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Consumer Choice in Competitive Location Models

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Al meu pare, per haver-me ensenyat com s'afronten els moments difícils.

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Contents

Chapter 1

Introduction

A well-known aphorism states, “the most important attributes of stores are location, location and location”. The research area of facility location - allocation has growth rapidly in the last decades. Economists, geographers, operations researchers and engineers have all contribute to this growth. The literature of facility location models is both large in size and rich in its applications.

Facility location - allocation models are often viewed in terms of their applicability to private versus public sector problems. This is principally due to the difference in the objective of each problem. While private sector problems generally aim to maximize profits or minimize monetary costs, public sector models search for optimality in some measure of utility (access, closeness, coverage) with a constraint on costs or resources. Another way of classifying the facility location - allocation models is as a function of its spatial representation; in the continuous location models, the potential locations of the facilities can be anywhere in the plane, while in the discrete location models, the facilities are allowed to locate at a finite set of possible locations on a network.

In today's highly competitive retail environment, firms can create competitive advantage by achieving a strong market presence by locating multiple stores in the same market. Locating multiple units in one market has a number of advantages. For one, it creates market presence so that all consumers in the market area have relatively easy access to a firm's stores. Second, it allows for managerial efficiencies as well as scales of economies in distribution, warehousing and transportation costs. Third, it increases the efficiency of advertising and promotional expenditures in the local market. In sum, concentrating outlets in an area can create synergy, improving the performance of individual outlets that are a part of a larger network. One approach to this field is that enshrined by Competitive Location literature in discrete space.

Competitive Location Literature in discrete space is a subgroup of location – allocation models that addresses the issue of optimally locating firms that compete for clients in space, and allocating the consumers to those locations based on the expected pattern of consumer behavior. A competitive location model is such that there is more than one firm competing in the spatial market and with interaction between them. The location decision of a firm will affect not only its market share, but also its competitors' market share (Serra and ReVelle, 1996). Traditionally, this literature has been successfully applied to locate public sector services where the main aim is to optimize some measure of service quality in terms of access (e.g. maximizing service coverage or minimizing average distance to the service). To date, this literature has assumed that consumers shop at the closest store supplying a specific product or service. However, one needs to ask whether this assumption reflects consumer behavior. It seems more realistic to admit that consumers do not merely consider distance when making choosing retail outlets.

Store-choice literature studies the key variables which a customer takes into account when he shops at a particular shop, and how these variables interact. This literature usually assumes that the consumer not only cares about which shop is the closest but also considers other variables in making the decision of patronizing a particular outlet.

A common classification of the store – choice literature states that this can be done in three groups. The first group includes models that rely on some normative assumption regarding consumer travel behavior. This hypothesis is to simple and is useful only on a limited number of applications. The second group uses information revealed by past behavior to understand the dynamics of retail competition and how consumers choose among alternatives shopping opportunities. The main problem of this second group is the context – dependence of its approach. To overcome this problem, the third group of models estimate consumer utility function from simulated choice data using information integration, conjoint or logit techniques.

A recent paper (Clarkon, et.al. 1996) has highlight the fact that firms prefers the revealed preference approach to model consumer store – choice behavior. This approach is preferred to normative models since it more faithfully reflects real consumer behavior, and to the direct utility approach because is simpler since it uses surveys and linear regression instead of conjoint, logit techniques or game theory.

This last statement sheds light on the next direction of competitive retail location literature, trying to include the revealed preference approach of store-choice theories in its models. It stand to reason that any retail location model should take into account the process underlying consumer’s choice stores. After the literature review (in **chapter 2**), I realize that continuous competitive location models for retail firms have already introduced the revealed preference approach of store – choice models in its models; while the discrete competitive one's have done it only for the profit maximization models. Then, up till now, there has not been a deep research for the incorporation of the revealed preference approach of store – choice model in the discrete competitive location models that have a maximum captured objective function, and this is the aim of this thesis.

To apply the location – allocation models to retail firms, it is necessary first to analyze the pattern of consumer travel behavior to existing facilities, and then apply the results to the allocation phase of location – allocation algorithms. This was done in the **third chapter** where I analyze the importance of consumer behavior with respect to distance in the optimality of locations obtained by a traditional discrete competitive location models. To do this, I consider different ways of taking into account distance based on several consumer choice theories. The deviations in demand captured, when the optimal locations of other models are used instead of the true ones, is computed to know how to introduce distance.

As we known from Store – Choice literature, consumers considers other variables, apart from distance, when deciding which shop patronize. As Clarkon, et.al. (1996) said, the best way to analyze it is the revealed preference approach of store – choice literature. Ghosh (1984) indicates that the best way to overcome the context dependence problem of these models is to establish a systematic empirical research of the impact of locational structure on choice, to be sure that this structure is taken into account in the allocation face of the location – allocation model. **Chapter 4** presents a methodology for determining which store attributes (other than distance) should be included in a new version of the Maximum Capture Discrete Competitive Location models to the retail sector, as well as how these parameters ought to be reflected. The revealed preference store choice model use to define this methodology is the Multiplicative Competitive Interaction model. **Chapter 4** presents also the application of this methodology to the supermarket sector in two different scenarios: Milton Keynes (in Great Britain) and Barcelona (in Spain).

After the introduction of consumer store – choice theories in the discrete competitive location models that have a maximum captured objective function, these models are improved with an additional element: the market threshold concept. Up till now, this concept has been introduced only in a deterministic way. In this thesis, this is introduced as an stochastic constraint. **Chapter 5** presents the New Chance – Constrained Maximum Capture Location Problem, which is a maximum capture model that takes into account the store-choice theories and has an stochastic threshold constraint.

Given that the models presented in this thesis (chapter 3 and chapter 5) are NP – hard problems, it is necessary to developed metaheuristics to solve all of them. **Chapter 6** presents a literature review of this methodology, the formulation of the two metaheuristics developed for this thesis and the results of their respectively computational experience.

Finally, **chapter 7** details some conclusions of the research developed in this thesis and highlight future research in this field.

Chapter 2

Literature Review

This chapter is divided into three main sections. In the first section, Section 2.1., a brief review of Store-Choice literature is provided. Section 2.2. reviews Location – Allocation models for retail planning. Last, Section 2.3. reviews the Location – Allocation models in a competitive environment.

2.1. Store – Choice Literature

Store-Choice literature tries to understand the consumer store-choice process. This literature studies the key variables that a customer takes into account when shopping at a particular shop, and how these variables interact. It is usually assumed that consumers not only cares about which shop is closest but also considers other variables in making their decision to patronise a particular establishment. The development of the consumer store-choice literature has been extensive and may be classified into three groups (Craig, et.al., 1984), as follows.

The first group includes models that rely on some **normative assumption** regarding consumer travel behaviour. The simplest model is based on the nearest-centre hypothesis; i.e., consumers patronise the nearest outlet that provides the required good or service. This hypothesis has not found much empirical support, except in areas where shopping opportunities are few and transportation is difficult.

This little empirical evidence suggested that consumers trade off the cost of travel with the attractiveness of alternative shopping opportunities. The first one to recognise this was Reilly in its “Law of Retail Gravitation” based on Newton’s Law of Gravitation[1] (1686). Reilly’s Law states that “the probability that a consumer patronises a shop is proportional to its attractiveness and inversely proportional to a power of distance to it” (Reilly, 1929). In fact, Reilly was the precursor of the spatial choice models known as “Gravity Models”. In the early stages, these models were non-calibrated in the sense that the parameters of the models have a priori assigned value. The best representative models of this group are the ones by Reilly (1929) and Converse (1949).

These non-calibrated gravity models have some limitations (Diez de Castro, 1997):

- They can only be applied to big stores like hypermarkets and shopping centres.
- They can only be applied when the consumer buys non-usual goods.
- They have a restrictive assumption that forces consumer’s zones to be assigned to only one shop.

[1] Newton’s Law of Gravitation studies the force between planets and stars in the universe. This law states that the force between two bodies is proportional to the product of the masses of the bodies and inversely proportional to the square of the distance between them.

<u>The second group</u> includes models that use the **revealed preference approach** to calibrate the "gravity" type of spatial choice models. These ones use information revealed by past behaviour to understand the dynamics of retail competition and how consumers choose among alternative shopping opportunities.

Huff (1964) was the first one to use the revealed preference approach to study retail store choice. The Huff probability formulation uses distance (or travel time) from consumer's zones to retail centres and the size of retail centres as inputs to find the probability of consumers shopping at a given retail outlet. He was also the first one to introduce the Luce axiom of discrete choice[2] in the gravity model. Using this axiom, consumers may visit more than one store and the probability of visiting a particular store is equal to the ratio of the utility of that store to the sum of utilities of all stores considered by the consumers.

However, the main critique to the Huff model is its over-simplification since it only considers two variables (distance and size) to describe consumer store-choice behaviour.

Nakanishi and Cooper (1974) extended Huff's model by including a set of store attractiveness attributes (rather than just one attribute employed in Huff's model). Attributes such as consumer opinion of store image, store appearance, and service level can be used, as well as objective measures such as travel distance and physical distance (Vandell & Carter, 1993). This more general statement was known as the Multiplicative[3] Competitive Interaction[4] (MCI model), which formulation states that:

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[2] Luce axiom applied to this case assumes that customers choose the optimal location option as a function of the utility of this option with respect to the level of utility of the other options.

[3] Note that this model becomes additive after the log-transformation is undertaken (see section 3.4.).

[4] The Competitve Interaction condition comes from the fact that in this model individuals select among alternatives probabilistically, in relation to the utilities offered by each choice alternative.

$$r_{ij} = \left(\prod_{k=1}^{s} A_{kij}^{\;b_k}\right) \Bigg/ \sum_{j\in J}\left(\prod_{k=1}^{s} A_{kij}^{\;b_k}\right) \qquad (1)$$

Where,

i, I = Index and set of consumers' zone.

j, J = Index and set of shops.

r_{ij} = The probability that consumers at location *i* will shop at shop *j*.

A_{kij} = The *k*-th attribute describing shop *j* attracting consumers from site *i*; *k=1,…,s*.

b_k = Parameters still not estimated, which reflect the sensitivity of consumers to the shop characteristics on the probability to shop at a particular shop.

Revealed preference methods overcome the problems of normative methods because consumers are not assigned exclusively to one shop, and the models can be applied to cases where consumers shopping habits are independent of store size. Despite these improvements, these models also have their drawbacks (Craig, et.al., 1984):

- They assume consumer utility function to be compensatory. But in reality consumers reject stores beyond a certain distance. Consumers may also reject stores unless they possess minimum levels of other attributes.
- Context dependence; i.e., the estimated parameters reflect the characteristics of existing stores in the area. For example, the parameters associated with characteristics on which the existing stores do not differ much would be low. This does not, however, imply that

such characteristics are unimportant to consumers but rather, that because of their similarity across stores, other variables are used to discriminate among them.

- The distance decay parameter (β) is highly dependent on the characteristics of the spatial structure. The implication is that in assessing the importance of location on store utilities, individuals consider not only the distance to that stores but also the relative distances to other stores in the area. The result is that consumers residing in different areas might differentially weight the impact of distance on store choice.

Finally, **the third group** includes the models that use **direct utility**. These models overcome the problem of context dependence, estimating consumer utility functions from simulated choice data using information integration, conjoint or logit techniques. Instead of observing past choices, these methods use consumer evaluations of hypothetical store descriptions to calibrate the utility function. The best representative model of this group is the one developed by Ghosh and Craig (1983) based on game theory.

Given that the aim of the thesis is the incorporation of one store-choice models into discrete competitive location models, **one of the previous store-choice models needs to be chosen**. The criterion used in making this choice is how well the resulting model can be implemented in the real world.

A recent paper (Clarkon, et.al., 1996) has analysed which location models are used by UK grocery retailers. The research shows that the procedure used by major grocery retailers operating within the UK does not rely on one approach but employs a combination of several. These different approaches were used in a sequence to maximise the overall effectiveness. Firms initially use checklist analysis to reduce the cost and time required to assess a large number of potential site locations before using the analogue approach, regression or a gravity model. Finally, the financial analysis decides which location is the most suitable for the new supermarket.

As it can be seen, theoretical models are applied to the real world as part of a wider analysis. The Clarkon's study also shows the fact that the most highly-developed models like MCI and Multiple Store Location (Achabal, et.al., 1982) are usually applied in a retailing context by US firms, but not by UK firms. The reason is that grocery retailers operating within the UK believed that the consumer spatial structure of shopping opportunities in the UK differs to the one found in the US.

The conclusions of the Clarkon paper show that firms prefer the revealed preference approach to the model consumer store-choice behaviour. This approach is preferred to normative models since it more faithfully reflects real consumer behaviour whilst the direct utility approach is simpler since it uses surveys and linear regressions instead of conjoint, logit techniques or game theory.

In the revealed preference approach, the most popular model is the MCI model (Craig, et.al., 1984). One of the practical problems of this model is that to date all the calibration had reflected the consumer spatial structure of shopping opportunities of the US market. The problem is overcome in this thesis because the empirical study is conducted in the UK and Spain. This means that calibration of the MCI in this case reflects British and Spanish Consumer Spatial structure.

2.2. Location – Allocation models

Of all elements of retail strategy, few are as important as the choice of locations from which to sell goods and services. While the location decision has always been critical, the rapid growth and expansion of multifacility networks in retailing has heightened the importance and complexity of the location decision.

In today's highly competitive retail environment, firms can create competitive advantage by achieving a strong market presence by locating multiple stores in the same market. Locating multiple units in one market has a number of advantages. For one, it creates market presence so that all consumers in the market area have relatively easy access to a firm's stores. Second, it allows for managerial efficiencies as well as scales of economies in distribution, warehousing and transportation costs. Third, it increases the efficiency of advertising and promotional expenditures in the local market. In sum, concentrating outlets in an area can create synergy, improving the performance of individual outlets that are a part of a larger network.

Organizing such networks, however, is a difficult task. Traditional methods of site selection, such as the *Analog Model* (Cohen and Applebaum, 1960) are not well suited for this purpose since they are limited to analyzing single-store locations. These single store procedures ignore the impact that an individual store may have on other outlets in the market area operated by the same firm. Establishing a network of two or more outlets, on the other hand, requires systematic evaluation of the impact of each store on the entire network of outlets operated by the firm and the consideration of the "system – wide store – location interactions" (p.8 of Achabal, et.al. (1982)).

The growth of multi-store networks has prompted the development of **location – allocation models for planning retail networks**. In the context of retail site locations, location – allocation models determine the best locations for new retail outlets on stated corporate objectives and allocations of consumers to those locations based on the expected pattern of consumer travel. The allocations depict the flows of consumers to stores and thus define each store's trade area. They are used to forecast sales and market share of outlets at different locations. Both the optimal location and the allocations must be determined simultaneously, since the optimal locations depend on consumer travel patterns (the allocations) and the travel patterns, in turn, depend on store locations.

Location – allocation models are a useful tool for retail locations planning because they can analyze both the location and allocation aspects of sitting and the system – wide impact of individual store locations. These models offer procedures for systematically evaluating store locations and finding sites that maximize corporate goals such as market share or profits. Location – allocation models can be used to develop a network of stores in a new market area, to expand an existing network or to relocate or close existing stores.

The earliest attempts at formulating location – allocation problems was done by Weber in Friedrich (1929), where he attempted to find the most efficient point of production for an industrial plant given the raw material sites and market locations. It was not until the early 1960's, however, that researchers came up with mathematical solution to the generalized Weber problem (Cooper, 1963). Since then, research on location – allocation modelling has increased rapidly, fuelled by advances in computing technology and the growing awareness of the applicability of these models to real world planning problems (for a review of the development of location – allocation models see Brandeau and Chiu (1989)).

The application of location – allocation models for retail planning is a relatively recent phenomenon, that can be grouped into five categories (Drezner, 1995): (1) p - median models, (2) covering models, (3) p - choice models, (4) consumer preference based models and (5) franchise models.

2.2.1. *p* – median models

The extension of the Weber problem to multiple supply points led, in the early 1960's, to the formulation to the well – known *p* – median model. The objective of the classic *p* – median problem (Hakimi, 1964) is to find the locations for a given number (*p*) of facilities that minimizes the average distance that separate consumers from their nearest facility. As is well known, the solution of this problem is the *p* weighted medians of the demand points represented on a graph (Hakimi, 1964).

To state the network version of the *p* – median problem, mathematically, w_i was defined as the demand for retail goods in the *i*-th demand zone, I as the set of demand zones, J as the set of feasible sites, and d_{ij} as the distance between *i* and feasible site *j*. The objective of the *p* – median problem can then be written as:

$$\text{Min } \sum_{i \in I} \sum_{j \in J} x_{ij} w_i d_{ij} \qquad (2)$$

subject to,

$$\sum_{j \in J} x_{jj} = p \qquad \forall i \in I \qquad (3)$$

$$x_{ij} \leq x_{jj} \qquad \forall i \in I, \forall j \in J \qquad (4)$$

$$\sum_{j \in J} x_{ij} = 1 \qquad \forall i \in I \qquad (5)$$

where,

$x_{ij} = 1$ if $d_{ij} = \min\{d_{ik} | k \in J\}$; 0 otherwise.

$x_{jj} =$ 1, if an outlet is opened at *j*; 0, otherwise

Constraints ensure that only p facilities are located, that no consumer is allocated to a site that has no outlets, and that all consumers are allocated to at least one outlet. x_{ij} operationalizes the allocation rule, since it defines the set of demand points served by each outlet. The variable takes the value of one when the outlet at j is closest to i; otherwise, it takes the value of zero. Thus only distance of each demand point to its nearest outlet is considered in the objective function. The objective function minimizes the distance separating consumers from their nearest facility.

Retail location models based on the p – median are generally useful in determining sites that maximizes the population's accessibility to retail services. In that case, it seems reasonable to use a normative assumption of store – choice behavior; that consumers visit the nearest outlet. This is likely to be the case for fast food outlets, public facilities, banks, post offices, and health facilities, to name just a few.

The two retail location models proposed by Goodchild (1984) and Hillsman (1984) are based on the p – median problem.

2.2.2. Covering models

A second type of location model of interest in retailing is the covering model. Covering models were originally developed for public sector location problems, such as the location of emergency medical and fire services; but the potential application of these techniques is much broader. They are important in designing multiunit networks for services oriented retail firms where access is a major determinant of patronage. The objective of covering models is to identify locations that provide potential users access to service facilities within a specified distance or travel time. This is important in situations where access plays a key role in determining the level of service utilization or the quality of the service delivered. For many convenience oriented retail facilities, such as movie theaters, banks, fast food restaurants, and ice cream parlors, most customers live near the outlet.

One of the first types of covering models to be proposed was the *Location Set Covering Problem (LSCP).* This model assumes that consumers residing beyond a specific maximum distance or travel time (S) from an outlet are not adequately served and, therefore, do not use the service. The objective of the set covering model is to find the minimum number and locations of facilities needed to serve all potential consumers within the specified distance or travel time (Toregas and ReVelle, 1972).

The goal of providing universal service may not be feasible because of the cost of operating a large number of outlets, making it necessary to tradeoff the cost of locating additional outlets with the potential revenue generated from incremental coverage. Then, instead of aiming for universal coverage, the retail manager may seek to maximize the amount of potential demand covered by a fixed number of service centers. This is the *Maximal Covering Location Problem (MCLP)* proposed by Church and ReVelle (1974). The MCLP can be written mathematically as follows:

$$\text{Max} \sum_{i \in I} w_i y_i \tag{6}$$

subject to,

$$\sum_{j \in J} x_j = p \tag{7}$$

$$\sum_{j \in N_i} x_j \geq y_i \qquad \forall i \in I \tag{8}$$

where,

y_i = 1 if demand point i is covered by an outlet (i.e., it has an outlet within a specific distance or time); 0 otherwise.

x_j = 1, if an outlet is opened at *j*; 0, otherwise

The critical operational variable of the model is the set N_i , defined for each demand point. The set designates the set of outlets within the specified distance or time from demand point *i* – the set of outlets that are considered accessible to that demand point. The objective of maximizing the demand that is covered is operationalized through the definitions of Y_i and constraint (8). This constraint dictates that Y_i is equal to 0 if N_i is empty. An uncovered zone does not contribute to the objective function since the corresponding Y_i is 0. The constraint (7) limits the number of outlets to a specified number, *p*. The objective function maximizes the amount of demand that is covered by *p* facilities accessibility criterion.

The MCLP has been the basis for a number of interesting developments; the Coherent Covering Location Problem (Serra, D., 1996) and the Probabilistic Maximal Covering Location – Allocation models for Congested systems (Marianov and Serra, 1998). Another line of research for the covering models, is the one that has tried to adapt the discrete MCLP model to the continuous space (Mehrez and Stulman (1982) and Church (1984)).

2.2.3. *p – choice* models

In location – allocation models, the allocation rule simulates consumers' shopping patterns. The implicit assumption underlying *p* – median models, for example, is that consumers travel to their nearest outlet and purchase their entire requirement of goods and services from that store. A store's market share is then given by the ratio of the demand within the store's proximal market area to the total demand. The assumption of nearest - centre travel is also made in covering models, with the additional stipulation that customers utilize the service only if the closest outlet is within a reservation distance. Summing up, both types of models use the *normative assumption* of the store – choice behaviour.

Implicit in the nearest centre allocation rule is the assumption rule is the assumption that all facilities provide the same assortment and level of service and charge similar prices. Thus facilities are distinguished only by their locations. In practice, however, even facilities providing similar types of services have varying assortments, prices and quality. This is another reason for consumers to visit facilities other than the nearest one.

The lack of support for the nearest centre hypothesis has led researchers to propose stochastic models of choice that incorporate distance (or travel time) as well as store characteristics such as service level and facility size. As seen before, an early proponent of this approach was Huff (1964), who proposed probabilistic choice rules based on the spatial interaction models; he was the first one to introduce the revealed preference approach to calibrate the "gravity" type of spatial choice models (as we have explained in epigraph 2.1.). This choice models can be written as:

$$P_{ij} = \frac{\dfrac{f(A_{ij})}{g(d_{ij})}}{\sum_{k \in K_i} \dfrac{f(A_{ij})}{g(d_{ij})}} \qquad (9)$$

where A_{ij} is a measure of attracting of facility *j* to consumer *i*, d_{ij} is the distance or travel time separating consumer *i* and facility *j* and K_i is the set of stores that are in consumer *i*'s choice set.

Two specific forms of equation (9) that are commonly used in retail research are the *Multiplicative Competitive Interaction (MCI)* model suggested by Nakanishi and Cooper (1974) and the *Multinomial Logit (MNL)* model (McFadden, 1974).

Probabilistic choice rules like that in equation (9) has been extensively used to predict the market share of facilities at different locations. The set of store attributes, A_{ij}, should include all relevant store and site characteristics hypothesized to influence consumer choice. For example, in their study of food retailing, Jain and Mahajan (1979) used factors as store size, the availability of credit card services, the number of checkout counters, whether the store was located at an intersection, in addition to distance, to explain the market share obtained by individual outlets. In addition to retail stores, choice models have been used to predict the market share of shopping centers (Weisbrod, 1984) and hospitals (Lowe and Sen, 1996). All these models belong to the group of revealed preference approach to calibrate the "gravity" type of spatial choice models.

The basic p – choice formulation states that:

$$\text{Max} \sum_{i \in I} \sum_{j \in J} w_i P_{ij} x_j \tag{10}$$

$$\text{s.t.} \sum_{j \in J} P_{ij} = 1 \quad \forall i \in I \tag{11}$$

where,

$P_{ij} =$ the probability that customers at i patronize facility at j.

$w_i =$ the number of customers at i.

$x_j =$ is 1 if a store is located at j; 0, otherwise.

From the formulation can be seen that the p – choice model is a generalized unsconstrained version of the p – median problem, because the all – or- nothing nature of consumer choice in the p – median problem is relaxed and is used a probabilistic choice rule. The objective function (10) of the p – choice model leads to a set of facilities that maximizes the number of customers served, or the firm's expected market share or the consumer's welfare, depending on the nature of the model.

The p – choice model has been the basis of a number of facility location problems, with different spatial characteristics. Hodgon (1981) developed a p – choice model in a discrete space, Achabal, et.al. (1982) developed one p – choice model in a competitive discrete space and Drezner (1994b) developed another one in a competitive continuous space.

Although the p – choice and p – median problems are similar in structure, they are generally produce very different solutions. The most important difference is that in the former, facilities no longer have well – defined geographic market areas. Each store's market area is a probabilistic surface that shows the probability of a customer from a given area patronizing that facility. The exact nature of this probability surface depends on the parameters of the spatial interaction model. Then, to apply the p – choice model in retail setting, the retail analysis *first needs to analyse the pattern of consumer travel to existing facilities, and then to apply the results from the empirical study in the allocation phase of location – allocation algorithms.*

2.2.4. Consumer Preference Based models

The fourth class of retail location – allocation models consists of models that directly incorporate consumer preferences. In these models, the allocation rule is based on consumer evaluation of hypothetical choice experiment, rather than observed choices. In other words, they used direct utility approach of store choice literature, instead of revealed – preferences ones. Consumers are asked to evaluate various choice scenarios, and their evaluations are used to predict choices in the allocation rule. Consumer preference based models overcome one major problem of the p – choice models just described – context dependency.

Including measures of locational structure in the calibrated choice functions is one remedy to the context dependency problem (Ghosh, 1984). *But such measures are difficult to implement without systematic empirical research on the impact of locational structure on choice*. Another approach is to assess consumer choice directly using structured choice experiments. Conjoint and information integration methods are typically used for this purpose. Conjoint analysis has been widely used in marketing studies to assess the impact of brand and service characteristics on consumer preferences (Witting and Cattin, 1989). Instead of analysing past choice, these models utilize consumer evaluations of hypothetical choice descriptions to calibrate the utility function. Properly conducted experimental studies can provide considerable insight into consumer behaviour and patronage decisions while removing the effects of the spatial context from the choices themselves.

To implement the conjoint method the first step is to identify the store characteristics (or attributes) that influence consumer preferences, and the levels that these attributes can potentially take. The impact of different attributes on preference is then assessed by calibrating a linear – additive function relating outlet characteristics to utility.

One of the earliest consumer preference based location models was proposed by Parker and Srinivasan (1976). They used conjoint analysis to determine how consumers evaluate different attributes of a primary care facility, including travel time. Later, other interesting applications of conjoint methods to retail choice have been Louviere (1984), Ghosh and Craig (1986) and Muñoz (1988).

2.2.5. Franchise models

Finally, there was a group of models developed for the specific problems of franchise firms. Franchising is a form of business in which the parent company (the franchisor) grants individual franchisees a license to engage in commerce using business practices, goods and services, and trademark in return for predetermined fees and royalties. Although the operations of franchised outlets are in many ways similar to other kinds of retail stores, a number of special considerations arise in making location decisions for franchise stores. In locating outlets, both the franchisor's and the franchisees' golas must be considered simultaneously. The basic models of this group are the ones developed by Pirkul, et.al (1987) and Ghosh and Craig (1991).

2.3. Competitive Facility Location models

One subgroup of location – allocation models deals with the location of plants, warehouses, retail and industrial or commercial facilities which operate in a competitive environment. All competitive locations models attempt to estimate the market share captured by each competing facility in order to optimise its location. The best location for a new facility is at the point at which its market share is maximized. For a survey of various competitive facility location models, see Eiselt, et.al. (1993).

The first modern paper on competitive facility location is generally agreed to be Hotelling's paper (1929) on duopoly in a linear market. Hotelling considered the location of two competing facilities on a segment (two ice – cream vendors along a beach trip). The distribution of buying power along the segment is assumed uniform and customers patronize the closest facility.

The following developments of competitive location models can be categorized in two groups, as a function of its spatial representation:

1 - ***Continuous competitive location models*** – where the potential location of the facilities can be anywhere in the plane.

2 - ***Discrete competitive location models*** – where facilities are allowed to locate at a finite set of possible locations on a network.

2.3.1. Continuous Competitive Location models

Continuous Competitive Location models for retail firms are an extension's of Hotelling's approach in the continuous planar space, where the retail firm is planning to open a chain of outlets in a market in which a competing chain already exists.

One first approach is the one that analyses the system – wide interactions among facilities. In these models, the allocation of customers to facilities is made using Hotelling's proximity assumption – each facility attracts the consumers closer to it. In these models, the market share attracted by each facility is calculated and then, the best locations for the new facilities are found. A good representative of these models is the location – allocation market share model (MSM) developed by Goodchild (1984). In that model, a retail firm is planning to open a chain of outlets in a market in which a competing chain already exists. The entering firm's goal is to maximize the total market share captured by the entire chain. A good review of these type of location – allocation models can be found in Ghosh and Harche (1993).

The Utility models are a *second approach* for retail firms. The utility models are predicted on consumer spatial choice models as well as on the premise that facilities of the same type are not necessarily comparable. The facilities vary in one or more of qualities which make up their total attractiveness to customers. Furthermore, varying importance assigned to each of these variables by different customers will result in a selective set of customers patronizing each. Utility models can be grouped in two groups: Deterministic Utility models and Random Utility models.

The Deterministic Utility model for competitive facility location in continuous space is introduced by Drezner (1994a). In this case, Hotelling's approach is extended by relaxing the proximity assumption. Consumers are known to make their choice of a facility based on factors other than distance alone. In this case, it is assumed that customers base their choice of a facility on facility attractiveness which is represented by a utility function. This utility function is a composite of facility attributes and the distance to the facility. The utility function represents the expected satisfaction. A distance differential is calculated based on the attractiveness difference between the competing facilities. It is suggested that a customer will patronize a better and farther facility as long as the extra distance to it does not exceed its attractiveness advantage; i.e., the calculated distance differential. A break – even distance is defined based on the distance differential. This break – even distance, therefore, is defined as the maximum distance that a customer will be willing to travel to a new facility based on his perception of its attractiveness and advantage, or disadvantage, relative to existing facilities. In this model, however, while customers are no longer assumed to patronize the closest facility, customers at a certain demand point are assumed to apply the same utility function. Therefore, they all patronize the same facility. The "all or nothing" property is maintained in this extension.

To address the "all or nothing" assumption of the Deterministic Utility model, a Random Utility model is introduced by Drezner and Drezner (1996). The Deterministic Utility model is extended by assuming that each customer draws his utility from a random distribution of utility functions. This assumption eliminates the “all or nothing” property since a probability that a customer will patronize a particular facility can be established and is no longer either 0 % or 100 %.

A *third approach* which follows the lines of store – choice theories, is the one that incorporate the gravity theories in the location – allocation continuous competitive models. In gravity models, a “customer selection rule” which depends on the attractiveness of the facility and the distance to it is used. The selection rule is probabilistic implying that the buying power of consumers located at a demand point is divided among the facilities and that the “all or nothing” property does not apply. Drezner presents two models to find the best location for a new facility (or multiple facilities) in a continuous space using the gravity model objective for single facility case (Drezner, 1994b) and for the location of multiple facilities (Drezner and Drezner, 2002a). Both models include the revealed preference approach of gravity models; specifically, the Huff’s facility floor area and Nakanishi and Cooper coefficient.

Finally, *another related approach* is the Central Place Theory (Christaller, 1933). It provides a framework for analysing the size and spacing of retail centres. The hierarchy of service centres represents differences in availability of goods and services of varying order. Customers are assumed to travel to the closest facility that offers the service or goods sought. Losch (1954) examined the interplay between range and threshold. The range is the maximal distance travelled to a facility, a spatial extent of centres. Christaller (1933) defined range as market area delineation and spatial coverage. The range is similar in concept to the break – even distance used in Drezner (1994a). Threshold is the total effective demand, or “critical mass”, required to support a particular facility. The ratio between the total demand and the threshold level determines the maximum number of facilities that can be profitably located in an area. While Christaller assumed that any place that offers a higher – order good will also offer all lower goods, Lösch relaxed this constraint. A more recent paper on central place theory is Beaumont (1987). A good review of this theory can be found in Ghosh and Rushton (1987).

2.3.2. Discrete Competitive Location models

From the late seventies, considerations on the interaction between competitive facilities in discrete space have been developed following several approaches. One of the first questions that has been addressed by several authors is the existence or (not) of a set of locations on the vertices of a network that will ensure a Nash equilibrium, that is, a position where neither firms have incentives to move. Wendell and McKelvey (1981) considered the location of two competitive firms with one server each and tried to find a situation where a firm would capture at least 50% of the market regardless of the location of its competitor. Results showed that there was not a general strategy for the firm that would ensure this capture if locating at vertices of the network. They did not develop a generic algorithm for finding solutions, but they looked at the possible locational strategies. They also examined the problem in a tree. Hakimi (1986) also analysed extensively the problem of competitive location on vertices and proved that, under certain mathematical conditions such as concave transportation costs functions, that there exists a set of optimal locations on the vertices of the network.

A similar problem was studied by Lederer and Thisse (1990). Their problem not only looked at the specification of a site but also at the sitting of a delivered price. They formulated the problem as a 2-stage game, where in the first stage both firms choose locations and in the second stage, they simultaneously set delivery prices schedules, and the result is that there is sub-game perfect Nash equilibrium. As Hakimi did, they proved that if firm's transport costs are strictly concave, then the set of locational choices of the firm is reduced to the vertices of the network. As a consequence, the location problem can be reduced to a 2 – median problem if social costs are minimized.

The problem of two firms competing in a spatial market has also been studied in the case where the market is represented by a tree. Eiselt (1992) proved that in such case there is not a sub – game perfect Nash equilibrium if both prices and locations are to be determined. Eiselt and Laporte (1993) extended the problem to the location of 3 facilities in a tree. They found that the existence of equilibria depended on the distribution of weights. In both models, firms are allowed to locate on the edges of the network.

The game-theoretical models presented so far restrict themselves to the location of firms with one facility each that compete against each other. Tobin and Friesz (1986) examined the case of a profit – maximizing firm that entered a market with several plants. They considered price and production effects on the market, since the increase in the overall production level from the opening of new plants in a spatial market stimulates reactions in the competitors. These reactions might affect not only production levels, but also prices and locations. A good review of these models can be found in Miller, et.al. (1996).

Another body of literature on competitive location in discrete space deals with the **sitting of retail convenience stores**. These types of stores are characterized by (a) a limited and very similar product offering across outlets, (b) similar store image across firms, and (c) similar prices. In this body of literature, we can find two main approaches, as a function of the type of objective function used.

The first approach correspond to the models that uses a profit maximisation objective. In this group, there are two basic models. The first one was developed by Ghosh and Craig

(1984).They considered the location of several retail facilities by two servers. The problem was to locate retail facilities in a competitive market knowing that a competing firm will also enter this market. They used a minimax approach, where the entering firm maximizes its profit given the best location of the competitor. Potential locations were restricted to the vertices of the network. The firm's objective was to maximize the net present value of its investment over a long – term planning horizon. The model did not allow location at the same site for both firms and did not examine the issues of ties. Gosh and Craig used a heuristic algorithm to obtain solutions. The model was also adapted to examine other strategies such as preemption, i.e., the identification of locations that are robust against competitive action. Other modifications included the relaxation of the number of stores that could be opened by each firm, and collusion by both servers.

In a similar model, Dobson and Karmarkar (1987) introduced the notion of stability in the location of retail outlets by two profit maximizing firms in a competitive market. Several integer programming models were developed to identify stable locations such that no competitors can enter the market and have profits given some rules on the competitive strategies. The models were solved using enumeration algorithms.

The second approach that examines competition among retail stores in a spatial market was the one developed by ReVelle (1986). The basic model was the Maximum Capture Problem (MAXCAP) (ReVelle, 1986). In essence, the MAXCAP problem seeks the location of a fixed number of stores (p stores) for an entering firm in a spatial market where there are other shops from other firms already competing for clients[5]. The spatial market is

[5] Without loss of generality, it is assumed that there is only one competing firm operating in the market (ReVelle, 1986).

represented by a network, where each node of the network represents a local market with a fixed demand. The location of the shops is limited to the nodes of the network and competition is based on distance (i.e., a market is "captured" by a given shop if there is no other shop closer to it). The objective of the entering firm is to maximise its market capture[6].

This model has been adapted to different situations. The first modification introduced shops that are hierarchical in nature and where there is competition at each level of the hierarchy (Serra, et. al., 1992). A second extension took into account the possible reaction from competitors to the entering firm (Serra and ReVelle, 1994). Finally, another modification of the MAXCAP problem introduced scenarios with different demands and / or competitor locations (Serra et.al. 1996). A good review of these models can be found in Serra and ReVelle (1996) and a real application of it in Serra and Marianov (1999).

All the previous discrete competitive location models assume that consumers patronize the closest shop. But as we have seen, earliest in this chapter, this hypothesis has not found much empirical support. To take this into account, several authors have tried to introduce the store-choice theories in its discrete competitive location models. However, this attempt has had a greater impact on the profit maximization approach of this type of models.

Karkazis (1989) considered two criteria that customers may use to decide which shop to patronize: a level criterion based on the preferences of a customer on the size of the facility and a distance criterion based on closeness to the store. He developed a model that would determine the location of a number of servers to enter the market when there are other firms

[6] This objective, given the assumptions on the characteristics of the retail stores, is almost equivalent to maximising profits (Hansen, et.al., 1987).

already operating in the market by maximizing the profit subject to a budget constraint. The problem was solved in a dynamic fashion since there is a trade – off between both criteria.

Achabal, et.al. (1982) developed the multiple store location (MULTILOC) model which extends the multiplicative competitive interaction (MCI) model to the multistore location problem. Its model has a profit maximization objective function. In the same line of research, recently, Santos-Peñate, et.al. (1996) have analysed the choice of the location and the optimal level of a service center's attractiveness for a firm that wants to enter in a market where the competitor's firm is already operating. The maximizing profit model was a modification of the traditional competitive location model using Huff's model and the Multiplicative Competitive Interaction model for consumer choice behaviour. The model was simply solved using a Greedy Adding procedure together with a Teitz and Bart algorithm.

Eiselt and Laporte (1989) presents a conditional location problem on a weighted network. To do this, they generalized ReVelle's finding of the MAXCAP formulation in order to include parameters based on Gravity models and Voronoi diagrams. The attraction functions were a simplified version of the gravity-type models, which was defined in terms of consumer facility distances and facility weights. The ratios of attractions of two facilities were defined in a normative way as the ratio of purchases made at the two respective facilities. The purpose of the paper was to locate an additional facility and determine simultaneously the optimal weight of that new facility. The model was solved using a simple procedure, which considers in turn all candidates' locations.

Serra, et.al. (1997) presents two new Maximum Capture models that overcome the all – or – nothing assumption, using different consumers decision rules. These rules are based on the proportional assumption, instead of the traditional all – or – nothing capture of the basic

MAXCAP model[7]. The models were solved using an exact method and a heuristic. Solutions were then compared to those obtained by the Maximum Capture Problem.

Summing up, up till now, we have seen that continuous competitive location models for retail firms have introduced the revealed preference approach of consumer choice; while, discrete competitive location model have done it only for the profit maximisation models, because the only attempt to do it for the maximum captured models is a model that analyses the single facility case using the normative approach of the simple gravity model .

[7] Although these decision rules are based on a proportional rule, they depend only on the distance or travel time variable.

Chapter 3

Integration between Consumer Choice and Discrete Competitive Location Models

The first issue for the incorporation of Consumer Choice Behavior theories in Discrete Competitive Location models, is the analysis of the best way to introduce distance or travel time. Do we have to take into account the various theories of consumer choice behavior to introduce distance in one or another way in the location models? How should we include distance in our location models?

To do this, I consider different ways of defining a key parameter of one basic competitive location model. This parameter will reflect the various ways of taking into account distance based on different Consumer Choice Behavior theories. The basic MAXCAP model (Maximum Capture model, ReVelle (1986)) considered the traditional assumption of *all or nothing capture* where consumers patronize the closest facility, only comparing its *distance to the closest facility for different chains*. In that model, the closest facility captures all the demand. Other models, such as, the Multiplicative Competitive Interaction model (MCI, Nakanishi and Cooper (1974)) and the Proportional Customer Preference model (Serra, et.al. (1997)) are based on a related distance.

The idea of related distance states that customers do not choose the chain, instead they select probabilities that are *functions of their distance (or travel time) to all outlets.* In these models, the demand captured in each node by each outlet is *proportional* in some fashion to the distance (or travel time) from node *i* to all the outlets, regardless of their ownership. The difference between both models is the incorporation of consumers' sensitivity to the distance (or travel time) involved. The MCI model introduces this parameter, while the Proportional Customer Preference model assumes that this sensitivity is equal to 1. Finally, the Partially Binary Preference model (Serra, et.al. (1997)) assumes that consumers patronize *the closest facility of the chosen chain*. In this case, the capture obtained in demand node *i* by each firm is *proportional* to the distance from node *i* to the closest facility.

After the application of the models to several numerical cases, I have been able to analyze whether the optimality of the locations substantially differ or not depending on the Consumer Behavior theory we take into account (see epigraph 3.2.).

3.1 The Models

In all the models, the basic problem states that a new firm (from now on Firm A) wants to enter with *p* servers in a market in order to obtain the maximum capture, given that it has to compete with *q* existing outlets. These competitors can belong to one or more firms, but without loss of generality it is assumed that there is only one competing firm (Firm B) operating in the market; as was assumed by ReVelle (1986).

These models study the location of retail facilities in discrete space. The models make the following assumptions:

- The spatial market is defined by a connected graph. At each vertex of the graph, there is a local market with a given number of consumers that generates a demand for the product.
- Potential locations for the services are also pre-specified (note that all outlets are allowed to locate only at the vertices of the graph).
- The customer wants to buy a unit of a specific product; i.e. we do not take into account multipurpose shopping behavior.
- Demand is totally inelastic.
- The product sold is homogeneous, in the sense that the customer goes to buy the same product at all the outlets.
- Price is set exogenously and consumers bear transportation costs.
- Unit costs are the same in all stores regardless of ownership.
- Both firms are profit maximizing.
- Under equal conditions (in terms of distance) the existing firm captures the demand[8].

Defining a key parameter of the models ρ_{ij} in a different way, I will be able to reflect the various ways of taking distance into account based on several Consumer Choice Behavior theories. In general terms, ρ_{ij} will reflect the proportion of demand captured by an outlet at *j* from a demand node *i*.

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[8] Note that here, I use the Hakimi assumption (1986) that states that in case of equal distance to the outlets from a node, demand is fully allocated to the existing firm. But this is not a key issue because it is easy to modify the models if I change this assumption (for example, the entrant firm will keep all the demand under equal conditions, or split the demand between both models).

3.1.1. Model 1: Maximum Capture model (MAXCAP).

I will use the Maximum Capture model (MAXCAP) of ReVelle (1986) in the P-median-like formulation as follows:

$$\text{MAX} \qquad Z^1 = \sum_{i \in I} \sum_{j \in J} a_i \ \rho_{ij} \ x_{ij} \tag{12}$$

Subject to

$$\sum_{j \in J} x_{ij} = 1 \qquad \forall i \in I \tag{13}$$

$$x_{ij} \leq x_{jj} \qquad \forall i \in I, \ \forall j \in J \tag{14}$$

$$\sum_{j \in J} x_{jj} = p \tag{15}$$

$$x_{ij} = \{0,1\} \quad x_{jj} = \{0,1\} \qquad \forall i \in I, \ \forall j \in J \tag{16}$$

Where the parameters are:

$i, I =$ Index and set of local markets that are located at the vertex of the graph.

$j, J =$ Index and set of potential locations for firm A's outlets.

$J^B (\in J) =$ The set of actual locations of the q firm B's outlet.

$d_{ij} =$ The network distances between local market i and an outlet in j.

$a_i =$ Demand at node i.

And the variables are defined as follows:

$x_{ij} = 1$, if demand node i is assigned to node j; 0, otherwise.

$x_{jj} =$ 1, if an outlet of firm's A is opened at node j; 0, otherwise.

The constraint set basically states that: constraint set (13) forces each demand node i to assign to only one facility. But for a demand node i to be assigned to a facility at j, there has to be a facility open at j; this is achieved by constraint set (14). Constraint (15) sets the number of outlets to be opened by firm A and constraint (16) is the integrality constraint of the decision variables.

The objective function defines the total capture that firm A can achieve with the siting of its *p* servers. And the term ρ_{ij} is the key parameter for my work. Basically, it will reflect the proportion of demand captured by an outlet at *j* from a demand node *i*. The definition of this parameter will depend on the Choice Consumer Behavior model I consider.

In the above case, the MAXCAP model uses the traditional view of **all or nothing capture** relative to the distance criterion. This is the general assumption where consumers patronize the closest shop. In other words, for each demand node, **consumers compare the distance between the closest firm A server and the closest firm B server.** Applied to this problem, an outlet of firm A located at *j* will capture all the demand in *i* if its distance to *i* is less than the distance between local market *i* and the closest B server. Thus, under this assumption the definition of ρ_{ij} is as follows:

$$\rho_{ij} = \ 1 \,, \text{ if } \ d_{ij} < d_{ib_i} ; \ 0 \,, \text{ otherwise,} \qquad (17)$$

where, d_{ib_i} is the distance from node *i* to the closest B server to *i*.

This criterion is a usual assumption in the most important Discrete Competitive Location models as ReVelle (1986), Serra et. al. (1992,1994,1996).

3.1.2. Models 2 and 3.

The next two models are based on the idea that the probability that a customer at location *i* will shop at retail facility *j* is a **relative function of its distance to all of its outlets.** The basic idea is that demand captured at each node by each outlet is proportional to the distance from node *i* to all the outlets, regardless of ownership. For these two cases, the formulation of the problem is as follows:

$$\text{MAX} \quad Z^{2,3} = \sum_{i \in I} \sum_{j \in J} a_i \, \rho_{ij} \; x_{i,j} \qquad (18)$$

Subject to

$$\sum_{j \in J} x_{ij} = p + q \qquad \forall i \in I \qquad (19)$$

$$x_{ij} \le x_{jj} \qquad \forall i \in I, \ \forall j \in J \qquad (20)$$

$$\sum_{j \in J} x_{jj} = p \qquad (21)$$

$$x_{ij} = \{0,1\}, x_{jj} = \{0,1\} \quad \forall i \in I, \ \forall j \in J \qquad (22)$$

This formulation is similar to the one in the P-median problem, except in the fact that I have reformulated constraint set (13): $\sum_{j \in J} x_{ij} = 1 \quad \forall i \in I$, the one that forces each demand node i to assign to only one facility. Instead, I use constraint set (19) that states that every demand node makes $p + q$ assignments to the p new and q existing outlets.

3.1.2.1. Model 2: Multiplicative Competitive Interaction (MCI) model

Consumer Choice Behavior Literature assumes that customers consider other variables apart from the distance to choose the facility they will patronize. The best example of this literature is the MCI model developed by Nakanishi and Cooper (1974).

In this thesis, I use the version of the MCI model offered by Jain and Mahajan (1979). They took into account that the characteristics of a retail facility could come from two sets. The first one includes the characteristics that are *independent* of the consumer's point of origin (e.g.: quality of product and services, in-store convenience level, price of the product, sales area in the store,). The other set includes the characteristics which are *dependent* on the consumer's point of origin (e.g.: distance or travel time).

Thus, the definition of r_{ij} based on this simple MCI modification is as follows:

$$r_{ij} = \left(\prod_{k=1}^{s} A_{kj}^{\ b_k}\right)\left(\prod_{e=1}^{r} B_{eij}^{\ b_e}\right) \Bigg/ \sum_{j=1}^{m}\left[\left(\prod_{k=1}^{s} A_{kj}^{\ b_k}\right)\left(\prod_{e=1}^{r} B_{eij}^{\ b_e}\right)\right] \qquad (23)$$

where,

r_{ij} = The probability that a customer at location *i* will shop at retail facility *j*. (The proportion of capture that an outlet in *j* will achieve by demand node *i*)

A_{kj} = The *k*-th attribute of the retail facility *j* which is independent of the consumer's point of origin; *k* = 1,...,*s*.

B_{eij} = The *e*-th attribute of the retail facility *j* which is dependent on the consumer's point of origin, *e* = 1,..., *r*.

m = *p* + *q*, total number of outlets in the market (*p* = Firm A servers, *q* = Firm B servers)

b_k, b_e = Empirically determined parameters, which reflect the sensitivity of the retail outlet characteristics on the probability of shopping at a particular store.

As the objective of the first part of the thesis is the analysis of how distance may be introduced in Discrete Competitive Location models, I can assume that all outlets are similar. Then, for my purposes the attributes of the outlets that are independent of the customer's point of origin can be assumed equal for all the outlets and equal to 1.

Assumption 1: $A_{kj} = 1 \Rightarrow \prod_{k=1}^{s} 1^{b_k} = 1$

I can also assume that the only relevant attribute dependent on the consumer's point of origin is the distance.

Assumption 2: $B_{eij} = d_{ij} \quad as \quad e = 1.$

As the distance is a disutility for the consumer, I will advance that b_e for e = distance (b_d , now on) will be negative; i.e., that utility declines as distance increases. To make comparisons easier, I will put this parameter in absolute value and remove the distance to the denominator.

At the same time, the summation of the denominator can be decomposed, using the definition $m = p + q$, as the summation of the distance to firm A 's located outlets (p) and the firm B's located outlets (q). Then, using the notation of the MAXCAP model and applying all these assumptions and simplifications, we can rewrite the definition of ρ_{ij}[9].

$$\rho_{ij} = \frac{1 / d_{ij}^{\,b_d}}{\sum_{j \in J} \left(1 / d_{ij}^{\,b_d} \right) x_{jj} + \sum_{j \in J^B} \left(1 / d_{ij}^{\,b_d} \right)} \qquad (24)$$

ρ_{ij} is the proportion of capture that an outlet in j will achieve at demand node i. The basic idea here is that customers select stores (regardless of the ownership) with probabilities that are inversely proportional to a function of their distances, taking into account of consumers' sensitivity to distance.

3.1.2.2. Model 3: Proportional Customer Preference model, an special case of the MCI model.

A new line of Consumer Behavior theory developed by Serra et.al. (1997) considers the existence of interaction among outlets that affect consumer decision. To reflect this behavior pattern, they develop a model assuming that all outlets compete for customers. A

[9] Note that this model is non-linear since there are x_{jj} variables in the denominator.

customer does not choose the chain; instead he selects stores with probabilities that are inversely proportional to functions of his distance to all the stores. Thus, the basic idea is that the demand captured in each node by each outlet is **proportional to the distance from node i to all the outlets, regardless of ownership**. This model can be considered as a special case of the MCI model, where $b_d = 1$. In this case, the definition of ρ_{ij} is as follows[10]:

$$\rho_{ij} = \frac{1/d_{ij}}{\sum_{j \in J}\left(1/d_{ij}\right)x_{jj} + \sum_{j \in J^B}\left(1/d_{ij}\right)} \qquad (25)$$

Where, ρ_{ij} is again the proportion of capture that an outlet in j will achieve by demand node i.

We should observe that the only difference in the definition of ρ_{ij} between this model and the MCI model is the parameter b_d (i.e. sensitivity of consumers to the distance attribute in the choice among outlets). The response sensitivity of location models to distance can therefore only be found by comparing the deviation from optimality of the locations found in both models.

3.1.3. Model 4: Partial Binary Preferences

In Serra et.al. (1997), a second model is defined which reflects partial binary customer preferences. This states that consumers patronize the **closest outlet of the chosen chain.** Thus, the proportion of times that he picks one outlet is inversely proportional to some function of the distance.

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[10] Note that this model is non-linear since there are x_{jj} variables in the denominator.

In this case, the capture obtained in demand node *i* by each firm is **proportional** to the distance from node *i* to the closest facility. Therefore, the definition of ρ_{ij} in this case is as follows[11]:

$$\rho_{ij} = \frac{d_{iB_i}}{d_{iA_i} + d_{iB_i}} \tag{26}$$

where,

d_{iA_i} = Distance from demand node *i* to the closest A server.

d_{iB_i} = Distance from demand node *i* to the closest B server (the competing firm)

ρ_{ij} = The proportion of capture that an outlet at *j* will achieve from demand node *i*.

Finally, it should be noted that the formulation in this case is equal to the MAXCAP formulation.

3.2. A Comparative Analysis of these models

3.2.1. Characteristics of the comparative analysis.

In the previous epigraph, I have presented several Discrete Competitive Location models. These models, by defining a key parameter ρ_{ij} of the models in a different way, reflect the various ways of taking distance into account based on several Consumer Choice Behavior theories.

Solving these models will provide the optimal locations for the entering firm in each case. But, how can I analyze whether the objective value associated with the optimal solution changes dramatically when applying these different models? To do so, I will compute the

[11] Note that since a_i is in the fraction's denominator of the parameter (i.e. in the objective), the model will always assign node *i* to the closest firm A's facility.

deviation in demand captured by each model and the optimal locations it provides and compare these with the optimal locations provided by other models.

$$\text{Deviation–}ij\text{:} \quad \left[Z^i(Loc_i) - Z^i(Loc_j)\right] \Big/ Z^i(Loc_i) \qquad \forall i \in I \;, \forall j \in J \qquad (27)$$

Where, $Z^i(Loc_i)$ is the demand captured by model *i* when the optimal location found by model *i* is used and $Z^i(Loc_j)$ is the demand captured by model *i* when the optimal location found by model *j* is used.

Analysis of these deviations will give an idea of the importance of introducing distances in different ways. Basically,

- *If these deviations are not significant* (i.e., are not different from zero at the, for example, 5% level of significance). The conclusion will be that it does not matter how distance is included because the demand captured by the optimal locations will be similar in all the models.
- *If these deviations are significant* (i.e., are different from zero at the 5% level of significance). The conclusion will be that before applying a location model (in order to find the optimal locations), we have to analyze which consumer behavior better represents the one analyzed. This prior analysis will tell us how to introduce distance in Competitive Location models.

The algorithm has been applied to several randomly generated networks. These networks have number of nodes *n* equal to 20,30 and 50. For each *n*, three different number of outlets are located so that *p*=2,3,4; while the number of the established firm outlets are pre-fixed *q* = 5 (the specific nodes in each case are the ones shown in table 3.1.). Finally, for each n and each p, ten networks were randomly generated. Therefore, a total of 90 networks were generated.

Table 3.1. Pre-fixed location for the outlets of firm B in each network case

	20-network	30-network	50-network
Nodes	4,7,11,17,19	4,7,17,22,27	4,21,22,36,38

For each of these networks, the location of the nodes $n \in (0,1000)^2$ were generated following a uniform distribution in a map of 1000 units * 1000 units. The Euclidean distances between nodes were computed. The neighborhood for each node is defined as the randomly (2-6) closest nodes using the Euclidean distance criterion. The demand in each node was randomly generated within the (800,1000) interval again following a uniform distribution.

In the MCI model, the sensitivity of consumers to quadratic distance is used in all the cases ($b_d = 2$).

For each 90 networks and for each model, optimal solutions were obtained by complete enumeration[12]. The deviation were computed using the optimal solutions found by complete enumeration.

3.2.2. Comparative results.

Tables 3.2., 3.3., 3.4. show the average (and the maximum) deviation in demand captured computed when the optimal locations of model *i* (Loc_i) are used, while the true model is *j*, for the networks of *20*-nodes, *30*-nodes, *50*-nodes respectively.

[12] Chapter 6 presents a Metaheuristic to solve these models in a reasonable computational time, as these models are NP – Hard and cannot be solved using the traditional methods of linear programming and branch and bound.

Table 3.2. Average (maximum) deviation for 20-nodes network

p	MODEL	Loc_1	Loc_2	Loc_3	Loc_4
2	*1*	0 %	27.24 % (53.21 %)	47.05 % (77.98 %)	13.19 % (35.15 %)
	2	8.51 % (20.42 %)	0 %	2 % (8.2 %)	4 % (11.4 %)
	3	8.94 % (18.02 %)	1.47 % (4.83 %)	0 %	5.7 % (12.2 %)
	4	4.26 % (9.24 %)	7.3 % (21.99 %)	17.66 % (30.15 %)	0 %
3	*1*	0 %	34.81 % (54.75 %)	34.82 % (54.75 %)	14.89 % (34.78 %)
	2	7.28 % (12.41 %)	0 %	0.36 % (1.04 %)	2.98 % (7.63 %)
	3	7.07 % (11.77 %)	0.32 % (1.83 %)	0 %	3.8 % (7.85 %)
	4	4.24 % (9.27 %)	16.5 % (31.84 %)	13.27 % (22.75 %)	0 %
4	*1*	0 %	34.6 % (76.09 %)	41.42 % (64.87 %)	17.1 % (30.57 %)
	2	9.79 % (12.43 %)	0 %	1.96 % (3.87 %)	6.15 % (12.2 %)
	3	8.63 % (13.65 %)	1.18 % (2.51 %)	0 %	5.71 % (11.06 %)
	4	8.47 % (11.35 %)	17.84 % (51.61 %)	20.73 % (37.78 %)	0 %

Table 3.3. Average (maximum) deviation for 30-nodes network

p	MODEL	Loc_1	Loc_2	Loc_3	Loc_4
2	*1*	0 %	15.65 % (34.96 %)	27.32 % (68.34 %)	10.93 % (19.43 %)
	2	5.75 % (12.01 %)	0 %	0.59 % (2.57 %)	0.94 % (3.71 %)
	3	6.27 % (11.39 %)	1.4 % (6.26 %)	0 %	2.64 % (7.49 %)
	4	3.01 % (9.39 %)	2.97 % (13.78 %)	6.85 % (23.86 %)	0 %
3	*1*	0 %	19.46 % (46.77 %)	35.17 % (72.08 %)	15.42 % (33.89 %)
	2	9.83 % (17.97 %)	0 %	1.78 % (5.68 %)	1.47 % (5.05 %)
	3	7.17 % (11.43 %)	0.97 % (3.25 %)	0 %	2.29 % (6.49 %)
	4	6.01 % (11.52 %)	5.17 % (17.43 %)	13.81 % (34.26 %)	0 %
4	*1*	0 %	26.25 % (52.78 %)	35.6 % (46.82 %)	11.75 % (19.02 %)
	2	8.75 % (19.12 %)	0 %	1.75 % (3.12 %)	2.75 % (5.87 %)
	3	7.14 % (13.87 %)	1.18 % (2.93 %)	0 %	3.58 % (7.92 %)
	4	6.76 % (14.78 %)	13.09 % (26.19 %)	18.96 % (34.25 %)	0 %

Table 3.4. Average (maximum) deviation for 50-nodes network

p	MODEL	Loc_1	Loc_2	Loc_3	Loc_4
2	*1*	0 %	13.02 % (27.21 %)	24.89 % (71.96 %)	14.11 % (32.92 %)
	2	7.36 % (12.36 %)	0 %	1.06 % (6.05 %)	0.82 % (2.42 %)
	3	5.71 % (10.78 %)	0.88 % (5.18 %)	0 %	1.22 % (3.24 %)
	4	3.76 % (8.67 %)	0.4 % (1.31 %)	6.81 % (45.3 %)	0 %
3	*1*	0 %	17.52 % (40.35 %)	31.27 % (51.95 %)	12.17 % (25.8 %)
	2	6.85 % (10.52 %)	0 %	1.59 % (3.84 %)	0.76 % (3.48 %)
	3	6.27 % (11.9 %)	1.35 % (3.79 %)	0 %	2.62 % (5.26 %)
	4	4.9 % (8.61 %)	2.36 % (9.18 %)	9.89 % (15.58 %)	0 %
4	*1*	0 %	22.88 % (44.34 %)	40.67 % (57.54 %)	12.55 % (24.72 %)
	2	5.75 % (11.99 %)	0 %	2.53 % (5.13 %)	2.29 % (4.12 %)
	3	5.24 % (7.79 %)	1.03 % (2.96 %)	0 %	3.58 % (8.19 %)
	4	4.46 % (7.93 %)	8.63 % (33.34 %)	14.87 % (28.63 %)	0 %

We can summarize these tables in two useful ones. Table 3.5. shows the average of all the deviations for all the networks generated.

Table 3.5. Average deviations for all the networks generated

MODEL	Loc_1	Loc_2	Loc_3	Loc_4
1	0 %	23.49 %	35.36 %	13.57 %
2	7.76 %	0 %	1.51 %	2.46 %
3	6.94 %	1.09 %	0 %	3.46 %
4	5.10 %	8.25 %	13.65 %	0 %

Table 3.6. shows the results from the statistical analysis of the significance of these deviations[13].

Table 3.6. Statistically significance of the deviations

MODEL	Loc_1	Loc_2	Loc_3	Loc_4
1	Deviation = 0	Deviation ≠ 0	Deviation ≠ 0	Deviation ≠ 0
2	Deviation ≠ 0[14]	Deviation = 0	Deviation = 0	Deviation = 0
3	Deviation = 0	Deviation = 0	Deviation = 0	Deviation = 0
4	Deviation = 0	Deviation ≠ 0	Deviation ≠ 0	Deviation = 0

From the previous tables, several conclusions can be extracted:

- The greatest deviation in demand captured is the one found when we use the optimal locations of model 2,3,4 while the true model is the first one (the traditional MAXCAP model). This behavior is constant for all the network size run. Specifically (using table 3.5.), the average deviation in capture by the use of optimal locations of model 2, model 3 and model 4 in relation to the use of the optimal location of model 1 (the true one's) is around 23.4 9%, 35.36 % and 13.57 % respectively. From table 3.6., I can conclude that these three deviations are significantly different from zero.

[13] We have used the normal distribution with 5% level of significance to contrast if the deviations are significantly equal or different to zero.

[14] With a level of significance of 4%, the deviation will become significantly equal to zero.

- The smallest deviation in demand captures is achieved in two cases. When we use the optimal location of model 2 when the true model is the third one and reversibly, when we use the optimal locations of model 3 while the true model is the second one. From table 3.5., the average deviation in demand captured by the use of optimal location of model 3 when the true one is the second model is 1.51 %. And reversibly, the deviation of using the optimal location of model 2 when the true model is the third one is 1.09 % on average. From table 3.6., I can conclude that both deviations are significantly equal to zero. Then, as the only difference between model 2 (MCI model) and model 3 (Proportional Customer Preference's model) is the introduction of the sensitivity of consumers to the quadratic distance in the choice among outlets, I conclude that the introduction of this sensitivity in the Competitive Location models is not important in terms of optimality.
- Finally, it seems that the use of the optimal locations of the traditional MAXCAP model produces the smallest deviation in demand captured. The average deviation in capture of using the optimal location of MAXCAP (Loc_1) in relation to the use of the optimal location of the true model 2, 3 and 4 are 7.76 %, 6.94% and 5.10% respectively on average (using table 3.5.). From table 3.6., we can conclude that the last two deviations are significantly equal to zero at a 5% significance level, while the first one is significantly equal to zero at a 4% significance level.

In essence, the deviations are, in general, significant. So, before applying one of the defined location models, we have to analyze which kind of consumer behavior we are dealing with. This analysis will tell us how to introduce distance in the location model. But if this analysis cannot be made or it is too costly, the location model that we will have to use is the traditional MAXCAP model (model 1) as this will give the smallest deviation in demand captured whatever the true model is.

3.3. An application to an example.

In this case, the four models have been applied to a 55-node network (Swain 1974, Figure 3.1.), where the total demand to capture is 3575. The demand at each node is indicated in the table 3.7.

Figure 3.1. 55 – node network (Swain, 1974)

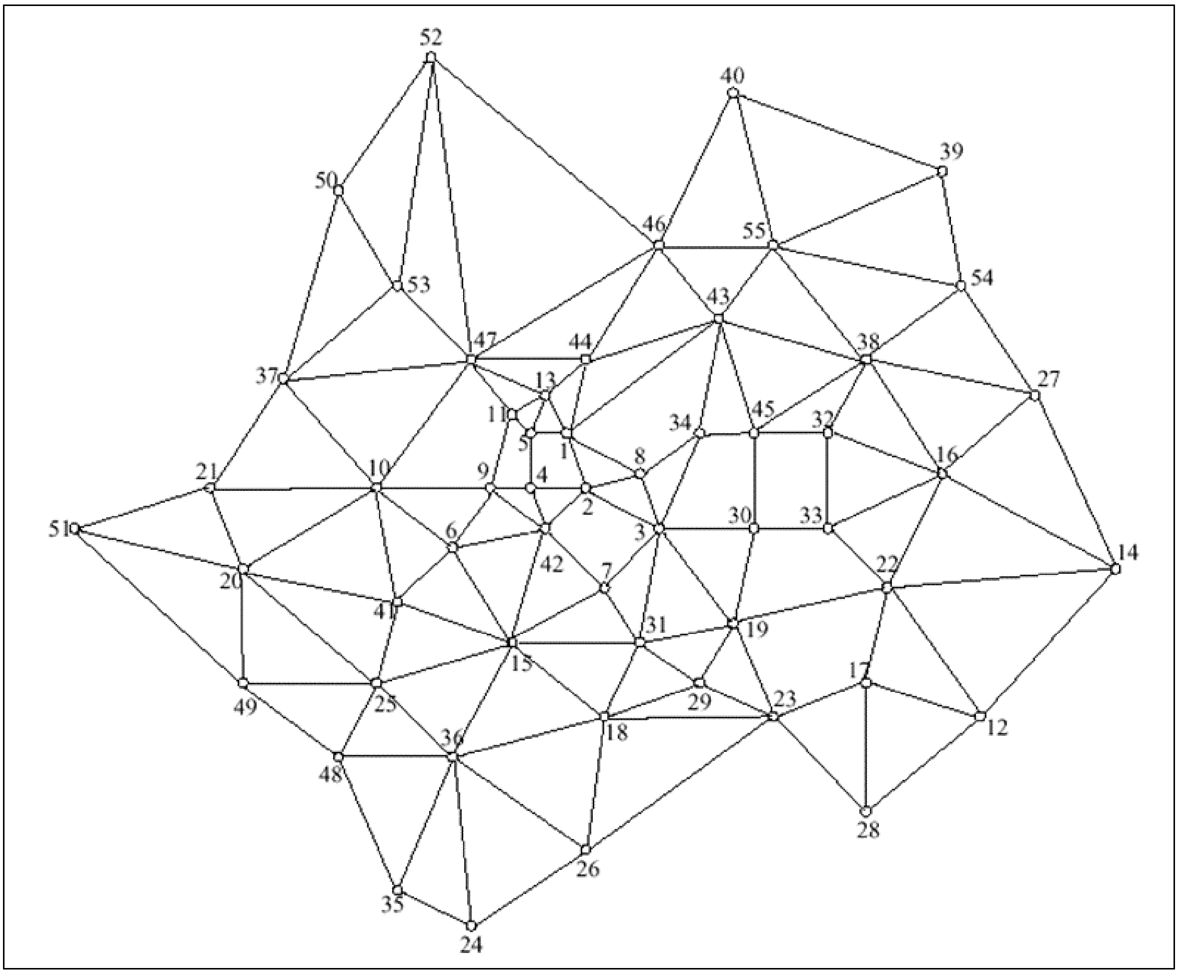

Table 3.7. 55-nodes network demand.

Node	Demand	Node	Demand	Node	Demand
1	120	*20*	77	*39*	47
2	114	*21*	76	*40*	44
3	110	*22*	74	*41*	43
4	108	*23*	72	*42*	42
5	105	*24*	70	*43*	41
6	103	*25*	69	*44*	40
7	100	*26*	69	*45*	39
8	94	*27*	64	*46*	37
9	91	*28*	63	*47*	35
10	90	*29*	62	*48*	34
11	88	*30*	61	*49*	33
12	87	*31*	60	*50*	33
13	87	*32*	58	*51*	32
14	85	*33*	57	*52*	26
15	83	*34*	55	*53*	25
16	82	*35*	54	*54*	24
17	80	*36*	53	*55*	21
18	79	*37*	51		
19	79	*38*	49		

As in the previous section, Firm B is already operating five outlets in the market. They are located at nodes 4, 21, 22, 36, 38. Three different scenarios are examined with regard to the number of outlets to be located by Firm A (p = 2, 3 and 4).

The total demand captured by the outlets located by Firm A and the optimal locations of these new outlets in each scenario and for each model in the 55-nodes network are presented in table 3.8.

Table 3.8. Demand captured and optimal locations in 55-nodes network

p	MODEL	Demand Captured	Optimal Locations
2	*1*	1462	5,42
	2	1468.587	2,4
	3	1392.797	2,4
	4	1402.409	5,3
3	*1*	1764	13,42,17
	2	1767.926	4,3,5
	3	1711.473	4,2,3
	4	1474.34	5,31,33
4	*1*	2000	5,17,30,42
	2	1979.321	3,4,5,7
	3	1952.43	2,3,4,5
	4	1462	5,31,33,41

From the previous tables, it can be pointed out that the optimal locations found by the Multiplicative Competitive Interaction model (model 2) and by the Proportional Customer Preference's model (model 3) are nearly the same ones. This fact explains why the deviations produced when using the locations of model 2 to evaluate the demand captured by model 3 and reversibly are the smallest ones. For example, in p = 2, there are no deviations in these cases (as is shown in Table 3.9.).

Tables 3.9., 3.10., 3.11. show the deviation in demand captured when the optimal locations of model i (Loc_i) are used, while the true model is j, when the entering firm wants to locate 2, 3 and 4 new outlets.

From these tables, it can be extracted the same conclusions found in computational experience. On the one hand, the greater deviation in demand captured is the one found in Table 3.9. using the optimal location of model 2 and 3 while the true model is the MAXCAP (model 1). These deviations are 63.54%. On the other hand, the use of the optimal locations of the traditional MAXCAP model is the one that produces the smallest deviation in demand captured. This deviation is less than 8.1% in all cases.

Table 3.9. Deviation for 55-nodes network. Case p = 2.

MODEL	Loc_1	Loc_2	Loc_3	Loc_4
1	0 %	63.54 %	63.54 %	16.76 %
2	6.97 %	0 %	0 %	3.59 %
3	5.42 %	0 %	0 %	3.03 %
4	4.64 %	31.15 %	31.15 %	0 %

Table 3.10. Deviation for 55-nodes network. Case p = 3.

MODEL	Loc_1	Loc_2	Loc_3	Loc_4
1	0 %	31 %	49.26 %	27.66 %
2	7.4 %	0 %	0.31 %	6.6 %
3	8.03 %	0.38 %	0 %	7.69 %
4	2.8 %	14.95 %	28 %	0 %

Table 3.11. Deviation for 55-nodes network. Case p = 4.

MODEL	Loc_1	Loc_2	Loc_3	Loc_4
1	0 %	31.05 %	33.45 %	16.8 %
2	6.26 %	0 %	0.42 %	5.8 %
3	7.35 %	0.78 %	0 %	8 %
4	3.6 %	20 %	17.77 %	0 %

Chapter 4

An Empirical Study of Store Choice Attributes in Competitive Location Models

The second issue for the incorporation of Consumer Choice Behavior theories in Discrete Competitive Location models, is the analysis of which store attributes (other than distance) should be included in competitive location models and how these could be incorporated.

In this fourth chapter, I am going to present a new methodology for determining which store attributes (other than distance) should be included in the Maximum Capture Model (MAXCAP) as well as how these parameters ought to be reflected. The parameter r_{ij} included in the MAXCAP model will be determined using the Multiplicative Competitive Interaction model. Specifically, the estimation of parameter r_{ij} will be performed to the supermarket sector in two scenarios: Milton Keynes (in Great Britain) and Barcelona (in Spain).

4.1. The methodology

The methodology presented and used in this chapter is shown in Figure 4.1.

Figure 4.1. Methodology

Step 1: *Design and development of a survey on consumer supermarket- choice behaviour*

↓

Step 2: *Estimation of the supermarket's key attributes through a Factor analysis applied to the survey database*

↓

Step 3: *Specification of the MCI model using the factors found in the previous analysis as variables*

↓

Step 4: *Calibration of the model (determination of significant supermarket factor attributes and estimation of sensitivity parameters) by applying of the ordinary least squares methods on the log-transformed centered form of the specified MCI equation*

4.1.1. First step: Survey

The first step of the paper is the design and development of a survey of consumer supermarket-choice behaviour. This is required since MCI is a revealed preference model. In other words, the model uses information revealed by past consumer behaviour to calibrate its parameters (as is explained in Chapter 2).

First of all, a questionnaire was design to be used in a personal interview survey. The main structure of the questionnaire included four parts: *introduction*, *general questions of shopping behaviour, specific questions of supermarket's attributes and demographic characteristics*.

A. INTRODUCTION

First of all an introduction was included to:

- Capture the attention of the respondents in the first minutes.
- Make clear the type of shopping activity covered by the questionnaire. Specifically, "shopping" was defined as the routine weekly or fortnightly shopping trip for food and groceries.

B. GENERAL QUESTIONS ON SHOPPING BEHAVIOUR

Questions 1&2

These questions determine the issue of multi-supermarket shopping; i.e., if consumers went to one or more supermarkets to do their shopping.

Questions 3&4

The answers of these questions give a general idea of which are the key supermarkets' attributes for consumers when choosing supermarkets.

Question 3 was an open-ended question on the reasons for choosing one supermarket to do the "shopping". This question allowed the consumer to express his opinion without being biased by responses to closed-ended questions.

In Question 4, consumers were asked to rank the main supermarket's attributes. These attributes were extracted from a paper (Burn, 1992) that reviewed the definition of store attributes by different authors. From this paper, nine dimensions of store attributes were identified: "easy to get these", hours of opening, consumer service, financial service[15], consumer account, quality of staff, price policy, quality of products and range of products.

C. SPECIFIC QUESTIONS ON SUPERMARKET'S ATTRIBUTES

This part of the questionnaire was the most important one. This general section of specific questions on supermarket's attributes was structured in blocks representing the main supermarket attributes groups. These blocks were the ones defined by London & Della (1998): *Location, Convenience, Customer Service, Merchandise and Prices.*

The split of this section into blocks was done to avoid the monotony of using a long question asking for the evaluation of a long list of attributes.

Consumers were asked to make scalar judgements in an interval on the importance of various supermarket attributes when choosing where to do their "shopping". The specific attributes in each block are the ones defined in London & Della (1988) and McGoldrick (1990). The attributes were measured in accordance with the procedures set out in the Marketing Scales handbook (Gordon, et.al., 1993).

Specifically, each of the blocks included the following:

Convenience

The convenience block included all the attributes relating to store location, accessibility, store layout, store atmosphere and opening hours. The importance of these attributes when choosing supermarket was evaluated on a 5-point scale.

Customer Service

This block includes customer service's attributes offered by supermarkets to attract consumers. In this case, financial services (supermarket club card and credit), services that

[15] Financial services were defined as the services offered by the supermarkets that had a bank (savings accounts, personal loans, mortgages, pensions, home and content insurance,...). This dimension was included only in the British questionnaire because Spanish supermarkets still do not offer these kind of service.

increase convenience (consumer service desk, home delivery, parents and baby facilities), checkout services (speed, express checkout), personnel services and additional services (petrol station, restaurant or cafeterias) were included. The importance of these attributes in choosing a supermarket was evaluated on a 5-point scale.

Merchandise

This block included three ways of judging merchandise (Davies & Flemmer, 1995):

- Quality of merchandise: the presence of well-known brands, fresh products and the perception of the standard of goods in the shop.
- Merchandise range: Both the width (number of different merchandise categories) and depth (number of examples within a particular category) held in stock.
- Price of merchandise

This group of attributes was also evaluated on a 5-point scale as a function in choice of supermarket.

Prices

A specific block of prices was included to analyse two specific dimensions of a low price policy:

- The evaluation of the importance gave by consumers to the low price policy image given by offers and sales advertising. In this case, the Marketing Scales Handbook (Gordon, et.al., 1993) gave a scale of 7-point.
- Whether low prices represent consumers' strongest preference. In this case, consumers were asked to choose one of the following statements on price policy: "The store has to have a low price policy", "The store has to have quality rather than low price policy" or "I consider unimportant the pricing policy of the store".

Location

This block is different to the previous ones because consumers were not asked for their reasons for choosing a supermarket. Rather, the aim was to glean information requires for determining variables.

The definition of ρ_{ij} and d_{ij} involves the determination of the origin and destination of the trip. The destination in this case is clear because it is the supermarket where customers had just done their shopping. But the point of origin is more difficult to ascertain. Most researchers assume that people always travel from home when they go shopping. But nowadays, given demographic changes (e.g., working women), the trip origin may be either home or the workplace. Thus, the first question of this block tries to analyse shoppers' point of origin. Specifically, the question asks the consumers about their usual origin's pattern of the trip when they went shopping[16]: i.e., if the trip is always started from home, if the trip is almost always started from home and sometimes from his workplace, etc. This question is complemented with two demographic questions that asked for the exact addresses of workplace and home.

Traditionally, distance has been considered one of the basic reasons for patronising one supermarket. Then, in this thesis, distance was computed both in terms of physical distance and travel time distance from home and workplace. Questions 2 and 3 ask for the distances from both places. The reason for doing so is that car use has made distance less important and shifted the emphasis towards travel time.

The importance of transport mode is another element, which is established in Question 4. Here, multiple-choice question was used to ascertain which means of transport customers most commonly use to get to the supermarket.

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[16] Note that I have assumed that there is only two possible origin for the trip. The reason is that these two are the most important ones and the adding of more options could complicate the analysis.

D. DEMOGRAPHIC CHARACTERISTICS

Finally, at the end of the questionnaire, some personal questions are asked:

- Questions 1&2 ask about the age and the number of persons in charged.
- Questions 3&4 ask about the exact location of home and workplace. These answers together with the first question on location defined the origin of the trips.

Summing up, Table 4.1. shows the objective of each question.

Table 4.1. Aim of each question in the questionnaire.

Question	Objective
B. General questions of shopping behaviour	
1&2	Multi-supermarket “shopping”
3&4	Reasons for choosing a supermarket
C. Specific questions of supermarket’s attributes	
Location	
1	Origin of trip
2	Physical distance
3	Travel time distance
4	Mode of transport used
Convenience	Determine key attributes (A_{ki*j})
Customer service	Determine key attributes (A_{ki*j})
Merchandise	Determine key attributes (A_{ki*j})
Prices	Determine key attributes (A_{ki*j})
D. Demographic characteristics	
1.	Age (Classification question)
2	Person in charge (Classification question)
3	Home (determine origin of the trip)
4	Workplace (determine origin of the trip)

The survey for this thesis was conducted in Spain and Great Britain. The only differences between both samples were the supermarkets involved. The type of survey, the questionnaire and the sample design were the same. This was so because the aim was to analyse the differences between Spanish and British consumer store-choice behaviour.

The target population in Great Britain was British supermarket shoppers. The sampling frame was shoppers at two supermarkets in the Food Centre of the Central Milton Keynes Shopping Centre. The two supermarkets located in this area are *Sainsbury*[17] and *Waitrose*[18]. The target population in Spain was the Spanish supermarket shoppers. The sampling frame was shoppers at two supermarkets in the centre of Barcelona. These are *Bon Preu*[19] *and Caprabo*[20].

In principle, the financial constraints of this study determined a sample of 200 consumers in each country. However operational problems in the British survey resulted in a sample of 99 consumers. Thus, the Spanish sample size gives a level of accuracy (confidence level) of ± 7.1 % (for all variables), while the British sample yields a level of accuracy of ± 10 % (for all variables).

The sample procedure selected in this case is a simple random sampling one. Additionally in this case, I split the sample size into different hours and days. The reason was that I wanted to avoid a sample biased toward only one type of supermarket customer (e.g. weekly and weekend shoppers). I therefore decided to conduct 60% of the interviews on

[17] Sainsbury’s supermarket is one of the oldest major foods retailing chains in Great Britain (established in 1869). Its supermarkets serve over 9 million customers a week at 391 supermarkets throughout the UK. The objectives of this group are to provide unrivalled value with regard to the quality of its products, prices and in range of choice offered (www.j-sainsbury.co.uk).

[18] This food retailing chain was created by John Lewis Partnership in 1955. They currently have 115 branches. Waitrose aims to combine the convenience of a supermarket with the expertise and service of a specialist shop (www. waitrose.co.uk).

[19] *Bon Preu, S.A* is a new food retailing chain with 42 supermarkets (basically in Catalonia). Its slogan “more and closer” reflects its strategy.

Wednesdays (all day) as a guide to weekly shopping habits and 40% on Fridays (afternoon & night) to give a picture of weekend consumers.

After conducting the fieldwork, the Spanish sample followed the previous *a priori* distribution. However operational problems with the British survey prevented this *a priori* distribution being followed. It also proved impossible to follow the a priori daily distribution, although it was possible to split the British distribution by supermarket patronised (59 Sainsbury consumers and 40 Waitrose consumers).

4.1.2. Second step: Estimation of supermarket's key attributes

When consumers choose one supermarket to shop, they have to evaluate a large number of attributes. In the questionnaire of this thesis, consumers were asked to evaluate the relative importance of a large number of supermarket's attributes. At this stage, store-choice behaviour can be seen as a large multi-attribute problem. But, I need a more parsimonious description of the data to assess a general store-choice behaviour. How can I do it?

A theoretical approach for handling multi-attribute judgement problems with a large number of attributes is the Hierarchical Information Integration approach (Louviere, 1984). This approach is based on the assumption that it is a reasonable strategy for consumers to organise individual decision attributes into clusters or sets. Consumers then evaluate and aggregate some property of each of the sets to reach an overall judgement. Moreover, this approach suggests that one could use factor analysis to determine the sets of attributes, and then use these sets as the basis for the hierarchical task.

[20] *Caprabo* is the Catalan food retailing chain with the highest growth rate over the last years. It currently has 246 supermarkets (mainly in Catalonia). Its strategy is to focus on customer service (Agustina, 1999).

As the supermarket choice behavior can be seen as a large multiattribute problem (Louviere & Gaeth, 1987), I can use the assumptions of the previous theoretical approach. Using them, the attributes evaluated in the surveys can be categorised into specific factors using Factors analysis.

Moreover, it can be pointed out that a recent research (Hutcheson and Moutinho, 1998) have used factor analysis and regression analysis to estimate the relative importance of each of the factors selecting supermarkets and the way in which they interact to determine the level of customer satisfaction.

4.1.3. Third step: Specification of the MCI model

After finding the key supermarket factor attributes, the next step is the specification of the MCI model. This specification involves the substitution of the A_{kij} variables of the MCI model, by the factors found in the previous factor analysis and two key variables related to distance[21] (physical distance[22] and travel time distance[23]).

The MCI version used in this thesis is the original version of Nakanishi and Cooper (Nakanishi and Cooper, 1974) which formulation states that:

[21] The ordinary least square theory states that the omission of relevant variables in a regression analysis could lead to biased estimators (i.e., a biased estimator is one where the estimated value is different from the true one). Then, in this case, the simplest distance variables have been included in the MCI specification to achieve unbiased estimators (although the thesis' aim is the determination of the store's attributes excluding distance variables).

[22] Physical distance is computed as the Manhattan rectilinear distance (because the scenario is a city) from the exact address of the origin to the supermarket in the Spanish survey. Due to some operational problems in the British survey, the physical distance in the British case has been computed with the answer to the second question of the location block: *How far is the store from your home / your workplace?*

[23] Travel time distance has been computed, in both cases, with the answers to the third question of the location block: *How long does it takes to get to the store from your home / your workplace?*

$$r_{ij} = \left(\prod_{k=1}^{s} A_{kij}^{\ b_k} \right) \Big/ \sum_{j=1}^{m} \left(\prod_{k=1}^{s} A_{kij}^{\ b_k} \right) \tag{28}$$

Where, at this stage,

r_{ij} = The probability that consumers at location *i* will shop at shop *j*. (i.e., The proportion of capture that a shop in *j* will achieve by consumers' zone *i*)

A_{kij} = The *k*-th attribute describing shop *j* attracting consumers from site *i*; in this case:

- The attributes' factors found by Factor analysis
- And two distance variables (physical distance and travel time distance from consumers' zone *i* to shop *j*).

i, I = Index and set of consumers' zone.

j, J = Index and set of shops.

b_k = Parameters still not estimated, which reflect the sensitivity of consumers to the shop characteristics on the probability to shop at a particular shop.

An assumption of the original Nakanishi and Cooper MCI model formulation restricts the estimation of the attribute's effect (b_k) to a single parameter reflecting aggregate market response to all shops alternatives. The use of such market wide parameters allows one to assess how each variable affects patronage but does not permit analysis of these influences for an individual shop (Black, et.al., 1985).

Given this assumption, the Nakanishi and Cooper estimation is not useful in most real cases. The reason is that a firm employing the MCI model usually wants to estimate its individual sensitivity parameters. This is a different case to the one studied in this thesis. Here, the variables and the sensitive parameters, which reflect aggregate market response to all shop alternatives, have been estimated. Following the same approach, Jain and

Mahajan (1979) estimated the original Nakanishi and Cooper MCI model for the food-retailing sector of a large US north-eastern metropolitan area.

4.1.4. Fourth step: Calibration of the MCI model

After specification of the MCI model, it only remains to calibrate the model to each specific scenario. The calibration involves two things:

- The identification of the significant attributes in each case (i.e., which attributes are significant to explain the supermarket choice in each scenario).
- The estimation of the sensitivity parameters (b_k) of consumers to the relevant supermarket factor-attributes (i.e., which level of importance is given to each significant attribute).

Nakanishi and Cooper (1974) showed that the MCI equation could be calibrated by the ordinary least square method on the log-transformed centered form of the equation. They also demonstrated that these estimations could be unbiased and efficient when sampling errors were negligible and specification errors were uncorrelated.

In practical terms, firstly, the original MCI equation[24] (equation (29)) is transformed into its log-transformed-centered form (equation (30)). And then, the ordinary least square method is applied to equation (30) to obtain the parameters' estimators.

$$r_{ij} = \left(\prod_{k=1}^{s} A_{kij}^{\ b_k} \right) V_{ij}^{*} \Bigg/ \sum_{j=1}^{m} \left(\prod_{k=1}^{s} A_{kij}^{\ b_k} \right) V_{ij}^{*} \qquad (29)$$

[24] Note that a disturbance term has to be included when the parameters of the model were estimated.

$$\ln\left(\frac{p_{ij}}{\hat{p}_i}\right) = \sum_{k=1}^{s} \ln\left(\frac{A_{kij}}{\hat{A}_{ki}}\right) + \ln\left(\frac{V_{ij}^{*}}{\hat{V}_i^{*}}\right) \qquad (30)$$

where,

$\hat{p}_i = \left(\prod_{j=1}^{m} p_{ij}\right)^{1/m}$ = Geometric mean of the probabilities of consumers at zone *i* shopping at *m*-shops.

$\hat{A}_{ki} = \left(\prod_{j=1}^{m} A_{kij}\right)^{1/m}$ = Geometric mean of *k*-th attributes of *m* shops evaluated by consumers at zone *i*.

$\hat{V}_i = \left(\prod_{j=1}^{m} V_{ij}^{*}\right)^{1/m}$ = Geometric mean of the specification error terms of *m* retail facilities.

Although this estimation seems operationally simple, there is a computational problem for the analyst: if consumers from any zone *i* (*i = 1...n)* do not shop at a shop *j* (*j = 1... m),* the resulting p_{ij} and the geometric mean, $\hat{p}_i$, for the consumers' zone will be equal to zero. In such an event, the transformation of the ratio $p_{ij} / \hat{p}_i$ will not be possible for parameter estimation (Jain and Mahajan, 1979). The practical solution is the creation of consumers' zone; each of this consumers' zone has to have consumers patronising all supermarket alternatives. For example, if the scenario has two supermarkets, each consumers' zone has to have consumers that shop in supermarket 1 and consumers that shop in supermarket 2.

In this study, this computational problem has one practical consequence: each database has individual consumers as cases. This implies that before applying ordinary least squares on the log-transformed centered form of the MCI equation, consumers' zones need to be created. Specifically, the consumers' zones have been created in such a way that consumers of both supermarkets[25] belong to it[26].

4.2. Analyses of data

4.2.1. Preliminary analyses

Preliminary analyses were undertaken on the Spanish and British samples. The results are detailed below.

4.2.1.1. Demographic characteristics

First, some classification questions issues were analysed. Some conclusions can be reached based on these analyses:

- 70.5 % of the Spanish sample and 61% of the British sample are females.
- More than half of the sample is working in both samples: in Spain (62%) and in Great Britain (56%).
- Finally, table 4.2. shows the age profile of the two samples.

[25] Note that the scenarios analysed in this thesis have two supermarkets.

[26] Note that the evaluation of each supermarket attribute for each consumer zone has been computed as the average evaluation of the customers of this specific supermarket in this specific demand node.

Table 4.2. Age profile of the samples

Age range	**Spanish sample**	**British sample**
18-27	15 %	19.4 %
28-37	22 %	25.5 %
38-47	20.5 %	25.5 %
48-57	23 %	18.4 %
58-67	14 %	7.1 %
68-80	5.5 %	4.1 %
Average	*44*	*41*

4.2.1.2. General shopping behaviour

Before the detailed questions about the supermarket's attributes, several questions related to general shopping behaviour were done. This section describes the analyses of data from these questions

First, a question on multi-supermarket "shopping" was asked. The aim here was to discover to what extent consumer did their shopping at just one supermarket. An article of Hogan (1996) states that 67.8% of the consumers did most of the grocery shopping at one store. Nowadays, supermarkets offer a range of services and products that *a priori* cover all shopping needs. This hypothesis is confirmed in both samples because 92% of Spaniards and 73% of Britons did their shopping at just one supermarket.

Second, two general questions were asked regarding reasons for choosing a given supermarket. The first one asked the consumers which things were considered important when choosing a supermarket. This open-ended question allowed the consumer to express his opinion without being biased by responses to closed-ended questions. The open-ended question was coded afterwards using the answers given by the consumers and was analysed as a multiresponse question.

From these analyses, the Spanish survey highlighted the fact that location was the most important attribute in Spain (31.5%), followed by price policy (12.65%), quality of products (12.4 %) and customer service (9.1%). While, for British consumers, price policy was the most important attribute (20.2%), followed by quality of products (19.4%), range of products (16.3%) and location (14.8%). The responses to these questions are summarized in Table 4.3.

Table 4.3. Breakdown of main supermarket attributes

Attributes	Spain	Great Britain
Location	31.5 %	14.8 %
Price policy	12.6 %	20.2 %
Quality products	12.4 %	19.4 %
Customer services	9.1 %	4.2 %
Range products	8.6 %	16.3 %
Special offers & Promotions	8.5 %	3.4 %
Convenience & Hours of opening	7.4 %	9.9 %
Staff	6.5 %	0.8 %
Layout	1.2 %	3.8 %
Fresh products	0 %	3.4 %
Others	2.2 %	3.8 %

In the second general question, consumers were asked to rank the main supermarket attributes. In the Spanish survey, there were 8 dimensions to be ranked (from very important 1 to not important 8); while in the British survey, there were 9 dimensions to be ranked (from very important 1 to not important 9)[27]. To summarise these rankings, the mean and standard deviation of each attribute were computed (Table 4.4.).

Table 4.4. Ranking of supermarket attributes[28]

IMAGE DIMENSION	Spanish sample		British sample	
	Mean	***Standard Deviation***	***Mean***	***Standard Deviation***
Convenience	1.62	1.17	2.98	1.98
Quality products	3.68	1.65	2.71	1.59
Range products	4.07	1.47	3.10	1.49
Price products	4.26	2.02	3.46	1.94
Staff	4.28	1.70	5.41	1.41
Hours of opening	4.80	2.24	5.03	1.73
Customer Service	6.15	1.71	5.80	1.80
Customer Account	7.16	1.35	8.12	1.11
Financial Service (only UK)	-	-	8.40	0.85

From the previous Table 4.4., it can be pointed out that, in the Spanish sample, convenience (1.62) was the most important characteristic for customers whilst financial services (7.16) was the least one. The other range of characteristics fell between these extremes in the following order: quality of products (3.68), range of products (4.07), price of products (4.26), staff (4.28), hours of opening (4.30) and customer service (6.15). The standard deviation scores also provided some useful information on the pattern of responses. Relatively low deviation scores were observed for items such as convenience

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[27] The British questionnaire includes the dimension of financial service (defined as the services offered by supermarkets that had a bank). While, the Spanish survey does not include it because Spanish supermarkets do not yet offer this type of service.

[28] A c^2 test to the frequencies of these variables shows that all of them are significant.

and financial services, whereas higher deviations were observed for items such as price products and hours. This result was to be expected, since the mean perceived importance of items is likely to be dependent, at least to some extent, on how the perceived importance of items differentiates the sample. For example, convenience was importance to most, if not all, respondents and was rated similarly by everyone. In contrast to this, some items appeal more to specific subgroups of the sample and therefore attract different ratings of importance, which increase the standard deviation measure. An example of this is hours of opening, which is not likely to be an important consideration for all respondents.

In the British sample, quality products (2.71) proved the most important characteristic whilst financial service (8.40) was the least one. The other characteristics fell between these two extremes in the following order: convenience (2.98), range of products (3.10), price of products (3.46), hours of opening (5.03), staff (5.41), customer service (5.80) and customer accounts (8.12). In this case, low deviation scores were obtained for items such as financial service and customer accounts; whereas higher deviations were observed for items such as convenience and price of products.

4.2.2 Determination of key supermarket attributes

4.2.2.1. Spanish case

Factor analysis[29] was applied to the Spanish survey. Eight factors were identified. These factors represented 68 percent of the variance of the 21 variables[30]. This percentage was acceptable given that the criterion of satisfactory percentage of variance explained in social science is 60 % (Hair, et.al., 1998).

[29] In this case, factors were extracted with component analysis and using Varimax rotation.

[30] Note that 5 variables were extracted in the reespecification because their communalities were less than 0.5.

The interpretation of the rotated factor matrix was supported by the fact that the minimum significance level for the factor loading in a sample size of 200 is 0.4 (using table 3.2., page 112, Hair, et.al., 1998). In other words, in a sample of size 200, the variables with factor loadings greater than 0.4 are considered significant.

The label and the significant factor loading variables (i.e., the variables with a factor loading greater than 0.4) of each factor are the ones shown in Table 4.5.

Table 4.5. Factors for Spanish survey

Variable	Characteristic	Factor loading
Factor 1: Accessibility by modes of transport		
Parksp	It is easy to park at the store	0.862
Publictsp	Easy access by Public Transport	0.715
Dpetrolsp	Petrol discounts	0.846
Dparksp	Parking discounts	0.815
Factor 2: Checkout and shopping assistance service		
Fchecksp	Fast checkout	0.780
Echecksp	Express checkout counters	0.703
Sassistsp	Shopping assistances are courteous and knowledgeable	0.666
Factor 3: Store design and physical facilities		
Crowdsp	No crowded store	0.572
Emovesp	It is easy to move around the store	0.811
Fprodsp	It is easy to find products (readable labels)	0.777
Factor 4: Club card facilities		
Clubcsp	Supermarket Club Card	0.790
Creditsp	The store lets you buy on credit	0.789
Pbrandsp	Store has products of all well known brands and own label ones	0.421
Factor 5: Quality and range of the merchandise		
Prangesp	Store has all basic products and a variety of special items	0.553
Pfreshsp	Store has fresh products	0.713
Pqualsp	Store has high quality products	0.765
Factor 6: Low price policy image		
Offersp	The store does a lot of “promotional offers”	0.892
Advertsp	The store does a lot of advertising of sales	0.869
Factor 7: Wider opening hours		
Omiddaysp	The store is open at noon	0.874
Olatesp	The store is open until late at night	0.864
Factor 8: Location		
Locatedsp	It is well located	0.777

The final step of the Factor analysis was the selection of the surrogate variables[31] of each factor. These surrogate variables were the representatives of the factors found and the ones used in the next regression analysis. In the Spanish case, for example, the first factor of "accessibility by modes of transport" was represented by the variable *parksp*[32] (i.e., "It is easy to park at the store") because was the variable with the higher factor loading. All Spanish surrogate variables were the ones presented in Table 4.6.

Table 4.6. Surrogate variables of the Spanish survey

Factor	Surrogate variable	Description of the factor
Factor 1	*Parksp*	Accessibility by modes of transport
Factor 2	*Fchecksp*	Checkout and shopping assistance service
Factor 3	*Emovesp*	Store design and physical facilities
Factor 4	*Clubcsp*	Club card facilities
Factor 5	*Pqualsp*	Quality and range of merchandise
Factor 6	*Offersp*	Low price policy image
Factor 7	*Omiddaysp*	Wideness of opening hours
Factor 8	*Locatedsp*	Location

The previous surrogate variables that represented the factor-attributes found were the key supermarket's attributes (A_{kij}) that would be included in the Spanish MCI model. As I have explained, additionally, the physical and travel time distance[33] were also introduced

[31] As the objective was the identification of appropriate variables for a subsequent application of the regression technique, a form of data reduction was applied. Given that the aim of this thesis was the practical use of the model (i.e., its replication) to locate supermarkets, the data reduction technique chose in this case was the surrogate variables. Surrogate form of data reduction examines the factor matrix and selects the variables with the highest factor loading on each factor to act as a surrogate variable that is representative of that factor (Hair, et.al., 1998).

[32] Note that it is easy to use a single surrogate variable instead of a linear combination of variables (i.e., Factor Scores).

[33] Physical distance and travel time distance from consumers' zone *i* to supermarket *j* in the Spanish scenario are represented by *dhousesp* and *timehsp* variables, respectively.

in the specification of the MCI model. Using the Spanish surrogate variables found and the distance variables, the specified MCI model in the Spanish scenario is the following one:

$$p_{ij} = \frac{Parksp_{ij}^{b1} * Fchecksp_{ij}^{b2} * Emovesp_{ij}^{b3} * C\,\mathrm{lub}\,csp_{ij}^{b4} * Pqualsp_{ij}^{b5}}{\sum_{j=1}^{m}(Parksp_{ij}^{b1} * Fchecksp_{ij}^{b2} * Emovesp_{ij}^{b3} * C\,\mathrm{lub}\,csp_{ij}^{b4} * Pqualsp_{ij}^{b5}}$$

$$\frac{* Offersp_{ij}^{b6} * Omiddaysp_{ij}^{b7} * Locatedsp_{ij}^{b8} * Dhousesp_{ij}^{b9} * Timehsp_{ij}^{b10}}{* Offersp_{ij}^{b6} * Omiddaysp_{ij}^{b7} * Locatedsp_{ij}^{b8} * Dhousesp_{ij}^{b9} * Timehsp_{ij}^{b10})} \qquad (31)$$

4.2.2.2. British case

Factor analysis was applied to the British survey. Eight factors were identified. These factors represented 77 percent of the variance of the 19 variables[34]. This percentage was acceptable given the criterion of satisfactory percentage of variance explained in social science is 60 % (Hair, et.al., 1998).

The interpretation of the rotated factor matrix was supported by the fact that the minimum significance level for the factor loading in a sample size of 99 (≈ 100) is 0.55 (using table 3.2., page 112, Hair, et.al., 1998). In other words, in a sample of size near 100, the variables with factor loadings greater than 0.55 are considered significant.

The label and the significant factor loading's variables (i.e., the variables with a factor loading greater than 0.55) of each factor are the ones shown in Table 4.7.

[34] Note that 8 variables were extracted in the reespecification.

Table 4.7. Factors for British survey

Variable	**Characteristic**	**Factor loading**
Factor 1: Low price policy image		
Lowpuk	Store always has sufficient stock	0.623
Offeruk	Store has fresh products	0.918
Advertuk	Store has high quality products	0.924
Factor 2: Store design and physical facilities		
Crowduk	No crowded store	0.751
Emoveuk	It is easy to move around the store	0.882
Fproduk	It is easy to find products (readable labels)	0.729
Factor 3: Quality and range of merchandise		
Pstockuk	Store always has sufficient stock	0.692
Pfreshuk	Store has fresh products	0.864
Pqualuk	Store has high quality products	0.832
Factor 4: Checkout and shopping assistance service		
Fcheckuk	Fast checkout	0.793
Echeckuk	Express checkout counters	0.841
Sassistuk	Shopping assistance are courteous and knowledgeable	0.661
Factor 5: Facilities for non-car customers		
Parkuk	It is easy to park at the store	-0.733
Publictuk	Easy access by Public transport	0.839
Homeduk	Home delivery	0.704
Factor 6: Wider opening hours		
Osundayuk	The store is open on Sunday	0.862
Olateuk	The store is open until late at night	0.859
Factor 7: Location		
Locateduk	It is well located	0.826
Factor 8: Facilities for car customers		
Dpetroluk	Petrol discounts	0.877

From the previous table, it can be pointed out that the fact that, in this case, two factors were created to represent the importance of modes of transport. Factor 5 represents the non-car customers' facilities, while factor 8 represents car customers' facilities. The polarisation of the British society between the car users and non-car users were shown by these two factors; specifically, by factor 5. The reason is that factor 5 included non-car users' variables ("Easy access by public transport" and "home delivery") with positive factors loading and, more important, a car user variable ("it is easy to park at the store") with negative factor loading. In other words, non-car users gave importance to non-car facilities and, at the same time, they did not give any importance to car facilities.

The final step of the Factor analysis was the selection of the surrogate variables of each factor. In the British case, for example, the sixth factor of "wider opening hours" was represented by the variable *osundayuk* (i.e., "the store is open on Sunday") because was the variable with the higher factor loading. All British surrogate variables are presented in Table 4.8.

Table 4.8. Surrogate variables of the British survey

Factor	Surrogate variable	Description of the factor
Factor 1	*Advertuk*	Low price policy image
Factor 2	*Emoveuk*	Store design and physical facilities
Factor 3	*Pfreshuk*	Quality and range of merchandise
Factor 4	*Echeckuk*	Checkout and shopping assistance service
Factor 5	*Publictuk*	Facilities for non-car customers
Factor 6	*Osundayuk*	Wideness opening hours
Factor 7	*Locateduk*	Location
Factor 8	*Dpetroluk*	Facilities for car customers

The previous surrogate variables that represent the factor-attributes found were the key supermarket's attributes (A_{kij}) that would be included in the British MCI model. As I have explained, additionally, the physical and travel time distance[35] were also introduced in the specification of the MCI model. Using the surrogate variables found and the distance variables, the specified MCI model in the British scenario is the following one:

$$p_{ij} = \frac{Advertuk_{ij}^{b_1} * Emoveuk_{ij}^{b_2} * Pfreshuk_{ij}^{b_3} * Echeckuk_{ij}^{b_4} * Publictuk_{ij}^{b_5}}{\sum_{j=1}^{m} (Advertuk_{ij}^{b_1} * Emoveuk_{ij}^{b_2} * Pfreshuk_{ij}^{b_3} * Echeckuk_{ij}^{b_4} * Publictuk_{ij}^{b_5}}$$

$$\frac{* Osundayuk_{ij}^{b_6} * Locateduk_{ij}^{b_7} * Dpetroluk_{ij}^{b_8} * Dhouseuk_{ij}^{b_9} * Timehuk_{ij}^{b_{10}}}{* Osundayuk_{ij}^{b_6} * Locateduk_{ij}^{b_7} * Dpetroluk_{ij}^{b_8} * Dhouseuk_{ij}^{b_9} * Timehuk_{ij}^{b_{10}})} \qquad (32)$$

4.2.3. Calibration of the MCI model to estimate p_{ij} in each scenario

The calibration of the model identifies, firstly, which of the relevant supermarket's attributes identified by consumers (in the factor analysis) are discriminatory supermarket choice. The calibration, also, estimates the consumers' sensitivity parameters to the significant (i.e., discriminatory) supermarket attributes.

4.2.3.1. Spanish case

Firstly, the consumers' zone was created from individual consumer responses, using two assumptions:

- First, the variable *"timehsp"* coded as interval was transformed to a numeric variable.

[35] Physical distance and travel time distance from consumers' *i* to supermarket *j* in the British scenario are represented by *dhouseuk* and *timehuk* variables, respectively.

- Second, consumers that went shopping exclusively from their workplace were excluded from the analysis. Given that only 11.5 % came exclusively from home, 177 consumers forming the initial sample were used to create the consumer zones.

  The reason of this exclusion is the purpose of the MCI model. Its main application is its replication in different zones to predict the market share capture of each supermarket in each zone. The model is estimated with a representative sample, and after this, it is extrapolated to the whole population by means of a census. Usually, this population census reflects the population that lives in these specific zones but not the people working there.

In the Spanish case, 15 zones were created. The next step was the computation of the new A_{kij} and p_{ij} for the consumer zone using the individual A_{ki*j} and the number of consumers in each zone[36].

The last computational transformation before the ordinary least squares (OLS) estimation was the log-centered transformation of the MCI equation. In this case, this transformation was:

$$\ln\left(\frac{p_{ij}}{\hat{p}_i}\right) = b_1 \ln\left(\frac{\mathrm{locatedsp}_{\mathrm{ij}}}{\widehat{\mathrm{locatedsp}}_{\mathrm{i}}}\right) + b_2 \ln\left(\frac{\mathrm{parksp}_{\mathrm{ij}}}{\widehat{\mathrm{parksp}}_{\mathrm{i}}}\right) + b_3 \ln\left(\frac{\mathrm{emovesp}_{\mathrm{ij}}}{\widehat{\mathrm{emovesp}}_{\mathrm{i}}}\right) + b_4 \ln\left(\frac{\mathrm{omiddaysp}_{\mathrm{ij}}}{\widehat{\mathrm{omiddaysp}}_{\mathrm{i}}}\right)$$

$$+ b_5 \ln\left(\frac{\mathrm{clubcsp}_{\mathrm{ij}}}{\widehat{\mathrm{clubcsp}}_{\mathrm{i}}}\right) + b_6 \ln\left(\frac{\mathrm{fchecksp}_{\mathrm{ij}}}{\widehat{\mathrm{fchecksp}}_{\mathrm{i}}}\right) + b_7 \ln\left(\frac{\mathrm{pqualsp}_{\mathrm{ij}}}{\widehat{\mathrm{pqualsp}}_{\mathrm{i}}}\right) + b_8 \ln\left(\frac{\mathrm{offersp}_{\mathrm{ij}}}{\widehat{\mathrm{offersp}}_{\mathrm{i}}}\right)$$

$$+ b_9 \ln\left(\frac{\mathrm{dhousesp}_{\mathrm{ij}}}{\widehat{\mathrm{dhousesp}}_{\mathrm{i}}}\right) + b_{10} \ln\left(\frac{\mathrm{timehsp}_{\mathrm{ij}}}{\widehat{\mathrm{timehsp}}_{\mathrm{i}}}\right) + \ln\left(\frac{V_{ij}^*}{\hat{V}_i}\right) \tag{33}$$

[36] Note that the A_{ki*j} used are the eight ones identified by the Factor analysis plus the physical and travel time distance.

Finally, the ordinary least squares were applied to the log-centered transformation form of the MCI[37]. The regression estimation for the Spanish survey states that:

$$\ln\left(\frac{p_{ij}}{\hat{p}_i}\right) = -2.989\ln\left(\frac{\mathrm{dhousesp_{ij}}}{\hat{\mathrm{dhousesp}}_{\mathrm{i}}}\right) + 0.858\ln\left(\frac{\mathrm{parksp_{ij}}}{\hat{\mathrm{parksp}}_{\mathrm{i}}}\right) + 1.645\ln\left(\frac{\mathrm{offersp_{ij}}}{\hat{\mathrm{offersp}}_{\mathrm{i}}}\right) \qquad (34)$$

The previous equation is the log-centered transformed form of the estimated Spanish MCI model. Using the parameters estimated in equation (34), the original MCI model for the Spanish scenario states that:

$$p_{ij} = \frac{dhousesp_{ij}^{-2.989} * parksp_{ij}^{0.858} * offersp_{ij}^{1.645}}{\sum_{j=1}^{m}\left[dhousesp_{ij}^{-2.989} * parksp_{ij}^{0.858} * offersp_{ij}^{1.645}\right]} \qquad (35)$$

where,

p_{ij} = The probability that a consumer at zone *i* will shop at shop *j*.

$dhousesp_{ij}$ = Physical distance from demand node *i* to the supermarket *j*.

$parksp_{ij}$ = Valuation by zone *i*'s consumers to "the accessibility by modes of transport" to the supermarket *j* (on a 5-point scale).

$offersp_{ij}$ = Valuation by zone *i*'s consumers to "the low price policy image" of supermarket *j* (on a 7-point scale).

Summing up, the calibration of the Spanish MCI model have identified:

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[37] The OLS procedure was applied using stepwise estimation. After the estimation, the statistical significance determines a R-square of 0.881 and an adjusted R-square of 0.868. Moreover, the t-tests of all three variables, except the constant, prove that all coefficients were significantly different from zero for a significant of 95%. Finally, an analysis of the residuals confirmed that the previous estimations were correct.

- *The discriminatory attributes to the Spanish scenario*

  Equation (35) shows that the probability of patronising the two Spanish supermarkets depends on three variables: "the physical distance from consumer's zone to the supermarket" (i.e., variable *dhousesp*), "the accessibility by modes of transport to the supermarkets" (i.e., variable *parksp*) and "the low price policy image" (i.e., variable *offersp*). In other words, the choice between both Spanish supermarkets depends only on these three attributes, because both supermarkets were very similar in the other relevant attributes.

- *The consumers' sensitivity parameters to the discriminatory supermarket attributes*

  In this case, the estimated parameters were –2.989 for the variables *dhousesp,* 0.858 for the variable *parksp* and 1.645 for the variable *offersp.* A positive sign of the sensitivity parameters indicates that a supermarket with higher levels of that attribute would have a higher probability of being patronised; while, a negative sign indicated that a supermarket with a higher level of that attribute would have a lower probability of being patronised. In this case, the supermarket with higher valuations of "accessibility by modes of transport" or "low price policy image" would achieve a higher capture of consumers (i.e., a higher probability (p_{ij})); while the further supermarket from consumers' zone would have a lower probability of being patronised.

4.2.3.2. British case

Firstly, the consumers' zone was created from individual consumer responses, using two assumptions:

- First, the variables *timehuk* and *dhouseuk* coded as interval were transformed to numeric variables.

- Second, consumers that went shopping exclusively from their workplace were excluded from the analysis[38].

Given the operational problems in the British survey, not all the interviewees did their usual “shopping” in the supermarket patronised in the survey. As the thesis’ aim was the analysis of the consumers’ supermarket choice in its usual “shopping”, the cases that did not comply with this condition were excluded[39]. Finally, a sample of 62 consumers was determined after the exclusion of the cases that did not comply with any of the previous conditions.

In the British case, 6 zones were created[40]. The next step was the computation of the new A_{kij} and p_{ij} for the consumers zone using the individual A_{ki^*j} and the number of consumers in each zone[41].

The last computational transformation before the ordinary least squares (OLS) estimation was the log-centered transformation of the MCI equation. In this case, this transformation was:

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[38] The justification to do this can be found in section 4.2.3.1.

[39] Note that, in the Spanish survey, all the interviewees were usual customers of the Spanish supermarkets. The reason was that, in this case, the interviewer confirmed that the interviewee did their usual shopping in that supermarket before begin the interview.

[40] The operational problems of the British sample did not allow knowing the exact address of the interviewees. Then, the zones were created using the zones described by the variable *dhouseuk* (i.e., “physical distance from consumer home to the supermarket”). The six zones created correspond to the six intervals defined in that variable (i.e., zone 1 includes consumers living within a radius of less than 2 kilometres round the two side-by-side supermarkets of the Food Centre).

[41] Note that the A_{ki^*j} used are the eight ones identified by the Factor analysis plus the physical and travel time distance.

$$\ln\left(\frac{p_{ij}}{\hat{p}_i}\right) = b_1 \ln\left(\frac{\mathrm{Advertuk_{ij}}}{\hat{\mathrm{Advertuk}}_{\mathrm{i}}}\right) + b_2 \ln\left(\frac{\mathrm{emoveuk_{ij}}}{\hat{\mathrm{emoveuk}}_{\mathrm{i}}}\right) + b_3 \ln\left(\frac{\mathrm{Pfreshuk_{ij}}}{\hat{\mathrm{Pfreshuk}}_{\mathrm{i}}}\right) + b_4 \ln\left(\frac{\mathrm{echeckuk_{ij}}}{\hat{\mathrm{echeckuk}}_{\mathrm{i}}}\right)$$

$$+ b_5 \ln\left(\frac{\mathrm{Publictuk_{ij}}}{\hat{\mathrm{Publictuk}}_{\mathrm{i}}}\right) + b_6 \ln\left(\frac{\mathrm{Osundayuk_{ij}}}{\hat{\mathrm{Osundayuk}}_{\mathrm{i}}}\right) + b_7 \ln\left(\frac{\mathrm{Locateduk_{ij}}}{\hat{\mathrm{Locateduk}}_{\mathrm{i}}}\right) + b_8 \ln\left(\frac{\mathrm{dpetroluk_{ij}}}{\hat{\mathrm{dpetroluk}}_{\mathrm{i}}}\right)$$

$$+ b_9 \ln\left(\frac{\mathrm{dhouseuk_{ij}}}{\hat{\mathrm{dhouseuk}}_{\mathrm{i}}}\right) + b_{10} \ln\left(\frac{\mathrm{timehuk_{ij}}}{\hat{\mathrm{timehuk}}_{\mathrm{i}}}\right) + \ln\left(\frac{V_{ij}^*}{\hat{V}_i}\right) \qquad (36)$$

As can be expected, all values of the variable "Dhouseuk" were zeros because the consumers' zones were created using the codes (i.e., the intervals) described by this variable. Then, this variable was excluded from equation (36) to be able to apply ordinary least squares efficiently (i.e., to find unbiased and efficient estimators).

Finally, the ordinary least squares were applied to the log-centered transformation form of the MCI[42]. The regression estimation for the Spanish survey states that:

$$\ln\left(\frac{p_{ij}}{\hat{p}_i}\right) = 2.163 \ln\left(\frac{\mathrm{Advertuk_{ij}}}{\hat{\mathrm{Advertuk}}_{\mathrm{i}}}\right) + 1.650 \ln\left(\frac{\mathrm{Pfreshuk_{ij}}}{\hat{\mathrm{Pfreshuk}}_{\mathrm{i}}}\right) \qquad (37)$$

The previous equation is the log-centered transformed form of the estimated British MCI model. Using the parameters estimated in equation (37), the original MCI model for the British scenario states that:

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[42] The OLS procedure was applied using stepwise estimation. After the estimation, the statistical significance determines a R-square of 0.864 and an adjusted R-square of 0.691. Moreover, the t-tests of both variables, except the constant, prove that all coefficients were significantly different from zero for a significant of 95%. Finally, an analysis of the residuals confirmed that the previous estimations were correct.

$$p_{ij} = \frac{advertuk_{ij}^{2.163} * pfreshuk_{ij}^{1.650}}{\sum_{j=1}^{m}\left[advertuk_{ij}^{2.163} * pfreshuk_{ij}^{1.650}\right]} \qquad (38)$$

where,

p_{ij} = The probability that a consumer at zone *i* will shop at shop *j*.

$Advertuk_{ij}$ = Valuation by zone *i*'s consumers to "low price policy image" of supermarket *j* (on a 7-point scale).

$Pfreshuk_{ij}$ = Valuation by zone *i*'s consumers to the "quality and range of merchandise" of supermarket *j* (on a 5-point scale).

Summing up, the calibration of the British MCI model have identified:

- *The discriminatory attributes to the British scenario*

  Equation (38) shows that the probability of patronising the two British supermarkets depends on two variables: "the low price policy image" (i.e., variable *advertuk*) and "quality and range of merchandise" (i.e., variable *pfreshuk*). In other words, the choice between both British supermarkets depends only on these two attributes, because both supermarkets were very similar in the other relevant attributes. For example, in this case, distance (i.e., travel time distance) was not significant to explain the supermarket choice because these two supermarkets are located side by side in the Food Centre of the Central Milton Keynes Shopping Centre.

- *The consumers' sensitivity parameters to the discriminatory supermarket attributes*

  In this case, these parameters were 2.163 for the variable *advertuk* and 1.650 for the variable *pfreshuk.* Here, the supermarket with higher valuation of "the low price policy image" or "quality and range of merchandise" would achieve a higher capture of consumers (i.e., a higher probability (p_{ij})).

4.2.4. A Discrete Competitive Location model for the supermarket sector

After applying the methodology, what we get is a new version of the maximum capture model for the specific supermarket sector which takes account of revealed consumer store-choice behaviour.

The maximum capture model (MAXCAP) resulted from this specific application of the methodology is the following one[43]:

$$\text{MAX } Z = \sum_{i \in I} \sum_{j \in J} a_i \ \rho_{ij} \ x_{ij} \tag{39}$$

Subject to

$$\sum_{j \in J} x_{ij} = p + q \ , \ \forall i \in I \tag{40}$$

$$x_{ij} \leq x_{jj} \, , \ \forall i \in I \, , \ \forall j \in J \tag{41}$$

$$\sum_{j \in J} x_{jj} = p \tag{42}$$

$$x_{ij} = \{0,1\} \ x_{jj} = \{0,1\} \ \forall i \in I \, , \forall j \in J \tag{43}$$

This formulation is similar to the one in the P-median problem (the one presented in epigraph 3.1.1.), except in two things:

- We have reformulate constraint (3): $\sum_{j \in J} x_{ij} = 1 \quad \forall i \in I$, the one that forces each consumers' zone *i* to assign to only one shop. Instead, we use constraint set (40) which states that every consumer zone makes $p + q$ assignments to the p new and q existing supermarket shops.

[43] The notation is the same to the one used in chapter 3.

- The parameter ρ$_{ij}$ is defined using the results found in the previous calibration of the Multiplicative Competitive Interaction model. Using this consumer store-choice model to define ρ$_{ij}$, the new version of MAXCAP model takes into account how consumers choose among alternative shopping opportunities.

  The calibration of the parameters of the ρ$_{ij}$ was performed separately for each country's database. Next, we present the two ρ$_{ij}$ (Spanish and British) values for use in the new MAXCAP model. The use of each will depend on the country where the model is applied.

  The Spanish $\boldsymbol{r_{ij}}$ resulted from the previous analysis states that:

$$p_{ij} = \frac{dhousesp_{ij}^{-2.989} * parksp_{ij}^{0.858} * offersp_{ij}^{1.645}}{\sum_{j=1}^{m}\left[dhousesp_{ij}^{-2.989} * parksp_{ij}^{0.858} * offersp_{ij}^{1.645}\right]} \qquad (44)$$

where,

p_{ij} = The probability that a consumer at zone *i* will shop at shop *j*.

$dhousesp_{ij}$ = Physical distance form demand node *i* to the supermarket *j*.

$parksp_{ij}$ = Valuation by zone *i*'s consumers to the accessibility by modes of transport to the supermarket *j* (on a 5-point scale).

$offersp_{ij}$ = Valuation by zone *i*'s consumers to the low price policy image of supermarket *j* (on a 7-point scale).

The British $\boldsymbol{r_{ij}}$ resulted from the previous analysis states that:

$$p_{ij} = \frac{advertuk_{ij}^{2.163} * pfreshuk_{ij}^{1.650}}{\sum_{j=1}^{m}\left[advertuk_{ij}^{2.163} * pfreshuk_{ij}^{1.650}\right]} \qquad (45)$$

where,

p_{ij} = The probability that a consumer at zone *i* will shop at shop *j*.

$Advertuk_{ij}$ = Valuation by zone *i*'s consumers to "low price policy image" of supermarket *j* (on a 7-point scale).

$Pfreshuk_{ij}$ = Valuation by zone *i*'s consumers to the "quality and range of merchandise" of supermarket *j* (on a 5-point scale).

4.3. A simple application of the new methodology.

In this section, I am going to present briefly the flow of a simple application of the new methodology presented in this chapter.

We can consider a scenario represented by a network. This network represents a small town in Great Britain and each node represent a consumer zone (i.e., neighbourhood zones). In this little town, there are several supermarkets located. A new supermarket chain wants to locate a store in that little town. The entering Supermarket Company decides to apply the MAXCAP methodology to find the optimal location of the new store. To do this, the company applied the following stages:

First Stage: Development of a survey of supermarket - choice behaviour

The first stage would be the development of a survey in this British town. In this survey, consumers would be asked to make judgements on the importance of various supermarket's attributes when choosing where to do their shopping.

To simplify the analysis, we could use the list of general attributes found in this thesis for the British case. In this case, the attributes evaluated in the survey do not need to be categorised into factors using Factor analysis because general attributes have been used from the beginning. In this case, the structure of the questionnaire would be the one presented in Figure 4.2.

Figure 4.2. Questionnaire Design

Consumers' zone of Origin: ____________________

Supermarket Patronised: _______________________

1. *What level of importance do you give to the following supermarket's attributes in choosing a store to do your "shopping"?*

Not Important	*Below Average Importance*	*Average Importance*	*Above Average Importance*	*Very Important*
1	2	3	4	5

1. Location	1	2	3	4	5
2. Low price policy image	1	2	3	4	5
3. Wideness of opening hours	1	2	3	4	5
4. Checkout and shopping assistance service	1	2	3	4	5
5. Store design and physical facilities	1	2	3	4	5
6. Quality and range of merchandise	1	2	3	4	5
7. Facilities for car customers	1	2	3	4	5
8. Facilities for non-car customers	1	2	3	4	5

2. *How far is the store from your home?* ____________________ (in kilometres)

3. *How long does it takes to get to the store from your home?* __________ (in minutes)

Second Stage: Calibration of the MCI model in this British scenario to determine the parameter r_{ij}

The calibration of the MCI involves:

- The computation of the new A_{kij} and p_{ij} for the consumers' zones using the individual A_{kij} and the number of consumers in each consumers' zones.
- The transformation of the MCI equation in its log-centered transformed form
- Finally, the application of the ordinary least square method to the previous log-centered transformed form of the MCI equation.

The calibration of the MCI model would give the estimated MCI for this small town scenario. Specifically, the calibration would identify which attributes are discriminatory in the choice between the supermarkets in that British town. Moreover, the calibration would also estimate the level of importance (i.e., sensitivity parameters) given by consumers to each of the previous discriminatory attributes.

Third Stage: Resolution of the MAXCAP model

Using the r_{ij} found in the previous stage, the new MAXCAP model can be solved[44].

The resolution of the MAXCAP model would give the optimal location for the new supermarket. Moreover, we could assume different levels of the significant key attributes for the new supermarket and find, in each case, the optimal location.

4.4. Limitations of the analyses

The main limitations of the analysis are the ones identified for revealed preference methods (Craig, et.al., 1984) and, specifically, for the MCI model used in this thesis. Now, I shall discuss these theoretical problems and their applicability to this real case.

- This model assumes consumer utility function to be compensatory. But really consumers reject stores beyond a certain distance and may also reject stores unless they meet threshold levels of other attributes.

  This problem does not apply here because the supermarket alternatives in both scenarios have a minimum level of all key attributes. Additionally, the supermarkets in both cases were closely enough to be alternative choices for all the consumers in the sample.

[44] The methodologies to solve this type of models are presented in chapter 6.

- The model is context dependent; i.e. the estimated parameters associated with characteristics on which the existing stores do not differ much would be low. This does not, however, imply that such characteristics are unimportant to consumers but rather that other variables are used to discriminate among them. This limitation applies to both scenarios. In the Spanish case, the ranking of supermarket attributes (identified in Section 4.2.1.2.) was convenience (location and access by transport mode), quality products, range of products and price products. Despite this ranking, the key discriminating variables between both Spanish supermarkets were convenience (distance and accessibility by transport mode) and price policy. This means that both supermarkets are very similar in terms of product quality and range. In a similar way, distance was not significant to explain the British supermarket choice because the two British supermarkets were located side by side in the Food Centre of the Central Milton Keynes Shopping Centre.
- The distance decay parameter (β_d) is highly dependent on the characteristics of the spatial structure.

  This limitation is also applicable to this study. Although both surveys were designed to be as similar as possible, it was not possible to overcome the issue of different spatial structure in both countries.

  - The Spanish scenario is the centre of Barcelona. Barcelona is a traditional Mediterranean city where supermarkets and grocery shops are located throughput the city.
  - The British scenario is the centre of Milton Keynes. Milton Keynes is a big residential area. Basically, its roundabouts and American style road network were designed to ensure that any part of the city would be within 15 minutes drive time. In terms of supermarkets, the city has a big shopping centre in the

middle of the city (called the Central Milton Keynes Shopping centre) and several small malls on the city outskirts. The two supermarkets used in the British survey are located side by side in the Central Shopping Centre.

Finally, there is a statistical limitation of the analysis. This is due to:

- The sample size: the Spanish survey had a sample size of 200 questionnaires, which gave a level of accuracy (confidence level) of ± 7.1%. The British survey had a sample size of 99 questionnaires, which gave a level of accuracy (confidence level) of ± 10%.
- The Spanish sample was distributed *a priori* as a function of the day of the week and the supermarket involved. The distribution chosen tries to avoid bias in choosing only one type of supermarket shopper (i.e., weekly or weekend one's). The British survey posed a problem in this respect. Operational difficulties meant the British survey could not be split as the basis of this *a priori* distribution. Likewise, we were able to establish the daily distribution of the sample afterwards. The British sample may therefore be biased toward one type of supermarket consumer.

Chapter 5

A New Chance – Constrained Maximum Capture Location Problem

After the introduction of Consumer Choice theories in the Discrete Competitive Location models, these models could be improved with an additional element. Up till now, the concept of market threshold has not been used so much in Discrete Facility Location Decision models. The threshold concept is particularly relevant to retail location, as it is widely recognized in the retail literature that states “there is a minimum size of a market below which a place will be unable to supply a central good … and is here termed the threshold sales level for the provision of that good from the center” (Berry and Garrison, 1958, p.111, as cited by Shonkwiler and Harris, 1996). In this case, a new model based on the basic Maximum Capture Model (MAXCAP) is presented. The new model, named Chance – Constrained Maximum Capture Problem, introduces two modifications:

- Firstly, the capture is determined by the gravity model proposed by HUFF (1964)[45], and not just based on proximity.
- Secondly, and new, stochastic threshold constraint is introduced. A facility can be open if the probability that the total demand assigned to that outlet was above the threshold level, is at least a desired probability.

5.1. Literature Review of Competitive Location models and Demand Entry Threshold

Several authors have recognised that there is a **demand entry threshold** and have introduced this concept in the facility location decision models in different ways.

Balakrishnan and Storbeck (1991) presents the McTHRESH model. This model addressed the issue of locating a given number of outlets so that market coverage was maximised within some predetermined range and the required threshold level of demands were maintained for all sites. In 1994, Current and Storbeck (1994) formulated a multiobjective model that selected franchise locations and identified individual franchise market areas. Constraints in their formulation guarantee that all francise locations were assigned at least a minimal threshold market area with sufficient demand to ensure economic survival.

Recently, Serra, ReVelle and Rosing (1999) presented a decision model for a firm that wished to enter a competitive market where several competitors were already located. The market was such that for each outlet there was a demand threshold level that had to be achieved in order to survive. In this model, the threshold constraint was deterministic and each facility must meet the threshold.

[45] A gravity model is introduced to take into account the first part of the research, but I consider the simplest gravity model to focus the analysis on the stochastic threshold constraint.

Finally, Drezner and Drezner (2002b) presented a location model based on the threshold concept. They assumed that the buying power at each community over the planning horizon was distributed according to some statistical distribution. Assuming that there was a minimum market share threshold to be captured, they introduced the threshold in the objective function. Their location objective become the minimisation of the probability of falling short of the required threshold.

In this chapter, I present a decision model for a retail firm with a stochastic threshold, but as a constraint.

5.2. The model

The basic model states that a new firm (from now on Firm A) wants to enter with *p* facilities in a market in order to obtain the maximum capture, given that it has to compete with *q* existing outlets[46], and subject to a threshold constraint that is stochastic.

This model studies the location of retail facilities in discrete space. The model takes the following assumptions:

- The spatial market is defined by a connected graph. At each vertex of the graph, there is a local market with a given number of consumers that generates a demand for the product.
- Potential locations for the services are also pre-specified (note that all outlets are allowed to locate only at the vertices of the graph).
- The customer wants to buy a unit of a specific product; i.e. we do not take into account multipurpose shopping behaviour.

[46] These competitors can belong to one or more firms, but without loss of generality it is assumed that there is only one competing firm (Firm B) operating in the market; as was assumed by ReVelle (1986).

- Demand is totally inelastic.
- The product sold is homogeneous, in the sense that the customer goes to buy the same product at all the outlets.
- Price is set exogenously and consumers bear transportation costs.
- Unit costs are the same in all stores regardless of ownership.
- Both firms are profit maximizing.
- The threshold level is defined as the minimum expected amount of demand necessary to cover costs or as the minimum number of customers required[47].
- Under equal conditions, the existing firm captures the demand, following Hakimi assumption (1986).
- The demand of each node is drawn from a multivariate normal distribution (i.e., the demand of each node a_i is normally distributed with mean m_i and standard deviation s_i). Note that according to the central limit theorem, it is not essential for these distribution to be normal if there are more that 30 nodes.
- The distributions of demand of two nodes are positively correlated or either uncorrelated (Drezner & Drezner, 2002b).
- The simple **gravity model** is used to define the capture. According to these models, "the probability that a consumer patronises a shop (or the proportion of demand capture form a node by one shop) is proportional to its attractiveness and inversely proportional to a power of distance to it" (Reilly, 1929). In this paper, the simple HUFF model is used[48] (Huff, 1964).

[47] Demand thresholds are usually measured in terms of population required to support one firm (Shonkwiler and Harris, 1996).

[48] The Huff probability formulation uses distance (or travel time) from consumer's zones to retail centers and the size of retail centers as inputs to find the probability of consumers shopping at a given retail outlet. He was also the first one to introduce the Luce axiom of discrete choice[48] in the gravity model. Using this axiom,

The integer programming formulation of the Chance – Constrained Maximum Capture Location problem is as follows:

$$\text{MAX } Z = \sum_{i \in I} \sum_{j \in J} a_i \ \rho_{ij} \ x_{ij} \qquad (46)$$

Subject to

$$\sum_{j \in J} x_{ij} = p + q \qquad \forall i \in I \qquad (47)$$

$$x_{ij} \le x_{jj} \qquad \forall i \in I, \forall j \in J \qquad (48)$$

$$P\left(\sum_{i \in I} a_i \, r_{ij} x_{ij} \ge T \right) \ge a \qquad \forall j \in J \qquad (49)$$

$$\sum_{j \in J} x_{jj} = p \qquad (50)$$

$$x_{ij} = \{0,1\} \quad x_{jj} = \{0,1\} \qquad \forall i \in I, \forall j \in J \qquad (51)$$

where the parameters are:

$i, I =$ Index and set of consumers' zones or nodes .

$j, J =$ Index and set of potential locations for entering firm.

J[B] *(Î J)* = The set of actual locations of the q outlets of the existing firm.

$p =$ Number of facilities to locate

$d_{ij} =$ The network distances between consumers' zone *i* and a shop in *j*.

r_{ij} = The probability that consumers at location *i* will shop at shop *j*. (i.e., The proportion of capture that a shop in *j* will achieve by consumers' zone *i*), based on HUFF model

consumers may visit more than one store and the probability of visiting a particular store is equal to the ratio of the utility of that store to the sum of utilities of all stores considered by the consumers.

$$r_{ij} = \frac{A_j / d_{ij}^b}{\sum_{j \in J^B} A_j / d_{ij}^b + \sum_{j \in J} A_j / d_{ij}^b * x_{jj}}$$

where A_j = The attractiveness of shop *j* (as in HUFF, the size of the shop)

$b =$ Distance decay parameter (as in HUFF, is equal to 2)

$T =$ Threshold demand level

a = Desired probability of satisfying the threshold level

a_i = Demand at consumers' zone *i*.

m_i = Mean of a_i

s_i = Standard deviation of a_i

And the variables are defined as follows:

x_{ij} = 1, if consumers' zone *i* is assigned to node *j*; 0, otherwise.

x_{jj} = 1, if a shop of firm's A is opened at node *j*; 0, otherwise.

The constraint set basically that: constraint set (47) states that every consumer zone makes *p* + *q* assignments to the *p* new and *q* existing outlets. But for a demand node *i* to be assigned to a facility at *j*, there has to be a facility open at *j*; this is achieved by constraint set (48). Constraints set (49) allows a facility to open at *j* only if the probability that the total demand assigned to node *j* was above than the threshold level, is at least the desired probability of satisfying this required threshold level. Constraint (50) sets the number of outlets to be opened by the entering firm and constraint (51) is the integrality constraint of the decision variables.

The objective function defines the total capture that the entering firm can achieve with the sitting of its p servers.

A deepest analysis of the deterministic equivalent of constraint set (49) states that:

For one constraint j, the mean of $\sum_{i\in I} a_i r_{ij} x_{ij}$ is equal to $\sum_{i\in I} m_i r_{ij} x_{ij}$ and the standard deviation

S_j is $\left[\bar{z}_j^T \, \mathrm{V} \, \bar{z}_j\right]^{1/2}$, where,

$$V = \begin{bmatrix} s_1^2 & \cdots & r_{1m} s_1 s_m \\ \vdots & \ddots & \vdots \\ r_{m1} s_m s_1 & \cdots & s_m^2 \end{bmatrix} \quad \text{and} \quad \bar{z}_j = \left(r_{1j} x_{1j}, \ \cdots \ , r_{mj} x_{mj}\right)$$

We have that $P\left(\sum_{i\in I} a_i r_{ij} x_{ij} \geq T\right) \geq a$ which is equivalent to $P\left(\sum_{i\in I} a_i r_{ij} x_{ij} \leq T\right) \leq 1 - a$

$$P\left(\frac{\sum_{i\in I} a_i r_{ij} x_{ij} - \sum_{i\in I} m_i r_{ij} x_{ij}}{S_j} \leq \frac{T - \sum_{i\in I} m_i r_{ij} x_{ij}}{S_j}\right) \leq 1 - a$$

$$P\left(Z \leq \frac{T - \sum_{i\in I} m_i r_{ij} x_{ij}}{S_j}\right) \leq 1 - a$$

$$F\left(\frac{T - \sum_{i\in I} m_i r_{ij} x_{ij}}{S_j}\right) \leq 1 - a$$

$$\frac{T - \sum_{i\in I} m_i r_{ij} x_{ij}}{S_j} \leq F_Z^{-1}(1 - a)$$

$$\frac{T - \sum_{i \in I} m_i r_{ij} x_{ij}}{S_j} \le K_{1-a}$$

$$\sum_{i \in I} m_i r_{ij} x_{ij} \ge T - K_{1-a} S_j$$

$$\sum_{i \in I} m_i r_{ij} x_{ij} + K_{1-a} S_j \ge T$$

Finally, we look at the covariance matrix S_j

- If a_i are completely dependent, then $V = \begin{bmatrix} s_1^2 & \cdots & s_1 s_m \\ \vdots & \ddots & \vdots \\ s_m s_1 & \cdots & s_m^2 \end{bmatrix}$, therefore

  $\left[\bar{z}_j^T V \bar{z}_j\right] = \left(\sum_{i \in I} r_{ij}^2 x_{ij}^2 \left[\sum_{k \in I} s_i s_k\right]\right)$ and $S_j = \left(\sum_{i \in I} r_{ij}^2 x_{ij}^2 \left[\sum_{k \in I} s_i s_k\right]\right)^{1/2}$. Then, the constraint becomes

  $$\sum_{i \in I} m_i x_{ij} + K_{1-a} \left(\sum_{i \in I} r_{ij}^2 x_{ij}^2 \left[\sum_{k \in I} s_i s_k\right]\right)^{1/2} \ge T$$ which *is not linear*.

- If a_i are completely independent, then $V = \begin{bmatrix} s_1^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & s_m^2 \end{bmatrix}$, therefore

  $\left[\bar{z}_j^T V \bar{z}_j\right] = \left(\sum_{i \in I} s_i^2 r_{ij}^2 x_{ij}^2\right)$ and then, the constraint becomes:

  $$\sum_{i \in I} m_i x_{ij} + K_{1-a} \left(\sum_{i \in I} s_i^2 r_{ij}^2 x_{ij}^2\right)^{1/2} \ge T$$ which *is not linear*.

*Then, constraint set (49) in general is a non-linear constraint***.** This non-linearity of the constraint set (49) don't allow to solve the model using the traditional methods of linear programming and branch and bound. Then, a metaheuristic model is used to solve it. The metaheuristic used to solve this model is presented in chapter 6.

5.3. An application to an example

The model was tested in the well-known Swain's (1974) 55-node network (figure 3.1.). The demand at each node follows a multivariate normal distribution, considering: m_i = the original demand of the Swain's network indicated in table 3.7., $s_i^{\,2} = \frac{m_i}{4}\left(uniform\ (0-1)\right)$ and r = 0. In this case, the total amount of demand to be captured is not always equal to 3575.

We need to pre-establish the value of the attractiveness of each shop. In this case, I assume that all the shops have the same attractiveness, ($A_i = 100$), regardless of node and ownership.

It is also assumed that there are five existing outlets. For each generated network, the location of the five existing outlets were found using the Teitz and Bart heuristic with a weighted total distance objective (i.e., minimised weighted by the population / demand of each node).

For the example, different scenarios were examined; which varies with respect to the number of outlets to be located by Firm A (p = 2, 3 and 4), and to the threshold level C: $C = b\left[{}^{pop}\!\big/_{(p+q)}\right]$ (where pop is the total amount of demand to be served; i.e., $pop = \sum_{i \in I} m_i$ and b is a threshold factor that was set to 0.3, 0.5 and 0.7).

The model was solved to optimality by using complete enumeration. Results are shown in tables 5.1., 5.2. and 5.3. In these tables, the locations and percentage[49] of demand captured by Firm B are computed before and after the entering of Firm A locates its outlets (using as objective function the one of the Chance – Constrained Maximum Capture Location Problem). Firm's A optimal locations and its percentage of demand capture are also computed. Finally, the following values are also computed for each scenario:

- *% Capture > T* = $\dfrac{(Capture\text{ - Threshold level})}{\text{Threshold level}} * 100$; the percentage of capture above the threshold level achieve by each Firm's A location.

- *% Constraint A.* = $\dfrac{\left(\sum_{i \in I} m_i r_{ij} x_{ij} + K_{1-a} S_j - Threshold\text{ Level}\right)}{Thershold\text{ Level}} * 100$; the percentage of threshold constraint accomplishment.

Note that the percentage of capture above the threshold level measures the accomplishment of the threshold constraint in the actual event. While the percentage of threshold constraint accomplishment measures this value in the general characteristics of the market.

[49] Note that the percentage of demand captured is computed instead the total amount because the scenario is stochastic; i.e. the total demand varies from one to other scenario. In this way, it is easy the comparison between scenarios.

Table 5.1. 55-nodes example (r = 0 and β=0.3)

		Firm B			Firm A			
β	**(p,q)**	**Outlet Location**	**Initial Capture**	**Final Capture**	**Outlet Location**	**Capture**	**% Capture > T**	**% Constraint A.**
0.3	**(2,5)**	17	16%	14%	4	18%	311%	240%
		41	22%	13%	5	12%	178%	228%
		38	13%	12%				
		31	18%	16%				
		5	30%	15%				
		Total Capture	***100%***	***70%***		***30%***		
0.3	**(3,5)**	12	12%	10%	4	15%	316%	284%
		41	22%	12%	5	13%	243%	243%
		38	14%	11%	31	11%	210%	209%
		31	21%	13%				
		5	31%	15%				
		Total Capture	***100%***	***61%***		***39%***		
0.3	**(4,5)**	17	13%	11%	3	11%	234%	245%
		25	25%	10%	4	14%	327%	266%
		38	12%	9%	5	11%	233%	238%
		3	25%	14%	25	8%	137%	150%
		5	25%	14%				
		Total Capture	***100%***	***57%***		***43%***		

Table 5.2. 55-nodes example (r = 0 and β=0.5)

		Firm B			Firm A			
β	**(p,q)**	**Outlet Location**	**Initial Capture**	**Final Capture**	**Outlet Location**	**Capture**	**% Capture > T**	**% Constraint A.**
0.5	**(2,5)**	22	16%	13%	3	17%	136%	94%
		25	21%	14%	5	15%	109%	124%
		38	11%	10%				
		31	21%	14%				
		6	31%	18%				
		Total Capture	***100%***	***69%***		***31%***		
0.5	**(3,5)**	16	16%	11%	3	11%	75%	91%
		41	25%	12%	4	14%	141%	110%
		23	15%	11%	5	14%	130%	100%
		3	18%	13%				
		2	25%	14%				
		Total Capture	***100%***	***61%***		***39%***		
0.5	**(4,5)**	22	17%	11%	3	13%	127%	96%
		20	17%	10%	4	15%	160%	124%
		31	24%	11%	5	10%	69%	107%
		38	11%	9%	31	9%	64%	74%
		5	32%	13%				
		Total Capture	***100%***	***54%***		***46%***		

Table 5.3. 55-nodes example (r = 0 and β=0.7)

		Firm B			Firm A			
β	**(p,q)**	**Outlet Location**	**Initial Capture**	**Final Capture**	**Outlet Location**	**Capture**	**% Capture > T**	**% Constraint A.**
0.7	**(2,5)**	12	10%	9%	5	18%	79%	77%
		25	20%	14%	31	12%	24%	29%
		31	24%	15%				
		38	14%	12%				
		5	32%	20%				
		Total Capture	***100%***	***70%***		***30%***		
0.7	**(3,5)**	22	19%	13%	4	16%	79%	62%
		25	21%	12%	5	11%	28%	51%
		43	11%	10%	31	11%	22%	25%
		31	20%	13%				
		5	28%	14%				
		Total Capture	***100%***	***62%***		***38%***		
0.7	**(4,5)**	22	19%	12%	3	14%	83%	53%
		20	15%	10%	4	14%	82%	65%
		38	11%	9%	5	11%	44%	54%
		18	19%	10%	18	8%	4%	19%
		5	36%	14%				
		Total Capture	***100%***	***54%***		***46%***		

From the previous tables, it can be pointed out the following:

- The percentage of total demand achieved by the entering firm is the same for a given number of outlets located, regardless threshold level. For example, when the entering firm locates 3 outlets, it captures the 39%, 39% and 38% of total demand, with β=0.3, 0.5 and 0.7 respectively.
- Obviously, the percentage of capture above the threshold level and the percentage of threshold constraint accomplishment achieve by each Firm's A, decrease with an increase of β value.

Finally, the robustness of the model is checked in the example. The model is solved several times in an specific scenario[50]. In each simulation of this scenario, demand nodes are randomly chosen following the fixed normal distribution defined and the model is solved to optimality by using complete enumeration.

Given the stochastic condition of the model, I want to check if the optimal locations vary in these different events of the demand nodes following a fixed normal distribution. The model was solved 200 times, and in all the cases, the optimal solution found was the same[51]. Therefore, I can conclude that the model is quite robust.

[50] The existing firm has five outlets located in nodes 5, 22, 25, 31, 38. The entering firm wants to locate 3 new outlets and the threshold level is defined as: $C = b\left[pop \big/ (p+q)\right]$ (where $pop = \sum_{i \in I} m_i$ and $b = 0.5$). All shops have the same attractiveness (A_i = 100) regardless of node and ownership. And finally, the demand at each node follows a multivariate normal distribution, considering: $m_i =$ the original demand of the Swain's network indicated in Table A1 of the appendix, $s_i^2 = \frac{m_i}{4}(uniform\ (0-1))$, and r = 0.

[51] The entering firm has to locate its outlets in nodes 1, 4 and 31.

Chapter 6

Metaheuristics to solve the models

6.1. Literature review of metaheuristics

The models presented in chapter 3 and chapter 5 are combinatorial optimization problems[52]. Many combinatorial problems are intractable and belong to the class of *NP*-Hard (non-deterministic polynomial-time complete) problems. In this case, the p-Median problem is *NP*-Hard on a general graph (Kariv and Hakimi, 1979). Moreover, in the model presented in chapter 5, the inclusion of a non-linear constraint reinforces the NP-Hard condition of the problem.

The common belief in this field is that no guaranteed polynomial time algorithm could ever be found to solve these inherently hard problems to optimality. Heuristics (or approximate algorithms) are considered one of the practical tools for solving hard combination optimization problems.

Several heuristics have been studied to solve the p-Median problem. Those heuristics can be grouped in two classes (Golden, et.al, 1980): construction algorithms and improvement algorithms. The

[52] Combinatorial Optimization problems are normally easy to describe but difficult to solve (Osman (1995))

former type tries to build a good solution from the beginning. In this group we can find the well-known greedy adding and greedy subtracting algorithm.

The second class of algorithms use a known starting solution and try to improve on it. The best representative of this group is the well-known Teitz and Bart (1968) one-opt heuristic. This method has been successful applied in, among others, Serra and Marianov (1996). But this heuristic has some problems. The first well-known problem is the possibility of finding only a locally optimal solution and the second one is a more recent one found by Rosing (1997). He has demonstrated that the solution provided by an interchange heuristic (p.e. the case of the Teitz and Bart heuristic applied to the p-Median problem) deteriorates, when either the number of demand nodes and / or the number of facilities to be located increases. This deterioration can be reflected both in the probability of finding the optimal solution and in the closeness of a typical solution to the optimal one.

Metaheuristics are the class of approximate methods, that have been developed since the mid – 1980's. They are designed to attack hard combinatorial optimization problems, where the classical heuristics have failed to be effective and efficient. Metaheuristics offer a general frame that allow for creating new hybrids by combining different concepts defined from: classical heuristics, artificial intelligence; biological evolution; natural phenomena; neural systems and statistical mechanisms. A good review of metaheuristics can be found in Osman (1995).

A metaheuristic is a process which applies a subordinate heuristic (or metaheuristic) at each step which has to be designed for each particular problem and then applies a controlling or overarching heuristic that modifies and redirects the subordinate heuristic. Although there is no guarantee of optimality of these methodologies, metaheuristics have proved highly successful in obtaining high quality solutions to many real world complex problems.

Metaheuristics can be grouped, doing an analogy to heuristics classification, in two classes: *problem – space methods and local search methods*.

6.1.1. Problem – Space metaheuristics

The problem – space methods are a class of heuristics superimposed on fast problem – specific constructive procedure. Its aim was to generate many different starting solutions that can be improved by local search methods. The best representatives are the Greedy Randomized Adaptive Search (GRASP) and recently, Ant System.

Greedy Randomized Adaptive Search (GRASP) method was developed by Feo and Resende (1989). It combines the power of greedy heuristics, randomization and local search procedures. GRASP is an iterative process, with each GRASP iteration consisting of two phases, a construction phase and a local search phase. The best overall solution is kept as the result. In the construction phase, a feasible solution is iteratively constructed, one element at a time. At each construction iteration, the choice of the next element to be added is determined by ordering all elements in a candidate list with respect to a greedy function. The function measures the (myopic) benefit of selecting each element. The heuristic is adaptive because the benefits associated with every element are updated at each iteration of the construction phase to reflect the changes brought on by the selection of the previous element. The probabilistic component of a GRASP is characterized by randomly choosing one of the best candidates in the list, but not necessarily the top candidate. The list of the best candidates is called the Restricted Candidate List (RCL). The restricted candidate list is determined by the application of two types of restrictions (Cardinality and Value) to the ordered candidate list. Basically, the cardinality restriction (BETA in my notation) restricts the initial length of the RCL; while the value restriction (ALPHA in my notation) restricts the candidates by the value of its greedy function. This choice technique allows different solutions to be obtained at each GRASP iteration, but this solution is not guaranteed to be locally optimal with respect to simple neighborhood definitions. Hence, the second GRASP phase (Local search phase) tries to improve each construction solution. Usually, a local optimize procedure such as a two-exchange is used in this second part.

GRASP has been applied successfully to several combinatorial problems such as: p-hub Location Problems (Klincewicz, 1992), Quadratic Assignment Problems (Li, et.al., 1994), Maximum Independent Set Problem (Feo, et.al., 1994), Satisfiability Problem (Resende and Feo, 1996) and Dense Quadratic Assignment Problems (Resende, et.al., 1996) and Vehicle Routing Problem (Kontoravdis and Bard, 1995).

The **Ant System** introduced by Colorni, Dorigo and Maniezzo (1991a, 1991b), Dorigo et.al. (1996), Dorigo and Di Caro (1999), is a cooperative search algorithm inspired by behavior of real ants. Ants lay down in some quantity an aromatic substance, known as pheromone, on their way to food. An ant chooses a specific path in correlation with the intensity of the pheromone. The pheromone trail evaporates over time if no more pheromone is laid down by other ants, therefore the best paths have more intensive pheromone and higher probability of being chosen. The Ant System approach associates pheromone trails to features of the solutions of a combinatorial problem, and can be seen as a kind of adaptive memory of the previous solutions. Solutions are iteratively constructed in a randomized heuristic fashion biased by the pheromone trails left by the previous ants. The pheromone trails, t_j, are updated after the construction of a solution, ensuring that the best features will have a more intensive pheromone.

Recently, Stützle (1997) have proposed an improved version of the Ant System, designated by MAX-MIN Ant System. The MAX-MIN ant system differs from the Ant System in the following way: only the best ant updates the trails in every cycle. To avoid stagnation of the search, i.e. ants always choosing the same path, Stüzle (1998a) proposed a lower and upper limit to the pheromone trail, $t_{\min}$ and $t_{\max}$, respectively.

Stützle and Hoos(1999), Stützle (1997,1998a) applied this procedure to Traveling Salesman Problem, Quadratic Assignment Problem and Flow-Shop Scheduling Problem; and Lourenço and Serra (2000) applied to the Generalized Assignment Problem.

6.1.2. Local Search metaheuristics

Local search methods form a general class of approximate metaheuristics based on the concept of exploring the vicinity of the current solution. Neighborhood solutions are generated by “a move generation mechanism”. These solutions are selected and accepted according to some pre-defined criteria. The best representatives of this group are *Simulated Annealing, Tabu Search and Genetic Algorithm*. A good review of them can be found in Pirlot (1992).

Simulated Annealing metaheuristic has its origins in statistical mechanisms. It was developed by Kirkpatrick, et.al. (1983) and first applied by Cerny (1985). The simulated annealing algorithm is based on the analogy between the annealing process of solids and the problem of solving combinatorial optimization problems. Simulated annealing applications in Operation Research are reviewed in Collins, et.al. (1988) and Koulamas, et.al. (1994).

Simulated annealing has been applied successfully to Quadratic Assignment Problem (Connolly, 1990), Vehicle Routing Problem (Osman, 1993), Capacitated Clustering Problem (Osman and Christofide, 1994) and Generalized Assignment Problem (Osman, 1995).

Tabu Search is a metaheuristic that guides local heuristic search procedures to explore the solution space beyond local optimality. It was introduced by Glover (1989, 1990). In essence, Tabu Search explores a part of the solution space by repeatedly examining all neighborhoods of the current solution, and moving to the best neighborhood even if this leads to a deterioration of the objective function. This approach tries to avoid being trapped in a local optimum. In order to avoid the cycling back to a solution that has recently been examined, nodes are inserted in a tabu list that is constantly updated. Additionally, several criterias of flexibility can be used in the tabu search including aspiration and diversification.

The aspiration criteria is used as insurance against restricting moves which would have led to finding high quality solutions. In other words, the aspiration criteria determines when a node can be move even if tabu. Usually, this criteria states that if a move produces a solution better than the best known solution (and the resulting solution is feasible), then the tabu status is disregarded and the move is executed.

Diversification criteria is utilized to escape from local optima and is achieved by using a long - term memory function. It allows a broader exploration of the solution space by starting from solutions that have not been well explored.

This method has been successfully applied to Flow Shop Scheduling (Taillard, 1990), Time Tabling Problem (Hertz, 1991), Vehicle Routing Problem (Gendreau, et.al., 1994) and Job Shop Scheduling Problem (Lourenço and Zwijnenburg, 1996). The method has also been successfully applied to a wide variety of location problems: p-hub Location Problems ((Klincewicz, 1992) and (Marianov, et.al., 1997)), (r | *Xp)*- Medianoid and (r | *p*)- Centroid Problems (Benati and Laporte, 1994), the Vehicle Routing Problem (Gendreau, et.al., 1994) and p-Median Problem (Rolland, et.al., 1996).

Genetic Algorithms are a class of adaptive search methods based on a highly abstract model of natural evolution. They were developed by Holland (1975) and only recently their potential for solving combinatorial optimization problems has been explored. The main difference with Simulated Annealing and Tabu Search, is that Genetic Algorithm deal with populations of solutions rather than with single solutions. The basic idea is maintain a population of candidate solutions which evolves under a selective pressure that favors the better solutions.

This method has been applied successfully to a wide range of problems. Several papers summarize this applications ((Goldberg, 1989) and (Reeves, 1995)).

All three local search methods have demonstrated its effectiveness for solving a problem. However, Pirlot (1992) in its tutorial recognized that Tabu Search is in general much faster than Simulated Annealing and Genetic Algorithm. Given that, I decide to use Tabu Search as a local search Metaheuristic in the metaheuristics applied in this thesis.

6.1.3. Metaheuristics for this research

Following recent applications, the metaheuristics developed in this thesis have two stages. In the first stage, a good initial solution is constructed using a problem – space metaheuristic. In the second one, the previous solution found is improved using a local search metaheuristic.

In this thesis, a metaheuristic based on **GRASP and Tabu Search** metaheuristics has been developed to solve **the models defined in chapter 3**. The formulation of this metaheuristic and its computational experience is presented in epigraph 6.2.

Given that the MAX-MIN Ant System has not been applied to any location problem, I decide to applied this Metaheuristic as the first step of the metaheuristic for the New

Chance – Constrained Maximum Capture Location problem (the new model presented in chapter 5). Then, in essence, I develop a **metaheuristic for the model defined in chapter 5** based on **MAX-MIN Ant System and Tabu Search** metaheuristics. The formulation of this metaheuristic and its computational experience is presented in epigraph 6.3.

6.2. A metaheuristic to solve the models

The Metaheuristic applied to the models presented in chapter 3 has two phases. In the first one, a good initial solution is constructed using GRASP; and in the second phase, the previous solution found is improved applying the well-known Tabu search heuristic, including the aspiration and diversification criterion (following Benati and Laporte (1994) application of Tabu Search)

6.2.1. Metaheuristic: Formulation.

In this epigraph, a formal description of the metaheuristic procedure is presented.

Notation

- LOCP is the set of Firm A locations.
- LOCQ is the set of Firm B locations.
- Z^m is the objective value of model m.

METAHEURISTIC GRASP + TABU search

PHASE 1: GRASP

1. Set K=1.

Construction Phase (Construct a Greedy Randomized Solution)

2. Let $i \rightarrow i + 1$ (LOCPi)

3. Compute Z^m for all nodes j not in $LOCP_i$ (or in $LOCQ_i$). Relabel the solution $LOCP_j$ in decreasing order of Z^m ($LOCP_j$). Relabel all vertices accordingly. Apply the Cardinality (BETA) and Value (ALPHA) Restrictions to construct the Restricted Candidate List.

4. Choose Randomly from among the elements of the Restricted Candidate List with each element having equal probability. Add this random choice to set $LOCP_i$.

5. If $i \leq P$, go to step 2.

Local Search phase: Teitz and Bart.

6. Start with the initial solution set $LOCP_i$ found in the construction phase. Compute $Z^m{}_c(LOCP_i)$.

7. Let $i \rightarrow i + 1$ (LOCPi)

8. Let $e \rightarrow e + 1$; $e \in$ (empty nodes).

9. Relocate $LOCP_i$ to an empty node. Compute $Z^m{}_{en}(LOCP_i)$. If e is the first empty node computed, set $Z^m{}_{STEP9}(LOCP_i) = Z^m{}_{en}(LOCP_i)$. If e is not the first empty node and if $Z^m{}_{en}(LOCP_i) > Z^m{}_{STEP9}(LOCP_i)$, keep the relocation and relabel $Z^m{}_{en}(LOCP_i)$ as $Z^m{}_{STEP9}(LOCP_i)$.

   Go to step 8 until all empty nodes are checked.

   If $i \leq P$, go to step 7.

9. If $Z^m{}_{STEP9}(LOCP) > Z^m{}_c(LOCP)$, then go to step 7.

10. If $K < MAXITER$, $K \rightarrow K+1$ and go to step 1. Updating the Best Solution found $Z^m{}_{BEST}(LOCP)$ in each GRASP iteration.

PHASE 2: TABU SEARCH (following Benati and Laporte (1994))

1. Let s = 0 (number of times, the diversification criteria is applied).

2. Let t = 0 (number of iterations of the TABU procedure).

3. Set $Z^m{}_0(LOCP) = Z^m{}_{BEST}(LOCP)$, the best solution found in GRASP. Set $LOCP_i{}^0$, for i=1,..,P the optimal locations found in GRASP.

4. $LOCP_i{}^0 \rightarrow LOCP_{i+1}{}^0$ (for each located node).

5. Consider all neighborhood nodes j of optimal location node i (i.e. $LOCP_{ngh\ j\,(Locp_i^0)}$).

   Let $LOCP_{ngh\ j\,(Locp_i^0)} \rightarrow LOCP_{ngh\ j+1(Locp_i^0)}$. Excahnge the facility from node $LOCP_i{}^0 \in$ LOCP to a neighborhood node $LOCP_{ngh\ j\,(Locp_i^0)} \notin$ LOCP. Compute the objective function in this new solution. Do it for all neighborhood nodes of node i. Relabel the solution $LOCP_{ngh\ j\,(Locp_i^0)}$ in decreasing order of $Z^m(LOCP_{ngh\ j\,(Locp_i^0)})$.

6. If ($Z^m(LOCP_{ngh\ j\,(Locp_i^0)}) > Z^m{}_{BEST}(LOCP_i{}^0)$) or $LOCP_{ngh\ j\,(Locp_i^0)}$ is not tabu, then set $Z^m{}_{BEST}(LOCP_i{}^0)= Z^m(LOCP_{ngh\ j\,(Locp_i^0)})$, the outlet is located in $LOCP_{ngh\ j\,(Locp_i^0)}$ and $LOCP_i{}^0$ is declared tabu until $t + \theta$, where θ is a pre-fixed value, and go to step 5.

   If all nodes visited are tabu and none improves the objective, then the model chooses the node with the lowest tabu tag ($t + \theta$) and lift the tabu status of $LOCP_{ngh\ j\,(Locp_i^0)}$).

   Go to step 5.

7. If t is less than a pre-fixed upper bound T; update the best solution found Z^{m}_{BEST}(LOCP), let $t \rightarrow t + 1$ and go to step 3.
8. If $s < s^{MAX}$ (maximum number of times, the diversification criteria is applied). The model starts a new procedure with an initial solution equal to the NP least visited nodes. Let $s \rightarrow s + 1$ and go to step 2.

Comments about the meta-heuristic:

- In step 6 of Tabu Search, tabu status can be canceled if this permits an improvement in the objective. This rule is called *the aspiration criterion*
- Step 8 of Tabu Search, states *the diversification criterion*, that allows a broader exploration of the solution space by starting from locations that have been less well explored.

6.2.2. Computational experience

6.2.2.1. Characteristics of the computational experience

The algorithm has been applied to several randomly generated networks. These networks have number of nodes *n* equal to 20,30 and 50. For each *n*, three different number of outlets are located so that *p*=2,3,4; while the number of the established firm outlets are pre-fixed *q* = 5 (the specific nodes in each case are the ones shown in table 3.1.). Finally, for each n and each p, ten networks were randomly generated. Therefore, a total of 90 networks were generated.

For each of these networks, the location of the nodes $n \in (0,1000)^2$ were generated following a uniform distribution in a map of 1000 units * 1000 units. The Euclidean distances between nodes were computed. The neighborhood for each node is defined as the randomly (2-6) closest nodes using the Euclidean distance criterion. The demand in each node was randomly generated within the (800,1000) interval again following a uniform distribution.

For each 90 networks and for each model, the metaheuristic solutions were compared to the optimal ones. Optimal solutions were obtained by complete enumeration.

In phase 1 (GRASP) of the metaheuristic, K was equal to 20. In step 3 of this phase 1, I set the cardinality restriction as thirty- percent of the network size and the value restriction as fifty- percent of the best candidate objective. In phase 2 (Tabu Search) of the algorithm, the stopping criterion imposed was T equal to 40. In step 3 of this phase 2, the θ was set equal to 5. In step 6 of this phase 2, the diversification criterion is applied starting four times with the less visited nodes. Finally, in the MCI model, the sensitivity of consumers to quadratic distance is used in all the cases ($b_d = 2$). The heuristic was programmed in FORTRAN77 and executed in a Pentium PC 133 with 16mb of RAM.

6.2.2.2. Results of the computational experience

The results of the behavior of the metaheuristic GRASP + TABU are shown in tables 6.1. and 6.2.

Column named "% GRASP optimal solutions" of table 6.1. shows the percentage of times that the optimal solution was found by phase 1 of the algorithm, making it unnecessary to execute phase 2. For example, for n = 20, p =3 and *model*[53] = 2; 80 % of the solutions found with this phase were optimal. Column named “% TABU optimal solutions” of table 6.1. shows the percentage of times that the optimal solution was found by phase 2 in the cases where the optimal solution was not found in the previous phase. For example, in all the runs of 20-network size, the TABU procedure (phase 2 of algorithm) found the optimal solution. Finally, the last column named “% total average deviation” indicates the average deviation from optimality where the algorithm failed to find the optimal solution. Only 6 solutions out of the 360 runs were non-optimal based on the comparison with complete enumeration. In general, the average deviation from optimality did not exceed 1%, except for the case when n = 30, p =4 and *model* = 1, where the deviation from optimality was equal to 7.6%.

Table 6.2. shows the average execution time in seconds spend per phases by global metaheuristic and by enumeration procedure. Notice that the algorithm becomes efficient when the network size is greater than 30 nodes and we have to locate 3 or more entering outlets. In these cases, the time spent by the algorithm is less than the one for the enumeration procedure. For example, in n = 50, p = 4, *model* = 2, the time spent by the algorithm is 21.733 seconds while the enumeration procedure spent 658.374 seconds to find the same solution. Although the average computing time of the metaheuristic increases with the number of nodes and the number of outlets, its relative advantage over enumeration will also increase.

[53] Models 1, 2, 3 and 4 are the models presented in chapter 3.

Table 6.1. Heuristic Performance of metaheuristic GRASP + TABU

N	(p,q)	Model	Optimal Solutions (%)		Total Average Deviation (%)
			GRASP	TABU	
20	**(2,5)**	**1**	100 %		
		2	100 %		
		3	80 %	100 %	
		4	100 %		
	(3,5)	**1**	100 %		
		2	80 %	100 %	
		3	70 %	100 %	
		4	70 %	100 %	
	(4,5)	**1**	70 %	100 %	
		2	30 %	100 %	
		3	10 %	100 %	
		4	10 %	100 %	
30	**(2,5)**	**1**	90 %	100 %	
		2	100 %		
		3	40 %	100 %	
		4	100 %		
	(3,5)	**1**	100 %		
		2	100 %		
		3	80 %	100 %	
		4	80 %	100 %	
	(4,5)	**1**	50 %	80 %	7.6 %
		2	50 %	80 %	0.78 %
		3	60 %	75 %	0.06 %
		4	90 %	100 %	
50	**(2,5)**	**1**	100 %		
		2	80 %	100 %	
		3	70 %	100 %	
		4	100 %		
	(3,5)	**1**	100 %		
		2	80 %	100 %	
		3	70 %	100 %	
		4	100 %		
	(4,5)	**1**	90 %	100 %	
		2	60 %	75 %	0.4 %
		3	30 %	71.43 %	0.202 %
		4	100 %		

Table 6.2. Time Performance of metaheuristic GRASP + TABU

N	(p,q)	Model	GRASP Average Time	TABU Average Time	GRASP+TABU Average Time	Enumeration Average Time
20	**(2,5)**	**1**	0.132	0.12	0.252	0.01
		2	1.259	1.336	2.595	0.141
		3	0.169	0.209	0.378	0.011
		4	0.18	0.116	0.296	0.012
	(3,5)	**1**	0.213	0.171	0.384	0.1
		2	1.535	2.295	3.83	1.088
		3	0.215	0.349	0.564	0.166
		4	0.203	0.197	0.4	0.01
	(4,5)	**1**	0.182	0.257	0.439	0.388
		2	1.403	3.389	4.792	5.376
		3	0.247	0.52	0.767	0.835
		4	0.162	0.269	0.431	0.406
30	**(2,5)**	**1**	0.287	0.175	0.462	0.056
		2	2.368	2.004	4.372	0.513
		3	0.34	0.319	0.659	0.073
		4	0.29	0.186	0.476	0.05
	(3,5)	**1**	0.45	0.298	0.748	0.45
		2	3.882	3.702	7.584	5.811
		3	0.66	0.555	1.215	0.901
		4	0.38	0.323	0.703	0.469
	(4,5)	**1**	0.418	0.416	0.834	3.275
		2	3.943	5.563	9.506	46.068
		3	0.682	0.833	1.515	7.092
		4	0.506	0.433	0.939	3.426
50	**(2,5)**	**1**	0.757	0.307	1.064	0.185
		2	7.898	3.442	11.34	2.426
		3	0.99	0.509	1.499	0.366
		4	0.923	0.342	1.265	0.227
	(3,5)	**1**	1.322	0.505	1.827	3.592
		2	13.529	6.369	19.898	47.37
		3	1.666	0.95	2.616	7.173
		4	1.623	0.554	2.177	3.871
	(4,5)	**1**	1.69	0.705	2.395	45.175
		2	11.896	9.837	21.733	658.374
		3	1.718	1.469	3.187	99.602
		4	2.231	0.763	2.994	48.73

6.3. A metaheuristic to solve the New Chance - Constrained Maximum Capture Location Problem

The Metaheuristic applied to the model presented in chapter 5 has two phases. In the first one, a good initial solution is constructed using MAX-MIN Ant System; and in the second phase, the previous solution found is improved applying the well-known Tabu search heuristic (following Benati and Laporte (1994) application of Tabu Search).

As MAX-MIN ANT SYSTEM has been never applied before to location models, I need to adapt this algorithm to the Chance – Constrained Maximum Capture Location Problem. To do it, I define t_j as the desirability of locating a shop in j. Initially, $t_j = \sum_{i \in I} \frac{A_j}{d_{ij}^{l}}$. The more attractive the index of a shop in *j* is, the more desired is the location of an outlet in that node.

The MAX-MIN Ant system is an iteratively procedure with three steps:

- In the first step of the iteratively procedure, a initial solution is constructed. To do this, the nodes are ordered with respect to the probability function defined by $p_j = \frac{t_j}{\sum_{l \in J} t_l}$.

  The initial solution is choose randomly, taking into account the probability distribution previously defined.
- The second step of the iteratively procedure tries to improve this initial solution by a local search method; in this case, applying a Teitz and Bart heuristic. In both steps, only feasible solutions are allowed.
- Finally, in the third step of the iteratively procedure, the pheromone trails are updated using the current solution in the following way: $t_j^{new} = r t_j^{old} + \Delta t_j$, where r, $0 < r < 1$,

is the persistence of the trail, i.e. 1 - r , represents the evaporation. The updated amount is $\Delta t_j = t_{\max} * Q$ if an outlet is located in j; 0, otherwise .

In this final stage, the MAX-MIN limits were checked and imposed $t_{\min} \leq t_j \leq t_{\max}, \forall j \in J$, if the updated pheromone falls outside the interval. In this case, the values of the parameters of the metaheuristic were set to Q = 0.05, r = 0.75, $t_{\max} = p * \max t_j$ and $t_{\min} = (1/p) * \min t_j$ (where, p is the number of outlets to locate).

The termination condition of this iteratively procedure is the number of total iterations.

6.3.1. Metaheuristic: Formulation

In this epigraph, a formal description of the metaheuristic procedure is presented.

Notation:

- LOCP is the set of Firm A locations.
- LOCQ is the set of Firm B locations.
- Z is the objective value of Chance – constrained maximum capture location problem.

<u>METAHEURISTIC MAX-MIN Ant System + TABU search</u>

PHASE 1: MAX-MIN Ant System

1. Initialize the parameters of Ant System and compute initial $t_j = \sum_{i \in I} \frac{A_j}{d_{ij}^l}$, for all nodes j of the network.

2. Let k $\rightarrow$ k + 1 (iterations of Ant System procedure)

First Step: Construct a good initial solution

3. Compute $p_j = \dfrac{t_j}{\sum_{l \in J} t_l}$, for all nodes j of the network.

4. Let i $\rightarrow$ i + 1 ($LOCP_i$)

5. Choose randomly a node to locate an outlet, taking into account that nodes follow the probability function defined in step 3.

6. If $i \leq P$, go to step 4.

7. Check the threshold constraint.
   - If this is not satisfied, go to step 4.
   - If this is satisfied, go to step 8.

Second Step: Local search phase; Teitz and Bart**.**

8. Start with the initial solution set $LOCP_i$ found in the first step. Compute $Z_c(LOCP_i)$.

9. Let i $\rightarrow$ i + 1 (LOCPi)

10. Let e $\rightarrow$ e + 1 ; e $\in$ (empty nodes).

11. Relocate $LOCP_i$ to an empty node. Compute $Z_{en}(LOCP_i)$ and check the threshold constraint.
    - If this is not satisfied, go to step 10.
    - If this is satisfied, then:
      - If e is the first empty node computed, set $Z_{STEP11}(LOCP_i) = Z_{en}(LOCP_i)$.
      - If e is not the first empty node and if $Z_{en}(LOCP_i) > Z_{STEP11}(LOCP_i)$, keep the relocation and relabel $Z_{en}(LOCP_i)$ as $Z_{STEP11}(LOCP_i)$.

    Go to step 10 until all the empty nodes are checked.

    If $i \leq P$, go to step 9.

12. If $Z_{STEP11}(LOCP) > Z_c(LOCP)$, then go to step 9.

Third Step: Update pheromone trails, using the current solution.

13. Compute the news t_j for all nodes j of the network, using the optimal locations found in the previous steps. Check also the max – min limits for all t_j (i.e. $t_{\min} \le t_j \le t_{\max}, \forall j$).

14. If K < MAXITER, set K $\rightarrow$ K+1 and go to step 2. Updating the Best Solution found Z_{BEST}(LOCP) in each MAX-MIN Ant system iteration.

PHASE 2: TABU SEARCH (following Benati and Laporte (1994))

1. Let s = 0 (number of times, the diversification criteria is applied).

2. Let t = 0 (number of iterations of the TABU procedure).

3. Set Z_0(LOCP) = Z_{BEST}(LOCP) , the best solution found in MAX-MIN Ant system. Set $LOCP_i^0$, for i=1,..,P the optimal locations found in MAX-MIN Ant system.

4. $LOCP_i^0 \rightarrow LOCP_{i+1}^0$ (for each located node).

5. Consider all neighborhood nodes j of optimal location node i (i.e. $LOCP_{ngh\ j\,(Locp_i^0)}$).

   Let $LOCP_{ngh\ j\,(Locp_i^0)} \rightarrow LOCP_{ngh\ j+1\,(Locp_i^0)}$. Exchange the facility from node $LOCP_i^0 \in$ LOCP to a neighborhood node $LOCP_{ngh\ j\,(Locp_i^0)} \notin$ LOCP.

   Check the threshold constraint in this new solution:

   - If this is satisfied, compute the objective function $Z(LOCP_{ngh\ j\,(Locp_i^0)})$.
   - If this is not satisfied, set $Z(LOCP_{ngh\ j\,(Locp_i^0)}) = 0$.

   Do it for all neighborhoods nodes of node i.. Relabel the solution $LOCP_{ngh\ j\,(Locp_i^0)}$ in decreasing order of $Z(LOCP_{ngh\ j\,(Locp_i^0)})$.

6. If ($Z(LOCP_{ngh\ j\,(Locp_i^0)})$) > $Z_{BEST}(LOCP_i^0)$) or $LOCP_{ngh\ j\,(Locp_i^0)}$ is not tabu, then set $Z_{BEST}(LOCP_i^0) = Z(LOCP_{ngh\ j\,(Locp_i^0)})$, the outlet is located in $LOCP_{ngh\ j\,(Locp_i^0)}$ and $LOCP_i^0$ is declared tabu until $t + \theta$, where θ is a pre-fixed value, and go to step 5.

   If all nodes visited are tabu and none improves the objective, then the model chooses the node with the lowest tabu tag ($t + \theta$) and lift the tabu status of $LOCP_{ngh\ j\,(Locp_i^0)}$).

   Go to step 5.

7. If t is less than a pre-fixed upper bound T; update the best solution found $Z_{BEST}(LOCP)$, let $t \rightarrow t + 1$ and go to step 3.

8. If $s < s^{MAX}$ (maximum number of times, the diversification criteria is applied). The model starts a new procedure with an initial solution equal to the NP least visited nodes. Let $s \rightarrow s + 1$ and go to step 2.

Comments about the meta-heuristic:

- In step 6 of Tabu Search, tabu status can be canceled if this permits an improvement in the objective. This rule is called *the aspiration criterion*
- Step 8 of Tabu Search, states *the diversification criterion,* that allows a broader exploration of the solution space by starting from locations that have been less well explored.

6.3.2. Computational experience

6.3.2.1. Characteristics of the computational experience

The algorithm has been applied to several randomly generated networks, having the number of nodes n equal to 35, 50 and 70. For each n, three different threshold level C were set using the following formula: $C = b\left[pop / (p+q) \right]$, where *pop* is the total amount of demand to be served, defined as $pop = \sum_{i \in I} m_i$; and b is a threshold factor that was set to 0.1, 0.2 and 0.3[54]. For the threshold constraint, we assumed α = 95% ⇒ Z_{1-a} = -1.645 (because, one-tailed test (left-tailed test) is applied).

It is assumed that there are five existing outlets. For each generated network, the location of the five existing outlets were found using the Teitz and Bart heuristic with a weighted total distance objective (i.e., minimised weighted by the population / demand of each node).

For each n, and each C, three different numbers of outlets of the entering firm were used; p = 2,3,4.

In this case, to generate the networks, the distributions of the demand nodes need to be established. I assume that the demand nodes follow a multivariate normal distribution $a_i \approx N(m_i, s_i^2)$. This distribution will be established in the following way: $m_i = uniform\ (50 - 100)$ and $s_i^2 = \frac{m_i}{4}\left(uniform\ (0 - 1)\right)$. We also give a priori value to the correlation between different demand nodes. This can be either unrelated or positively related (i.e., $r = 0$ or 0.1, as in Drezner and Drezner 2002b)

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[54] The computation of C is made a posteriori, when the distribution of the demand nodes is established.

Finally, the value of the attractiveness of each shop ($A_i = uniform\ (60,100)$) need also to be pre-established. It can be assumed that the attractiveness level represents the size of the shops .

Summing up, for each n, each b , each p and each r; ten networks were randomly generated. Therefore, a total of 540 networks were generated.

In phase 1 (MAX – MIN ant system) of the metaheuristic, the termination condition was set equal to 30 iterations. In phase 2 (Tabu Search) of the algorithm, the stopping criterion imposed was T equal to 40. In step 3 of this phase 2, the θ was set equal to 5. In step 6 of this phase 2, the diversification criterion is applied starting four times with the less visited nodes.

Optimal solutions were obtained using complete enumeration. The heuristic was programmed in FORTRAN and executed in Pentium III 450 Mhz with 128 mb of RAM.

6.3.2.2. Results of the computational experience.

The results of the behavior of the metaheuristic MAX-MIN ant system + TABU search are shown in tables 6.3.,6.4.,6.5. and 6.6.

Tables 6.3. and 6.4. presents the heuristic performance in the unrelated cases and in the positively related cases, respectively. In these tables, the percentages of optimal solutions are presented in the column labeled “optimal solutions”. If at least a no optimal solution is found among the ten runs, the average deviation from optimality in both stages of the metaheuristic are presented at the two last columns named “Max-Min average deviation” and “Total average deviation”. In this case, it can be noticed that the stochastic condition of the model arises the difficulty to find the optimal solutions. With the metaheuristic, a near-optimal solutions were found with a minimal deviation.

- **r = 0**. 41 out of 270 runs were non-optimal based on the comparison with complete enumeration. The maximum average deviation from optimality did not exceed 3.3%.
- **r = 0.1**. 29 out of 270 runs were non-optimal based on the comparison with complete enumeration. The maximum average deviation from optimality did not exceed 3.9 %.

In both tables, an additional column named “Lack of solution” has been included. These columns represent the percentages of cases without a feasible solution; in other words, a network where the entering firm cannot find a solution that satisfied all the constraints; included the threshold constraint. It can be noticed that this lack of solution appears in table 6.4. with an r = 0.1. We can deduce, from previous models without an stochastic threshold constraint, that this constraint is the one no satisfied in these cases. A statistical interpretation of this result can be that a greater correlation means a greater S_j and, as $K_{1-\alpha}$ is negative, the threshold constraint is more difficult to achieve. An economic interpretation of this output could be the following: correlation between demand nodes can be interpreted as that a higher demand power of one node implies a higher demand power of the others nodes. In this scenario, the established firm will capture more demand, by an initial assumption[55], and then, the entering firm will have more problems to find its outlets locations which satisfy the threshold constraint.

[55] Under equal conditions, the existing firm captures the demand (Hakimi, 1986).

Table 6.3. **Heuristic Performance of metaheuristic MAX-MIN Ant System + TABU (r = 0)**

N	β	(p,q)	Optimal Solutions	Lack of solution	Max-Min Average Deviation (%)	Total Average Deviation (%)
35	0.1	(2,5)	100 %		5.10 %	
		(3,5)	100 %		6.91 %	
		(4,5)	70 %		8.52 %	1.29 %
	0.2	(2,5)	90 %		4.57 %	1.05 %
		(3,5)	80 %		8.13 %	2.02 %
		(4,5)	80 %		7.82 %	1.13 %
	0.3	(2,5)	100 %		2.26 %	
		(3,5)	90 %		6.30 %	0.42 %
		(4,5)	70 %		9.16 %	2.90 %
50	0.1	(2,5)	100 %		4.29 %	
		(3,5)	100 %		4.29 %	
		(4,5)	70 %		7.41 %	2.15 %
	0.2	(2,5)	90 %		4.05 %	1.70 %
		(3,5)	80 %		4.47 %	1.46 %
		(4,5)	70 %		6.80 %	2.04 %
	0.3	(2,5)	100 %		1.54 %	
		(3,5)	90 %		6.05 %	1.40 %
		(4,5)	70 %		6.23 %	1.06 %
70	0.1	(2,5)	90 %		1.50 %	0.10 %
		(3,5)	80 %		1.71 %	1.20 %
		(4,5)	70 %		4.99 %	0.49 %
	0.2	(2,5)	90 %		1.01 %	3.30 %
		(3,5)	90 %		1.58 %	1.90 %
		(4,5)	70 %		5.90 %	0.49 %
	0.3	(2,5)	90 %		1.28 %	1.27 %
		(3,5)	90 %		2.59 %	1.72 %
		(4,5)	70 %		3.65 %	0.91 %

Table 6.4. Heuristic Performance of metaheuristic MAX-MIN Ant System + TABU (r = 0.1)

n	*b*	*(p,q)*	*Optimal Solutions*	*Lack of solution*	*Max-Min Average Deviation (%)*	*Total Average Deviation (%)*
35	**0.1**	**(2,5)**	100 %		5.41 %	
		(3,5)	80 %		5.67 %	2.36 %
		(4,5)	80 %		8.56 %	1.11 %
	0.2	**(2,5)**	100 %		4.85 %	
		(3,5)	100 %		8.27 %	
		(4,5)	70 %		5.87 %	1.66 %
	0.3	**(2,5)**	90 %		4.58 %	3.39 %
		(3,5)	100 %		6.34 %	
		(4,5)	80 %		8.92 %	0.86 %
50	**0.1**	**(2,5)**	100 %		1.91 %	
		(3,5)	90 %		6.03 %	0.42 %
		(4,5)	70 %		5.81 %	1.24 %
	0.2	**(2,5)**	90 %		3.15 %	0.78 %
		(3,5)	80 %		4.94 %	3.81 %
		(4,5)	70 %		5.44 %	2.23 %
	0.3	**(2,5)**	100 %	20 %	21.94 %	
		(3,5)	90 %	30 %	1.88 %	0.36 %
		(4,5)	80 %	20 %	23.36 %	0.62 %
70	**0.1**	**(2,5)**	100 %		0.36 %	
		(3,5)	80 %	10 %	2.89 %	1.23 %
		(4,5)	70 %	20 %	2.77 %	2.31 %
	0.2	**(2,5)**	100 %	40 %	0.21 %	
		(3,5)	90 %	70 %	1.99 %	0.73 %
		(4,5)	100 %	100 %		
	0.3	**(2,5)**	100 %	100 %		
		(3,5)	100 %	100 %		
		(4,5)	100 %	100 %		

Tables 6.5. and 6.6. presents the time performance of the metaheuristic in the unrelated cases and in the positively related cases, respectively. These tables show the average execution time in seconds spend per phases by global metaheuristic and by enumeration procedure.

The average computing time of the heuristic is similar, maintaining the others parameters equal, for a network assuming r = 0 and r = 0.1. Notice that the algorithm becomes very useful when we have to locate 3 or more entering outlets, regardless of the constraint level. In these cases, the time spent by the algorithm is less than the one for the enumeration procedure. For example, in n = 70, p = 4, r = 0 and β =0.2, the time spent by the algorithm is 21.44 seconds while the enumeration procedure spent 4451.67 seconds to find the same solution. Although the average computing time of the heuristic increased with the number of nodes and the number of outlets, as expected, its relative advantage over enumeration will also increase.

Table 6.5. Time Performance of metaheuristic MAX-MIN Ant System + TABU (r = 0)

N	*b*	*(p,q)*	*Max-Min Average Time*	*TABU Average Time*	*TOTAL Average Time*	*Enumeration Average Time*
35	**0.1**	**(2,5)**	0.20	1.34	1.55	0.44
		(3,5)	0.34	3.08	3.42	7.80
		(4,5)	0.36	5.51	5.87	86.58
	0.2	**(2,5)**	0.36	1.34	1.70	0.46
		(3,5)	0.32	3.11	3.43	7.78
		(4,5)	0.33	5.60	5.93	86.59
	0.3	**(2,5)**	0.19	1.33	1.52	0.46
		(3,5)	0.35	3.07	3.42	7.79
		(4,5)	0.38	5.61	5.99	86.60
50	**0.1**	**(2,5)**	1.14	2.31	3.45	1.62
		(3,5)	1.16	5.44	6.60	40.40
		(4,5)	1.42	10.00	11.41	659.86
	0.2	**(2,5)**	0.60	2.29	2.88	1.60
		(3,5)	0.91	5.38	6.29	40.39
		(4,5)	1.77	9.86	11.63	659.84
	0.3	**(2,5)**	0.74	2.30	3.04	1.61
		(3,5)	1.01	5.38	6.39	40.39
		(4,5)	1.08	9.70	10.77	659.78
70	**0.1**	**(2,5)**	4.02	3.96	7.98	5.41
		(3,5)	8.33	9.18	17.50	192.03
		(4,5)	8.52	16.65	25.17	4458.96
	0.2	**(2,5)**	2.66	3.93	6.59	5.40
		(3,5)	5.11	9.14	14.25	191.70
		(4,5)	5.03	16.41	21.44	4451.67
	0.3	**(2,5)**	3.44	3.96	7.40	5.40
		(3,5)	4.29	9.25	13.53	191.76
		(4,5)	11.82	16.88	28.70	4452.62

Table 6.6. Time Performance of metaheuristic MAX-MIN Ant System + TABU (r = 0.1)

n	*b*	*(p,q)*	*Max-Min Average Time*	*TABU Average Time*	*TOTAL Average Time*	*Enumeration Average Time*
35	**0.1**	**(2,5)**	0.23	1.33	1.55	0.46
		(3,5)	0.57	3.08	3.64	7.80
		(4,5)	0.37	5.62	5.99	86.58
	0.2	**(2,5)**	0.28	1.31	1.60	0.47
		(3,5)	0.32	3.13	3.45	7.80
		(4,5)	0.51	5.55	6.06	86.59
	0.3	**(2,5)**	0.22	1.32	1.55	0.45
		(3,5)	0.45	3.08	3.53	7.80
		(4,5)	0.57	5.64	6.21	86.64
50	**0.1**	**(2,5)**	1.66	2.31	3.97	1.61
		(3,5)	1.11	5.31	6.42	40.39
		(4,5)	1.50	9.83	11.33	660.78
	0.2	**(2,5)**	1.17	2.31	3.48	1.62
		(3,5)	1.74	5.16	6.90	40.39
		(4,5)	3.35	9.67	13.01	659.72
	0.3	**(2,5)**	2.42	2.16	4.58	1.61
		(3,5)	2.85	4.99	7.84	40.40
		(4,5)	4.16	8.97	13.13	659.98
70	**0.1**	**(2,5)**	5.70	3.76	9.46	5.39
		(3,5)	9.75	8.47	18.22	191.98
		(4,5)	14.12	16.56	30.68	4463.64
	0.2	**(2,5)**	8.96	3.53	12.49	5.40
		(3,5)	12.30	7.74	20.05	191.66
		(4,5)	18.12	14.02	32.13	4459.10
	0.3	**(2,5)**	8.12	3.40	11.52	5.42
		(3,5)	13.66	7.74	21.40	191.67
		(4,5)	17.87	13.98	31.85	4459.01

Chapter 7

Conclusions

This thesis presents the incorporation of the Revealed Preference approach of Store - Choice models in the Discrete Competitive Location models that have a Maximum Capture objective function. Up till now, the Discrete Competitive Location models for retail sector had done few intents to overcome the unreal assumption of consumers choosing the closest shop. Obviously, Retail Location models should take into account the process underlying consumer’s choice stores, to be useful in real world. The consumer store choice behavior is analyzed by a line of research called Store Choice Literature. This literature, usually applied in Marketing, revels that consumers not only cares about which shop is the closest, but also considers other variables in making the decision of patronizing a particular outlet.

To do this work, first, in **chapter 3**, I have analyzed how important is the precise method of including distance in Discrete Competitive Location Models. To do this, I have considered different ways of defining a key parameter of one basic Discrete Competitive Location model. This parameter reflects the various ways of taking into account distance based on different Consumer Choice Behavior theories. The basic Maximum Capture model (MAXCAP) uses the traditional view of all or nothing capture by outlets, where consumers compare distance to the closest facility of the other chain. The Multiplicative Competitive

Interaction (MCI) model and the Proportional Customer Preference model are based on the same idea: proportional capture where consumers select stores to patronize with probabilities that are functions of their distance to all the outlets. The difference between both models is the introduction in the MCI model of the sensitivity of consumers to quadratic distance. Finally, the Partial Binary Preference model assumes that consumers patronize the closest facility of the chosen chain and the capture is proportional.

In order to analyze if the optimality of locations changes dramatically when applying these different models that reflect different assumptions on consumer travel behavior, I have computed the deviation in demand captured by the use of the optimal location of the true model in relation to the use of the optimal locations of the other models. The deviation have been computed in 90 generated networks and in the well – known 55 – nodes network of Swain (1974). From these computations, it can be conclude:

- The greatest deviation in demand captured is the one found when the MAXCAP assumption is the true one, but we use the optimal locations found by the other models.
- The introduction of consumers sensitivity to the quadratic distance is unimportant in optimality terms since the smallest deviations in demand captured are the ones between the MCI model and Proportional Customer Preference model.
- In general, the deviations are significant; which suggests that prior analysis of consumer travel behavior is needed in order to decide how best to include distance. If this analysis cannot be made or it is too costly, the results indicate that the best assumption to use is the one behind the traditional MAXCAP model as it is the one which gives the smallest deviation in demand captured (on average, less than 10%) whatever the true one is.

The second issue for the incorporation of Consumer Choice Behavior theories in Discrete Competitive Location models, is the analysis of which stores attributes (other than distance) should be included in these models. **Chapter 4**, presents a new methodology to determine which store attributes should be included and how these parameters ought to be reflected, as these will be different in each scenario.

The methodology involves the determination of the parameter r_{ij} (i.e., the proportion of capture that a shop in *j* will achieve by consumer's zone *i*) included in the Maximum Capture Models, using the Multiplicative Competitive Interaction (MCI) model. The methodology has been performed to the supermarket sector in two scenarios: Milton Keynes (in Great Britain) and Barcelona (in Spain). This methodology involves basically four steps. First of all, it is necessary to design and develop a survey of consumer supermarket – choice behavior. Secondly, the supermarket's key attributes will be estimated through a Factor analysis applied to the survey database. Third, the factors found in the previous analysis will be used as a variables for the specification of the MCI model. And finally, the MCI model (i.e. parameter r_{ij}) will be calibrated by applying the ordinary least squares methods on the log – transformed centered form of an specified MCI equation.

After the introduction of Consumer Choice Theories in the Discrete Competitive Location models, I have improved these models with an additional element. **Chapter 5** has presented the New Chance – Constrained Maximum Capture Location Problem, which introduces, a part of a gravity model, the issue of minimum requirements to survive in a given spatial setting. This threshold requirement has been introduced as an stochastic constraint. The model presented in **chapter 5** is particularly relevant to private retail sector setting because it takes into account two real characteristics of the market. First of all, the capture is determined by a gravity model, which is a revealed preference model; and secondly, the model includes a threshold constraint which reflex the fact that a facility cannot be open if the demand captured is below a threshold level.

The behavior of the model has been computed in the well known example of Swain. From these analyses, I can conclude that the model is quite robust to the stochastic condition and it behaves as it was expected.

Finally, **chapter 6** presents two metaheuristics to solve the models presented in chapter 3 and chapter 5, as they belong to the NP-Hard problems. Following recent applications, the metaheuristics developed in this thesis are composed by two stages; in the first stage, a good initial solution is constructed using a Problem – Space Metaheuristic. And in the second one, the previous solution is improved using a Local Search Metaheuristic.

A Metaheuristic based on GRASP and TABU Search metaheuristics has been developed to solved the models defined in chapter 3. This metaheuristic behaves very well in finding the optimal locations, as only 6 of the 360 runs computed were non-optimal and the average deviation from optimality did not exceed 1%, except in one case where the deviation was equal to 7.6%. The algorithm also becomes efficient in terms of computational time, when the network size is greater than 30 nodes or, 3 or more entering outlets need to be located.

The metaheuristic developed to solve the New Chance – Constrained Maximum Capture Location Problem is one based on on MAX – MIN Ant System and TABU system. It is the first time that the MAX – MIN Ant system is adapted to solve a location problem. As it was expected, the stochastic condition of the model has arisen the difficulty to find the optimal solutions. With this metaheuristic, near-optimal solutions have been found with a minimal deviation (for example, for uncorrelated cases, 41 out of 270 runs were non-optimal based on the comparison with complete enumeration, and the maximum average deviation from optimality did not exceed 3.3 %). The algorithm also becomes efficient in terms of computational time, when 3 or more entering outlets need to be located.

Although the research presented in this thesis will be useful to the retail sector, the rapid change of the business environment highlights new lines of research for this field. **Future research** will focus on the effect of *e-commerce* in the retail location decisions. This can be analyzed from two perspectives. First of all, models can be modified to take into account that consumer store-choice behavior can be changed with the possibility of shopping from their computer. Secondly, up till now, all the Store Location models were based on the assumption that consumers go to the shop. With the e-commerce, this assumption is not absolutely true because, in this new business environment, part of the business is done in the reverse way; i.e. the stores go to the consumer’s houses.

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References

- Achabal, D., Gurr, W.L. and Mahajan, V. "MULTILOC: A Multiple Store Location Decision Model"**.** *Journal of Retailing,* Summer 1982, v58, pp5-25.
- Agustina, L. "¿Alcanzará Caprabo a Pryca y Continente?". *Actualidad Económica,* 17th to 23th of May, 1999.
- Ashish, S. and Smith, T.E. *Gravity Models of Spatial Interaction Behavior*. New York: Springer-Verlag. 1997.
- Balakrishnan, P. and Storbeck, J. "Mctresh: Modeling Maximum Coverage with Threshold Constraint". *Environment & Planning B,* 18, 1991, pp. 459-472.
- Beaumont, J.R. "Location Allocation Models and Central Place Theory", *Spatial Analysis and Location Allocation Models,* A. Ghosh and G. Rushton, Editors, NY, Van Nostrand Reinhold, 1987.
- Benati, S and Laporte, G. "Tabu Search algorithms for the (r | X_p)- Medianoid and (r | p)-Centroid Problems". *Location Science*, 2(4), 1994, pp. 193-204.
- Berry, B.J. and Garrison, W. "Recent developments of Central Place Theory". *Papers and Proceedings of the Regional Science Association,* 4, 1958, pp.107-120.
- Black, W.C., Osthind, L.E. and Westbrook, R.A. "Spatial Demand Models in an Intrabrand Context"**.** *Journal of Marketing,* v49, Summer 1985, pp.106-113.

- Brandeau, M.L. and Chiu, S.S. "An Overview of Representative Problems in Location Research". *Management Science,* 35, 1989, pp. 645 – 674.
- Burn, D. J. "Image Transference and Retail Site Selection"**.** *International Journal of Retail & Distribution Management,* Sept-Oct, v20 n5, 1992, pp. 38-46.
- Cerny, V. "A thermodynamic approach to the traveling salesman problem: an efficient simulated annealing algorithm". *Journal of Optimization Theory and Application,* 45, 1985, pp. 41-51.
- Christaller, W. *Central Places in Southern Germany,* 1933, translated in 1966, Prentice – Hall, NJ.
- Church, R. "The Planar Maximal Covering Location Problem". *Journal of Regional Science,* Vol. 24 (2), 1984, pp. 185-201.
- Church, R. and ReVelle, C. "The Maximal Covering Location Problem". *Papers of the Regional Science Association,*32, 1974, pp. 101-118.
- Clarkon, R.M., Clarke-Hill, C.M. and Robinson, J. "UK Supermarket Location Assessment". *International Journal of Retail & Distribution Management,* v24 n6, 1996, pp. 22-33.
- Cohen, S. and Applebaum, W. "Evaluating Store Sites and Determining Store Rents". *Economic Geography,* 36, 1960, pp. 1-35.
- Collins, N.E., Eglese, R.W. and Golden, B.L. "Simulated Annealing – An annotated bibliography". *American Journal of Mathematic and Management Sciences,*vol. 9, 1988, pp. 209-307.
- Colorni, A. Dorigo, M. and Maniezzo, V. "Distributed optimisation by Ant Colonies", *Proceeding of ECAL91 – European Conference on Artificial Life:* Elsevier Publishing, Paris, France, 1991, pp. 134-142.

- Colorni, A. Dorigo, M. and Maniezzo, V. "The Ant System: Optimisation by a Colony of Cooperating Agents", *IEEE Transactions on Systems, Man and Cybernetics – Part B,* 26, 1, 1991, pp. 29-41.
- Connolly, D. "An improved annealing scheme for the QAP". *European Journal of Operational Research,* 46, 1990, pp. 93-100.
- Converse, P.D. "New Laws of Retail Gravitation". *Journal of Marketing* 14, 1949, pp.379-384.
- Cooper, L. "Location – allocation Problems". *Operations Research,* 11, 1963, pp. 331-343.
- Craig, C.S., Ghosh, A and McLafferty, S. "Models of the Retail Location Process: A Review". *Journal of Retailing,* v60, n1, Spring 1984, p5-36.
- Current, J. and Storbeck, J. "A Multiobjective Approach to Design Franchise Outlet Networks", *Journal of Operational Research Society,* 45, 1994, pp.71-81.
- Davies, J.B. and Flemmer, M. Consumer Behaviour in the European Union. *In the book: The internationalisation of Retailing edited by Gary Akehmist and Nicholas Alexander,* Frank Cass London, 1995, p177-190.
- Diez de Castro, E. *Distribución Comercial.* McGRaw Hill, second edition, 1997.
- Dobson, G. and Karmarkar, U. "Competitive Location on a Network", *Operations Research,* 35, 1987, pp. 565 – 574.
- Dorigo, D. C. "The ant colony optimization meta-heuristic". In D.Corne, M.Dorigo and F.Glover (eds), *News Ideas in Optimisation,* 1999, MsGraw-Hill.
- Dorigo, M. Maniezzo, V. and Colorni, A.. "The ant system: Optimisation by a colony of cooperating agents", *IEEE Transactions on Systems, Man and Cybernetics –* Part B, 26(1), 1996, pp. 29-42.

- Drezner, T. “Locating a single new facility among existing unequally attractive facilities”. *Journal of Regional Science*, vol. 34, nº2, 1994a, pp. 237-252.
- Drezner, T. “Optimal Continuous Location of a Retail Facility, Facility Attractiveness, and Market Share: an Interaction Model”. *Journal of Retailing*, 70, 1994b, pp. 49-64.
- Drezner, Z. *Facility Location: A survey of Applications and Methods.* Springer Series in Operation Research, 1995.
- Drezner, T. and Drezner, Z. “Competitive Facilities: market share and location with random utility”. *Journal of Regional Science*, vol. 36, nº1, 1996, pp. 1-15.
- Drezner, T. and Drezner, Z. “Location of Multiple Retail Facilities in a Continuous Space”, recently in European Journal of Operation Research, 2002a.
- Drezner, T. and Drezner, Z. “A Threshold – Satisfying Competitive Location Model”. In August 2002 (2002b) in the Journal of Regional Science.
- Eiselt, H.A. “Hotelling’s Duopoly on a Tree”, *Annals of Operations Research*,40, 1992, pp. 195-207.
- Eiselt, H.A. and Laporte, G. “Location of a new facility on a linear market in the presence of weights”. *Asia-Pacific Journal of Operational Research* 5, 1988, pp.160-165.
- Eiselt, H.A. and Laporte, G. “The Maximum Capture Problem in a Weighted Network“. *Journal of Regional Science* 29(3), 1989, pp. 433-439.
- Eiselt, H.A. and Laporte, G. “The Existence of Equilibria in the 3-facility Hotelling Model in a Tree”, *Transportation Science*, 27, 1993, pp. 39-43.
- Eiselt, H.A., Laporte, G. and Thisse, J.H. “Competitive Location Models: a Framework and Bibliography”, *Transportation Science*,27, 1993, pp. 44-54.

- Feo, T. and Resende, M. "A probabilistic heuristic for a computationally difficult set covering problem". *Operations Research Letters* 8, 1989, pp. 67-71.
- Feo, T. and Resende, M. "Greedy Randomized Adaptive Search Procedures". *Journal of Global Optimization.* 6, 1995, pp. 109-133.
- Feo, T; Resende, M and Smith, S.H. "A Greedy Randomized Adaptive Search procedure for maximum independent set". *Operations Research* 42(5), 1994, pp. 860-878.
- Friedrich, C.J. *Alfred Weber's Theory of Location of Industries,* University of Chicago Press, 1929, Chicago.
- Friez, T., Miller, T. and Tobin, R. "Competitive Network Facility location Models: A Survey". *Papers of the Regional Science Association*, v65, 1988, p47-57.
- Gendreau, M.; Hertz, Z. and Laporte, G. "A Tabu Search Heuristic for the Vehicle Routing Problem". *Management Science* 40(10), 1994, pp.1276-1289.
- Ghosh, A. "Parameter Nonstationarity in Retail Choice Models", *Journal of Business Research*,12, 1984, pp. 425-36.
- Ghosh, A. and Craig, C.S. "Formulating Retail Location Strategy in a Changing Environment". *Journal of Marketing*, v47, 1983, pp. 56-68.
- Ghosh, A. and Craig, C.S. "A Location Allocation Model for Facility Planning in a Competitive Environment"**.** *Geographical Analysis*, 16, 1984, pp. 39-51.
- Ghosh, A. and Craig, C.S. "An Approach to Determining Optimal Locations for New Services"**.** *Journal of Marketing Research*, 23, 1986, pp. 354-362.
- Ghosh, A. and Craig, C.S. "FRANSYS: a Franchise Location Model"**.** *Journal of Retailing,* 67, 1991, pp. 212-234.

- Ghosh, A. and Harche, F. “Location – allocation Models in the Private Sector: Progress, Problems, and Prospects”**.** *Location Science,* 1, 1993, pp. 81-106.
- Ghosh, A. and McLafferty, S. *Location Strategies for Retail and Service Firms,* Lexington, Mass., Lexington Books, 1987.
- Ghosh, A. and Rushton, S. “Progress in Location Allocation Modelling” *Spatial Analysis and Location Allocation Models,* A- Ghosh and G. Rushton, Editors, NY, Van Nostrand Reinhold, 1987.
- Glover. F. “Tabu Search, part I“. *ORSA Journal of Computing,* 1, 1989, pp.190-206.
- Glover. F. “Tabu Search, part II“. *ORSA Journal of Computing,* 2, 1990, pp. 4-32.
- Goldberg, D.E. *Genetic algorithms in search, Optimisation and Machine Learning,* Addison – Wesley, New York, 1989.
- Goodchild, M.F. “ILACS: a Location Allocation Model for Retail Site Selection”. *Journal of Retailing,* 60, 1984, pp. 84-100.
- Golden, B., Bodin, L., Doyle,T. and Stewart, J. “Approximate Traveling Salesman Algorithm“. *Operation Research* 28, 1980, pp. 694-711.
- Gordon, C., Bruner, I and Hensel, P.J. *Marketing Scales Handbook. A Compilation of Multi-Item Measures.* American Marketing Association, Chicago, Illinois, USA, 1993.
- Hair, J., Anderson, R., Tatham, R. and Black, W. *Multivariate Data Analysis.* FifthEdition, Prentice Hall International, Inc., 1998.
- Hakimi, S.L. “Optimal Location of Switching Centers and the Absolute Centers and Medians of a Graph“. *Operations Research* 12, 1964, pp. 450-459.
- Hakimi, S.L. “P-median theorems for competitive location“. *Annals of Operation Research* 5, 1986, pp. 79-88.

- Hakimi, S.L. “Locations with Spatial Interactions: Competitive Locations and Games”. In *Discrete Location Theory*, New York, 1990, pp. 439-478.
- Hansen, P., Labbé, M., Peeters, D. and Thisse, J.F. “Facility Location Analysis”**.** *Fundamentals of Pure and Applied Economics*, v22, 1987, pp. 1-70.
- Hertz, A. “Tabu search for large scale time tabling problems”. *European Journal of Operational Research*, 51, 1991, pp. 39-47.
- Hillsman, E.L. “The *p* – median Structure as a Unified Linear Model for Location – allocation Analysis”, *Environment and Planning A*, 16, 1984, pp. 305-318.
- Hodgson, M.J. “Toward more realistic allocation in location-allocation models: an interaction approach“. *Environment and Planning A* 10, 1978, pp. 1273-1285.
- Hodgson, M.J. “A Location – Allocation Model Maximizing Consumers’ Welfare“. *Regional Studies* 15, 1981, pp. 493-506.
- Hogan, B.A. “What drivers shoppers’ decisions supermarket News”**.** *Supermarket News*, May 6, 1996, v46, n19, pp. 50-52.
- Holland, J.H. *Adaptation in Natural and Artificial Systems.* The University of Michigan Press, Ann Arbor, U.S.A, 1975.
- Hotelling, H. “Stability in Competition”, *Economic Journal*, 1929, Vol. 39, pp 41-57.
- Huff, D. “Defining and Estimating a Trading Area“. *Journal of Marketing* 28, 1964, pp. 34-38.
- Hunt, J. “The Big Squeeze (Food Retailing in the United Kingdom)”. *Grocer*, v219 n7308, 1997, pp. 38-44.
- Hutcheson, G.D. and Moutinho, L. “Measuting Preferred Store Satisfaction Using Consumer Choice Criteria as a Mediating Factor”**.** *Journal of Marketing Management*, v14, 1998, pp. 705-720.

- Jain, A.K. and Mahajan, V. “Evaluating the Competitive Environment in Retailing using Multiplicative Competitive Interactive Model“. *Research in Marketing*, v2, 1979, pp. 217-235.
- Kariv, O. and Hakimi, S.L. “An Algorithmic Approach to Network Location Problems II: The p-Medians“. *SIAM Journal on Applied Mathematics.* 37, 1979, pp. 539-560.
- Karkazis, J. “Facilities location in a competitive environment: A promethee based multiple criteria analysis“. *European Journal of Operation Research,* 42, 1989, pp. 294-304.
- Kinnear, T.C. and Taylor, J.R. *Marketing Research: An applied Approach.* McGraw-Hill Series in Marketing, 1987.
- Kirkpatrick, S., Gelatt, C.D. and Vecchi, P.M. “Optimisation by Simulated Annealing”, *Science*, 220, 1983, pp. 671-680.
- Klincewicz, J.G. “Avoiding Local Optima in the p-Hub location problem using Tabu Search and GRASP“. *Annals of Operations Research,* 40, 1992, pp. 283-302.
- Kontoravdis, G. and Bard, J.F. “A GRASP for the Vehicle Routing Problem with Time Windows”, *ORSA Journal on Computing,* Vol. 7, 1995, pp. 10-23.
- Koulamas, C., Antony, S.R. and Jean, R. “A survey of simulated annealing application to operation research problems”, *OMEGA*, 22, 1994, pp. 41-56.
- Lederer, P.J. “Duopoly Competition in Networks”. *Annals of Operations Research,* v6, 1986, pp. 99-109.
- Lederer, P.J. and Thisse, J-F. “Competitive Location on Networks Under Delivered Pricing”, *Operations Research Letters,* 9, 1990, pp. 147 – 153.

- Lehmann, D. R., Gupta, S. and Steckel, J.H. *Marketing Research.* Addison-Wesley, 1998.
- Li, Y.; Pardalos, P.M. and Resende, M. "A Greedy Randomized Adaptive Search Procedure for the Quadratic Assignment Problem". *DIMACS Series in Discrete Mathematics and Theoretical Computer Science.* Vol. 16, 1994, pp. 237-261.
- London, D and Della, A.J. Chapter 20: Purchasing Processes**.** *Consumer Behaviour: Concepts and Applications.* McGraw-Hill Series in Marketing, 1988.
- Lösch, A. *The Economics of Location,* Yale University Press, New Haven, 1954.
- Lourenço, H.R. and Zwijnenburg, M. Combining the large – step optimization with tabu search: application to the job – shop scheduling problem, in *Meta-heuristics: Theory and Applications.* I.H. Osman and J.P. Kelly, Kluwer Academic Publishers, 1996.
- Lourenço, H. and Serra, D. "Adpative Search Heuristics for The Generalised Assignment Problem". *Economic and Business Working Paper number 288,* 1998, Universitat Pompeu Fabra, Barcelona.
- Louviere, J. J. and Gaeth, G. J. "Decomposing the Determinants of Retail Facility Choice Using the Method of Hierarchical Information Integration: A Supermarket Illustration". *Journal of Retailing,* v63 n1, spring 1987, pp. 25-48
- Louviere, J.J. "Using Discrete Choice Experiments and Multinomial Logit Choice Models to Forecast Trial in Competitive Retail Environment: a Fast Food Restaurant Illustration", *Journal of Retailing,* 60, 1984a, pp. 81-107.
- Louviere, J.J. "Hierarchical Information Integration: A New Method for the Design and Analysis of Complex Multiattribute Judgement Problems". *Advances in*

Consumer Research, Provo Utah: Association for Consumer Research, 1984b, pp. 148-155.

- Lowe, J.M. and Sen, A. “Gravity model applications in health planning: analysis of an urban hospital market”. *Journal of Regional Science*, vol. 36, nº3, 1996, pp.437 – 461.
- Marianov, V and Serra, D. “Probabilistic, Maximal Covering Location – Allocation Models for Congested Systems”. *Journal of Regional Science*, Vol. 38 (3), 1998, pp. 401-24.
- Marianov, V., Serra, D., and ReVelle, C. “Location of Hubs in a Competitive Environment“. *European Journal of Operational Research*, Vol. 114, Iss. 2, Ap. 16, 1999, pp. 363 - 371.
- Mehrez, A. and Stulman, A. “The Maximal Covering Location Problem with Facility Placement on the Entire Plane”. *Journal of Regional Science*, Vol. 22 (3), 1982, pp. 361-65.
- McFadden, D. “Conditional Logit Analysis of Quantitative Choice Behavior”, *Frontiers in Economics,* P. Zarembkar, Editor, New York: Academic Press, 1974.
- McGoldrick, P.J. *Retail Marketing.* McGraw-Hill Book Company, 1990.
- Miller, T.; Friesz, T. and Tobin, R. *Equilibrium Location of Networks.* NY, Springer, 1996.
- Muñoz, J. “Competitive Location of Two New Facilities with Rectilinear Distance”, B. Pelegrin, Editor, *Proceedings of IIIrd Meeting of the Euro Working Group on Locational Analysis,* Sevilla, Spain, 1988.

- Nakanishi, M. and Cooper, L. “Parameter Estimation for a Multiplicative Competitive Interaction Model - Least Squares Approach“. *Journal of Marketing Research*, Vol. XI (August 1974); pp. 303-11.
- Okabe, A., Boots, B. and Sugihara, K. *Spatial Tessellations - Concepts and Applications of Voronoi Diagrams*. Chischester, England: John Wisley & Sons, 1992.
- Orgel, D. “Consuming Issues: Responses to a SN Consumer Survey show Preferences about Supermarket Shopping and Reveal Merchandising Opportunities”. *Supermarket News,* August 25, v47 n34, 1997, pp. 1-4.
- Osman, I.H. “Metastrategy simulated annealing and tabu search algorithms for the vehicle routing problems”. *Annals of Operations Research,* 41, 1993, pp.421 – 451.
- Osman, I.H. “Heuristics for the generalized assignment problem: simulated annealing and tabu search approaches”. *OR Spektrum,* 17, 1995, pp. 211-225.
- Osman, I.H. and Christofides, N. “Capacitated clustering problems by hybrid simulated annealing and tabu search”. *International Transaction in Operational Research,* 1, 1994, pp. 317-336.
- Osman, I.H. and Kelly, J.P. “Meta-heuristics: Theory and Applications“. Kluwer Academic Plubishers, 1995.
- Pau, J. and Navasmes, R. *Manual de Logística Integral*. Diaz de Santos, 1998.
- Parker, B.R. and Srinivasan, V. “A Consumer Preference Approach to the Planning of Rural Primary Health-Care Facilities”, *Operations Research,* 24, 1976, pp. 991-1029.

- Pirkul, H., Narashimhan and P. De "Firm Expansion Through Franchising: a Model and Solution procedure", *Decision Sciences,* 18, 1987, pp. 631-45.
- Pirlot, M. "General Local Search heuristics in Combinatorial Optimisation: A tutorial". JORBEL, *Belgian Journal of Operations Research, Statistics and Computer Science,* Vol. 32, nº 1-2, 1992 pp.7-67.
- Reeves, C.R. Genetic Algorithms and combinatorial optimisation, in *Applications of Modern Heuristics Methods,* Ed. V. Rayward – Smith, Alfred Waller Ltd, in association with UNICOM, Henley – on – Thames, 1995.
- Reilly, W.J. *The Law of Retail Gravitation.* New York, Knickerbocker Press, 1929.
- Resende, M. and Feo, T. "A GRASP for Satisfiability". *DIMACS Series in Discrete Mathematics and Theoretical Computer Science.* Vol. 26, 1996.
- Resende, M., Pardalos, P. and Li, Y. "Algorithm 754: Fortran Subroutine for Approximate Solution of Dense Quadratic Assignment Problems Using GRASP". *ACM Transactions on Mathematical Software.* Vol. 22, nº 1, March 1996, pp. 104-118.
- ReVelle, C. "The Maximum Capture or " Sphere of Influence" Location problem: Hotelling revisited on a Network ". *Journal of Regional Science,* Vol.26, nº2 1986, pp. 343-358.
- ReVelle, C. and Swain, R. "Central Facility Location". *Geographical Analysis,* 1970, pp.30-42.
- Rolland, E.; Schilling, D.A. and Current, J.R. "An efficient Tabu Search procedure for the p-Median problem". *European Journal of Operation Research* 1996(2), *pp*.329-342.

- Rosing, K.E. “An Empirical investigation of the Effectiveness of a Vertex Substitution Heuristic”. *Environment and Planning: Series B – Planning & Design* 1997, vol 24, Iss 1, pp 59-67.
- Rosing, K.E. and ReVelle, C. “Heuristic Concentration: A Metaheuristic for Combinatorial Location Problems“. *Studies in Locational Analysi,*. n9, 1996, pp.109-112.
- Rosing, K.E. and ReVelle, C. “Heuristic Concentration: Two Stage Solution Construction“. *European Journal of Operations Research* 1997, pp.75-86.
- Rosing, K.E., ReVelle, C.S., Rolland, E., Schilling, D.A. and Current, J.R. “Heuristic Concentration and Tabu Search: A Head to Head Comparison“. *European Journal of operational Research* 1998, pp. 104(1).
- Santos-Peñate, D.R.; Suárez-Vega, R. and Dorta-González, P. “Localización competitiva con criterios basados en funciones de atracción”. *XXIII Congreso Nacional de Estadística e Investigación Operativa.* Valencia 11-14 de marzo de 1997.
- Serra, D. “The Coherent Covering Location Problem”. *Papers in Regional Science,* Vol. 75 (1), 1996, pp. 79-101.
- Serra, D and Marianov , V. “The p-Median problem in a Changing Network: The Case of Barcelona”. *Location Science*. Vol. 6, nº4, 1999, pp. 383-394.
- Serra,D. and ReVelle, C. “Market Capture by two Competitors: The Preemptive Location Problem“. *Journal of Regional Science*, 1994, Vol. 34, nº4, pp.549-561.
- Serra, D and ReVelle, C. “Competitive Location in networks”, in Z. Drezner (ed.): *Facility Location. A survey of Applications and Methods*, 1996, Springer.

- Serra, D., Marianov, V. and ReVelle, C. "The Hierarchical Maximum Capture Problem". *European Journal of Operational Research,* 1992, 62, 3.
- Serra, D., Ratick, S and ReVelle, C. "The Maximum Capture problem with uncertainty". *Environment and Planning B,* 1996, v62, pp.49-59.
- Serra, D., ReVelle, C. and Rosing, K.E. "Surviving in a Competitive Spatial Market: The Threshold Capture Model". *Journal of Regional Science,* Vol. 39, nº4, 1999, pp. 637-652.
- Serra, D., Eiselt, H.A., Laporte, G. and ReVelle, C. "Market Capture Models under Various Customer Choice Rules". *Environment and Planning B,* 26(5), 1999, pp. 141-150.
- Shonkwiler, J. and Harris, H.. "Rural Retail Business Thresholds and Interdependencies", *Journal of Regional Science,* 36, 1996, pp. 617-630.
- Stüzle, T. "An ant approach for the flow shop problem". *In proceeding of the 6th European Congress on Intelligent Techniques & Soft Computing (EUFIT'98),* 3, 1998, pp. 1560-1564.
- Stüzle, T. "Local Search Algorithms for Combinatorial Problems – Analysis, Improvements, and New Applications". PhD thesis, Department of Computer Science, Darmstadt University of Technology, Germany, 1998.
- Stüzle, T. "MAX-MIN Ant System for the Quadratic Assignment Problem". *Technical Report AIDA-97-4, 1997, FG Intellektik, TU Darmstadt, Germany.*
- Stüzle, T. and Hoos, H. "Max-Min Ant System and Local Search for Combinatorial Optimisation", in S.Voβ, S. Martello, I.H. Osman and C. Roucairol (eds.), *Meta-Heuristics: Trends in Local Search paradigms for Optimisation,* 1999, Kluwer Academic Publishers, pp. 313-329.

- Swain, R. “A parametric descomposition algorithm for the solution of uncapacited location problems“. *Management Science,* 21, 1974, pp. 189-198.
- Taillard, E. “Some efficient heuristic methods for the flow shop sequencing problem”. *European Journal of Operational Research* 47, 1990, pp. 65-74.
- Teitz, M.B. and Bart, P. “Heuristic methods for estimating the generalized vertex median of a weighted graph“. *Operations Research,* 16, 1986, pp. 955-965.
- The European editorial, “Hyper Growth in Food Sales (Small Retail Traders lose out to Supermarkets in Europe)”, *The European,* n411, April 6, 1998, pp. 61.
- Tobin, R and Friesz, T. “Spatial Competition Facility Location Models: Definition, Formulations and Solution Approach”. *Annals of Operations Research*, v6, 1986, pp. 49-74.
- Toregas, C.R. and ReVelle, C. “Optimal Location Under Time or Distance Constraints”, *Papers of the Regional Science Association,* 28, 1972, pp. 133-144.
- Vandell, K.D. and Carter, C.C. “Retail Store Location and Market Analysis: A Review of the Research”. *Journal of Real Estate Literature,* n1, 1993, pp. 13-45.
- Weisbrod, G.E., Parcells R.J. and Kern, C. “A Disaggregate Model for Predicting Shopping Area Market Attraction”, *Journal of Retailing,* 60, 1984, pp. 65-83.
- Wendell, R.E. and McKelvey, R.D. “New Perspectives in Competitive Location Theory”, *European Journal of Operational Research,* 6, 1981, pp. 174-182.
- Witting, D.R. and Cattin, P. “Commercial Use of Conjoint Analysis: an Update”, *Journal of Marketing,* 53, 1989, pp. 91-96.