Industrial Organization, Trade and Social Capital

Doctoral Thesis

by

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Academic Year 2006/2007

Dipòsit legal: B.53871-2007
ISBN: 978-84-691-1751-4

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Contents

Acknowledgements

I am extremely grateful to my advisor Antonio Cabrales for constant encouragement and very helpful suggestions and discussions. I also owe much to Antonio Ciccone, Jaume Ventura, Gino Gancia and to seminar and workshop participants at UPF, who discussed with me part of this work and were important for the development of many ideas. I am also grateful to Urs Schweizer, Avner Shaked and Jörg Oechssler, who were important for me during the time I spent at the University of Bonn, as well as to my current colleagues in Padua, whose support and comments I enjoyed over the last two years. In Bonn I entered the European Doctoral Program in Quantitative Economics (EDP), from whose various Jamborees across Europe I also benefited much.

One chapter of my dissertation is based on an article written with Angelo Antoci and Pier Luigi Sacco, and now published in Benedetto Gui and Robert Sugden (eds.), 'Economics and Social Interaction: Accounting for Interpersonal Relations', Cambridge: Cambridge University Press, 2006. Another chapter is a substantial revision of an article written with Facundo Albornoz, and appeared as University of Birmingham, Dept. of Economics Discussion Paper N. 12, 2005. Angelo and Pier Luigi have been important for the 'social side' of my research and Facundo for the 'trade side'. They are friends who shared with me the enthusiasm and the ups and downs of joint research.

Several other friends have accompanied me through these years and helped my work through useful discussions. First among them is Volker Reinthaler, with whom I arrived to Barcelona and shared the joy of living in this city, besides a number of fiestas in our flat. Very important were also Katharina Greulich, Chiara Forlati, Burkhard Schipper and several other friends and colleagues, especially from Spain, Italy, Germany and Argentina: thanks to all of them. Helpful suggestions came as well from Benedetto Gui and Robert Sugden, from various conference participants and from a few anonymous referees.

Finally, many more people shared with me these years on the personal side, helping me with constant support, helpful critiques, love and patience. In particular I want to thank my parents and my brother (who also shared with me a great time in Barcelona), my various flatmates and my great vecinas in C/ Nápoles 195, and especially Cecilia Tossounian, who danced with me a hard tango in Barcelona and now shares my life in Italy (to Facundo I am not only indebted as a co-author and friend, but also for introducing me to her!).

Abstract

This thesis applies game theoretic tools to the investigation of the dynamic effects of several forms of local interaction externalities. It consists of three chapters. The first is devoted to study repeated interaction among oligopolits in markets with asymmetric information; the second focuses on interaction among monopolistic competitors facing foreign competition and on their learning externalities; and the third investigates the patterns of private and social capital accumulation arising from individual choices in an environment with social interaction externalities.

In particular, the first chapter, "Competition and Reputation", investigates a dynamic oligopoly model with endogenous entry, quality selection of an experience good and repeated market interaction, in which prices may be used as quality signals. In this setting, reputation and competition may drive out of the market those firms that do not comply with their quality promises. One may thus presume that competitive pressure improves average market quality. I show that the opposite may be true. Cheating firms may enter the market, fool even rational consumers, and exit the market when discovered, implying a failure of the basic reputation mechanism and a likely increasing time path of prices. Markets for closer substitutes tend to have a lower initial average quality and less trusting consumers, whereas the number of competitors has no clear relationship with average quality.

The second chapter, "Trade Policy and Industrial Structure", presents an international trade model with heterogeneous firms and dynamic learning externalities, and studies how the relative costs and benefits of protection and liberalization depend on the initial industrial structure. In particular, it shows that under certain conditions the relative benefits from protection increase in the number and quality of local backward firms, but decrease in the quality of local advanced firms.

The last chapter, "Economic Growth and Social Development", develops a growth model with both private and social capital accumulation and investigates the conditions under which these two processes may either move in the same direction or conflict with each other. In particular, it focuses on strategic complementarity in social participation and on its dynamic effects on social capital accumulation, finding that a growing economy may fall into a social poverty trap.

Chapter 1

Competition and Reputation

1.1 Introduction

Consider two situations: the purchase of an experience good and advance payment trade (for instance in e-commerce). One thing they have in common is that the buyer must trust the seller: confidence is needed that either the effective quality (say, the good's duration) corresponds to the expected (and paid for) one, or that the good will indeed be delivered at the promised time and with the promised characteristics. Lack of trust may cause loss of efficient trade opportunities. In general terms, one may say that compliance with promises by the seller depends on punishment threats and on efficiency payments (on stick and carrot). Such incentives may be provided by an external authority, say, the state with its judicial system. Yet, in some cases compliance is hard to verify and, even when it is verifiable, state enforcement is generally costly. Alternatively, market interaction might autonomously shape the adequate incentives. One obvious way is through repeated interaction and reputation mechanisms. For instance, if buyers pay a price premium for high quality, if they are informed on past compliance and never buy from a seller who cheated on quality in the past, the present value of the stream of future profits granted by compliance may be higher than the one period deviation gain that can be obtained by cheating consumers, so that the seller is indeed induced to be trustworthy.

This mechanism has been proposed by Klein and Leffler (1981) and formally investigated by Shapiro (1983), who shows that low and rising prices guarantee high quality in a competitive market, because premiums for high quality ensure that no firm has an incentive to cut on quality and cheat the market, but competition for such premiums induces firms to set initially low, loss-making prices, which correspond to an investment in reputation, to which later profits are the normal market return. Yet, this result relies on consumers' expectations on new products' quality, which are on average wrong: it is the fact that consumers (irrationally) mistrust new products' quality that forces firms to signal high quality through low prices. Milgrom and Roberts (1986) overcome this limitation and show that a monopolist may signal the exogenous quality of a newly introduced experience good to rational consumers through low and rising prices, possibly used together with dissipative advertising. In their model, prices are initially low, because consumers' initial uncertainty about their own preferences lowers initial demand, and in turn initial demand is positively related to the subsequent demand from repeated purchase. With no uncertainty about preferences, Bagwell and Riordan (1991) argue that high and declining prices may guarantee high quality, because in principle a high quality monopolist might separate itself from a low quality one through either low or high initial prices, but only high introductory prices are intuitive, in the sense of Cho and Kreps (1987), and should therefore be expected; moreover, prices decline over time as information about quality spreads among consumers, reducing the signaling distortion. Linnemer (2002) extends Bagwell and Riordan (1991) by allowing the monopolist to use both price and advertising signals, and shows that only (high) prices are used for new products, both signals are used at intermediate levels of information diffusion and no signaling is needed for mature products. Hertzendorf and Overgaard (2001) and Fluet and Garella (2002) neglect information diffusion and repeated purchase, but extend the analysis from monopoly to duopoly[1]

One limitation of these approaches, which do not rely on consumers' irrationality, is that they restrict attention to exogenously given market structures, namely monopoly and duopoly, and they do not allow firms to choose quality. Thus, one cannot use these models to study suppliers' incentive to comply with promises and how market structure and reputation interact with one another. To tackle these questions, I display a full-fledged dynamic oligopoly model, in which both market structure and sellers' trustworthiness are determined in equilibrium.

Research in this direction has not progressed much in two decades for the obvious reason that such a model is complicated. Yet, on one side, sorting out these complications seems to be necessary, if one thinks that the degree of market competition may be fundamental to determine incentives for high quality provision[2]. Indeed, competition may both lower monopoly rents, and

[1]Hertzendorf and Overgaard (2001) consider a vertically differentiated duopoly, in which production costs are constant across qualities (and normalized to zero), and show that the high quality producer uses (high) price signals alone when quality differentials are high, and both price and advertising when quality differentials are low. Fluet and Garella (2002) consider the four possible combinations of the two firms' quality, assume that high quality is more costly to produce than low quality, and characterize the parameter space for which separating equilibria exist and the kind of signal mix that is used. In both papers quality is not chosen by firms, but rather randomly determined by Nature.

[2]A clear indication that, when prices signal quality, competition may have non standard effects comes from Overgaard (1994), who shows that potential entry by an uninformed high quality firm may inefficiently distort the incumbent price upwards, because it strengthens a low quality incumbent's incentive to convince both consumers and the potential entrant that its quality is high, in order to deter entry, and therefore forces a high quality incumbent to distort its price further up to effectively separate. In the present work entry is simultaneous,

thus weaken the carrot, and offer buyers more alternatives, and thus strengthen the stick. Moreover, competition itself depends on entry and exit, which depend on expected profits and therefore on returns to reputation and on reputation building costs (besides standard entry and exit costs). On the other side, this paper shows that one can proceed a long way into the analysis of such a dynamic oligopoly model, and that results have a clear economic interpretation.

In order to focus on market structure and quality choice, I consider price signals of quality alone and disregard the role of advertising and brand name investment. I consider a game with four stages: entry, quality selection (of an experience good) and twice repeated market interaction (repetition allows for reputation accumulation). In this game, the relationship between competition and reputation is hard to assess in general terms, because it crucially depends on consumers' expectations and on firms' introductory pricing strategies, so multiple equilibria are possible. I restrict attention to the case in which low quality products cannot be sold at profitable prices if recognized. Then separation through high prices becomes impossible, because for low quality firms it is always profitable to mimic such prices; and the possibility of separation through low prices depends on whether the quality differential is sufficiently high as to grant enough profits from repeat purchase as to compensate initial losses. In either case, consumers observe a unique introductory market price. For low quality differentials, this price is uninformative and consumers do not know whether it is set by a high or a low quality firm. Yet, being rational, consumers correctly anticipate equilibrium average market quality, and therefore reduce their demand when they expect an initially low average market quality.

There are many ways in which firms' strategies may specify introductory prices (on and off the equilibrium path of play). As a consequence, different equilibria may result. Unfortunately, usual equilibrium refinements, developed for signaling games, cannot be straightforwardly used, and the application of more general refinements turns out to be prohibitively complex[3]. For this reason, I consider the simplest class of introductory pricing strategies, which requires all firms to set the same price on and off the equilibrium path of play, and the most intuitive one, which requires all firms to pool on the introductory price that is most profitable for high quality firms, given that low quality ones mimic them. For each of these possibilities, and for both low and high quality differentials, a unique equilibrium exists, which allows for comparative statics on the effects of the various parameters on all the endogenous variables.

To mention just a few results, under high quality differentials, low and rising prices guarantee high quality. When quality differentials are low, high and decreasing prices may guarantee high quality, but under intuitive introductory prices the reputation mechanism tends to fail and some firms in equilibrium choose low quality and cheat rational consumers. When cheating firms are dis-

the number of entrants endogenous and firms choose their quality, which makes the model substantially different from Overgaard (1994).

[3] Yet, it can be shown that a reasonable application of forward induction arguments, in the spirit of the intuitive criterion introduced by Cho and Kreps (1987), thus not eliminate this multiplicity.

covered, they are forced out of the market, whereas high quality firms compete for repeat purchase at the mature stage of the market interaction.

This result offers a new explanation to the shake-out phenomenon commonly observed by the product life cycle literature, for instance by Gort and Klepper (1982) and Klepper (1996). This phenomenon is usually attributed to supply rather than demand factors, namely sellers' experimentation with new technologies and varieties, only some of which will turn out to be successful. By contrast, the explanation proposed here relies on information diffusion among buyers and on strategic cheating by sellers.

When this shake-out occurs, the time path of prices is rising, since rational consumers correctly anticipate the lower initial average quality, and therefore their demand rises over time. Interestingly, a higher competitive pressure, due to the fact that different varieties are closer substitutes, reduces initial average market quality and consumers' trust, because it reduces relatively more high quality than low quality firms' profits, due to the fact that high quality firms compete repeatedly. By contrast, no clear relationship between the number of entrants and average quality and trust emerges, because entry by a firm of a given quality makes favors the relative profitability of the other quality[4].

1.2 Model

1.2.1 Structure

I consider a game with the following four-stage structure. At stage one $N \in \mathbb{N} \cup \{\infty\}$ firms simultaneously decide whether to enter the market or not. Each entering firm pays a fixed entry cost $\zeta > 0$, which is sunk after entry, and chooses a different variety of an experience good. Varieties are imperfect substitutes. The number of firms who enter the market is denoted by n. At stage two the n firms on the market simultaneously choose whether to produce high or low quality. The result of these choices is a vector $\mathbf{z} \in \{0,1\}^n$, with $z_j = 1$ meaning that firm j has chosen high quality. Denote $h = \sum_{j=1}^{n} z_j$ the number of high quality firms. Once decided, the quality level remains the same in the two following market stages. To simplify and concentrate only on asymmetric information on consumers' side, I assume that, once chosen, a firm's quality becomes known to all firms on the market, but not to consumers. Consumers may learn a firm's quality either through direct experience with its products or by information extraction from equilibrium price signals. At stage three firms and consumers interact on the market for the first time. They move sequentially: first, firms simultaneously choose prices, determining a price vector $\mathbf{p^1} \in \mathbb{R}^n_+$. Next, having observed $\mathbf{p^1}$, consumers (indeed, a representative consumer) decide how much to demand to each firm, determining the demand vector $\mathbf{q^1} \in \mathbb{R}^n_+$. Stage four is analogous to stage three, but consumer now have additional information: if

[4] Huck et al. (2006) find in an experiment that competition fosters trust. The present paper emphasizes the importance to distinguish between competitive pressure, due to higher product substitutability, and competition in the sense of the number of competitors.

they have consumed a positive quantity of a firm's product, they are fully informed about its quality. Again, first firms simultaneously choose prices and determine the new price vector $\mathbf{p^2} \in \mathbb{R}^n_+$ and then consumers choose the new demand vector $\mathbf{q^2} \in \mathbb{R}^n_+$.

1.2.2 Preferences and technology

Preferences are assumed in such a way as to generate linear demands. This is done by extending a model first presented by Shubik and Levitan (1980) and recently used by Motta (2004). I assume the following expected utility function:

$$U(\mathbf{q}, \mathbf{e}) = \sum_{j=1}^{n} \alpha(e_j) q_j - \frac{n}{2(1+\mu)} \left[\sum_{j=1}^{n} q_j^2 + \frac{\mu}{n} \left(\sum_{j=1}^{n} q_j \right)^2 \right] + y, \tag{1.1}$$

where $\mathbf{e} \in [0,1]^n$ is a vector of beliefs, i.e., its elements are the probability attributed by the representative consumer to the fact that each good is of high quality, conditional on information about previous play of the game (which I omit to write for notational simplicity): $e_j = \Pr\{z_j = 1\}$; $\alpha(e_j)$ reflects the utility value attributed to good j's expected quality, defined as $\alpha(e_j) = \beta + e_j \gamma$, where $\beta \geq 0$ and $\gamma \geq 0$ are parameters: β captures the value attributed to a unit of a low quality good and γ the additional value of a unit of a high quality good; $\mu \in [0, \infty)$ is a parameter capturing the degree of substitutability between different varieties; y is a perfectly competitive outside good, introduced only to make partial equilibrium analysis justified. This representation extends Shubik and Levitan (1980) and Motta (2004) by allowing for imperfect observability and product-specific quality, two features which are absent in the baseline model (which may be seen as corresponding to the special case in which $\forall j,\ \alpha(e_j) = \alpha > 0$, a known parameter).

One feature of this model is that (at interior consumers' choices) market size does not depend upon either the degree of substitutability or the number of products, but only upon average expected quality and average price. To see this, notice that maximization with respect to $\mathbf{q}$ of (2.1) under an exogenous income constraint (and given prices and beliefs) implies the system of n FOC's $\frac{\partial U}{\partial q_j} = \alpha(e_j) - \frac{1}{1+\mu}\left(nq_j + \mu \sum_{i=1}^{n} q_i\right) = p_j$, by inverting which one obtains each product's demand

$$q_j(\mathbf{p}, \mathbf{e}, n) = \frac{1}{n} \left\{ \left[\frac{n + \mu(n-1)}{n} \right] [\alpha(e_j) - p_j] - \frac{\mu}{n} \sum_{i \neq j} [\alpha(e_k) - p_k] \right\}, \tag{1.2}$$

or, in matrix notation, $\mathbf{q}(\mathbf{p}, \mathbf{e}, n) = E(n) \cdot [\alpha(\mathbf{e}) - \mathbf{p}]$, where $E(n)$ is an $n \times n$ matrix with elements $E_{ii}(n) = \frac{n+\mu(n-1)}{n^2}$ and $E_{ik}(n) = -\frac{\mu}{n^2}$, $\alpha(\mathbf{e})$ is the vector of $\alpha(e_j)$'s and $\mathbf{p}$ is the price vector[5]. The size of the market is then $Q(\mathbf{p},\mathbf{e},n) \equiv \sum_{j=1}^{n} q_j(\mathbf{p},\mathbf{e},n) = \frac{1}{n}\sum_{j=1}^{n}[\alpha(e_j) - p_j] = \bar{\alpha} - \bar{p}$. In the special case in which all products are expected to be of the same quality α and have the same price p, individual demands are then clearly $q_j = \frac{\alpha - p}{n}$.

Although I later let consumers' beliefs depend upon the previous history of play, it is useful to see how firms with constant returns to scale react to this demand under exogenous quality expectations. In this case, the general expression of Nash equilibrium prices can be calculated[6]. If all firms have the same marginal cost c and all products are expected to be of the same quality $\alpha(e) > c$, this expression simplifies to

$$p^E(n,e,c) = \frac{n\alpha(e) + [n + \mu(n-1)]c}{2n + \mu(n-1)}, \tag{1.3}$$

which converges to c as $\mu \to \infty$ and further simplifies to the usual monopoly price $\frac{\alpha(e)+c}{2}$ if $n = 1$.

To later consider deviations from equilibrium, notice that if firm j manages to convince consumers that it is the only one offering high quality, i.e., if $e_j = 1$ and $\forall i \neq j,\ e_i = 0$, then $\forall n > 0$ and $\forall \mathbf{p}$ such that $p_j < \alpha(1)$ and $p_i \geq \alpha(0)\ \forall i \neq j$, it holds that $q_j(\mathbf{p},\mathbf{e},n) = \frac{1+\mu}{n+\mu}[\alpha(1) - p_j]$ and $\forall i \neq j,\ q_i(\mathbf{p},\mathbf{e},n) = 0$[7].

All goods are produced with a constant returns to scale technology, with higher quality being more expensive to produce. Marginal costs of low and high quality are $c_L \geq 0$ and $c_H > c_L$, respectively. Firms can always exit the market if it is in their interest to do so.

1.2.3 Parameter restrictions

Assumption 1.1. $\alpha(1) > c_H$

Assumption 1.1 is needed to ensure that high quality firms receive positive demand in equilibrium.

Assumption 1.2. $\alpha(0) = c_L$

Under perfect information, Assumption 1.2, which equalizes the intrinsic utility of low quality goods and their production cost, makes demand for low

[5] The procedure to invert the system of FOC's is analogous to that described in Motta (2004, pp. 578-579), and is not reported here. Clearly, equation (1.2) characterizes demand only when all the FOC's hold with equality.

[6] Let $\mathbf{c}$ be the vector of marginal costs. Nash equilibrium prices are $p_j^E(n,\mathbf{e},\mathbf{c}) = \frac{n+\mu(n-1)}{[2n+\mu(2n-1)][2n+\mu(n-1)]}\left\{\left[\frac{n^2(1+\mu)}{n+\mu(n-1)} + n + \mu(n-1)\right]\alpha(e_j) + n(2+\mu)c_j + \mu\sum_{i\neq j}[c_i - \alpha(e_i)]\right\}$.

[7] The reason why firm j's demand depends on n is that, although j is the only one selling a positive quantity, it is not the only one initially on the market. Consumers are 'tempted' by the other goods, although they do not buy them: the presence of other firms posting prices and offering their products reduces the marginal utility derived from the j's good, so that j is able to sell at p_j a lower quantity than it would, at the same price, if it were alone on the market (i.e., if $n = 1$). Technically, only j's FOC holds with equality, whereas all the other ones hold with strict inequality. Notice that, given $n > 1$ and $\mathbf{p}$, j's demand increases in μ, since a higher degree of substitutability reduces consumers' temptation from different goods.

quality goods insufficient even for the profitable entry of a single low quality monopolist, since its demand would be positive only at prices strictly below marginal cost. This implies that, under imperfect information, firms can profitably produce goods only as long as they manage to convince consumers of their high quality (or count to recoup initial losses in the future). Assumption 1.2 makes Assumption 1.1 equivalent to $\gamma > c_H - c_L$, so that the utility difference between high and low quality is higher than their cost difference. This implies that an expected increase in firm j's competitors' quality makes them more competitive and therefore forces j to react by lowering its own price.

1.2.4 Equilibrium

I look for a pure strategy weak perfect Bayesian equilibrium (WPBE) of the entire game. Since several equilibria are possible, depending on how consumers form expectations on quality based on observed prices, I restrict attention to a simple class of belief functions (specified below) and to pure strategy equilibria, in which all firms set the same prices in the first stage of market interaction. Indeed, if either all firms choose the same quality or if firms with different quality set different prices, at a WPBE consumers are never fooled, contrary to what happens in many markets, to explain whose functioning the present analysis is devoted. I order firms by assigning lower indices to high quality firms. I start solving the model by backward induction.

1.3 Analysis

1.3.1 Stage 4: second market interaction

When consumers choose demand in the last move before the game ends, they are fully informed about the quality of goods on the market, because they have already experienced them. In other words, beliefs are $e_j^2(\mathbf{p^1}, \mathbf{q^1}, \mathbf{p^2}) = z_j$ if $q_j^1 > 0$[8]. I assume $e_j^2 = 0$ if $q_j^1 = 0$, to rule out the possibility that a firm finds it optimal to produce only at stage 4. Thus consumers never buy either goods indexed $j > h$, because they know these goods are of low quality, or goods they have not experienced in the first stage of market interaction, because they expect them to be of low quality. Anticipating this behavior, all low quality firms exit the market, together with all firms that did not previously sell a positive quantity. High quality firms stay on the market and, taking as given other firms's prices, set their own profit maximizing price. This leads to the following proposition.

Proposition 1.1. ***(unique stage 4 equilibrium)***
For any $n \geq 0$, $h \in \{0, \ldots, n\}, \mathbf{p^1}$ and $\mathbf{q^1}$, there exists a unique pure strategy Nash equilibrium in the corresponding stage 4 subgame, at which all firms remaining on the market receive strictly positive demand: all firms that at stage

[8] The superscript 2 is due to the fact that beliefs are relevant only in the two stages of market interaction and stage 4 is the second one.

3 did not sell anything (if any) exit the market; among firms that at stage 3 sold positive quantities, low quality ones (if any) exit the market and high quality ones (if any) set prices, receive demand and make profits as follows: $\forall j \in \{1,...,h\}: \ q_j^1 > 0, \quad p_j^2 = p^E(h,1,c_H) = p^2(h), \ q_j^2 = q_j(\mathbf{p^2},\mathbf{e^2},n) = q^2(h)$ *and* $\pi_j^2 = \pi^2(h)$*, where*

$$p^2(h) = \frac{h\alpha(1) + [h + \mu(h-1)]c_H}{2h + \mu(h-1)}, \tag{1.4}$$

$$q^2(h) = \frac{h + \mu(h-1)}{h[2h + \mu(h-1)]}[\alpha(1) - c_H], \tag{1.5}$$

$$\pi^2(h) = \frac{h + \mu(h-1)}{[2h + \mu(h-1)]^2}[\alpha(1) - c_H]^2. \tag{1.6}$$

Proof. Suppose first all firms produce a positive quantity at stage 3, so their quality is known at stage 4. By Assumption 1.2, once its quality is known, any low quality firm receives zero demand if it stays on the market, at any price (weakly) higher than its marginal cost, independently of what other firms do. Thus, whatever $h \in \{0,\ldots,n\}$, low quality firms indeed choose to exit and high quality firm's best response (if $h \geq 1$) is indeed to stay on the market and set their profit maximizing price. See Motta (2004), formulae (8.65) and (8.66) on p. 569, for the precise expression of high quality firms' choices.

Now, by the assumption that $e_j^2 = 0$ if $q_j^1 = 0$, any firm, which does not produce at stage 3, is forced out of the market at stage 4 as well, since it is regarded as a low quality firm. □

1.3.2 Stage 3: first market interaction

At stage 3 (first market interaction) firms set prices $\mathbf{p^1}$, consumers observe them, formulate beliefs on each firm's quality and then choose demand. While at stage 4 any collection of previous histories of play identifies a proper subgame, this is not the case at stage 3, because, for any n, any price vector $\mathbf{p^1} \in \mathbb{R}^n_+$ identifies one information set for the representative consumer, independently of $\mathbf{z} \in \{0,1\}^n$. To analyze the game, we need to specify each firm's introductory price after any possible $n > 0$ and $\mathbf{z} \in \{0,1\}^n$, since this identifies any possible information set at which firms may be called to set prices. Moreover, recall that firms' quality is not randomly determined by Nature, but is rather chosen by each firm at stage 2. If such quality choice is in pure strategies, and if consumers can identify each firm and correctly anticipate its equilibrium strategy, then, even after observing a pooling price at stage 3 (at least as long as this price is along the equilibrium path of play), consumers would be able to precisely anticipate each firm's quality. Yet, given firms' initial symmetry to consumers' eyes, it is more interesting and reasonable to assume that, upon observing a pooling price (at least on the equilibrium path of play), consumers may extract information about average quality, but cannot precisely identify which firm has chosen which quality[9].

I first show that no pure strategy equilibria exist, in which both low and high quality firms are present and choose different introductory prices. Hence, I subsequently focus on equilibria with pooling introductory prices.

Proposition 1.2. ***(no equilibria with separation)***
There exist no pure strategy weak perfect Bayesian equilibria, in which, along the equilibrium path of play, at stage 3 both high and low quality firms are present on the market and set two different prices.

Proof. Suppose such an equilibrium exists. By observing two prices on the market, consumers infer each firm's quality. Then the result follows from the proof of Proposition 1.1. □

Beliefs

It is convenient to specify consumers' beliefs in terms of a prior e^0, which is updated upon observation of the introductory price vector, because such prior may be interpreted as the degree of consumers' initial trust in firms' product quality, which is assumed to be common knowledge[10].

When n firms post their price at stage 3, and consumers have a prior $e^0 \in [0,1]$, the space of possible posterior beliefs is the set of all functions $\mathbf{e^1} : \mathbb{R}^n_+ \times [0,1] \to [0,1]^n : (\mathbf{p^1}, e^0) \mapsto \mathbf{e^1}(\mathbf{p^1}, e^0)$, mapping prices and prior to quality expectations. In order to study equilibria with pooling introductory prices, I restrict attention to a specific simple class of beliefs, which is especially likely to support such equilibria: $\forall n > 0$, $\forall j \in \{1, ..., n\}$, $\forall \mathbf{p^1} \in \mathbb{R}^n_+$, $\forall e^0 \in [0,1]$,

$$e^1_j(\mathbf{p^1}, e^0) = \begin{cases} e^0 & , \text{ if } \exists p^1 \in (c_L, \alpha(1)) : \ \forall i, \ p^1_i = p^1 \\ 1 & , \text{ if } p_j \leq c_L \\ 0 & , \text{ otherwise} \end{cases} \tag{1.7}$$

This means that, upon observing a pooling introductory price (in the range of profitable prices for low quality firms), consumers receive no information from price signals and do not revise their prior[11].

[9] To make this idea formally consistent with equilibrium, one may either let firms choose their quality in mixed strategies, or allow Nature to initially randomly choose, for any possible n, a permutation of the n firms (drawn from a uniform distribution), and assume that consumers cannot observe this move by Nature. This second route is chosen here and it means that any firms setting the same price are initially indistinguishable for consumers.

[10] While in principle consumers might have a set of priors, one for each firm, this would not be coherent with the above assumption, that firms are initially indistinguishable to consumers' eyes. Hence, I assume that prior beliefs are the same for all goods: $\forall j \in \{1, ..., n\}, \ e^0_j = e^0 \in [0,1]$.

[11] Observe that, if the information structure is changed to make each firm only aware of its own chosen quality, then, after specifying firms' beliefs about other firms quality, stage 3 fits the definition of a multistage (continuation) game with observable actions given by Fudenberg and Tirole (1991, pp. 331-333), for which they define a Perfect Bayesian equilibrium (PBE). In this case, beliefs (1.7) violate requirement B(iii) of a PBE, because, starting from a pooling price $p^1 \in (c_L, \alpha(1))$, a deviation by $j \neq i$ changes consumers' posterior beliefs on i's quality.

If a firm's price is (weakly) lower than c_L, then consumers conclude it must be a high quality firm. Indeed, at such introductory price no firm makes positive initial profits, but only a high quality firm may expect to recoup initial losses at stage 4, whereas a low quality firm, expecting to exit at stage 4, would never stay on the market unless at stage 3 it can price above its marginal cost.

Further, beliefs (1.7) entail a high degree of distrust in price signals (not to be confused with trust in product quality, captured by the prior), in the sense that, if different prices are observed on the market, consumers interpret any price, at which low quality firms could make profits, as a trial to cheat them, and hence expect low quality. Indeed, if low quality firms anticipated that consumers are going to interpret a certain price deviation as a signal of high quality, they would deviate themselves to such price, as long as it is above their marginal cost. That is why no deviation above c_L is a credible signal.

In summary, I am considering an environment in which consumers do not trust price signals, because they are too easy to imitate, unless they convey the information that a firm is indeed willing to incur losses to build a good reputation, losses that low quality firms could never recoup.

Finally, notice that, differently from what happens in a Bayesian game, the prior is not implied by an exogenous type distribution. If in equilibrium n^* firms enter the market, h^* of them choose high quality, and all of them set the same introductory price $p^1 \in (c_L, \alpha(1))$, then beliefs (1.7) imply $e_j^1(\mathbf{p^1}, e^0) = e^0 \ \forall j$, so Bayes' rule implies $e^0 = \frac{h^*}{n^*}$ (if quality is chosen in mixed strategies, then in equilibrium $e^0 = \frac{E(h^*)}{n^*}$, where expectations are based on firms' strategies)[12].

Introductory prices, quantities and profits

As already mentioned, firms' strategies specify an introductory price p^1 for any possible stage 3 information set, at which they move, thus $\forall n > 0$, $\forall \mathbf{z} \in \{0,1\}^n$. Yet, to study equilibria with pooling introductory prices, it is enough to summarize $\mathbf{z}$ through h[13]. Further, since e^0 is common knowledge, introductory prices may depend on it. Thus, I consider each firm's strategy as specifying an introductory price function of the form $p^1(n, h, e^0)$. When all potential entrants adopt the same $p^1(n, h, e^0)$, I call it a pooling introductory price function. Observe that in this case consumers' quality expectations at stage 3 are the same for all firms and, using (1.7), can be written as $e^1(p^1, e^0)$, where $p^1 = p^1(n, h, e^0)$.

Consider a pooling introductory price function $p^1(n, h, e^0)$. Given $e^0 \in [0, 1]$, at any stage 3 information set such that $n > 0$ and $h \in \{0, \dots, n\}$, denoting $p^1 = p^1(n, h, e^0)$ and $e^1 = e^1(p^1, e^0)$, if $p^1 < \alpha(e^1)$, then each firm j on the market expects to receive demand according to (1.2) as $q_j(\mathbf{p^1}, \mathbf{e^1}, n) = q^1(p^1, e^1, n)$

Yet, when considering stage 3 together with stage 2, as we have to do here, this is not a (multistage) game with observable actions anyway, so the definition of PBE in Fudenberg and Tirole (1991) does not apply. Further, firms are informed of other firms' quality, so that their signal might indeed be revealing some information they possess about others.

[12] If in equilibrium p^1 does not belong to $(c_L, \alpha(1))$, then Bayes' rule imposes no restrictions on the prior, since e^0 does not affect posterior beliefs along the equilibrium path of play.

[13] This will imply some slight abuse of language when talking about firms' stage 3 information sets. Such imprecisions are immaterial to the analysis and simplify the exposition.

(where $\mathbf{p^1}$ and $\mathbf{e^1}$ denote the $n \times 1$ vectors whose elements are all p^1 and e^1, respectively), so that initial profits for low and high quality firms are $\pi_L^1(p^1, e^1, n)$ and $\pi_H^1(p^1, e^1, n)$, respectively, where

$$q^1(p^1, e^1, n) = \frac{\alpha(e^1) - p^1}{n}, \tag{1.8}$$

$$\pi_L^1(p^1, e^1, n) = (p^1 - c_L) \cdot q^1(p^1, e^1, n), \tag{1.9}$$

$$\pi_H^1(p^1, e^1, n) = (p^1 - c_H) \cdot q^1(p^1, e^1, n), \tag{1.10}$$

Overall profits

From equations (1.6), (1.7), (1.9), (1.10), letting again $p^1 = p^1(n, h, e^0)$ and $e^1 = e^1(p^1, e^0)$, define low and high quality firms' overall expected profits at a pooling price $p^1 < \alpha(e^1)$ as $\pi_L(n, h, p^1, e^1) \equiv \pi_L^1(p^1, e^1, n)$ and $\pi_H(n, h, p^1, e^1) \equiv \pi_H^1(p^1, e^1, n) + \pi^2(h)$, respectively. At any price $p^1 \geq \alpha(e^1)$, any firm receives zero demand and expects zero overall profits: in this case, let $\pi_L(n, h, p^1, e^1) = 0$ and $\pi_H(n, h, p^1, e^1) = 0$. If $n > 1$, denote a firm's overall deviation profits, if it sets $p \neq p^1$ when all other firms set p^1, as $\pi'_L(n, h, p, p^1, e')$ and $\pi'_H(n, h, p, p^1, e')$, for a low and a high quality firm respectively, where e' is still derived from (1.7), but taking into account that the deviating firm prices at p and all the other ones price at p^1. In particular, if $n \geq h > 0$, $e^0 > 0$ and $p^1 > c_L$, then by deviating to $p = c_L$, a high quality firm earns overall deviation profits $\pi'_H(n, h, c_L, p^1, 1) = -(c_H - c_L)\left(\frac{1+\mu}{n+\mu}\right)\gamma + \pi^2(1)$.

Sequential rationality

To be part of a WPBE, a pooling introductory price function must be sequentially rational.

Definition 1.1. ***(sequentially rational pooling introductory prices)*** *Suppose everybody expects that at stage 4 the unique pure strategy Nash equilibrium is played. A pooling introductory price function $p^1(n, h, e^0)$ is sequentially rational given beliefs (1.7), with $e^0 \in [0, 1]$, if at any stage 3 information set at which firms choose (hence, $\forall n > 0$, $\forall h \in \{0, \ldots, n\}$), it specifies a price p^1, which is sequentially rational (given such beliefs and at the corresponding information set) for each firm on the market, provided all other firms set the same price (i.e., $\forall p \neq p^1$ it holds that, if $h < n$, then $\pi_L(n, h, p^1, e^1) \geq \pi'_L(n, h, p, p^1, e')$, and if $h > 0$, then $\pi_H(n, h, p^1, e^1) \geq \pi'_H(n, h, p, p^1, e')$).*

In particular, pooling introductory prices must be sequentially rational for a monopolist. Lemma 1.1 in the Appendix shows that a high quality monopolist has no sequentially rational introductory prices if initial trust is low (but different from zero). In this case, it must incur initial losses if it wants to sell a positive quantity. But then it has no profit maximizing initial price, since, from any price at which demand is strictly positive, a slight price increase would reduce initial losses. This implies the following result.

Proposition 1.3. ***(either high or zero trust)***
Given beliefs (1.7), if $e^0 \in \left(0, \frac{c_H - c_L}{\gamma}\right]$*, then no pooling introductory price function is sequentially rational.*

Proof. It follows from Lemma 1.1. □

Proposition 1.3 restricts the initial levels of trust, for which we may find sequentially rational pooling introductory price functions, to $e^0 \in \{0\} \cup \left(\frac{c_H - c_L}{\gamma}, 1\right]$. Given this, necessary and sufficient conditions for a pooling price function to be sequentially rational are stated in Lemma 1.2 in the Appendix. Under some parameters, there are many functions satisfying these conditions, and this remains true even after trying to reduce their number by eliminating untenable beliefs through forward induction arguments. Under other parameters, there are none. For this reason, from now on I restrict attention to two special cases, for which clear results may be obtained: one in which there are infinitely many potential entrants and one in which their number is limited[14]. In particular, I consider the following two alternative assumptions.

Assumption 1.3. $N < \frac{(c_H - c_L)\gamma}{\pi^2(1)}$

Assumption 1.4. $N = \infty$ *and* $\gamma \geq \bar{\gamma}_2$

These assumptions have opposite implications for the profitability of the investment in reputation and for the incentive to deviate in price in order to escape competition. Assumption 1.3 implies that $\forall n \in \{1, \ldots, N\}$, $\forall h \in \{1, \ldots, n\}$, it holds that $\pi_H(n, h, c_L, 1) < 0$ and that, if $p^1 > c_L$, then $\pi'_H(n, h, c_L, p^1, 1) < 0$. By contrast, Assumption 1.4 implies that $\forall n > 0$, $\forall h \in \{1, \ldots, n\}$, it holds that $\pi_H(n, h, c_L, 1) > 0$ and that, if $p^1 > c_L$, then $\pi'_H(n, h, c_L, p^1, 1) > 0$. Observe that Assumption 1.3 is equivalent to $\gamma < \bar{\gamma}$, where the threshold $\bar{\gamma} \equiv \frac{N + 2\left(1 + \sqrt{N+1}\right)}{N}(c_H - c_L)$ is decreasing in N, increasing in $(c_H - c_L)$ and satisfies $\bar{\gamma} < \bar{\gamma}_2$[15]. Moreover, Assumptions 1.2 and 1.4 imply Assumption 1.1.

An infinite pool of potential entrants may be a good assumption to study potentially large markets, in which the incentive provided by repeated purchase remains substantial even as the number of competitors grows to infinity. In markets in which this is not the case, non trivial equilibrium results may be obtained only if we limit by assumption the number of potential entrants (and Assumption 1.3 is just a particularly convenient way of doing it). The following proposition makes this point clear.

Proposition 1.4. ***(no sequential rationality for low*** γ***, if*** $N = \infty$***)***
Suppose $N = \infty$*. If* $\gamma < \bar{\gamma}_2$*, then no pooling introductory price function* $p^1(n, h, e^0)$ *is sequentially rational.*

[14] The difference in results is due to the fact that the set of off equilibrium paths of play is substantially different in these two cases, and the notion of WPBE requires sequential rationality at any information set.

[15] For instance, for $N = 3$, $\bar{\gamma} = 3(c_H - c_L)$, for $N = 8$, $\bar{\gamma} = 2(c_H - c_L)$, and for $N = 15$, $\bar{\gamma} = \frac{5}{3}(c_H - c_L)$.

Proof. If $N = \infty$ and $e^0 \in \{0\} \cup \left(\frac{c_H - c_L}{\gamma}, 1\right]$, then, given $\gamma < \bar{\gamma}_2$, the conditions stated in Lemma 1.2 are violated for n and h sufficiently high, since in this case the following holds: if $p^1(n, h, e^0) = c_L$, then $\pi_H(n, h, c_L, 1) < 0$ (see the definition of $\bar{\gamma}_2$); and if $p^1(n, h, e^0) > c_L$, then deviating to $p = c_L$ is profitable (see the proof of Proposition 1.8). □

If high quality is not sufficiently important to consumers, as the number of initial entrants diverges, coordination on a price higher than c_L is impossible, since any high quality firm would prefer to deviate to c_L, thus monopolizing the market; at the same time, coordination at c_L is also impossible, since at that price future profits are not sufficient to compensate a high quality firm's initial losses, so that deviation to a whatever high price granting zero demand, and thus no losses, would be profitable.

Proposition 1.4 implies that with infinitely many potential entrants an equilibrium may exist only if γ is sufficiently high as to grant the profitability of the investment in reputation, no matter how many firms enter the market. For lower values of γ, investing in reputation may be profitable when the number of competitors is low, but not when it is high, so that coordination on $p^1 = c_L$ is not sequentially rational for high n and h. In this case, coordination on prices higher than c_L is also impossible, since any high quality firm would like to deviate to c_L and escape high competition. Thus for intermediate values of γ, no pooling introductory price is profitable for high quality firms when too many competitors enter the market. For even lower values of γ, and this is the case considered by Assumption 1.3, investing in reputation is never profitable, not even for a high quality monopolist. Let me first consider this latter case.

Introductory prices under low γ

Proposition 1.5. ***(no investment in reputation)***
Let Assumption 1.3 hold. If a pooling introductory price function $p^1(n, h, e^0)$ is sequentially rational, then $\forall (n, h): \; N \geq n \geq h \geq 1, \quad \forall e^0 \in \{0\} \cup \left(\frac{c_H - c_L}{\gamma}, 1\right]$, $p^1(n, h, e^0) > c_L$.

Proof. Given $N < \frac{(c_H - c_L)\gamma}{\pi^2(1)}$, high quality firms would deviate from a pooling introductory price equal to c_L, since at that price they make negative profits. Indeed, $\forall (n, h) \; : \; N \geq n \geq h \geq 1, \quad \pi_H(n, h, c_L, 1) \leq \pi_H(N, 1, c_L, 1)$; and $\pi_H(N, 1, c_L, 1) < 0 \iff N < \frac{(c_H - c_L)\gamma}{\pi^2(1)}$. □

Proposition 1.5 asserts that when high quality is not very important to consumers, repeated purchase is not enough remunerative to justify the initial losses incurred to invest in reputation. Although investing in reputation is not profitable, several pooling introductory price functions are sequentially rational, as can be easily checked from Lemma 1.2. The intuition for this multiplicity is straightforward. If consumers were to believe any price signal that can be profitably imitated by low quality firms, these latter firms would obviously send the same signal. Thus, price signals are not credible, unless they cannot be profitably imitated. This would be the case of prices (weakly) below c_L, but under low γ such prices are not profitable. When consumers distrust price signals so much, it becomes profitable for firms to coordinate on a single price, which yields positive profits to all firms (or initial losses for high quality firms, later recouped through future gains). Provided firms can coordinate, there may be several profitable pooling prices. Yet pooling prices are uninformative signals, so that consumers' demand depends on their initial trust e^0. If they distrust product quality so much that they are never induced to try it, the natural consequence is that no firm finds it profitable to start selling, so all markets remain closed. Finally, if there is a high quality monopolist, obviously no coordination issue arises.

The simplest class of sequentially rational pooling introductory price functions, is the one in which firms set the same price both on and off the equilibrium path of play, provided that $e^0 > 0$ and $n > 1$ (so that in this case the price schedule is flat in n and in h)[16].

Definition 1.2. ***(flat pooling introductory price functions)***
Let the pooling introductory price function $p^1_\delta(n, h, e^0)$ be defined as follows. $\forall e^0 \in [0, 1]$, $p^1_\delta(1, 0, e^0) = p^E(1, e^0, c_L)$, $p^1_\delta(1, 1, e^0) = p^E(1, e^0, c_H)$ *and* $\forall n \in \{2, \ldots, N\}$, $\forall h \in \{0, \ldots, n\}$, $p^1_\delta(n, h, e^0) = p^1_\delta(e^0)$, *where*

$$p^1_\delta(e^0) = c_L + \delta e^0. \tag{1.11}$$

The parameter $\delta \in [0, \gamma)$ is common knowledge (as all parameters). It captures firms' price responsiveness to consumers' trust and it yields a simple parametrization of firms' ability to coordinate on higher or lower prices.

Let $\check{\delta} \equiv \frac{(c_H - c_L)^2}{(c_H-c_L)^2 + \frac{4N[N+\mu(N-1)][\alpha(1)-c_H]^2}{[2N+\mu(N-1)]^2}}\gamma$.

Proposition 1.6. ***(seq. rationality of flat introductory prices)***
Let Assumption 1.3 hold. The pooling introductory price function $p^1_\delta(n, h, e^0)$ is sequentially rational $\forall e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$ if and only if $\delta \in [\check{\delta}, \gamma)$.

Proof. By Proposition 1.3, $e^0 \in \{0\} \cup \left(\frac{c_H - c_L}{\gamma}, 1\right]$. By Proposition 1.5, $e^0 > 0$, because $p^1_\delta(0) = c_L$. Given $e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$ and $n > 1$, the worst possible case for high quality firms if when $n = h = N$ and $e^0 = \frac{c_H - c_L}{2\delta}$. In this case, high quality firms' overall profits are positive if and only if $\delta \geq \check{\delta}$. Thus the conditions of Lemma 1.2 hold $\forall e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$ if and only if $\delta \in [\check{\delta}, \gamma)$. □

So, given beliefs (1.7) and Assumption 1.3, flat introductory price schedules are sequentially rational for any trust level such that $\alpha(e^0) > c_H$, if and only if they specify sufficiently high initial prices. The reason is that, in case initial prices are loss making for high quality firms, the higher they are, the lower initial

[16] Observe that the following definition may be simplified by amending beliefs (1.7) for the case of $n = 1$, but this is not of much interest, given the focus of this paper.

losses, and thus the easier it is to recoup them through repeated purchase. A variant of Proposition 1.6 is the following: under the same assumptions, $\forall\delta \in [\check{\delta}, \gamma)$, $p^1_\delta(n, h, e^0)$ is sequentially rational if and only if $e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$. Further, $\forall\delta \in [\check{\delta}, \gamma)$, the variant of $p^1_\delta(n, h, e^0)$, in which all firms exit the market if $e^0 = 0$, is sequentially rational if and only if $e^0 \in \{0\} \cup \left(\frac{c_H - c_L}{\gamma}, 1\right]$.

It can be shown that an application of forward induction reasoning is not sufficient to rule out the multiplicity of sequentially rational pooling introductory price functions of the kind $p^1_\delta(n, h, e^0)$, so that these are indeed robust rational price schedules. Yet one may find the idea that firms set the same price both on and off the equilibrium path of play excessively restrictive and little intuitive. Consider then the most intuitive price schedule, by which, whenever this is rational, low quality firms mimic the price strategy of high quality ones, by setting $p^E(n, e^0, c_H)$.

Definition 1.3. ***(intuitive pooling introductory price function)*** *Let the pooling introductory price function $p^1_E(n, h, e^0)$ be defined as follows. $\forall e^0 \in [0, 1]$, firms set $p^1_E(1, 0, e^0) = p^E(1, e^0, c_L)$ and $p^1_E(n, h, e^0) = p^E(n, e^0, c_H)$ in any other case.*

Proposition 1.7. ***(seq. rationality of intuitive introductory prices)*** *Let Assumption 1.3 hold. The pooling introductory price function $p^1_E(n, h, e^0)$ is sequentially rational if and only if $e^0 \in \{0\} \cup \left(\frac{c_H - c_L}{\gamma}, 1\right]$.*

Proof. Given the restrictions on e^0, which follow from Proposition 1.3, the conditions of Lemma 1.2 always hold for $p^1_E(n, h, e^0)$ under Assumption 1.3. □

So the intuitive introductory price schedule, whereby low quality oligopolists act as if they had high quality goods, is sequentially rational, given beliefs (1.7) and Assumption 1.3, when trust is either zero or high. The same would be true for the variant of $p^1_E(n, h, e^0)$, according to which, if $e^0 = 0$, then all markets remain closed.

Introductory prices under high γ

When γ is high, pricing at the initially loss making price c_L is profitable for high quality firms. Although any pooling introductory price $p^1 \in (c_L, \alpha(1))$ is strictly preferred by all firms to an equilibrium in which they initially pool on $p^1 = c_L$, it holds that, whenever h and n are high, the only sequentially rational pooling introductory price is precisely $p^1 = c_L$. The reason is that the temptation to monopolize future gains is so high, that any firm would deviate from a profitable pooling introductory price and would rather incur the initial losses necessary to grant future monopoly, thus creating a sort of Prisoner's Dilemma situation.

Proposition 1.8. ***(investment in reputation under high competition)*** *Let Assumption 1.4 hold. $\forall h > 1$, $\forall e^0 \in [0, 1]$ and for any pooling introductory price function $p^1(n, h, e^0)$, $\exists\, \bar{n}(h, e^0) < \infty$ such that, if $p^1(n, h, e^0)$ is sequentially rational, then it must specify, $\forall n > \bar{n}(h, e^0)$, $p^1(n, h, e^0) = c_L$.*

Proof. For any $h > 1$ and $e^0 \in [0,1]$, as n diverges, the LHS in (1.12) converges to zero, whereas the RHS remains bounded away from zero. □

As was true for low values of γ, here too there is a multiplicity of sequentially rational pooling introductory price functions. First observe that no functions in the class $p^1_\delta(n,h,e^0)$ are sequentially rational, given beliefs (1.7), under Assumption 1.4, since in this case a high quality monopolist facing zero consumers' trust has a unique sequentially rational price, namely c_L.

Proposition 1.9. ***(seq. rationality of flat introductory prices)***
Let Assumption 1.4 hold. The pooling introductory price function $p^1_\delta(n,h,e^0)$, with the only modification of letting $p^1_\delta(1,1,0) = c_L$, is sequentially rational if and only if $e^0 \in \{0\} \cup \left(\frac{c_H - c_L}{\gamma}, 1\right]$ and $\delta = 0$.

Proof. It follows from Proposition 1.8 and Lemma 1.2. □

So the only sequentially rational flat introductory price schedule, given beliefs (1.7) and under Assumption 1.4, has oligopolists always investing in reputation. I shall call this the 'investment in reputation' pooling introductory price function and denote it $p^1_R(n,h,e^0)$.

Definition 1.4. *($p^1_R(n,h,e^0)$)*
Let the pooling introductory price function $p^1_R(n,h,e^0)$ be defined as follows. $p^1_R(1,1,0) = c_L$; $\forall e^0 \in (0,1]$, $p^1_R(1,1,e^0) = p^E(1,e^0,c_H)$; $\forall e^0 \in [0,1]$, $p^1_R(1,0,e^0) = p^E(1,e^0,c_L)$ and $\forall n > 1$, $\forall h \in \{0,\ldots,n\}$, $p^1_R(n,h,e^0) = c_L$.

Proposition 1.9 tells us that $p^1_R(n,h,e^0)$ is sequentially rational, given beliefs (1.7) and Assumption 1.4, under either zero or high trust.

In light of Proposition 1.8, it is clear that the intuitive introductory price schedule considered above cannot be sequentially rational, given beliefs (1.7) and under Assumption 1.4, since rational prices must eventually be equal to c_L. Corollaries 1.1 and 1.2 to Lemma 1.2 in the Appendix study in detail sequential rationality of $p^E(n,e^0,c_H)$ under Assumption 1.4. Essentially, they show that, for $p^E(n,e^0,c_H)$ to be sequentially rational, competitive pressure must not be too high, because otherwise deviating to c_L becomes profitable for high quality firms, since it permits to monopolize the market at both stage 3 and 4. Then consider the following modification of the function $p^1_E(n,h,e^0)$ introduced above, obtained by allowing low quality firms to mimic high quality firms' prices, unless high competitive pressure makes deviation to c_L profitable. I shall call this the 'mimicry' pooling introductory price function and denote it $p^1_M(n,h,e^0)$.

Definition 1.5. *($p^1_M(n,h,e^0)$)*
Let the pooling introductory price function $p^1_M(n,h,e^0)$ be defined as follows. $\forall e^0 \in [0,1]$, $\forall (n,h)$: $n \geq h > 1$, if $\pi'_H(n,h,c_L,p^E(n,e^0,c_H),1) > \pi_H(n,h,p^E(n,e^0,c_H),e^0)$, then $p^1_M(n,h,e^0) = c_L$. In any other case, firms set $p^1_M(n,h,e^0) = p^1_E(n,h,e^0)$.

Let the thresholds $\bar{\mu}$ and $\tilde{\gamma}\left(2, 2, p^E(2,1), 1\right)$ be defined as in the Appendix. Corollary 1.2 implies the following remark, which tells us that it is meaningful to study the 'mimicry' introductory price schedule only if the competitive pressure generated by consumers' taste for quality and by the degree of product substitutability is not too high.

Remark 1.1. *The two functions $p^1_R(n, h, e^0)$ and $p^1_M(n, h, e^0)$ are different if and only if $\mu \leq \bar{\mu}$ and $\gamma \leq \tilde{\gamma}\left(2, 2, p^E(2,1), 1\right)$.*

Proposition 1.10. ***(sequential rationality of** $p^1_M(n, h, e^0)$**)***
Let Assumption 1.4 hold. Assume $\mu \leq \bar{\mu}$ and $\gamma \leq \tilde{\gamma}\left(2, 2, p^E(2,1), 1\right)$. Then $p^1_M(n, h, e^0)$ is sequentially rational, given beliefs (1.7), if and only if $e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$.

Proof. It follows from Lemmas 1.1 and 1.2, Corollary 1.1 and Remark 1.1. □

So when under Assumption 1.4 mimicry is indeed possible, the 'mimicry' introductory price schedule is sequentially rational, given beliefs 1.7, only for high levels of trust. Under the assumptions of Proposition 1.10, the variant of the 'mimicry' introductory price schedule, in which $\forall n \geq 1,\ p^1_M(n, 1, 0) = c_L$, is sequentially rational, given beliefs (1.7), if and only if $e^0 \in \{0\} \cup \left(\frac{c_H - c_L}{\gamma}, 1\right]$.

1.3.3 Stage 2: quality choice

At stage 2 the n firms on the market simultaneously choose whether to specialize their technology to produce high or low quality goods. This choice determines whether their marginal cost in the two subsequent periods will be c_H or c_L, respectively. Indeed, firms' strategies must specify such quality choice at any possible stage 2 information set[17]. This yields a sequence (a vector if $N < \infty$) of quality choice profiles $\mathbf{z}(n, e^0)$, whose sequential rationality has to be studied.

Given a pooling introductory price function $p^1_\varphi(n, h, e^0), e^0)$, let $\pi^\varphi_i(n, h, e^0) \equiv \pi_i\left(n, h, p^1_\varphi(n, h, e^0), e^1(p^1_\varphi(n, h, e^0), e^0)\right)$, for $i \in \{L, H\}$.

Definition 1.6. ***(sequentially rational quality choices)***
Suppose everybody expects that at stage 3 all firms pool on a given introductory price function $p^1_\varphi(n, h, e^0)$, and that at stage 4 the unique pure strategy Nash equilibrium is played. A sequence of quality choice profiles $\mathbf{z}(n, e^0)$ is sequentially rational if $\forall n > 0$, the number of high quality firms $h(n, e^0) = \sum_{j=1}^{n} z_j(n, e^0)$ satisfies the following properties:

[17] Order the N potential entrants so that if n of them enter the market, it is the first ones who do. Any potential entrant i's strategy should specify a quality choice for any $n \geq i$ (i.e., whenever it enters the market) and for any possible permutation of n. To simplify, I assume symmetry in position-contingent quality choice sequences, so that for any potential entrant i, its strategy specifies, $\forall n \geq i,\ \forall j \in \{1, \ldots, n\},\ z^i_j(n)$, i.e., its quality choice whenever it enters the market, together with $(n-1)$ other firms, and is assigned position j in a permutation. Symmetry also implies that $\forall n > 0,\ \forall i, j, k \in \{1, \ldots, n\},\ z^i_j(n) = z^k_j(n) = z_j(n)$. Thus, $\forall n > 0$, consumers face the same vector $\mathbf{z}(n)$, independently of the particular permutation of n.

– *if* $h(n) > 0$*, then* $\pi_H^\varphi\left(n, h(n, e^0), e^0\right) \geq \pi_L^\varphi\left(n, h(n, e^0) - 1, e^0\right)$*;*
– *and if* $h(n, e^0) < n$*, then* $\pi_L^\varphi\left(n, h(n, e^0), e^0\right) \geq \pi_H^\varphi\left(n, h(n, e^0) + 1, e^0\right)$*.*

Quality choice under low γ

Consider first the case in which the utility difference between high and low quality is not much higher than their cost difference, i.e., let Assumption 1.3 hold. The following analysis shows that similar results for sequentially rational quality choice profiles are obtained, independently of whether pooling introductory prices are set according to the flat or to the intuitive price schedule.

Flat prices

Consider first the variant of the flat pooling introductory price schedule $p_\delta^1(n, h, e^0)$, in which all firms exit the market if $e^0 = 0$. As shown above, under Assumption 1.3 and given $\delta \in \left(\check{\delta}, \gamma\right)$, this function is sequentially rational if and only if $e^0 \in \{0\} \cup \left(\frac{c_H - c_L}{\gamma}, 1\right]$.

For $e^0 = 0$, firms are indifferent between choosing either quality and exiting the market, since in any case they expect to exit at the next stage. Thus in particular market exit if $e^0 = 0$ is sequentially rational at stage 2. This is not surprising, since, when consumers distrust the market so much as to expect all goods to be worthless, and when firms would have to sustain too high initial losses, in order to convince consumers of their high quality, no firm has an incentive to stay on the market.

Lemmas 1.3 and 1.4 investigate sequential rationality of quality choice profiles when $e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$. Together, they imply that under Assumption 1.3 a monopolist, who is going to set its full information monopoly price, taking as given consumers' trust and expectations, chooses to produce low quality. The reason is that a low γ makes the additional demand granted by re-purchase of a high quality product insufficient to compensate for its additional production cost. More interesting are the implications for the case of true oligopoly, which may be summarized as follows, letting $\tilde{\delta}_H(N)$ and $\tilde{\delta}_L(N)$ be defined as in Lemma 1.3.

Proposition 1.11. ***(under low γ and flat prices, trust may worsen market quality and competition has ambiguous effects on it)***
Let Assumption 1.3 hold. Assume that $e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$*, that prices are set according to the pooling introductory price function* $p_\delta^1(n, h, e^0)$ *at stage 3, with* $\delta \in \left(\check{\delta}, \gamma\right)$*, and that the unique Nash equilibrium is played at stage 4.*

1. *If introductory prices are very high, relative to consumers' trust (i.e., if* $\delta \geq \tilde{\delta}_H(N)$*), then* $\forall n \in \{2, \ldots, N\}$*,* $\forall e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$*,* $h(n, e^0) = n$ *characterizes sequentially rational quality choice profiles.*

2. *If introductory prices are very low, relative to consumers' trust (i.e., if $\delta \leq \tilde{\delta}_L(N)$), then $\forall n \in \{2, \ldots, N\}$, $\forall e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$, $h(n, e^0) = 0$ characterizes sequentially rational quality choice profiles.*

3. *For intermediate levels of introductory prices, full high quality and full low quality may be sequentially rational when the number of competitors is low (depending on whether introductory prices are high within this range and consumers' trust is low, or the other way around, respectively), but when competition is sufficiently strong, sequentially rational quality choices imply initial coexistence on the market of both high and low quality firms, with average market quality falling into an interval of length $1/n$, whose boundaries decrease in consumers' trust.*

Proof. It follows from Lemmas 1.3 and 1.4, and from the monotonicity properties of the thresholds of δ and of the functions f and g there specified. □

In words, if at stage 3 oligopolists coordinate on a sufficiently high flat price schedule, relative to consumers' trust, so that initial market demand is close to zero, then each of them makes initial profits (or losses) close to zero. This makes it convenient to choose high quality and gain stage 4 profits, whatever small they may be (i.e., independently of the fierceness of market competition and of consumers' initial trust). Thus high introductory prices are essentially a means of reducing firms' temptation to choose low quality and cheat consumers.

If the flat pooling introductory price is sufficiently low, relative to consumers' trust, then Assumption 1.3 implies that profits from repeated purchase of high quality goods are lower than the initial profit advantage obtained by low quality producers, again independently of the initial number of oligopolists and of consumers' initial trust.

If the flat pooling introductory price falls into an intermediate range, relative to consumers' trust, then firms' quality choice depends on both competition and consumers' trust. In this case, initial market demand is sufficiently high as to make the temptation to choose low quality substantial. Since initial demand is increasing in consumers' trust, so is the temptation to cheat consumers.

For high prices (within this intermediate range), at low levels of competition universal adoption of high quality is the unique sequentially rational choice, independently of consumers' trust; but when the number of competitors is sufficiently high, full high quality is only sequentially rational for low trust levels, but not any more for high trust levels. The reason is that, provided that all entrants are expected to choose high quality, if their number increases, then each firm's temptation to choose low quality increases as well. This is due to the fact that a high quality firm competes twice, whereas a low quality firm competes only once. At stage 3 the relative profitability of high vs. low quality increases after entry of additional high quality firms, because low quality firms lose more profits from a reduction in demand than high quality firms initially do. At stage 4 the reverse is true, because low quality firms earns zero profits independently of the degree of competition, whereas high quality firms are obviously penalized by an increase in their number. Precisely because low quality firms' profits do not change at stage 4, this latter effect is stronger than the former one, so that entry of additional high quality firms shifts the overall relative profitability in favor of low quality. Thus, when trust is high, at a certain point a new entrant finds it more profitable to choose low quality. The higher the number of entrants, the lower the threshold of trust, below which full adoption of high quality remains sequentially rational.

The converse reasoning implies that, for low prices (within the intermediate range), when n is low, the unique sequentially rational choice has $\forall e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$, $h(n, e^0) = 0$; but when n is sufficiently high, universal adoption of low quality is only sequentially rational if trust is high, but not any more if trust is low. Provided that all entrants are expected to choose low quality, if their number increases, then each firm's temptation to choose high quality increases. Indeed, if it does so, it monopolizes the market at stage 4, and thus makes the same stage 4 profits independently of the increase in competition. By contrast, higher competition reduces its stage 3 profits by the same amount, independently of what quality it chooses. So higher competition increases the relative profitability of high quality, and when trust is low, at a certain point a new entrant finds it more profitable to choose high quality. The higher the number of entrants, the higher the threshold of trust, above which full adoption of low quality remains sequentially rational.

For intermediate prices (within the intermediate range), sequentially rational quality choice profiles imply the initial coexistence on the market of both high and low quality firms, for any admissible trust level and number of oligopolists. Generically for all admissible combinations of n and e^0, there is a unique number $h(n, e^0)$ of high quality firms, which makes the quality choice profile sequentially rational[18]. The resulting average quality is decreasing in trust, since higher trust always makes low quality more profitable, relative to high quality.

In turn, average quality need not be a monotonic function of n, since increases in $h(n, e^0)$ and in n need not be simultaneous and proportional. Indeed, suppose that, given e^0 and n, sequentially rational choices determine a given number $h(n, e^0)$ of high quality firms. Then it must be the case that high quality firms earn (weakly) higher profits than low quality ones, since otherwise they would deviate; but it must also be the case that, if one firm deviates from low to high quality, then high quality firms' profits become lower than low quality firms' profits (which, under the flat price schedule, do not change after such deviation). Now suppose these n firms keep playing the same way, but a new entrant arrives. If it rationally chooses high quality, then it shifts relative profitability in favor of low quality[19]. Sequential rationality of the new entrant's choice grants that the other n firms' choices remain sequentially rational, so

[18]Genericity here refers to the admissible combinations of n and e^0.

[19]The relative advantage to high quality decreases faster at stage 4 than it increases at stage 3, because high quality firms' profits decrease at both stages of market interaction, whereas low quality firms are hurt by higher competition only at the initial stage.

that $h(n+1, e^0) = h(n, e^0) + 1$ and average quality goes up after the increase in competition. If yet an additional firm enters the market, now it is likely that, taking as given the other $(n+1)$ firms' choices, it rationally chooses low quality, since the relative profitability of low quality has increased. If it does so, then $h(n+2, e^0) = h(n+1, e^0)$ and average quality goes down[20]. Thus, sequentially rational quality choices may result in oscillations of average quality as competition increases, although the range of these oscillations shrinks as n rises.

Intuitive prices

Now consider the variant of the intuitive pooling introductory price schedule $p_E^1(n, h, e^0)$, in which all firms exit the market if $e^0 = 0$. As shown above, under Assumption 1.3 this function is sequentially rational if and only if $e^0 \in \{0\} \cup \left(\frac{c_H - c_L}{\gamma}, 1\right]$.

As was true under flat introductory prices, for $e^0 = 0$ firms are indifferent between choosing either quality and exiting the market. Thus in particular market exit if $e^0 = 0$ is sequentially rational at stage 2.

Lemma 1.5 investigates sequential rationality of quality choice profiles when $e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$. It first shows that a monopolist chooses low quality when the additional demand granted by re-purchase of high quality goods is not sufficient to compensate for their additional cost. When $n > 1$, sequentially rational quality choices depend on the initial level of consumers' trust and competition in the following way.

Proposition 1.12. ***(under low γ and intuitive prices, trust worsens market quality and competition has ambiguous effects on it)***
Let Assumption 1.3 hold. Assume that prices are set according to the pooling introductory price function $p_E^1(n, h, e^0)$ at stage 3, and that the unique Nash equilibrium is played at stage 4. $\forall n \in \{2, \ldots, N\}$, $\exists \tilde{e}_B(n), \tilde{e}_T(n) : \frac{c_H - c_L}{\gamma} < \tilde{e}_B(n) < \tilde{e}_T(n)$, such that sequentially rational quality choices depend on consumers' trust in the following way.

1. *Low trust implies full high quality, i.e., if $e^0 \in \left(\frac{c_H - c_L}{\gamma}, \tilde{e}_B(n)\right)$, then $h(n, e^0) = n$ characterizes sequentially rational quality choice profiles.*

2. *High trust and low competition may imply full low quality, i.e., if $e^0 \in (\tilde{e}_T(n), 1]$ (which is an empty interval for high n), then $h(n, e^0) = 0$ characterizes sequentially rational quality choice profiles.*

3. *If either trust is at intermediate levels or both trust and competition are sufficiently high, i.e., if $e^0 \in [\tilde{e}_B(n), \tilde{e}_T(n)]$ (for high n, $\tilde{e}_T(n) > 1$), then*

[20] This shifts relative profitability in favor of high quality, because low quality firms lose more profits from a reduction in demand at stage 3, whereas, given $h(n+2, e^0) = h(n+1, e^0)$, the relative advantage to high quality firms at stage 4 is unaffected. So the reasoning may go on.

sequentially rational quality choice profiles are characterized by initial coexistence on the market of both high and low quality firms. Average market quality falls into an interval of length $1/n$*, whose boundaries decrease in consumers' trust.*

Proof. It follows from Lemma 1.5 and the fact that $\tilde{e}_B(n)$ and $\tilde{e}_T(n)$ there defined are respectively decreasing and increasing. □

Low trust implies low initial demand, so that high quality is always more profitable than low quality, independently of the number of competitors, since it yields profits from repeated purchase. High trust makes the initial profit advantage of low quality firms higher, the more so, the lower the number of competitors, so that in particular a combination of high trust and low competition may induce all firms to cheat the market. For intermediate trust levels, given n, average quality decreases in trust; yet, given e^0, average market quality needs not be a monotonic function of n. The reason is that entry of additional low quality firms makes high quality relatively more profitable, and entry of additional high quality firms tends to make low quality relatively more profitable. Thus, although a proportional increase in n and h raises the relative profitability of low quality, so that, if we take a sufficiently long sequence of n, a decreasing trend in average quality, as determined by sequentially rational choices, should be detectable, this may not be the case when N is low, since then discrete jumps in n and h may determine non monotonic jumps in average quality, with no observable trend.

Quality choice under high γ

Consider now the case in which the utility difference between high and low quality is much higher than their cost difference, i.e., let Assumption 1.4 hold. Under low γ, the above analysis has shown that for firms facing distrustful consumers it is rational ro exit the market at stage 2. This does not hold any more under high γ. The following general result holds, independently of the specific way in which firms choose sequentially rational introductory prices.

Proposition 1.13. ***(distrust grants high quality with low and rising prices)***
Let Assumption 1.4 hold. If $e^0 = 0$*, then at any WPBE supported by beliefs (1.7), at which* $n > 0$ *markets are open, all firms on the market choose high quality and subsequently set* $p^1 = c_L$ *and* $p^2 = p^2(n) > c_L$*.*

Proof. Given Assumptions 1.2 and 1.4 and beliefs (1.7) with $e^0 = 0$, a low quality firm expects to make zero profits at any sequentially rational introductory price (i.e., at any $p^1 \geq c_L$); a high quality firm has a unique sequentially rational introductory price, $p^1 = c_L$, by setting which it grants itself strictly positive profits. Thus all firms choose high quality. Equilibrium pricing strategies must specify, $\forall(n,h) : n > 0,\ n \geq h \geq 0$ (i.e., both on and off the equilibrium path of play), $p^2 = p^2(n)$, $p^1 = c_L$ if $h > 0$, and $p^1 \geq c_L$ if $h = 0$. Along the equilibrium path of play, if $n > 0$, then $h = n$ implies $p^1 = c_L$ and $p^2 = p^2(n)$. □

Proposition 1.13 confirms the result obtained by Shapiro (1983), that consumer's initial distrust forces firms to offer high quality at prices that are initially below marginal cost, since they have to invest in reputation, and later above marginal cost, since the initial investment pays off. Yet in Shapiro's equilibrium consumers are not fully rational, since their initial expectations turn out to be on average wrong. In contrast, under the present distinction between initial trust and posterior beliefs, consumers' distrust forces high quality firms to lower their initial price to a level that is unprofitable for low quality ones, thus convincing consumers of their high quality. So initial distrust is reconciled with correct equilibrium quality expectations.

To study what happens when consumers' trust is higher, I restrict attention to the comparison between $p_R^1(n,h,e^0)$ and $p_M^1(n,h,e^0)$.

Proposition 1.14. ***(full high quality with investment in reputation)*** *Let Assumption 1.4 hold. If* $e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$ *and firms price according to* $p_R^1(n,h,e^0)$ *at stage 3 and play the Nash equilibrium at stage 4, then a sequence of quality choice profiles* $\mathbf{z}(n,e^0)$ *is sequentially rational, given beliefs (1.7), if and only if* $\forall n > 0,\ h(n,e^0) = n$.

Proof. Let $n > 1$. Since $\forall h \in \{0,\dots,n\},\ p_R^1(n,h,e^0) = c_L$, Assumption 1.4 implies that for $h > 0$, $\pi_H^R\left(n,h,e^0\right) > \pi_L^R\left(n,h-1,e^0\right) = 0$; and for $h < n$, $\pi_L^R\left(n,h,e^0\right) = 0 < \pi_H^R\left(n,h+1,e^0\right)$. Now let $n = 1$. Since $p_R^1(1,1,e^0) = p^E(1,e^0,c_H)$ and $p_R^1(1,0,e^0) = p^E(1,e^0,c_L)$, $\pi_H^R\left(1,1,e^0\right) > \pi_L^R\left(1,0,e^0\right) \iff e^0 < \frac{c_H - c_L}{\gamma} + \frac{[\alpha(1)-c_H]^2}{(c_H-c_L)\gamma}$, which always holds under Assumption 1.4, because the latter implies $\frac{[\alpha(1)-c_H]^2}{(c_H-c_L)\gamma} > 1$. □

Not surprisingly, Proposition 1.14 shows that choosing high quality is the only sequentially rational choice for firms that are going to invest in reputation, since, anticipating that they are going to set low introductory prices, which are not profitable for low quality firms, it would be inconsistent to choose low quality.

Let $\bar{e}$ and $\hat{e}$ be the solutions by e^0, in the range $e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$, of $\pi_H(2,2,c_L,1) = \pi_L(2,2,p^E(2,e^0),e^0)$ and of $\pi'_H(2,2,c_L,p^E(2,e^0),1) = \pi_H(2,2,p^E(2,e^0),e^0)$, respectively. Let $\tilde{e}$ be defined so that, for $e^0 > \frac{c_H - c_L}{\gamma}$, $\pi'_H(3,3,c_L,p^E(3,e^0),1) > \pi_H(3,2,p^E(3,e^0),e^0) \iff e^0 < \tilde{e}$.

Proposition 1.15. ***(full high quality with no investment in reputation)*** *Let Assumption 1.4 hold. Suppose* $e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$, $\mu \le \bar{\mu},\ \gamma \le \tilde{\gamma}(2,2,p^E(2,1),1)$, *firms price according to* $p_M^1(n,h,e^0)$ *at stage 3 and they play the Nash equilibrium at stage 4. Then a necessary condition for a sequence of quality choice profiles* $\mathbf{z}(n,e^0)$ *to be sequentially rational, given beliefs (1.7), is that* $\forall n > 0,\ h(n,e^0) = n$. *For the wide majority of parameters (i.e., when* $\hat{e} > \bar{e}$ *and* $\tilde{e} > 1$, *which always holds, unless both* μ *and* γ *are very low),*

1. *if* $e^0 \in \left(\frac{c_H - c_L}{\gamma}, \bar{e}\right]$*, then* $h(n, e^0) = n$ *is a sufficient condition for sequential rationality and it implies* $\forall n \geq 2,\ p^1_M(n, n, e^0) = c_L$*;*
2. *if* $e^0 \in (\bar{e}, \hat{e})$*, then no* $\mathbf{z}(n, e^0)$ *is sequentially rational;*
3. *if* $e^0 \in [\hat{e}, 1]$*, then* $h(n, e^0) = n$ *is a sufficient condition for sequential rationality and it implies* $\forall n \geq 3,\ p^1_M(n, n, e^0) = c_L$*.*

Proof. For any parameter constellation, $\tilde{\gamma} > \max\{\bar{e}, \hat{e}\}$. When μ is low, there is a small range of γ, close to $\bar{\gamma}_2$, for which it is possible that $\tilde{e} \leq 1$ and $\hat{e} \leq \bar{e}$. Under such parameters, we are in case 1 of Lemma 1.6 if and only if either $n \geq 4$ or $n = 3$ and $e^0 \in [\hat{e}, \tilde{e})$. For any other parameter constellation, i.e., for the wide majority of parameters, $\tilde{e} > 1$ and $\hat{e} > \bar{e}$. Under such parameters we are in case 1 of Lemma 1.6 if and only if either $n \geq 3$. In this case, by Lemma 1.6, high quality is the only sequentially rational choice in markets with either 1 or at least 3 firms. If $e^0 < \hat{e}$, then for $n = 2$ we are in case 2 of Lemma 1.6. Then sequential rationality requires $h(2, e^0) = 2$. Condition $\pi_H(2, 2, c_L, 1) \geq \pi_L(2, 2, p^E(2, e^0), e^0)$ holds if and only if $e^0 \leq \bar{e}$. So, $h(2, e^0) = 2$ is sequentially rational if $e^0 \in \left(\frac{c_H - c_L}{\gamma}, \bar{e}\right]$, whereas it is not if $e^0 \in (\bar{e}, \hat{e})$. If $e^0 \in [\hat{e}, 1]$, then for $n = 2$ we are in case 3 of Lemma 1.6. In this case, if $h(2, e^0) \in \{0, 1\}$, then sequential rationality is impossible, because deviating to high quality is profitable when $h(2, e^0) = 0$ and because condition (a) does not hold for $h(2, e^0) = 1$ (it never holds in case 3 for $h(n, e^0) = n - 1$); in turn, $h(2, e^0) = 2$ is sequentially rational. Price choices then follow from the definition of $p^1_M(n, n, e^0)$. An analogous procedure shows that $\forall n > 0,\ h(n, e^0) = n$ is still a necessary condition for sequential rationality for low μ and γ. The only difference is that under such parameters, when trust is high, we have $p^1(n, n, e^0) = p^E(n, e^0, c_H)$ for $n \in \{1, 2, 3\}$ rather than just for $n \in \{1, 2\}$. □

Proposition 1.15 shows that the strategy of investing in reputation is not the only one that may grant full high quality. Indeed, when the difference between high and low quality is sufficiently important to consumers, relative to the cost advantage granted by low quality, as captured by Assumption 1.4, profits from repeated purchase may easily exceed the one shot profit advantage obtained by cheating the market. Indeed, $p^E(n, e^0, c_H)$ is the most favorable pooling introductory price function for high quality firms and so for them $p^1_M(n, h, e^0)$ is the most favorable sequentially rational pooling introductory price function. Hence, when firms are going to price according to $p^1_M(n, h, e^0)$, the initial profit advantage granted by low quality is minimum. In this case high quality is the only possible sequentially rational choice.

Proposition 1.15 also shows that, under $p^1_M(n, h, e^0)$, sequential rationality of the universal adoption of high quality does not in general hold for any trust level $e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$, since it does not hold for an intermediate range of trust within this interval. There is a tension between the conditions granting sequential rationality of price choices and of quality choices, and such conditions need not always be compatible.

Finally, Proposition 1.15 makes clear that, under Assumption 1.4, and given (sequentially rational) universal adoption of high quality, $p_M^1(n,n,e^0) \neq p_R^1(n,n,e^0)$ only in the case of a duopoly with high trust. Whenever $n \geq 3$, Assumption 1.4 makes the incentive to initially deviate from any pooling price $p^1 \neq c_L$ to $p = c_L$ (and thus monopolize the market) so strong, that c_L remains the only sequentially rational pooling introductory price. This, in turn, makes the incentive to choose high quality very strong.

1.3.4 Stage 1: entry and consistency

At stage 1 firms decide whether to enter the market or not. Since, given beliefs (1.7) and e^0, under any of the above considered sequentially rational continuations of the game, overall expected profits, for both high and low quality firms, are decreasing in n, the sequentially rational number of entrants is uniquely determined in each case as the highest integer such that each firm's overall expected profits are higher than the fixed entry cost ζ. This yields n as an increasing function of $e^0 \in \{0\} \cup \left(\frac{c_H - c_L}{\gamma}, 1\right]$.

At a WPBE beliefs must be consistent with strategies along the equilibrium path of play. Whenever equilibrium introductory pooling prices imply that beliefs are determined by initial trust, consistency imposes equality in equilibrium between trust and average expected quality.

Low γ

When the utility difference is not much higher than the cost difference between high and low quality, under either the flat or the intuitive introductory price schedule, nothing grants the joint existence of a sequentially rational pure strategy quality choice profile and of an initial level of consumers' trust, which, under beliefs (1.7), makes quality expectations consistent with equilibrium strategies. Yet, in either case this is only due to an integer problem, which is easily solved by allowing just one firm to choose quality in mixed strategies along the equilibrium path of play. Lemmas 1.7 and 1.8 make this precise and thus grant equilibrium existence. The analytical expressions of the equilibrium number of entrants and of expected average quality can be obtained, but this requires solving a third degree polynomial equation, whose solution is essentially unreadable. Yet, comparative statics on n is easy to draw, based on our previous results.

Proposition 1.16. ***(equilibrium and comparative statics for low*** γ***)***
Let Assumption 1.3 hold and suppose that ζ *is sufficiently low as to allow equilibrium entry by a number of firms higher than* $\frac{\gamma}{c_H - c_L}$. *Suppose further that beliefs are given by (1.7) and that firms play the unique Nash equilibrium at stage 4, pool on either the flat or the intuitive introductory price function at stage 3 (in case of flat prices, with* $\delta \in (\check{\delta}, \gamma)$ *sufficiently high as to make* $\bar{e}_M(n) > \frac{c_H - c_L}{\gamma}$*), make sequentially rational quality choices at stage 2, with the possibility that one firm randomizes along the equilibrium path of play, and keep entering until expected profits exceed entry costs. Then there exists only one level of initial*

consumers' trust, which makes beliefs consistent with strategies along the equilibrium path of play. So under these assumptions a WPBE exists, is unique and has the following properties.

1. *The equilibrium number of entrants is increasing in N, decreasing in ζ, μ and c_H, trend increasing in γ, and under the flat pooling introductory price function its relationship with δ has no stable sign.*

2. *If firms pool on a very high flat introductory price function, then in equilibrium universal adoption of high quality is granted, consumers' trust is high ($e^0 = 1$), and prices decrease over time.*

3. *If firms pool on a lower (but not too low) flat introductory price function, then in equilibrium average expected quality and consumers' trust are decreasing in c_H, trend increasing in δ, trend decreasing in μ, unrelated to N and their relationship with ζ and γ has no stable sign. As the market matures, there is a shake-out, after which prices rise if δ is low and fall if δ is high.*

4. *If firms pool on the intuitive introductory price function, then in equilibrium average expected quality and consumers' trust are trend increasing in c_H, trend decreasing in μ, unrelated to N and their relationship with ζ and γ has no stable sign. As the market matures, there is a shake-out, after which prices rise.*

Proof. Existence and uniqueness of a WPBE follow from Lemmas 1.7 and 1.8. No WPBE with flat pooling introductory prices and low δ exists, since this would imply that too many firms rationally choose low quality, so that consistent beliefs make trust so low, that $p^1_\delta(n, h, e^0)$ is not sequentially rational any more. The above assumption of a sufficiently high δ eliminates this concern.

1. The number of potential entrants matters only when entry costs are so low that all N firms enter the market. For higher costs, it does not affect the equilibrium number of entrants. An increase in entry costs obviously reduces the number of entrants. An increase in the elasticity of substitution makes competition tougher at stage 4 under $p^1_\delta(n, h, e^0)$, and both at stage 3 and 4 under $p^1_E(n, h, e^0)$. This reduces firms' profits and therefore the number of entrants. A rise in the production cost of high quality obviously discourages high quality production, and thus decreases consumers' confidence and reduces the size of the market, allowing entry by a lower number of firms[21]. A rise in γ expands consumers' demand and allows more firms to enter the market. Yet, as shown below, a rise in γ may be associated to a reduction in trust, which reduces consumers' demand and thus works in the opposite direction. Numerical analysis shows that the former effect tends to prevail, with the number of entrants increasing stepwise in γ, but that this trend is compatible with occasional stepwise

[21] I do not make comparative statics exercises on c_L, since, under Assumption 1.2, their interpretation would be unclear

falls of n after an increase in γ. An increase in δ under flat introductory prices has two effects: first, given n and e^0, it reduces or increases firms' initial profits, depending on whether initial prices fall into an elastic or inelastic region of demand, respectively; second, given n, it favors high over low quality. The first effect increases the equilibrium number of entrants for low δ and reduces it for high δ; the second effect would raise equilibrium trust and average expected quality if the number of entrants remained constant, but when a rise in δ triggers a change in n, equilibrium trust and average expected quality may either rise or fall, determining a variable sign of the equilibrium relationship between δ and n.

2. In case of high flat pooling introductory prices, the result follows from Proposition 1.11. The fact that prices decrease over time follows from numerical analysis.

3. Consider now lower flat pooling introductory prices (yet with δ high enough as to grant that equilibrium trust is sufficiently high to make the flat pooling introductory price function sequentially rational). The negative impact of a rise in c_H on equilibrium average expected quality is obvious. The non monotonic impact of δ on average expected quality has been discussed above. Notwithstanding the possible fluctuations, numerical analysis shows a clear increasing trend in the relationship between δ and equilibrium average expected quality. As shown above, a rise in μ lowers the number of entrants. Given n, it favors low over high quality, since it does not affect relative profitability at stage 3, whereas it reduces high quality firms' profits at stage 4. Jumps in n make this effect discontinuous, with possible upwards jumps in trust after a rise in μ, but numerical analysis shows a clear decreasing trend in the relationship between μ and equilibrium average expected quality. The number of potential entrants is irrelevant for equilibrium average expected quality. Higher barriers to entry reduce the number of entrants, and this in turn expands the interval $[\bar{e}_M(n), \bar{e}_N(n)]$ in which, according to Lemma 1.4, average expected quality must fall in an asymmetric quality choice equilibrium. Yet, within this range, equilibrium trust may increase as well as decrease in ζ. A higher γ has two effects: on one side, as shown above, it tends to (stepwise) increase the equilibrium number of entrants; on the other side, within those intervals in which n does not change, it raises or lowers average expected quality and trust depending on whether $\delta < [\gamma + (c_H - c_L)]/2$ or $\delta > [\gamma + (c_H - c_L)]/2$, respectively (because in Lemma 1.4 $\frac{\partial g}{\partial \gamma} < 0 \iff \delta < \frac{\gamma + (c_H - c_L)}{2}$). Moreover, when a rise in γ triggers a change in the equilibrium number of entrants, it also makes average expected quality and trust change discontinuously. Finally, it is possible to find numerical examples of both an increasing and a decreasing time path of prices, depending on whether δ is low or high, respectively.

4. Now consider intuitive pooling introductory prices. A rise in c_H has two effects (besides that of reducing the number of entrants). First, it tends

to make low quality relatively more profitable, since this becomes comparatively cheaper to produce. Second, it raises firms' prices. Since low quality firms' introductory prices are already distorted upwards with respect to their full information optimum, and since moreover they set a higher markup than high quality firms initially do, the profit loss may be higher for low quality than for high quality firms, so that equilibrium average expected quality may go up after a rise in c_H. Numerical analysis indicates that indeed this tends to be the case. The elasticity of substitution, the number of potential entrants, entry costs and consumers' preference for high quality, all have the same effects on equilibrium average expected quality, and for the same reasons, under intuitive prices as under flat prices (in the case of γ, this is due to the fact that in Lemma 1.5 $\frac{\partial \tilde{g}}{\partial \gamma} < 0 \iff e^0 > \frac{2(c_H - c_L)}{\gamma + 2(c_H - c_L)}$, i.e., given n, and provided that equilibrium trust is sufficiently high, a rise in γ makes high quality relatively more profitable, thus fostering an increase in average quality and in trust; but this, in turn, raises the size of the market and makes more firms enter, with counterbalancing effects, since a proportional increase in n and h makes low quality relatively more profitable). Finally, whenever average expected quality is initially lower than 1 (as is the case in all the numerical exercises performed), either with certainty or with positive probability there is a shake-out, and since prices are decreasing in the number of firms on the market, their time path is increasing.

□

While all parameters have the expected impact on the equilibrium number of entrants, more interesting is to discuss their effect on equilibrium average expected quality, trust and prices.

If all firms pool on a high flat introductory price schedule, then they make consumers fully confident that the quality of their products is high. Indeed, at such prices initial demand is very high, so that the initial profit advantage from cheating the market is very low, and it is more than compensated by the subsequent profit advantage, which accrues to firms complying with high quality promises due to repeated purchase. This result is in line with Bagwell and Riordan (1991), who show that a monopolist uses high and declining prices to signal high quality in markets with information diffusion. In markets with an endogenous number of firms, the intuition that high and declining prices make consumers confident in good quality still holds, as long as firms' strategies specify a sufficiently high flat pooling introductory price. Yet, it does not hold any more if firms pool on intuitive introductory prices, since in this case some firms rationally choose low quality, mimic the pricing strategy of high quality firms, are able to initially cheat the market, but, upon discovery, are forced out of the market, and prices rise over time both because the number of competitors is lower and because average quality, consumers' confidence and hence demand are higher.

Most parameters have an analogous effect on average expected quality and on trust, independently of whether firms pool on intuitive introductory prices or on flat a introductory price schedule, which is not so high as to guarantee full high quality. In particular, average quality and quality expectations tend to be lower in markets in which products are closer substitutes, because this erodes the price premium that motivates firms to produce high quality goods. More specifically, at the equilibrium levels of n, h and e^0, a rise in μ lowers high quality firms' profits at stage 4 (which is the same as their profit advantage, since low quality firms exit the market), whereas its effect at stage 3 depends on how introductory prices are set: under flat prices, it has no effect one either high or low quality firms' profits; under intuitive prices, it lowers high quality firms' profits and raises low quality firms' profits (because it reduces their upward prices distortion and allows them to sell more). In either case, it makes low quality relatively more profitable, and this effect tends to remain predominant even after considering the equilibrium decrease in n and in e^0. To have a concrete example of this, think for instance of many souvenir markets for tourists. More broadly, a higher competitive pressure due to higher product substitutability imposes a trade-off to consumers between lower prices and lower average quality.

The number of potential entrants, entry costs and consumers' preference for high quality (within the range allowed by the fact that we are considering the case in which γ is low, relative to $c_H - c_L$), have no clear effect on average quality and on quality expectations. While this is intuitive for N and ζ, since these parameters directly affect the number of entrants, but not the relative profitability of high vs. low quality, it is surprising for γ. Indeed, the first intuition would be that a higher consumers' preference for high quality should raise average market quality. This effect is at work, but there are also counterbalancing effects: given n and e^0, a higher number of high quality firms increases the relative profitability of low quality, since it decreases high quality firms' stage 4 profits; moreover, if consumers expect a higher average quality, the size of the market increases and this may raise the number of entrants; additional entrants may then be tempted to choose low quality, precisely because it has become relatively more profitable; yet, if this happens and is anticipated, average quality and trust may decrease. Thus, it is not at all straightforward that a higher consumers' preference for high quality translates into higher average market quality.

The only parameter that has a different impact on average quality and expectations under the flat and under the intuitive pooling introductory price schedule, besides δ, is c_H. It does not surprise that δ tends to raise average quality and trust under flat prices, since higher introductory prices favor high over low quality. Under flat prices, a rise in c_H, besides reducing the number of entrants, has the only effect of making high quality less profitable. Under intuitive prices, by contrast, this effect is counterbalanced by the price increase induced by a rise in c_H. Higher prices favor high over low quality at stage 3 and low over high quality at stage 4, with the former effect that tends to prevail on the latter.

High γ

When the utility difference is much higher than the cost difference between high and low quality, equilibrium existence and properties under the pooling introductory price functions $p^1_R(n,h,e^0)$ and $p^1_M(n,h,e^0)$ are easy to derive, based on our previous results. Since they are very similar, I state them formally only for $p^1_R(n,h,e^0)$[22].

Proposition 1.17. ***(equilibrium and comparative statics for high*** γ***)*** *Let Assumption 1.4 hold. The strategy profile $(n, h(n), p^1_R(n,h,e^0))$, together with the unique Nash equilibrium at stage 4 and with beliefs (1.7), is WPBE if and only if $e^0 \in \{0\} \cup \left(\frac{c_H - c_L}{\gamma}, 1\right]$ and the following holds.*

1. *If $\zeta \leq \frac{(1+\mu)[\alpha(1)-c_H]^2}{(2+\mu)^2}$, then $n = \infty$. Then along the equilibrium path of play $h = n,\ p^1 = c_L,\ p^2 = \frac{\alpha(1)+(1+\mu)c_H}{2+\mu}$.*

2. *If $\zeta \in \left(\frac{(1+\mu)[\alpha(1)-c_H]^2}{(2+\mu)^2}, \frac{(c_L-c_H)\gamma}{2} + \frac{(1+\mu)[\alpha(1)-c_H]^2}{(2+\mu)^2}\right]$, then n is the highest natural number such that equilibrium profits exceed entry costs, i.e., $\pi_H(n,n,c_L,1) = \frac{(c_L-c_H)\gamma}{n} + \frac{[n+\mu(n-1)][\alpha(1)-c_H]^2}{[2n+\mu(n-1)]^2} \geq \zeta$. Then n is increasing in γ and decreasing in μ and $(c_H - c_L)$, and along the equilibrium path of play $2 \leq n < \infty,\ h = n,\ p^1 = c_L,\ p^2 = p^2(n)$.*

3. *If $\zeta \in \left(\frac{(c_L-c_H)\gamma}{2} + \frac{(1+\mu)[\alpha(1)-c_H]^2}{(2+\mu)^2}, (c_L - c_H)\gamma + \frac{[\alpha(1)-c_H]^2}{4}\right]$, then $e^0 = 0$ and $n = 1$. Then along the equilibrium path of play $h = 1,\ p^1 = c_L,\ p^2 = p^2(1)$.*

4. *If $\zeta \in \left((c_L - c_H)\gamma + \frac{[\alpha(1)-c_H]^2}{4}, \frac{[\alpha(1)-c_H]^2}{2}\right]$, then $e^0 = 1$ and $n = 1$. Then along the equilibrium path of play $h = 1,\ p^1 = p^2 = p^2(1)$.*

5. *If $\zeta > \frac{[\alpha(1)-c_H]^2}{2}$, then all markets are closed.*

Proof. It follows from Propositions 1.8, 1.9, 1.13 and 1.14, with some algebra. □

In words, the number of entrants is decreasing in entry costs, and it passes from infinity to zero as as ζ rises. Whenever markets are open, all firms choose high quality, and whenever they compete with other firms, they invest in reputation, initially pricing at low quality firms' marginal cost and then raising prices at their perfect information optimum. Given this pricing strategy, consistency of consumers' beliefs does not impose any restriction on their initial trust level, except for the particular case in which entry costs are so high that just a

[22] Results under $p^1_M(n,h,e^0)$ are analogous in terms of equilibrium strategies, although they may imply different restrictions on initial trust. In particular, equilibrium strategies are different only when $e^0 = 1$ and in equilibrium $n = 2$, in which case $h = 2$ and $p^1 = p^2 = p^2(1)$. This follows from Propositions 1.8, 1.10, 1.13, 1.15 and Lemma 1.6.

monopolist enters the market. In all other cases, firms' low introductory prices convince consumers that quality is high, independently of initial trust.

This is in line with Shapiro (1983), who shows that at a competitive equilibrium firms signal product quality through low and rising prices, and that premiums for high quality products in the mature phase of market interaction are just the normal market return to the initial investment in reputation. The same intuition works in the present oligopoly model with endogenous entry, as long as the utility difference between high and low quality is high, relative to their cost difference. Yet, as shown above, this intuition stops working when the cost of producing high quality rises relative to its utility, because in that case convincingly low price signals are unprofitable (i.e., the investment in reputation is too expensive), high and declining prices may guarantee quality and make consumers confident, and it is also possible (indeed, likely, according to intuitive forward induction arguments) that both high and low quality firms initially sell at a low, uninformative price, consumers are initially cautious, because they anticipate that, although rational, they are going to be fooled by some firms, and prices rise after information diffusion clears the market of cheating firms.

In summary, when investing in reputation is cheap, low and rising prices guarantee high quality. When investing in reputation is expensive, high and decreasing prices may guarantee high quality, but it is likely that the reputation mechanism fails to grant high quality production by all firms. In this case a number of interesting phenomena emerge, like rational consumers who are fooled by some firms, a demand-driven shake-out, an increasing time path of market prices and a negative impact of the competitive pressure due to product substitutability on average market quality and consumers' trust.

1.4 Conclusion

I have displayed a linear demand oligopoly model, in which firms endogenously decide whether to enter the market and whether to specialize on high or low quality products, and then repeatedly interact to sell experience goods to consumers, who are able to precisely discover a firm's product quality only after the first purchase, but who are sufficiently rational to form correct expectations about average market quality. Although introductory prices may be used as signals of quality, consumers do not trust them if such signals are too easy to imitate. This creates a strong incentive for firms to pool on the same introductory price, independently of their quality. This coordination incentive, in turn, creates the scope for multiple pooling price equilibria.

Results are presented for the simplest class of pricing rules (in which firms set the same introductory price on and off the equilibrium path of play) and for the most intuitive pricing rule (in which low quality firms mimic high quality firms' introductory price and, given this, high quality firms' introductory price is the most profitable they can set). It is possible to show that an application of forward induction to the simplest class of pricing rules, which adapts the Intuitive Criterion to the present context, does not rule out the multiplicity of sequentially rational pooling introductory prices. On the other hand, the intuitive pricing rule is labeled intuitive precisely because not only it survives the adaptation of the Intuitive Criterion, but it also considers that, since high quality firms would like to be recognized and low quality firms would not like it, the former have a sort of advantage, so that it is intuitive to expect that they set the introductory price that is best for them, given that they are going to be imitated.

Under either the simplest class of introductory pricing rules and the most intuitive pricing rule, both the case in which investing in reputation is cheap and the case in which it is expensive are studied. In the former case the incentive provided by repeated purchase is sufficient to make the reputation mechanism work well, so that low and increasing prices guarantee that all firms produce high quality goods. In the latter case, high quality may be guaranteed by high and decreasing prices under the simplest class of pricing rule, but the most likely outcome is that the market initially hosts both high and low quality firms, and that they pool on the same introductory price, which is therefore uninformative. In this case, a number of interesting results emerge: first, rational consumers know that they are going to be fooled by some firms at the early stage of market interaction, and therefore do not trust firms' initial quality claims very much; second, cheating firms exit the market when discovered and this tends to raise both average quality and prices at the mature stage of market interaction; third, higher product substitutability raises competitive pressure, but tends to lower, besides the number of entrants, both prices and average market quality.

These main results have been derived under the assumption that low quality products cannot be profitably sold under perfect information. This allows to focus on equilibria with pooling introductory prices, which are the only ones in which actual cheating occurs and consumers are not able to distinguish a firms' product quality based on its price. Relaxing this assumption makes the analysis of equilibria with pooling introductory prices significantly and unnecessarily more complicated, although it would be necessary to investigate markets in which different qualities are sold at different prices, with no cheating. Preliminary investigation indicates that this extension promises to yield new interesting results, but that it also poses new subtleties and therefore requires a separate work.

The above results have been obtained under the assumption of either very cheap or very expensive reputation investment, in order to grant that either such investment is always profitable, independently of the degree of market competition, or it is always unprofitable. The analysis of the intermediate case, in which investing in reputation may be profitable when competition is low but not when it is high, would make derivation and presentation of results unnecessarily cumbersome, without adding much to intuition. It is to be expected that, if entry costs are high and the equilibrium number of entrants is low, results would be analogous to those obtained here for the case of very cheap reputation investment; in turn, if entry costs are low and the equilibrium number of entrants is high, results would resemble those obtained here in the case of very expensive reputation investment.

Appendix

Definitions

For $h > 0$ and $p^1 < \alpha(e^1)$, let $\tilde{n}(h, p^1, e^1) \equiv \frac{(c_H - p^1)[\alpha(e^1) - p^1]}{\pi^2(h)}$ be the solution by n of $\pi_H(n, h, p^1, e^1) = 0$, so that at the pooling introductory price $p^1 < \alpha(e^1)$ high quality firms expect positive overall profits if and only if $n \geq \tilde{n}(h, p^1, e^1)$[23].

For $h > 0$ and $p^1 > c_L$, let $\bar{n} \equiv \left[\frac{(1+\mu)(c_H - c_L)\gamma}{\pi^2(1)} - \mu\right]$ be the solution by n to $\pi'_H(n, h, c_L, p^1, 1) = 0$, so that for a high quality firm deviating from the pooling price $p^1 > c_L$ to $p = c_L$ yields strictly positive overall profits if and only if $n > \bar{n}$[24].

For $h > 1$ and $p^1 \in (c_L, \alpha(e^0))$, let $\tilde{\gamma}(n, h, p^1, e^0)$ be implicitly defined by the solution by γ, in the range $\gamma > c_H - c_L$, to $\pi_H(n, h, p^1, e^1) = \pi'_H(n, h, c_L, p^1, 1)$, so that for a high quality firm deviating from a pooling price $p^1 \in (c_L, \alpha(e^0))$ to $p = c_L$ is profitable if and only if $\gamma > \tilde{\gamma}(n, h, p^1, e^0)$[25].

Let $\bar{\gamma}_1$ and $\bar{\gamma}_2$ be implicitly defined by the solutions by γ, in the range $\gamma > c_H - c_L$, of $\tilde{n}(1, c_L, 1) = 1$ and of $\lim_{n\to\infty} \frac{n}{\tilde{n}(n, c_L, 1)} = 1$, respectively[26]. This means that at the introductory price $p^1 = c_L$ a high quality monopolist expects positive overall profits if and only if $\gamma \geq \bar{\gamma}_1$. Moreover, at the pooling introductory price $p^1 = c_L$, high quality firms expect strictly positive overall profits for any $n > 0$ and $h \in \{1, \ldots, n\}$, if and only if $\gamma \geq \bar{\gamma}_2$.

Let $\bar{\mu}$ be implicitly defined by the solution by μ, in the range $\mu > 0$, of $\tilde{\gamma}\left(2, 2, p^E(2,1), 1\right) = \bar{\gamma}_2$, so that, when more than one high quality firm is initially present on the market, pricing at $p^E(n, e^0, c_H)$ may be sequentially rational under Assumption 1.4 only if $\mu \leq \bar{\mu}$[27].

Let $\bar{h}$ be the highest h such that $\pi_H(h, h, p^E(h,1), 1) \geq \pi'_H(h, h, c_L, p^E(h,1), 1)$, so

---

[23]For $h > 0$ and $p^1 < \alpha(e^1)$, the function $\tilde{n}(h, p^1, e^1)$ is increasing in h, increasing in p^1 if $\alpha(e^1) \leq c_H$, inverted U shaped in p^1 if $\alpha(e^1) > c_H$, increasing in e^1 if $p^1 < c_H$ and increasing in e^1 if $p^1 > c_H$. $\forall n > 0$, $\forall h \in \{1, \ldots, n\}$, $\forall e^0 \in [0,1]$, $\forall p^1 < \alpha(e^1)$, if γ is sufficiently high, then $n \geq \tilde{n}(h, p^1, e^1)$.

[24]If γ is sufficiently high, then $\forall n > 1,\ n > \bar{n}$

[25]To see that the defining equation admits a unique solution in the required range, look at expression (1.12). If n, h, e^0 and $p^1(n, h, e^0)$ were given and independent of γ, the LHS would be a linear, increasing and strictly positive function of γ (to verify it, take the limit for $p^1 \to c_L$): deviating to $p = c_L$ is always initially costly. The RHS would be a quadratic function of γ, converging to zero as $\gamma \to (c_H - c_L)$ and increasing above this value: the gains from future monopoly increase quadratically in γ, since both price and quantity increase. The function $\tilde{\gamma}(n, h, p^1, e^0)$ is decreasing in n and in h, converges to $(c_H - c_L)$ as n diverges, is increasing in $p^1 \in \left(c_L, p_H^M(e^0)\right)$ and decreasing in $p^1 \in \left(p_H^M(e^0), \max\{c_H, \alpha(e^0)\}\right)$, and is increasing in e^0 for $p^1 > c_H$ and decreasing in e^0 for $p^1 \leq c_H$.

[26]The precise expression of $\bar{\gamma}_2$ is $\bar{\gamma}_2 = \left[\frac{\mu^2 + 6(1+\mu) + (2+\mu)\sqrt{\mu^2 + 8(1+\mu)}}{2(1+\mu)}\right](c_H - c_L)$. Its definition comes from observing that $n \geq \tilde{n}(h, c_L, 1)$ holds for any $n > 0$ and $h \in \{1, \ldots, n\}$, if and only $n \geq \tilde{n}(n, c_L, 1)$ holds for any $n > 0$. In turn, since $\tilde{n}(n, c_L, 1)$ is always strictly positive and $\frac{n}{\tilde{n}(n, c_L, 1)}$ is decreasing in n, this holds if and only if $\lim_{n\to\infty} \frac{n}{\tilde{n}(n, c_L, 1)} \geq 1$. Observe that $\bar{\gamma}_2 > \bar{\gamma}_1$, that $\gamma \geq \bar{\gamma}_2$ implies $\bar{n} < 1$, and that $\bar{\gamma}_2$ is a continuous and unboundedly increasing function of both μ and $(c_H - c_L)$.

[27]This solution exists and is unique, due to the fact that $\tilde{\gamma}\left(2, 2, p^E(2,1), 1\right) = \left[\frac{2(1+\mu)(4+\mu)^2 + (2+\mu)\mu^2 + 2(4+\mu)\sqrt{(1+\mu)^2(4+\mu)^2 + (1+\mu)(2+\mu)\mu^2}}{(2+\mu)\mu^2}\right](c_H - c_L)$ is a continuous and decreasing function of μ, diverging as $\mu \to 0$, and to the above seen properties of $\bar{\gamma}_2$. Explicit calculation yields $\bar{\mu} \simeq 6.3$.

that, for a high quality firm, deviating from the pooling price $p^E(n, e^0, c_H)$ to $p = c_L$ is profitable if $h > \bar{h}$.

Sequentially rational pooling introductory price functions

Lemma 1.1. ***(either high or zero trust)***
A high quality monopolist has no sequentially rational introductory price, given beliefs (1.7), if $e^0 \in \left(0, \frac{c_H - c_L}{\gamma}\right]$.

Proof. Under Assumption 1.2, $\alpha(e^0) \in (c_L, c_H] \iff e^0 \in \left(0, \frac{c_H - c_L}{\gamma}\right]$. In this case, if a high quality monopolist sets $p^1 \geq \alpha(e^0)$, it receives zero demand and makes zero overall profits. This is not sequentially rational, because there exists an introductory price $p \in (c_L, \alpha(e^0))$, sufficiently close to $\alpha(e^0)$, which grants strictly positive overall profits. In turn, $\forall p^1 < \alpha(e^0)$, $\exists p \in (p^1, \alpha(e^0))$, which grants strictly lower initial losses and the same future profits, and hence strictly higher overall profits. □

Lemma 1.2. ***(seq. rational pooling introductory price functions)***
A pooling introductory price function $p^1(n, h, e^0)$ *is sequentially rational given beliefs (1.7), with* $e^0 \in \{0\} \cup \left(\frac{c_H - c_L}{\gamma}, 1\right]$*, if and only if it satisfies the following conditions.*

1. $\forall n > 0$, $\forall h \in \{0, \dots, n\}$, $p^1(n, h, e^0) \geq c_L$.
2. *If* $n = 1$, $h = 0$ *and* $e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$, *then* $p^1(1, 0, e^0) = p^E(1, e^0, c_L)$.
3. *If* $n = 1$ *and* $h = 1$, *then*
   (a) *if* $e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$, *then* $p^1(1, 1, e^0) = p^E(1, e^0, c_H)$;
   (b) *if* $e^0 = 0$ *and* $\gamma < \bar{\gamma}_1$, *then* $p^1(1, 1, 0) > c_L$;
   (c) *if* $e^0 = 0$ *and* $\gamma > \bar{\gamma}_1$, *then* $p^1(1, 1, 0) = c_L$.
4. *If* $n > 1$, *then*
   (a) *if* $h \in \{1, \dots, n\}$, *then* $p^1(n, h, e^0) = c_L \Rightarrow n \geq \tilde{n}(h, c_L, 1)$;
   (b) *if* $h \in \{1, \dots, n\}$ *and* $e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$, *then* $p^1(n, h, e^0) \geq \alpha(e^0) \Rightarrow n \leq \bar{n}$;
   (c) *if* $h \in \{1, \dots, n\}$ *and* $e^0 = 0$, *then* $p^1(n, h, e^0) > \alpha(e^0) \Rightarrow n \leq \bar{n}$;
   (d) *if* $h = 1$, *then* $p^1(n, 1, e^0) \in (c_L, \alpha(e^0)) \Rightarrow \pi_H(n, h, p^1(n, 1, e^0), e^0) \geq 0$;
   (e) *if* $h \in \{2, \dots, n\}$, *then* $p^1(n, h, e^0) \in (c_L, \alpha(e^0)) \Rightarrow \pi_H(n, h, p^1(n, 1, e^0), e^0) \geq 0$ *and*

$$[p^1(n, h, e^0) - c_H] \left\{ \frac{e^0\gamma - [p^1(n, h, e^0) - c_L]}{n} \right\} + (c_H - c_L) \left(\frac{1 + \mu}{n + \mu} \right) \gamma$$
$$\geq \left\{ \frac{1}{4} - \frac{h + \mu(h - 1)}{[2h + \mu(h - 1)]^2} \right\} [\gamma - (c_H - c_L)]^2. \quad (1.12)$$

Proof. 1. If, for some $n > 0$ and $h \in \{0, \ldots, n\}$, $p^1(n, h, e^0) < c_L$, then at the corresponding information set any firm would strictly gain by deviating to $p = c_L$.

2. Given Assumption 1.2, $e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$ implies $\alpha(e^0) > c_L$. In this case, under beliefs (1.7) a low quality monopolist faces exogenous quality expectations e^0, whatever price it may choose in the interval $(c_L, \alpha(e^0))$. Only if it chooses its optimal monopoly price in this interval, no profitable deviations are possible[28].

3. For a high quality monopolist,

   (a) if $\alpha(e^0) > c_H$, then the above argument applies;

   (b) if $e^0 = 0$, then overall profits are zero at any price $p^1(1, 1, 0) > c_L$. Deviating from any such price to $p = c_L$ is not profitable if and only if $\gamma \leq \bar{\gamma}_1$. In turn, if $p^1(1, 1, 0) = c_L$, deviating to some $p > c_L$ is not profitable if and only if $\gamma \geq \bar{\gamma}_1$;

   (c) see the above argument.

4. When several firms initially enter the market and a pooling introductory price $p^1(n, h, e^0) \geq c_L$ is expected, low quality ones have no profitable deviations. Any high quality firm (if present) may guarantee itself zero overall expected profits through a deviation to $p > c_L$; if it deviates to $p \leq c_L$, it monopolizes the market at both stages 3 and 4, but it makes initial losses (so that the best such deviation is to $p = c_L$).

   (a) If $h > 0$, the pooling price is $p^1(n, h, e^0) = c_L$ is sequentially rational if and only if $\pi_H(n, h, c_L, 1) \geq 0 \iff n \geq \tilde{n}(h, c_L, 1)$.

   (b) If $h > 0$ and $e^0 > 0$, at any pooling price $p^1(n, h, e^0) \geq \alpha(e^0)$, all firms expect zero demand and zero overall profits. By deviating to $p = c_L$, a high quality firm earns overall deviation profits $\pi'_H(n, h, c_L, p^1(n, h, e^0), 1) = -(c_H - c_L)\left(\frac{1+\mu}{n+\mu}\right)\gamma + \pi^2(1)$. Such deviation is not profitable if and only if $\pi'_H(n, h, c_L, p^1(n, h, e^0), 1) \leq 0 \iff n \leq \bar{n}$.

   (c) This holds by the same argument as above.

   (d) If $h = 1$, at stage 4 the unique high quality firm monopolizes the market, and makes the same profits, independently of whether at stage 3 it sticks to a pooling price $p^1(n, h, e^0) \in (c_L, \alpha(e^0))$ or it deviates to $p = c_L$. Yet, such deviation yields strictly lower initial profits (indeed, strictly higher initial losses), so it is not profitable. Hence, a pooling price $p^1(n, h, e^0) \in (c_L, \alpha(e^0))$ is sequentially rational in this case if and only if $\pi_H(n, h, p^1(n, 1, e^0), e^0) \geq 0$.

   (e) If $h > 1$, then a pooling price $p^1(n, h, e^0) \in (c_L, \alpha(e^0))$ is sequentially rational if and only if both $\pi_H(n, h, p^1(n, h, e^0), e^0) \geq 0$ and $\pi_H(n, h, p^1(n, h, e^0), e^0) \geq \pi'_H(n, h, c_L, p^1(n, h, e^0), 1)$. (1.12) is just the explicit expression of this last condition.

□

[28]Observe that, here and below, in the cases not explicitly considered no additional constraints are imposed. So for $e^0 = 0$, any $p^1(1, 0, e^0) \geq c_L$ is sequentially rational at the corresponding information set.

Corollary 1.1. ***(sequential rationality of** $p^E(n,e^0,c_H)$**)***
Let Assumption 1.4 hold. At a stage 3 information set such that $n > 0$ and $h \in \{0,\dots,n\}$, and given beliefs (1.7), with $e^0 \in \left(\frac{c_H-c_L}{\gamma},1\right]$, suppose all firms set $p^E(n,e^0,c_H)$.

1. *If $n = 1$, then $p^E(n,e^0,c_H)$ is sequentially rational if and only if $h = 1$.*
2. *If $n > 1$, then $p^E(n,e^0,c_H)$ is sequentially rational if and only if either $h \in \{0,1\}$ or $\pi_H(n,h,p^E(n,e^0,c_H),e^0) \geq \pi'_H(n,h,c_L,p^E(n,e^0,c_H),1)$.*

Proof. This follows from Lemma 1.2, since Assumption 1.4 implies that $\pi_H(n,h,p^E(n,e^0,c_H),e^0) \geq 0$ holds $\forall(n,h): n \geq h > 1$, $\forall e^0 \in \left(\frac{c_H-c_L}{\gamma},1\right]$. □

Corollary 1.2. ***(no seq. rationality of** $p^E(n,e^0,c_H)$ **for high** μ, γ, h **or** n**)***
Let Assumption 1.4 hold.

1. *If either $\gamma > \tilde{\gamma}\left(2,2,p^E(2,1),1\right)$ or $\mu > \bar{\mu}$, then $\forall(n,h): n \geq h > 1$, $\forall e^0 \in \left(\frac{c_H-c_L}{\gamma},1\right]$, $p^1(n,h,e^0) = p^E(n,e^0,c_H)$ is not sequentially rational at the corresponding stage 3 information set.*
2. *If $\mu \leq \bar{\mu}$ and $\gamma \leq \tilde{\gamma}\left(2,2,p^E(2,1),1\right)$, then*
   - *(a) the maximum number $\bar{h}$ of high quality firms, such that, $\forall h > \bar{h}$, $\forall n \geq h$, $\forall e^0 \in \left(\frac{c_H-c_L}{\gamma},1\right]$, $p^E(n,e^0,c_H)$ is not sequentially rational at the corresponding stage 3 information set, is finite and satisfies $\bar{h} \geq 2$;*
   - *(b) $\forall h \in \{2,\dots,\bar{h}\}$, the maximum number $\bar{n}(h)$ of high quality firms, such that, $\forall n \geq \bar{n}(h)$, $\forall e^0 \in \left(\frac{c_H-c_L}{\gamma},1\right]$, $p^E(n,e^0,c_H)$ is not sequentially rational at the corresponding stage 3 information set, is finite, decreasing in h, and satisfies $\bar{n}(h) \geq h$.*

Proof. This follows from Lemma 1.2, Corollary 1.1, and four observations. First, $\forall(n,h): n \geq h > 1$, $\forall e^0 \in \left(\frac{c_H-c_L}{\gamma},1\right]$, we have $\tilde{\gamma}\left(n,h,p^E(n,e^0,c_H),e^0\right) \leq \tilde{\gamma}\left(2,2,p^E(2,1),1\right)$. Second, $\bar{\gamma}_2 > \tilde{\gamma}\left(2,2,p^E(2,1),1\right) \iff \mu > \bar{\mu}$. Third, $\gamma \leq \tilde{\gamma}\left(2,2,p^E(2,1),1\right) \Rightarrow \bar{h} \geq 2$. Fourth, $\forall e^0 \in \left(\frac{c_H-c_L}{\gamma},1\right]$, $\tilde{\gamma}\left(n,h,p^E(n,e^0,c_H),e^0\right)$ is decreasing in n and h, and $\lim_{h\to\infty}\tilde{\gamma}\left(h,h,p^E(h,e^0),e^0\right) = (c_H - c_L) < \bar{\gamma}_2$, so there must exist $\bar{h} \in [2,\infty)$ with the above mentioned properties. The same kind of argument applies to $\bar{n}(h)$. □

These last two corollaries assert that when high quality is sufficiently important to consumers, and several high quality firms are initially present on the market, future monopoly profits are so much higher than initial monopoly losses that deviating to $p = c_L$ is always profitable. This is always the case, for any γ satisfying Assumption 1.4, when the elasticity of substitution is high, since this makes competition fierce and monopoly more attractive. In turn, a low elasticity of substitution, by alleviating competitive pressure, shifts the threshold of γ, above which $p^E(n,e^0,c_H)$ stops being sequentially rational, above $\bar{\gamma}_2$, so that, under Assumption 1.4, if γ is not too high, then there exist $(n,h,e^0): n \geq h > 1$, $e^0 \in \left(\frac{c_H-c_L}{\gamma},1\right]$, for which $p^E(n,e^0,c_H)$ is sequentially rational. Indeed, in this case sequential rationality holds as long as the competitive pressure is not too high, i.e., as long as n and h are not too high.

Sequentially rational quality choices

Let $f(x,n) \equiv \frac{(1+\mu)x-\frac{\mu}{n}}{[(2+\mu)x-\frac{\mu}{n}]^2}$ and $g(e^0) \equiv \frac{(c_H-c_L)(\gamma-\delta)e^0}{[\alpha(1)-c_H]^2}$. The functions f and g have the following properties: $f(\frac{1}{n},n) = \frac{n}{4}$; $f(1,n) = \frac{n[n+\mu(n-1)]}{[2n+\mu(n-1)]^2}$; given $n \geq 1$, $f(x,n)$ is decreasing in $x \in \left[\frac{1}{n},1\right]$ and $f\left(x+\frac{1}{n},n\right)$ defines a new function of x, which is just a leftwards shift of $f(x,n)$ by $1/n$; given $x \in \left[\frac{1}{n},1\right]$, $f(x,n)$ is decreasing in n; given $x \in [0,1]$, $f\left(x+\frac{1}{n},n\right)$ defines a new function of n, which is increasing; given $h \in \{1,\ldots,n\}$, $f(\frac{h}{n},n)$ defines a new function of n, which is increasing; $g(e^0)$ is increasing.
Let $\tilde{\delta}_H(n) \equiv \gamma - \frac{n[n+\mu(n-1)][\alpha(1)-c_H]^2}{[2n+\mu(n-1)]^2(c_H-c_L)}$, $\tilde{\delta}_L(n) \equiv \gamma - \frac{n[\alpha(1)-c_H]^2\gamma}{4(c_H-c_L)^2}$,
$\hat{\delta}_H(n) \equiv \gamma - \frac{n[n+\mu(n-1)][\alpha(1)-c_H]^2\gamma}{[2n+\mu(n-1)]^2(c_H-c_L)^2}$ and $\hat{\delta}_L(n) \equiv \gamma - \frac{n[\alpha(1)-c_H]^2}{4(c_H-c_L)}$.
Let $\bar{e}_B(n) \equiv \frac{n[n+\mu(n-1)][\alpha(1)-c_H]^2}{[2n+\mu(n-1)]^2(c_H-c_L)(\gamma-\delta)}$ and $\bar{e}_T(n) \equiv \frac{n^2}{4(c_H-c_L)(\gamma-\delta)}$.

Lemma 1.3. ***(thresholds for flat prices)***
Let Assumption 1.3 hold and assume $N \geq 8$. Then $\forall n \in \{2,\ldots,N\}$,
$0 < \bar{e}_B(n) < \bar{e}_T(n)$, $\quad 0 < \tilde{\delta}_L(n) < \hat{\delta}_L(n) < \hat{\delta}_H(n) < \tilde{\delta}_H(n)$ *and*

1. *if $\delta > \tilde{\delta}_H(n)$, then $\bar{e}_B(n) > 1$;*
2. *if $\delta \in \left(\hat{\delta}_H(n),\tilde{\delta}_H(n)\right]$, then $\bar{e}_B(n) \in \left(\frac{c_H-c_L}{\gamma},1\right]$ and $\bar{e}_T(n) > 1$;*
3. *If $\delta \in \left(\hat{\delta}_L(n),\hat{\delta}_H(n)\right]$, then $\bar{e}_B(n) \leq \frac{c_H-c_L}{\gamma}$ and $\bar{e}_T(n) > 1$;*
4. *if $\delta \in \left(\tilde{\delta}_L(n),\hat{\delta}_L(n)\right]$, then $\bar{e}_B(n) \leq \frac{c_H-c_L}{\gamma}$ and $\bar{e}_T(n) \in \left(\frac{c_H-c_L}{\gamma},1\right]$;*
5. *if $\delta \leq \tilde{\delta}_L(n)$, then $\bar{e}_T(n) \leq \frac{c_H-c_L}{\gamma}$.*

Proof. The above thresholds are defined so that, $\forall n \in \{2,\ldots,N\}$, $g(\bar{e}_B(n)) = f(1,n)$; $g(\bar{e}_T(n)) = f(\frac{1}{n},n)$; $f\left(\frac{1}{n},n\right) > g\left(\frac{c_H-c_L}{\gamma}\right) \iff \delta > \tilde{\delta}_L(n)$; $f(1,n) < g(1) \iff \delta < \tilde{\delta}_H(n)$; $f\left(\frac{1}{n},n\right) < g(1) \iff \delta < \hat{\delta}_L(n)$; $f(1,n) < g\left(\frac{c_H-c_L}{\gamma}\right) \iff \delta < \hat{\delta}_H(n)$; and under Assumption 1.3 and $N \geq 8$, it is immediate to verify the above chain of inequalities. Observe that $\tilde{\delta}_L(n)$ and $\hat{\delta}_L(n)$ are decreasing; $\tilde{\delta}_H(n)$ and $\hat{\delta}_H(n)$ are increasing; $\bar{e}_B(n)$ is decreasing and $\bar{e}_T(n)$ is increasing. □

Let $\bar{\delta}(n) \equiv \gamma - \frac{n^2[\alpha(1)-c_H]^2}{4(c_H-c_L)}$. Let $x_1(n,e^0)$ be the solution of $g(e^0) = f\left(x+\frac{1}{n},n\right)$, by $x > 0$, given $e^0 \in [\bar{e}_B(n),\bar{e}_T(n)]$; $x_2(n,e^0)$ be the solution of $g(e^0) = f(x,n)$, by $x > \frac{1}{n}$, given $e^0 \in [\bar{e}_B(n),\bar{e}_T(n)]$; $\bar{e}_M(n)$ be the solution of $g(x) = f\left(x+\frac{1}{n},n\right)$, by $x > 0$, given $\delta \leq \tilde{\delta}_H(n)$; and $\bar{e}_N(n)$ be the solution of $g(x) = f(x,n)$, by $x > \frac{1}{n}$, given $\delta \in [\bar{\delta}(n),\tilde{\delta}_H(n)]$.

Lemma 1.4. ***(quality choice under low γ, flat prices and $e^0 > 0$)***
Let Assumption 1.3 hold. Assume that $e^0 \in \left(\frac{c_H-c_L}{\gamma},1\right]$ and that prices are set according to the pooling introductory price function $p^1_\delta(n,h,e^0)$ at stage 3, with $\delta \in \left(\check{\delta},\gamma\right)$, and that the unique Nash equilibrium is played at stage 4. Assume as well $N \geq 8$. Then $\forall e^0 \in \left(\frac{c_H-c_L}{\gamma},1\right]$, the unique sequentially rational quality choice for a monopolist is low quality. $\forall n \in \{2,\ldots,N\}$,

1. *if $e^0 \in \left(\frac{c_H-c_L}{\gamma},\bar{e}_B(n)\right)$, then the unique sequentially rational quality choice profile has $h(n,e^0) = n$;*

2. *if $e^0 \in [\bar{e}_B(n), \bar{e}_T(n)]$, then a quality choice profile is sequentially rational if and only if $\frac{h(n,e^0)}{n} \in [x_1(n,e^0), x_2(n,e^0)]$; in particular, $x_1(n,e^0)$ and $x_2(n,e^0)$ are both decreasing in e^0; $x_1(n,e^0)$ is increasing in n; $x_2(n,e^0)$ is decreasing in n; $\bar{e}_M(n)$ is increasing in n; if $\delta \geq \bar{\delta}(n)$, then $\bar{e}_N(n)$ is decreasing in n, the two intervals $[x_1(n,e^0), x_2(n,e^0)]$ and $[\bar{e}_M(n), \bar{e}_N(n)]$ are both contained in $[0,1]$, may or not overlap, satisfy $x_2(n,e^0) - x_1(n,e^0) = \frac{1}{n}$, $\bar{e}_N(n) - \bar{e}_M(n) < \frac{1}{n}$, $[\bar{e}_M(n+1), \bar{e}_N(n+1)] \subset [\bar{e}_M(n), \bar{e}_N(n)]$, and*
   (a) $e^0 > \bar{e}_N(n) \Rightarrow x_2(n,e^0) < e^0$;
   (b) $e^0 \in [\bar{e}_M(n), \bar{e}_N(n)] \Rightarrow e^0 \in [x_1(n,e^0), x_2(n,e^0)]$;
   (c) $e^0 < \bar{e}_M(n) \Rightarrow x_1(n,e^0) > e^0$;
3. *if $e^0 \in (\bar{e}_T(n), 1]$, then the unique sequentially rational quality choice profile has $h(n,e^0) = 0$.*

Proof. Notice first that $\bar{\gamma} \leq 2(c_H - c_L) \iff N \geq 8$. Under Assumption 1.3 and $N \geq 8$, a monopolist pricing according to $p^1_\delta(n,h,e^0)$ strictly prefers low to high quality $\forall e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$ (to be precise, weakly for $N = 8$ and $\gamma = \bar{\gamma}$, and strictly in any other case)[29]. Given $n \in \{2, \ldots, N\}$ and $e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$, sequential rationality of the quality choice profile $\mathbf{z}(n,e^0)$ may be conveniently rewritten in terms of the above defined functions f and g:

$$f\left(\frac{h}{n}, n\right) \geq g(e^0) \geq f\left(\frac{h+1}{n}, n\right), \tag{1.13}$$

where only the first inequality matters if $h = n$ and only the second one if $h = 0$. Lemma 1.3 establishes, $\forall n \in \{2, \ldots, N\}$, whether and how the two intervals $\left(\frac{c_H - c_L}{\gamma}, 1\right]$ and $(\bar{e}_B(n), \bar{e}_T(n))$ intersect.

1. If $e^0 \in \left(\frac{c_H - c_L}{\gamma}, \bar{e}_B(n)\right)$, then the second inequality of (1.13) is always violated for $h < n$, whereas the first one always holds for $h = n$.
2. If $e^0 \in [\bar{e}_B(n), \bar{e}_T(n)]$, then condition (1.13) holds if and only if $\frac{h(n,e^0)}{n} \in [x_1(n,e^0), x_2(n,e^0)]$. The monotonicity properties of the boundaries of this interval follow from those of f and g, and the same is true for whether $e^0 \in [x_1(n,e^0), x_2(n,e^0)]$ or not. Since the length of the $[x_1(n,e^0), x_2(n,e^0)]$ interval is $1/n$, an integer $h(n,e^0) \in \{0, \ldots, n\}$ such that (1.13) holds exists[30]. Notice that $e^0 \in [\bar{e}_B(n), \bar{e}_T(n)] \Rightarrow \delta \leq \tilde{\delta}_H(n)$, so that $\bar{e}_M(n)$ in this case is well defined. In turn, $\bar{e}_N(n)$ is well defined if and only if $\delta \geq \bar{\delta}(n)$, since this condition is equivalent to $f(\frac{1}{n}) \geq g(\frac{1}{n})$. Since $\bar{\delta}(n)$ is decreasing, $\delta \geq \bar{\delta}(n)$ implies that $\bar{e}_N(n')$ is well defined $\forall n' > n$. The fact that $[\bar{e}_M(n), \bar{e}_N(n)]$ is a sequence of shrinking intervals, each contained in the previous one, again follows from the monotonicity properties of f and g[31].

[29] The assumption of $N \geq 8$ plays no other role than to simplify the exposition of a monopolist's choice.

[30] If $n \cdot x_1(n,e^0) \in \{0, \ldots, n-1\}$, then both $h(n,e^0) = n \cdot x_1(n,e^0)$ and $h(n,e^0) = n \cdot x_1(n,e^0) + 1$ satisfy condition (1.13). In any other case, the integer $h(n,e^0)$ such that (1.13) holds is unique.

[31] Observe that $\bar{e}_M(n)$ and $\bar{e}_N(n)$ are decreasing in μ and $(c_H - c_L)$, and increasing in δ and γ.

3. If $e^0 \in (\bar{e}_T(n), 1]$, then the first inequality of (1.13) is always violated for $h > 0$, whereas the second one always holds for $h = 0$.

□

Let $\tilde{f}(x,n) \equiv \frac{[2n+\mu(n-1)][(1+\mu)x-\frac{\mu}{n}]}{[n+\mu(n-1)][(2+\mu)x-\frac{\mu}{n}]^2}$ and $\tilde{g}(e^0) \equiv \frac{(c_H-c_L)[\alpha(e^0)-c_H]}{[\alpha(1)-c_H]^2}$. The functions $\tilde{f}$ and $\tilde{g}$ have the following properties: $\tilde{f}(\frac{1}{n}, n) = \frac{n[2n+\mu(n-1)]}{4[n+\mu(n-1)]}$; $\tilde{f}(1,n) = \frac{n}{2n+\mu(n-1)}$; given $n \geq 1$, $\tilde{f}(x,n)$ is decreasing in $x \in \left[\frac{1}{n}, 1\right]$ and $\tilde{f}\left(x + \frac{1}{n}, n\right)$ defines a new function of x, which is just a leftwards shift of $\tilde{f}(x,n)$ by $1/n$; given $x \in \left[\frac{1}{n}, 1\right]$, $\tilde{f}(x,n)$ is decreasing in n; given $x \in [0,1]$, $\tilde{f}\left(x + \frac{1}{n}, n\right)$ defines a new function of n, which tends to be increasing; given $h \in \{1, \ldots, n\}$, $\tilde{f}(\frac{h}{n}, n)$ defines a new function of n, which is increasing; $g(e^0)$ is increasing; $\forall n \in \{1, \ldots, N\}$ it holds that $\tilde{f}\left(\frac{1}{n}, n\right) > \tilde{g}\left(\frac{c_H-c_L}{\gamma}\right)$ and that, if $N \geq 3$, then $\tilde{f}(1,n) < \tilde{g}(1)$.
Let $\tilde{e}_B(n) \equiv \frac{c_H-c_L}{\gamma} + \frac{n[\alpha(1)-c_H]^2}{[2n+\mu(n-1)](c_H-c_L)\gamma}$ and $\tilde{e}_T(n) \equiv \frac{c_H-c_L}{\gamma} + \frac{n[2n+\mu(n-1)][\alpha(1)-c_H]^2}{4[n+\mu(n-1)](c_H-c_L)\gamma}$.
Let $\tilde{x}_1(n,e^0)$ and $\tilde{x}_2(n,e^0)$ be the solutions of $\tilde{g}(e^0) = \tilde{f}\left(x + \frac{1}{n}, n\right)$ and $\tilde{g}(e^0) = \tilde{f}(x,n)$, by $x > 0$, given $e^0 \in [\tilde{e}_B(n), \tilde{e}_T(n)]$, respectively.
Let $\tilde{e}_M(n)$ be the solution by $x > 0$ of $\tilde{g}(x) = \tilde{f}\left(x + \frac{1}{n}, n\right)$; and $\tilde{e}_N(n)$ be the solution by $x > \frac{1}{n}$ of $\tilde{g}(x) = \tilde{f}(x,n)$.

Lemma 1.5. ***(quality choice under low γ, intuitive prices and** $e^0 > 0$**)***
Let Assumption 1.3 hold. Assume that $e^0 \in \left(\frac{c_H-c_L}{\gamma}, 1\right]$, that prices are set according to the pooling introductory price function $p_E^1(n,h,e^0)$ at stage 3, and that the unique Nash equilibrium is played at stage 4. Assume as well $N \geq 8$. Then $\forall e^0 \in \left(\frac{c_H-c_L}{\gamma}, 1\right]$, the unique sequentially rational quality choice for a monopolist is low quality. $\forall n \in \{2, \ldots, N\}$,

1. *if $e^0 \in \left(\frac{c_H-c_L}{\gamma}, \tilde{e}_B(n)\right)$, then the unique sequentially rational quality choice profile has $h(n,e^0) = n$;*
2. *if $e^0 \in [\tilde{e}_B(n), \tilde{e}_T(n)]$, then a quality choice profile is sequentially rational if and only if $\frac{h(n,e^0)}{n} \in [\tilde{x}_1(n,e^0), \tilde{x}_2(n,e^0)]$; in particular, $\tilde{x}_1(n,e^0)$ and $\tilde{x}_2(n,e^0)$ are both decreasing in e^0; $\tilde{x}_1(n,e^0)$ and $\tilde{e}_M(n)$ tend to be increasing in n; $\tilde{x}_2(n,e^0)$ and $\tilde{e}_N(n)$ are decreasing in n; the two intervals $[\tilde{x}_1(n,e^0), \tilde{x}_2(n,e^0)]$ and $[\tilde{e}_M(n), \tilde{e}_N(n)]$ are both contained in $[0,1]$, may or not overlap, satisfy $\tilde{x}_2(n,e^0) - \tilde{x}_1(n,e^0) = \frac{1}{n}$ and $\tilde{e}_N(n) - \tilde{e}_M(n) < \frac{1}{n}$, and*
   (a) $e^0 > \tilde{e}_N(n) \Rightarrow \tilde{x}_2(n,e^0) < e^0$;
   (b) $e^0 \in [\tilde{e}_M(n), \tilde{e}_N(n)] \Rightarrow e^0 \in [\tilde{x}_1(n,e^0), \tilde{x}_2(n,e^0)]$;
   (c) $e^0 < \tilde{e}_M(n) \Rightarrow \tilde{x}_1(n,e^0) > e^0$.
3. *if $e^0 \in (\tilde{e}_T(n), 1]$, then the unique sequentially rational quality choice profile has $h(n,e^0) = 0$.*

Proof. Under Assumption 1.3 and $N \geq 8$, a monopolist pricing according to $p_E^1(n,h,e^0)$ strictly prefers low to high quality $\forall e^0 \in \left(\frac{c_H-c_L}{\gamma}, 1\right]$[32]. Given $n \in \{2, \ldots, N\}$ and

[32]As in Lemma 1.4, the assumption of $N \geq 8$ plays no other role here than to simplify the exposition of a monopolist's choice.

$e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$, sequential rationality of the quality choice profile $\mathbf{z}(n, e^0)$ may be conveniently rewritten in terms of the following condition:

$$\tilde{f}\left(\frac{h}{n}, n\right) \geq \tilde{g}(e^0) \geq \tilde{f}\left(\frac{h+1}{n}, n\right), \tag{1.14}$$

where only the first inequality matters if $h = n$ and only the second one if $h = 0$. $\forall n \in \{2, \dots, N\}$, $\tilde{e}_B(n)$ and $\tilde{e}_T(n)$ are the solutions by $e^0 > 0$ of $\tilde{f}(1, n) = \tilde{g}(e^0)$ and $\tilde{f}\left(\frac{1}{n}, n\right) = \tilde{g}(e^0)$, respectively; and the following holds.

1. If $e^0 \in \left(\frac{c_H - c_L}{\gamma}, \tilde{e}_B(n)\right)$, then the second inequality of (1.14) is always violated for $h < n$, whereas the first one always holds for $h = n$.

2. If $e^0 \in [\tilde{e}_B(n), \tilde{e}_T(n)]$, then condition (1.14) holds if and only if $\frac{h(n,e^0)}{n} \in [\tilde{x}_1(n, e^0), \tilde{x}_2(n, e^0)]$. The monotonicity properties of the boundaries of this interval, as well as those of $[\tilde{e}_M(n), \tilde{e}_N(n)]$, follow from those of $\tilde{f}$ and $\tilde{g}$, and the same is true for whether $e^0 \in [\tilde{x}_1(n, e^0), \tilde{x}_2(n, e^0)]$ or not[33]. Since the length of the $[\tilde{x}_1(n, e^0), \tilde{x}_2(n, e^0)]$ interval is $1/n$, an integer $h(n, e^0) \in \{0, \dots, n\}$ such that (1.14) holds exists[34].

3. If $e^0 \in (\tilde{e}_T(n), 1]$ (but observe that this interval is empty for high n), then the first inequality of (1.14) is always violated for $h > 0$, whereas the second one always holds for $h = 0$.

□

Lemma 1.6. ***(seq. rat. quality choice under** $p^1_M(n, h, e^0)$ **and** $e^0 > 0$)*
Let Assumption 1.4 hold. If $e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$, $\mu \leq \bar{\mu}$, $\gamma \leq \tilde{\gamma}(2, 2, p^E(2, 1), 1)$ *and firms price according to* $p^1_M(n, h, e^0)$ *at stage 3 and play the Nash equilibrium at stage 4, then a sequence of quality choice profiles* $\mathbf{z}(n, e^0)$ *is sequentially rational, given beliefs (1.7), if and only if the following conditions are satisfied:* $h(1, e^0) = 1$ *and for* $n > 1$,

1. *if* $\pi'_H(n, n, c_L, p^E(n, e^0, c_H), 1) > \pi_H(n, n-1, p^E(n, e^0, c_H), e^0)$, *then* $h(n, e^0) = n$;

2. *if* $\pi_H(n, n, p^E(n, e^0, c_H), e^0) < \pi'_H(n, n, c_L, p^E(n, e^0, c_H), 1) \leq \pi_H(n, n-1, p^E(n, e^0, c_H), e^0)$, *then* $h(n, e^0) = n$ *and* $\pi_H(n, n, c_L, 1) \geq \pi_L(n, n, p^E(n, e^0, c_H), e^0)$;

3. *if* $\pi'_H(n, n, c_L, p^E(n, e^0, c_H), 1) \leq \pi_H(n, n, p^E(n, e^0, c_H), e^0)$, *then either* $h(n, e^0) = n$, *or* $0 < h(n, e^0) < n$ *and the following holds:*

   (a) $\pi_H(n, h(n)+1, p^E(n, e^0, c_H), e^0) < \pi'_H(n, n, c_L, p^E(n, e^0, c_H), 1) \leq \pi_H(n, h(n), p^E(n, e^0, c_H), e^0)$,

   (b) $\pi_L(n, h(n), p^E(n, e^0, c_H), e^0) \geq \pi_H(n, h(n) + 1, c_L, 1)$.

Proof. If either $n = 1$ or $n > 1$ and $\pi'_H(n, h, c_L, p^E(n, e^0, c_H), 1) > \pi_H(n, h-1, p^E(n, e^0, c_H), e^0)$, then $p^1_M(n, h, e^0) = p^1_R(n, h, e^0)$ and the result follows from Proposition 1.14. Now consider $n \geq h > 1$ and $\pi'_H(n, h, c_L, p^E(n, e^0, c_H), 1) \leq \pi_H(n, h-1, p^E(n, e^0, c_H), e^0)$. If $\pi_H(n, h, p^E(n, e^0, c_H), e^0) < \pi'_H(n, h, c_L, p^E(n, e^0, c_H), 1)$, then $p^1_M(n, h, e^0) = c_L$, so that $h < n$ cannot be sequentially rational, since $p^1_M(n, h+1, e^0) = c_L$ and low quality

[33] Observe that $\tilde{e}_M(n)$ and $\tilde{e}_N(n)$ are decreasing in μ, increasing in $(c_H - c_L)$, and increasing in γ when they are high, but decreasing in γ when they are low

[34] If $n \cdot x_1(n, e^0) \in \{0, \dots, n-1\}$, then both $h(n, e^0) = n \cdot x_1(n, e^0)$ and $h(n, e^0) = n \cdot x_1(n, e^0) + 1$ satisfy condition (1.14). In any other case, the integer $h(n, e^0)$ such that (1.14) holds is unique.

is a strictly dominated choice; $h = n$ is sequentially rational if and only if e^0 is strictly lower than 1 and sufficiently close to $\frac{c_H - c_L}{\gamma}$, since $p^1_M(n, h-1, e^0) = p^E(n, e^0, c_H)$ and deviating to low quality is profitable for $e^0 = 1$, because $\pi^N_L(n, n-1, 1) = \pi_L(n, n-1, p^E(n,1), 1) = \pi^1_L(p^E(n,1), 1, n) > \pi^2(n) > \pi_H(n, n, c_L, 1) = \pi^N_H(n, n, 1)$, but it is not for $e^0 \to \frac{c_H - c_L}{\gamma}$, because then $\pi_L(n, n-1, p^E(n,1), 1) \to 0$. Notice that deviation gains are continuous in e^0. If $\pi'_H(n, h, c_L, p^E(n, e^0, c_H), 1) \leq \pi_H(n, h, p^E(n, e^0, c_H), e^0)$, then $p^1_M(n, h, e^0) = p^1_M(n, h-1, e^0) = p^E(n, e^0, c_H)$ and $\pi^N_H(n, h, e^0) > \pi^N_L(n, h-1, e^0)$ always holds under Assumption 1.4. In this case, if $h < n$, then $\pi^N_L(n, h, e^0) \geq \pi^N_H(n, h+1, e^0)$ never holds for $\pi'_H(n, h, c_L, p^E(n, e^0, c_H), 1) \leq \pi_H(n, h+1, p^E(n, e^0, c_H), e^0)$, because then $p^1_M(n, h+1, e^0) = p^E(n, e^0, c_H)$, for $h = n-1$ we know from above that $\pi^N_L(n, n-1, e^0) < \pi^N_H(n, n, e^0)$, and if it is profitable to deviate to high quality for $h = n-1$, then it is profitable to do it for any $h < n-1$; in turn, if $\pi'_H(n, h, c_L, p^E(n, e^0, c_H), 1) > \pi_H(n, h+1, p^E(n, e^0, c_H), e^0)$, then $p^1_M(n, h+1, e^0) = c_L$ and $\pi^N_L(n, h, e^0) \geq \pi^N_H(n, h+1, e^0)$ does not hold for $e^0 \to \frac{c_H - c_L}{\gamma}$, because then $\pi_L(n, h, p^E(n, e^0, c_H), e^0) \to 0$, but it may hold or not for higher e^0, depending on parameters. □

Trust and consistent expectations

Let $\bar{h}(n)$ be the unique integer such that $\frac{\bar{h}(n)}{n} \in \left(\bar{e}_N(n) - \frac{1}{n},\, \bar{e}_N(n)\right]$.

Lemma 1.7. ***(consistency under low γ and flat prices)***
Under the assumptions of Lemma 1.4, suppose that in equilibrium $n \in \{2, \ldots, N\}$ firms enter the market and allow one firm to randomize between high and low quality. Assume that δ is sufficiently high as to make $\bar{e}_M(n) > \frac{c_H - c_L}{\gamma}$. Then both sequential rationality at stages 2, 3 and 4 and consistency of beliefs (1.7) along the equilibrium path of play simultaneously hold if and only if $\bar{h}(n)$ firms choose high quality in pure strategies and

1. *if $\frac{\bar{h}(n)}{n} \in [\bar{e}_M(n),\, \bar{e}_N(n)]$, then $(n - \bar{h}(n))$ firms choose low quality in pure strategies and $e^0 = \frac{\bar{h}(n)}{n}$;*
2. *if $\frac{\bar{h}(n)}{n} \in \left(\bar{e}_N(n) - \frac{1}{n},\, \bar{e}_M(n)\right)$, then $(n - \bar{h}(n) - 1)$ firms choose low quality in pure strategies,*

$$e^0 = \frac{n[\bar{h}(n) + 1 + \mu\bar{h}(n)][\alpha(1) - c_H]^2}{[2\bar{h}(n) + 2 + \mu\bar{h}(n)]^2 (c_H - c_L)(\gamma - \delta)} \tag{1.15}$$

and the mixing firm chooses high quality with probability $w = ne^0 - \bar{h}(n)$.

Proof. Under the above assumptions, consistency of beliefs requires $e^0 = \frac{\bar{h}(n)}{n}$ if all firms play in pure strategies, with $\bar{h}(n)$ of them choosing high quality, and $e^0 = \frac{E(h)}{n} = \frac{\bar{h}(n)+w}{n}$ if one firm chooses high quality with probability $w \in (0,1)$, $\bar{h}(n)$ firms choose high quality in pure strategies and $(n - \bar{h}(n) - 1)$ firms choose low quality in pure strategies. By Lemma 1.4, given this, sequential rationality of quality choices implies that either $\delta > \tilde{\delta}_H(n)$ (i.e., $\bar{e}_B(n) > 1$ and $\bar{e}_N(n) > 1$), or $e^0 \in [\bar{e}_M(n), \bar{e}_N(n)]$. In the former case, consistency and sequential rationality imply $e^0 = 1$ and $h(n,1) = n$. In the latter case, if δ is so low that $\bar{e}_M(n) \leq \frac{c_H - c_L}{\gamma}$, then a WPBE with flat pooling introductory prices may not exist, because consistent beliefs may require $e^0 \leq \frac{c_H - c_L}{\gamma}$, in which case $p^1_\delta(n, h, e^0)$ is not sequentially rational. By contrast, if δ is sufficiently high as to grant $\bar{e}_M(n) > \frac{c_H - c_L}{\gamma}$, as assumed above, then sequential rationality of $p^1_\delta(n, h, e^0)$ is not an issue. In this case, if $\frac{\bar{h}(n)}{n} \in [\bar{e}_M(n), \bar{e}_N(n)]$, then we have consistency and sequential rationality in pure strategies. If this does not hold and one firm chooses high quality with probability $w \in (0, 1)$, this firm must be indifferent between the two qualities. Given the other firms' pure strategy quality choices (i.e., given that $\bar{h}(n)$ other firms choose high quality and $(n - \bar{h}(n) - 1)$ choose low quality), this condition may be expressed as $g(e^0) = f\left(\frac{\bar{h}(n)+1}{n}, n\right)$, which, solved for e^0, yields (1.15). Firms choosing low quality in pure strategies then expect exactly the same profits as the mixing firm. In turn, firms choosing high quality in pure strategies expect slightly higher profits, because with probability w there will be $\bar{h}(n) + 1$ competitors at stage 4, but with probability $(1 - w)$ there will be only $\bar{h}(n)$. Yet, no firm has a profitable deviation, since if $\bar{h}(n) + 1$ firms were to choose high quality in pure strategies, then expected profits would be lower than in equilibrium. Then w is determined by the consistency equation $e^0 = \frac{\bar{h}(n)+w}{n}$, and this indeed grants that $e^0 \in [\bar{e}_M(n), \bar{e}_N(n)]$). Finally, Lemma 1.4 implies that for any other number of firms choosing high quality in pure strategies and any other specification of w, sequential rationality of quality choices and consistency of beliefs cannot hold at the same time. □

Let $\tilde{h}(n)$ be the unique integer such that $\frac{\tilde{h}(n)}{n} \in \left(\tilde{e}_N(n) - \frac{1}{n},\, \tilde{e}_N(n)\right]$.

Lemma 1.8. ***(consistency under low γ and intuitive prices)***
Under the assumptions of Lemma 1.5, suppose that in equilibrium $n \in \{2, \ldots, N\}$ firms enter the market and allow one firm to randomize between high and low quality. Then both sequential rationality at stages 2, 3 and 4 and consistency of beliefs (1.7) along the equilibrium path of play simultaneously hold if and only if $\tilde{h}(n)$ firms choose high quality in pure strategies and

1. *if $\frac{\tilde{h}(n)}{n} \in [\tilde{e}_M(n),\, \tilde{e}_N(n)]$, then $(n - \tilde{h}(n))$ firms choose low quality in pure strategies and $e^0 = \frac{\tilde{h}(n)}{n}$;*
2. *if $\frac{\tilde{h}(n)}{n} \in \left(\tilde{e}_N(n) - \frac{1}{n},\, \tilde{e}_M(n)\right)$, then $(n - \tilde{h}(n) - 1)$ firms choose low quality in pure strategies,*

$$e^0 = \frac{c_H - c_L}{\gamma} + \frac{n[2n + \mu(n-1)][\tilde{h}(n) + 1 + \mu\tilde{h}(n)][\alpha(1) - c_H]^2}{[n + \mu(n-1)][2\tilde{h}(n) + 2 + \mu\tilde{h}(n)]^2(c_H - c_L)\gamma} \tag{1.16}$$

and the mixing firm chooses high quality with probability $w = ne^0 - \tilde{h}(n)$.

Proof. The proof is analogous to that of Lemma 1.7, with $\tilde{h}(n)$, $\tilde{e}_M(n)$ and $\tilde{e}_N(n)$ in place of $\bar{h}(n)$, $\bar{e}_M(n)$ and $\bar{e}_N(n)$, respectively; with, by Lemma 1.5, sequential rationality implying $e^0 \in [\tilde{e}_M(n), \tilde{e}_N(n)]$; and with (1.16) being a re-writing of $\tilde{g}(e^0) = \tilde{f}\left(\frac{\tilde{h}(n)+1}{n}, n\right)$. □

Chapter 2

Trade Policy and Industrial Structure

2.1 Introduction

How and when should an industrial structure be protected? Answers to this question go back into to the early stages of development of contemporary industrialized countries and have motivated a plethora of work in international trade and vigorous disagreements among economists and policy makers. Consensus in this area has been alternating over time between the polar cases of infant industry protection and trade liberalization.

At the core of the development strategy of several countries lies an imitation and replication process, whereby more sophisticated technologies, available in advanced countries, are copied or reverse engineered. See for example Lewis (1954); Hirschman (1968); Kim and Nelson (2000) and Kim (1997). This enables developing countries to produce goods that are highly substitutable to those produced by advanced economies and that, although produced at lower quality levels (at least temporarily), rely on cost advantage to be competitive. This high substitutability aspect of internationally competing goods produced at different quality levels tends to be overlooked by the recent literature on trade with heterogeneous firms. One reason is that vertical differences are modelled in terms of cost rather than quality[1]. While both choices yield a version of comparative advantages, the latter one allows to focus in a natural way on the different degree of substitutability between two varieties (horizontal heterogeneity) and two qualities of the same variety (vertical heterogeneity). We argue that imitation-driven industrialization makes such distinction relevant and explore its implications for trade policy and industrial development. This sheds light on the different patterns of industrialization followed by successful South-Eastern Asian and unsuccessful Latin American countries.

We first analyze a static, small open economy, general equilibrium model with

[1] This holds both in the models developed along the path set by Melitz (2003) and by Bernard et al. (2003), and in the literature on trade and innovation, such as Aghion et al. (2005). Instead of investigating innovation effort, we focus on the evolution of the industrial structure.

heterogenous firms. We rule out any gains from trade due to either love of variety or increasing returns. This way, gains from trade for the small economy derive from vertical differentiation alone, because it allows to substitute higher quality imported goods for lower quality, domestically produced ones.

Our first result is that whether protection is better than liberalization, or the other way around, crucially depends on the degree of vertical heterogeneity. Domestic vertical heterogeneity increases gains from trade and therefore liberalization becomes preferable when it is sufficiently high. In this case, local champions can emerge and compete internationally, and it is desirable to let international competition substitute high quality imported goods for low quality domestic ones. By contrast, protection tends to be preferable where the local industrial structure is more homogeneous, since in this case gains from trade are lower. A crucial role for this result is played by trade costs. Our second result is that the lower transportation costs, the lower is the degree of heterogeneity required for free trade to be better. Thus globalization makes free trade a better policy for a wider range of industrial structures. This may also help understand why the sign of the correlation between tariffs and growth changes over time (Clemens and Williamson, 2004).

We next extend the static framework to include dynamic learning externalities. If externalities arise from localized network interaction with other firms, liberalization adds dynamic costs to static trade costs if, when forcing several industries to close, it brings about a loss of relevant learning externalities. The relevance of learning should be out of doubt. Amsden (1989) already observed: "If industrialization first occurred in England on the basis of invention, and if it occurred in Germany and the United States on the basis of innovation, then it occurs now among 'backward' countries on the basis of learning" (Amsden, 1989, p. 4). The fact that learning externalities are highly localized has been recently emphasized by the literature on innovation and growth. The strong implications this has for trade are investigated by Baldwin and Robert-Nicoud (2006) in the context of Melitz's (2003) model. Yet, although localized learning externalities are plausible and relevant, they are not necessarily the main source of learning. Thus, beyond their intrinsic interest, our reason for focusing on them is that they deliver a dynamic version of the infant industry argument and we can assess its sensitivity to the specific characteristics of a country's initial industrial structure, defined by the distribution of its firms' quality. Our focus on both vertical and horizontal heterogeneity thus allows us to articulate in a novel way Baldwin and Robert-Nicoud's (2006) analysis.

As was true in the static model, our dynamic analysis confirms that, the higher trade costs, the narrower the range of initial industrial structures for which liberalization is preferable, and in particular the higher the initial degree of vertical heterogeneity necessary for this to hold. When there are two types of firms, technologically backward and technologically advanced, the relative gains to initial protection increase with the quality of backward firms (the cost of protection becomes lower), decrease with the quality of advanced firms (a lower level of heterogeneity reduces the benefits from free trade) and increase with the mass of backward firms (the loss of learning externalities would be higher). In essence, a rather homogeneous industrial structure, with all firms at a similar (and not too wide) quality gap from the frontier, is more worth protecting than a heterogeneous industrial structure, with a few very backward sectors and many relatively advanced ones.

We finally discuss how farsighted a policy maker should be to choose the right policy, and observe that this also changes with the initial industrial structure. Thus some countries are more exposed to policy mistakes than others, due to a stronger temptation to follow a policy that is better in the short run, but worse in the long run.

The remainder of this paper is organized as follows. Sections 2 and 3 introduce the model and discuss its static equilibria. Section 4 introduces the leaning dynamic and presents simulation results. Section 5 concludes.

2.2 Model

We consider a small open economy, populated by a measure 1 of identical individuals, each endowed with 1 unit of labor, which is supplied inelastically in a competitive labor market, and where a continuum $[0,1]$ of goods are produced. Good 1 is a consumption good, produced and sold by a perfectly competitive representative firm; goods $[0,1)$ are intermediate goods, produced and sold by monopolistic firms.

The representative consumer maximizes

$$U = \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho} \right)^t \ln c(t), \tag{2.1}$$

where $\rho \in (0,1)$ is the intertemporal discount rate and $c(t)$ is consumption at time t. Since we do not consider intertemporal transfer, the solution to this problem reduces to the solution of the static problem, so that demand for good 1 (the consumption good) at time t is $y^d(1,t) = \frac{E(t)}{p(1,t)}$, where $E(t)$ denotes aggregate income and $p(1,t)$ the price of good 1, both at time t. Aggregate income $E(t) = W(t)+\Pi(t)+T(t)$ is equal to the sum of aggregate wage income $W(t)$, which under full employment simply equals the wage rate $w(t)$, aggregate profits $\Pi(t) = \int_0^1 \pi(m,t)\,\mathrm{d}m$, where $\pi(m,t)$ denotes firm m's profits at time t, and aggregate tariff revenue on imports $T(t)$, which will be specified below.

Each intermediate good $m \in [0,1)$ is produced with labor according to the decreasing returns to scale technology

$$y(m,t) = L(m,t)^\alpha, \tag{2.2}$$

with $\alpha \in (0,1)$. Even though we make this assumption for simplicity, it goes in line with the the empirical evidence suggesting that manufacturing firms in developing countries do not enjoy scale economies (Tybout, 2000)[2].

Intermediate goods, which are both horizontally and vertically differentiated, are the only input in the production of the unique consumption good, which is produced according to the constant returns to scale technology

$$y(1,t) = \left[\int_0^1 h(m,t)^{\frac{\sigma-1}{\sigma}} \mathrm{d}m \right]^{\frac{\sigma}{\sigma-1}}, \tag{2.3}$$

[2] Simplicity comes from the fact that decreasing returns allow us to easily introduce vertical differentiation in a small open economy model. Decreasing returns reinforce trade costs in our model, because specialization implies production on a higher, less efficient scale. Yet, since we explicitly introduce trade costs in the form of transportation costs, decreasing returns are not crucial for our results and most of them would hold even if we assumed a technology with initially increasing and eventually decreasing returns to scale. With globally non-decreasing returns they would probably still hold, but we would have to explicitly model the number of trading countries.

where $\sigma > 1$ captures the elasticity of substitution between any two different varieties of intermediate goods and $h(m,t)$ is the 'effective input' of good m at time t.[3].

The 'effective input', which may be either bought locally or imported from the rest of the world, is given by its quantity multiplied by its quality:

$$h(m,t) = \begin{cases} x(m,t)v(m,t) & , \quad \text{if it is bought locally} \\ x(m^*,t)v(m^*,t) & , \quad \text{if it is imported} \end{cases} \tag{2.4}$$

where $x(m,t)$ denotes local quantity and $v(m,t)$ the quality of the domestically produced good m, and m^* is a perfect substitute to m, produced in the rest of the world at the quality frontier $v(m^*,t)$.

Local intermediate goods directly compete with their foreign perfect substitutes. Taking into account the presence of an import tariff $\tau(t) \geq 0$ (applying to landed import and the same for each variety at a given time) and of transport (or adoption) costs of the iceberg type $a \geq 0$, which render the buyer price of an imported intermediate good equal to $p(m^*,t)[1+\tau(t)](1+a)$, the final good producer decides whether to buy locally or to import according to the best quality/price ratio. The set of locally acquired inputs, and indeed of domestic intermediate good producers who are active at all, is $D(t) = \left\{m \in [0,1) : \frac{v(m,t)}{p(m,t)} \geq \frac{v(m^*,t)}{p(m^*,t)[1+\tau(t)](1+a)}\right\}$, where $p(m,t)$ denotes the price of good m at time t set by its local producer. Therefore, defining the threshold function

$$p_H(m,t) \equiv \frac{v(m,t)}{v(m^*,t)} p(m^*,t)[1+\tau(t)](1+a), \tag{2.5}$$

we have $D(t) = \{m \in [0,1) : \ p(m,t) \leq p_H(m,t)\}$. Goods $m \in [0,1) \setminus D(t)$ are not produced domestically and their foreign perfect substitutes are imported.

Calling $M(m^*,t)$ the quantity of good m^* imported at time t implies that the aggregate revenue from an import tariff is

$$T(t) = \int_{[0,1)\setminus D(t)} \tau(t) p(m^*,t)(1+a) M(m^*,t) \mathrm{d}m^*.$$

A similar production structure for the world economy implies that the final good producer in the rest of the world will be willing to import intermediate good m from our small economy only if the quality/price ratio is competitive. Letting $\tau^*(t)$ be the foreign import tariff at time t, the set of exportable intermediate goods for our small economy is $F(t) = \left\{m \in [0,1) : \frac{v(m,t)}{p(m,t)[1+\tau^*(t)](1+a)} \geq \frac{v(m^*,t)}{p(m^*,t)}\right\}$ which defines the threshold function

$$p_L(m,t) \equiv \frac{v(m,t)}{v(m^*,t)} \frac{p(m^*,t)}{[1+\tau^*(t)](1+a)}, \tag{2.6}$$

and therefore we have $F(t) = \{m \in [0,1) : \ p(m,t) \leq p_L(m,t)\}$.

We assume the rest of the world immediately responds reciprocally to the tariff choice of the domestic economy, by imposing the same import tariff ($\tau^*(t) = \tau(t)$)[4].

[3]Together with perfect competition, it is equivalent to assuming that each consumer assembles and consumes a bundle of traded intermediates.

[4]We regard this assumption as the most meaningful to study the dynamic effects of trade policy in the context of a small open economy model: keeping the tariff set by the rest of the world fixed would be dynamically implausible, but for a deeper analysis of the tariff choice problem of the rest of the world a different, more complicated, two country (or n country) model would be better suited than our small open economy model. Yet our interest is not

Equations (2.5) and (2.6) then show that a higher level of tariff protection allows a greater number of domestic intermediate good producers to survive, but at the same time reduces the number of them who may profitably export.

We now drop for notational simplicity the time index. The domestic producer of intermediate good $m \in [0,1)$ receives a local demand $x(m)$ and a foreign demand $x^*(m)$, so the total demand she receives is $y^d(m) = x(m) + x^*(m)$[5]. Letting P be the hedonic price aggregator, local demand is

$$x(m) = \begin{cases} p(m)^{-\sigma} \left[v(m)P\right]^{\sigma-1} p(1)y(1) & , \quad \text{if } p(m) \leq p_H(m) \\ 0 & , \quad \text{if } p(m) > p_H(m) \end{cases} \tag{2.7}$$

At price $p(m) = p_L(m)$, local production of intermediate goods is assumed to be first absorbed by local demand and then exported for the reminder. Since we are dealing with a small open economy, foreign demand is infinitely elastic at $p_L(m)$:

$$x^*(m) = \begin{cases} \in \left[x\left(m\,|p_L(m)\right), \infty\right) & , \quad \text{if } p(m) = p_L(m) \\ 0 & , \quad \text{if } p(m) > p_L(m) \end{cases} \tag{2.8}$$

When $p(m) > p_H(m)$, good m is not bought locally and its perfect substitute m^* is imported. Local demand for imports are

$$M(m^*) = \begin{cases} \left[p(m^*)(1+a)(1+\tau)\right]^{-\sigma} \left[v(m^*)P\right]^{\sigma-1} p(1)y(1) & , \quad \text{if } p(m) > p_H(m) \\ 0 & , \quad \text{if } p(m) \leq p_H(m) \end{cases} \tag{2.9}$$

While equation (2.8) follows from our assumptions, equations (2.7) and (2.9) are obtained from cost minimization given the technology described in (2.3) and (2.4).

The term P that appears in (2.7) and in (2.9) is a price index corresponding to the marginal cost of production of good 1. It is determined taking into account the fact that prices must be weighted by quality and that there is the possibility to import intermediate goods:

$$P = \left\{ \int_0^1 \left[\frac{p^F(m)}{v^F(m)}\right]^{1-\sigma} \mathrm{d}m \right\}^{\frac{1}{1-\sigma}},$$

where

$$p^F(m) = \begin{cases} p(m) & , \quad \text{if } p(m) \leq p_H(m) \\ p(m^*)(1+\tau)(1+a) & , \quad \text{if } p(m) > p_H(m) \end{cases}$$

and

$$v^F(m) = \begin{cases} v(m) & , \quad \text{if } p(m) \leq p_H(m) \\ v(m^*) & , \quad \text{if } p(m) > p_H(m) \end{cases}.$$

Merging the previous equations yields total demand for good $m \in [0,1)$:

on strategic trade policy, but rather on the interaction of different industrial structures and dynamic learning, and on its implications for policy. We feel that our assumption reaches a good compromise between plausibility and simplicity.

[5]When necessary, we will write $x\left(m\,|p_L(m)\right)$ to denote local demand of good m at price $p_L(m)$ (and analogously for other prices), but we drop the price for notational simplicity whenever this does not create confusion.

$$y^d(m) = \begin{cases} \in [x(m\,|p_L(m)\,), \infty) & , \quad \text{if } p(m) = p_L(m) \\ x(m) & , \quad \text{if } p(m) \in (p_L(m), p_H(m)] \\ 0 & , \quad \text{if } p(m) > p_H(m) \end{cases} \tag{2.10}$$

Finally, recalling that D is the set of active domestic intermediate good producers, the overall demand for labor is

$$L^d = \int_D [y(m)^{\frac{1}{\alpha}}]\mathrm{d}m. \tag{2.11}$$

We now solve the firms' profit maximization problem. The final good producer operates in a perfectly competitive market and thus sells its product at its marginal cost:

$$p(1) = P.$$

The case of intermediate good producers is somewhat more complicated. Recall that they are monopolists facing a discontinuous demand function. They first decide whether to produce or not and then, if they produce, they establish their optimal quantity of production under the constraints imposed by technology (equation 2.2) and demand (equation 2.10). Defining the two thresholds $y_H(m) \equiv x(m\,|p_L(m)\,)$ and $y_L(m) \equiv x(m\,|p_H(m)\,)$, using inverse demand and letting $R(y(m))$ be the revenues and $C(y(m))$ the cost, one gets a profit function $\pi(m) = R(y(m)) - C(y(m))$, which is twice differentiable almost everywhere, it is continuous but not differentiable at $y_H(m)$, it is discontinuous at $y_L(m)$, and it is twice differentiable and concave within each of the ranges determined by these two thresholds, but it is not globally concave[6]. Therefore, the usual condition of equality between marginal cost (MC) and marginal revenue (MR) is neither sufficient nor necessary to ensure optimality. Rather, the following result holds.

For either sufficiently low or sufficiently high w, there exists a unique (local and global) profit maximizing quantity; for intermediate wage levels there may exist two local optima, one involving production just for the domestic market and one also involving exports. In such case a firm's choice is determined by direct comparison of the profitability of these two strategies.

This is formally stated in Lemma 2.1.

Lemma 2.1. ***(intermediate firms' optimal choice)*** *$\forall m \in [0, 1)$, there exist positive thresholds $w_0(m)$, $\widehat{w_1}(m)$, $\widetilde{w_1}(m)$ and $\overline{\tau}$, such that*

1. *If $w \leq \frac{\sigma-1}{\sigma}\widehat{w_1}(m)$, then m produces $y_E(m) = [\frac{\alpha}{w}p_L(m)]^{\frac{\alpha}{1-\alpha}}$.*
2. *If $w \in (\frac{\sigma-1}{\sigma}\widehat{w_1}(m), max\{\widehat{w_1}(m), \widetilde{w_1}(m)\})$, then m's choice depends on a combination of wage and protection level.*
   - *For $\tau \leq \overline{\tau}$, we have two cases:*
     - *if $w < \widetilde{w_1}(m)$, then m compares $\pi(y_E(m))$ and $\pi(y_M(m))$;*
     - *if $w \geq \widetilde{w_1}(m)$, then m compares $\pi(y_E(m))$ and $\pi(y_L(m))$.*
   - *For $\tau > \overline{\tau}$, we have again two cases:*
     - *if $w < \widehat{w_1}(m)$, then m compares $\pi(y_E(m))$ and $\pi(y_M(m))$;*

[6] For the details see Appendix A and Albornoz and Vanin (2005).

– *if* $w \geq \widehat{w_1}(m)$*, then* m *produces* $y_M(m)$.

3. *If* $w \in [max\{\widehat{w_1}(m), \widetilde{w_1}(m)\}, w_0(m)]$*, then* m *produces* $y_L(m)$.
4. *If* $w > w_0(m)$*, then* m *stays inactive.*

Proof. See Appendix A. □

Given this result, in order to characterize the equilibria of the model, it may become necessary to first identify a candidate equilibrium and then check whether the optimality conditions established by Lemma 2.1 are satisfied.

2.3 Static Equilibria

We define an equilibrium as a collection of prices and quantities such that consumers maximize utility, producers maximize profits and all markets clear. We call an equilibrium symmetric when firms with the same quality level make the same choices. We restrict our attention to symmetric equilibria. We first discuss the symmetric equilibrium of our economy under autarky. We next let our small economy be open.

To keep the general equilibrium analysis as simple as possible, we make the following assumptions on initial conditions.

Assumption 2.1. *A fraction* u *of local intermediate good producers begins with a 'bad' quality, i.e., with a quality gap w.r.t. the international quality frontier. The remaining fraction* $(1-u)$ *starts with no quality gap*[7]*. Formally,* $\exists\, u, \beta \in [0,1] : \forall m \in [0,u),\ v(m,0) = \beta v(m^*,0)$ *and* $\forall m \in [u,1),\ v(m,0) = v(m^*,0)$.

We can thus study two dimensions of the initial industrial structure: horizontal one, given by the proportion u of 'bad' firms, and the vertical one, given by their quality gap $1-\beta$ w.r.t. the international quality frontier.

Assumption 2.2. *We normalize at the beginning the international quality frontier for each sector:* $\forall m^* \in [0,1),\ v(m^*,0) = v^*(0)$.

Since over time both 'good' and 'bad' firms may learn, and the international quality frontier moves, we define the ratio of local to international quality at time t, $\beta_L(t) \equiv \frac{v(L,t)}{v^*(t)}$ and $\beta_H(t) \equiv \frac{v(H,t)}{v^*(t)}$, for 'bad' and 'good' firms, respectively[8]. We also denote by $p_L(L,t)$ and $p_L(H,t)$ the lower price threshold and by $p_H(L,t)$ and $p_H(H,t)$ the higher price threshold, for the two types of firms at a given point in time.[9]

2.3.1 Equilibrium under autarky

Proposition 2.1. ***(Autarkic symmetric equilibrium)*** *There exists a unique symmetric equilibrium under autarky.*

Proof. See Appendix A. □

[7]In the remainder of the paper we indifferently refer to these two groups of firms as 'backward' and 'advanced', 'low quality' and 'high quality', or 'bad' and 'good', respectively.

[8]Thus Assumption 1 means $\beta_L(0) = \beta$ and $\beta_H(0) = 1$. In some cases we relax assumption 1 to allow for $\beta_H(0) \leq 1$.

[9]Observe that $p_H(L,t) < p_L(H,t) \Leftrightarrow \frac{\beta_L(t)}{\beta_H(t)} < \frac{1}{\{[1+\tau(t)](1+a)\}^2}$, where $\frac{\beta_L(t)}{\beta_H(t)}$ denotes the ratio of 'bad' firms' quality to 'good' firms' quality.

From the proof of Proposition 2.1 production and consumption patterns in the autarkic equilibrium are:

$$y_A(1) = \left[uv(L)^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}} + (1-u)v(H)^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}\right]^{\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}} \tag{2.12}$$

$$y_A(L) = \left\{u + (1-u)\left[\frac{v(L)}{v(H)}\right]^{-\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}\right\}^{-\alpha} \tag{2.13}$$

$$y_A(H) = \left\{u\left[\frac{v(L)}{v(H)}\right]^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}} + (1-u)\right\}^{-\alpha} \tag{2.14}$$

The autarkic consumption level $y_A(1)$ is a decreasing function of u and an increasing function of both $v(L)$ and $v(H)$ (and therefore, given $v(H)$, of the domestic 'bad' to 'good' quality ratio). It is also an increasing function of σ, since a higher elasticity of substitution allows a more intensive use of 'good' inputs and a less intensive use of 'bad' ones. As it was to be expected, $y_A(1)$ does not depend on either τ or a.

The autarkic production patterns of intermediate good producers have the following properties: $y_A(L) < 1 < y_A(H)$; both $y_A(L)$ and $y_A(H)$ are increasing functions of u; $y_A(L)$ is increasing in $\frac{v(L)}{v(H)}$; $y_A(H)$ is decreasing in $\frac{v(L)}{v(H)}$; $y_A(L)$ is decreasing in σ; $y_A(H)$ is increasing in σ. Thus the difference in production between 'good' and 'bad' domestic firms increases with the quality gap between them, and a higher elasticity of substitution yields a more intensive use of high quality inputs (confirming analytically the intuition given above). Again, τ and a do not affect autarkic production patterns.

2.3.2 Equilibrium in the rest of the world

When we open our small economy, we consider the equilibrium in the rest of the world as determined under autarky. Taking the final good produced abroad at time $t = 0$ as numeraire, i.e., setting $p(1^*, 0) = 1$, Assumption 2 and the definition of the price index P^* imply that, letting $p^*(t)$ be the common price of all intermediate goods produced abroad at time t, the initial foreign marginal cost of producing the final good is $P^*(0) = \frac{p^*(0)}{v^*(0)} = p(1^*, 0) = 1$, so that $p^*(0) = v^*(0)$. Our derivation of the autarkic equilibrium then implies that for any $t \geq 0$, foreign consumption is $y(1^*, t) = v^*(t)$, the common quantity of all intermediate goods produced abroad is $y^*(t) = 1$, prices are $p(1^*, t) = P^*(t) = 1$, $p^*(t) = v^*(t)$, and the wage rate is $w^* = \frac{\alpha(\sigma-1)}{\sigma}v^*(t)$.

2.3.3 Equilibrium for the small open economy

In the open economy, the sharp international competition implied by the perfect substitutability of intermediate goods at different quality levels, combined with the presence of heterogeneous local producers, significantly complicates the (symmetric) general equilibrium analysis of the model. Since we consider two types of domestic intermediate good producers, each of which has three basic alternatives (stay closed, serve just the local market or also export), and since it is easy to show that 'bad' firms cannot profitably export when 'good' ones do not, and cannot profitably stay open unless also 'good' ones can, there exist six types of structurally different potential symmetric equilibria, summarized in the following table.

	Type of symmetric eq.	'Good firms'	'Bad firms'
EE	**Export and export**	sell locally and export	sell locally and export
ES	**Export and survive**	sell locally and export	just sell locally
ED	**Export and die**	sell locally and export	stay closed
SS	**Survive and survive**	just sell locally	just sell locally
SD	**Survive and die**	just sell locally	stay closed
DD	**Die and die**	stay closed	stay closed

In Appendix B we provide a detailed analytical discussion of the issue of existence and uniqueness of each type of equilibrium. In that discussion, and in the remainder of the paper, we take initial foreign consumption as numeraire. The following proposition summarizes our results:

Proposition 2.2. *The following results hold.*

- *If an ED equilibrium exists, then it is unique and its consumption and production patterns are*

$$
\begin{aligned}
y_{ED}(1) &= \frac{(1-u)^{1-\alpha} p_L(H)}{P_{ED} - u\tau(1+a)[(1+a)(1+\tau)]^{-\sigma} P_{ED}^{\sigma}}, \quad \text{where} \\
P_{ED} &= \left\{ u[(1+a)(1+\tau)]^{1-\sigma} + (1-u)[(1+a)(1+\tau)]^{\sigma-1} \right\}^{\frac{1}{1-\sigma}} \\
y_{ED}(L) &= 0 \\
y_{ED}(H) &= (1-u)^{-\alpha}
\end{aligned}
$$

- *If tariff protection is sufficiently high, then there exists an SS equilibrium with the same production and consumption patterns as in autarky, namely those described by equations (2.12), (2.13) and (2.14). We call it henceforth 'autarky-like SS equilibrium'.*
- *For some parameter values, there exists a different SS equilibrium, which we call 'limit price SS equilibrium', whose consumption and production patterns are*

$$
\begin{aligned}
y_{SS}(1) &= \left[uv(L)^{-\frac{1}{\alpha}} + (1-u)v(H)^{-\frac{1}{\alpha}} \right]^{-\alpha} \\
y_{SS}(L) &= \left\{ u + (1-u) \left[\frac{v(L)}{v(H)} \right]^{\frac{1}{\alpha}} \right\}^{-\alpha} \\
y_{SS}(H) &= \left\{ u \left[\frac{v(L)}{v(H)} \right]^{-\frac{1}{\alpha}} + (1-u) \right\}^{-\alpha}
\end{aligned}
$$

- *No other type of symmetric equilibrium exists.*

Proof. See Lemmata 2.2 to 2.7 in Appendix B. □

The intuition behind these results is as follows. No EE and ES equilibria exist, because their high demand for labor would push up wages too much to allow even 'good' firms to profitably export. Recall that, because in our model we rule out any gains to trade due to specialization or product differentiation, the only competitive advantage of domestic firms is labor costs.

For some parameter values, an ED equilibrium exists and it is unique. In other words, in a symmetric equilibrium of this economy, exporting is only compatible with the existence of some inactive local firm. Exit of backward firms reduces labor demand and therefore allows advanced firms to enjoy a cost advantage and therefore to export. Parameter restrictions come from the fact that, for 'good' firms to profitably export when 'bad' ones find it optimal to stay closed, the quality gap between them must be sufficiently high.

Since in an SS equilibrium there is no international trade, taking time 0 foreign consumption as a numeraire opens the possibility that, for some values of the parameters (in particular, of the tariff), there is an entire range of one price compatible with equilibrium. Under autarky, taking a numeraire was sufficient to uniquely determine all prices. Yet for an open economy, when the numeraire is taken in the foreign economy, and there is no international trade, one price in the domestic SS equilibrium remains analytically undetermined. Every value of that price then defines a potential SS equilibrium, and one has to check whether nobody has an incentive to deviate. We perform this check and find that there may exist a continuum of SS equilibria, corresponding to values of the undetermined price within a given interval. We show that this is true both for the SS equilibrium with autarkic production quantities and for that with higher quantities and limit pricing[10]. We further show that in both cases any equilibrium in the corresponding range displays the same production quantities and consumption levels, independently of the particular price chosen in the equilibrium interval[11].

As far as SD equilibria are concerned we give conditions for them to exist and find numerically that they are never satisfied. Notice in any case that such equilibria are not very interesting from an economic point of view. Finally, we prove that no DD equilibrium exists, because there would be excess supply of labor.

2.3.4 Autarky versus Free Trade

Let us now compare, when an ED equilibrium exists, its consumption level with the autarkic one: i.e, $y_A(1)$ with $y_{ED}(1)$ as to identify for which industrial structures an ED equilibrium under free trade exists and yields a higher consumption than autarky. The best way to think of this comparison (and indeed the way that gives an ED equilibrium its best chances) is as one between the the two polar cases of high protection, which

---

[10]In the working paper version of this paper we show that if, for a given tariff value, both an 'autarky-like SS equilibrium' and a 'limit price SS equilibrium' exist, then the former Pareto-dominates the latter. This is intuitive because, algebraically, a 'limit price SS equilibrium' corresponds to the autarkic equilibrium that would hold if there were no possibility of substitution between different intermediate inputs. We also show that the 'limit price SS equilibrium' may exist for lower tariff values, for which the 'autarky-like SS equilibrium' does not exist.

[11]Therefore, given that our focus is on production and consumption patterns, in our numerical simulations we resolve this multiplicity issue by picking up one specific value for the undetermined price. For mathematical convenience, we take the undetermined price to be $p(L)$ in the former case and w in the latter case and, from the respective intervals where SS equilibria exist, we pick up the mean value of $p(L)$ and the highest value of w. While this is clearly arbitrary, it is useful to stress once again that it has no consequences on the determination of production and consumption patterns, which is what we are interested in.

isolates the economy from the rest of the world, and of free trade, in the sense of zero tariff.

Defining $K \equiv \left\{ \frac{1-u}{u} \left[\left(\frac{u(1+a)^{2(1-\sigma)}+(1-u)}{1-u} \right)^{\frac{1}{\alpha+\sigma(1-\alpha)}} - 1 \right] \right\}^{\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}}$, equations (2.12) and (2.15) yield

$$y_{ED}(1) > y_A(1) \iff \frac{v(L)}{v(H)} < K. \qquad (2.15)$$

Notice that $0 < K < 1$ and that K is decreasing in a[12]. Therefore, we have the following proposition.

Proposition 2.3. *If under free trade an ED equilibrium exists, then it is Pareto-superior to the autarkic equilibrium if and only if the industrial structure displays sufficient vertical heterogeneity.*

Proof. The result immediately follows from equation (2.15). □

Remark 2.1. *As long as an ED equilibrium under free trade exists, a reduction in transportation costs, which is a simple way of thinking of globalization, makes free trade preferred to autarky for a wider range of industrial structures.*

Remark 2.2. *As we have noticed above, under free trade an ED equilibrium exists only if the industrial structure is sufficiently heterogeneous. Therefore, heterogeneity of the domestic industrial structure plays the double role of generating gains from trade and of allowing them to be reaped in equilibrium.*

2.4 Simulation Exercises on Dynamics

We turn now to the dynamic analysis of the model. It is a well known fact that the dynamic benefits of any trade policy depend on firms' learning curves (see Melitz (2005) for a modern study of this question[13]). In a general formulation, the quality and quantity of own production, as well as aggregate production in the local industrial network, imports and exports, are all potential factors that help technological progress through learning externalities (leaving aside, to remain in the present framework, human capital accumulation, R&D investment and so on). Which of these factors are the main engines of learning will determine the dynamic optimality of trade policy. For example, in the presence of learning by doing or of relevant cross-country learning externalities, either through imports or through exports, international trade and specialization would obviously favor dynamic learning. In turn, localized learning externalities set a dynamically favorable case for protection.

The empirical literature has not yet offered a clear verdict about the main sources of learning in different sectors and countries, so that any specific assumption on the

[12] To see that $K < 1$ calculate it for $a = 0$ and then observe that in that case a sufficient condition for $K < 1$ is $1 - (1-u)^{\frac{1}{\alpha+\sigma(1-\alpha)}} < u$, which is always satisfied for $u < 1$, due to strict convexity of the left hand side, to continuity and to equality of the two sides for $u = 0$ and $u = 1$.

[13] Melitz focuses on competitive firms producing imperfect substitutes at home and abroad, and on the role of quotas. We rather emphasize market power, perfect substitutability across quality and the role of tariffs.

learning curve is to some degree arbitrary[14]. Since we want to place our analysis in the debate about the validity of the infant industry argument, we incorporate the idea that initial protection may be desirable by assuming that in its absence potential learning externalities may be lost. We thus focus on knowledge spillovers due to interaction within the local industrial network. Recently, Baldwin and Robert-Nicoud (2006) have introduced the same idea (namely, that learning externalities increase in the number of locally active firms) in Melitz's (2003) model and have shown that under this extension freer trade may be detrimental to growth, even if it initially raises productivity. Coherently with their finding, our assumptions also yield a growth detrimental effect of trade in our framework. Yet, while their interest is then devoted to explore the effects of different learning externalities, we rather investigate how the dynamic costs and benefits to protection depend on the initial industrial structure, are affected by globalization and expose different countries to policy mistakes.

Specifically, we assume the following learning dynamic

$$v(m,t+1) = v(m,t) + v(m,t)^{\varphi}\left[\int_{D(t)} y(i,t)v(i,t)\,\mathrm{d}i\right]^{1-\varphi-\epsilon} \quad (2.16)$$

where $\varphi \in (0,1)$ and $\epsilon \in (0, 1-\varphi)$ are parameters. The rest of the world learns according to the same dynamic, with its respective variables. Concavity of the learning function (granted by our parameter restrictions) implies, all else equal, a tendency to converge to the technological frontier. If free trade forces initially inefficient sectors out of the market, it may destroy a potentially important base for future development. Observe that the networking effect (captured by the term in brackets) depends on the production of domestic firms in efficiency units: while it is possible to learn something from any firm, one learns more from technologically more advanced partners.

We study the following two policies

1. Free Trade, under which $\tau(t) = 0$ for all $t \geq 0$;
2. Temporary Protection, which requires selecting, at each point in time, the minimum tariff that is necessary to keep all domestic firms active[15].

These policies allow us to compare an outward-oriented development strategy, more associable to contemporaneous consensus, with an import substitution strategy (especially aimed at protecting infant industries), which was a common recommendation between World War II and mid Seventies.

We compare these two policies when Free Trade gives rise to an ED equilibrium at any point in time and when the Paretian ranking of their outcome is reversed when we pass from the static analysis of the initial industrial structure to the dynamic analysis over an infinite time horizon[16]. This tends to happen when static trade costs are

[14] Trade theory has considered both the implications of cross-country but industry-specific learning externalities, as in Krugman (1987), and of externalities which are both industry and country-specific, as in Brezis et al. (1993). Knowledge spillovers have been found to be highly localized in the empirical literature on innovation and growth, for instance by Keller (2002) and by Bottazzi and Peri (2003). In the same literature, Jones (1995) and Segerstrom (1998) have stressed that learning functions are concave in own knowledge, which in our context means that good products are harder to improve than bad ones.

[15] Protection here is termed temporary because, due to concavity of the learning function, and therefore to convergence, the minimum tariff necessary to keep all domestic firms active converges to zero in finite time.

[16] If, under Free Trade, at some point in time no ED equilibrium exists, or even no symmetric

very low. Therefore, we initially carry out our simulations assuming no transportation costs ($a = 0$). From our assumptions it is immediate to derive the following result.

For any initial industrial structure, for which Free Trade is initially Pareto-superior to Temporary Protection from a static point of view, it holds that

- there exists a discount rate $\bar{\rho} > 0$, such that, for any $\rho < \bar{\rho}$, the present discounted value of the stream of consumption obtained in a sequence of SS equilibria under Temporary Protection is higher than that obtained in a sequence of ED equilibria under Free Trade[17];
- consequently, for any value of $\rho < \bar{\rho}$, there exists a time $\bar{t}$, such that the partial sum of the difference in discounted utility between the two policies is negative until $\bar{t}$ and positive afterwards.

In light of this result, one way of comparing Free Trade and Temporary Protection across different initial industrial structures is to ask how $\bar{t}$ changes with initial conditions. This way is interesting because in several cases it is reasonable to assume that policy makers are myopic, in the sense that, although aware of the representative consumer's time discount rate, they only plan over a finite horizon[18]. In the following table we compare the values of $\bar{t}$ for four different initial industrial structure and two degrees of patience[19].

$\bar{t}$	$u = 0.2$ $\beta_L(0) = 0.3$	$u = 0.2$ $\beta_L(0) = 0.7$	$u = 0.7$ $\beta_L(0) = 0.3$	$u = 0.7$ $\beta_L(0) = 0.7$
$\rho = 0.05$	30	12	22	7
$\rho = 0.1$	∞	14	39	7

The main message conveyed by this table is that, given consumers' patience, the planning horizon necessary to appreciate the dynamic advantages to protection (where they exist) is highly sensitive to the initial industrial structure[20]. In particular, quite intuitively, $\bar{t}$ is increasing in vertical backwardness $(1 - \beta_L(0))$, because the costs of protection have to be borne for more time before convergence makes its dynamic advantages prevail. More surprisingly, it is decreasing in horizontal backwardness

equilibrium exists at all, then the comparison is either trivial or impossible. If, in turn, one policy is better than the other both statically (given the initial industrial structure) and dynamically, then the analysis is again trivial. Finally, if at some time t for $\tau(t) = 0$ both an ED and an SS equilibrium exist, then we focus on the former under Free Trade and on the latter under Temporary Protection. Observe that our welfare measure is always given by equation (2.1).

[17] In all of our numerical simulations we find that, if an ED equilibrium under Free Trade is statically superior to an SS equilibrium under Temporary Protection for the initial industrial structure, then under Free Trade at each point in time along the entire dynamic there exists an ED equilibrium. Thus, existence of ED equilibria in this case is not an issue. Recall that SS equilibria always exist for a sufficiently high tariff.

[18] A similar comparison might be done in terms of $\bar{\rho}$ rather than of $\bar{t}$, without considering any myopic policy maker. Qualitative results would obviously be the same.

[19] Parameters are set at the following values: $\beta_H(0) = 1$, $p^*(0) = v^*(0) = 1$, $a = 0$, $\sigma = 4$, $\varphi = 0.3$ and $\epsilon = 0.1$. The term ∞ appears because under the quadruple $(u, \beta_L(0), \beta_H(0), \rho) = (0.2, 0.3, 1, 0.1)$ both the gains from trade and the discount rate are too high to make Temporary Protection dynamically preferable to Free Trade.

[20] Obviously, the gains from protection increase with the level of patience (the lower ρ, the lower $\bar{t}$).

(u), because, although a wider mass of backward firms raises the cost of Temporary Protection, it raises even more the cost of Free Trade, when such policy, by driving a greater number of firms out of business, substantially shrinks the industrial network and therefore surviving firms' development potential. In terms of the old debate on infant industries, the payoff to protection is higher when there are many backward firms, but it becomes smaller when these firms are very backward.

When also domestic advanced firms start with an initial quality gap from the international frontier ($\beta_H(0) < 1$), we find that, contrary to what one could expect, it now takes a shorter time to appreciate the dynamic superiority of Temporary Protection (obviously, when it exists). While at first sight surprising, this result is explained by the fact that a lower $\beta_H(0)$ implies a higher homogeneity of the initial industrial structure, which, as discussed above, reduces the relative gains to Free Trade.

When transportation or adoption costs increase, this obviously reduces the gains from trade and thus favors protection and reduces $\bar{t}$, but it does not alter the way $\bar{t}$ depends on the initial industrial structure[21]. If we interpret again globalization as a reduction of a, these results may help explain changes in the consensus on the benefits of protecting backward firms: with lower transport costs, the horizon over which Temporary Protection appears superior becomes longer, so that the ability of such policy to command political consensus decreases.

2.5 Conclusions

In this paper we investigate the static and dynamic effects of trade policy on industrial structure in a context characterized by perfect substitutability between domestic and foreign varieties. This characteristic is common in developing economies and is consistent with many facts associated to trade liberalization: replacement of low quality inputs by better quality imports (Amiti and Konings, 2005), higher exit than entry resulting in a reduction of the mass of active firms (Alvarez and Vergara, 2006; Eslava et al., 2005), entry of surviving firms into the export market (Bernard et al., 2003, among others) and exporters' supply of higher quality products (Kraay et al., 2002). We contend that vertical differentiation is a relevant (and overlooked) dimension to be considered when assessing the effects of any trade policy.

Our contribution to the debate between supporters of an outward-oriented development strategy, more associable to contemporaneous consensus, and of an import substitution strategy (especially aimed at protecting infant industries), which was a common recommendation between World War II and mid Seventies, consists in arguing that the choice should be context-dependent.

We find that free trade is preferred to autarky when an industrial structure is sufficiently heterogeneous. The level of heterogeneity required for free trade to Pareto-dominate temporary protection increases with transport costs. We also find that transport costs reduce the optimality of free trade in a dynamic setting. These results may help explain changes in the consensus on the benefits of protecting backward firms: with lower transport costs, the horizon over which temporary protection appears superior becomes longer, so that the ability of such policy to command political consensus decreases.

[21]To have a numerical feeling, with $u = 0.8$, $\beta_L(0) = 0.3$, $\beta_H(0) = 1$ and $\rho = 0.05$, passing from $a = 0$ to $a = 0.1$ makes $\bar{t}$ pass from 19 to 8. With $\beta_L(0) = 0.6$ these two values become 10 and 3, respectively.

A main result emerging from our analysis is that the benefits of protection depend upon the level of backwardness in the following way: for a given mass of backward firms, the relative gains from protection increase with the quality of backward firms (the cost of protection is lower) and decrease with the quality of advanced firms (a lower level of heterogeneity reduces the benefits from free trade). On the other hand, for given production quality levels, the relative advantage of protection increases with the mass of backward firms. According to these results, for instance, the gains to protection are much higher for a quite homogeneous, not too backward industrial structure than for a heterogeneous one, with a few very backward firms and many relatively advanced firms.

Our findings do not constitute an overall assessment of the relative desirability of temporary protection vs. free trade. Rather, they specify how the dynamic costs and benefits of these two policies depend on several characteristics of the country to which they are applied, of its development process, and of the world trading environment. We thus see this work as a starting point for a new wave of careful and critical research on an old theme, rather than as a point of arrival.

Appendix A: Proofs of Lemma 2.1 and Proposition 2.1

Proof of Lemma 2.1

When deciding, each firm m considers other firms' choice and all equilibrium variables as given. Revenues and costs are

$$R(y(m)) = \begin{cases} 0 & , \quad \text{if } y(m) < y_L(m) \\ y(m)^{\frac{\sigma-1}{\sigma}} [v(m)P]^{\frac{\sigma-1}{\sigma}} [p(1)y(1)]^{\frac{1}{\sigma}} & , \quad \text{if } y(m) \in [y_L(m), y_H(m)) \\ p_L(m)y(m) & , \quad \text{if } y(m) \geq y_H(m) \end{cases} \quad \text{and}$$

$C(y(m)) = wy(m)^{\frac{1}{\alpha}}$, respectively.

Therefore, marginal revenues and marginal costs are

$$MR(y(m)) = \begin{cases} 0 & , \quad \text{if } y(m) < y_L(m) \\ \frac{\sigma-1}{\sigma} y(m)^{-\frac{1}{\sigma}} [v(m)P]^{\frac{\sigma-1}{\sigma}} [p(1)y(1)]^{\frac{1}{\sigma}} & , \quad \text{if } y(m) \in [y_L(m), y_H(m)) \\ p_L(m) & , \quad \text{if } y(m) \geq y_H(m) \end{cases} , \tag{2.17}$$

$$MC(y(m)) = \frac{w}{\alpha} y(m)^{\frac{1-\alpha}{\alpha}}. \tag{2.18}$$

Observe that MC is concave if $\alpha \in \left(\frac{1}{2}, 1\right)$, convex otherwise. Observe further that $\lim\limits_{y(m) \searrow y_L(m)} MR(y(m)) = \frac{\sigma-1}{\sigma} p_H(m)$ and $\lim\limits_{y(m) \nearrow y_H(m)} MR(y(m)) = \frac{\sigma-1}{\sigma} p_L(m)$.

Notice that MC and MR do not necessarily cross. Four cases are possible:

1. If, for any $y(m) \geq 0$, $MC(y(m)) \geq MR(y(m))$, then firm m is either not active or, if and only if $\pi(y_L(m)) \geq 0$, it sells

$$y_L(m) = p_H(m)^{-\sigma} [v(m)P]^{\sigma-1} p(1)y(1)$$

at $p_H(m)$.

2. If $MC(y(m))$ and $MR(y(m))$ cross only once for strictly positive quantities, and if they cross in the open interval between $y_L(m)$ and $y_H(m)$, i.e., if $MC(y_L(m)) < \lim_{y(m)\searrow y_L(m)} MR(y(m))$ and $MC(y_H(m)) \geq \lim_{y(m)\searrow y_H(m)} MR(y(m))$, then there exists a unique (global) profit maximizer, $y_M(m) \in (y_L(m), y_H(m))$. Such quantity is entirely sold on the local market at price $p_M(m)$. Given that within this range, the equality between MC and MR is sufficient to ensure optimality, we can derive from (2.18) and (2.17) that:

$$y_M(m) = (\frac{\sigma-1}{\sigma}\frac{\alpha}{w})^{\frac{\alpha\sigma}{\alpha+\sigma(1-\alpha)}}[v(m)^{\sigma-1}P^{\sigma}y(1)]^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}} \tag{2.19}$$

and

$$p_M(m) = (\frac{\sigma}{\sigma-1}\frac{w}{\alpha})^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}}[v(m)^{\sigma-1}P^{\sigma}y(1)]^{\frac{1-\alpha}{\alpha+\sigma(1-\alpha)}} \tag{2.20}$$

3. If between $y_L(m)$ and $y_H(m)$ $MC(y(m))$ lies below $MR(y(m))$, and crosses it afterwards, i.e., if $MC(y_H(m)) \leq \lim_{y(m)\nearrow y_H(m)} MR(y(m))$, then there exists a unique (global) profit maximizer, $y_E(m) > y_H(m)$. Such quantity is sold at price $p_L(m)$, partly on the local market, which absorbs $y_H(m)$, and for the remaining part, $y_E(m) - y_H(m)$, it is exported. In this case the choice to export induces marginal cost pricing, which yields

$$y_E(m) = [\frac{\alpha}{w}p_L(m)]^{\frac{\alpha}{1-\alpha}}.$$

4. If either $MC(y(m))$ and $MR(y(m))$ cross twice for strictly positive quantities or if they cross once, but $MC(y(m))$ lies above $MR(y(m))$ between $y_L(m)$ and $y_H(m)$, i.e., if $MC(y_H(m)) > \lim_{y(m)\searrow y_L(m)} MR(y(m))$ and $MC(y_H(m)) < \lim_{y(m)\searrow y_H(m)} MR(y(m))$, then there exist two positive local maximizers, one in which firm m sells exclusively on the local market, choosing either $y_M(m)$ or $y_L(m)$, and one in which it also exports, choosing $y_E(m)$. Its choice in this case cannot be determined *a priori* at the present stage, but has to be determined in equilibrium by comparison of the two local maxima.

Explicit calculation allows us to find the thresholds mentioned in Lemma 2.1. Let us define

$$\begin{aligned}
w_0(m) &\equiv p_H(m)^{\frac{\alpha+\sigma(1-\alpha)}{\alpha}[v(m)^{\sigma-1}P^{\sigma}y(1)]^{\frac{\alpha-1}{\alpha}}}, \\
\widehat{w_1}(m) &\equiv \alpha[(1+\tau)(1+a)]^{-2\frac{\alpha+\sigma(1-\alpha)}{\alpha}}w_0(m), \\
\widetilde{w_1}(m) &\equiv \alpha\frac{\sigma-1}{\sigma}w_0(m), \\
\overline{\tau} &\equiv \frac{1}{1+a}\left(\frac{\sigma}{\sigma-1}\right)^{\frac{\alpha}{2[\alpha+\sigma(1-\alpha)]}} - 1.
\end{aligned}$$

These thresholds are defined such that

- $\pi(m|y_L(m)) \geq 0 \iff w \leq w_0(m)$,
- $MC(y_L(m)) = \lim_{y(m)\searrow y_L(m)} MR(y(m)) \iff w = \widetilde{w_1}(m)$,

- $MC(y_H(m)) = \lim_{y(m)\nearrow y_H(m)} MR(y(m)) \iff w = \widehat{w_1}(m)$
- $\widetilde{w_1}(m) > \widehat{w_1}(m) \iff \tau > \overline{\tau}$.

Given this, Lemma 2.1 just amounts to a re-writing of the results obtained above.

It is also easy to show that $w_0(m)$ is greater than both $\widetilde{w_1}(m)$ and $\widehat{w_1}(m)$, that all of them are increasing functions of $v(m)$, and that therefore, at a given w, firms with a very low quality will remain inactive, firms with intermediate quality will produce to serve the domestic market, and firms with a very high quality will also export.

Proof of Proposition 2.1

In closed economy, there is no competition with the rest of the world, which means that intermediate good producers face a continuous demand with no threshold effects. Then the general equilibrium is easy to derive. From the final good market we know that $p(1) = P$. Equilibrium in the intermediate goods market ($y(m) = x(m)$, $m = L, H$, according to (2.7)) and in the labor market ($L^d = 1$, according to (2.11)) then yield $y(1)$ as a function of P and of the prices of low and high quality intermediate goods, $p(L)$ and $p(H)$, respectively. The definition of the price index P in (2.2) then yields $y(1)$ as a function of the prices of intermediate goods alone. Such prices are determined by $p(m) = p_M(m)$, $m = L, H$, according to (2.20). This yields the wage rate w as a function of $p(L)$ and $p(H)$. Substituting for w, we can therefore express $p(H)$, w, P and $y(1)$, $y(L)$, $y(H)$, all as functions of $p(L)$ alone. In particular, we find that the real part of the equilibrium is independent from the nominal part: defining a variable $A \equiv \left\{ u \left[\frac{v(L)}{v(H)}\right]^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}} + (1-u) \right\}$, which is decreasing in u and increasing in the domestic 'bad' to 'good' quality ratio $\frac{v(L)}{v(H)}$, we have $y(1) = v(H) A^{\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}}$, $y(L) = \left[\frac{v(L)}{v(H)}\right]^{\frac{\alpha(\sigma-1)}{\alpha+\sigma(1-\alpha)}} A^{-\alpha}$ and $y(H) = A^{-\alpha}$. The nominal part is defined by $p(H) = \left[\frac{v(L)}{v(H)}\right]^{\frac{(\alpha-1)(\sigma-1)}{\alpha+\sigma(1-\alpha)}} p(L)$, $w = \frac{\alpha(\sigma-1)}{\sigma} A^{1-\alpha} \left[\frac{v(L)}{v(H)}\right]^{\frac{(\alpha-1)(\sigma-1)}{\alpha+\sigma(1-\alpha)}} p(L)$ and $p(1) = P = A^{\frac{1}{1-\sigma}} \left[\frac{v(L)}{v(H)}\right]^{\frac{(\alpha-1)(\sigma-1)}{\alpha+\sigma(1-\alpha)}} \frac{p(L)}{v(H)}$. Taking one good as numeraire, for instance setting $p(1) = 1$, completes the characterization of the unique general equilibrium.

Appendix B: Existence and uniqueness of symmetric equilibria

Given the discontinuities and non convexities of the model we prove existence in two steps: first, we provide an analytical characterization of a candidate symmetric equilibrium of a given type, by assuming that every agent in the economy behaves in a specific way and by imposing that, given this, all markets clear; second, we study the conditions under which the candidate equilibrium is indeed an equilibrium, i.e., the conditions under which nobody wants to deviate. This second step amounts to checking whether the optimality conditions spelled out in Lemma 2.1 are satisfied in the candidate equilibrium. In what follows, we give the general expressions that are necessary for the proofs. Explicit calculations are given in Albornoz and Vanin (2005).

Lemma 2.2. ***(non existence of 'export and export' equilibria)*** *There does not exist any symmetric equilibrium such that every intermediate good producer both serves the domestic market and exports.*

Proof. Suppose there exists a symmetric equilibrium such that both types of firms, besides serving the local market, also export. They would produce $y(m) = \left[\frac{\alpha}{w} p_L(m)\right]^{\frac{\alpha}{1-\alpha}}$ and sell it at $p(m) = p_L(m)$, for $m = L, H$. Labor market equilibrium $L^d = u\left[\frac{\alpha}{w} p_L(L)\right]^{\frac{1}{1-\alpha}} + (1-u)\left[\frac{\alpha}{w} p_L(H)\right]^{\frac{1}{1-\alpha}} = 1$ would then determine the wage rate $w = \alpha\left[u p_L(L)^{\frac{1}{1-\alpha}} + (1-u) p_L(H)^{\frac{1}{1-\alpha}}\right]^{1-\alpha}$. Plugging w into the expressions for $y(L)$ and $y(H)$ yields these variables as functions of the parameters only. Observing that $P = [(1+a)(1+\tau)]^{-1} = p(1)$, it is then immediate to calculate intermediate goods producers' profits and add them to the aggregate wages to derive nominal national income $E = w + u\pi(L) + (1-u)\pi(H) = \frac{w}{\alpha}$. Equilibrium in the final good market then yields $y(1) = \left[u p_L(L)^{\frac{1}{1-\alpha}} + (1-u) p_L(H)^{\frac{1}{1-\alpha}}\right]^{1-\alpha} [(1+a)(1+\tau)]$. This immediately yields a contradiction since, given these values, it is immediate to prove that intermediate firms' production is entirely absorbed by domestic demand, so that, contrary to the hypothesis, there are no exports. □

Lemma 2.3. ***(non existence of 'export and survive' equilibria)*** *There does not exist any symmetric equilibrium such that high quality firms both serve the domestic market and export, and low quality firms just serve the domestic market.*

Proof. Suppose an ES equilibrium exists. Advanced firms sell at $p_L(H)$ their production whereas Backward firms may sell either at $p_M(L)$ or $p_H(L)$. We then have two subcases:

1. $p(L) = p_M(L)$
   Solving for P and $y(1)$ at the candidate equilibrium we obtain

$$P = \left\{1 - u\left[\frac{y(1)}{v(L)}\right]^{\frac{1-\sigma}{\sigma}}\left[\frac{1}{u} - \frac{1-u}{u}\left(\frac{\alpha}{w} p_L(H)\right)^{\frac{1}{1-\alpha}}\right]^{\frac{\alpha(\sigma-1)}{\sigma}}\right\}^{\frac{1}{\sigma-1}} \frac{(1-u)^{\frac{1}{1-\sigma}}}{[(1+a)(1+\tau)]} \tag{2.21}$$

   and

$$y(1) = \left\{u[y(L)v(L)]^{\frac{\sigma-1}{\sigma}} + (1-u)[y(H)v(H)]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}} \tag{2.22}$$

   From (2.7) we know that local demand for advanced firms equals

$$x(H) = p(H)^{-\sigma} v(H)^{\sigma-1} P^{\sigma} y(1) \tag{2.23}$$

   Plugging equations (2.21) and (2.22) into (2.23) leads to a contradiction, since we obtain $x(H) = y(H)$. This means that input supply of advanced firms equals local demand and therefore there are no exports.

2. $y(L) = y_L(L)$
   In this case, backward firms sell their production at the limit price: $p(L) = p_H(L)$. A similar procedure leads to the same contradiction: $x(H) = y(H)$ and therefore under this candidate equilibrium, exporting is not optimal for advanced firms and therefore the candidate is not an equilibrium.

□

Lemma 2.4. ***(existence and uniqueness of 'export and die' equilibria)*** *For some parameter values there exists a symmetric equilibrium such that high quality firms both serve the domestic market and export, whereas low quality firms stay closed and the corresponding goods are imported. If it exists, such equilibrium is unique and its consumption level is*

$$y(1) = \frac{(1-u)^{1-\alpha} p_L(H)}{P - u\tau(1+a)[(1+a)(1+\tau)]^{-\sigma} P^{\sigma}}, \tag{2.24}$$

where

$$P = \left\{u[(1+a)(1+\tau)]^{1-\sigma} + (1-u)[(1+a)(1+\tau)]^{\sigma-1}\right\}^{\frac{1}{1-\sigma}} \tag{2.25}$$

Proof. Suppose there exists an ED equilibrium. Advanced firms would produce $y(H) = [\alpha p_L(H)]^{\frac{\alpha}{1-\alpha}} w^{\frac{\alpha}{\alpha-1}}$ and sell it at $p_L(H)$. Labor market equilibrium $L^d = (1-u)y(H)^{\frac{1}{\alpha}} = 1$ yields $w = (1-u)^{1-\alpha}\alpha p_L(H)$. Equation (2.25) follows from the fact that backward firms stay closed and the corresponding goods are imported, and therefore their price for the domestic buyer includes both transportation costs and tariff. Profits are $\pi(H) = p_L(H)y(H) - wy(H)^{\frac{1}{\alpha}} = \frac{p_L(H)(1-\alpha)}{(1-u)^{\alpha}}$. As τ applies to landed imports, aggregate tariff revenue is $T = \tau p^*(1+a)Im$ where $Im = uP^{\sigma}y(1)[(1+a)(1+\tau)p^*]^{-\sigma}(v^*)^{\sigma-1}$ as stated by equation (2.9). Having determined $w, \pi(H)$ and T, we can now compute $E = w + (1-u)\pi(H) + T$. Equilibrium in the final good market $E = \frac{y(1)}{P}$ then yields $y(1)$ as stated in equation (2.24). This determines a unique candidate equilibrium. Therefore, if such equilibrium indeed exists, uniqueness is trivially proved. In Albornoz and Vanin (2005) we characterize the (necessary and sufficient) conditions under which nobody has incentive to deviate from the candidate equilibrium, that is, under which this is indeed an equilibrium, and provide abundant numerical examples of parameter constellations for which such conditions are satisfied. □

Lemma 2.5. ***(existence of 'survive and survive' equilibria)*** *For any parameter constellation, if τ is sufficiently high, then there exist symmetric equilibria such that both high quality and low quality firms are only active on the local market. In such equilibria, each intermediate good producer m can either produce the same quantity as under autarky, $y_M(m)$, or produce the higher quantity $y_L(m)$. If both high quality and low quality firms produce $y_M(m)$, then the consumption level is*

$$y(1) = \left[uv(L)^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}} + (1-u)v(H)^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}\right]^{\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}} \tag{2.26}$$

If they both produce $y_L(m)$, then the consumption level is

$$y(1) = \left[uv(L)^{-\frac{1}{\alpha}} + (1-u)v(H)^{-\frac{1}{\alpha}}\right]^{-\alpha} \tag{2.27}$$

Proof. Suppose that an SS equilibrium exists. From the final good market we know that $p(1) = P$. Equilibrium in the intermediate goods market and in the labor market yields
$y(1) = P^{-\sigma}\left\{u\left[p(L)^{-\sigma}v(L)^{\sigma-1}\right]^{\frac{1}{\alpha}} + (1-u)\left[p(H)^{-\sigma}v(H)^{\sigma-1}\right]^{\frac{1}{\alpha}}\right\}^{-\alpha}$. From Lemma 2.1 it is immediate to see that any intermediate firm m who chooses to sell only to the domestic market, has only two possible optimal choices: it either produces $y_M(m)$ and

sells it at $p_M(m)$, or it produces $y_L(m)$ and sells it at $p_H(m)$. Such quantities and prices are defined in equations (2.19), (2.20), and, through (2.7), by $y_L(m) \equiv x(m|p_H(m))$ and (2.5), respectively.

Since there are two types of intermediate goods producers, we have four possible combinations of their choices. Only for expositional purposes, we restrict attention to the two cases in which either $y(L) = y_M(L)$ and $y(H) = y_M(H)$, or $y(L) = y_L(L)$ and $y(H) = y_L(H)$.

Consider first the former case, i.e., $y(L) = y_M(L)$ and $y(H) = y_M(H)$. The definition of the price index P in (2.2) allows to write $y(1)$ as a function of $p(L)$ and $p(H)$ alone. Then (2.20) yields the wage rate w as a function of them. Substituting for w, we can therefore express $p(H)$, w, P, $y(L)$, $y(H)$, $y(1)$, all as functions of $p(L)$ alone. In particular, defining a variable $A \equiv u\left[\frac{v(L)}{v(H)}\right]^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}} + (1-u)$, which is decreasing in u and increasing in the domestic quality gap $\frac{v(L)}{v(H)}$, we find the following expressions:

$$
\begin{aligned}
p(H) &= \left[\frac{v(L)}{v(H)}\right]^{\frac{(\sigma-1)(\alpha-1)}{\alpha+\sigma(1-\alpha)}} p(L) \\
w &= \alpha\frac{\sigma-1}{\sigma}A^{1-\alpha}\left[\frac{v(L)}{v(H)}\right]^{\frac{(\sigma-1)(\alpha-1)}{\alpha+\sigma(1-\alpha)}} p(L) \\
P &= \frac{A^{\frac{1}{1-\sigma}}}{v(H)}\left[\frac{v(L)}{v(H)}\right]^{\frac{(\sigma-1)(\alpha-1)}{\alpha+\sigma(1-\alpha)}} p(L) \\
y(L) &= \left[\frac{v(L)}{v(H)}\right]^{\frac{\alpha(\sigma-1)}{\alpha+\sigma(1-\alpha)}} A^{-\alpha} \\
y(H) &= A^{-\alpha}
\end{aligned}
$$

and $y(1)$ is given by (2.26), which is the same as (2.12).

This proves that, if the equilibrium considered in this case exists, then its level of production of both final and intermediate goods is univocally determined, independently of $p(L)$. We now have to make sure that this candidate equilibrium is indeed an equilibrium, in the sense that nobody wants to deviate. This is going to determine a set of values of $p(L)$, for each of which such an equilibrium exists. In principle, this set can either be empty, or be a singletone, or have cardinality higher than one. We find two results: first, for τ sufficiently high this set is not empty. To see this, recall that the autarkic equilibrium always exists. Second, when it is not empty, this set is an interval, that is, there exists an interval of values of $p(L)$, for each of which there exists a symmetric SS equilibrium. Any such equilibrium displays the same production quantities as under autarky. For our purposes, thus, this multiplicity is more apparent than real, and it is due to the fact that we take time 0 foreign consumption as numeraire, but that in equilibrium there is no international trade. This means that the choice of the numeraire is not sufficient to pin down all equilibrium prices, but still the characteristics of the rest of the world influence our small economy, because they determine the range in which no agents wants to deviate. For instance, an SS equilibrium in which production quantities are the same as under autarky exists for the following constellation of parameters: $v^* = 1, v(H) = 1, v(L) = 0.8, u = 0.5, a = 10\%, \alpha = 0.9, \sigma = 4$ and $\tau > 85\%$.

Consider now the second possibility, i.e., $y(L) = y_L(L)$ and $y(H) = y_L(H)$. Intermediate goods prices are $p(L) = p_H(L)$ and $p(H) = p_H(H)$, so that $p(1) = P = (1+a)(1+\tau)$. Then intermediate goods market equilibrium and labor market equilibrium imply that $y(1)$ is given by (2.27). Knowing this, also $y(L)$ and $y(H)$ are univocally determined. Once again, we have a free price, in this case w. Therefore, each value of w defines a candidate equilibrium, and we have to check for which values of w nobody has an incentive to deviate (so that prices and quantities indeed constitute an equilibrium). In any such equilibrium, production quantities are the same, so that, if multiple such equilibria exist, once again for our purposes multiplicity is more apparent than real. For instance, an SS equilibrium in which both high and low quality firms produce more than under autarky exists for the following constellation of parameters: $v^* = 1, v(H) = 1, v(L) = 0.8, u = 0.5, a = 10\%, \alpha = 0.9, \sigma = 4$ and $\tau > 10\%$. □

Lemma 2.6. ***(non existence of 'survive and die' equilibria)*** *There does not exist any symmetric equilibrium such that high quality firms are only active on the local market and backward firms stay closed.*

Proof. Assume an SD equilibrium exists. Solving for y(1) we obtain $y(1) = v(H)(1-u)^{\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}} \left\{ 1 - \left[\frac{u^{\frac{1}{\sigma-1}}}{\Gamma} \right]^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{1-\sigma}}$, where $\Gamma \equiv [(1+a)(1+\tau)] - \tau(1+a)[(1+a)(1+\tau)]^{\sigma}$.

Observe that $\Gamma > u^{\frac{1}{\sigma-1}}$ is a necessary and sufficient condition for $y(1) > 0$. This means that this equilibrium may exist only for sufficiently low values of τ, which push backward firms out of the domestic market. Given that advanced firms are only active on the domestic market, they can either produce quantity $y_M(H)$ and sell it at price $p_M(H)$, or they can produce $y_L(H)$ and sell it at the limit price $p_H(H)$. We therefore have two cases.

1. For $y(H) = y_M(H)$, in Albornoz and Vanin (2005) we characterize the (necessary and sufficient) conditions for this to be an equilibrium. Despite careful and systematic numerical exploration of the parameter space, we have never found parameter values for which such conditions are satisfied.
2. For $y(H) = y_L(H)$, notice that this holds if and only if $(1-u)^{-\alpha} = p_H(H)^{-\sigma} v(H)^{\sigma-1} P^{\sigma} y(1)$, which after some algebra becomes

$$\Gamma = \frac{1}{u} \tag{2.28}$$

   Since Γ is decreasing in τ and for $\tau = 0$ we have $\Gamma = 1 + a$, where $a \in [0, 1)$ is small, a sufficient condition for (2.28) NOT to hold is $\frac{1}{u} > 1 + a$, which we assume.

□

Lemma 2.7. ***(non existence of 'die and die' equilibria)*** *There cannot exist any symmetric equilibrium such that both high and low quality firms are inactive.*

Proof. Trivially, if such an equilibrium existed, domestic labor demand would be zero, but then the domestic labor market would not clear. □

Chapter 3

Economic Growth and Social Development

3.1 Introduction

In the long run, individual and aggregate well-being depend on both material growth and social and cultural development. Although this has perhaps always been true, systematic and sustained material growth has been absent for most of human history, with some positive and negative exceptions (see, e.g., Goodfriend and McDermott, 1995). Instead, since the Industrial Revolution, a significant fraction of the world has kept growing at a positive rate, accumulating physical capital, developing better and better technologies, and accumulating human capital. Indeed, these processes have captured most of economists' attention, whereas social and cultural dynamics have remained at the margin of economic analysis. In recent years, however, an increasing number of economists have begun to pay attention to the interplay between material growth and social development.

When material needs have been satisfied to a substantial degree, as is the case in advanced economies, well-being depends to an increasing extent upon social factors, like social environment, individual relative position and social status, and the ability to construct and enjoy meaningful and satisfactory relations with other people[1]. Social status has already received a great deal of attention by economists. Here, we rather focus on the social environment and the enjoyment of social relations, building on the notions of 'social capital' and 'relational goods'.

The present contribution proposes a simple growth model with private and social capital accumulation. We investigate whether these two processes move in the same direction or whether they conflict with each other, and show that both outcomes are possible, depending on the parameters and initial conditions of the economy. Taking into account the effects of these dynamics on the consumption of both private and relational goods, we draw conclusions about well-being that apply to advanced economies. Section 2 clarifies the concepts and motivates our set-up. Sections 3 and 4 introduce the static and dynamic versions of our model. Section 5 concludes.

[1]Sacco et al. (2006) provide an extensive discussion of these issues.

3.2 Motivation

Social capital is the collection of those productive assets which are incorporated in the social structure of a group (rather than in physical goods and single human beings, as physical and human capital) and which allow cooperation among its members to reach common goals. If we bear in mind that the group considered may range from being very small to including the whole of society, this definition of social capital encompasses most of the definitions to be found in the literature. At one extreme, some scholars even define social capital as an individual asset, but we prefer to consider it as a collective asset, in order to emphasise its interpersonal nature[2]. Examples of social capital range from trust to effective civic norms and to the networks of voluntary associations typical of civil society. A peculiar feature of social capital is that it is not accumulated through a standard mechanism of individual investment, since most of its benefits are not privately appropriable[3]. Rather, or at least to a much greater extent, it is accumulated through social participation in group activities. This participation may only partially be regarded as an investment, since it is, perhaps mainly, an activity that entails the simultaneous production and consumption of a particular kind of goods, namely, relational goods.

Relational goods display two peculiar features: they cannot be enjoyed alone, but exist only inasmuch as they are shared; and their production and consumption very often cannot be separated: relational goods are produced and consumed at the same time through participation in some social activity with other people[4]. Examples range from going out with friends to participating in a choir, a football club, a voluntary organization, and so on.

We focus on two aspects of the relationship between relational goods and social capital. On one hand, a higher social capital increases the return to the time spent in social participation. For instance, it is easier and more rewarding to participate in an association with people whom we trust and who share our values and norms, and in a social context characterised by a rich network of associative opportunities; similarly, it is more rewarding to go out with friends with whom we share a higher capital of confidence, long-lasting relations and common norms, and in a context that offers many options for socially enjoyed leisure. In other words, social capital may be seen as an input in the production of relational goods[5]. On the other hand, higher social

[2]Glaeser et al. (2002) call 'social capital' the 'social' component of human capital. Since we distinguish social capital from human capital, we do not follow their approach. DiPasquale and Glaeser (1999) define individual social capital as an individual's connections to others, and argue that it is important for private provision of local amenities and of local public goods. This is in line with our focus, although we emphasize more the role of aggregate social participation.

[3]Glaeser et al. (2002) make the opposite point, namely, that social capital accumulation responds to incentives to investment in exactly the same way as human capital. Indeed, this result is natural if one defines social capital as a component of human capital, but it does not hold if one considers social capital as a group asset rather than as an individual asset.

[4]The concept of relational goods is due to Uhlaner (1989). Corneo and Jeanne (1999) refer to them as to socially provided private goods and study their interplay with social status and growth. (ed.) Gui (2000) provides a number of interesting contributions on the interpersonal dimension of economic interaction.

[5]Much of the literature on social capital also stresses its positive impact on the productivity of traditional private goods. We do not examine this effect here, thus making our point sharper: as in our framework, a problem of under-accumulation of social capital exists, this problem will become even worse if we also consider the effect of social capital on private production. We discuss this point in more detail in the concluding section.

participation brings about social capital accumulation as a by-product. For instance, trust (or empathy) may be reinforced and generalised through social interactions (if individuals do not behave opportunistically). Likewise, high social participation may lead to the formation of new associations, while continuing to feed the existing ones.

Social participation is an activity intrinsically characterised by external effects (generally speaking, there is no market in which other people's participation may be bought, and even less is there a market for social capital). If other people's participation is low, or if the level of social capital is low, the time spent in participating is little productive, and it becomes worthwhile to shift to private activities, that is, to ones which yield private goods. For instance, if my friends do not have time to go out with me, or if they do go out with me but the environment does not offer any interesting social opportunities, I may decide to spend my time better watching television or reading a book. Indeed, Corneo (2005) presents striking empirical evidence that the time devoted to watching television and working are positively correlated across countries, and proposes an explanation based on the substitution between privately enjoyed and socially enjoyed leisure (i.e., between some private goods and relational goods). While our work is quite close in spirit to Corneo's paper, the main difference is that we analyse the dynamics of private and social capital accumulation, whereas he displays a simple static model with multiple equilibria.

Specifically, we propose here a model in which a reduction in social participation implies at the same time an increase in labour supply and a substitution of private for relational goods. On one hand, such a shift stimulates the economy[6]; on the other, it generates a negative externality on the productivity (in terms of relational goods) of social participation. Dynamically, this change has a negative effect on social capital accumulation, whereas the sign of its effect on private capital accumulation depends on whether total savings increase (together with private production) or decrease (due to a more than proportional increase in private consumption)[7]. Theoretically, private and social capital may be both positively or negatively correlated[8].

Both ideas - that private growth brings about social development, and that it generates social disruption - are supported by long-standing traditions of thinking. We do not attempt to reconstruct this fascinating intellectual debate here, but limit ourselves to referring to Hirsch (1976) as a representative of the view that private growth may entail negative social externalities. In particular, Hirsch argues that growth makes individual time constraints increasingly binding, thereby inducing a shift from time-intensive activities (among which there is indeed social participation) to time-saving ones (among which there are many forms of private consumption – e.g., fast food)[9]. We emphasise here that this kind of shift may even reinforce private growth.

The idea that negative externalities, either on the natural or the social environment, may foster growth was first studied within an evolutionary framework by Antoci (1996) and Antoci and Bartolini (1997). The environmental economics literature on this subject has subsequently been rapidly expanding. For instance, Bartolini and Bonatti (1997) and several other contributions have further explored the basic idea

[6]While most private goods enter the GDP, most relational goods do not.

[7]While this is consistent with an interpretation of private capital in terms of physical capital, an extension of the concept to include human capital would not alter the picture significantly.

[8]See also the empirical findings of Putnam (1995, 2000) about the rise and decline of US social capital.

[9]See Becker (1965) for a pioneering economic analysis of time allocation.

within a neoclassical framework[10]. While our work is closely related to theirs, the main departure consists in our focus on social capital accumulation, which depends on social participation, whereas the above literature, although it mentions the possibility of a sociological interpretation, is more focused on natural resources, which are typically subject to a spontaneous flow of renewal.

In two companion papers (Antoci et al., 2007a,b), we explore a similar framework, respectively with the tools of evolutionary game theory and of neoclassical economics. In both studies we find that growing economies may fall into social poverty traps, defined as situations in which, although material wealth is high, social poverty forces down overall well-being. For the sake of simplicity, in those models we consider the dynamics of only one asset, social capital. Here, we extend an analysis to include the accumulation of private capital. One might expect that, once the latter is taken into account, possibly together with the positive externalities it causes, material growth may be strong enough, from the point of view of well-being, to more than compensate its negative social externalities. Indeed, we show that this may but need not be the case, and that whether it happens or not depends on the parameters of preferences and technology. An interesting result is that impatience may increase steady-state well-being, since it reduces inefficient over-accumulation of private capital[11].

3.3 Static model

We now present a simple growth model with private and social capital accumulation. Since some of the basic insights may be appreciated even in a static framework, we first introduce a static version, in which private and social capital are considered as exogenously given in some strictly positive amount, and then introduce their dynamics (in continuous time).

Preferences and technology

We model an economy populated by a continuum of identical, infinitely lived individuals, of size normalised to 1, whose utility depends on three goods: a private consumption good C, used to satisfy basic needs (e.g., food and clothes); a relational good B (e.g., enjoying time with friends); and a private consumption good C_s that serves as a substitute for the relational good (e.g., a luxury good). Instantaneous preferences are described by the utility function $u(C, B, C_s) = \ln C + a \ln(B + bC_s)$, where $a > 0$ is the elasticity of substitution between basic needs satisfied by C on one hand and needs satisfied by either B or C_s on the other, and $b > 0$ is the marginal rate of substitution between B and C_s[12].

The key point is how individuals decide to allocate their time (they are endowed with one unit) between social participation, labour and private consumption, besides the allocation of the latter between the two private goods. Since it is out of our focus, we disregard the allocation of time to C and C_s, simply assuming that both

[10] Among recent contributions, see Antoci and Bartolini (2004) for an evolutionary one and Bartolini and Bonatti (2004) for a neoclassical one.

[11] A similar result is also found in the above-mentioned environmental economics literature, since in that case too growth is the result of a failure of coordination.

[12] The assumption of perfect substitutability between B and C_s greatly simplifies the mathematics. Relaxing this assumption may have non-obvious economic consequences and make closed-form solutions hard to obtain. We simulated a version of this model with the hypothesis of imperfect substitution, but did not gain any interesting insight.

require income but not time; on the contrary, B may only be enjoyed if an individual spends time in social participation. Time must therefore be allocated between social participation (fraction s) and labour (fraction $1 - s$). A single individual considers average social participation $\bar{s} = \int_0^1 s(i)\mathrm{d}i$ in the economy as exogenously given.

We assume a backyard technology[13], by which identical individuals produce private goods for their own consumption using labour and private capital K, according to production function $Y = (1 - s)^{\epsilon} K^{1-\epsilon} A$, where $\epsilon \in (0, 1)$ is a parameter. Term $A \equiv (1 - \bar{s})^{\sigma} \bar{K}^{\vartheta}$ captures a positive externality in production, which can be due to either the observability of other people's production or the availability of help when needed. Average private capital $\bar{K} = \int_0^1 K(i)\mathrm{d}i$ is considered as exogenously given by each individual and, consequently, the same is true for the whole term A (σ and ϑ are strictly positive parameters).

Besides private capital, our economy is characterised by the presence of social capital K_s. Social capital is not the private property of any individual, but is rather an endowment of the entire economy, that each single individual considers as exogenous.

The quantity of relational good B enjoyed by the representative individual is a function of his own social participation, average social participation and social capital, all of which are indispensable factors: $B = s^{\alpha} \bar{s}^{\beta} K_s^{\gamma}$, where $\alpha,\ \beta,\ \gamma > 0$.

The maximisation problem of the representative individual, and symmetric Nash equilibria

The problem solved by the representative individual is:

$$\max_{s,C,C_s} u(C, B, C_s) = \ln C + a \ln(s^{\alpha} \bar{s}^{\beta} K_s^{\gamma} + bC_s) \quad \text{s.t.} \tag{3.1}$$

$$C + C_s = Y = (1 - s)^{\epsilon} K^{1-\epsilon} (1 - \bar{s})^{\sigma} \bar{K}^{\vartheta}, \quad C, C_s \geq 0, \quad s \in [0, 1]. \tag{3.2}$$

A symmetric Nash equilibrium (SNE) is a triplet (s^*, C^*, C_s^*) that solves problem (3.1) under constraints (3.2), given that every other individual in the economy chooses s^*, so that, in particular, $\bar{s} = s^*$.

It is easy to show that there is always an SNE with no social participation. To see this, let $\tilde{s} \equiv 0$, $\tilde{C} \equiv \frac{1}{1+a} K^{1+\vartheta-\epsilon}$, and $\tilde{C}_s \equiv \frac{a}{1+a} K^{1+\vartheta-\epsilon}$.

Proposition 3.1. *The triplet $(\tilde{s}, \tilde{C}, \tilde{C}_s)$ is always an SNE, that is, for any parameter constellation, there exists an SNE with no social participation*[14].

In this equilibrium, no time is devoted to social interaction, since each individual believes that every other one will spend his entire amount of time working, thus rendering social participation not worthwhile.

To be able to investigate analytically the existence of an SNE in which $s > 0$, we make the following simplifying assumption.

Assumption 3.1. $\alpha + \beta = \epsilon + \sigma = \varphi < 1$*: this implies that, at an SNE, the elasticity of relational goods with respect to social participation equals the elasticity of private production with respect to labour; we call this elasticity φ, and assume that the two functions are concave ($\varphi < 1$)*[15].

[13] This simplifying assumption allows us to rule out any concern about market structure.

[14] All proofs are given in the Appendix.

[15] The equality plays no other role than to enable us to derive an analytic solution, whereas the assumption that $\varphi < 1$ allows a strictly positive equilibrium social participation, even for a low ratio of social over private capital.

Proposition 3.2. *Under Assumption 3.1, there exists a unique SNE with strictly positive social participation, namely, the triplet $(\hat{s}, \hat{C}, \hat{C}_s)$, defined as follows:*

$$\text{Case (a)} : K_s < h(K) : \qquad \begin{cases} \hat{s} \equiv \frac{1}{1+\left(\frac{b\epsilon K^{1+\vartheta-\epsilon}}{\alpha K_s^{\gamma}}\right)^{\frac{1}{1-\varphi}}} \\ \hat{C} \equiv \frac{1}{b(1+a)} \hat{s}^{\varphi} K_s^{\gamma} + \frac{1}{(1+a)} (1-\hat{s})^{\varphi} K^{1+\vartheta-\epsilon} \\ \hat{C}_s \equiv \frac{a}{(1+a)} (1-\hat{s})^{\varphi} K^{1+\vartheta-\epsilon} - \frac{1}{b(1+a)} \hat{s}^{\varphi} K_s^{\gamma} \end{cases} \tag{3.3}$$

$$\text{Case (b)} : K_s \geq h(K) : \qquad \begin{cases} \hat{s} \equiv \frac{a\alpha}{a\alpha+\epsilon} \\ \hat{C} \equiv (1-\hat{s})^{\varphi} K^{1+\vartheta-\epsilon} \\ \hat{C}_s \equiv 0 \end{cases} \tag{3.4}$$

where $h(K) \equiv \left[\left(\frac{\epsilon b}{\alpha}\right)^{\varphi} (ab)^{1-\varphi} K^{1+\vartheta-\epsilon}\right]^{\frac{1}{\gamma}}$.

Note that $\hat{s}$ is an increasing function of K_s and α and a decreasing function of K. We will come back to the interpretation of these findings in the context of the dynamic specification of the model. Observe also that cases (a) and (b) of Proposition 3.2, defined as $K_s < h(K)$ and $K_s \geq h(K)$, respectively, identify an economy in which social capital is scarce and, respectively, abundant relative to private capital. Whatever the economy's (exogenous) endowment of the two forms of capital, Proposition 3.2 gives us the SNE with participation $(\hat{s}, \hat{C}, \hat{C}_s)$ as a function of them and of the parameters.

Proposition 3.3. *Let Assumption 3.1 hold.*

In case (a), there are both parameter constellations for which the SNE with positive social participation $(\hat{s}, \hat{C}, \hat{C}_s)$ Pareto-dominates the SNE with no participation $(\tilde{s}, \tilde{C}, \tilde{C}_s)$, and other ones for which the reverse is true.

In case (b), let $g(K) \equiv \left\{ \left[\frac{(a\alpha+\epsilon)^{\frac{1+a}{a}}}{(1+a)^{\frac{1+a}{a\varphi}}} \right]^{\varphi} (ab)^{1-\varphi} K^{1+\vartheta-\epsilon} \right\}^{\frac{1}{\gamma}}$. *For any parameter constellation, the SNE with participation $(\hat{s}, \hat{C}, \hat{C}_s)$ Pareto-dominates the one with no participation $(\tilde{s}, \tilde{C}, \tilde{C}_s)$ if and only if $K_s > g(K)$, the reverse being true when $K_s < g(K)$.*

Proposition 3.3 tells us that in economies relatively poor in social capital and rich in private capital (meaning $K_s < h(K)$), which of the two equilibria Pareto-dominates the other depends on the parameters of preferences and technology; but when social capital is abundant enough relative to private capital ($K_s \geq h(K)$), it eventually (that is, for $K_s > g(K)$) becomes more efficient to devote a positive fraction of time to social participation, thereby foregoing some (or all) luxury consumption but enjoying relational goods. Since, for any parameter constellation and for any strictly positive endowment of both forms of capital, both equilibria are present, it is possible that, due to coordination failure, an economy becomes stuck in the Pareto-inferior equilibrium. The limitation of Proposition 3.3 is that it does not tell us anything about the sources of the relative abundance of private versus social capital. To investigate this aspect, we have to turn to the dynamic specification of our model.

However, before doing this, a further comment may be made on the externalities which drive the story of this static model. Since both average social participation and average labour time are supposed to exert positive external effects (on the production of the relational good and of the private goods, respectively), it is not *a priori* clear whether, overall, social participation displays positive or negative spillovers[16]. In general, in this game there tend to be positive spillovers from social participation when social capital is high relative to private capital, whereas they are, overall, negative when the reverse is true[17].

Remark 3.1. *Under Assumption 3.1, since, generically, in the SNE with positive participation* $(\hat{s}, \hat{C}, \hat{C}_s)$ *spillovers are present, this equilibrium is inefficient even when it Pareto-dominates the SNE with no participation* $(\tilde{s}, \tilde{C}, \tilde{C}_s)$[18].

Remark 3.1 tells us that the common result that, in the presence of non-internalised externalities, even the best SNE is generally inefficient, also applies to our case.

3.4 Dynamic model

In the dynamic specification of the model, preferences and technology are the same as above, with the only difference that now private and social capital are endogenously determined. The dynamics of the representative individual's private capital is given by $\dot{K} = Y - C - C_s - \eta K$, where $\eta \geq 0$ is the private capital depreciation rate[19].

Social capital is not accumulated through a process of investment; rather, its stock increases when a high average social participation brings about a high average enjoyment of the relational good (denoted $\bar{B} = \int_0^1 B(i)\mathrm{d}i$). Since relations deteriorate over time if individuals do not actively take care of them, we also assume that K_s depreciates at a rate $\delta > 0$. We can thus summarise the dynamics of social capital as $\dot{K}_s = f(\bar{B}) - \delta K_s$, where f is a strictly increasing function[20]. The more rewarding social participation is in terms of relational goods, the more it contributes to social capital accumulation[21].

For the sake of simplicity, we make the following assumptions.

Assumption 3.2. $\eta = 0$*: we ignore private capital depreciation.*

Assumption 3.3. $f(x) \equiv x$*: this means that* $\dot{K}_s = \bar{B} - \delta K_s$*.*

[16] According to Cooper and John's (1988) terminology, social participation has positive (negative) spillovers if an increase in average social participation raises (decreases) individual utility, i.e., if $\frac{\partial u(C,B,Y-C)}{\partial \bar{s}}$ is positive (negative).

[17] Formally, under the reasonable assumption that $\beta, \sigma < 1$, which is even weaker than Assumption 3.1, $\frac{\partial u(C,B,Y-C)}{\partial \bar{s}} > 0 \;\Leftrightarrow\; \beta s^{\alpha} \bar{s}^{\beta-1} K_s^{\gamma} > b\sigma(1-s)^{\epsilon}(1-\bar{s})^{\sigma-1} K^{1+\vartheta-\epsilon}$, i.e., when K_s is high relative to K, s is high and $\bar{s}$ is low.

[18] Precisely, in the SNE $(\hat{s}, \hat{C}, \hat{C}_s)$ there are positive spillovers when $\alpha < \frac{\beta\epsilon}{\sigma}$ and negative ones when the reverse is true. There are no spillovers only in the non-generic case in which $\alpha = \frac{\beta\epsilon}{\sigma}$. Remark 3.1 then follows from Proposition 2 of Cooper and John (1988).

[19] For notational simplicity, we omit the time index $t \in \Re_+$.

[20] The idea that non-material forms of capital may be accumulated through a 'consumption' activity rather than through investment, although unconventional in economics, is neither new (it goes back to Aristotle's analysis of ethical virtues, the influence of which is to be found in the discussion of relational goods by Nussbaum (1986)) nor surprising (e.g., knowledge, which is accumulated through the use of knowledge itself).

[21] This specification seems a good first approximation for both main forms of social capital, namely trust and social norms on one hand, and association networks on the other, since the ability of both of them to prosper and expand crucially depends on the reward they yield to the people involved, and this reward consists to a high degree of relational goods. The reason it is a first approximation is that material rewards may also play a role: we discuss this point in the concluding section.

Assumption 3.4. *$\epsilon > \vartheta$ and $\gamma < 1$: this means that we do not allow either K or K_s to grow steadily at a strictly positive rate.*

Assumption 3.2 is an innocent one. Assumption 3.3 is only made for the sake of analytical simplicity[22]. Assumption 3.4 means that, in our model, there is no engine for endogenous growth.

The representative individual's maximisation problem

Let $r > 0$ be the inter-temporal discount rate. The representative individual chooses at time $t = 0$ how to allocate, at present and at any point in the future, his own time to participation and labour, and his private production to subsistence and luxury consumption on one hand, and investment in new private capital on the other, in order to maximise lifetime utility. At any given point in time, his control variables are therefore s, C and C_s (which must respect $C, C_s \geq 0$ and $s \in [0, 1]$). A strategy is a time path of controls. Initial stocks of social capital (K_s^0) and private capital (K^0, assumed to be the same for every individual: $K^0 = \bar{K}^0$) are exogenously given. When choosing his strategy, the representative individual regards as exogenously given the strategies of the rest of the population. Since the time path of social capital and population averages are independent of any single individual's strategy, this amounts to taking the entire future path of $K_s, \bar{K}$ and $\bar{s}$ as given. The set of variables that the representative individual considers as predetermined at any point in time therefore includes these three variables and the state variable that is under his own direct control, namely, his own private capital K[23]. In short, taking for granted the constraints imposed by technology and by the set of admissible controls, the representative individual's problem may be written as follows[24]:

$$\max_{s,C,C_s} \int_0^\infty u(C, B, C_s)\mathrm{e}^{-rt}\mathrm{d}t = \int_0^\infty [\ln C + a\ln(s^\alpha \bar{s}^\beta K_s^\gamma + bC_s)]\mathrm{e}^{-rt}\mathrm{d}t \quad \text{s.t.} \tag{3.5}$$

$$\dot{K}_s = \bar{s}^{\alpha+\beta} K_s^\gamma - \delta K_s, \tag{3.6}$$

$$\dot{K} = (1-s)^\epsilon K^{1-\epsilon} A - C - C_s, \qquad A \equiv (1-\bar{s})^\sigma \bar{K}^\vartheta. \tag{3.7}$$

Symmetric Nash equilibrium

An SNE of this economy is a strategy (that is, a time path of the controls s^*, C^* and C_s^*) that solves problem (3.5) under constraints (3.6)-(3.7), given that every other individual in the economy chooses the same strategy.

[22] In principle, there is no reason for the 'gross investment' in social capital to be exactly equal to the average benefit from social participation, although it is an increasing function of the latter; however, this specification is by far the easiest one. For instance, all our results would still hold if we assumed $f(x) \equiv \psi x,\ \psi \in (0, 1)$.

[23] Notice that, in the absence of uncertainty and in the impossibility for the representative individual to affect population averages (which eliminates any incentive to behave strategically to influence other people's future choices), in the present model considering open loop vs. closed loop strategies makes no difference.

[24] Recall that all the variables here (both the controls s, C, C_s and the predetermined variables $K_s, K, \bar{K}, \bar{s}$, as well as the rates of change $\dot{K}_s$ and $\dot{K}$) should appear with a time index t, omitted for notational simplicity. The maximisation is taken over the time path of the three controls, with a slight abuse of notation.

We now study the SNE of our economy and its dynamic properties[25]. In order to maintain the analytical tractability of the static version also in the dynamic version of the model, we modify Assumption 3.1 into the following one.

Assumption 3.5. $\alpha+\beta=\epsilon+\sigma=\varphi=1$*: this implies that, at any SNE, the relational good is obtained as a linear function of social participation, and private production as a linear function of labour.*

Proposition 3.4. *Let Assumptions 3.2 to 3.5 hold, and consider an SNE of the economy. At any point in time the curve:*

$$K_s = \left(\frac{\epsilon b}{\alpha} K^{1+\vartheta-\epsilon}\right)^{\frac{1}{\gamma}}, \tag{3.8}$$

separates in the (K, K_s) *plane the region in which* $s > 0$ *and* $C_s = 0$ *that in which* $s = 0$ *and* $C_s > 0$ *(see figure 1)*[26].

Precisely, in the two regions, s *and* C_s *are chosen as follows*[27]*:*

$$\text{Case (a)}: K_s < \left(\frac{\epsilon b}{\alpha} K^{1+\vartheta-\epsilon}\right)^{\frac{1}{\gamma}} : \quad \begin{cases} s = 0 \\ C_s = \frac{a}{\lambda} \end{cases}, \tag{3.9}$$

$$\text{Case (b)}: K_s > \left(\frac{\epsilon b}{\alpha} K^{1+\vartheta-\epsilon}\right)^{\frac{1}{\gamma}} : \quad \begin{cases} s = \min\left\{1, \frac{a\alpha}{\epsilon\lambda K^{1+\vartheta-\epsilon}}\right\} \\ C_s = 0 \end{cases}. \tag{3.10}$$

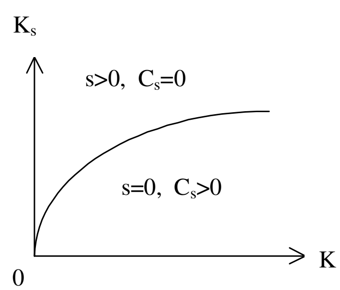

Figure 1

[25] Notice that, although the representative individual *ex ante* (i.e., when deciding) considers the future time path of $\bar{s}$ and $\bar{K}$ as exogenous, *ex post* (i.e., at an SNE) it turns out to be equal to that of his own values s^* and K.

[26] The difference between $\varphi < 1$ and $\varphi = 1$ is that the latter assumption rules out the possibility of a strictly positive equilibrium choice of social participation for low values of social capital relative to private capital (i.e., in the lower region of figure 1). For high values, the choice of $s = 0$ is still an SNE, but not an interesting one, since the resulting dynamics are trivial. Therefore, we only examine the case in which individuals coordinate on the equilibrium with $s > 0$ in the upper region of figure 1.

[27] λ is the shadow price of K. All variables are considered at time $t \in \Re_+$. Observe that the conditions spelled in this proposition do not just hold at steady states, but rather at any point in time.

Case (a) identifies a situation in which, at a given point in time, social capital is scarce relative to private capital, so that, rather than spending time in social participation, the returns of which are low, in equilibrium it is better to choose a high labour supply, which has a high return, and to substitute a high consumption of private goods for the relational good.

On the contrary, case (b) captures a situation of relative scarcity of private capital as compared with social capital. In equilibrium, social interaction (besides subsistence consumption) is the basic source of individual well-being. On one hand, labour productivity is too low to make it worthwhile to work more in order to substitute some private consumption for the relational good; on the other hand, the social environment is rich in opportunities and makes returns on social participation high. The difference between cases (a) and (b) shows why we observe large differences in the patterns of time allocation across countries of comparable size and private capital stock: indeed, these differences may be due to the presence of different relative stocks of private and social capital.

Fixed points

Exploiting Proposition 3.4, we are now able to characterise the dynamic properties of our economy. In particular, we focus attention on the fixed points by stating the next proposition[28], where we define:

$$K^* \equiv \left(\frac{1-\epsilon}{r}\right)^{\frac{1}{\epsilon-\vartheta}}, \tag{3.11}$$

$$K_s^* \equiv 0, \tag{3.12}$$

$$K^{**} \equiv \left[\frac{\epsilon(1-\epsilon)}{r(\epsilon+a\alpha)}\right]^{\frac{1}{\epsilon-\vartheta}}, \tag{3.13}$$

$$K_s^{**} \equiv \left[\frac{a\alpha}{\delta(\epsilon+a\alpha)}\right]^{\frac{1}{1-\gamma}}. \tag{3.14}$$

Proposition 3.5. *In the plane (K, K_s), point (K^*, K_s^*) is always a fixed point of the economy. Such point is locally saddle-path stable.*

There exists at most one more fixed point, namely (K^{**}, K_s^{**}). It is a fixed point if and only if:

$$\frac{a\alpha}{\delta(\epsilon+a\alpha)} > \left(\frac{\epsilon b}{\alpha}\right)^{\frac{1-\gamma}{\gamma}} \left[\frac{\epsilon(1-\epsilon)}{r(\epsilon+a\alpha)}\right]^{\frac{(1-\gamma)(1+\vartheta-\epsilon)}{\gamma(\epsilon-\vartheta)}}. \tag{3.15}$$

If this condition is met, (K^{**}, K_s^{**}) is locally saddle-path stable.

Remark 3.2. *Straightforward calculations show that $K^{**} < K^*$.*

Remark 3.2 emphasises the fact that, when both fixed points are present, private capital is lower in the fixed point in which social capital is higher.

Remark 3.3. *For given values of the other parameters, condition (3.15) holds if δ and b are low enough and r, α and a are high enough.*

[28] For expositional purposes, we do not mention here the steady-state values of λ, that are in any case uniquely determined.

Remark 3.3 tells us that the fixed point at which social capital is positive exists when:

δ **is low:** social capital does not depreciate too fast (an intuitive condition);

r **is high:** individuals are not too patient: while impatience clearly reduces private accumulation, it may foster social capital accumulation, to the extent that this reflects the external effects of relational consumption;

α **is high:** the relational good is sufficiently a private and not too much of a public good, i.e., its enjoyment is sufficiently sensitive to one's own contribution;

a **is high:** enough weight is attributed to the needs satisfied by either the relational good or its private substitute (again, an intuitive condition);

b **is low:** the balance between the relational good and its private substitute as a means of satisfying preferences for non-subsistence goods is not excessively in favour of the private substitute[29].

It is interesting to speculate on the meaning of such parameters in terms of real world examples. One might argue, for instance, that a high degree of individual mobility is associated with a high δ, the social capital depreciation rate[30]. It is true that mobility gives rise to many 'weak' ties, which are indeed a form of social capital[31]; but they are also a form which depreciates quickly. More generally, individual mobility may make many forms of previously accumulated social capital unproductive (in relational terms). From this point of view, we might speculate that a steady state with high social capital is more likely to exist in Europe than in the US, precisely because individual mobility is lower in the former than in the latter.

Another interesting discussion concerns parameter α, which captures the relative degree to which the relational good is a private rather than a public good[32]. We may associate a high and a low α, respectively, to more active forms of social participation, in which my return crucially depends on my contribution (say, organizing an event and enjoying its success), and to more passive ones, in which my benefit mostly depends on other people's contribution (say, attending the same event as a member of the audience). Another interpretation is that a high α reflects an open context, where integration is easy and my benefits from participation (suppose I am a newcomer or an immigrant) depend to a high degree upon my own choice, whereas a low α reflects closed contexts, where integration is difficult and I may be excluded anyway, despite my efforts to participate.

Therefore, while individual mobility (both geographical and social) may increase the depreciation rate of social capital, it may also render relational goods more private

[29]To have a numerical intuition, let us parameterise the model in a simple way, so that $a = b = 1$, $\alpha = \epsilon = 0.5$, $\vartheta = 0.1$, $\gamma = 0.8$. In this case, if social capital depreciation rate δ is, for instance, 10%, then condition (3.15) is met even at a discount rate r of 1%. If we lower γ to 0.5, then, with the same $\delta = 10\%$, condition (3.15) fails to be met up to a discount rate r of 8%, whereas it is met for $r \geq 9\%$.

[30]Schiff (1999, 2002) analyses the clear-cut difference between the two traditional forms of factor mobility, namely migration and trade, which becomes apparent once we consider their different impact on social capital.

[31]Granovetter (1973) makes the point that weak ties may be economically very important, since they are often the vehicle of new information.

[32]Note that B is a pure public good if $\alpha = 0$, in which case any private incentive to social participation is absent. On the other hand, B is a pure private good if $\beta = 0$, that is, under Assumption 3.5, if $\alpha = 1$. In general terms, relational goods are an intermediate case between private and public goods.

and less public. This second effect would then probably favour the US over Europe as regards the existence of a steady-state with high social capital. However, as we show in the next section, the crucial point is not just whether this steady state exists (e.g., to follow our illustrative speculation, that it exists both in the US and in Europe), but rather whether it is more or less desirable than the other one.

Analysis of well-being

Let us now consider, when both fixed points exist, i.e., under condition (3.15), which one is Pareto-superior. Let u^* and u^{**} be the representative individual's utility in fixed points (K^*, K_s^*) and (K^{**}, K_s^{**}), respectively.

Proposition 3.6. *Assume that condition (3.15) is satisfied. Then fixed point (K^{**}, K_s^{**}) Pareto-dominates (K^*, K_s^*), i.e., $u^{**} > u^*$, if, ceteris paribus, δ is low enough and r and γ are high enough. The reverse is true if δ is high enough and r and γ are low enough.*

Proposition 3.6 tells us that the same two forces, impatience and low social capital depreciation rate, that let (K^{**}, K_s^{**}) be a fixed point, also make it Pareto-superior. Moreover, as it is natural to expect, high elasticity γ of the relational good to social capital contributes to the comparative efficiency of the fixed point with positive social capital[33].

When fixed point (K^{**}, K_s^{**}) Pareto-dominates (K^*, K_s^*) and the economy becomes stuck in the latter, it may be described as a social poverty trap[34]. The convergence to such a trap may have two basic causes: it may be due to a low initial endowment of social relative to private capital (for instance, Russia)[35]; or to the general problem posed by externalities, the presence of which may lead to inefficient private choices. If the outcome is an over-accumulation of private capital, at the expense of social capital and individual and social well-being, we may say that private growth and social development come into conflict with each other, and that it would be efficient to increase social participation and decrease labour supply, sacrificing some accumulation of private capital, but gaining in terms of an improved social environment. Of course, this remains true only if fixed point (K^{**}, K_s^{**}) Pareto-dominates (K^*, K_s^*) and the economy becomes stuck in the latter; since the former fixed point is also locally stable, the economy will converge to it if its initial endowment of social capital is high enough[36]. When convergence to fixed point (K^{**}, K_s^{**}) takes place from below along

[33] Let us consider again the simple parameterisation $a = b = 1$, $\alpha = \epsilon = 0.5$, $\vartheta = 0.1$, $\gamma = 0.8$. In this case, $u^{**} - u^* = \frac{3}{2}\ln r - 4\ln\delta - 4\ln 2$, which, for instance, is positive for $\delta = 10\%$ and $r = 3\%$, as well as for any lower social capital depreciation rate and higher discount rate. If $\delta = 5\%$, then $u^{**} > u^*$, even at a discount rate of 1%. If we lower γ to 0.5, then $u^* > u^{**}$ for any reasonable value of δ and r.

[34] The use of an infinitely lived agent model may lead to underestimating the consequences of social impoverishment. If, as stressed by Coleman (1988, 1990), social capital is relevant for children's identity formation and for the creation of human capital, the consequences are likely to be more serious than pointed out here.

[35] Rose (1998) considers in detail how the centralisation of the Soviet Union may have eroded ample forms of social capital, inducing individuals to rely on a narrow circle of family ties, which represents at the same time a response to the state of affairs and a social trap which inhibits the mechanism of social development.

[36] More precisely, if initial endowment (K^0, K_s^0) is close enough to (K^{**}, K_s^{**}). Note that even this case, although more favourable, does not solve the problem of externalities.

both dimensions, social development and economic growth move together[37].

Instead, we have seen that (K^*, K_s^*) may Pareto-dominate fixed point (K^{**}, K_s^{**}) if the 'social technology' is 'bad' (high δ and low γ), and if individuals are very patient (low r). Moreover, we have shown that, in the same conditions, (K^{**}, K_s^{**}) may even fail to be a fixed point. In the first case, (K^{**}, K_s^{**}) should be regarded as a situation in which individuals devote too much time to socially enjoyed leisure, while working and saving too little to reach a more efficient steady state. In the second case, since there is no alternative, there is no comparative discussion.

3.5 Conclusions

The present contribution sheds light on the interplay between the private and social components of well-being in a scenario in which both private and social capital are present, relational goods play a role, and their substitutability with some private goods is taken into account.

We first present a static model, in which social and private capital are constant at some exogenously given stock. This model displays two equilibria: a privately oriented one, in which labour time and private production are high and relational goods are substituted by private goods, and a socially oriented one, in which labour supply is low and social participation high, so that, besides private consumption, relational goods become a key determinant of well-being. If social capital is low relative to private capital, the privately oriented equilibrium tends to be Pareto-superior; if the reverse is true, the socially oriented equilibrium definitely becomes more efficient. Since equilibrium selection is a matter of coordination, it is possible for the economy to become stuck in the Pareto-inferior equilibrium.

The static model does not explain the determinants of the relative endowment of social and private capital. We confront this issue in the dynamic extension of the model, in which we assume that private capital is accumulated, as usual, through savings, while the accumulation of social capital is a by-product of the generation of relational goods. The most interesting case is when parameters are such that the dynamics of the system admit two fixed points: one in which there is only private capital, and another in which both forms of capital are present. We further discuss the conditions in which the latter steady-state Pareto-dominates the former and show that, in this case, both equilibria are saddle-path stable, so the system can converge to the Pareto-inferior one. In this case, we witness a conflict between economic growth and social development, since growth drives the economy into a social poverty trap. If, instead, the economy converges to the Pareto-superior fixed point, we may have economic growth and social development moving in the same direction. The distinction between these two cases once again depends upon the initial relative endowment of private and social capital, but also upon the technology of social interaction and the degree of impatience.

Some of the assumptions under which we derive our results deserve a short discussion. First, we assume, for simplicity, that neither social capital matters for the production of private goods, nor private capital for relational goods. An interesting future extension could include these cross influences[38]. Second, while we consider positive learning-by-doing externalities in private production, we do not allow them to

[37] However, recall that, because of Assumption 3.4, neither private growth nor social development may be endogenously sustained forever.

[38] See, along similar lines, Bartolini and Bonatti (2004).

be strong enough to generate endogenous growth. This is another possible extension of the model. Third, we assume that private consumption does not require time, so that all leisure time is devoted to social participation. Although unrealistic, we make this modelling choice because, generally speaking, social participation is a more time-intensive activity than private consumption[39]. Fourth, the assumption that the 'gross investment' in social capital is exactly equal to the average production of the relational good could easily be generalised (for instance, by assuming that only a fraction of the relational good produced accumulates as social capital), without changing any of the results of the model: it has simply been dictated by notational economizing. Lastly, perfect substitutability and Assumptions 3.1 and 3.5 are crucial to obtain simple analytical solutions[40]. Relaxing Assumption 3.1 to some extent would not alter the results of the static model, although it would preclude the possibility of expressing them in closed form[41]. As regards Assumption 3.5, a comparison with Antoci et al. (2007a) allows us to conjecture that its main effect is to rule out a repulsive fixed point that separates the two stable ones. Since our mathematical findings are supported by a clear economic intuition, we are quite confident in their general validity.

The model and its results may aid better understanding of a number of concrete situations. For instance, it is widely recognised that, besides more traditional economic fundamentals, social factors played important roles in the crises of both Russia and Argentina. In cases, past political history was responsible for widespread disruption of the essential structures of civil society. In terms of our model, this amounts to a sharp reduction in the stock of social capital. Indeed, both countries faced a very low ratio of social to private capital, a situation to which individuals reacted by shifting to privately oriented strategies, thus worsening the problem. While the present version of our model allows us clearly to understand these dynamics and their socially disruptive consequences, the above extension to include the effects of social capital on private productivity may help in explaining the relatively low success of the private sectors of these economies.

Another quite different situation which may be explained by our model is the case of successful professionals, who devote much of their time to working, earning high incomes, and consuming great quantities of luxury goods, but who have poor social lives and are overall dissatisfied. Casual observation tells us that this is quite a common case in advanced societies. In terms of our model, this is precisely what one would observe along the convergence path towards a social poverty trap. Moreover, when this situation is widespread, individual reactions tend to be to invest even more time in private activities, thus exacerbating the problem. Again, pursuing this application rigorously would probably require an extension of the model towards a non-homogeneous society or an asymmetric equilibrium. Our hope is that our contribution may serve as a starting point for future research.

[39] Clearly, the consumption of some private substitutes of the relational good (e.g., watching television) is also time-intensive, so that an interesting extension would be to take this into account, along the lines set by Corneo (2005).

[40] Recall that Assumption 3.1, made for the static model, implies the equality in equilibrium of the degree of concavity of the relational good and of private production as functions, respectively, of social participation and labour; Assumption 3.5, made for the dynamic model, implies a linear specification for both functions in equilibrium.

[41] More precisely, this would be the case if one just assumed $\alpha + \beta < 1$ and $\epsilon + \sigma < 1$ without requiring them to be equal.

Appendix

Proof of Proposition 3.1

Using the production function and the budget constraint to substitute for C_s, and calling $v(s, C) = u(C, B, Y - C)$, we can re-write problem (3.1)-(3.2) as:

$$\max_{s,C} \; v(s, C) = \tag{3.16}$$
$$= \ln C + a \ln\{s^{\alpha}\bar{s}^{\beta}K_s^{\gamma} + b[(1-s)^{\epsilon}(1-\bar{s})^{\sigma}K^{1+\vartheta-\epsilon} - C]\} \quad \text{s.t.}$$
$$C \geq 0, \qquad (1-s)^{\epsilon}(1-\bar{s})^{\sigma}K^{1+\vartheta-\epsilon} - C \geq 0, \qquad s \in [0,1]. \tag{3.17}$$

If $0 < C < Y$, the FOCs of this problem are:

$$\frac{\partial v}{\partial C} = 0, \tag{3.18}$$
$$\frac{\partial v}{\partial s} \leq 0, \qquad s\frac{\partial v}{\partial s} = 0, \qquad 0 \leq s \leq 1. \tag{3.19}$$

Equation (3.18) immediately yields:

$$C = \frac{1}{b(1+a)}[s^{\alpha}\bar{s}^{\beta}K_s^{\gamma} + b(1-s)^{\epsilon}(1-\bar{s})^{\sigma}K^{1+\vartheta-\epsilon}], \tag{3.20}$$

which, substituted into inequality (3.19), after rearranging, yields:

$$\frac{(1-s)^{1-\epsilon}}{s^{1-\alpha}} \leq \frac{b\epsilon(1-\bar{s})^{\sigma}K^{1+\vartheta-\epsilon}}{\alpha\bar{s}^{\beta}K_s^{\gamma}}, \text{ with equality if s} > 0, \quad 0 \leq s \leq 1. \tag{3.21}$$

When $\bar{s} = 0$, we have $B = 0$ whatever the individual choice of s. Hence, the optimal individual response to $\bar{s} = 0$ is to choose $s = 0$. The rest of the proposition follows from equation (3.20) and from the production function, which also shows that, for $s = \bar{s} = 0$, constraint $0 \leq C \leq Y$ is not binding.

Proof of Proposition 3.2

The value of $\hat{s}$ in case (a) follows from equation (3.21) after applying the SNE condition $\bar{s} = s$ and Assumption 3.1. The values of $\hat{C}$ and of $\hat{C}_s$ then follow from equation (3.20) and from the budget constraint. The definition of function h, and therefore the distinction between case (a) and case (b), follows from equation (3.21) and the production function, setting $C = Y$. When this constraint is binding, i.e., in case (b), the representative individual sets $\frac{\partial v}{\partial s} = -\frac{\partial v}{\partial C}\frac{\partial Y}{\partial s}$. The values of $\hat{s}$, $\hat{C}$, $\hat{C}_s$ in case (b) follow from this equation, conditions $C = Y$, $s = \hat{s}$, and the budget constraint.

Proof of Proposition 3.3

Let $\tilde{u}$ and $\hat{u}$ be the representative individual's utility in the two SNE $(\tilde{s}, \tilde{C}, \tilde{C}_s)$ and $(\hat{s}, \hat{C}, \hat{C}_s)$, respectively.

Consider first case (a). Using condition $K_s < h(K)$, it is easy to show that $\hat{u} - \tilde{u} > (1+a)\varphi \ln\left(\frac{\alpha}{\alpha+ab^2\epsilon}\right) + (1-a)\ln b$, and that this term is $\simeq 0.41$ for $a = \frac{1}{9}$, $b = 3$, $\alpha = \epsilon = 0.5$, $\beta = \sigma = 0.3$; therefore, for such parameters $\hat{u} > \tilde{u}$. Analogously, we show that $\hat{u} - \tilde{u} < (1+a)\ln\left[\left(\frac{\epsilon}{\alpha+ab^2\epsilon}\right)^{\varphi} a^{1-\varphi} + 1\right] + (1-a)\ln b$, where this term is $\simeq -0.21$ for $a = 0.5$, $b = 0.1$, $\alpha = \epsilon = 0.5$ and $\beta = \sigma = 0.3$; therefore, for such parameters, $\tilde{u} > \hat{u}$.

Consider now case (b). The definition of function g comes from a straightforward substitution of the equilibrium values in the utility function. Note that g is strictly increasing. A comparison between functions h and g shows that a sufficient condition for $\hat{u} > \tilde{u}$ to hold is $\frac{b\epsilon}{\alpha} > \frac{(a\alpha+\epsilon)^{\frac{1+a}{a}}}{(1+a)^{\frac{1+a}{a\varphi}}}$.

Proof of Proposition 3.4

The current Hamiltonian value for problem (3.5) under constraints (3.6)-(3.7) is

$$\begin{aligned} H &= \ln C + a\ln(s^{\alpha}\bar{s}^{\beta}K_s^{\gamma} + bC_s) + \lambda[(1-s)^{\epsilon}K^{1-\epsilon}A - C - C_s] + \\ &\quad +\mu[\bar{s}^{\alpha+\beta}K_s^{\gamma} - \delta K_s]. \end{aligned} \tag{3.22}$$

By the maximum principle, we have:

$$\dot{K} = \frac{\partial H}{\partial \lambda} = (1-s)^{\epsilon}K^{1-\epsilon}A - C - C_s, \tag{3.23}$$

$$\dot{\lambda} = r\lambda - \frac{\partial H}{\partial K} = \lambda[r - (1-\epsilon)(1-s)^{\epsilon}K^{-\epsilon}A], \tag{3.24}$$

$$\dot{K}_s = \frac{\partial H}{\partial \mu} = \bar{s}^{\alpha+\beta}K_s^{\gamma} - \delta K_s. \tag{3.25}$$

We omit the dynamics of μ, the 'shadow price' of social capital, since equations (3.23) to (3.25) are independent of it, due to the fact that the representative individual considers both $\bar{s}$ and K_s (and therefore $\dot{K}_s$) as exogenous.

The first-order conditions are:

$$\frac{\partial H}{\partial C} = \frac{1}{C} - \lambda = 0, \qquad C > 0, \tag{3.26}$$

$$\frac{\partial H}{\partial C_s} = \frac{ab}{s^{\alpha}\bar{s}^{\beta}K_s^{\gamma} + bC_s} - \lambda \leq 0, \qquad C_s\frac{\partial H}{\partial C_s} = 0, \qquad C_s \geq 0, \tag{3.27}$$

$$\begin{aligned} \frac{\partial H}{\partial s} &= \frac{a\alpha s^{\alpha-1}\bar{s}^{\beta}K_s^{\gamma}}{s^{\alpha}\bar{s}^{\beta}K_s^{\gamma} + bC_s} - \epsilon\lambda(1-s)^{\epsilon-1}K^{1-\epsilon}A \leq 0, \\ & s\frac{\partial H}{\partial s} = 0, \qquad s \in [0,1]. \end{aligned} \tag{3.28}$$

Note that s and C_s cannot both be set at zero. Thus, either condition (3.27) or condition (3.28) must hold with equality.

The transversality condition for private capital is:

$$\lim_{t\to\infty} \mathrm{e}^{-rt}\lambda(t)K(t) = 0. \tag{3.29}$$

Substituting Assumption 3.5 and equilibrium conditions $\bar{s} = s$ and $\bar{K} = K$ into equations (3.26) to (3.28), we obtain:

$$C = \frac{1}{\lambda}, \tag{3.30}$$

$$\frac{\partial H}{\partial C_s} = \frac{ab}{sK_s^{\gamma} + bC_s} - \lambda \leq 0, \qquad C_s\frac{\partial H}{\partial C_s} = 0, \qquad C_s \geq 0, \tag{3.31}$$

$$\frac{\partial H}{\partial s} = \frac{a\alpha K_s^{\gamma}}{sK_s^{\gamma} + bC_s} - \epsilon\lambda K^{1+\vartheta-\epsilon} \leq 0, \qquad s\frac{\partial H}{\partial s} = 0, \qquad s \in [0,1]. \tag{3.32}$$

Inequality $\frac{\partial H}{\partial C_s} \leq 0$ may be re-written in the form $\frac{a}{sK_s^\gamma + bC_s} - \frac{\lambda}{b} \leq 0$.

For $K_s > 0$, inequality $\frac{\partial H}{\partial s} \leq 0$ may be re-written in the form $\frac{a}{sK_s^\gamma + bC_s} - \frac{\epsilon K^{1+\vartheta-\epsilon}}{\alpha K_s^\gamma}\lambda \leq 0$.

Hence, if $\frac{\epsilon K^{1+\vartheta-\epsilon}}{\alpha K_s^\gamma} > \frac{1}{b}$, $\frac{\partial H}{\partial C_s} = 0$ and $\frac{\partial H}{\partial s} < 0$ hold, so that the representative individual's equilibrium choice is such that $C_s > 0$ and $s = 0$. If, on the contrary, $\frac{\epsilon K^{1+\vartheta-\epsilon}}{\alpha K_s^\gamma} < \frac{1}{b}$, then we have $C_s = 0$ and $s > 0$. If, lastly, $\frac{\epsilon K^{1+\vartheta-\epsilon}}{\alpha K_s^\gamma} = \frac{1}{b}$, we remain with one equation for two unknowns, and the choice of C_s and s is indeterminate. The remainder of Proposition 3.4 follows from a straightforward substitution in equations (3.31) and (3.32).

Proof of Proposition 3.5

For case (a), i.e., in condition (3.9), the equilibrium dynamics of our economy are described by:

$$\dot{K} = K^{1+\vartheta-\epsilon} - \frac{1+a}{\lambda}, \tag{3.33}$$

$$\dot{\lambda} = \lambda[r - (1-\epsilon)K^{\vartheta-\epsilon}], \tag{3.34}$$

$$\dot{K}_s = -\delta K_s. \tag{3.35}$$

For case (b), i.e., in condition (3.10), if $\frac{a\alpha}{\epsilon\lambda K^{1+\vartheta-\epsilon}} \leq 1$,[42] the equilibrium dynamics are:

$$\dot{K} = K^{1+\vartheta-\epsilon} - \left(1 + \frac{a\alpha}{\epsilon}\right)\frac{1}{\lambda}, \tag{3.36}$$

$$\dot{\lambda} = \lambda\left[r - (1-\epsilon)\left(K^{\vartheta-\epsilon} - \frac{a\alpha}{\epsilon\lambda K}\right)\right], \tag{3.37}$$

$$\dot{K}_s = K_s^\gamma\left(\frac{a\alpha}{\epsilon\lambda K^{1+\vartheta-\epsilon}} - \delta K_s^{1-\gamma}\right). \tag{3.38}$$

The analytical determination of (K^*, K_s^*) and (K^{**}, K_s^{**}) follows from a straightforward substitution in the systems (3.33) to (3.35) and (3.36) to (3.38), setting the LHS of each equation at zero. (K^*, K_s^*) satisfies the condition of case (a): $K_s^* < \left(\frac{\epsilon b}{\alpha}K^{*1+\vartheta-\epsilon}\right)^{\frac{1}{\gamma}}$, and is thus indeed a fixed point. (K^{**}, K_s^{**}) is a fixed point if and only if it satisfies the condition of case (b): $K_s^{**} > \left(\frac{\epsilon b}{\alpha}K^{**1+\vartheta-\epsilon}\right)^{\frac{1}{\gamma}}$. Equation (3.15) is simply a re-writing of this condition.

The stability properties are determined as follows. The Jacobian matrix of the system (3.33) to (3.35), evaluated at (K^*, K_s^*), is:

$$A = \begin{bmatrix} (1+\vartheta-\epsilon)K^{\vartheta-\epsilon} & \frac{1+a}{\lambda^2} & 0 \\ (1-\epsilon)(\epsilon-\vartheta)\lambda K^{\vartheta-\epsilon-1} & 0 & 0 \\ 0 & 0 & -\delta \end{bmatrix}.$$

One eigenvalue is therefore $-\delta < 0$, and the other two have opposite signs, since the determinant of the sub-matrix obtained from A by deleting the third row and the

[42] Since we are interested in the fixed points of these dynamics, we do not consider, in case (b), the possibility that $\frac{a\alpha}{\epsilon\lambda K^{1+\vartheta-\epsilon}} > 1$, since in this case $\dot{K} = -\frac{1}{\lambda}$ and there is no fixed point. Note, moreover, that this possibility is not a relevant one, since it means that individuals do not work at all, and derive their private consumption only from 'eating' their existing stock of private capital.

third column is negative. Therefore, if (K, K_s) is initially close enough to (K^*, K_s^*), there exists a single initial value of λ that puts the representative agent on the stable arm (which, in turn, has dimension 2).

Observe now that the Jacobian matrix of the system (3.36) to (3.38), evaluated at (K^{**}, K_s^{**}), is such that $\frac{\partial \dot{K}}{\partial K_s} = \frac{\partial \dot{\lambda}}{\partial K_s} = 0$ and $\frac{\partial \dot{K}_s}{\partial K_s} = -\delta(1-\gamma) < 0$. Therefore, the latter value is one of the eigenvalues of the Jacobian matrix and the other two have opposite signs, since the determinant of the sub-matrix is negative:

$$B = \begin{bmatrix} \frac{\partial \dot{K}}{\partial K} & \frac{\partial \dot{K}}{\partial \lambda} \\ \frac{\partial \dot{\lambda}}{\partial K} & \frac{\partial \dot{\lambda}}{\partial \lambda} \end{bmatrix}.$$

To see this, observe that $\frac{\partial \dot{K}}{\partial K} = (1+\vartheta-\epsilon)K^{\vartheta-\epsilon} > 0$, $\frac{\partial \dot{K}}{\partial \lambda} = \left(1+\frac{a\alpha}{\epsilon}\right)\frac{1}{\lambda^2} > 0$, $\frac{\partial \dot{\lambda}}{\partial K} = -(1-\epsilon)\lambda\left[-(\epsilon-\vartheta)K^{\vartheta-\epsilon-1} + \frac{a\alpha}{\epsilon\lambda K^2}\right]$, $\frac{\partial \dot{\lambda}}{\partial \lambda} = -\frac{a\alpha(1-\epsilon)}{\epsilon K \lambda} < 0$.

It is then easy to obtain $\operatorname{Det} B = -\frac{(1-\epsilon)(\epsilon-\vartheta)}{\lambda^2 K^2}\left(1+\frac{a\alpha}{\epsilon}\right) < 0$.

Proof of Proposition 3.6

In order to calculate u^*, observe first that, since we are in case (a), $s = 0$ and $u^* = \ln C + a \ln bC_s$. From equations (3.30) and (3.31), it follows immediately that $C = \frac{1}{\lambda}$ and $C_s = \frac{a}{\lambda}$, so that $C_s = aC$. Equations (3.33) and (3.11) then imply $C = \frac{1}{1+a}K^{*1+\vartheta-\epsilon} = \frac{1}{1+a}\left(\frac{1-\epsilon}{r}\right)^{\frac{1+\vartheta-\epsilon}{\epsilon-\vartheta}}$ and $C_s = \frac{a}{1+a}\left(\frac{1-\epsilon}{r}\right)^{\frac{1+\vartheta-\epsilon}{\epsilon-\vartheta}}$. Therefore, $u^* = \ln\frac{1}{1+a} + a\ln\frac{ab}{1+a} + (1+a)\frac{1+\vartheta-\epsilon}{\epsilon-\vartheta}\ln\frac{1-\epsilon}{r}$ is easily yielded.

Let us now calculate u^{**} in an analogous way. Since we are in case (b), $C_s = 0$ and $u^{**} = \ln C + a \ln sK_s^{\gamma}$. Remember that in the fixed point $\lambda K^{1+\vartheta-\epsilon} = 1 + \frac{a\alpha}{\epsilon}$, equations (3.13) and (3.30) yield $C = \frac{1}{\lambda} = \frac{K^{**1+\vartheta-\epsilon}}{1+\frac{a\alpha}{\epsilon}} = \frac{\left[\frac{\epsilon(1-\epsilon)}{r(\epsilon+a\alpha)}\right]^{\frac{1+\vartheta-\epsilon}{\epsilon-\vartheta}}}{1+\frac{a\alpha}{\epsilon}}$ and equation (3.32) yields $s = \frac{a\alpha}{\epsilon\lambda K^{1+\vartheta-\epsilon}} = \frac{a\alpha}{\epsilon+a\alpha}$. Since K_s is given by equation (3.14), we obtain $u^{**} = \ln\frac{\epsilon}{\epsilon+a\alpha} + \frac{1+\vartheta-\epsilon}{\epsilon-\vartheta}\ln\frac{\epsilon(1-\epsilon)}{r(\epsilon+a\alpha)} + a\ln\frac{a\alpha}{\epsilon+a\alpha} + a\frac{\gamma}{1-\gamma}\ln\frac{a\alpha}{\delta(\epsilon+a\alpha)}$.

Proposition 3.6 follows from an analysis of the following expression[43]:

$$\begin{aligned} u^* - u^{**} &= \ln\frac{\epsilon+a\alpha}{\epsilon+a\epsilon} + \frac{1+\vartheta-\epsilon}{\epsilon-\vartheta}\left[a\ln(1-\epsilon) - a\ln r + \ln\frac{\epsilon+a\alpha}{\epsilon}\right] + \\ &\quad + a\ln\frac{\epsilon+a\alpha}{\alpha+a\alpha} + a\ln b + a\frac{\gamma}{1-\gamma}\left[\ln\delta + \ln\frac{\epsilon+a\alpha}{a\alpha}\right]. \end{aligned} \tag{3.39}$$

[43] Note that the term in the last square brackets is negative if $\delta < \frac{a\alpha}{\epsilon+a\alpha}$, that $\frac{\gamma}{1-\gamma}$ increases rapidly with γ, and that the absolute value of $\ln r$ is a decreasing function of r.

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