5, from the whole the MF PANEL FAVAR

Note: Posterior distributions of impulse response functions to an estimated shock of one standard deviation using short-long restrictions, as in Equation 2.5. Median (solid line) and 90% probability density intervals (shaded area) based on 10,000 draws.

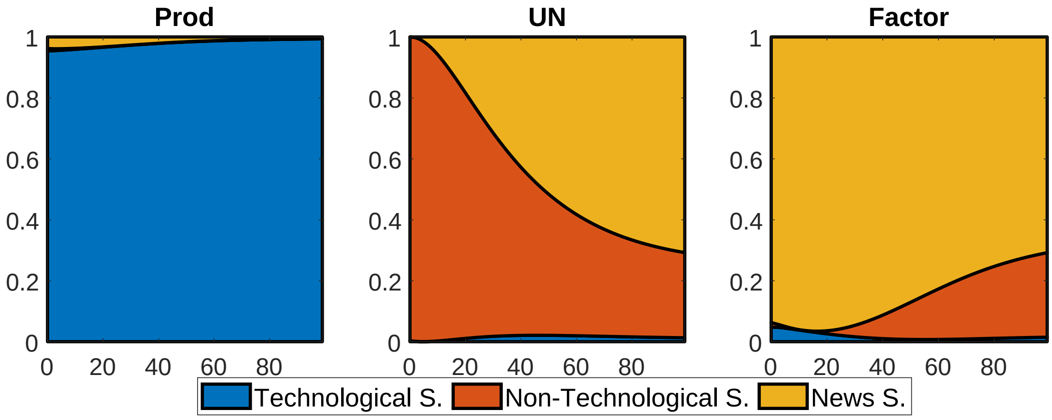


Figure 2.C.2: Variance decomposition at different frequencies

Note: The colored areas represent the point-wise median cumulative contributions of each identified shock to the forecast error variance contributions of each variable at horizons j = 0, 1, . . . , 100 using joint short and long-run restrictions as in equation 2.5.

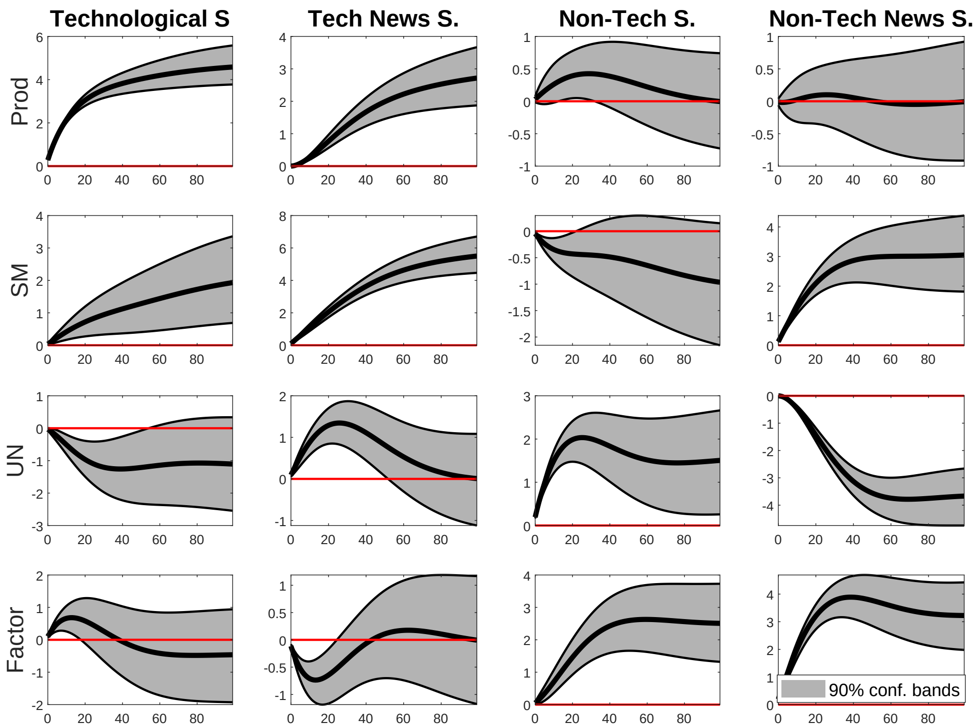


Figure 2.C.3: Impulse response functions from the whole the MF PANEL FAVAR with stock prices

Note: Posterior distributions of cumulative impulse response functions to an estimated shock of one standard deviation using short-long restrictions, as in Equation 2.6. Median (solid line) and 90% probability density intervals (shaded area) based on 10,000 draws.

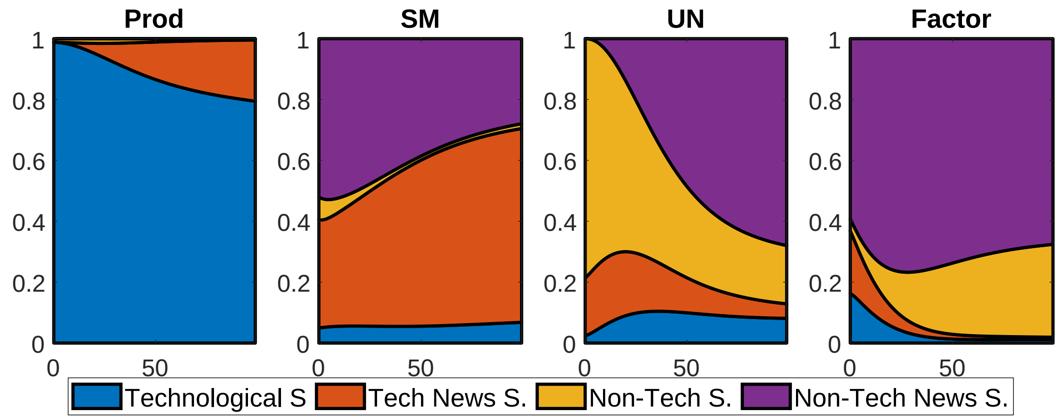


Figure 2.C.4: Variance decomposition at different frequencies

: The colored areas represent the point-wise median cumulative contributions of each identified shock to the forecast error variance contributions of each variable at horizons j = 0, 1, . . . , 100 using joint short and long-run restrictions as in equation 2.6.

Appendix 2.D Robustness Figures

In this section, we present different the results of several extensions to the baseline model specification. We include figures using (i) 7 and 9 lags in the MF Panel Favar model, (ii) changing the long-run horizon imposed at the identification schemes to 4.1 years (50 periods) and 12.5 years (150 periods), (iii) enlarging the model with more variables investment, (iv) control for the extreme observations from the COVID-19 outbreak, (v) generate dummy data to show that the identification scheme does not generate spurious results, and (vi) simulating data from the theoretical model.

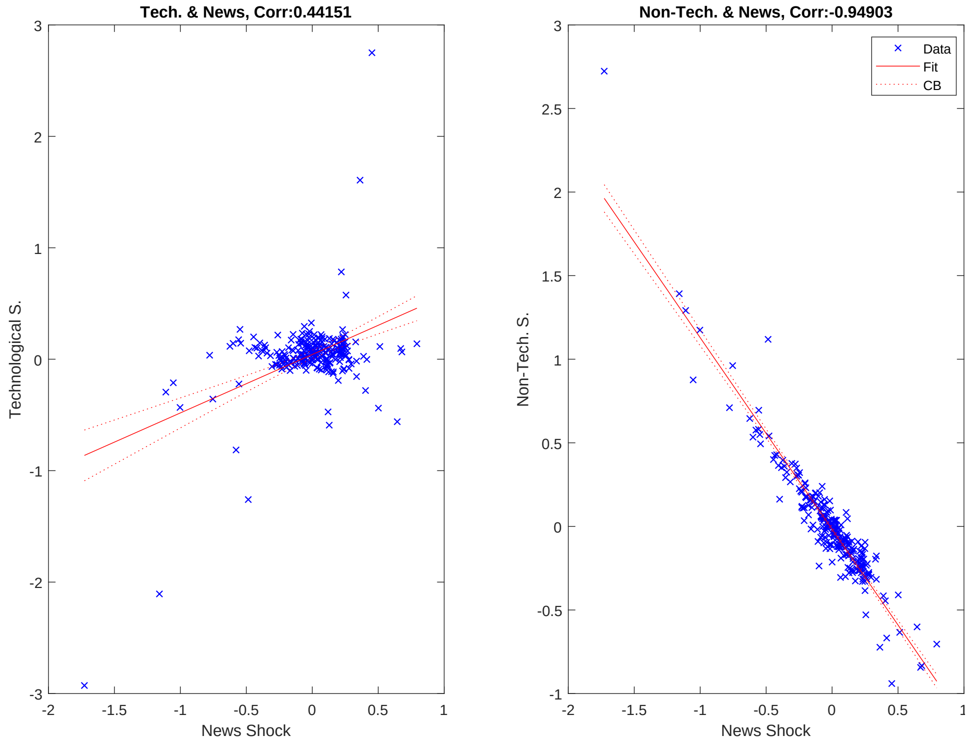


Figure 2.D.1: Plot of w_{3t} against $\tilde{w}_{1t}$ - left - and on the right w_{3t} against $\tilde{w}_{2t}$ in the MF Panel VAR with 7 lags.

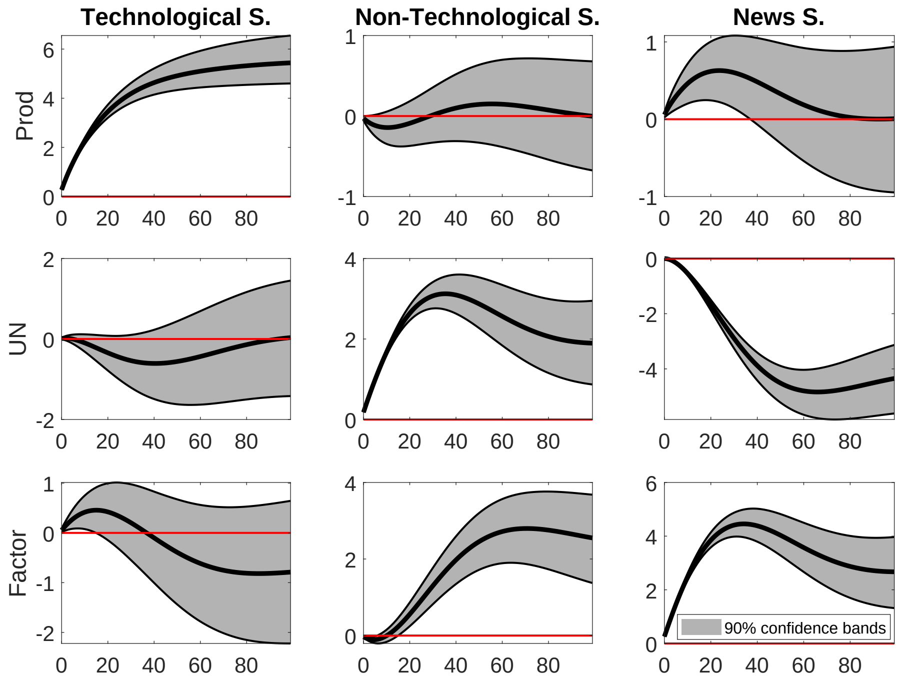


Figure 2.D.2: Impulse response functions from the whole the MF Panel VAR with 7 lags.

Note: Posterior distributions of cumulative impulse response functions to an estimated shock of one standard deviation using short-long restrictions, as in Equation 2.5. Median (solid line) and 90% probability density intervals (shaded area) based on 10,000 draws.

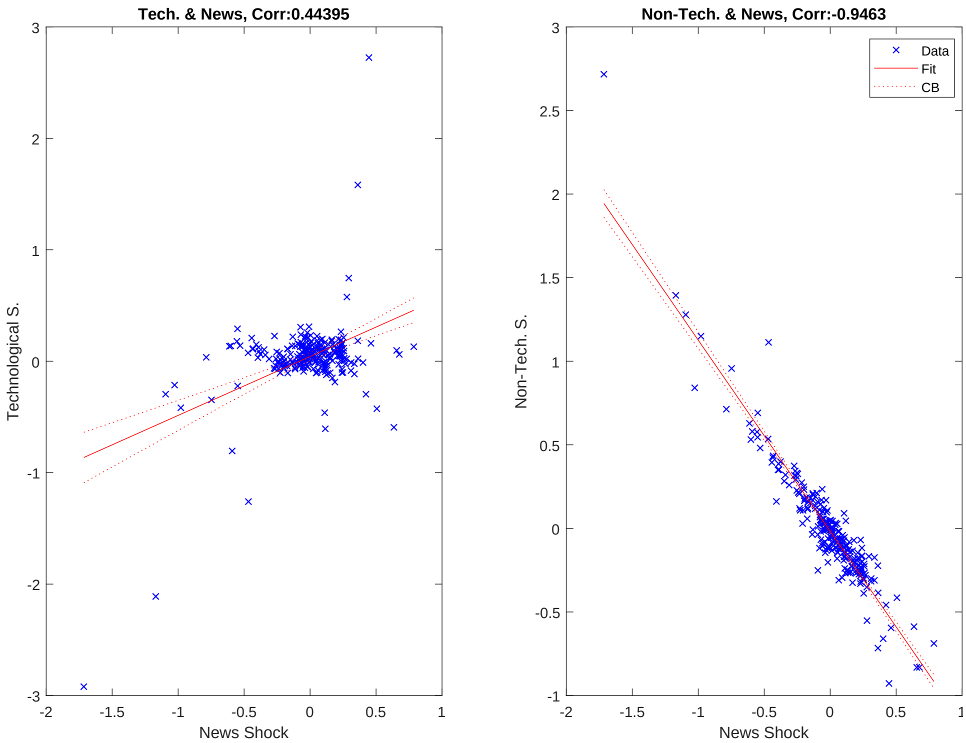


Figure 2.D.3: Plot of w_{3t} against $\tilde{w}_{1t}$ - left - and on the right w_{3t} against $\tilde{w}_{2t}$ in the MF Panel VAR with 9 lags.

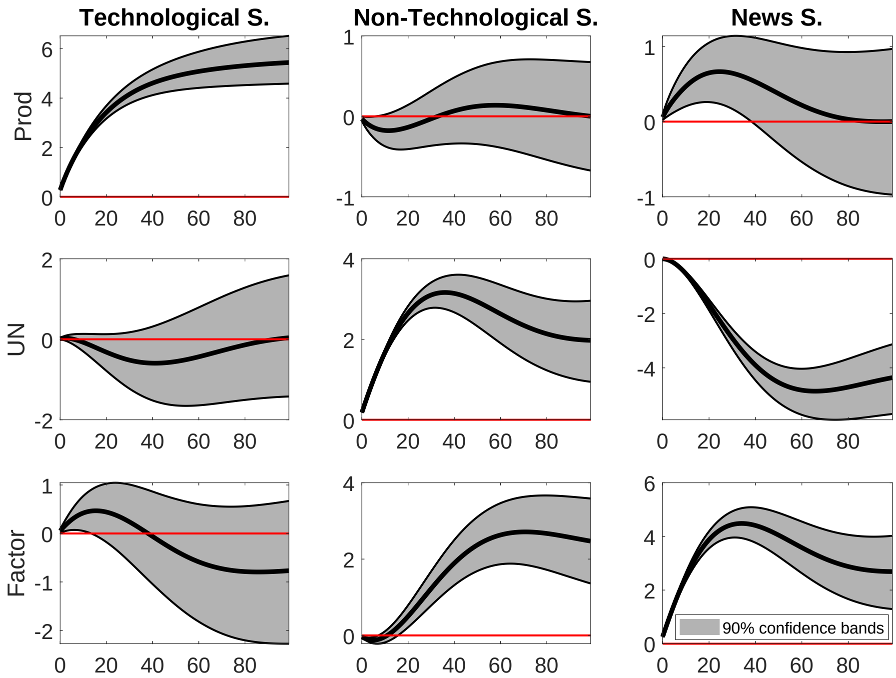


Figure 2.D.4: Impulse response functions from the whole the MF Panel VAR with 9 lags.

Note: Posterior distributions of cumulative impulse response functions to an estimated shock of one standard deviation using short-long restrictions, as in Equation 2.5. Median (solid line) and 90% probability density intervals (shaded area) based on 10,000 draws.

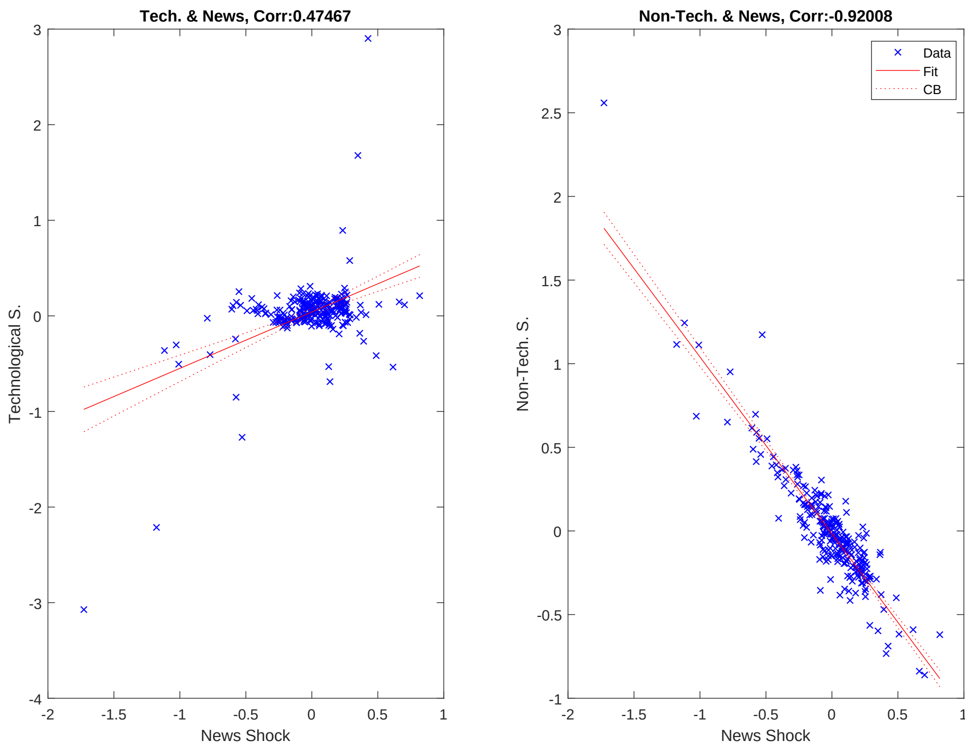


Figure 2.D.5: Plot of w_{3t} against $\tilde{w}_{1t}$ - left - and on the right w_{3t} against $\tilde{w}_{2t}$ in the MF Panel VAR with 5 lags. Long-run shocks are imposed to be neutral at 50 periods horizon.

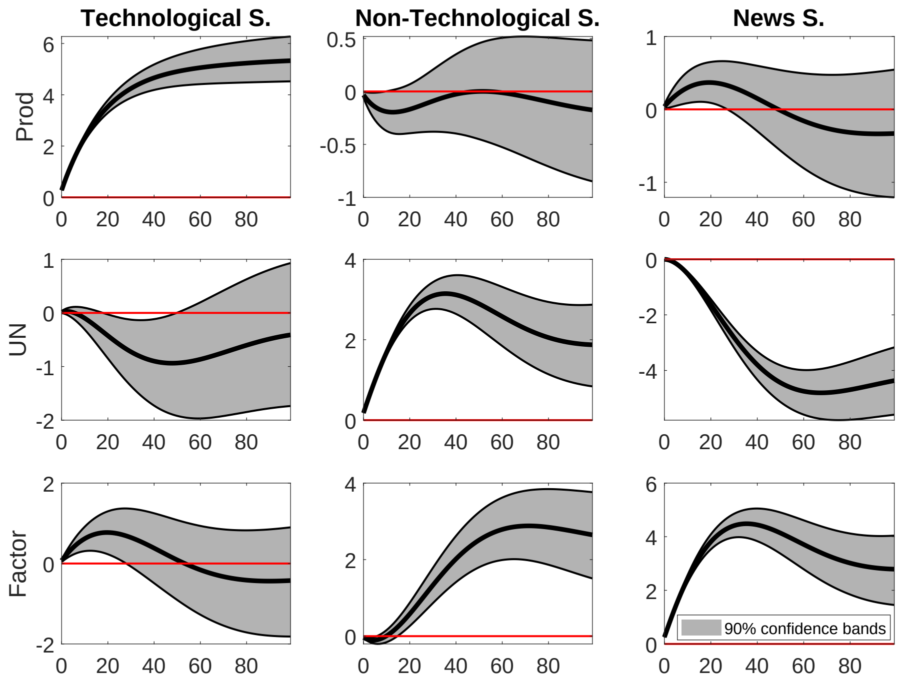


Figure 2.D.6: Impulse response functions from the whole the MF PANEL FAVAR imposing long-run horizon at 50 periods.

Note: Posterior distributions of cumulative impulse response functions to an estimated shock of one standard deviation using short-long restrictions, as in Equation 2.5. Median (solid line) and 90% probability density intervals (shaded area) based on 10,000 draws.

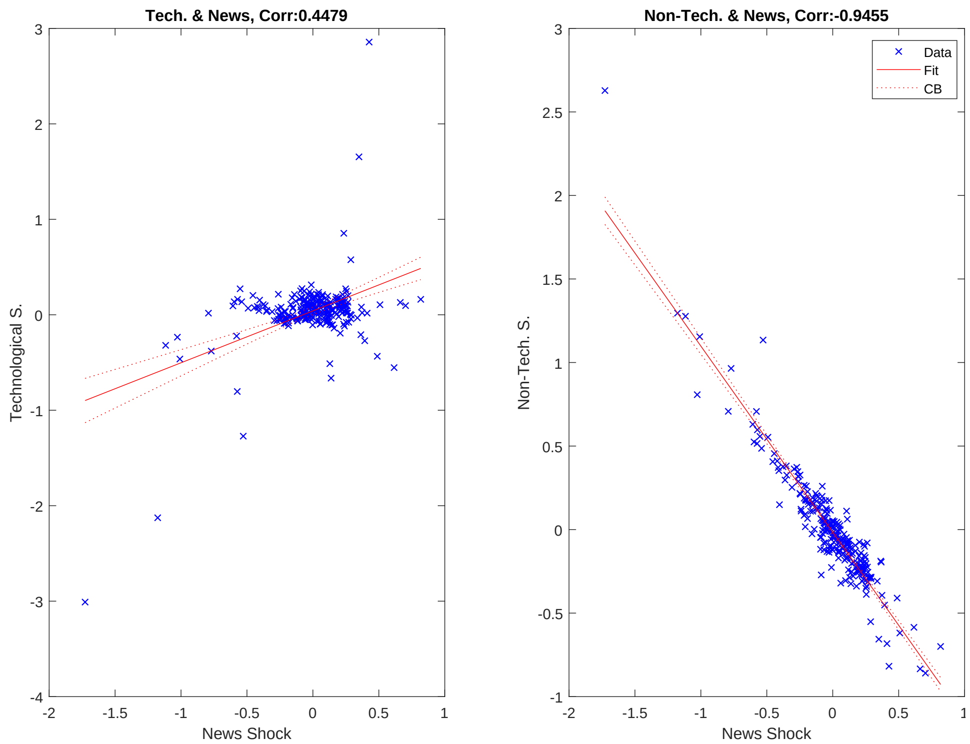


Figure 2.D.7: Plot of w_{3t} against $\tilde{w}_{1t}$ - left - and on the right w_{3t} against $\tilde{w}_{2t}$ in the MF Panel VAR with 5 lags. Long-run shocks are imposed to be neutral at 150 periods horizon.

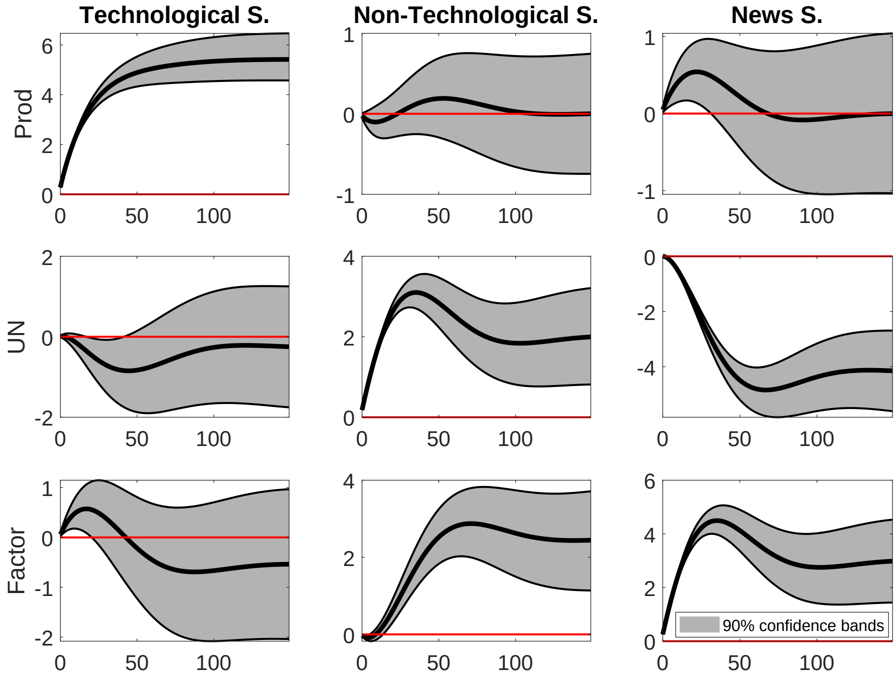


Figure 2.D.8: Impulse response functions from the whole the MF PANEL FAVAR imposing long-run horizon at 150 periods.

Note: Posterior distributions of cumulative impulse response functions to an estimated shock of one standard deviation using short-long restrictions, as in Equation 2.5. Median (solid line) and 90% probability density intervals (shaded area) based on 10,000 draws.

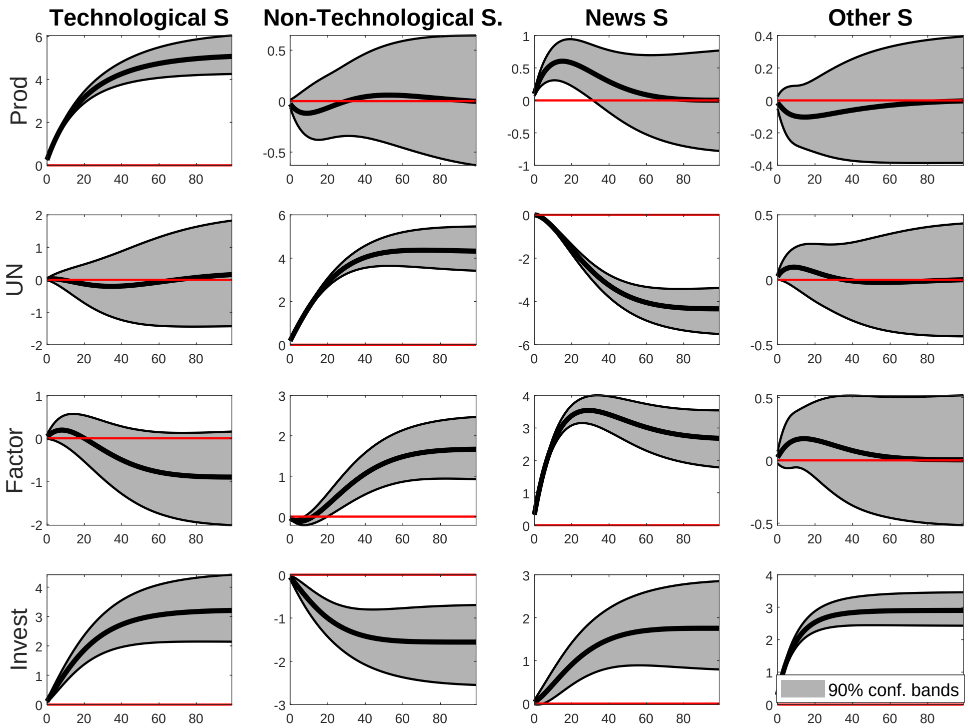


Figure 2.D.9: Impulse response functions from the whole the MF PANEL FAVAR with investment

Note: Posterior distributions of cumulative impulse response functions to an estimated shock of one standard deviation using short-long restrictions, as in Equation 2.6. Median (solid line) and 90% probability density intervals (shaded area) based on 10,000 draws.

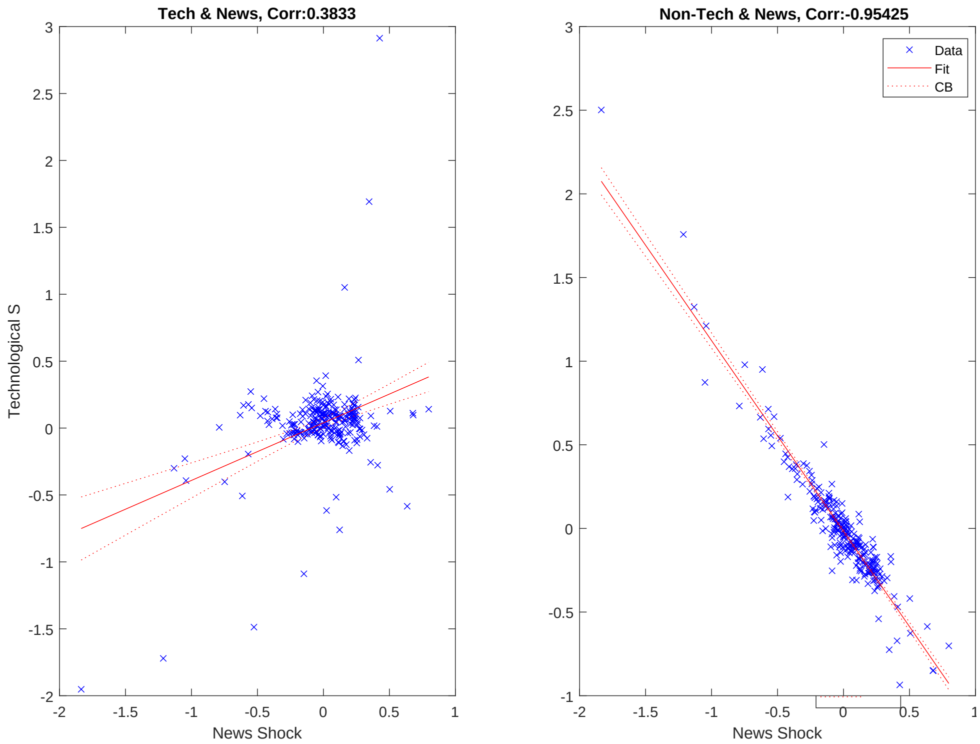


Figure 2.D.10: Plot of w_{3t} against $\tilde{w}_{1t}$ - left - and on the right w_{3t} against $\tilde{w}_{2t}$ in the MF Panel VAR excluding observations from March, April and May 2020

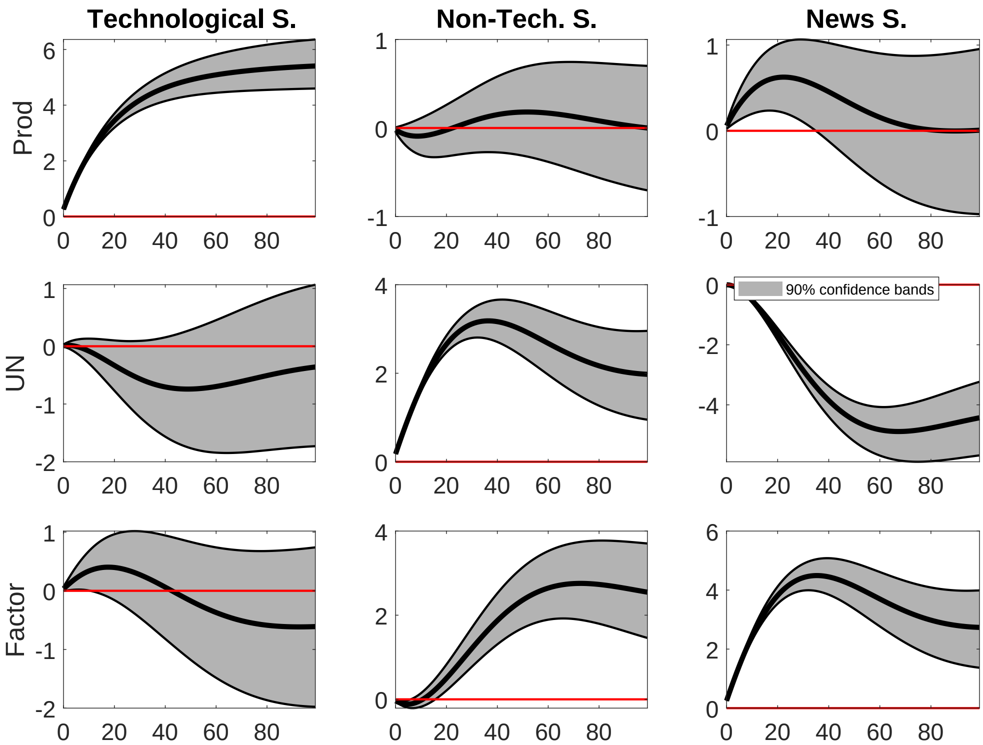
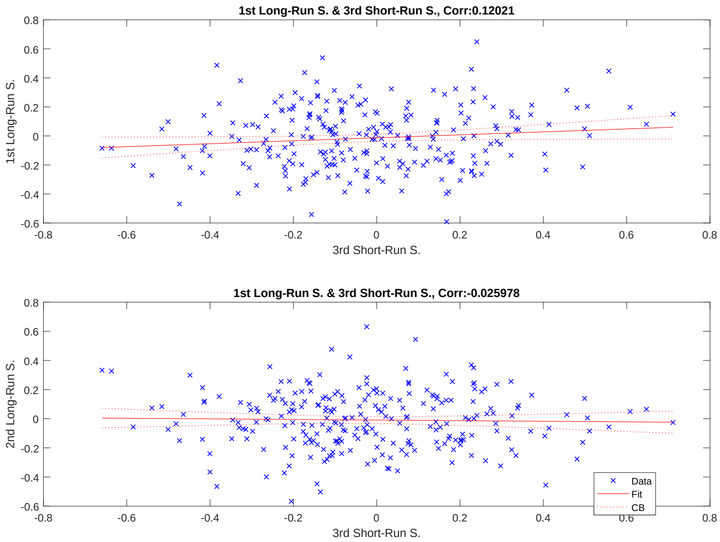
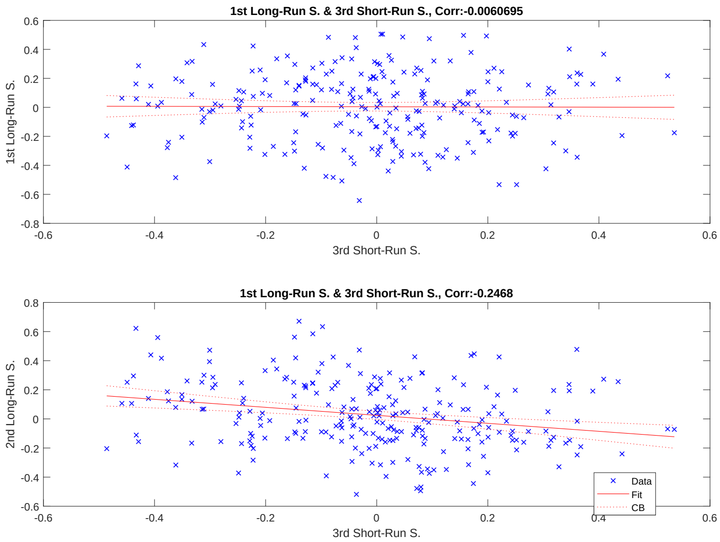


Figure 2.D.11: Impulse response functions from the whole the MF PANEL FAVAR with excluding observations from March, April and May 2020

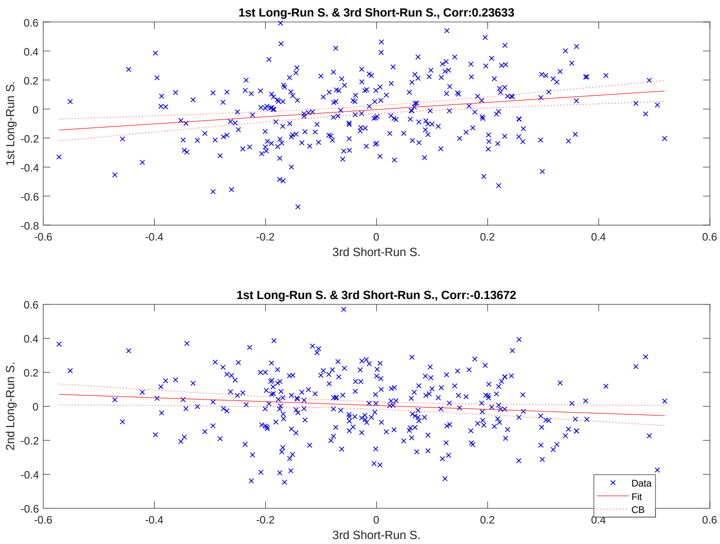
Note: Posterior distributions of cumulative impulse response functions to an estimated shock of one standard deviation using short-long restrictions, as in Equation 2.6. Median (solid line) and 90% probability density intervals (shaded area) based on 10,000 draws.



Simulated data, group 1.



Simulated data, group 2.



Simulated data, group 3.

Figure 2.D.12: Plot of w_3 against $\tilde{w}_1$ - left - and $\tilde{w}_2$ -right. Shocks are obtained from the trivariate specification with five lags.

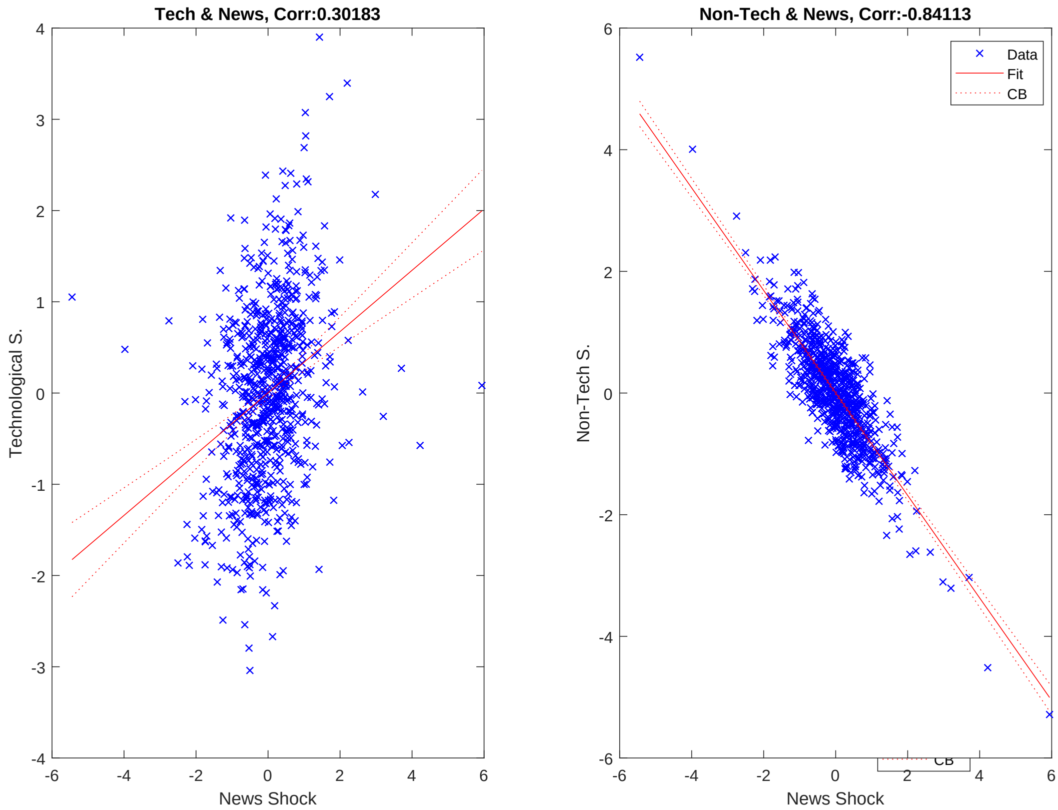


Figure 2.D.13: Plot of w_{3t} against $\tilde{w}_{1t}$ - left - and on the right w_{3t} against $\tilde{w}_{2t}$, using data generated from the theoretical model. The three shocks are active.

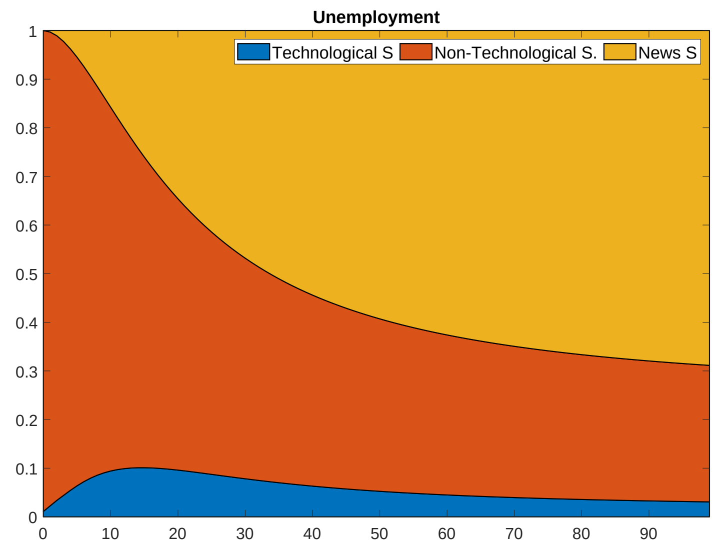


Figure 2.D.14: Variance decomposition at different frequencies of the unemployment rate - using data generated from the theoretical model. The three shocks are active.

Note: The colored areas represent the point-wise median cumulative contributions of each identified shock to the forecast error variance contributions of each variable at horizons j = 0, 1, . . . , 100 using joint short and long-run restrictions as in equation 2.5.

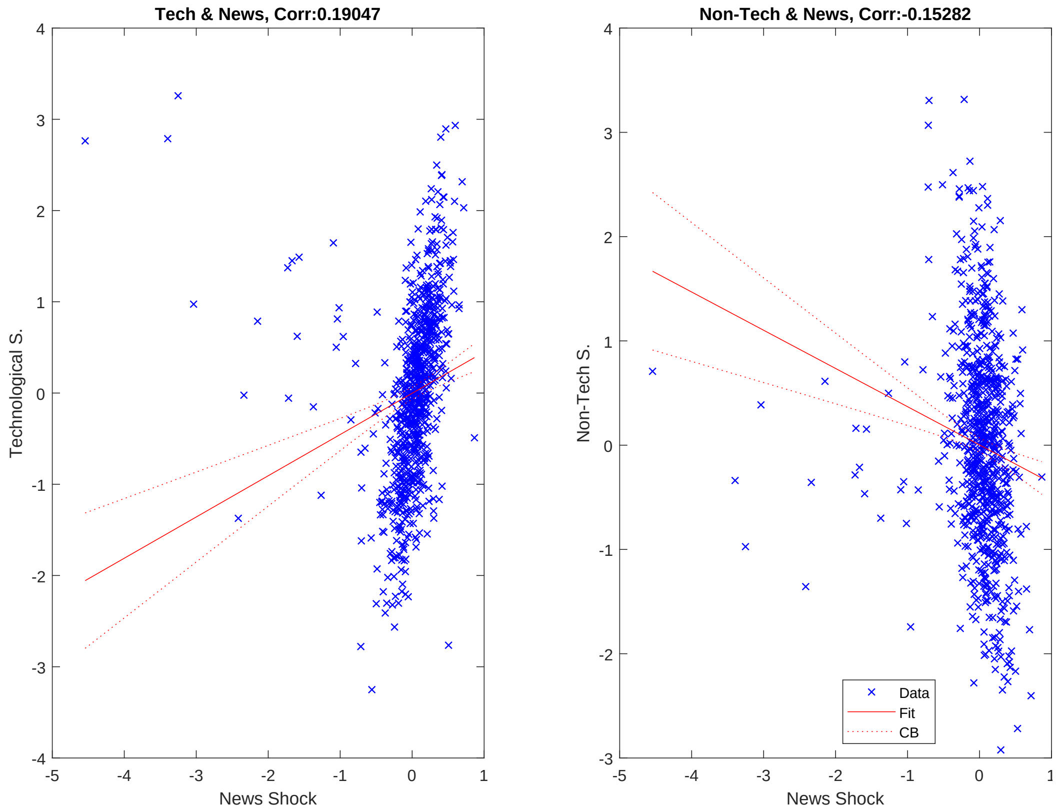


Figure 2.D.15: Plot of w_{3t} against $\tilde{w}_{1t}$ - left - and on the right w_{3t} against $\tilde{w}_{2t}$, using data generated from the theoretical model. Only technological and non-technological shocks are active.

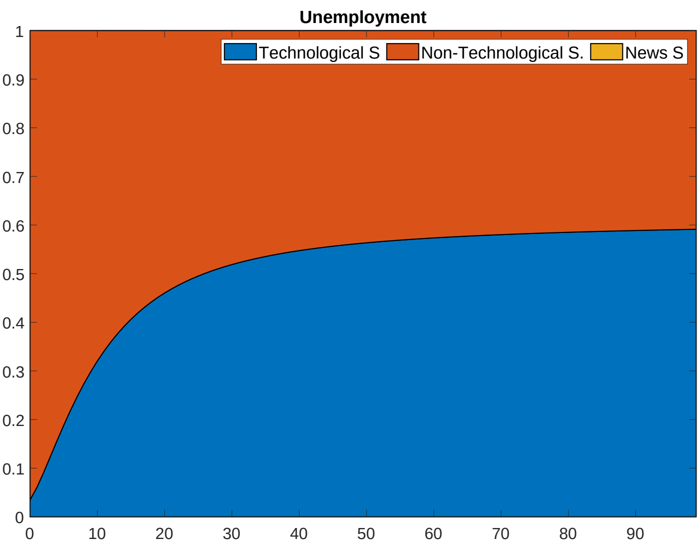


Figure 2.D.16: Variance decomposition at different frequencies of the unemployment rate - using data generated from the theoretical model. Only technological and non-technological shocks are active.

Note: The colored areas represent the point-wise median cumulative contributions of each identified shock to the forecast error variance contributions of each variable at horizons j = 0, 1, . . . , 100 using joint short and long-run restrictions as in equation 2.5.

Appendix 2.E Linearization of the Job Creation Condition

We linearize the job creation condition applying a first-order Taylor polynomial of this equation at the steady state $\theta = \overline{\theta}$ and $y = \overline{y} = 1$. The job creation condition is represented by the following equation:

$$\frac{c}{\beta q(\theta_t)} = (1-\alpha)(E_t y_{t+1} - b) + \frac{(1-\lambda)c}{q(\theta_{t+1})} - \alpha c E_t \theta_{t+1}. \tag{2.35}$$

We take the first-order Taylor polynomial of each component of the previous equation:

$$\frac{c}{\beta q(\theta_t)} = \frac{c}{\beta q(\overline{\theta})} - \frac{c}{\beta q(\overline{\theta})^2} \frac{\partial q(\overline{\theta})}{\partial \overline{\theta}} (\theta_t - \overline{\theta}) \tag{2.36}$$

$$(1-\alpha)(E_t y_{t+1} - b) = (1-\alpha)(1-b) + (1-\alpha)(E_t y_{t+1} - b) \tag{2.37}$$

$$\frac{c(1-\lambda)}{q(E_t\theta_{t+1})} = \frac{c(1-\lambda)}{q(\overline{\theta})} - \frac{c(1-\lambda)}{q(\overline{\theta})^2} \frac{\partial q(\overline{\theta})}{\partial \overline{\theta}} (E_t\theta_{t+1} - \overline{\theta}) \tag{2.38}$$

$$\alpha c E_t \theta_{t+1} = \alpha c \overline{\theta} + \alpha c (E_t \theta_{t+1} - \overline{\theta}) \tag{2.39}$$

We can write equation (35) as:

$$\begin{aligned} \frac{c}{\beta q(\overline{\theta})} - \frac{c}{\beta q(\overline{\theta})^2} \frac{\partial q(\overline{\theta})}{\partial \overline{\theta}} (\theta_t - \overline{\theta}) &= (1-\alpha)(1-b) + (1-\alpha)(E_t y_{t+1} - b) \\ &+ \frac{c(1-\lambda)}{q(\overline{\theta})} - \frac{c(1-\lambda)}{q(\overline{\theta})^2} \frac{\partial q(\overline{\theta})}{\partial \overline{\theta}} (E_t\theta_{t+1} - \overline{\theta}) - \alpha c \overline{\theta} + \alpha c (E_t \theta_{t+1} - \overline{\theta}) \end{aligned} \tag{2.40}$$

We subtract the steady state of equation (35) from both sides of equation (40):

$$\begin{aligned} -\frac{c}{\beta q(\overline{\theta})^2} \frac{\partial q(\overline{\theta})}{\partial \overline{\theta}} (\theta_t - \overline{\theta}) &= (1-\alpha)(E_t y_{t+1} - b) \\ &- \frac{c(1-\lambda)}{q(\overline{\theta})^2} \frac{\partial q(\overline{\theta})}{\partial \overline{\theta}} (E_t\theta_{t+1} - \overline{\theta}) - \alpha c (E_t \theta_{t+1} - \overline{\theta}) \end{aligned} \tag{2.41}$$

In the next step, we plug the functional form of $q(\overline{\theta}) = \mu\overline{\theta}^{-\nu}$

$$\frac{c\nu\overline{\theta}^{\nu-1}}{\beta\mu}(\theta_t - \overline{\theta}) = (1-\alpha)(E_t y_{t+1} - b) \tag{2.42}$$
$$-\frac{c(1-\lambda)\nu\overline{\theta}^{\nu-1}}{\mu}(E_t\theta_{t+1} - \overline{\theta}) - \alpha c(E_t\theta_{t+1} - \overline{\theta})$$

Then θ_t can be written as:

$$\theta_t = \phi_0 + \phi_1 E_t y_{t+1} + \phi_2 E_t \theta_{t+1} \tag{2.43}$$

where

$$\phi_0 = \overline{\theta} - \phi_2\overline{\theta} - \phi_1 \tag{2.44}$$
$$\phi_1 = \frac{(1-\alpha)\beta\mu}{c\nu\overline{\theta}^{\nu-1}} \tag{2.45}$$
$$\phi_2 = \beta(1-\lambda) - \frac{\beta\alpha\mu}{\nu\overline{\theta}^{\nu-1}} \tag{2.46}$$

Next, we plug in the previous equation, the expectation if the productivity in the next period, that under rational expectations is $E_t y_{t+1} = (1-\rho) + \rho y_t$. Therefore, we come up with:

$$\theta_1 = \hat{\phi}_0 + \hat{\phi}_1 y_{t-1} + \hat{\phi}_2 E_t\theta_{t+1} + \hat{\phi}_1\rho^{-1}\epsilon_t, \tag{2.47}$$

where

$$\hat{\phi}_0 = \phi_0 + (1-\rho)(1+\rho)\phi_1 \tag{2.48}$$
$$\hat{\phi}_1 = \rho^2\phi_1 \tag{2.49}$$
$$\hat{\phi}_2 = \phi_2 \tag{2.50}$$
$$\tag{2.51}$$

Appendix 2.F Rational Expectation Coefficients

The Rational Expectation Equilibrium correspond to fixed points of the T-mapping. The T-mapping of this model is represented by the following vector:

$$T(\hat{A}_t, \hat{B}_t) = [\hat{\phi}_0 + \hat{\phi}_2\hat{A}_t + \hat{\phi}_2(1-\rho)\hat{B}_t, \hat{\phi}_2\rho\hat{B}_t + \hat{\phi}_1]. \tag{2.52}$$

Therefore, the REE is defined by the set of coefficients $(\overline{A}, \overline{B})$ such that

$$\begin{pmatrix} \overline{A} \\ \overline{B} \end{pmatrix} = \begin{pmatrix} \hat{\phi}_0 \\ \hat{\phi}_1 \end{pmatrix} + \begin{pmatrix} \hat{\phi}_2 & \hat{\phi}_2(1-\rho) \\ 0 & \hat{\phi}_2\rho \end{pmatrix} \begin{pmatrix} \overline{A} \\ \overline{B} \end{pmatrix} \tag{2.53}$$

Solving the previous system, we come up with

$$\overline{B} = \frac{\hat{\phi}_1}{1-\hat{\phi}_2\rho}, \tag{2.54}$$

$$\overline{A} = \frac{1}{1-\hat{\phi}_2}[\hat{\phi}_0 + \hat{\phi}_2(1-\rho)\frac{\hat{\phi}_1}{1-\hat{\phi}_2\rho}]. \tag{2.55}$$

The coefficient $\overline{C}$ in ecuation (19) is just a function of $\overline{B}$:

$$\overline{C} = \hat{\phi}_1(\rho^{-1} + \frac{\hat{\phi}_1}{1-\hat{\phi}_2\rho}). \tag{2.56}$$

We can now define E-stability for determining the stability of the REE under least squares learning. E-stability determines the stability of the REE under a stylized learning rule in which the PLM parameters (A, B) are adjusted slowly in the direction of the implied ALM parameters. The REE $(\overline{A}, \overline{B})$ is E-stable if small displacements from $(\overline{A}, \overline{B})$ are returned to $(\overline{A}, \overline{B})$ under this rule. It follows that the REE is E-stable if and only if the eigenvalues of $\begin{pmatrix} \hat{\phi}_2 & \hat{\phi}_2(1-\rho) \\ 0 & \hat{\phi}_2\rho \end{pmatrix}$ are < 1. The two eigenvalues are $\lambda_1 = \hat{\phi}_2$ and $\lambda_2 = \hat{\phi}_2\rho$.

3

Labor Market Dynamics and Imperfect Market Knowledge: A Comparative Study

Marta García Rodríguez

Abstract

This paper examines labor market dynamics within dynamic and stochastic search and matching models, focusing on the implications of deviations from full information rational expectations to solve existent puzzles in the labor market. This study employing a unified simulation framework to introduces a comparative analysis of models by Menzio (2022), Di Pace et al. (2021), and García-Rodríguez (2023) to assess how these models replicate observed labor market fluctuations and forecast data from professional forecasters. The findings indicate that all three models successfully address the amplification puzzle by matching the relative fluctuations in labor market variables to the average labor productivity. However, the models by Menzio (2022) and Di Pace et al. (2021) fail to resolve the propagation and expectational puzzles. They neither reduce the contemporaneous correlation between labor market tightness and average labor productivity, nor do they align the co-variation between wage forecasts from professional forecasters and average labor productivity. In contrast, García-Rodríguez's model more effectively matches these aspects, offering a superior representation of labor market dynamics.

Keywords: Subjective Expectations, labour markets, search and matching frictions.

JEL Classification: E24; E32; D83; J64

3.1 Introduction

Deviations from full information rational expectations (FIRE) have emerged as a potential mechanism for boosting the volatility of labor market variables in standard search and matching models (DMP) applied to the business cycle. This paper studies and compares the ability of DMP models that incorporate departures from rational expectations to explain certain stabilization facts observed in the US labor market, as well as in expectational survey data. The models cover situations in which agents do not know the wage equilibrium equation or the exogenous process of productivity.

A prominent theory, proposed by Menzio (2022), posits that workers misperceive the true productivity process, maintaining stubborn beliefs about economic fundamentals. Apart from that, they know the complete model. In Menzio's model, workers erroneously assume that aggregate productivity is constant over time, irrespective of underlying fluctuations. This results in static expectations regarding job prospects and wages, unresponsive to shifts in economic fundamentals. Firms, on the other hand, possess rational expectations and full knowledge of how workers form their expectations. Since the firm knows that the worker's beliefs cannot be changed, it has no choice but to accommodate them. Notably, in this model, workers make systematic errors without engaging in a learning process. Menzio evaluates the model abiliy to generate the elasticity of the market tightness, unemployment and vacancy rates with respect to aggregate productivity. However, he does not simulate or compute moments related to the labor market. This paper fills this gap.

On the other hand, agents may operate under limited knowledge of market behavior, exemplified by scenarios where they lack insight about some market outcomes, and try to learn about it using past data. Following this line, García-Rodríguez (2023), in the first chapter of the thesis, relaxes the standard assumption that agents have perfect knowledge about the wage function obtained from the standard Nash bargaining process, (hereinafter referred to as GR). She proposes a model framed under internal rationality where firms and workers use the minimum state variable solution to forecast future wages. Agents are equipped with a perceived law of motion (PLM) that is well specified, i.e., nest a REE of interest. However, economic agents, like econometricians, may fail to correctly specify the actual law of motion (ALMs), even asymptotically. Agents may adopt PLMs for forecasting, which may diverge from ALMs. For example, agents may include only a subset of the state variables in their forecast rule. This case is addressed in this framework by Di Pace et al. (2021), henceforth DMZ. As in the previous paper, they assume that agents do not know the wage process and propose an adaptive learning framework where restrict PLMs of wages to those which do not depend on productivity. Particularly, they assume that agents use autoregressive models to form wage expectations.

Two additional distinctive features set apart the papers of GR and DMZ beyond the formation of wage expectations. First, GR's builds on the adaptive learning literature but maintains the rationality of the agents. Importantly, it is also specific about beliefs system that the agents have in the economy. The adaptive learning literature does not specify what agents' views are on the evolution of macro-variables. They only equip them with a recursion, which tracks some moments of the variable. If beliefs are not fully specified in the model, then why, exactly, agents must form expectations according to a given recursion and how this relates to rational behavior is unclear. These distinctive modeling aspects embody the Internal Rationality framework as developed by Adam and Marcet (2011). Di Pace et al. (2021) do not specified the system of beliefs of agents. Therefore, GR's paper provides microfundations for previous adaptative learning papers on unemployment dynamics. Second, to quantitatively evaluate the learning model, GR employs formal structural estimation based on the method of simulated moments to see how well it matches labor market moments individually, a technique not used by DMZ. This paper adopts GR's methodology to estimate and evaluate the three models under consideration.

The key contributions and findings of this study can be summarized as follows: First, this paper employs a unified framework to simulate and evaluate the three models mentioned. Second, while all three models demonstrate the ability to generate pronounced labor market fluctuations, García-Rodríguez's (2023) model outperforms both the Di Pace et al. (2021) and Menzio (2022) models in some dimensions. Specifically, the model significantly reduces the correlation between productivity and labor market tightness, and accurately replicates observed fluctuations in real wages. This contrasts with the heightened relative wage fluctuations produced by DMZ approach and the opposing effect observed in the Menzio's model. The ability to reduce the correlation between productivity and labor market tightness is attributed to the incorporation of learning mechanisms—a feature absent in Menzio's model. Additionally, I found that the DMZ model yields identical results when adaptive learning is disregarded. Furthermore, my model aligns closely with the co-movements between survey wage expectations coming from professional forecasters and labor productivity, offering enhanced explanatory power. Wage expectations comove with labor productivity, but the comovements are significantly lower than the one implies by RE. The PLM proposed by GR is consistent with these findings, contrasting with the assumptions of DMZ and Menzio, who posit that wage expectations remain unresponsive to labor productivity.

The rest of the paper is organized as follows. Section 2 describes and compares the models. Section 3 presents the calibration of the models and summarizes the main results. Section 4 concludes.

3.2 DMP Model under Imperfect Knowledge

In this section, I compare search-theoretic models of the labor market within a dynamic stochastic framework, exploring variations in agents' expectations formation as proposed in the literature. Specifically, I investigate three scenarios in which agents deviate from full information rational expectations: I. Workers believe that aggregate productivity is always equal to its normal value and they form expectations about the tightness of the labor market, the probability of finding a job, and the wage they will earn once they find a job by computing the equilibrium outcomes of a hypothetical labor market without aggregate productivity shocks, as proposed by Menzio (2022).[1] II. Both firms and workers lack knowledge of the wage equilibrium equation and rely on a PLM that takes the minimum state variable solution suggested by García-Rodríguez (2023). III. Firms and workers, unaware of the wage equilibrium equation, assume wages follow an autoregressive process AR(1), as proposed by Di Pace et al. (2021).

3.2.1 Environment

Time is discrete and the population is normalized to one. The models share the same assumptions as Shimer (2005) regarding preferences, technology, and search frictions. The number of matches, resulting from a firm posting vacancies, v_t, is determined by the function $m(u,v)$, where u represents the unemployment rate. I denote $\theta_t = \frac{v_t}{u_t}$, f_t and q_t as the labor market tightness, job finding and vacancy filling rates, respectively.

The is one source of aggregate fluctuations, which are the shocks to labor productivity, ϵ_t. The labor productivity takes the form of stationary AR(1) in logs:

$$\ln(y_t) = (1-\rho)\ln(\overline{y}) + \rho\ln(y_{t-1}) + \epsilon_t,\ 0 < \rho < 1. \tag{3.1}$$

[1]Menzio (2022) introduces an alternative scenario not explored in this paper, wherein workers observe the actual productivity y of the firm and, if different from the steady state of productivity $\overline{y}$, they rationalize $y - \overline{y}$ as a permanent firm-specific component of productivity.

Where ρ measures the persistence and $\bar{y}$ the unconditional mean or steady state of productivity.

At the beginning of period t, the employment rate is n_t. A fraction λ of employed workers are then separated from their jobs at a exogenous and constant rate. Thus, employment evolves according

$$n_{t+1} = (1-\lambda)n_t + q(\theta_t)v_t. \tag{3.2}$$

Workers There is a continuum of identical, risk neutral workers with total measure one and an infinite horizon. These workers can either be employed or unemployed in each period. An employed worker earns a wage w_t at t, and faces a probability λ of losing his job in the subsequent period. Conversely, an unemployed worker receives unemployment benefits l and has a probability $f(\theta_t)$ of finding a job in the next period. Workers compute the net surplus of the match, that is used to bargain the wage with the firm if the match is realized. This surplus is the difference between the value of being employed and unemployed. The computation of such surplus depends on how agents form expectation.

Let m_t denote the vector containing variables that, in the workers' minds, summarize the best forecast of future outcomes, and $E_t^{\mathcal{P}^k}$ represents the conditional expectation, measured with a subjective probability measure formed by workers, denoted by k. Notice that m_t and $E_t^{\mathcal{P}^k}$ will differ across models. The workers' share of the total surplus is given by:

$$\mathcal{W}(m_t) - \mathcal{U}(m_t) = w_t - l + \beta\left(1-\lambda-f(\theta_t)\right)E_t^{\mathcal{P}^k}\left(\mathcal{W}(m_{t+1}) - \mathcal{U}(m_{t+1})\right). \tag{3.3}$$

Where $\mathcal{U}(.)$ is the present value of unemployment, $\mathcal{W}(.)$ is the one of the employment.

Model's assumptions and implications In Menzio's model, the way that workers form expectations implies: $m_t = m_{t+1} = \bar{y}$. Therefore, they perceive θ and w as functions of $\bar{y}$. As a result, their expectations regarding these labor market variables are solely based on $\bar{y}$, without consideration for temporal variations. In the models proposed by DMZ and GR, the formulation of expectations by individual workers is characterized by two main features: First, individual workers have a model that forecast correctly the true evolution of θ. Second, they do not know the wage equilibrium equation, but they have a specific belief system related to it. I define the specific belief system regarding wage within these models in Sections 3.2.2 and 3.2.3, respectively, offering a detailed description of how wages are forecast based on their underlying frameworks.

Firms The economy is populated by a mass of infinity firms. To hire workers, firms post vacancies, v_t, that are subject to a per unit cost c. In addition, firms pay its workers a wage w_t determined by a Nash bargaining process described below. $E^{\mathcal{P}^f}$ denotes the subjective expectations of firms, f. A representative firm maximizes profits, (Π), by solving

$$\Pi(m_t) = \max_{v_t \geq 0} \quad y_t n_t - w_t n_t - c v_t + E_t^{\mathcal{P}^f} \ \Pi(m_{t+1}) \tag{3.4}$$

subject to

$$n_{t+1} = (1-\lambda) n_t + q(\theta_t) v_t. \tag{3.5}$$

This framework assumes that individual firms know the law of motion of n.

Optimality Conditions The firm's optimal plan is characterized by the first order condition, together with the envelop condition with respect to n_t.

$$E_t^{\mathcal{P}^f} \mathcal{J}_{t+1} = \frac{c}{\beta q(\theta_t)}, \tag{3.6}$$

$$\mathcal{J}_t = y_t - w_t + \beta(1-\lambda) E_t^{\mathcal{P}^f} \mathcal{J}_{t+1}. \tag{3.7}$$

where $\mathcal{J}_t = \frac{\partial \Pi(m_t)}{\partial n_t}$ represents the marginal value of having an additional worker employed at the firm. The two previous equations give us the actual tightness of the labor market, and it is a function of firms expectations about future productivity and wages.

Notice that in Menzio's model, which I refer to with the superscript z, the expected tightness of the labor market by the worker differ from the one by the firm, and is given by,

$$E_t^{\mathcal{P}^{k,z}} \mathcal{J}(w(\overline{y}), \overline{y}) = \frac{c}{\beta q(\theta(\overline{y}))}. \tag{3.8}$$

Wages Upon meeting, a worker and a firm bargain over the wage. The bargaining game follows the alternating-offer protocol of Binmore et al. (1986). Without loss of generality, the wage negotiation starts with a wage proposal from the worker. If the firm accepts the worker's demand, the game ends. Conversely, if the firm declines the offer, there exists a chance for the negotiation to either end, with a probability of $1 - exp(-\mu_1 \Delta)$, or proceed, with a probability of $exp(-\mu_1 \Delta)$, where $\mu_1 > 0$ and $\Delta > 0$. A break at this point means the end of the negotiation. However, if negotiations go forward, it then becomes the firm's turn to present a wage counteroffer. Should this offer be accepted by the worker, the negotiations come to an end. An agreement from the worker at this stage concludes the negotiation. Rejection leads to a potential breakdown of the negotiation with a probability of $1 - exp(-\mu_2 \Delta)$, and a continuing negotiation with a probability of $exp(-\mu_2 \Delta)$, with $\mu_2 > 0$. This alternating sequence of wage proposals and counterproposals occurs until a mutual agreement is achieved or until the negotiation process unequivocally ceases. As standard in the bargaining literature, I will focus on the outcome of the bargaining game in the limit for $\Delta \to 0$. The unique perfect equilibrium for the model approaches the solution for

$$\max_{w_t} \quad [\mathcal{W}(m_t) - \mathcal{U}(m_t)]^{\alpha} \, \mathcal{J}_t^{1-\alpha} \ , \tag{3.9}$$

where $\alpha = \frac{\mu_1}{\mu_1 + \mu_2}$ represents the bargaining power of the worker and $1 - \alpha = \frac{\mu_2}{\mu_1 + \mu_2}$ denotes the bargaining power of the firm.

Menzio demonstrates that in his model the equilibrium wage is primarily influenced by the worker's beliefs. Recognizing the misalignment in beliefs, firms anticipate workers' negotiation strategies, leading to an equilibrium wage that remains insensitive to both the firm's productivity shifts and variations in the workers' unemployment value due to aggregate productivity changes. Then, the equilibrium wage is "rigid" in this case.[2]

$$w_t^z = \alpha \bar{y} + (1 - \alpha)(1 - \beta)\mathcal{U}(\bar{y}). \tag{3.10}$$

Conversely, in the models by GR and DMZ, both parties hold aligned expectations regarding productivity and future wages, resulting in an equilibrium wage follows the standard equation[3]

$$w_t^i = \alpha(y_t + c\theta_t^i) + (1 - \alpha)b, \quad \text{for } i = d, m. \tag{3.11}$$

[2]Hall (2005) reaches a similar conclusion through an alternative approach, positing that wages are not the result of negotiations between workers and firms but are instead set according to a social norm. This norm remains unchanged despite fluctuations in aggregate productivity, leading to wage rigidity.

[3]Both models assume that firms and workers form wage expectations using the same model but they don't know that before going to the bargaining process.

Where d and m refers to the DMZ and GR model, respectively. This wage adjusts in response to productivity shifts and labor market tightness. Notice that the impact of labor market tightness on wage adjustments is further influenced by how agents project future wages, which will be detailed in the in the subsequent sections.

Linearization and Equilibrium equations To ilustrate and highlight the distinctions between the models, I linearize two pivotal equilibrium equations: the job creation condition and the wage equilibrium equation. $\tilde{x}_t$ represents absolute deviations from the steady state, $\tilde{x}_t = x_t - \overline{x}$. I normalize the steady state of the productivity. [4]

Firms determine their vacancy posting strategy by combining and linearizing equations (3.6) and (3.7). This process yields the labor market tightness $\tilde{\theta}_t$:

$$\tilde{\theta}_t = \frac{1}{\phi} E_t^{\mathcal{P}f} \sum_{j=1}^{\infty} [\beta(1-\lambda)]^j [\frac{\tilde{y}_{t+j} - \tilde{w}_{t+j}}{1-\lambda}], \tag{3.12}$$

where $\phi = \frac{-cq'(\overline{\theta})}{q(\overline{\theta})^2} = \frac{c}{A}\overline{\theta}^{-\nu}(1-\nu)$.

Wage determination within this framework for each model, i, relies on the linearization of equation (3.11), leading to:

$$\tilde{w}_t^i = \alpha(\tilde{y}_t + c\tilde{\theta}_t^i) \quad \text{for } i = d, m. \tag{3.13}$$

Notice that under Menzio's assumptions, $\tilde{w}_t^z = 0$, implying $\tilde{w}_{t+j}^z = 0$ for all future periods.

In contrast, in the models proposed by GR and DMZ feature agents who are informed about the productivity process, but lack knowledge of the equilibrium wages at the time of making their wage forecast. Based on that, it is important to understand how agents view the wage process. That is summarized in the perceived law of motion (PLM) of wages. This is one difference between the model of GR and the model of DMZ, $\tilde{w}_{t+j}^m \neq \tilde{w}_{t+j}^d$, and consequently, $\tilde{w}_t^m \neq \tilde{w}_t^d$. Details on how agents formulate their wage forecasts will be elaborated in the following sections.

[4] Appendix 3.A shows in detail the linearization process of the two equations mentioned above together with the remaining equilibrium equations.

3.2.2 Beliefs Formation under Private Information: The case of Di Pace et al. (2021)

Following DMZ, agents do not internalize the effect of productivity on the wages when they form wage expectations. This assumption belongs to an environment characterized by partial information, which leads to the existence of hidden state variables from the perspective of agents. By applying the theoretical framework established by Marcet and Sargent (1989), I formulate and compute the equilibrium for this version of the search and matching model with partial information and hidden state variables.

Consider a vector of relevant variables defined as:

$$z_t = \begin{bmatrix} \tilde{y}_t \\ \tilde{w}_t \end{bmatrix}. \tag{3.14}$$

To determine the labor market tightness, agents need to forecast future wages and productivity. Although the process for productivity is assumed to be known by the agents, they rely on a subset of z for wage forecasts:

$$z_{at} = e_a z_t, \tag{3.15}$$

where e_a is a selector matrix, $z_{at} = \tilde{w}_t^d$.[5] In this framework, agents are endowed with a PLM for wages, expressed as:

$$\tilde{w}_t = b\tilde{w}_{t-1} + \varepsilon_t \tag{3.16}$$

where ε is a i.i.d. shock. This PLM indicates that agents' forecasts of future wages are based solely on past wages, omitting the direct influence of productivity on wage adjustments.[6]

Learning Algorithm Agents update their beliefs of wages over time, updating their coefficient every period as new data become available. The least squared learning algorithm can be written in

[5] z_{at} is equivalent to m_t that I defined in section 2.1 in the DMZ model.

[6] Di Pace et al. (2021) also propose a PLM that takes an AR(2). The results from the simulations don't differ significantly.

recursive terms as:

$$\hat{b}_t = \hat{b}_{t-1} + gR_{t-1}^{-1}\tilde{w}_{t-2}(\tilde{w}_{t-1} - \hat{b}_{t-1}\tilde{w}_{t-1}) \tag{3.17}$$
$$R_t = R_{t-1} + g(\tilde{w}_{t-1}\tilde{w}_{t-1} - R_{t-1}).$$

Where $\hat{b}_t$ denotes the coefficient estimate for the current period and $0 < g < 1$ represents the constant gain, determining the rate at which older observations are discounted. To determine the tightness of the market, agents make forecasts about future wages and future productivity. I can split equation (3.12) in two parts. On one hand, agents know the process of the productivity, therefore:

$$\beta E_t^{\mathcal{P}f} \sum_{j=1}^{\infty} \beta^{j-1}(1-\lambda)^{j-1}\tilde{y}_{t+j} = \beta \frac{\rho}{1-(1-\lambda)\beta\rho}\tilde{y}_t \tag{3.18}$$

On the other hand, according to the PLM in deviation from the steady state, agents forecast future wages in period t, from period $t+1$ onward, using previous wages.[7] Therefore, the infinity forecast of wages is given by

$$\beta E_t^{\mathcal{P}f} \sum_{j=1}^{\infty} \beta^{j-1}(1-\lambda)^{j-1}\tilde{w}_{t+j} = \beta \frac{\hat{b}_t^2}{1-\beta(1-\lambda)\hat{b}_t}\tilde{w}_{t-1} \tag{3.19}$$

Equation (3.12) can be write in the following way:

$$\tilde{\theta}_t = \frac{\beta}{\phi}[\psi_y \tilde{y}_t - \psi_w \tilde{w}_{t-1}], \tag{3.20}$$

where $\phi = \frac{c(1-\nu)A\overline{\theta}^{\nu-2}}{\overline{q}^2}$, $\psi_y = \frac{\rho}{1-\beta(1-\lambda)\rho}$, and $\psi_w = \frac{b^2}{1-\beta(1-\lambda)b}$. Using equations (3.20) and (3.13), I can calculate the realized wages, the actual law of motion for wages (ALM), implied by the PLM (3.16) as follows:

$$\tilde{w}_t = (\alpha + \frac{c}{\phi}\beta\Phi_y)\tilde{y}_t - \alpha(\frac{c}{\phi}\beta\Phi_w)\tilde{w}_{t-1}. \tag{3.21}$$

Where $\Phi_y = \frac{\rho}{1-\beta(1-\lambda)\rho}$ and $\Phi_w = \frac{b^2}{1-\beta(1-\lambda)b}$.

[7] Standard timing assumption in learning literature, to avoid problems of simultaneous determination of forecast and endogenous variables.

Therefore, the ALM for $\tilde{w}_t$ is given by:

$$\tilde{w}_t = T(b)z_{t-1} + V\epsilon_t, \tag{3.22}$$

where $T(b) = \left[\rho\alpha(1+\frac{\psi_y c\beta}{\phi}) \quad -\alpha(\frac{\psi_w c\beta}{\phi})\right]$ is the T-mapping and $V = \alpha(1+\frac{\psi_y c\beta}{\phi})$. Notice that from the ALM of wages, the realized wages are a function of productivity and previous wages. However, the PLM is just use previous wages. Therefore, the model that agents have in mind to form wage expectations is misspecified.

If the ALM of $\tilde{w}_t$ is (3.22), the linear least-squared projection of $\tilde{w}_t$ on $\tilde{w}_{t-1}$ is given by:

$$E(\tilde{w}_t|\tilde{w}_{t-1}) = S(b)\tilde{w}_{t-1}, \tag{3.23}$$

where

$$S(b) = T(b)[M_w^{-1}M_{w,z}]'. \tag{3.24}$$

Where $M_w = E[\tilde{w}_{t-1}]^2$ and $M_{w,z} = E[\tilde{y}_{t-1}, \tilde{w}_{t-1}]'\tilde{w}_{t-1}$. The operator *S(b)* maps the perception b into the projection coefficient *S(b)*.[8]

Definition A rational expectations equilibrium with partial information is a coefficient b that satisfies $b = S(b)$. Thus, a rational expectations equilibrium or the long-run equilibrium is a fixed point of the mapping *S*. Let us denote such equilibrium $b^{REE,d}$. Notice that this concept of a rational expectations equilibrium is relative to the fixed information set z_{at} specified by the model builder.

Since the agent's equation of wages differs from the truth and his beliefs evolve over time, it is important to check if the learning rule induce instability in the state evolution. In this framework, the E-stability conditions are analized as follows: [9]

$$\Omega = \left.\frac{\partial[S(b)-b]}{\partial b}\right|_{b=b_f} < 0. \tag{3.25}$$

[8]See appendix 3.B for more details.

[9]If g is small enough, the local stability conditions are the same than assuming decreasing gain, $g = \frac{1}{t-1}$

If the eigenvalue is real and negative, the condition for local stability of the learning mechanisms is satisfied.[10]

Figure (3.2.1) shows the S-mapping function, which is a decrease concave function. Values of the coefficient b, on the right hand side of the fixed point, indicate that agents expect wages above their realization and vice versa. Leaving productivity aside, the higher the wage expectation, the lower the number of vacancies that firms open up, because their expected profits are lower. This makes the market tighter, which in turn reduces the probability of finding a job. When firms and workers negotiate wages, throughout the bargaining process, in the presence of lower expected profits and a lower probability of finding a job, wages tend to fall and the other way around. This logic underlying the model is what explains the mean reversion mechanism. Moreover, the concave shape of the operator implies that in the presence of deviations from rational expectations, convergence is fast due to the larger size of the errors.

This whole analysis on the finding of T-mapping and S-mapping is missing in Di Pace et al. (2021). They compare the moments of the misspeficied learning model with the ones under FIRE.[11] Whereas in such environments, the model never converges to the FIRE. The natural question that arrives here is if a model where all agents agree to use the same inaccurate model in the formation of wage expectations can generate amplification in a DMP model. This is something that I discuss in the section 3.2.

3.2.3 Correctly Specified Beliefs: The case of García-Rodríguez (2023)

In GR's model, agents have the following PLM which they use to make forecast of wages:

$$\begin{aligned} \tilde{w}_t &= d_t \tilde{y}_{t-1} + \epsilon_t, \\ d_t &= d_{t-1} + \nu_t. \end{aligned} \tag{3.26}$$

Where $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$ and $\nu_t \sim \mathcal{N}(0_{2,1}, \sigma_\nu^2 I_2)$ are independent of each other. This PLM considers a fundamental or minimal state variable solution with unobserved coefficients.[12]

[10]With the calibration proposed in section 3.1., I found that the model is local stable.

[11]In Appendix 3.C, I formulate the FIRE of this model.

[12]The learning describes in this section is not exactly the same than in the core paper of García-Rodríguez (2023). In that case, they learn about the level of wages instead of the absolute deviation of wages. However, it is equivalent to the case where GR equips agents with the true constant coefficients, and agents just learn about the coefficient that goes with the productivity, proposed in a robust exercise. Also, in this version, I mute the belief shock.

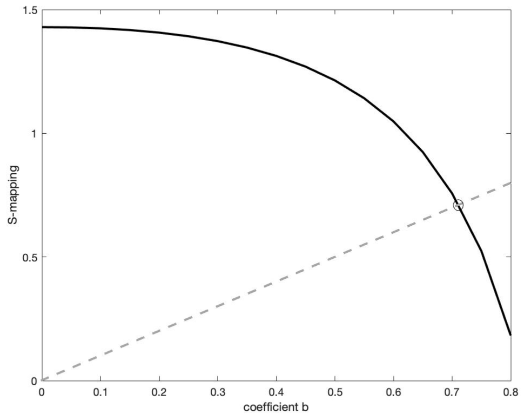


Figure 3.2.1: S-mapping

Note: The thick black line represents the operator, Smapping, which is a concave function. The dashed black line thought the function is the 45 degree line. The ellipse represents the RE point under limited information,which is the fixed point of the S-mapping.

Agents update $\hat{D}_t$ according to the recursive least squares (RLS) algorithm:

$$\hat{d}_t = \hat{d}_{t-1} + gR_t^{-1}\tilde{y}_{t-1}[w_{t-1} - \hat{d}'_{t-1}\tilde{y}_{t-1}],$$
$$R_t = R_{t-1} + g(\tilde{y}_{t-1}\tilde{y}'_{t-1} - R_{t-1}). \quad (3.27)$$

Where $\hat{d}_t$ represent the estimated coefficients.

Taking into account the previous PLM, agents forecast wages and determine the labor market tightness:

$$\tilde{\theta}_t = \frac{\beta}{\phi}[\rho(\psi_y - \psi_{wy})\tilde{y}_{t-1} + (\psi_y - \psi_{wy})\epsilon_t]. \quad (3.28)$$

Where $\phi_y = \frac{\rho}{1-\beta(1-\lambda)\rho}$ and $\phi_{wy} = \frac{\hat{d}_t}{1-\beta(1-\lambda)\hat{d}_t}$.

The ALM for wages in this case, is the following:

$$\tilde{w}_t = T_y(\hat{d}_t)\tilde{y}_{t-1} + T_\epsilon(\hat{d}_t)\epsilon_t. \quad (3.29)$$

where T_y and T_ϵ are functions of the estimated coefficients of the PLM.[13] Notice that in this case, the ALM of wages is a function of y_{t-1} as the PLM.

Definition A rational expectations equilibrium is a coefficient d that satisfies $d = T(d)$. Thus a rational expectations equilibrium is a fixed point of the mapping T. Let us denote such an equilibrium $d^{REE,m}$.

3.3 Quantitative Analysis

This section evaluates the quantitative performance of the search and matching models under deviation from rational expectations. The approach to assessing the models' performance is a mixed strategy: I undertake a formal estimation and validation of the models, employing a hybrid calibration strategy that leverages the Method of Simulated Moments (MSM). This method is specifically designed to analyze the effectiveness of the models in capturing the key empirical moments described in Table 3.3.3, thus providing an overall understanding of their explanatory power in relation to the characteristics of the observed data.

It should be noted that this analytical strategy coincides with the methodology adopted by GR. In contrast, DMZ do not use the Simulated Moment Method, and Menzio does not simulate the model. By comparing these models against specific data moments, I aim to shed light on their relative strengths and limitations in explaining labor market dynamics under deviations from rational expectations.

3.3.1 Estimation of the Model

This section describes the calibration/estimation of the model parameters. The parameterization strategy is threefold: a subset of the parameters is selected from the literature, another subset is picked from the US data, and the rest is estimated by MSM.

Specifically, the vector $\hat{Z} = [\beta, \lambda, \alpha, \overline{y}, \nu]$ is obtained directly from the literature. I normalize time to one-quarter. Following the literature, I assume that the matching function is Cobb-Douglas. Without loss of generality, the steady state of productivity is normalized to 1. The value of the discount factor β is set to generate an annual real interest rate of approximately 5%. The value of the separation

[13] For exact formula for T_y, and T_ϵ and the derivations see Appendix 3.D.

rate is set following Shimer (2005), who suggests a quarterly separation rate of 0.10.

Parameter	Description	Value	Source
β	discount factor	0.99	r=0.05
λ	separation rate	0.10	Shimer (2005)
α	bargaining power worker	0.50	Hosios rule: $\alpha = 1 - \nu$
ν	elasticity of matching function	0.50	standard
$\overline{y}$	steady state productivity	1.00	Normalization
σ_ϵ	st. dev. of productivity shocks	0.0058	Data
ρ	persistency of productivity	0.73	Data

Table 3.3.1: Calibrated quarterly parameters from literature and data 1990Q1-2020Q1

I set the value of the elasticity of the matching function at 0.5 in line with the literature. This value lies within the plausible interval of [0.5 0.7] as surveyed by Petrongolo and Pissarides (2001). Following Hosios (1990), I set the bargaining power of the worker to 0.5. Using US data, I set the standard deviation and persistence of the productivity process to match the empirical behavior of labor productivity from 1990 to 2020. I find a quarterly autocorrelation and standard deviation of 0.7518 and 0.0058, respectively.

Defining $Z = [c, A, g, b]$ as the vector of parameters to be estimated using an extension of the Simulated Method of Moments. These parameters are estimated to match the first 11 statistics reported in Table 3.3.3. I chose these moments because are the ones that standard search and matching models struggles to generate.[14] The MSM estimator is given by

$$\min_{Z} \quad (\hat{\mathcal{S}} - \tilde{\mathcal{S}}(Z))'\hat{\Sigma}_{\mathcal{S}}^{-1}(\hat{\mathcal{S}} - \tilde{\mathcal{S}}(Z)) \ , \tag{3.30}$$

where $\tilde{\mathcal{S}}(Z)$ is the vector of empirical moments to be matched, $\hat{\mathcal{S}}$ is the model moments counterpart and $\hat{\Sigma}_{\mathcal{S}}$ is the weighting matrix, which determines the relative importance of each statistic deviation from its target. I use a diagonal weighting matrix whose diagonal is composed of the inverse of the estimated variances of the data statistics.[15] Model-implied statistics are generated through a Montecarlo experiment with 10000 realizations. I formally test the hypothesis that any individual model statistics differ from its empirical counterpart.

[14] I include functions of moments, instead of pure moments. I target 12 functions of moments. See appendix C4 in the first chapter of this thesis.

[15] In practice the estimated variances of the data moments, $\hat{\mathcal{S}}$ is used. The variances are obtained using a Newey-West estimator and the delta method as in Adam et al. (2017).

Parameter	Description	GR	Di Pace	Menzio
c	cost of open a vacancy	0.45	0.44	0.03
A	efficiency matching technology	0.97	0.57	2.5
g	constant gain	0.009	0.03	-
b	unemployment benefits	0.75	0.4	0.45

Table 3.3.2: Estimated quarterly parameters from MSM

3.3.2 Statistical Properties

This section presents the estimation outcomes of the models under consideration. Table 3.3.3 compares statistics from the US labor market data spanning from the first quarter of 1990 to the first quarter of 2020 against those generated by the three models proposed by Menzio (2022), Di Pace et al. (2021) and GR. Each model was simulated over 120 quarters (T=120) for 10,000 iterations to calculate the average of the desired statistics as deviations from the steady state, facilitating comparison to earlier studies. The statistics considered include the relative standard deviation of various labor market variable with respect to the standard deviation of labor productivity, correlations among labor variables and market tightness, autocorrelations of labor market tightness and wages, and the Beveridge curve relationship between unemployment and vacancies. Finally, the concluding rows present non-targeted coefficients from regression analyses comparing survey wage expectations from professional forecasters with actual wage outcomes against the productivity.[16] The second column in Table 3.3.3 outlines the empirical labor market moments, while the subsequent columns detail the moments and t-statistics for GR's, Menzio's, and DMZ's models, respectively.

Table 3.3.3 shows that all models adeptly align the relative standard deviations of labor market variables with empirical data, thus effectively addressing the amplification puzzle. However, the models significantly diverge in two dimensions: the relative standard deviation of wages and the contemporaneous correlation between labor market tightness and average labor productivity. GR's model stands out for its ability to reduce this contemporaneous correlation, achieving a closer match with empirical observation. Conversely, both Menzio's and DMZ's models produce a contemporaneous correlation approaching unity. Additionally, Menzio's model exhibits wage rigidity, failing to replicate the empirical standard deviation in wages, while DMZ's model predicts overly volatile wages. Finally, a further analysis of the regression coefficients from the Rational Expectations test reveals that GR's model aligns more closely with empirical data, whereas DMZ's model significantly

[16]For more details, refer to section 3 in the first chapter of the thesis.

	Data	Menzio	t-stat	GR	t-stat	Di Pace et al	t-stat
$\sigma_{\tilde{u}}/\sigma_{\tilde{y}}$	11.952	7.420	2.188	7.060	2.362	9.171	1.342
$\sigma_{\tilde{v}}/\sigma_{\tilde{y}}$	13.221	17.289	-2.195	17.994	-2.275	12.505	0.386
$\sigma_{\tilde{\theta}}/\sigma_{\tilde{y}}$	24.713	20.215	1.145	27.705	-0.762	20.498	1.073
$\sigma_{\tilde{w}}/\sigma_{\tilde{y}}$	1.737	0.000	7.550	1.551	0.807	3.914	-9.467
$\rho(\tilde{y}_t,\tilde{\theta}_t)$	-0.040	1.000	-2.420	0.314	-0.823	0.975	-2.362
$\rho(\tilde{v}_t,\tilde{\theta}_t)$	0.984	0.9335	5.541	0.966	1.995	0.961	2.503
$\rho(\tilde{u}_t,\tilde{\theta}_t)$	-0.980	-0.963	-1.585	-0.957	-2.141	-0.925	-5.070
$\rho(\tilde{w}_t,\tilde{\theta}_t)$	0.780	0.00	2.221	0.866	-0.245	0.999	-0.623
$\rho(\tilde{\theta}_{t-1},\tilde{\theta}_t)$	0.941	0.754	11.32	0.896	2.731	0.897	2.639
$\rho(\tilde{w}_{t-1},\tilde{w}_t)$	0.826	0.000	25.791	0.772	1.716	0.839	-0.406
$\rho(\tilde{u}_t,\tilde{v}_t)$	-0.927	-0.815	-5.567	-0.866	-3.027	-0.888	-1.961
b^E	0.15	0.000	-	0.482	-3.451	2.116	-20.492
b^R	0.75	0.000	-	0.83	-0.398	3.854	-11.836

Table 3.3.3: Labor Market Statistics

Data moments are computed over the period 1990Q1: 2020Q1. Moments have been computed as averages over 1000 simulations. b^R is the coefficients of regression 5 and b^E is the coefficients of regression 6 running in Section 3 of the first chapter of the thesis. Survey data: European Commission from 1990-2020. t-ratios are defined as $\sqrt{T}$(data moment-model moment)/(estimated standard deviation of the model moment).

overestimates the coefficients.

An intriguing aspect to explore is the contribution of learning mechanisms on the outcomes observed in the models developed by GR and DMZ. To investigate this, I intend to neutralize the learning effect by setting the gain to zero and simulate the models at their respective rational expectations equilibria, particularly, at the fixed point of the S-mapping for DMZ's model and the T-mapping for GR's. Insights from Table 3.3.4 reveal a stark contrast: the phenomena observed in GR's model heavily rely on the mechanism of learning, underscoring the pivotal role played by the learning gain. On the contrary, when the learning gain is nullified in DMZ's framework, the results remain unchanged. This suggests that the observed amplification effect within DMZ's framework stems solely from agents utilizing a misspecified model for expectation formation, with the learning process itself being inconsequential. In other words, if all agents agree to use the same inaccurate model in the formation of wage expectations, the model can solve the amplification puzzle. This distinction marks another notable difference between the two mechanism to generate the amplification in the model.

	Learning GR	REE GR	Learning DMZ	REE-PI DMZ
$\sigma_{\tilde{u}/\sigma_{\tilde{y}}}$	7.060	1.553	9.171	8.938
$\sigma_{\tilde{\theta}/\sigma_{\tilde{y}}}$	27.205	6.512	20.498	19.905
$\sigma_{\tilde{w}/\sigma_{\tilde{y}}}$	1.551	0.492	3.914	3.834
$\rho(\tilde{y}_t, \tilde{\theta}_t)$	0.314	0.999	0.975	0.988
$\rho(\tilde{u}_t, \tilde{\theta}_t)$	-0.957	-0.976	-0.925	-0.932
$\rho(\tilde{w}_t, \tilde{\theta}_t)$	0.866	0.999	0.999	0.999
$\rho(\tilde{u}_t, \tilde{v}_t)$	-0.866	-0.679	-0.888	-0.897

Table 3.3.4: Labor Market Statistics: Learning vs REE/REE-PI

Note: Data moments are computed over the period 1990Q1: 2020Q1. Moments have been computed as averages over 1000 simulations. Columns 3 and 4 set the parameter gain to zero.

3.4 Conclusions

This paper has explored the intricacies of dynamic and stochastic search and matching models within the context of deviations from rational expectations by examining how different modeling approaches, namely those proposed by Menzio (2022), Di Pace et al. (2021), and García-Rodríguez (2023) (as introduced in the first chapter of this thesis), generate labor market dynamics. Through quantitative analysis and simulation, I have dissected the relative strengths and limitations of each model in replicating key empirical moments of the U.S. labor market from 1990Q1 to 2020Q1.

The findings reveal that all models adeptly align the relative standard deviations of labor market variables with empirical data, effectively addressing the amplification puzzle that challenges standard search and matching models. Notably, the models diverge in their handling of wage rigidity and the contemporaneous correlation between labor market tightness and average labor productivity. GR's model, in particular, exhibits a superior capability to closely match empirical observations, especially in reducing the contemporaneous correlation between these variables and in reproducing the observed relative standard deviation in real wages. Additionally, GR's mechanism better matches some properties of forecast data on wage growth coming from professional forecasters.

In addition, this analysis delves into the role of learning mechanisms in shaping the outcomes of the models developed by GR and DMZ. By setting the learning gain to zero and simulating the models at their rational expectations equilibria, a stark contrast emerges: GR's outcomes depend heavily on learning mechanisms, while the DZM model's outcomes are unaffected by neutralizing learning effects. This underscores a crucial insight that the amplification observed in DMZ's framework results from the utilization of a misspecified model for expectation formation rather than from the learning process itself.

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Appendix

Appendix 3.A Linearizations

To see the logic method consider the equation:

$$x_{t+1} = f(x_t),$$

where f is a possibly complicated nonlinear function. A first-order Taylor polynomial of this equation at the steady state $x_t = \overline{x}$ gives:

$$x_{t+1} \approx f(\overline{x}) + f'(x)(x_t - \overline{x}) = f(\overline{x}) + f'(x)(\tilde{x}_t).$$

I start with the linearization of equation (**??**):

$$\frac{c}{q(\overline{\theta})} - (\theta_t - \overline{\theta})\frac{cA(\nu-1)\overline{\theta}^{\nu-2}}{q(\overline{\theta})^2} = \beta E_t^{sf} \sum_{j=1}^{\infty} [\beta(1-\lambda)]^{j-1}[\overline{y} + \tilde{y}_{t+j} - \overline{w} - \tilde{w}_{t+j}].$$

I plug the equation in the steady state in the previous expression, that is:

$$\frac{c}{q(\overline{\theta})} = \beta E_t^{sf} \sum_{j=1}^{\infty} [\beta(1-\lambda)]^{j-1}[\overline{y} - \overline{w}].$$

And we come up with the following expression:

$$-(\theta_t - \overline{\theta})\frac{cA(\nu-1)\overline{\theta}^{\nu-2}}{q(\overline{\theta})^2} = \beta E_t^{sf} \sum_{j=1}^{\infty} [\beta(1-\lambda)]^{j-1}[\tilde{y}_{t+j} - \tilde{w}_{t+j}].$$

To linearize equation (**??**), we subtract the steady state of the wages on both sides:

$$w_t - \overline{w} = \alpha(y_t + c\theta_t) + (1-\alpha)b - \overline{w}.$$

Where $\overline{w} = \alpha(y + c) + (1-\alpha)b$ and I come up with the following expression:

$$\tilde{w}_t = \alpha(\tilde{y}_t + c\tilde{\theta}_t).$$

To determine the vacancies, I know that $v_t = \theta_t u_t$, we assume that $\hat{x} \approx \frac{x_t - \overline{x}}{\overline{x}}$, therefore:

$$\overline{v}(1+\hat{v}_t) = \overline{\theta}(1+\hat{\theta}_t)\overline{u}(1+\hat{u}_t)$$

Plugging the steady state, $\overline{v} = \overline{\theta}\overline{u}$, we come up with:

$$(1+\hat{v}_t) = (1+\hat{\theta}_t)(1+\hat{u}_t) = 1+\hat{\theta}_t+\hat{u}_t$$

By assuming that $\tilde{u}_t$ and $\tilde{\theta}$ are by assumption close to zero, its product will be negligably different from zero.

$$\frac{v_t - \overline{v}}{\overline{v}} = \frac{\theta_t - \overline{\theta}}{\overline{\theta}} + \frac{u_t - \overline{u}}{\overline{u}}$$

Therefore:

$$\tilde{v}_t = \overline{\theta}\tilde{u}_t + \overline{u}\tilde{\theta}_t$$

The process of linearization of the law of motion for employ (8) is:

$$\overline{n}(1+\hat{n}_{t+1}) = (1-\lambda)\overline{n}(1+\hat{n}_t) + \overline{vq}(1+\hat{v}_t)(1+\hat{q}_t)$$

$$\overline{n} + \overline{n}\hat{n}_{t+1} = (1-\lambda)\overline{n} + (1-\lambda)\overline{n}\hat{n}_t + \overline{vq}(1+\hat{q}_t+\hat{v}_t)$$

I subtract n on the left and $(1-\lambda)n + vq$ on the right to obtain:

$$\overline{n}\hat{n}_{t+1} = (1-\lambda)\overline{n}\hat{n}_t + \overline{vq}\hat{q}_t + \overline{vq}\hat{v}_t$$

$$\hat{n}_{t+1} = (1-\lambda)\hat{n}_t + \frac{\overline{vq}}{\overline{n}}\hat{q}_t + \frac{\overline{vq}}{\overline{n}}\hat{v}_t$$

Rewriting the previous equation in absolute deviations of the steady state, $\hat{x}_t = \frac{\tilde{x}_t}{x}$ and rearrange terms, i come up with the previous expression:

$$\tilde{n}_{t+1} = (1-\lambda)\tilde{n}_t + \overline{v}\tilde{q}_t + \overline{q}\tilde{v}_t$$

Finally, I going to linearize the matching function:

$$\overline{m}(1+\hat{m}_t) = A\overline{v}^{\nu}(1+\nu\hat{v}_t)\overline{u}^{1-\nu}(1+(1-\nu)\hat{u}_t)$$

I divide by $\overline{m}$ on the left side and by $Av_t^{\nu}u_t^{1-\nu}$ on the right side:

$$(1+\hat{m}_t) = (1+\nu\hat{v}_t)(1+(1-\nu)\hat{u}_t) = 1+(1-\nu)\hat{u}_t+\nu\hat{v}_t$$

I rewrite in terms of absolute deviations from steady state and a rearrange the terms to come up with the following expression:

$$\tilde{m}_t = (1-\nu)\tilde{u}_t\overline{\theta q} + \nu\overline{q}\tilde{v}_t.$$

Appendix 3.B S-mapping in Di Pace et al. (2021)

The starting point is the actual law of motion for the wages:

$$\tilde{w}_t = T(b)z_{t-1} + V\epsilon_t, \tag{3.31}$$

where $T(b) = \begin{bmatrix}\rho\alpha(1+\frac{\psi_y c\beta}{\phi}) & -\alpha(\frac{\psi_w c\beta}{\phi})\end{bmatrix}$ is the T-mapping and $V = \alpha(1+\frac{\psi_y c\beta}{\phi})$.

The linear least-squares projection of w_t on w_{t-1} can be express in the following way:

$$\begin{aligned} E(\tilde{w}_t|\tilde{w}_{t-1}) &= T_{11}E(\tilde{y}_{t-1}|\tilde{w}_{t-1}) + T_{12}(b)\tilde{w}_{t-1} \\ &= T_{11}\frac{cov(\tilde{y}_{t-1},\tilde{w}_{t-1})}{(var(\tilde{w}_{t-1}))}\tilde{w}_{t-1} + T_{12}(b)w_{t-1} \\ &= \begin{bmatrix}T_{11} & T_{12}\end{bmatrix}\begin{bmatrix}\frac{cov(\tilde{y}_{t-1},\tilde{w}_{t-1})}{(var(\tilde{w}_{t-1}))} & 1\end{bmatrix}'\tilde{w}_{t-1} \\ &= S(b)\tilde{w}_{t-1} \end{aligned} \tag{3.32}$$

Appendix 3.C FIRE of Di Pace et al. (2021)

In this section, I analyse an environment in which the agent use all the relevant variables, z, to forecast wages. In this framework agents know that productivity affect to the wages. Therefore, the PLM in this benchmark economy is the followings:

$$\tilde{w}_t = c_w \tilde{w}_{t-1} + c_y \tilde{y}_t + \varepsilon_t \tag{3.33}$$

Taking into account the previous PLM, agents forecast wages and determine the labor market tightness:

$$\tilde{\theta}_t = \frac{\beta}{\phi}[\psi_y \tilde{y}_t - \psi_w \tilde{w}_{t-1} - \psi_{wy} \tilde{y}_t], \tag{3.34}$$

Where $\Phi_y = \frac{\beta\rho}{1-\beta(1-\lambda)\rho}$,$\Phi_w = \frac{\hat{c}_w^2}{1-\beta(1-\lambda)\hat{c}_w}$ and $\Phi_{wy} = \frac{\hat{c}_y}{1-\beta(1-\lambda)\hat{c}_y}$.

The ALM for wages is the following:

$$\tilde{w}_t = \alpha(1 + \frac{c\beta}{\phi}(\psi_y - \psi_{wy}))\tilde{y}_t - \alpha(\frac{\psi_w c\beta}{\phi})\tilde{w}_{t-1}. \tag{3.35}$$

Therefore, $T(C) = \left[\alpha(1 + \frac{c\beta}{\phi}(\psi_y - \psi_{wy})) \quad -\alpha(\frac{\psi_w c\beta}{\phi})\right]$ is the T-mapping of this economy and $C = [c_y, c_w]'$ is the vector of coefficients.

Definition A rational expectations equilibrium with full information is a matrix $C = [c_y, c_w]$ that satisfies $C = T(C)$. Thus a rational expectations equilibrium is a fixed point of the mapping T. Let us denote such an equilibrium $b_w^{f,REE}$ and $b_y^{f,REE}$. Notice that this concept of a rational expectations equilibrium is relative to the full information set z_t.

Relative Standard Deviation	$\sigma_{\tilde{u}/\sigma_{\tilde{y}}}$	$\sigma_{\tilde{v}/\sigma_{\tilde{y}}}$	$\sigma_{\tilde{\theta}/\sigma_{\tilde{y}}}$	$\sigma_{\tilde{n}/\sigma_{\tilde{y}}}$	$\sigma_{\tilde{w}/\sigma_{\tilde{y}}}$
	1,41	1,81	3,11	0,10	1.10
corr($\tilde{u}$,$\tilde{x}$)	$\tilde{y}$	$\tilde{v}$	$\tilde{\theta}$	$\tilde{n}$	$\tilde{w}$
	-0,95	-0,93	-0,95	-1	-0.95
Autocorrelations	$\tilde{u}$	$\tilde{v}$	$\tilde{\theta}$	$\tilde{n}$	$\tilde{w}$
	0,93	0,82	0,91	0,93	0,89

Table 3.C.1: Some Statistics of the REE model with full information

Note: $\tilde{y}$,$\tilde{u}$, $\tilde{v}$ and $\tilde{\theta}$ denote labor productivity, unemployment, vacancies and labor market tightness, respectively. All variables denotes the percentage deviation of the variable from its steady-state. The model calibration is the one used in the main model for the Di Pace et al model by setting $g = 0$.

Fixed points of T		Fixed points of S
c_w^f	c_y^f	b^f
0	0,9440	0.713
Eigenvalues of Ω		
-1	-5,5169	-

Table 3.C.2: REE full information vs partial information

Note: To calculate the fixed point of S-mapping I start the learning algorithm with a prior of zero, I simulate it 1000000, in an economy without shocks. It is the point to where it converges.

Appendix 3.D T-Mapping in García-Rodríguez (2023)

Let's take the linearized equation of wages and plug the job creation condition determined by the PLM (3.26):

$$\begin{aligned}
\tilde{w}_t &= \alpha(\tilde{y}_t + c\tilde{\theta}_t) \\
&= \alpha(\rho\tilde{y}_{t-1} + c\tilde{\theta}_t) + \alpha\epsilon_t \\
&= \alpha\rho\tilde{y}_{t-1} + \alpha c[\frac{\beta}{\phi}[\rho(\psi_y - \psi_{wy})\tilde{y}_{t-1}] + \alpha c\frac{\beta}{\phi}(\psi_y - \psi_{wy})\epsilon_t + \alpha\epsilon_t \\
&= \alpha\rho(1 + \frac{\beta c}{\phi}(\psi_y - \psi_{wy}))\tilde{y}_{t-1} + \alpha(\frac{\beta c}{\phi}(\psi_y - \psi_{wy}) + 1)\epsilon_t \\
&= \underbrace{\alpha\rho(1 + \frac{\beta c}{\phi}(\frac{\rho}{1-\beta(1-\lambda)\rho} - \frac{\hat{d}_t}{1-\beta(1-\lambda)\hat{d}_t}))}_{T_y(\hat{d}_t)}\tilde{y}_{t-1} + ... \\
&... + \underbrace{\alpha(\frac{\beta c}{\phi}(\frac{\rho}{1-\beta(1-\lambda)\rho} - \frac{\hat{d}_t}{1-\beta(1-\lambda)\hat{d}_t}) + 1)}_{T_\epsilon(\hat{d}_t)}\epsilon_t.
\end{aligned} \tag{3.36}$$