Essays in Information and Asset Pricing

Francesco Sangiorgi
DEPARTMENT OF ECONOMICS
UNIVERSITAT POMPEU FABRA
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Supervisor: José Marin

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Foreword

This thesis is about the formation and equilibrium properties of asset prices when agents participating in financial markets are asymmetrically informed. I build on the seminal contributions of Hellwig (1980a), Grossman and Stiglitz (1980), Verrecchia (1982a), Kyle (1985) and Kyle (1989), and explore three main applications, that fall into the category of behavioral finance, markets for information and asset pricing.

Methodologically, the first two chapters share the feature that the initial allocation of information among agents in the model arises endogenously instead of being imposed exogenously. It is well known that the equilibrium properties of models with asymmetric information strongly depend on the assumed initial distribution of information. In the applications I consider, the exogenous information assumption is not without loss of generality and conclusions derived in such setting might be not robust, or even misleading, once the initial allocation is allowed to arise endogenously. In both these chapters, as is common in most of the market microstructure literature, I focus on informational efficiency issues and abstract from the pricing of risk. In the third chapter instead, the allocation of information is exogenous but the consequences of price formation and efficiency for risk premia are explicitly taken into account.

In the first chapter I analyze a model with heterogenous agents, with the purpose of embedding in a the classic rational expectations framework some of the recent findings of the behavioral finance literature. In particular, some agents are assumed to be overconfident and overestimate their ability to interpret private signals, while the rest of the market has rational expectations. Each agent can freely decide to acquire some costly information. The main finding of this analysis is an irrelevance result: the equilibrium price that arises is observationally equivalent to a market in which all agents are rational. The mechanism driving the result is the externality in the valuation of information: overconfident aggressive trading makes prices more informative, reducing the incentives for rational traders to acquire information, offsetting the overconfident initial price impact. Nevertheless, overconfidence does affect other properties of the equilibrium; for instance trading volume is higher than in a rational economy.

The second chapter extends the information sales problem of Admati and Pfleiderer

(1986) to non-competitive markets. The optimal information allocation takes on a particularly simple form: (i) sell to as many agents as possible very imprecise information; (ii) sell to a restrict group of traders signals as precise as possible. The optimality of one form or the other is driven by the tradeoff between maximizing interim profits and ex-ante risk-sharing. As noise trading per unit of risk-tolerance of the bidders becomes small, the exclusivity contracts (ii) dominates the large scale rumors in (i). Several comparative statics results get reversed once the information in the market is endogenous. The literature argues that limit-order markets are more informative than those driven by market-orders, due to the fact that traders facing smaller execution-price risk trade more aggressive on their information. Precisely due to this aggressiveness, the seller of information gives traders in limit-order markets less information, which in equilibrium actually reverses the standard result. Similarly, under imperfect competition the seller chooses to sell more precise information than under perfect competition, because investors' strategic behavior internalizes their trade impact on prices. As a consequence, in the endogenous information allocation prices are more informative in the imperfectly competitive setting than under perfect competition, in sharp contrast to the common belief.

In the third chapter I explore the implications of informational efficiency on the pricing of risk. I investigate how the degree of asymmetric information influences the risk premium under the assumption of large informed speculators trading against a risk-averse market. When large (i.e. strategic) informed traders are faced by a smaller number of uninformed, a situation which we associate to more asymmetric information, they are forced to trade less on their information not to dissipate their profits. As a result of lower endogenous volume of informed trading, prices convey less information and the risk faced by uninformed traders increases, resulting in higher risk premium. From an empirical perspective, the analysis provides an alternative reason for why an increase in the investor base should result in lower cost of capital, on top of risk sharing considerations, providing a rationale for differentials in risk premia that are only related to market microstructure issues.

Chapter 1
Overconfidence and market efficiency with heterogeneous agents

joint with Diego García and Branko Urošević
We study financial markets in which both rational and overconfident agents coexist and make endogenous information acquisition decisions. We demonstrate the following

irrelevance result: when a positive fraction of rational agents (endogenously) decides to become informed in equilibrium, prices are set as if *all* investors were rational, and as a consequence the overconfidence bias does *not* affect informational efficiency, price volatility, rational traders' expected profits or their welfare. Intuitively, as overconfidence goes up, so does price informativeness, which makes rational agents cut their information acquisition activities, effectively undoing the standard effect of more aggressive trading by the overconfident. The main intuition of the paper, if not the irrelevance result, is shown to be robust to different model specifications.

Chapter 2
Information sales and strategic trading

joint with Diego García
We study information sales in financial markets with strategic risk-averse traders. Our main result establishes that the optimal selling mechanism is one of the following two: (i) sell to as many agents as possible very imprecise information; (ii) sell to a single agent a signal as precise as possible. As noise trading per unit of risk-tolerance becomes large, the "newsletters" or "rumors" associated with (i) dominate the "exclusivity" contract in (ii). The optimal information sales contracts share similar qualitative behavior in models where agents can submit market-orders as well as more general limit-orders. On the other hand, models with imperfect competition and those where competitive behavior is assumed yield qualitatively different equilibria. The endogeneity of the information allocation creates new comparative statics across markets and models: the model with market-order has equilibrium prices that are more informative than with limit-orders, and the model with imperfect competition yields more informative prices than its competitive counterpart. These results are driven by the seller of information offering more precise signals when agents submit market-orders and when they act strategically.

Chapter 3
Information and Expected Returns with Large Informed Traders

This paper investigates the relationship between asymmetric information and the required return under the assumption of large informed speculators trading against a risk-averse market. When informed traders are faced by a smaller number of uninformed

ones, a situation which we associate with more asymmetric information, they have an incentive to restrict the quantity they trade because the price impact of their trades is high. As a result of lower endogenous informed trading volume, prices convey less information and the risk faced by uninformed traders increases, which results in a higher risk premium

Chapter 1

Overconfidence and market efficiency
with heterogeneous agents

1.1   Introduction

Bounded rationality of economic agents participating in financial markets has been a subject of intense scrutiny in the last decade (see, for example, Thaler (1992), Thaler (1993), and Shleifer (2000)). One such well-documented behavioral pattern is investor overconfidence.[1] Our paper contributes to the emerging literature on the effects of behavioral biases in financial markets by studying the reaction of rational agents to the degree of overconfidence of a set of irrational traders. To the best of our knowledge, this is the first paper that simultaneously adopts two important features of real financial markets: 1) coexistence of rational and overconfident traders, and 2) endogenous information acquisition by agents.[2] In particular, we extend the existing literature by analyzing the impact that the presence of heterogenous (i.e. rational and overconfident) traders has on informational efficiency of prices, willingness of agents to acquire information, market liquidity, and performance and welfare of rational (and overconfident) agents.

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[1] For an excellent review on psychological literature on overconfidence see Odean (1998) and references therein. For empirical evidence on overconfidence in financial markets see Barber and Odean (2001), Glaser and Weber (2003), and Statman, Thorley, and Vorkink (2003), among many others.

[2] DeLong, Shleifer, Summers, and Waldmann (1990), DeLong, Shleifer, Summers, and Waldmann (1991), Shleifer and Vishny (1997), and Bernardo and Welch (2001), among others, demonstrate that irrational traders may have long-term viability and can coexist with rational traders. For an opposite result, where behavioral agents are driven out of the market, see Sandroni (2005).

Most of the existing models with overconfidence assume exogenous distribution of information among the economic agents. Such simplification is not innocuous: since traders' overconfidence impacts the market precisely through the incorrect interpretation of their private signals on the fundamental value of the traded asset, the effects of overconfidence in the economy may crucially depend on the distribution of information among the agents. It seems natural, therefore, not to specify a priori the information that different agents possess, but to instead allow it to arise endogenously. We first show that overconfidence will reduce rational agents' incentives to gather information within the standard competitive rational expectations paradigm (Hellwig, 1980b). In this setup we show that a simple condition on the primitives of the model exists under which overconfidence has no price impact, and as a consequence has no impact on informational efficiency, price volatility, as well as welfare and expected profits of rational agents. None of these properties are affected by the presence of overconfident traders (and coincide with the values in the purely rational economy) if the degree of overconfidence in the economy is below a certain threshold.

To gain intuition for this result we first recall that overconfident traders, by overestimating the precision of their signal, trade more aggressively on their private signals than rational traders. In doing so, more information is revealed by the price. Rational agents react to such anticipated behavior of the overconfident by scaling down their own demand for information, aiming to neutralize the negative externality imposed by overconfidence on the rational agents' expected profits and welfare. This "reaction" can be observed only when rational traders are free to decide whether or not to become informed. Thus, endogeneity of information acquisition is crucial for this result to hold.

Nevertheless, investors heterogeneity does influence other properties of the equilibrium. The presence of overconfidence leads to a *decrease* in the overall informed population as opposed to an increase (as argued elsewhere in the literature). Moreover, overconfident traders earn higher expected profits than rational traders but achieve a worse risk return trade-off, providing a new testable implication. Finally, an economy with overconfident agents will always exhibit a higher trading volume than if all agents were rational, a result well established theoretically as well as empirically (see Barber and Odean, 2001, for example).

Within the class of competitive models, the irrelevance result for informational efficiency is shown to be robust to different assumptions regarding the information gathering technology: when agents can choose the precision of the signal they purchase (as in Verrecchia, 1982a), and when the error term in the private signal is perfectly correlated among agents (as in Grossman and Stiglitz, 1980). We further show that the main intuition from the paper, that rational agents will cut down information acquisition activities the more overconfident agents there are in the market, is robust to the competitive as-

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sumption. In particular, we extend the Kyle (1985) framework to accommodate for rational and overconfident agents. Within this framework, but with exogenous information structure, Odean (1998) and Benos (1998) show that overconfidence increases price informativeness and liquidity. We show that if information acquisition activities are endogenous this may no longer be the case - a result with a similar flavor to the irrelevance proposition discussed above.[3] Our analysis therefore suggests that the effects of overconfidence are more subtle than what the literature portraits.

Several recent theoretical studies focus on the effects of overconfidence on key features of financial markets, as well as on the performance of overconfident traders.[4] Kyle and Wang (1997), Odean (1998) and Benos (1998) consider models with informed insiders and noise traders submitting market orders and find that overconfidence leads to an increase in trading volume, market depth and price informativeness. Both Kyle and Wang (1997) and Benos (1998) allow for rational agents in their models, but information acquisition decisions are fixed in both models.[5] Odean (1998), heuristically, argues that the introduction of rational traders to his model "would mitigate but not eliminate the effects of overconfident traders"(see Odean, 1998, Model I). Rubinstein (2001) summarizes the effects of overconfidence by stating that "[overconfidence] does create a positive externality for passive investors who now find that prices embed more information and markets are deeper than they should be." We show that precisely due to this externality, rational agents will reduce their information gathering activities, and that, indeed, this can eliminate the standard positive effect of overconfidence on price informativeness.

The paper is organized as follows. Section 1.2 presents a competitive model with endogenous information acquisition. The irrelevance result is developed in detail in section 1.3. Section 1.4 considers various extensions, where we argue that the results discussed in the paper are robust to the types of financial market model we consider in the main body of the paper. Section 1.5 concludes. Proofs are relegated to the Appendix.

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[3]In non-competitive models it is virtually impossible to get the irrelevance result that we uncover in the competitive framework due to the discreteness of strategic models.

[4]See Caballé and Sàkovics (2003), Daniel, Hirshleifer, and Subrahmanyam (1998), Daniel, Hirshleifer, and Subrahmanyam (2001), and Scheinkman and Xiong (2003) for some recent work.

[5]In Model III, Odean (1998) allows traders can decide to purchase a single piece of costly information. The author finds that in an economy with *only* overconfident traders, a greater degree of overconfidence leads to a larger fraction of traders that would decide to become informed in equilibrium. In contrast to our paper, Odean (1998) does not model rational traders.

1.2 The model

The basic model in this paper extends the standard one period rational expectations model with endogenous information acquisition (see Hellwig (1980b) and Verrecchia (1982a)) to the setting in which overconfident (irrational) economic agents coexist with rational ones. In particular, we assume that a measure $m_o \in (0,1)$ of the trader population is of the type o (overconfident), while the measure $m_r = 1 - m_o$ is of the type r (rational). All traders in the economy have CARA preferences with risk aversion parameter τ, i.e. their utility function, defined over the terminal wealth, is $u(W_i) = -e^{-\tau W_i}$. There are two assets in the economy: a riskless asset (the numeraire) in perfectly elastic supply (its gross return is, without loss of generality, normalized to 1), and a risky asset with payoff X and random supply Z. Without loss of generality we normalize initial wealth to zero. Letting θ_i denote the number of units of the risky asset bought by agent i, and letting P_x denote its price, we have that the final wealth of a trader i is given by $W_i = \theta_i(X - P_x)$.

Each trader can decide to purchase a noisy signal about the payoff of the risky asset, which we will denote by $Y_i = X + \epsilon_i$, at a cost $c > 0$. Therefore, the information set of uninformed trader i, which we denote by $\mathcal{F}_i$, consists of the risky asset price P_x, while for the informed the information set contains, also, the signal. Formally, we will denote an informed agent's information set by $\mathcal{F}_I$ (the σ-algebra generated by (Y_i, P_x)) and an uninformed agent's information set by $\mathcal{F}_U$ (the σ-algebra corresponding to the risky asset price P_x). All random variables X, Z and ϵ_i are independent Gaussian random variables, defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, with zero mean and variances equal, respectively, to σ_x^2, σ_z^2 and σ_ϵ^2. We further normalize the payoff of the risky asset X so that $\sigma_x^2 = 1$.

In the basic setup, the only difference between the two types of traders is that type o incorrectly believe that the variance of the signal σ_ϵ^2 is equal to $b_\epsilon^{-1}\sigma_\epsilon^2$, where $b_\epsilon > 1$. Thus, traders of type o overestimate the precision of the signal, and higher values of b_ϵ are associated with higher degrees of overconfidence. In contrast, traders of type r correctly estimate the precision of the signal (for such traders $b_\epsilon = 1$). Type $j = o, r$ expectations are denoted as $\mathbb{E}^j$. Here, agents of type r compute the expectations vis-a-vis the true measure (we denote $\mathbb{E}^r$ as $\mathbb{E}$ for brevity), while the agents of the type o, those with a behavioral bias, compute their expectations, denoted by $\mathbb{E}^o$, using the probability measure that underestimates the variance of the signal (i.e. that uses $b_\epsilon^{-1}\sigma_\epsilon^2$ instead of σ_ϵ^2).[6]

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[6]We treat the overconfidence bias of agents as exogenous. In principle, if the overconfident could participate in multiple trading rounds they could update their estimate of the precision of the signal by observing past performance. In this case rational learning could eliminate their bias. See Hirshleifer and Luo (2001) for a discussion of this point; Daniel, Hirshleifer, and Subrahmanyam (1998) and Gervais

Every trader in the economy is a price-taker and knows the structure of the market. In particular, each type $j = o, r$ knows that the other type has different beliefs about the precision of the signal.[7] The timing in the model is as follows. For each type $j = o, r$, a fraction λ_j of the respective population decides to acquire a signal. Once that decision is made, each trader submits the demand schedule for the risky asset conditional on her information set ($\mathcal{F}_I$ or $\mathcal{F}_U$). The price is set to clear the market. Finally, the fundamental value of the risky asset is revealed and the endowments consumed.

The next definition is standard.

Definition 1. *An equilibrium in the economy is defined by a set of trading strategies θ_i and a price function $P_x : \Omega \to \mathbb{R}$ such that:*

1. *Each agent i of type j chooses her trading strategy so as to maximize her expected utility given her information set $\mathcal{F}_i$:*

$$\theta_i \in \arg\max_{\theta} \mathbb{E}^j \left[u\left(W_i\right) | \mathcal{F}_i \right]. \tag{1.1}$$

2. *The market clears:*

$$m_o \Theta_o + m_r \Theta_r = Z; \tag{1.2}$$

where $\Theta_j = \frac{1}{m_j} \int_0^{m_j} \theta_i di$ is the per capita (average) trade by the type j agents $(j = o, r)$.

The setup thus far closely parallels Diamond (1985), which is a special variation of the model discussed in Verrecchia (1982a).[8] For expositional simplicity we introduce two basic assumptions regarding the information technology.

Definition 2. *We call an information technology non-trivial if $C(\tau)^{-1} b_\epsilon > \sigma_\epsilon^2$, where $C(\tau) \equiv e^{2c\tau} - 1$.*

Definition 3. *We say that the information technology satisfies the no free lunch condition if $\Lambda^* \leq 1$, where*

$$\Lambda^* = \frac{1}{m_r} \left(\tau \sigma_\epsilon \sigma_z \sqrt{C(\tau)^{-1} - \sigma_\epsilon^2} - m_o b_\epsilon \right). \tag{1.3}$$

and Odean (2001) for models in which agents learn about their own abilities; and Zábonίc (2004) for a rational model in which a bias in self-assessment arises endogenously.

[7]In equilibrium, traders properly deduce the fraction of the population of each trader type that, in equilibrium, becomes informed. This is consistent with the bulk of the literature in rational expectations models (see Squintani, 2006, and the references therein).

[8]The main difference from those models is that we relax their assumption that there are only rational agents in the economy. In section 1.4.1 we further argue that the reduced-form model of Diamond (1985) is isomorphic to the model of Verrecchia (1982a) for an open set of the model's primitives.

Definition 2 requires that the information technology has a sufficiently high price-to-quality ratio so that some traders find it optimal to invest in information acquisition activities. If the condition did not hold no agent would ever become informed in equilibrium. Definition 3 plays the opposite role. In particular, when $\Lambda^* \geq 1$ the equilibrium at the information acquisition stage will be such that all agents, rational and overconfident, find it optimal to acquire information. The label "free-lunch" comes from a slightly different interpretation of the source of information. In particular, consider a model where a seller of information charges some price c for the signal (see Admati and Pfleiderer, 1986). From the definition of the equilibrium in the next section it will become clear that such seller of information will never choose c that would violate $\Lambda^* \leq 1$.[9] The variable Λ^* will play a crucial role in the discussion that follows. In essence, the equilibrium in the model will depend crucially on whether the constant Λ^* is positive or not. We further discuss the role of these assumptions on the model's primitives in the next section.

1.3   Equilibrium prices

This section solves for the competitive equilibrium with information acquisition, and derives main results of the paper including the irrelevance result. Throughtout this section, we assume that the information technology is non-trivial and does not allow free lunch.

1.3.1   The competitive equilibrium with information acquisition

As is customary in models with endogenous information acquisition, the model is solved in two stages: we first determine the equilibrium asset price function by taking λ_j as exogenously fixed; then we go back to the information acquisition stage and find the equilibrium values for λ_j, thus completing the specification of equilibrium.

Lemma 1. *For given values of $\lambda_j \geq 0$, the competitive equilibrium price P_x is given by the expression $P_x = \hat{a}X - \hat{d}Z$, where the coefficients $\hat{a}$ and $\hat{d}$ satisfy:*

$$\frac{\hat{a}}{\hat{d}} \equiv \gamma = \frac{1}{\tau \sigma_\epsilon^2} \left(\lambda_o m_o b_\epsilon + \lambda_r m_r \right); \tag{1.4}$$

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[9]Indeed, it can be seen that charging c such that $\Lambda^* > 1$ would be strictly dominated by charging $\hat{c}$ such that $\Lambda^* = 1$. Thus, such seller of information would be "leaving money on the table," and Definition 3 rues out this case.

$$\hat{d} = \frac{1 + \dfrac{\gamma}{\tau \sigma_z^2}}{\gamma + \dfrac{\gamma^2}{\tau \sigma_z^2} + \dfrac{1}{\tau}}. \tag{1.5}$$

The informational content of price, or simply market efficiency, is measured by the conditional variance of the fundamental asset value given the market price. From Lemma 1 it follows that this quantity is given by:

$$\mathrm{var}\left(X|P_x\right) = \left(1 + \frac{\gamma^2}{\sigma_z^2}\right)^{-1}. \tag{1.6}$$

The smaller the conditional variance (1.6), the more information is revealed by the price in equilibrium. Since the information revealed by the price monotonically increases in γ, comparative statics of γ encapsulate everything we need to know about the dependence of (1.6) on the parameters measuring the overconfidence in the economy. When λ_j are exogenously fixed we obtain

$$\frac{d\gamma}{db_\epsilon} = \frac{m_o \lambda_o}{\tau \sigma_\epsilon^2} \geq 0. \tag{1.7}$$

From (1.7) it follows that, when λ_o is exogenous and positive, an increase in the intensity of overconfidence b_ϵ raises the amount of information revealed by the price. The intuition for this result is the same as in Odean (1998), namely, the more overconfident traders are, the more aggressively they trade on their information, which makes the price more informative.

The next Lemma characterizes the equilibrium with endogenous information acquisition.

Lemma 2. *The equilibrium with information acquisition belongs to one of the following two classes:*

(a) *If the parameters of the model are such that $\Lambda^* > 0$, a fraction (possibly all) of the rational agents and all overconfident agents become informed: in equilibrium $\lambda_o^* = 1$ and $\lambda_r^* = \Lambda^*$.*

(b) *If the parameters of the model are such that $\Lambda^* \leq 0$, a fraction (possibly all) of the overconfident traders becomes informed and no rational trader becomes informed: in equilibrium $\lambda_r^* = 0$ and (in the interior solution)*

$$\lambda_o^* = \frac{\tau \sigma_z}{m_o} \sqrt{k_\epsilon \sigma_\epsilon^2 \left(C(\tau)^{-1} - k_\epsilon \sigma_\epsilon^2\right)} \tag{1.8}$$

Lemma 2 shows that depending on the values of the primitives that characterize the economy, different types of equilibria may endogenously arise: traders who decide to acquire the signal and become informed can be either only a fraction of overconfident traders, all overconfident but no rational traders, all overconfident and a fraction of rational traders, or all traders in the economy. The relevant property of the equilibrium is that rational traders become informed only if all overconfident traders are informed.[10] This result is intuitive since overconfident overestimate the precision of the signal, and therefore it cannot be that some rational trader decides to become informed and an overconfident does not.[11]

Fixing other parameter values, region $\Lambda^* > 0$ arises when: (i) degree of overconfidence $m_o b_\epsilon$ is sufficiently small; (ii) information acquisition costs c are sufficiently low and/or the variability of the aggregate supply shock σ_z is large; (iii) values of the risk-aversion τ and signal precision σ_ϵ^2 are intermediate. The first two conditions are rather intuitive: if there are many overconfident agents, or their bias is too high, they will crowd out the rational agents, and we are back to the setting where the overconfident are the marginal buyers of information. If the cost is low or the noise large, traders find information acquisition activities more attractive, eventually making the rational traders (marginal) buyers of information. The third result comes from the dual role that those two parameters, risk-aversion and signal precision, play in this type of competitive models. On one hand they affect the value of becoming informed: more risk-tolerant agents are willing to pay more for a signal, and more precise signals are more valuable to agents. At the same time these parameter values affect the information revealed by prices: more risk-tolerant agents, or agents with more precise signals, trade more aggressively thereby exacerbating the negative externality of their trades. It can be shown that this second effect dominates for small values of τ and σ_ϵ^2, which pushes down the fraction of informed agents towards zero. At the same time, as both τ and σ_ϵ^2 grow without bound agents eventually have no incentives to buy information, and again we do not satisfy the $\Lambda^* > 0$ condition.

1.3.2 Irrelevance result and comparative statics

In the following Proposition we state the main irrelevance result on overconfidence.

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[10]The fact that the overconfident will always buy the signal before the rational agents do is independent of the strong parametric assumptions of this paper. It follows from Blackwell's theorem on comparisons of information structures that overconfident agents will assign a higher value to a given signal. We thank an anonymous referee from highlighting this.

[11]In the existing literature with overconfidence and asymmetric information, it is typically argued that those traders that do not buy the information are those that value it properly (see, for instance, Odean (1998), page 1907 and Daniel, Hirshleifer, and Subrahmanyam (2001), page 928). Lemma 2 formalizes this argument in the class of models we study.

Proposition 1. *If $\Lambda^* > 0$ then overconfidence is irrelevant for the parameters of the equilibrium price function, and as a consequence for informational efficiency, price volatility and rational traders expected profits and welfare. These quantities are equal to those that would endogenously arise in a fully rational economy, i.e. the equilibrium is independent of the overconfidence parameters b_ϵ and m_o.*

We can interpret $\Lambda^* = 0$ as an irrelevance threshold and think of this result in the following way. Compare two economies characterized by a common set of primitives (variances and risk aversion): one in which $m_o = 0$ (fully rational economy) and one in which $m_o > 0$, i.e., in which a positive measure of overconfident traders interacts with rational traders. The above Proposition states that as long as the degree of overconfidence in the economy, as measured by $m_o b_\epsilon$, is not too large[12] the two economies will have identical asset prices. While previous studies argue that overconfidence is costly to society, (see, for instance, Odean, 1998), Proposition 8 gives the conditions under which the process of competitive trading itself is a mechanism able to prevent overconfidence from affecting the informational efficiency of the price, *and* the welfare and profits of the rational traders. In this case overconfidence can be costly only to the overconfident.

This result obtains because of the reaction on the part of rational traders to the presence of overconfidence. From the equilibrium equation for γ in (2.88), we have that for $\Lambda^* > 0$

$$\frac{d\gamma}{db_\epsilon} = \frac{1}{\tau \sigma_\epsilon^2} \left(m_o + m_r \frac{d\lambda_r^*}{db_\epsilon} \right). \tag{1.9}$$

The first term, $m_o / \tau \sigma_\epsilon^2$, is the standard term stemming from more aggressive trading by the overconfident agents as b_ϵ increases. The second term, which measures the (negative) reaction of the rational population to the increase in overconfidence, is what drives the irrelevance result. A simple inspection of (1.3), and noting that $\lambda_r^* = \Lambda^*$, yields that γ is indeed independent of the overconfidence parameter b_ϵ.[13] In turn, this implies that the parameters of the equilibrium price function (see equations (2.88) and (1.5)) do not depend on overconfidence parameters and are given by the same quantities as in the fully rational economy. As a consequence, the same is true for the unconditional variance, expected utilities and the expected profits of the rational traders.

To gain some intuition on why the reaction of rational traders *exactly* offsets overconfidence, notice that when $\Lambda^* > 0$, the rational traders are the marginal buyers of information, and the equilibrium fraction of informed rational traders (λ_r^*) is set to equate informed and uninformed expected utilities. In the Appendix it is shown that

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[12] Note that the condition $\Lambda^* > 0$ is equivalent to requiring $m_o b_\epsilon$ to be below the threshold value $\tau \sigma_\epsilon \sigma_z \sqrt{C(\tau)^{-1} - \sigma_\epsilon^2}$.

[13] Similarly, differentiating (1.3) with respect to m_o one can see that γ does not depend on m_o either.

this condition is equivalent to

$$e^{-2\tau c}\text{var}\left(X|P_x,Y_i\right)^{-1} = \text{var}\left(X|P_x\right)^{-1}; \qquad (1.10)$$

where the two conditional variances only depend on the amount of noise of the economy σ_z^2, the precision of agents' signals σ_ϵ, and the equilibrium parameter γ. When the rational agents are the marginal buyers of information (1.10) needs to hold as an equality, and therefore it must be that $d\gamma/db_\epsilon = d\gamma/dm_o = 0$, which in turn implies the reaction in λ_r^* described above. The presence of overconfidence is perceived by rational traders as an "exogenous" effect on price informativeness, which in turn affects the relative expected utility of informed versus uninformed. Since in equilibrium expected utilities must be equal, and the overconfidence parameters (m_o, b_ϵ) enter into (1.10) only *indirectly* via γ, the equilibrium condition on information acquisition requires λ_r^* to adjust in such a way that the net effect on γ is identically zero.[14] In contrast, a marginal change in one of the other "fundamental" primitives of the model $(\sigma_z^2, \sigma_\epsilon^2, \tau, c)$, does imply an adjustment in λ_r^* to equate expected utilities, but because these parameters enter *directly* into (1.10), this adjustment will affect the equilibrium price coefficients.

On the other hand, as long as $\Lambda^* > 0$ is satisfied, the two economies (the fully rational and the one with overconfidence) will exhibit some interesting differences, described in the next Proposition.

Proposition 2. *If $\Lambda^* > 0$ then: (i) the measure of informed traders is lower that what would be observed in a fully rational economy; (ii) overconfident traders earn higher expected profits than rational traders, although the Sharpe ratios of their portfolios are lower; and (iii) expected trading volume is increasing in parameters of overconfidence.*

We will discuss these three results in order. Result (i) is surprising. In fact, it goes in the opposite direction of what previous literature finds: Odean (1998), for example, considers a model where overconfident traders can decide to acquire a single piece of information, and finds that too many of them are willing to buy it. We find that the measure of informed traders, both rational and overconfident, is lower than in the corresponding rational economy. This is rather intuitive: when m_o or b_ϵ increases, γ remains constant, but since the overconfident reveal more of their signal than rational traders, now a smaller measure of informed is sufficient to sustain a given level of γ.

Result (ii) follows by noting that the overconfident take higher risks (without real-

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[14] For the same reason, the same result can be generated in an economy with agents with two different risk-aversion parameters, say $\bar\tau > \underline\tau$. If the high risk-aversion agents are the marginal buyers of information, then changes in the risk-aversion parameter $\underline\tau$ will not affect price informativeness. Therefore, these results can be viewed as a precise statements under which the weak inequalities in Verrecchia (1982a), in terms of the effects of risk-aversion on price informativeness, hold as equalities.

izing it) by trading more aggressively on their information, which in turn yields higher expected profits.[15] Differently from an agent who is simply less risk averse, the overconfident incorrectly weights the market price in his trading strategy, which yields a portfolio with higher volatility and a lower Sharpe ratio (with respect to a rational agent). The result that overconfident achieve a worse risk return trade-off provides a new testable implication, and is in contrast to models in which the overconfident are better off, using the true probability measure, than the rational agents.[16]

Result (iii) confirms the robustness of previous findings on the effect of overconfidence on trading volume. Namely, an increase in the degree of overconfidence $m_o b_\epsilon$ enhances expected trading volume. On one hand the trading volume of the overconfident goes up, due to their higher responsiveness to their information. The rational agents, as a group, trade less as overconfidence rises: even though the trading strategies of informed and uninformed rational agents are unchanged, the fraction of informed rational agents is decreasing in overconfidence, and thereby total trading volume for the rational agents is reduced. The proposition shows that the effect on the overconfident dominates the later effect, and trading volume is indeed increasing in $m_o b_\epsilon$. Our conclusions are consistent with the bulk of the empirical evidence on trading volume and overconfidence, while at the same time showing that some properties of asset prices may actually be independent of overconfidence.

Above the irrelevance threshold,[17] only a fraction of overconfident and no rational traders become informed in equilibrium. Going back to the expression for γ, which measures price informativeness, we see that in that case:

$$\frac{d\gamma^*}{db_\epsilon} = \frac{1}{\tau\sigma_\epsilon^2}\left(m_o\frac{d\left(\lambda_o^* b_\epsilon\right)}{db_\epsilon}\right). \tag{1.11}$$

Now there are two effects that influence γ, the direct effect through higher information

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[15]The result in the Proposition refers to the comparison between overconfident and rational *informed* traders. Rational uninformed trade on less precise information, and achieve lower expected profits but the same expected utility of their informed colleagues. This makes the comparison between informed and uninformed expected profits of risk averse agents uninteresting.

[16]See Kyle and Wang (1997) and Dubra (2004) for some examples from the literature, as well as the discussion in section 1.4.3.

Hirshleifer and Luo (2001) propose an evolutionary model in which the replication of rational and overconfident is assumed to be increasing in the profitability (expected profits) of their strategies. According to this evolutionary mechanism, overconfident always survive in the long run. In their model traders are risk averse and assumed to be all informed. But when some traders find it optimal not to become informed, the comparison of expected profits might not be the appropriate measure of performance (risk matters for expected utility). Hence, the result that overconfident earn higher expected profits but lower Sharpe ratios could provide a new (negative) argument for the evolutionary selection of overconfident traders in financial markets.

[17]That is, when $m_o b_\epsilon \geq \tau\sigma_\epsilon\sigma_z\sqrt{C(\tau)^{-1} - \sigma_\epsilon^2}$.

revelation by the informed (overconfident) agents, plus the change in the fraction of informed agents. It can be easily verified from (1.8) that the product $\lambda_o^* b_\epsilon$ is increasing in b_ϵ, therefore increasing information revelation.[18] A higher value of γ in turn implies that the impact of noise on the equilibrium price is reduced, and so are noise traders expected losses (and therefore other traders' expected profits and welfare). This illustrates the fact that in order to capture the effects that we described in Propositions 8 and 7 it is necessary to consider a model with heterogeneous agents, where rational agents coexist together with overconfident traders.

1.4 Extensions

In this section of the paper we consider several models in which we illustrate the robustness of the previous results. We study more general information acquisition technologies, a version of the Grossman and Stiglitz (1980) model, and an imperfectly competitive market (as in Kyle (1985)). We argue that the main results of the previous section, in particular the fact that price informativeness is unaffected by overconfidence, is robust across these three rational expectations models.

1.4.1 General information acquisition technologies

Consider now the following variation of the basic model. Agents can obtain signals of the type $Y_i = X + \epsilon_i$, with $\epsilon_i \sim \mathcal{N}(0, 1/p)$. In order to obtain such signals traders need to pay the price, in units of the numeraire, equal to $c(p)$. We assume that $c(p) \geqslant 0$, $c'(p) > 0$ and $c''(p) \geq 0$, $\forall\ p > 0$. Thus, the cost of their signal is increasing and convex in its precision. In this way we extend the basic model to allow for more general information gathering technologies. The overconfident, as before, erroneously believe to receive signals, after paying the cost $c(p_o)$, with precision $b_\epsilon p_o$ for some $b_\epsilon > 1$.

The competitive equilibrium in this variation of the model is defined as in section 1.2. The equilibrium in information acquisition is characterized by fractions of informed agents λ_r^* and λ_o^*, and precision levels p_r^* and p_o^*, such that: (1) no uninformed agent would want to become informed; (2) no informed agent would be better off by choosing other precision levels $p \neq p^*$, or by becoming uninformed.[19] The equilibrium in information acquisition follows Verrecchia (1982a), with the additional considerations that may

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[18]It should be noted that in general λ_o^* may not be increasing in b_ϵ. For large values of b_ϵ the negative externality imposed by the informed on price informativeness may actually make λ_o^* decreasing in b_ϵ. See the discussion on non-monotonicity relationships in this type of REE models following Lemma 2.

[19]Note that since in principle we do not exclude the case $c(0) > 0$ we must allow for this possibility separately in the analysis.

arise if $\lambda_r^* \neq 1$.[20]

For the purpose of characterizing the equilibrium, define the following function of the primitives:

$$\Lambda_{GI}^* = -\frac{m_o b_\epsilon p_o^*}{m_r p_r^*} + \frac{1}{m_r p_r^*} \sqrt{\frac{\tau \sigma_z^2 \left(e^{-2c(p_r^*)\tau} - 2\tau c'(p_r^*)\right)}{c'(p_r^*)}}; \quad (1.12)$$

where p_o^* and p_r^* are defined in the Appendix. The next Proposition describes the equilibrium in such economy.

Proposition 3. *When traders can choose a signal of arbitrary precision, then the fraction of rational informed traders is given by: a) $\lambda_r^* = \Lambda_{GI}^*$ if $\Lambda_{GI}^* \in (0,1)$; b) $\lambda_r^* = 1$ if $\Lambda_{GI}^* \geq 1$; c) $\lambda_r^* = 0$ if $\Lambda_{GI}^* \leq 0$. The irrelevance result in Proposition 8 holds if $\Lambda_{GI}^* \in (0,1)$.*

If the parameters of the model are such that $\Lambda_{GI}^* \in (0,1)$, then an interior fraction of rational agents becomes informed. The interpretation of Λ_{GI}^* as an irrelevance threshold is similar to the basic model: for Λ_{GI}^* to be positive it must be that

$$m_o b_\epsilon < \frac{1}{p_o^*} \sqrt{\tau \sigma_z^2 \left(e^{-2c(p_r^*)\tau} - 2\tau c'(p_r^*)\right) c'(p_r^*)^{-1}}, \quad (1.13)$$

where the left-hand side of the above expression can be interpreted as the degree of overconfidence, and the term on the right as some threshold level. The intuition of the irrelevance result goes back to the usual expression for the relative price coefficients γ, which in this case takes on the form

$$\gamma = \frac{m_o b_\epsilon p_o^*}{\tau} + \frac{m_r \lambda_r^* p_r^*}{\tau}. \quad (1.14)$$

so that the impact of overconfidence is given by

$$\frac{d\gamma}{db_\epsilon} = \frac{m_o p_o^*}{\tau} + \frac{m_o b_\epsilon}{\tau} \frac{dp_o^*}{db_\epsilon} + \frac{m_r p_r^*}{\tau} \frac{d\lambda_r^*}{db_\epsilon} + \frac{m_r \lambda_r^*}{\tau} \frac{dp_r^*}{db_\epsilon}. \quad (1.15)$$

The impact of overconfidence on price revelation is driven by the standard first two terms (more aggressive trading by the overconfident plus more information acquisition on their part), plus the two other terms which measure the response by rational agents to the higher levels of overconfidence. In the Appendix we show that when $\lambda_r^* \in (0,1)$,

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[20] The assumptions in Verrecchia (1982a) imply that equation (1.39) in the Appendix never binds. In our symmetric model this means that either all agents become informed, or none does, as we show in the proof.

then rational traders react by scaling down the demand for information via the second term (response in the equilibrium fraction of informed traders) in a way that offsets the first two terms given by the increase of overconfidence, and the fourth term (response in the equilibrium precision) is equal to zero. On the other hand, if $\lambda_r^* = 1$, then the third term is equal to zero and the offsetting effect comes from the fourth term, i.e. $dp_r^*/db_\epsilon < 0$, but is smaller in magnitude than the positive effect resulting from more aggressive trading by the uninformed, and therefore overconfidence will increase price informativeness.

1.4.2 Correlated signals

To inspect the robustness of our main result on overconfidence and informational efficiency, we further consider the case in which every informed agent gets a signal $Y_i = X + \epsilon_i$ with $\epsilon_i = \epsilon$, $\forall i$, i.e. a competitive economy where agents get signals whose errors are perfectly correlated. All other assumptions regarding the structure of the market are unchanged with respect to section 1.2. This variation of the model is a direct extension of the model of Grossman and Stiglitz (1980), and allows us to argue that independence of the signals does not drive any of the results derive thus far.[21]

Prices are conjectured to be of the form $P_x = \hat{a}\left(Y - \gamma^{-1}Z\right)$. Prices now transmit information, but do not aggregate it, and therefore the noise of the signal appears in the equilibrium price. Notice that in this model γ is again the relevant parameter for market efficiency, since

$$\text{var}(X|P_x) = \frac{\sigma_\epsilon^2 + \dfrac{\sigma_z^2}{\gamma^2}}{1 + \sigma_\epsilon^2 + \dfrac{\sigma_z^2}{\gamma^2}} \tag{1.16}$$

and that (1.16) is monotonically decreasing in γ. Furthermore, as we show in the proof of Proposition 4, in equilibrium we have that

$$\gamma = \frac{1}{\tau\sigma_\epsilon^2}\left(\lambda_o m_o b_\epsilon + \lambda_r m_r\right); \tag{1.17}$$

where λ_j denotes, as before, the fractions of agents that are informed. Equation (1.16) and (1.17) immediately imply that when λ_j are exogenous, an increase in overconfidence b_ϵ raises the amount of information revealed by the price.

We next turn to describing the equilibrium at the information acquisition stage.

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[21]One can show that the irrelevance result holds for imperfectly correlated signals, i.e. signal structures of the form $Y_i = X + \epsilon + \epsilon_i$, where ϵ denotes a common error term, and the ϵ_i's are i.i.d., which subsumes the model in section 1.2 and the one currently being discussed.

Define Λ^*_{GS} as

$$\Lambda^*_{GS} = \frac{1}{m_r}\left(\tau\sigma_\epsilon\sigma_z\sqrt{\frac{(1-C(\tau)\sigma^2_\epsilon)}{(1+\sigma^2_\epsilon)C(\tau)}} - m_o b_\epsilon\right). \tag{1.18}$$

The next Proposition characterizes the equilibrium with endogenous information acquisition of perfectly correlated signals.

Proposition 4. *The equilibrium with information acquisition belongs to one of the following two classes:*

(a) *If the parameters of the model are such that $\Lambda^*_{GS} > 0$, a fraction (possibly all) of the rational agents and all overconfident agents become informed. In particular $\lambda^*_o = 1$ and $\lambda^*_r = \Lambda^*_{GS}$.*

(b) *If the parameters of the model are such that $\Lambda^*_{GS} \leq 0$, a fraction (possibly all) of the overconfident traders becomes informed, but none of the rational agents, $\lambda^*_r = 0$.*

If $\Lambda^*_{GS} > 0$ then overconfidence is irrelevant for informational efficiency, that is, γ is equal to what would endogenously arise in a fully rational economy.

The equilibrium with endogenous information acquisition shares the same properties of the basic model: rational traders will become informed only if all overconfident are informed. The intuition for the irrelevance result is identical to the case where signals were independent: the rational traders, when they are the marginal buyers of information, scale back their information acquisition activities (less of them become informed), and this exactly offsets the standard effect of higher price informativeness stemming from more overconfidence.

This shows that the result on the irrelevance of overconfidence for market efficiency is robust to other types of information structure in the market. It should be remarked that other variables of interest, and in particular the price function itself, do depend on the level of overconfidence b_ϵ, in contrast to the case studied in section 1.3. This dependence goes much along the same lines as in Odean (1998) (Model III) and will not be reported here for brevity.

1.4.3 An imperfectly competitive model

In order to further analyze the effects of overconfidence in markets populated by both rational and overconfident agents we now turn to study a multi-agent version of the Kyle (1985) model. The main departure point from the previous section is the fact that all

agents are "large", in the sense that their trades affect prices. We recall that Odean (1998) and Benos (1998) showed that the introduction of overconfidence increases market depth.[22] We show below that this result depends critically on the fact that informed agents are overconfident: once we allow for rational traders and endogenous information acquisition a higher degree of overconfidence can make some rational agents drop out of the market, thereby decreasing market liquidity.

We consider a finite-agent economy, where all traders observe a signal of the form $Y_i = X + \epsilon_i$, where $X \sim \mathcal{N}(0,1)$ denotes the final payoff of the risky asset, and $\epsilon_i \sim \mathcal{N}(0, \sigma_\epsilon^2)$. For simplicity all signals' errors ϵ_i are assumed to be independent. There are m overconfident agents, who erroneously believe that the variance of their signal's estimation error is actually $k_\epsilon \sigma_\epsilon^2$, where $k_\epsilon < 1$.[23] In addition to overconfident agents, n rational traders exist in the economy. These agents estimate the precision of their private signal correctly. In order to abstract from risk-aversion effects we let both overconfident and rational traders be expected profits maximizers. On top of these two types of agents, there are also noise traders in the market, who submit orders that we denote by U, where $U \sim \mathcal{N}(0, \sigma_u^2)$.

As usual in this type of models, prices are set by a risk-neutral market maker, who is assumed to be competitive (i.e. earns zero expected profits in equilibrium). Namely, the market maker sets prices equal to the expected value of the fundamental, conditional on total order flow. We let θ_i denote the trading strategy of agent i. All traders and the market maker are assumed to know the structure of the market, in particular they rationally anticipate the trading strategies of other types of traders, given their exogenously specified biases. The following definition formalizes the notion of an equilibrium in this type of model.

Definition 4. *An equilibrium in the economy is defined by a set of trading strategies θ_i and a price function $P_x : \Omega \to \mathbb{R}$ such that:*

1. *Each agent i chooses her trading strategy so as to maximize her expected profits given her signal Y_i:*

$$\theta_i \in \arg\max_\theta \pi_i = \mathbb{E}^i \left[\theta_i (X - P_x) | Y_i \right]; \tag{1.19}$$

   where if agent i is overconfident the expectation is taken under the beliefs that $\epsilon_i \sim \mathcal{N}(0, k_\epsilon \sigma_\epsilon^2)$, whereas if agent i is rational $\epsilon_i \sim \mathcal{N}(0, \sigma_\epsilon^2)$.

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[22]The analysis is also similar to Kyle and Wang (1997), although the emphasis in that paper is on the commitment benefits of overconfidence.

[23]In the previous notation, $b_\epsilon = 1/k_\epsilon$

2. The market maker breaks even:

$$P_x = \mathbb{E}[X|\omega], \tag{1.20}$$

where ω denotes the total order flow, i.e. $\omega = \sum_{i=1}^{n+m} \theta_i + U$.

The following lemma characterizes the equilibrium price and trading strategies.[24]

Lemma 3. *The equilibrium price and trading strategies are linear in ω and Y_i respectively, i.e. price is given by $P_x = \lambda\omega$, rational agents' trading strategies are $\theta_i = \beta_r Y_i$ and those of the overconfident are $\theta_i = \beta_o Y_i$, where*

$$\beta_r = \frac{\eta}{1 + 2\sigma_\epsilon^2}; \qquad \beta_o = \frac{\eta}{1 + 2k_\epsilon\sigma_\epsilon^2}; \tag{1.21}$$

$$\lambda^{-1} = \eta\left(1 + \frac{n}{1 + 2\sigma_\epsilon^2} + \frac{m}{1 + 2k_\epsilon\sigma_\epsilon^2}\right); \tag{1.22}$$

$$\eta^2 = \sigma_u^2 \left(\frac{n(1 + \sigma_\epsilon^2)}{(1 + 2\sigma_\epsilon^2)^2} + \frac{m(1 + (2k_\epsilon - 1)\sigma_\epsilon^2)}{(1 + 2k_\epsilon\sigma_\epsilon^2)^2}\right)^{-1}. \tag{1.23}$$

A necessary and sufficient condition for an equilibrium to exist is that (1.23) defines a positive real number.[25]

As expected, the overconfident agents trade more aggressively than the rational. This is simply due to the fact that these agents believe their information to be more precise than that of the rational. It should nonetheless be noted that the trading aggressiveness of the overconfident is no longer a simple function of their behavioral bias: it now depends, through the market maker price setting, on the market wide variable η, which is itself a non-monotonic function of the bias measure b_ϵ. The following proposition is immediate.

Proposition 5. *If the number of informed agents m and n are exogenously fixed, then market depth is increasing in overconfidence.*

The proposition highlights the robustness of the positive effect of overconfidence on market liquidity, when information is exogenously fixed, reported elsewhere in the literature (Odean, 1998; Benos, 1998). Compared to a purely rational economy, financial markets with overconfident will exhibit higher market depth.

We now turn to study the incentives to acquire information by rational agents. In particular, we fix the number (and information) of the overconfident, and allow a large

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[24]The Lemma extends Benos (1998), who considers the extreme case in which $k_\epsilon = 0$.

[25]In the analysis that follows we will always assume this condition to be satisfied.

number of rational agents to purchase a signal of precision $1/\sigma_\epsilon^2$ for a cost c. We let n^* denote the largest n^* such that $\pi_r(n^*) \geq c$, i.e. n^* denotes the largest number of rational agents such that if n^* of them are informed it is still profitable for them to acquire information. This is the natural outcome of a standard Nash equilibrium in information acquisition in this type of setting.

The following proposition shows that the same forces that were in action in the competitive models play a role in this version of the Kyle (1985) model for moderate levels of overconfidence.

Proposition 6. *Given* m, *let* n^* *be determined endogenously. For moderate levels of overconfidence,* n^* *is weakly decreasing in overconfidence. As a result, market depth can decrease as a function of overconfidence.*

The result in Proposition 6 highlights the robustness of the main effect which drives the irrelevance result of previous sections:[26] rational agents' incentives to gather information are reduced when overconfidence appears. As discussed in Benos (1998), an increase in overconfidence (given m and n) has two opposite effects on the aggressiveness of rational traders: a market liquidity effect and a strategic substitution effect. The first one is related to the increase in market depth, which causes rational traders be more aggressive; the second is related to the increase in the aggressiveness of the overconfident, which leads rational traders to trade less. When overconfidence is not too severe the second effect dominates, reducing expected trading profits of rational traders.[27] This can in turn force some of them to drop out of the market and reduce market depth.[28] One can view this result in light of the benefits of overconfidence as a commitment device, discussed in Kyle and Wang (1997) and Benos (1998). Namely, if there is heterogeneity with respect to commitment power, those agents that lack commitment will have less incentives to invest in information, compared to the economy where all agents lack this commitment power. This in turn can make the market less liquid.

1.5   Conclusion

This paper considers a model in which rational traders coexist with overconfident ones. We have shown that endogenizing the information acquisition decision generates new

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[26]In the finite-agent economies, such an irrelevance result is impossible to obtain, due to the discreteness of the model.

[27]In particular, a sufficient condition for n^* to be weakly decreasing in overconfidence is that $2k_\epsilon\sigma_\epsilon^2 > 2\sigma_\epsilon^2 - 1$, which is clearly satisfied as $k_\epsilon \to 1$ or when $2\sigma_\epsilon^2 - 1 < 0$.

[28]Consider the following numerical example: $\sigma_\epsilon^2 = 1/5$; $\sigma_u^2 = 2$; $c = 0.1$; $m = 2$. One can easily verify that for $k_\epsilon = 0.5$ the model implies $n^* = 3$ and $\lambda^{-1} \approx 3.8$, while for $k_\epsilon = 0.4$ the model implies $n^* = 2$ and $\lambda^{-1} \approx 3.6$.

predictions on the effects of overconfidence on asset prices, with respect to models with exogenous information distribution. In particular, there exist economies in which the equilibrium price corresponds to what would endogenously arise in a rational expectations equilibrium. The rational agents react to the presence of overconfident agents by reducing their information acquisition activities, since the returns to informed trading are reduced when overconfident agents trade more aggressively and thereby reveal more of their information through prices. This reaction offsets the impact of the overconfident on asset prices. On the other hand, we show that other asset pricing variables are impacted be overconfidence: trading volume is higher in the presence of overconfident traders, confirming empirical findings in the literature. Our results yield further insights into the interaction of overconfidence, information acquisition and price revelation in financial markets.

Appendix

Proof of Lemma 1.

By standard techniques, it is straightforward to see that the average trade by the overconfident can be written as

$$\Theta_o = m_o \lambda_o \frac{b_\epsilon}{\tau \sigma_\epsilon^2} X + (\lambda_o q_o + (1 - \lambda_o) w) P_x \tag{1.24}$$

where $w = (1/\tau)(\gamma(1/d - \gamma)/\sigma_z^2 - 1)$ and $q_o = w - (1/\tau) b_\epsilon / \sigma_\epsilon^2$. Similarly the average trade by the rational agents is given by

$$\Theta_r = m_r \lambda_r \frac{1}{\tau \sigma_\epsilon^2} X + (\lambda_r q_r + (1 - \lambda_r) w) P_x \tag{1.25}$$

where $q_r = w - (1/\tau)/\sigma_\epsilon^2$ Using the market clearing condition (3.2) we obtain two equilibrium conditions from which (2.88) and (1.5) follow. $\square$

Proof of Lemma 2.

An informed overconfident agent t gets ex ante expected utility[29]

$$\mathbb{E}^o [u(W_t)] = -\sqrt{\frac{\text{var}^o(X | Y_t, P_x)}{\text{var}^o(X - P_x)}} e^{\tau c} \tag{1.26}$$

and an informed rational t agent has expected utility

$$\mathbb{E} [u(W_t)] = -\sqrt{\frac{\text{var}(X | Y_t, P_x)}{\text{var}(X - P_x)}} e^{\tau c}. \tag{1.27}$$

On the other hand, an uninformed t agent (rational or overconfident)[30] expected utility is given by

$$\mathbb{E} [u(W_t)] = \mathbb{E}^o [u(W_t)] = -\sqrt{\frac{\text{var}(X | P_x)}{\text{var}(X - P_x)}}. \tag{1.28}$$

For each class of traders (rational or overconfident), the equilibrium fraction of informed traders is set to equate informed and uninformed expected utilities. If such equality does not hold for any value of λ between zero and one, then the equilibrium

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[29]The ex-ante utility expressions follow from Admati and Pfleiderer (1987).

[30]Notice that unconditional variances in (1.28) do not involve the random variable ϵ, hence are equal for rational and overconfident

fraction of informed traders corresponds to the corner solution of one (zero) if the informed (uninformed) achieves higher expected utility. From (1.27) and (1.28), it follows that a rational agent will buy information if

$$-\text{var}(X|Y_t, P_x)^{1/2} e^{\tau c} \geq -\text{var}(X|P_x)^{1/2}. \tag{1.29}$$

If this inequality is satisfied, then it must be that (1.26) is greater than (1.28), since $\text{var}^o(X|Y_t, P_x) < \text{var}(X|Y_t, P_x)$. This in turn implies the corner solution $\lambda_o^* = 1$. Condition (1.29) can be expressed more explicitly as

$$\left(1 + \frac{\gamma^2}{\sigma_z^2}\right) e^{2\tau c} \leq \left(1 + \frac{\gamma^2}{\sigma_z^2} + \frac{1}{\sigma_\epsilon^2}\right). \tag{1.30}$$

In the interior solution $\lambda_r^* \in (0, 1)$, the above inequality holds as an equality. Substituting γ from (2.88), using $\lambda_o^* = 1$ and solving for λ_r^* we find the expression in the Lemma.[31]

For parameter values such that $\Lambda^* \leq 0$, none of the rational agents would choose to be informed,[32] so $\lambda_r^* = 0$. An overconfident agent will buy information if

$$-\text{var}^o(X|Y_t, P_x)^{1/2} e^{\tau c} \geq -\text{var}^o(X|P_x)^{1/2}. \tag{1.31}$$

When the above inequality binds as an equality, using γ from (2.88), the fact that $\lambda_r^* = 0$, writing explicitly (1.31) and solving for λ_o^* gives the expression in the Lemma. When the inequality in (1.31) is strict, then $\lambda_o^* = 1$. Finally, notice that Definition 2 rules out the case in which condition (1.31) is violated. $\square$

Proof of Proposition 1.

Substituting λ_r^* and λ_o^* from Lemma 2, and using (1.3) in expression (2.88) for γ, we have that

$$\begin{aligned}
\gamma &= \frac{1}{\tau \sigma_\epsilon^2} (\lambda_o^* m_o b_\epsilon + \lambda_r^* m_r) = \frac{1}{\tau \sigma_\epsilon^2} \left(m_o b_\epsilon + m_r \frac{1}{m_r} \left(\tau \sigma_\epsilon \sigma_z \sqrt{C(\tau)^{-1} - \sigma_\epsilon^2} - m_o b_\epsilon \right) \right) \\
&= \frac{\sigma_z}{\sigma_\epsilon} \sqrt{C(\tau)^{-1} - \sigma_\epsilon^2}.
\end{aligned}$$

Therefore, γ is independent of the overconfidence parameters (m_o, b_ϵ). Further note that the price coefficient d only depends on b_ϵ through γ (see equation (1.5)). Therefore

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[31]Notice that Definition 3 rules out the the case in which the inequality in (1.29) is strict, but it does not rule out the limiting case in which $\lambda_r = 1$.

[32]In particular, if $\Lambda^* < 0$, then condition (1.29) would be violated for any $\lambda_r \geq 0$, implying $\lambda_r^* = 0$.

the price function is independent of (m_o, b_ϵ). Price volatility (simply defined as $\text{var}(P_x) = \hat{a}^2 + \hat{d}^2\sigma_z^2$) and rational traders expected utilities ((1.27) and (1.28)), only depend on (m_o, b_ϵ) via the price coefficients. The same can be shown for rational expected profits, defined (net of the cost of information) for agent i as $\mathbb{E}[\theta_i(X - P_x)]$. This completes the proof. $\square$

Proof of Proposition 2.

The measure of informed traders, $m_o\lambda_o^* + m_r\lambda_r^*$, is decreasing in overconfidence when $\Lambda^* > 0$, since in this case $\lambda_o^* = 1$ and from expression (1.3) we have that

$$m_o + m_r\Lambda^* = m_o + \left(\tau\sigma_\epsilon\sigma_z\sqrt{C(\tau)^{-1} - \sigma_\epsilon^2} - m_o b_\epsilon\right) \tag{1.32}$$

The above expression valued at $b_\epsilon = 1$ corresponds to the measure of informed traders in a fully rational economy, and is decreasing in b_ϵ.

For expected profits, a direct computation shows that for an overconfident informed agent i's trading strategy can be expressed as $\theta_i = b_\epsilon\kappa(Y_i - P_x) + wP_x$, with $\kappa = 1/(\tau\sigma_\epsilon^2)$. It is immediate that we can write the expected profits of an overconfident informed agent as $\pi_o \equiv \mathbb{E}[\theta_i(X - P_x)] = \kappa D + \pi_u$, where $\pi_u = \mathbb{E}[wP_x(X - P_x)]$ are the expected profits of uninformed agents, and $D = \mathbb{E}[(X - P_x)^2]$.[33] Setting $b_\epsilon = 1$ recovers the trading strategy and expected profits for rational informed agents. It is immediate that overconfident agents earn higher expected profits than the rational traders. Furthermore, note that the variance of the profits of the overconfident agents can be expressed as $v_o \equiv \text{var}[\theta_i(X - P_x)] = v_u + b_\epsilon^2\kappa^2 F + 2b_\epsilon\kappa G$, where $G = \text{cov}[(X - P_x)^2, wP_x(X - P_x)]$, and $F = \text{var}[(Y_i - P_x)(X - P_x)]$. Making the dependence of π_o and v_o on b_ϵ explicit, the statement in the proposition reduces to showing that $S(b_\epsilon) \equiv \pi_o(b_\epsilon)/\sqrt{v_o(b_\epsilon)}$ satisfies $S(1) > S(b_\epsilon)$ for all $b_\epsilon > 1$. Some tedious but straightforward calculations show that $S(b_\epsilon)$ actually achieves a maximum at $b_\epsilon = 1$, which is sufficient for the claim in the proposition.

Trading volume is measured in ex-ante terms, as the number of shares that are expected to be traded in the market. Each trader's expected trading volume, T_i, is given by the expectation of the absolute value of his trading strategy, i.e. $T_i = \mathbb{E}[|\theta_i|]$. Expected trading volume is defined as $V = \int_i T_i di$, where the index of integration runs

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[33]Notice that we abstract from the cost of information, which does not affect any of the results that follow

through all agents (overconfident and rational). Some simple calculations[34] show that

$$V = \sqrt{\frac{2}{\pi}} \left[m_o \sqrt{w^2 \mathrm{var}(P_x) + 2Ab_\epsilon + Bb_\epsilon^2} + m_r \left(\lambda \sqrt{w^2 \mathrm{var}(P_x) + 2A + B} + (1-\lambda)\sqrt{w^2 \mathrm{var}(P_x)} \right) \right];$$
(1.33)

where $A = w^2 d^2 \sigma_z^2 / \sigma_\epsilon^2$ and $B = (1/(\tau\sigma_\epsilon^2))^2 (\sigma_\epsilon^2 + \mathrm{var}(X - P_x))$. Noting that the trading strategies of the rational agents, in the equilibrium under consideration, are independent of b_ϵ, we have that

$$\frac{\partial V}{\partial b_\epsilon} = \sqrt{\frac{2}{\pi}} m_o \left[\frac{A + Bb_\epsilon}{\sqrt{w^2 \mathrm{var}(P_x) + 2Ab_\epsilon + Bb_\epsilon^2}} - \sqrt{\mathrm{var}(w^2 \mathrm{var}(P_x) + 2A + B)} + \sqrt{\mathrm{var}(w^2 \mathrm{var}(P_x))} \right].$$
(1.34)

In order to see that the above quantity is positive for all b_ϵ the reader can verify (after some tedious calculations) that $\frac{\partial V}{\partial b_\epsilon}$ is indeed positive when evaluated at $b_\epsilon = 1$, and that $\frac{\partial^2 V}{\partial b_\epsilon^2} > 0$. This completes the proof. $\square$

Proof of Proposition 3.

An informed rational agent will choose p_r so as to maximize

$$\mathbb{E}\left[u(W_t)\right] = -\sqrt{\frac{\mathrm{var}(X|Y_t, P_x)}{\mathrm{var}(X - P_x)}} e^{\tau c(p_r)}$$
(1.35)

where the above conditional variance depends on p_r, namely

$$\mathrm{var}(X|Y_t, P_x) = \left(1 + \frac{\gamma^2}{\sigma_z^2} + p_r \right)^{-1}.$$
(1.36)

When maximizing (1.35) agents take the parameters of the price function as given. The first-order condition of (1.35) with respect to p_r yields

$$2\tau c'(p_r^*) \left[1 + \frac{\gamma^2}{\sigma_z^2} + p_r^* \right] = 1$$
(1.37)

Similarly, an informed overconfident agent will choose p_o^* such that

$$2\tau c'(p_o^*) \left[1 + \frac{\gamma^2}{\sigma_z^2} + b_\epsilon p_o^* \right] = 1$$
(1.38)

It is straightforward to show, as in Lemma 2, that no rational agent will become informed

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[34]Using the fact that if $x \sim N(0, \sigma^2)$, then $\mathbb{E}\left[|x|\right] = \sqrt{\frac{2\sigma^2}{\pi}}$.

unless all overconfident choose to do so. As in the main body of the text we focus then on the case where $\lambda_o^* = 1$. Equating the expected utilities of a rational informed (1.35) and a rational uninformed agent we get

$$e^{2\tau c(p_r^*)}\left(1 + \frac{\gamma^2}{\sigma_z^2}\right) = \left(1 + \frac{\gamma^2}{\sigma_z^2} + p_r^*\right) \tag{1.39}$$

where γ is given by (1.14). Substituting (1.14) and (1.37) into (1.39) we get a quadratic equation for λ_r, whose unique non negative solution yields $\lambda_r^* = \Lambda_{GI}^*$.

The above argument yields the equilibrium value for λ_r^* as long as $\Lambda_{GI}^* \in (0,1)$. Otherwise the equilibrium λ_r^* is characterized by corner solutions ($\lambda_r^* = 0$ if $\Lambda_{GI}^* \leq 0$ and $\lambda_r^* = 1$ if $\Lambda_{GI}^* \geq 1$). Assume now that the parameters are such that $\Lambda_{GI}^* \in (0,1)$ and therefore $\lambda_r^* = \Lambda_{GI}^*$. Substituting (1.12) into (1.14) it is easy to see that (1.14) is not a direct function of b_ϵ since the first term of (1.14) cancels out with the first term in (1.12). Therefore $d\gamma/db_\epsilon = 0$ as long as $dp_r/db_\epsilon = 0$. The last condition can be verified by substituting (1.14) into (1.37): since γ is not directly a function of b_ϵ then the first-order condition for p_r is not a function of b_ϵ neither. This yields the result that if $\lambda_r^* = \Lambda_{GI}^*$ then $d\gamma/db_\epsilon = 0$.

On the other hand, now suppose that $\lambda_r^* = 1$, i.e. constraint (1.39) does not bind and all rational agents find it optimal to become informed. Applying the implicit function theorem to (1.37) we have

$$\frac{dp_r^*}{db_\epsilon} = -\frac{m_o}{m_r \sigma_\epsilon^2}\left(\frac{4c'(p_r^*)\gamma/\sigma_z^2}{4c'(p_r^*)\gamma/\sigma_z^2 + 2\tau c'(p_r^*)/m_r + 2\tau c''(p_r^*)\left(\text{var}(X|Y_i, P_x)m_r\right)^{-1}}\right). \tag{1.40}$$

Given the assumption on the cost function, i.e. $c'(p^*) > 0$ and $c''(p^*) \geq 0$, the fraction in parenthesis in the above expression is less than 1. Then, it can be easily checked by substituting (1.40) into (1.15) that in this case $d\gamma/db_\epsilon > 0$. $\square$

Proof of Proposition 4.

The proof closely follows those of Lemma 1 and 2. The aggregate trade by the overconfident is

$$\Theta_o = m_o\left(\lambda_o\frac{\mathbb{E}^o(X|Y, P_x) - P_x}{\tau\text{var}^o(X|Y, P_x)} + (1 - \lambda_o)\frac{\mathbb{E}^o(X|P_x) - P_x}{\tau\text{var}^o(X|P_x)}\right); \tag{1.41}$$

whereas for the rational agents

$$\Theta_r = m_r\left(\lambda_r\frac{\mathbb{E}(X|Y, P_x) - P_x}{\tau\text{var}(X|Y, P_x)} + (1 - \lambda_r)\frac{\mathbb{E}(X|P_x) - P_x}{\tau\text{var}(X|P_x)}\right). \tag{1.42}$$

Substituting for the conditional expectations and variances (in particular note that for the informed agents their signal Y is now a sufficient statistic for X, i.e. they do not condition their trade on price) and using the market clearing condition $\Theta_o + \Theta_r = Z$ yields (1.17).

The description of the equilibrium at the information acquisition stage follows as in Lemma 2, where $\text{var}(X|P_x)$ is now given by (1.16), and $\text{var}(X|Y, P_x) = \text{var}(X|Y) = 1 + 1/\sigma_\epsilon^2$. Solving for λ_r^* and λ_o^* yields the statements in the Proposition.

Using the expression for γ from (1.17) we have that when $\Lambda_{GS}^* > 0$

$$\begin{aligned}
\gamma &= \frac{1}{\tau \sigma_\epsilon^2}(\lambda_o m_o b_\epsilon + \lambda_r m_r) = \frac{1}{\tau \sigma_\epsilon^2}\left(m_o b_\epsilon + m_r \frac{1}{m_r}\left(\tau \sigma_\epsilon \sigma_z \sqrt{\frac{(1 - C(\tau)\sigma_\epsilon^2)}{(1 + \sigma_\epsilon^2)C(\tau)}} - m_o b_\epsilon\right)\right) \\
&= \frac{\sigma_z}{\sigma_\epsilon}\sqrt{\frac{(1 - C(\tau)\sigma_\epsilon^2)}{(1 + \sigma_\epsilon^2)C(\tau)}}
\end{aligned}$$

Therefore, γ is independent of the overconfidence parameters (m_o, b_ϵ). This completes the proof. $\square$

Proof of Lemma 3.

Each agent maximizes his expected trading profits, $\pi_i = \theta_i \mathbb{E}[(X - P_x)]$, i.e. for the rational agents

$$\max_{\theta_i} \quad \theta_i \mathbb{E}(X|Y_i) - \lambda \theta_i^2 - \theta_i \lambda[(n-1)\beta_r + m\beta_o]\mathbb{E}(X|Y_i); \tag{1.43}$$

which yields the optimal trading strategies

$$\theta_i = \frac{(\lambda^{-1} - (n-1)\beta_r - m\beta_o)}{2(1 + \sigma_\epsilon^2)}Y_i \equiv \beta_r Y_i. \tag{1.44}$$

Similarly for the overconfident traders we have

$$\theta_i = \frac{(\lambda^{-1} - n\beta_r - (m-1)\beta_o)}{2(1 + k_\epsilon \sigma_\epsilon^2)}Y_i \equiv \beta_o Y_i. \tag{1.45}$$

Some simple manipulations of (1.44) and (1.45) yields (1.21) for some constant η that satisfies

$$\eta + n\beta_r + m\beta_o = \lambda^{-1}. \tag{1.46}$$

It is straightforward to see, given the standard properties of normally distributed

random variables, that $\mathbb{E}[X|\omega] = \lambda\omega$, where

$$\lambda = \frac{n\beta_r + m\beta_o}{(n\beta_r + m\beta_o)^2 + (n\beta_r^2 + m\beta_o^2)\sigma_\epsilon^2 + \sigma_u^2}. \tag{1.47}$$

Using (1.47) with (1.46), (1.44) and (1.45) yields the expression for the equilibrium value for λ, namely equation (1.22). $\square$

Proof of Proposition 5.

If we let $\lambda^{-1}(k_\epsilon)$ denote the market depth as a function of the overconfidence bias, we have that $\lambda^{-1}(1) < \lambda^{-1}(k_\epsilon)$, for all $k_\epsilon < 1$.[35] The result directly follows from partially differentiating (1.22) with respect to k_ϵ. Taking into account the condition for the existence of the equilibrium, it is easy to verify that $d\lambda^{-1}/dk_\epsilon < 0$. $\square$

Proof of Proposition 6.

It is straightforward to compute the expected trading profits at equilibrium, $\pi_r = \mathbb{E}[\theta_i(X - P_x)]$, by the informed rational agents, which are given by

$$\pi_r = \frac{(1 + \sigma_\epsilon^2)}{(1 + 2\sigma_\epsilon^2)^2}\eta\xi \tag{1.48}$$

where

$$\xi = \left(1 + \frac{n}{1 + 2\sigma_\epsilon^2} + \frac{m}{1 + 2k_\epsilon\sigma_\epsilon^2}\right)^{-1} \tag{1.49}$$

Therefore we have

$$\frac{d\pi_r}{dk_\epsilon} = \frac{(1 + \sigma_\epsilon^2)}{(1 + 2\sigma_\epsilon^2)^2}\left(\frac{d\eta}{dk_\epsilon}\xi + \eta\frac{d\xi}{dk_\epsilon}\right) \tag{1.50}$$

It is easy to verify that

$$\mathrm{sign}\left(\frac{d\eta}{dk_\epsilon}\right) = \mathrm{sign}\left(-1 + 2\sigma_\epsilon^2(1 - k_\epsilon)\right); \tag{1.51}$$

and that $d\xi/dk_\epsilon > 0$. It follows that for $d\pi_r/dk_\epsilon > 0$ a sufficient condition is $2k_\epsilon\sigma_\epsilon^2 > 2\sigma_\epsilon^2 - 1$. The second result follows immediately by considering small changes in the overconfidence parameter when the constraint $\pi_r(n^*) \geq c$ binds. $\square$

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[35]This generalizes Benos (1998), who showed $\lambda^{-1}(1) < \lambda^{-1}(0)$.

Chapter 2

Information sales and strategic trading

2.1 Introduction

This paper explores the allocation of information that arises when information is sold to a set of strategic risk-averse traders. A monopolist seller chooses how information gets distributed in the market so as to maximize consumer surplus. We study the problem across different types of markets (limit-order versus market-order), allowing for rather general allocations of information. For tractability, the paper uses the standard CARA/Gaussian models of Kyle (1985) and Kyle (1989). These models allow for strategic trading with multiple informed *risk-averse* agents, both important features of the problem, and can be extended to accommodate rather general allocations of information. In essence, we take the information sales problem of Admati and Pfleiderer (1986), and extend it to non-competitive markets. Framing the problem in such a parametric setting gives us some analytical tractability in a share auction model with common values,[1] which allows us to link our results to both the market microstructure literature as well as the literature on auctions with endogenous informational asymmetries.

Our main finding in a limit-order market is that the optimal sales of information take on a particularly simple form: (i) sell to as many agents as possible very imprecise information; (ii) sell to a single agent a signal as precise as possible. Whether one form or the other dominate depends on the noise trading per unit of risk-tolerance of the bidders: as this becomes small, the *exclusivity* contract (ii) dominates the large scale

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[1]Share auctions, first studied in Wilson (1979), allow bidders to receive fractional amounts of the good for sale. Examples in financial markets abound, from auctions of Treasury securities to auctions of equity stakes.

newsletters or *rumors* associated with (i). We show that the optimality is driven by the tradeoff between maximizing interim profits and ex-ante risk-sharing. The rumours equilibria maximizes ex-ante risk-sharing by splitting the information in such a way that agents hold very small risky portfolios, at the cost of introducing competition and noise in the information. The exclusivity contract maximizes expected trading profits, at the cost of leaving ex-ante risk-sharing gains untapped. These types of contracts seem to compare well with some of the types of sales we see in markets for information: many financial services firms do sell newsletters that seem to have little informational content,[2] and many financial consulting services are associated with exclusivity contracts. Therefore, our paper contributes to the literature by showing how particularly simple sale strategies are optimal in a particular auction market driven by asymmetric information considerations.

We show that the optimal information sales with market-orders exhibits a similar duality as in the case of limit-orders. For low values of the risk tolerance per unit of noise, the seller finds it optimal to sell to as many agents as possible very noisy signals. In this equilibrium the informational properties of asset prices are identical to those that arise under limit-orders. On the other hand, for high values of risk tolerance per unit of noise, the information seller may decide to sell to a small number of traders very accurate signals. Intuitively, when the agents cannot submit limit-orders risk-sharing becomes an issue even when traders have a signal with infinite precision: noise traders can always move prices against a well-informed agent. This makes the information seller choose a finite number of agents instead of having an exclusive contract with one them, as in the limit-order model.

Our analysis yields three other interesting results. First, the paper provides an example of a large auction market where imperfect competition yields different equilibria than a purely competitive equilibrium concept. In particular, we complement the examples in Kyle (1989) and Kremer (2002),[3] by providing a simple economic problem where the type of limiting economy studied in these papers arises (with precision vanishing as the number of informed agents increases). Second, we find that assuming perfect competition at the trading stage the monopolist seller will always choose to sell noisy newsletters to traders, i.e. the exclusivity contracts that arise when traders are strategic are never optimal if one assumes competitive behavior. Intuitively, it is the fact that the monopolist trader fully internalizes his trades' impact on price (partially with multiple traders) which drives the dominance of the exclusive contracts versus the newsletters. Thereby, models with and without the price-taking assumption yield *qualitatively* different impli-

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[2]See, for example, Graham and Harvey (1996), Jaffe and Mahoney (1999) and **?**.

[3]See pages 339-340 of Kyle (1989), and in particular Theorem 9.1. Kremer (2002)'s counterexample concludes his paper.

cations.[4] Finally, we find that several comparative statics results get reversed once the information in the market is endogenous. The literature argues that limit-order markets are more informative than those driven by market-orders, due to the fact that traders facing smaller execution-price risk trade more aggressive on their information. (Brown and Zhang, 1997) Precisely due to this aggressiveness, the seller of information gives traders in limit-order markets less information, which in equilibrium actually reverses the standard result and yields less informative prices in limit-order markets. Similarly, under imperfect competition the seller chooses to sell more precise information than under perfect competition. This is due to the fact that investors' strategic behavior internalizes their trade impact on prices. Under the optimal information sales prices are more informative in the imperfectly competitive setting than under perfect competition, in sharp contrast to the common belief, which further highlights the importance of endogenizing the allocation of information in this class of models.

2.1.1 Related literature

Our paper is closely related to previous work on information sales, as well as the literature on mutual funds and analysts.[5] We model information sales as *direct*, in the sense defined in Admati and Pfleiderer (1986). The paper by Admati and Pfleiderer (1988), who study information sales in a Kyle (1985) framework is perhaps the closest to our model. Admati and Pfleiderer (1988) show that in the context of *photocopied* noise (see Admati and Pfleiderer, 1986) a monopolistic seller of information would like to sell to finite number of traders, depending on the risk-aversion of the traders. We extend their analysis by allowing the information seller to add *personalized* noise, which as argued in Admati and Pfleiderer (1986) and Dridi and Germain (2000) is potentially more beneficial in terms of dampening the effects of competition between traders. We go further than these papers by studying information sales across different market settings, allowing the monopolist to choose among a larger class of allocations of information.

The second strand of the literature to which our paper contributes is that of share auctions and information aggregation.[6] The main contribution of our paper is to endog-

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[4]This is in sharp contrast to the results in Kovalenkov and Vives (2004), who argue that the competitive and strategic models we study (in particular the competitive and strategic versions of Kyle, 1989) have similar equilibrium properties. In our application the models are truly different, both quantitatively and qualitatively, even when there are large numbers of agents.

[5]Some early work includes Admati and Pfleiderer (1986), Admati and Pfleiderer (1987), Admati and Pfleiderer (1988), Admati and Pfleiderer (1990), Benabou and Laroque (1992), Fishman and Hagerty (1995). See Ross (2000), Biais and Germain (2002), Veldkamp (2003), Germain (2003), Morgan and Stocken (2003), García and Vanden (2003), Massa (2004a), García and Urosević (2003), Massa (2004b), and Cespa (2004) for some recent contributions.

[6]Some classic papers on information aggregation include Wilson (1977), Milgrom (1979), and Mil-

enize the allocation of information in two special types of share auctions with risk-averse buyers.[7] Prices in the share auctions we study indeed aggregate the diverse pieces of information that agents receive from the monopolist seller. On the other hand, the type of information received by agents is a non-trivial function of the number of informed agents in the equilibrium that yields optimal rumors, in sharp contrast to most of the limiting equilibria studied in the literature, where signals' precision are typically held constant as new traders are added to the auction. Our results highlight the importance of endogenizing the allocation of information when studying issues of information aggregation.

2.2   The model

In this section we present the main ingredients of our economy with endogenous asymmetric information. In essence we extend Kyle (1989) by analyzing how the private information possessed by traders arises from a simple model of information sales.

2.2.1   The share auction setting

We assume, for usual tractability reasons, that all agents have CARA preferences with a risk aversion parameter r. Thus, given a final payoff W_i, each agent i derives the expected utility $\mathbb{E}[u(W_i)] = \mathbb{E}[-\exp(-rW_i)]$. There is a large number of uninformed traders who participate in the stock market alongside the traders who can become informed. This makes the specification of the economy, post information sales, identical to the one discussed in Kyle (1989) under the assumption of free entry of uninformed speculators.[8]

There are two assets in the economy: a risk-less asset in perfectly elastic supply, and a risky asset with a random final payoff $X \in \mathbb{R}$ and variance normalized to 1. All random variables in the economy are defined on some probability space $(\Omega, \mathcal{F}, \mathbb{P})$, and unless stated otherwise, are normally distributed, uncorrelated, and have zero mean. There is random aggregate supply Z of the risky asset. This variable is the usual driver

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grom (2000). See Pesendorfer and Swinkels (1997), Pesendorfer and Swinkels (2000), Hong and Shum (2004), Jackson and Kremer (2004), Reny and Perry (2003) and Cripps and Swinkels (2004) for some recent work on the area.

[7]The work on information acquisition in auctions in general is a rapidly growing field. See Matthews (1984), Hausch and Li (1991), Guzman and Kolstad (1997), Persico (2000), Moresi (2000), Gaier and Katzman (2002), Jackson (2003).

[8]This assumption plays no role on the actual results of the paper, but it significantly simplifies the presentation of the model, as well as the comparison of the equilibria to the model of Kyle (1985) in section 2.4.1.

in preventing private information to be revealed perfectly to other market participants.[9] We let σ_z^2 denote the variance of Z. We use θ_i to denote the trading strategy of agent i, i.e. the number of shares of the risky asset that agent i acquires. With this notation, the final wealth for agent i is given by $W_i = \theta_i(X - P_x)$, where P_x denotes the price of the risky asset.[10]

2.2.2   The monopolist information seller

There is a single agent who has perfect knowledge about the payoff from the risky asset X, whom we shall refer to as the information seller. We will focus on *direct* sales of personalized information (see Admati and Pfleiderer, 1986), i.e. the case where the seller of information gives agent i a signal of the form $Y_i = X + \epsilon_i$, with ϵ_i i.i.d. and $s_\epsilon \equiv \mathrm{var}(\epsilon_i)^{-1}$ denoting the precision of the signals offered by the monopolist. We allow the seller of information to ration the market, i.e. to sell to m agents, and to freely choose the signal quality s_ϵ. Essentially, we allow the monopolist to sell different pieces of her information to different agents.

The timing is as follows. The monopolist seller of information contacts m agents and offers them signals as specified above for a price c. If an agent accepts he pays the fee c, and next period he receives the signal Y_i, which he will use to make his portfolio decision. If an agent declines he trades as an uninformed investor when financial markets open. Traders are not allowed to resell the information they receive to other traders and the precision of the signals is assumed to be contractible. The type of information sales we are considering can be thought as "subscriptions" to some future advice, for which trades pay some ex-ante price c, and later get to observe information about the risky assets. Figure 2.1 sketches the major stages of the model.

We should emphasize that all the assumptions on the information sellers of Admati and Pfleiderer (1986) are in place. In particular, there is no reliability problem between the information seller and the buyers, in the sense that she can commit to truthfully revealing the signal Y_i that she promised. Furthermore, the information seller is not allowed to trade on her information.[11] Our goal is not to generalize the information

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[9]We note that the size of the aggregate supply is fixed, in sharp contrast with much of the literature, where some notion of large noise is introduced (Hellwig, 1980a; Verrecchia, 1982b; Admati, 1985; García and Urosević, 2003).

[10]We normalize here, as is customary in the literature, the agents' initial wealth and the risk-free rate to zero. These assumptions are innocuous since the model contains only one period of trading and agents are assumed to have CARA preferences. In addition, there are no borrowing or lending constraints imposed on the agents.

[11]Following the discussion in Admati and Pfleiderer (1988), we conjecture that allowing the information seller to trade would only condition some of our results on the optimal sales. For high values of the risk-tolerance per unit of noise parameter the information seller would choose to not sell her

sales model studied in Admati and Pfleiderer (1986), but rather to study their model under different assumptions on the market structure, in particular breaking away with the competitive assumption.

2.2.3 The equilibrium at the trading stage

A linear rational expectations equilibrium is defined, as in Kyle (1989), by a linear function $P_x : \Omega \to \mathbb{R}$ such that (i) agents' trading strategies are optimal given their information set and their impact on prices, (ii) markets clear in all states. We should emphasize that the agents do *not* act as price takers - they anticipate the dependence of prices on their trading strategies (see Kyle, 1989, for details of the equilibrium concepts used).

We follow Kyle (1989) and search for equilibria where prices are linear functions of the primitives in the economy, namely the vector $(X, \{\epsilon\}_{i=1}^{m}, Z)$. We let θ_i denote the number of units of the stock that agent i trades. Without loss of generality, due to the CARA/normal setup, we characterize informed agents' trading strategies by two positive constants (β, γ), defined by $\theta_i = \beta Y_i - \gamma P_x$, for $i = 1, \ldots, m$. The parameter β measures the intensity of trading on the basis of private information, whereas γ is the intensity with which they trade as a function of price (including their optimal response ot its informational content). From the market-clearing condition follows that prices will be of the form

$$P_x = \lambda \left(\beta \sum_{i=1}^{m} Y_i - Z \right); \tag{2.1}$$

for some $\lambda > 0$. Kyle (1989) characterized the equilibrium in this economy for a fixed number of informed traders m and information precision s_ϵ. In particular, he showed that with enough uninformed speculators in the market a linear equilibrium always exists and is unique. Kyle (1989)'s equilibrium characterization (see Theorem 8.1) is via the solution to the non-linear equation

$$\kappa \sqrt{\frac{\phi}{(m-1)s_\epsilon(1-\phi)}} = 1 - 2\phi - \frac{\phi(1+ms_\epsilon)}{(m-1)(1+\phi m s_\epsilon)}; \tag{2.2}$$

where ϕ is defined as the proportion of the $m-1$ traders information revealed by prices to an informed agent, namely

$$\text{var}(X|Y_i, P_x)^{-1} \equiv \tau_i = 1 + s_\epsilon + \phi(m-1)s_\epsilon. \tag{2.3}$$

information and just trade on her own account (analogous to selling to one single agent), whereas for low values she would choose to sell to as many agents as possible.

Two aspects of the model will be crucial for the following analysis: the informational content of the equilibrium price and the strategic behavior of informed traders. The informational variable ϕ defined above is a measure of the informational content of equilibrium prices, as is ψ defined via[12]

$$\text{var}(X|P_x)^{-1} \equiv \tau_u = 1 + \psi m s_\epsilon. \tag{2.4}$$

The variable ψ measures the fraction of the informed agents' precision that is revealed by prices. It is useful to define the product of the number m of signals sold times the precision in each signal, $y \equiv m s_\epsilon$, as the "stock of private information" in the economy.

Following Kyle (1989), we also define the informational incidence parameter

$$\zeta \equiv \beta\lambda\text{var}(X|Y_i, P_x)^{-1}/s_\epsilon. \tag{2.5}$$

This variable is related to strategic considerations in the following way: ζ measures the change in the price that obtains when an informed agent's valuation of the risky asset goes up by one dollar as a result of a larger realization of his signal Y_i. At its extreme values, $\zeta = 1/2$ corresponds to the case of risk neutral monopoly, $\zeta = 0$ to a perfectly competitive market.

Of course these variables are interrelated: as speculators internalize their price impact, in equilibrium the fraction of private information revealed by prices is never more than one half, i.e $\psi \leq 1/2$. On the other hand information revelation clearly affects the value of ζ; in equilibrium (see Lemma 4 in the Appendix) we have $\zeta = \psi\tau_i/\tau_u$, i.e. the informational incidence parameter is a fraction of the relative conditional precisions of the informed and uninformed.

The goal of this paper is to endogenize the allocation of information among market participants, namely the number of informed m and the quality of their information s_ϵ as the outcome of a prior stage in which information is optimally sold via a financial intermediary,[13] and to characterize the corresponding equilibrium properties.

2.2.4 The monopolist's problem

A monopolist seller of information would charge a price c that makes each of the agents just indifferent between accepting the monopolist's offer or trading as an uninformed

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[12]The advantage of the variable ψ is the it is well defined for all m, whereas ϕ can only be used for $m \geq 2$.

[13]Although most of the discussion that follows focuses on the monopolist case, none of the qualitative conclusions we draw from the model are affected by the potential existence of competition among information providers.

agent (earning no trading profits). As a consequence, the profits earned by the seller of information from a particular allocation depend on the number of traders m to which information is sold, and on the value of signals to traders, given by the certainty equivalent of wealth.[14]

Computing the ex-ante expected utility for informed agents in the equilibrium described in the previous section we have that the monopolist's problem reduces to

$$\max_{m\in\{1,\dots,N\},s_\epsilon\in\mathbb{R}_+} \mathcal{C}(m,s_\epsilon) = \frac{m}{2r}\log\left(1+2r\mathbb{E}[\chi]\right); \tag{2.6}$$

such that (2.2) holds, where

$$\chi_i = \mathbb{E}[\pi_i|Y_i,P_x] - \frac{r}{2}\mathrm{var}(\pi_i|Y_i,P_x) \tag{2.7}$$

and $\pi_i = \theta_i(X - P_x)$ denotes the trading profits of agent i.[15]

Consumer surplus $\mathcal{C}$ is a concave function of $\mathbb{E}[\chi]$, the expected interim certainty equivalent. Note that χ_i is precisely the objective function that trader i maximizes at the interim stage (this is $t = 2$ in Figure 2.1). The monopolist seller controls $\mathbb{E}[\chi]$ by choosing both s_ϵ and m. More precise signals generate smaller interim discounts due to risk, the second term in (2.7). On the other hand, the expected profits component may or may not increase in signals' precision: too much information can induce agents into trading very aggressively, and as a consequence eliminate their trading profits.

We conclude the model formulation by highlighting a different interpretation of the consumer surplus function defined above. In particular, the certainty equivalent for a single trader in ex-ante terms can be written as

$$\mathcal{U}_i = \frac{1}{2r}\log\left(1+2r\mathbb{E}[\chi]\right) = \frac{1}{2r}\log\left[1+\left(\frac{\tau_i}{\tau_u}-1\right)\frac{(1-2\zeta)}{(1-\zeta)^2}\right].$$

The above expression highlights the difference between the strategic model we are studying, and that of competitive models, studied by Admati and Pfleiderer (1986, 1987). In the standard competitive REE models, the value of information is simply given by the above expression setting $\zeta = 0$, reducing to the relative informational advantage over the uninformed traders. Using a strategic solution concept the certainty equivalent also depends on ζ, the informational incidence parameter. This is rather intuitive - other things equal traders prefer to face smaller price impact.

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[14]Assuming initial homogenous beliefs and the same risk aversion for all traders guarantees that traders value a given piece of information in the same way.

[15]In (2.6) we drop the i subscript in χ as in the expecatation the idiosyncratic part disappears.

Next section endogenizes the allocation of information in the economy by considering the solution to (2.6), namely the choice of the number of agents the monopolist sells information to, m, and the quality of the signal she offers, s_ϵ (or equivalently, the number of traders m and the total stock of private information y). We remark that, given the symmetry assumption and the fact that the information allocation is endogeneous, the only two primitives in (2.6) are the risk-aversion of each trader r and the total amount of aggregate noise σ_z. Moreover, by inspection of (2.6) and (2.2) one can check that it is the product of these two variables that affects the optimal sales. Although r and σ_z have different effects on total profits, the optimal information sales are determined solely by $\kappa \equiv r\sigma_z$, the noise per unit of risk-tolerance in the economy.

2.3 Optimal information sales

2.3.1 General considerations

The problem in (2.6) is driven by the interaction of strategic trading, externalities in the valuation of information and risk sharing considerations. Before exploring the full problem, it is worthwhile to look at it considering one of the two decision variables at the time. If the monopolist sells information to only one trader, then it is optimal not to add any noise to the signal, i.e. to set $s_\epsilon = \infty$ (this is proven in Proposition 1 below). This is rather intuitive: in absence of competition the informed trader fully internalizes the price impact of his trades, and the seller maximizes the value of the signal giving him full information. On the other hand, if she were to sell perfect information to more than one trader, these would compete away their profits to zero (see Kyle, 1989, Theorem 7.5). With a perfect signal traders can hedge completely noise-trader risk Z using strategies that are measurable with respect to P_x. As a consequence, if perfectly informed, speculators compete very aggressively on the trading opportunity, making the price reveal all their private information. The seller can avoid this outcome by adding noise to the signals she sells, as in Admati and Pfleiderer (1986): adding noise dampens the competition problem and informed traders earn positive profits.[16] With respect to the perfectly informed single trader though, traders with imperfect signals only partially distinguish price movements related to fundamentals from those related to noise-trading. The higher this risk the higher the discount in the interim certainty equivalent, reducing the value of information for each trader at the ex ante stage.

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[16]The motivations for adding noise in Admati and Pfleiderer (1986) and in our paper are related, but not identical. For instance, the seller would never sell perfect information to a single trader in the competitive model of Admati and Pfleiderer (1986). In section 2.4.2 we further compare our results to a case closely related to Admati and Pfleiderer (1986), where agents act as price takers.

Now consider the case in which the total stock of private information y is finite and fixed. Increasing the number of informed traders induces a clear trade-off in monopolist's profits (or, equivalently, consumer's surplus). On the one hand it improves welfare by means of better risk sharing: the more informed traders the less the noise-trading risk to be borne by each of them in ex-ante terms. On the other hand it decreases welfare because of higher competition: for a given total stock of private information y more traders trade in aggregate more aggressively. As more information gets incorporated in the price the informational advantage over the market is reduced, decreasing expected profits. Risk sharing gains and competition are respectively decreasing and increasing in the number of traders, explaining why, generally speaking (i.e., for intermediate values of risk aversion and stock of private information), consumer's surplus increase in the number of traders for low m and decrease for large m.

Summarizing, there are two opposing forces. On one hand the information seller wants to maximize interim consumer surplus, by providing agents with high aggregate profits and low trading risk. On the other hand, trading profits depend on the realization of noise-trading demand, and risk averse traders discount this uncertainty from an ex ante point of view (when the price for information is paid). At the *interim* stage a single perfectly informed trader faces no risk and is fully able to take advantage of his information, so concentrating the information allocation maximizes interim certainty equivalent. Multiple imperfectly informed traders compete over profits but share the noise-trading risk, so dispersing the information allocation by spreading noisy signals among different agents improves ex-ante risk-sharing. When solving her problem (2.6), the monopolist seller of information weights these forces when choosing the number of newsletters m and their precision s_ϵ.

2.3.2 Optimal exclusivity contracts and noisy newsletters

In this section we study the problem in (2.6) for open sets around the risk-neutral and large risk-aversion cases.

Proposition 7. *There exists $\underline{\kappa}$ such that for all $\kappa < \underline{\kappa}$, the monopolist optimally sells to a single agent, $m^* = 1$, and sets $s_\epsilon^* = \infty$, i.e. tells the agent what he knows. In this case, informational efficiency and monopolist's profits satisfy:*

$$\text{var}(X|P_x)^{-1} = 2, \tag{2.8}$$

$$\mathcal{C}_1(r, \sigma_z) \equiv \frac{1}{2r} \log\left(1 + r\sigma_z\right). \tag{2.9}$$

The Proposition establishes that if risk-aversion or noise trading in the auction are

small, the damaging effects of competition are high enough that the monopolist optimally sells her information to one trader. In the risk-neutral case the monopolist's problem reduces to that of maximizing expected profits, since the objective function, taking limits in (2.6) is simply $m\mathbb{E}[\pi_i]$. The proof of the Proposition starts showing that in the $m = 1$ case the trader's certainty equivalent is increasing in his signal precision s_ϵ. Given that the informed speculator will be a monopolist at the trading stage, he will optimally internalize the price impact of his trades. As a consequence, the information seller maximizes the value of the information she sells by giving him full information. It further solves for the optimal stock of information y^* that the monopolist chooses to sell for $m \geq 2$. A direct comparison of aggregate profits for different m shows that competition only lowers aggregate profits. Rather intuitively, for low κ risk-sharing gains are negligible with respect to the costs induced by competition and noisy signals. Therefore, the concentrated information allocation with $m = 1$ is optimal. This coincides with the results in Admati and Pfleiderer (1988) for the case where $r = 0$, although here the allocations with $m \geq 2$ are allowed to have personalized noise.[17]

Under the optimal information sales, half of the information of the seller gets impounded into prices, i.e. the conditional volatility of the risky asset is exactly $1/2$ the unconditional volatility, irrespective of the level of noise trading. This result follows from the fact that the speculator's effective risk aversion is zero as he receives a signal with no noise and can submit limit orders. As a consequence, he optimally adjusts his trading strategy so as to offset any variation in noise trading and keep the optimal amount of information revelation constant, as the risk neutral monopolist in Kyle (1985).

Next Proposition describes the allocation of information that arises with a large number of traders, and establishes its optimality when the monopolist faces an economy with highly risk-averse traders and/or an asset with large amounts of noise.

Proposition 8. *There exists some $\bar{\kappa}$ such that for all $\kappa > \bar{\kappa}$ the monopolist's problem (2.6) is solved for $m^* = N$. As $N \uparrow \infty$, the optimal stock of private information sold is*

$$y^* = \frac{1 - 2\psi_\infty^2}{\psi_\infty(1 - \psi_\infty)}; \tag{2.10}$$

where ψ_∞ is the unique real solution in $[0, 1/2]$ to

$$\psi_\infty^4 - \psi_\infty^3 + \frac{(\kappa^2 - 2)}{8}\psi_\infty^2 + \frac{1}{2}\psi_\infty - \frac{1}{8} = 0. \tag{2.11}$$

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[17]We discuss the relationship between our model and that of Admati and Pfleiderer (1988) at more length in section 2.4.3.

As $N \uparrow \infty$, informational efficiency and monopolist's profits satisfy:

$$\text{var}(X|P_x)^{-1} = 1 + \frac{1 - 2\psi_\infty^2}{(1 - \psi_\infty)}; \tag{2.12}$$

$$\mathcal{C}_\infty(r, \sigma_z) \equiv \frac{(1 - 2\psi_\infty^2)(1 - 2\psi)}{2r\psi_\infty(1 - 2\psi_\infty^2 + 1 - \psi_\infty)}. \tag{2.13}$$

The Proposition shows that the solution for the monopolist's problem when either risk-aversion or the amount of noise trading in the economy are large is to sell to as many agents as possible very imprecise signals. More risk averse speculators trade less aggressively on information, and more noise trading makes prices less informative, so for κ large the negative effects of competition become negligible. The monopolist could still sell perfect information to a single trader, maximizing the interim certainty equivalent, but the ex ante value of information would be highly discounted due to large risk aversion or large risk. As a consequence, risk sharing gains dominate competition effects, driving the optimality of selling to N agents for large noise trading per unit of risk-tolerance.

Condition (2.11) implies that the fraction of information that prices reveal, ψ_∞, is a decreasing function of κ. As a consequence, (from (2.10)) the stock of private information sold y^* is increasing in κ. The higher noise trading per unit of risk tolerance, the lower information leakage, the lower the dilution of the value of information. Hence, the seller optimally increases the amount of private information sold. The net effect on price informativeness, as illustrated in Figure 2.3, is negative - price informativeness is decreasing in noise trading per unit of risk-tolerance in the optimal diffuse information allocation. The optimal allocation of information with large number of traders does indeed resemble noisy newsletters, as individual precision in each trader's signal vanishes in the large N limit. The comparison with the exclusivity contract is interesting as in that case the speculator receives infinite precision wile here the total amount of private information sold, y^*, is finite. Nevertheless, informational efficiency is always greater than in the exclusivity contract case, one can verify from (2.12) that $\text{var}(X|P_x)^{-1} > 2$ for $\kappa > \bar{\kappa}$ and finite. Prices aggregate the information dispersed in the economy and reveal more than under the equilibrium with a monopolist trader.

We also remark that the monopolist sells signals in such a way as to have a large number of informed agents monopolistically competing against each other as in the leading example of section 9 of Kyle (1989). The Proposition therefore shows the conditions under which such economic setting arises endogenously. Even in the large N limit, when agents are "small" in terms of their informational advantage, they internalize their price impact, i.e. the equilibrium $\zeta(\kappa) > 0$ for all $\kappa > \bar{\kappa}$ and finite.

2.3.3 The general case

After establishing that the optimal solution is non-interior for two open sets of $\kappa \in \mathbb{R}_+$, we further analyze the problem in this section to assess how tight the bounds $[0, \underline{\kappa})$ and $(\bar{\kappa}, \infty)$ actually are. Proposition 7 only establishes the existence of an open set $[0, \underline{\kappa}]$, whereas in Proposition 8 it is unclear if the bound $\bar{\kappa}$ has a finite limit if we let $N \uparrow \infty$. The general problem in (2.6), for an arbitrary κ, is particularly challenging analytically due to its non-linear nature.[18]

Nonetheless, one can simplify the problem by fixing m and studying the optimal stock of information y that the monopolist would like to sell. As discussed in section 2.3.1, when $m = 1$ it is optimal to sell as precise a signal as possible, irrespective of κ. Form the optimality condition of the monopolist it is possible to show that, fixing m, the solution to the monopolist problem (2.6) satisfies $y = x/\phi$, where x solves the quadratic equation

$$(m-2)(1-\phi)x^2 + (1+\phi(1-\phi)+(m-1)\phi(2\phi-1))x + \phi^2 + (m-1)(2\phi^2-1) = 0, \quad (2.14)$$

where ϕ is determined by the equilibrium condition (2.2). The optimal stock of information y, given the equilibrium variable ϕ, is independent of κ. This is very useful numerically, since it implies that the solution to the problem fixing m can be characterized by a single non-linear equation for ϕ, namely (2.2) evaluated at $y^*(m, \phi)$ as given by (2.14).

We argue next that indeed the functions $\mathcal{C}_1(r, \sigma_z)$ and $\mathcal{C}_\infty(r, \sigma_z)$ as defined in (2.9) and (2.13) yield an upper bound for the profits of the monopolistic seller of information. Figure 2.2 plots the profits obtained by the monopolist from selling to $m = 2, \dots, 40$ (dotted lines), as well as the profit functions corresponding to $m = 1$ and $m = \infty$ (solid lines). For each m we solve the single non-linear equation that characterizes the equilibrium ψ at the optimal y, and compute the profits, as well as other equilibria quantities. As Figure 2.2 makes clear, the profit functions with $m = 1$ and $m = N$ (for N large), form an upper envelope that dominates any allocation of information to m informed agents.

The following Theorem summarizes our main findings.

Theorem 1. *There exists κ^* such that Proposition 8 and Proposition 7 hold with $\bar{\kappa} = \underline{\kappa} = \kappa^* \approx 1.74$.*

Figures 2.3 and 2.4 plots the equilibrium values of ζ and $\text{var}(X|P_x)^{-1}$ as a function of κ. For $\kappa < \kappa^*$, a single trader gets perfect information, monopoly power is at it's

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[18]For instance, there exist open sets of κ such that optimal profits as a function of m exhibit both local maximum and minimum which are not the global maximum (or minimum).

maximum and the speculator minimizes information leakage; as a consequence we have in equilibrium $\zeta = 1/2$, $\text{var}(X|P_x)^{-1} = 2$. When κ is close to zero, the equilibrium condition (2.2) forces ζ to be at its upper bound of $\zeta = 1/2$. As long as $\kappa > 0$, the monopolist can choose to lower the informational incidence parameter to some interior value, at the cost of giving a finite stock of information (rather than reveal X perfectly). We find that for $\kappa > \kappa^*$ the monopolist optimally sells to an infinite number of agents very imprecise signals: in this range we have $\zeta(\kappa) < 1/2$ and $\text{var}(X|P_x)^{-1} = 1 + \frac{1-2\psi(\kappa)^2_\infty}{1-\psi(\kappa)_\infty} \geq 2$. Moreover, both are are monotonically decreasing functions of κ: increasing either risk aversion or noise trading in this range has a qualitative effect that is similar to the partial equilibrium setting in which the allocation of information is exogenous.[19]

The discontinuity in the equilibrium prices at κ^* creates a rationale for regime shifts between the $m = 1$ and $m = N$ cases. Namely, consider a repeated sequence of economies (identical and independent from each other), where the noise-trading parameter σ_z follows some Markov chain and $r = 1$. As this process crosses the κ^* barrier the allocation of information in the economy will shift from one type of equilibrium to the other. This will in turn generate regimes with different asset pricing properties, depending on whether σ_z is above or below κ^*. For example, in periods of high noise-trading prices will be more informative, and aggregate trading profits lower, whereas with low noise-trading price informativeness drops (to its lower bound).

2.4 Market structure, trading behavior and information sales

In this section we extend our analysis to a setting where agents can execute market-orders instead of limit-orders. We show that the bang-bang nature of the solution does not change, although now the seller of information may choose to sell to a finite number of agents for high values of the risk-tolerance per unit of noise trading parameter. These agents are offered perfect information, as in the low κ equilibria of section 2.3. Next, we study our model under the assumption of non-strategic, i.e. price-taking, behavior by the traders. We find that the model yields significantly different answers in this case: the monopolist seller of information will always sell to as many agents as possible very imprecise information, even for high values of risk-tolerance per unit of noise. We finish this section by discussing more general allocations of information, as well as the possibility that the monopolist seller of information has a noisy signal.

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[19]We notice that in the monopolist competition model discussed in Kyle (1989), as the cost of information is kept fixed, price informativeness is an increasing function of the amount of noise trading (see Kyle, 1989, Theorem 10.5).

2.4.1 Market orders

In this section we study how the particular market structure used throughout section 2.3 affects optimal sales of information. Instead of allowing traders to submit demand schedules, we study the case where traders submit market-orders to a risk-neutral market-maker, who sets the price according to weak form efficiency

$$P_x = \mathbb{E}[X|\omega];$$

where ω denotes total order flow. Using the previous notation, $\omega = \sum_{i=1}^{m} \theta_i + Z$. We remark that, since prices are a linear function of ω, the informational content of prices and order flow is the same. More critically, in the Kyle (1985) model agents are not allowed to condition their trades on prices, so their optimization problem is over trading strategies θ_i that are Y_i-measurable. Lemma 5 in Appendix A describes the equilibrium in this variation of the model in detail.

As in section 2.3, the consumer surplus, for a given information allocation of with m informed traders who have signals of precision s_ϵ, is given by (2.6), where the interim certainty equivalent is now

$$\chi_i = \mathbb{E}[\pi_i|Y_i] - \frac{r}{2}\text{var}(\pi_i|Y_i) \tag{2.15}$$

with $\pi_i = \theta_i(X - P_x)$ denoting the trading profits of agent i. As in the previous analysis, consumer surplus is a concave function of the expected value of the interim certainty equivalent χ_i. In contrast to the limit-order model, the agents interim certainty equivalent depends only on their private information Y_i, since neither order flow ω nor the asset price P_x is in their information sets. Notably, the conditional variance of profits, even when the agents has perfect information, is non-zero, since noise trader demand will randomly move prices.

The information seller's problem is, analogously to the case of limit-orders, to maximize consumer surplus

$$\max_{m\in\{1,...,N\},s_\epsilon\in\mathbb{R}_+} \mathcal{C}(m, s_\epsilon) = \frac{m}{2r}\log\left(1 + 2r\mathbb{E}[\chi_i]\right); \tag{2.16}$$

subject to the equilibrium constraint, given by (2.24) in the appendix. The residual uncertainty that traders face with market orders makes the information sales problem more challenging. In fact, contrary to the limit orders case, residual uncertainty prevents effective risk aversion of informed traders from vanishing as the precision of the signal increases. The reason being that, once the order is submitted, traders are exposed to the risk that liquidity traders may push the price against them. Therefore, risk averse

traders do not compete away their profits even if perfectly informed on the one hand, and value the reduction in risk that comes with precision on the other. As a consequence, the seller may find it optimal to sell precise signals to more than one trader.

The next Proposition presents open sets parameter statements for the market-order model, as Propositions 7 and 8 did for the limit-order model.

Proposition 9. *(a) There exists $\underline{\kappa}$ such that for all $\kappa < \underline{\kappa}$ the monopolist optimally sells to one agent, $m^* = 1$, and gives him perfect information, $s_\epsilon = \infty$;*

- *There exists $\bar{\kappa}$ such that for all $\kappa > \bar{\kappa}$ the monopolist optimally sets $m^* = N$, with $\lim_{N\uparrow\infty} s_\epsilon N = y^* \in \mathbb{R}_+$, with y^* given as in (2.10).*

The risk-neutral case, $\kappa = 0$, can be solved explicitly, and mirrors the conclusion of the limit-order model. As shown in Dridi and Germain (2000), the monopolist seller of information may want to add some noise to her information, in particular if $m \geq 4$. Furthermore, their expressions and analysis immediately imply that consumer surplus is maximized concentrating the information in one single trader, as in Admati and Pfleiderer (1988). The novel feature of the Proposition is the fact that consumer surplus is higher for the $m = N$ equilibria for sufficiently large κ. The optimality of the rumous equilibria is driven by the ex-ante risk-sharing gains - the main difference is the extent of these gains. Furthermore, the proposition shows that the equilibrium price informativeness coincides with that in the limit-order model, i.e. the models behave identically in terms of the informational content of prices.

The general case is again challenging analytically, since equilibrium with risk-averse traders in a Kyle (1985) can only be characterized via a non-linear equation (Subrahmanyam, 1991). As in the limit-order model, we solve the model numerically. For each m, we solve for the optimal s_ϵ and the equilibrium price, obtaining the maximum consumer surplus for each m. We do this for a fine grid of values for κ, and report the resulting consumer surplus for different m in Figure 2.5. Whether it is optimal to set $s_\epsilon = \infty$ or not depends on both m and κ. For $m \leq 3$ it is never optimal to add any noise, so that $s_\epsilon = \infty$ is always optimal. For $m \geq 4$, on the other hand, the seller of information would like to sell noisy signals, $s_\epsilon < \infty$ if and only if $\kappa \leq \bar{\kappa}_m$, where κ_m is increasing in m. Rather intuitively, for a fixed m, the monopolist gives the agents her information if and only if the noise per unit of risk tolerance is sufficiently high.

Comparing Figure 2.5 to Figure 2.2, we see that the upper envelope now consists on the fragments of six different profit lines, those that encompass $m \leq 5$ and the m large case. For low values of κ, we find that the monopolist seller of information prefers to give her information to a finite number of agents. The optimality of the $m = 1$ solution though disappears. The intuition for this is driven by the fact that

even after giving an agent perfect information he will face risk at the trading stage. Thereby risk-sharing gains at the interim stage are always possible so long as agents are risk-averse. Furthermore, and precisely due to this trading risk, the economy allows for having multiple informed agents with perfect information - in contrast to the limit-order model here profits are bounded away from zero for $m \geq 2$.

We summarize the above discussion in the following theorem.

Theorem 2. *For $\kappa < \hat{\kappa} \approx 3.1$, the monopolist sells signals with no noise to a finite number of agents (at most five).[20] For $\kappa \geq \hat{\kappa}$ it is optimal to sell signals with vanishing precision to an infinite number of traders.*

As in the analysis in section 2.3, the problem's solution is of the bang-bang nature: (i) either to concentrate the information in the hands of a few traders, or (ii) to disperse it to a large number of them, giving each of them a very noisy signal.

In terms of comparing the solution to the limit order model, we see that for low values of κ, the allocation of information in the models coincide: optimal information sales involve $m = 1$ and $s_\epsilon = \infty$. On the other hand, the model with market-orders has interior optima for m for an open set of values of κ.[21] Figures 2.3 and 2.4 plots equilibrium values of $\text{var}(X|P_x)^{-1}$ and ζ as a function of κ. As more traders receive private information, monopoly power decreases monotonically and informational efficiency increases. The allocation of information for κ large coincides with the corresponding equilibrium in the limit-order model.

2.4.2 Competitive behavior

All our previous results were derived assuming agents acted strategically when trading, i.e. they anticipated the effect of their trading on equilibrium asset prices. Of course, one could solve the model under the alternative competitive assumption, that is, assuming that agents act like price takers.[22] The previous discussion highlights the fact that with a finite amount of noise σ_z, one cannot get away assuming competitive behavior: in the two classes of equilibria discussed in the previous section agents were partially internalizing their trading strategies' effect on prices, namely $\zeta > 0$. Although the equilibria will have different characterizations, it is not clear to what extent the competitive assumption will

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[20]Namely: for $\kappa < \kappa_1 \approx 0.19$ it is optimal to have $m = 1$; for $\kappa_1 < \kappa < \kappa_2 \approx 0.81$, it is optimal to have $m = 2$; for $\kappa_2 < \kappa < \kappa_3 \approx 1.56$, $m = 3$ maximizes profits; for $\kappa_3 < \kappa < \kappa_4 \approx 2.38$, the seller sets $m = 4$; and for $\kappa_4 < \kappa < \hat{\kappa}$, the seller sets $m = 5$.

[21]This is true when treating m as a continuous variable.

[22]As shown in Kyle (1989) and García and Urosević (2003) this assumption is innocuous in large economies, where noise, measured by σ_z, grows with the number of agents N.

affect the qualitative aspects of the optimal contracts. The next Proposition shows that indeed it does: assuming perfect competition the exclusive contracts are never optimal.

Proposition 10. *If agents act as price takers, the optimal sales satisfy $m^* = N$, i.e. the information seller sells to as many agents as she can. Furthermore, as $N \rightarrow \infty$ she sets*

$$y^* = \frac{\kappa^2 \psi_\infty^c}{(1 - \psi_\infty^c)^3} \tag{2.17}$$

where ψ_∞ solves

$$-1 + 3\psi_\infty^c + (\kappa^2 - 3)(\psi_\infty^c)^2 + (1 + \kappa^2)(\psi_\infty^c)^3 = 0; \tag{2.18}$$

Profits satisfy

$$\lim_{N \uparrow \infty} \Pi(m, s_\epsilon) = \frac{1}{2\tau} \frac{(1 - \psi_\infty^c)^2}{\psi_\infty^c (2 + \psi_\infty^c)}. \tag{2.19}$$

Under perfect competition the information seller always chooses to sell to as many agents as possible, controlling the damaging effects of information leakage by giving agents very imprecise signals. The optimality of selling to a single trader disappears. This implies that it is critical to model the price impact of traders in our application. Rather intuitively, it is precisely the fact that the monopolist trader internalizes his trades' impact on prices that drives the optimality of the exclusivity contract in section 2.3. The result has a similar flavor to that in Admati and Pfleiderer (1986), where it is shown, in a large market with perfectly competitive traders, that the seller of information would always optimally sell to all agents. Proposition 10 highlights that it is not the particular structure of the large market in Admati and Pfleiderer (1986) that drives their result, but rather the competitive assumption.[23]

It is also interesting to note how equilibrium prices are affected across the equilibria discussed in Proposition 10 and the one from our previous section. The next Corollary highlights some striking differences.

Corollary 3. *For any value of the primitives κ, equilibrium prices at the optimal sales are less informative if agents act as price takers than if they act strategically. Moreover, price informativeness is increasing in κ.*

The first result may be surprising, since as Kyle (1989) convincingly shows, strategic trading makes agents more cautious with their trades, thereby having less informative

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[23]Of course, the large market that Admati and Pfleiderer (1986) study, following Hellwig (1980a), actually does exhibit competitive behavior even if a non-competitive solution concept is used (Kyle, 1989; García and Urosević, 2003).

prices (see Theorem 7.1 in Kyle, 1989). The intuition for our result is that the information seller will give agents more informative signals under imperfect competition precisely because these traders, in contrasts to competitive traders, will marginally internalize the effect of their trades on prices, i.e. trade less aggressively on their information. In terms of the information revealed by prices this later effect dominates the usual effect of less aggressive trading by the informed.

Moreover, in the competitive case we have that, contrary to the exogenous information model, informational efficiency is *increasing* in risk aversion. The intuition can be grasped by noting that for low risk aversion the seller is forced to sell very noisy signals to control for the dilution in the value of information via information leakage. Risk tolerant agents therefore end up with noisier signals, which makes the equilibrium prices more informative as risk-aversion is increased. This is not only in contrast with the exogenous information model, but also with the model with strategic traders, where for $\kappa > \kappa^*$ we have that price informativeness decreases in κ. We conclude that the price-taking assumption eliminates the $m = 1$ equilibria and generates contrary comparative static results with respect to the solution where agents anticipate their impact on prices.

2.4.3 More general information structures

Throughout the paper, we have assumed that the monopolist seller of information markets signals of the form $Y = X + \epsilon_i$ to the agents, where the set $(\epsilon_i)_{i=1}^m$ are i.i.d. with $\epsilon_i \sim \mathcal{N}(0, \sigma_\epsilon^2)$. This class of signals are what Admati and Pfleiderer (1986) refer to as allocations with "personalized noise," in contrast to the case where the signals ϵ_i are perfectly correlated (as in Admati and Pfleiderer, 1988). Two natural questions arise: (i) is this information structure without loss of generality? (ii) how does the equilibrium change if the monopolist did not observe X, but rather a noisy signal of the form $X + \delta$, with $\delta \sim \mathcal{N}(0, \sigma_\delta^2)$? The answer to (i) is critical, since our analysis does not subsume the case studied in Admati and Pfleiderer (1988), where the market structure is as in section 2.4.1, but signals are perfectly correlated. Furthermore, addressing (ii) is also important in terms of checking how robust the corner solutions we have found actually are.

In the most general case, the monopolist seller would have at her disposal allocations of information among m traders of the form $Y_i = X + \delta + \epsilon_i$, with $\epsilon = (\epsilon_i)_{i=1}^m \sim \mathcal{N}(0, \Sigma_\epsilon)$, where her choice variable is the positive semi-definite variance-covariance matrix Σ_ϵ. In principle, her choices are therefore how many agents to sell to, m, as well as the elements of this matrix, a total of $m + m(m-1)/2$ entries, a rather daunting optimization problem. In order to gain some intuition, and at least a partial answer to (i) above, we let $\sigma_\delta = 0$ and study numerically the problem for a fixed m, when the monopolist seller chooses

Σ_ϵ to be a symmetric matrix. Essentially this allows us to address to what extend the optimal solution changes when $m > 1$. The characterization of the equilibrium, fixing m and Σ_ϵ is standard. For a fixed m, and under symmetric allocations, the monopolist profits are a function of the actual variance of ϵ_i, σ_ϵ^2, as well as the correlation between signals sold to different traders, which we shall denote by ρ. One can verify that the profits are a decreasing function of ρ: rather intuitively, the monopolist is better off selling signals with low (or negative) correlation (conditional on X), since the heterogeneity in the information reduces the competition among agents. Furthermore, profits are increasing in m for all $\kappa > \kappa^* \approx 1.74$, i.e. the main qualitative result in Theorem 1 is robust to more general allocations of information. Finally, we note that as $N \uparrow \infty$ the optimal allocations of information converge to those stated in Proposition 8. Rather intuitively, the monopolist seller likes to sell negatively-correlated signals to agents, but since Σ_ϵ needs to be positive semi-definite, as $m \uparrow \infty$ the optimal correlation tends to zero.

Having shown that personalized allocations are indeed optimal, we now turn to discussing the assumption that the monopolist information is perfect, i.e. in the notation introduced above, how does the optimal sales of information change if $\sigma_\delta^2 > 0$? Following our previous analysis, we solve the model fixing m, endowing agents with signals of the form $Y_i = X + \delta + \epsilon_i$, with $\epsilon_i \sim \mathcal{N}(0, \sigma_\epsilon^2)$ i.i.d. We then calculate the profits for the monopolist under the given equilibrium, and solve for the optimal amount of noise σ_ϵ^2 for each m. We find that the nature of the solution, as stated in Theorem 1 does not change: for $\kappa > \kappa_n^*(\sigma_\delta)$ the monopolist optimally chooses to sell to $m = N$ agents, with $\lim_{N \uparrow \infty} s_\epsilon = 0$. On the other hand, for $\kappa < \kappa_n^*(\sigma_\delta)$ the monopolist may now optimally sell to a finite number of agents, much as in the case of market-orders discussed in section 2.4.1.[24] Intuitively, once $\sigma_\delta^2 > 0$ the trading strategies for the agents are risky, so for finite κ, even if small, the solution with a finite m may dominate the $m = 1$ allocation due to risk-sharing gains. Furthermore, when the monopolist has a noisy signal she may choose to sell to $m > 1$ agents signals with no noise added.[25] Finally, we find that the breakpoint κ_n^* changes, namely we have $\kappa_n^* > \kappa^*$, so that noisy signals make the parameter space for which noisy newsletters are optimal smaller.

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[24]For example, if $\kappa = 2$ profits are maximized for $m = 2$, with $\sigma_\epsilon^2 = 0.18$.
[25]Consider the following parameter values: $\sigma_\delta^2 = 10$, $\kappa = 0.1$. The optimal allocation of information satisfies $m = 2$, with $\sigma_\epsilon^2 = 0$.

Appendix A

Following Kyle (1989), we describe the equilibrium in terms of the parameters ψ and ζ, rather than the actual price coefficients. We classify the three models we study as (1) strategic traders who can use limit orders (price-contingent trades); (2) strategic traders who can only use market orders; (3)competitive traders who submit limit orders. Comparisons of (1) and (3) yield differences driven by the competitive assumption, whereas (1) and (2) give insights with respect to the effect of the market structure.

Lemma 4 (Strategic traders, limit orders). *The equilibrium is characterized by ψ and ζ that satisfy*

$$\frac{\zeta}{\tau_i} = \frac{\psi}{\tau_u} \tag{2.20}$$

$$\kappa\sqrt{\frac{\psi}{m(1-\psi)s_\epsilon}} = m\frac{1-2\zeta}{1-\zeta}\frac{(1-\psi)}{(m-\psi)}. \tag{2.21}$$

The certainty equivalent of the informed speculator's profit is given by

$$\frac{1}{2r}\log\left(1+\left(\frac{\tau_i}{\tau_u}-1\right)\frac{(1-2\zeta)}{(1-\zeta)^2}\right) = \frac{1}{2r}\log\left(1+\left(\frac{\zeta-\psi}{\psi}\right)\frac{(1-2\zeta)}{(1-\zeta)^2}\right), \tag{2.22}$$

where

$$\tau_i = 1 + \frac{y\left(1+\psi(m-2)\right)}{m-\psi}, \quad \tau_u = 1 + \psi y.$$

Next Lemma presents the equilibrium prices and expected utilities in a Kyle (1985) model where agents are risk-averse and received signals as in section (2.2.2).

Lemma 5 (Strategic traders, market orders). *The equilibrium is characterized by ψ and ζ that satisfy*

$$\frac{\zeta}{\tau_i} = \frac{\psi}{\tau_u}, \tag{2.23}$$

$$\kappa\sqrt{\frac{\psi}{(1-\psi)ms_\epsilon}} = \frac{(1+\psi s_\epsilon)\tau_u(1-2\zeta)}{\tau_i\tau_u - s_\epsilon(1-\psi)^2}. \tag{2.24}$$

The certainty equivalent of the informed speculator's profit is given by

$$\frac{1}{2r}\log\left(1+\left(\frac{\tau_\pi}{\tau_u}-1\right)\frac{(1-2\zeta)}{(1-\zeta)^2}\right), \tag{2.25}$$

where

$$\tau_\pi = \frac{\tau_i\tau_u^2}{\tau_i\tau_u - s_\epsilon(1-\psi)^2}; \quad \tau_i = 1 + s_\epsilon; \quad \tau_u = 1 + \psi m s_\epsilon.$$

The last Lemma states the equilibrium coefficients and expected utilities in the perfectly competitive market of Hellwig (1980a).

Lemma 6 (Competitive traders, limit orders)**.** *When $m \geq 2$, the equilibrium under perfect competition is characterized by ϕ that satisfies*

$$\frac{(m-1)s_\epsilon}{\sigma_z^2 r^2} = \frac{\phi}{(1-\phi)^3}. \tag{2.26}$$

The certainty equivalent of the informed speculator's profit is given by

$$\Pi(m) = \frac{1}{2r} \log \left(1 + \frac{\tau\beta(1-\phi)}{1 + \psi m s_\epsilon} \right) \tag{2.27}$$

where β is given by $\beta = \sqrt{\frac{\phi\sigma_z^2 s_\epsilon}{(m-1)(1-\phi)}}$ and ψ via $\phi = (m-1)\psi/(m-\psi)$.

When $m = 1$, the equilibrium is characterized by

$$\psi = \frac{s_\epsilon}{s_\epsilon + \sigma_z^2 \tau^2}$$

The certainty equivalent of the informed speculator's profit is given by

$$\Pi(m) = \frac{1}{2r} \log \left(\frac{(1-\psi)s_\epsilon}{1 + \psi s_\epsilon} \right). \tag{2.28}$$

Proof of Lemma 4.

We solve for the equilibrium in Kyle (1989) setup with large number of uninformed speculators following Bernhardt and Taub (2006).[26] The procedure consists in solving an equivalent model in which informed speculators submit market orders conditioning on the price and assuming weak form efficiency:

$$P = \mathbb{E}[X|\omega] = \lambda\omega, \tag{2.29}$$

where ω is the order flow:

$$\omega = \sum_{i=1}^{m} \theta_i(Y_i, \omega) + Z, \tag{2.30}$$

and $\theta_i(Y_i, \omega) = \beta Y_i - \gamma\omega$ is speculator i market order. We notice that observing the

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[26]More precisely, we extend their results to the case of risk aversion and to a different signal structure. The reader can verify that the system of equations in Lemma 4, that characterizes the endogenous variables ψ and ζ as a function of the exogenous parameters (κ, y, m), corresponds to the results in Kyle (1989).

order flow is informationally equivalent to observing

$$g(P) \equiv \frac{\omega(1+m\gamma)}{\beta m} = X + \frac{1}{m}\left(\sum_{i=1}^{m}\epsilon_i + \frac{Z}{\beta}\right). \tag{2.31}$$

Computing (2.29) using (2.31) yields

$$\lambda = \frac{\beta m s_\epsilon\left(1+\gamma m\right)}{\beta^2 m(1+mse)+\sigma_z^2 s_\epsilon}, \tag{2.32}$$

and

$$\tau_u \equiv \text{var}[x|\omega]^{-1} = 1 + m s_\epsilon \psi, \tag{2.33}$$

where

$$\psi = \frac{\beta^2 m}{\beta^2 m + \sigma_z^2 s_\epsilon}. \tag{2.34}$$

For speculator i, the order flow can be written as

$$
\begin{aligned}
\omega &= \theta_i(Y_i,\omega) + \sum_{j=1,j\neq i}^{m}\theta_j(Y_j,\omega) + Z \Leftrightarrow \\
\omega &= \frac{\theta_i(Y_i,\omega) + \beta \sum\limits_{j=1,j\neq i}^{m} Y_j + Z}{1+(m-1)\gamma},
\end{aligned}
$$

hence a speculator price impact is given by

$$P_\theta \equiv \frac{\partial P}{\partial \theta_i} = \frac{\lambda}{1+(m-1)\gamma}. \tag{2.35}$$

Speculator i chooses θ_i in order to maximize expected utility. The first order condition gives

$$\theta_i = \frac{\mathbb{E}[X|Y_i,\omega] - P}{r\tau_i^{-1} + P_\theta}. \tag{2.36}$$

To compute conditional moments in (2.36), notice that for speculator i, the price is informationally equivalent to the signal $g_i(P)$, defined as

$$g_i(P) \equiv \frac{\omega(1+(m-1)\gamma) - \theta_i}{\beta(m-1)} = X + \frac{1}{(m-1)}\left(\sum_{j=1,j\neq i}^{m}\epsilon_j + \frac{Z}{\beta}\right). \tag{2.37}$$

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Using (2.37), speculator i conditional precision and conditional expectation are given by

$$\tau_i \equiv Var^i[x]^{-1} = 1 + s_\epsilon + s_\epsilon(m-1)\phi, \tag{2.38}$$

where

$$\phi = \frac{(m-1)\beta^2}{(m-1)\beta^2 + \sigma_Z^2 s_\epsilon}, \tag{2.39}$$

and

$$\mathbb{E}[x|Y_i, \omega] = \frac{s_\epsilon Y_i + s_p g_i(P)}{\tau_i} \tag{2.40}$$

$$= \frac{s_\epsilon Y_i + s_\epsilon \phi \frac{\omega(1+(m-1)\gamma)-\theta_i}{\beta}}{\tau_i}. \tag{2.41}$$

Substituting conditional moments into (2.36) and rearranging gives

$$\theta_i = \frac{1}{r\tau_i^{-1} + P_\theta + \frac{\phi s_\epsilon \tau_i^{-1}}{\beta}} \left(\frac{s_\epsilon}{\tau_i} Y_i - (P_\theta - \frac{s_\epsilon \phi}{\tau_i \beta}) \frac{\lambda}{P_\theta} \omega \right) \tag{2.42}$$

Matching coefficients of the above expression with the conjectured strategy yields the following expressions for the undetermined coefficients β, γ :

$$\gamma = \frac{\lambda \tau_i \beta}{s_\epsilon} - \frac{\lambda \phi}{P_\theta}, \tag{2.43}$$

$$\beta = \frac{s_\epsilon(1-\phi)}{r + P_\theta \tau_i}. \tag{2.44}$$

Using (2.32), (2.35), (2.38) and (2.39) the price impact (2.35) can be written as

$$P_\theta = \frac{m\beta s_\epsilon}{(m-1)\beta^2(1+ms_\epsilon) + \sigma_\epsilon^2 s_\epsilon}. \tag{2.45}$$

The equilibrium is fully described by the system of 8 equations (2.32) (2.33), (3.43), (2.38), (2.39), (2.43), (2.44) and (2.45). Let us introduce a new parameter, denoted ζ, defined as the "informational incidence parameter"; ζ measures the (dollar) price variation that can be attributed to a dollar increase in the the informed speculator conditional expectation as a result of a larger realization of his signal. Notice that the

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speculator FOC can be written as

$$\theta_i = \frac{\mathbb{E}[X|Y_i,\omega] - P_\theta(\beta \sum_{j \neq i}^{m} Y_j + Z)}{r\tau_i^{-1} + 2P_\theta},$$

hence, by its definition we have

$$\zeta \equiv \frac{\partial P}{\partial \mathbb{E}[X|Y_i,\omega]} = \frac{\partial P}{\partial \theta_i} \frac{\partial \theta_i}{\partial \mathbb{E}[X|Y_i,\omega]} = \frac{P_\theta \tau_i}{r + 2P_\theta \tau_i}. \tag{2.46}$$

We are now interested in defining the equilibrium as a function of the two endogenous parameters ζ and ψ. For this purpose, using (2.46) and (2.44) we can express

$$\frac{\zeta}{\tau_i} \equiv \frac{P_\theta}{\tau_i P_\theta + \frac{s_\epsilon(1-\phi)}{\beta}}; \tag{2.47}$$

using the definitions of P_θ and ϕ in (2.45) and (2.39),

$$\frac{P_\theta}{\tau_i P_\theta + \frac{s_\epsilon(1-\phi)}{\beta}} = \frac{m\beta^2}{m\beta^2(1+ms_\epsilon) + \sigma_\epsilon^2 s_\epsilon}; \tag{2.48}$$

using the definitions of ψ and τ_u in (3.43) and (2.33),

$$\frac{\psi}{\tau_u} = \frac{m\beta^2}{m\beta^2(1+ms_\epsilon) + \sigma_\epsilon^2 s_\epsilon}. \tag{2.49}$$

The last three equations imply the first equilibrium condition

$$\frac{\zeta}{\tau_i} = \frac{\psi}{\tau_u}. \tag{2.50}$$

Using definitions of τ_i and τ_u and (3.43) to eliminate ϕ from τ_i, (2.50) can be expressed as

$$s_\epsilon = \frac{(m-\psi)(\zeta-\psi)}{\psi m(1 + \psi(m-2) - \zeta(m-\psi))}. \tag{2.51}$$

To derive the second equilibrium condition (2.21), using (2.46) we can rewrite (2.44) as

$$\beta = \frac{s_\epsilon}{r} \frac{1-2\zeta}{1-\zeta}(1-\phi); \tag{2.52}$$

then using (3.43) and (2.39) to eliminate β and ϕ from (2.52) yields the result. Therefore, (2.51) and (2.21) define the equilibrium in the endogenous ζ and ψ as a function of the

exogenous (m, s_ϵ, κ).

The value of information for each trader is given by the certainty equivalent of wealth for an informed speculator. Given an insider demand θ_i and profits $\pi_i = \theta_i (X - P)$, interim welfare is given by $\mathbb{E}\left[-\exp\left(-r\pi_i\right)|Y_i, P_x\right]$. Using standard results on the integration of exponential of quadratic forms of normal random variables, the expression for the certainty equivalent reduces to

$$\frac{1}{2r} \log\left(1 + r\left(V^u[X - P] - V^i[X - P]\right) \frac{2P_\theta + rV^i[X - P]}{(P_\theta + rV^i[X - P])^2}\right). \tag{2.53}$$

As informed speculators condition both on signals and prices, in this case we can use (2.46) and definitions of τ_i, τ_u to rewrite (2.53) as (2.22).

Proof of Lemma 5

Assuming weak form efficiency:

$$P = \mathbb{E}[X|\omega] = \lambda\omega, \tag{2.54}$$

where ω is the order flow:

$$\omega = \sum_{i=1}^{m} \theta_i(Y_i) + u, \tag{2.55}$$

and $\theta_i(Y_i, \omega) = \beta Y_i$ is speculator i market order. Given the conjectured strategy, the market maker's conditional expectation is based on

$$\omega = \beta \sum_{i=1}^{m} Y_i + u = m\beta\left(X + \frac{1}{m}\left(\sum_{i=1}^{m} \epsilon_i + \frac{u}{\beta}\right)\right).$$

Hence, computing (2.54) yields

$$\lambda = \frac{\psi s_\epsilon}{\tau_u \beta}, \tag{2.56}$$

where

$$\tau_u \equiv \text{var}[x|\omega]^{-1} = 1 + m s_\epsilon \psi, \tag{2.57}$$

and

$$\psi = \frac{\beta^2 m}{\beta^2 m + \sigma_u^2 s_\epsilon}. \tag{2.58}$$

Notice that for speculator i, the order flow can be written as

$$\omega = \theta_i(Y_i) + \sum_{j=1, j\neq i}^{m} \theta_j(Y_j) + u,$$

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so that a speculator price impact is given by

$$P_\theta \equiv \frac{\partial P}{\partial \theta_i} = \lambda. \tag{2.59}$$

Speculator FOC gives

$$\theta_i = \frac{\mathbb{E}[X - P|Y_i]}{r\tau_\pi^{-1} + P_\theta}. \tag{2.60}$$

As speculator i only observes his private signal, the conditional moments of the payoff are given by

$$\mathbb{E}^i[X] \equiv \mathbb{E}[X|Y_i] = \frac{s_\epsilon Y_i}{\tau_i}, \tag{2.61}$$

$$\tau_i \equiv \text{var}[X|Y_i]^{-1} = 1 + s_\epsilon; \tag{2.62}$$

while the conditional moments of the profits are:

$$\mathbb{E}^i[X - P] = \mathbb{E}^i[X](1 - P_\theta(m-1)\beta) - P_\theta\theta_i, \tag{2.63}$$

$$\tau_\pi \equiv \text{var}[X - P|Y_i]^{-1} = \left(\tau_i^{-1}(1 - P_\theta(m-1)\beta)^2 + P_\theta\left(\frac{\beta^2(m-1)}{s_\epsilon} + \sigma_u^2\right)\right)^{-1} \tag{2.64}$$

using (2.56) and rearranging, the last formula simplifies into

$$\tau_\pi = \frac{\tau_i \tau_u^2}{\tau_i \tau_u - s_\epsilon(1-\psi)^2}. \tag{2.65}$$

Substituting (2.63) into speculator's FOC and rearranging gives

$$\theta_i = \frac{s_\epsilon(1 - P_\theta(m-1)\beta)}{\tau_i\left(r\tau_\pi^{-1} + 2P_\theta\right)}Y_i \tag{2.66}$$

and solving for β gives

$$\beta = \frac{s_\epsilon(1 + s_\epsilon\psi)\tau_\pi}{\tau_i\tau_u(r + 2P_\theta\tau_\pi)}, \tag{2.67}$$

where in the second equality we made use of (2.56).

Defining the parameter ζ as in the limit-order case:

$$\zeta \equiv \frac{\partial P}{\partial\mathbb{E}[x|Y_i]} = \frac{\partial P}{\partial\theta_i}\frac{\partial\theta_i}{\partial\mathbb{E}[x|Y]} = \frac{P_\theta\tau_\pi}{r + 2P_\theta\tau_\pi}. \tag{2.68}$$

Using (2.68) we can express (2.67) as

$$\beta = \frac{s_\epsilon(1 + s_\epsilon\psi)\zeta}{\tau_i\tau_u P_\theta}. \tag{2.69}$$

Using (2.56) and (2.59) to eliminate β in the l.h.s. of (2.69) and rearranging, we get the first equilibrium condition

$$\frac{\zeta}{\tau_i} = \frac{\psi}{1 + \psi s_\epsilon}; \tag{2.70}$$

that can be solved for s_ϵ giving

$$s_\epsilon = \frac{\zeta - \psi}{\psi(1 - \zeta)}. \tag{2.71}$$

The second equilibrium condition is obtained from (2.67) in the following way: use (2.58) to eliminate β in the l.h.s., and (2.68) and (2.65) to eliminate respectively P_θ and τ_π in the r.h.s., yielding

$$\kappa\sqrt{\frac{\psi}{(1 - \psi)m s_\epsilon}} = \frac{(1 + \psi s_\epsilon)\tau_u(1 - 2\zeta)}{\tau_i\tau_u - s_\epsilon(1 - \psi)^2}. \tag{2.72}$$

To derive the certainty equivalent expression in (2.25), as informed speculators condition only on signals, using (2.68) and definitions of τ_π, τ_u into (2.53) yields the desired result.

Appendix B

As it is going to be used in what follows, we rewrite the two equilibrium conditions as:

$$y = \frac{(m - \psi)(\zeta - \psi)}{\psi(1 + \psi(m - 2) - \zeta(m - \psi))}, \tag{2.73}$$

and combining the above expression and (2.21) in order to eliminate y,

$$\frac{\kappa}{m(1 - \zeta)} \sqrt{\frac{(m - \psi)(\zeta - \psi)(1 + \psi(m - 2) - \zeta(m - \psi))}{(1 - \psi)^2}} = \frac{(\zeta - \psi)}{\psi} \frac{1 - 2\zeta}{(1 - \zeta)^2} \tag{2.74}$$

Next we state and proof two useful results[27]

Result 1 as $\frac{\kappa^2}{y} \to 0$, we have $\zeta \to 1/2$.

Proof. Immediate by taking limits in (2.21).

Result 2: As $y \to \infty$, the certainty equivalent in (2.22) vanishes if $m \geq 2$.

Proof. By Result 1 we have $\zeta \to 1/2$, and taking limits as $\zeta \to 1/2$ and $y \to \infty$ in (2.73) we have that $\psi \to \frac{m-2}{2(m-2)+1}$. These limits imply that the l.h.s. of (2.74) is zero. Comparing the r.h.s. of (2.74) and (2.22) yields the result.

Proof of Proposition 7

We first show that the monopolist sets $s_\epsilon = \infty$ when $m = 1$. Using (2.73)-(2.74) to eliminate ψ and ζ in (2.22), we have that, for any $\kappa \geq 0$, the monopolist problem reduces to

$$\max_{s_\epsilon} \quad \Pi(1) = \max_{s_\epsilon}, \frac{1}{2r} \log \left(1 + r\sigma_z \frac{\sqrt{r^2 \sigma_z^2 + 4s_\epsilon(1 + s_\epsilon)} - r\sigma_z}{2(1 + s_\epsilon)}\right). \tag{2.75}$$

It is easy to verify that the above function is strictly increasing in s_ϵ , so that the optimal solution is to set $s_\epsilon = \infty$. Using Result 1, the fact that $s_\epsilon \to \infty$ implies $\psi \to 0$ from (2.73). Moreover (2.73) also implies that in this case $y\psi \to 1$, and therefore that $var(X|P_x)^{-1} = 2$. Furthermore, taking limits in (2.75) we have that when the monopolist sells to a single agent her profits are given by

$$\Pi(1) = \frac{1}{2r} \log \left(1 + r\sigma_z\right). \tag{2.76}$$

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[27]These are basically equivalent to Theorem 7.2 and Theorem 7.5 in Kyle (1989).

When $r \downarrow 0$ we obviously have that[28]

$$\lim_{r \downarrow 0} \Pi(1) = \frac{\sigma_z}{2} \tag{2.77}$$

Next we show that the allocation $s_\epsilon = \infty$ and $m = 1$ is indeed optimal for an open interval of κ around zero. Making use of (2.74) and Result 1, and taking limits as $r \downarrow 0$ in (2.6) one can verify that the problem for $\kappa = 0$ reduces to

$$\max_{m,\psi} \quad \Pi(m, \psi) = \frac{\sigma_z}{2} \sqrt{\frac{(m - \psi)(1 - 2\psi)(\psi - (m-2)(1 - 2\psi))}{(1 - \psi)^3}}, \tag{2.78}$$

such that the constraint (2.73) holds with $\zeta = 1/2$. Using the constraint into the above expression to eliminate ψ, we get

$$\Pi(m, \psi) = \frac{\sigma_z}{2} \sqrt{\frac{y}{1 + y}} h(m, y),$$

where

$$h(m, y)^2 = \frac{1 + 2y + 2(2 + y(2 + y))m - (2 + y)^2 m^2 + (m(2 + y) - 1)\sqrt{(1 + 2y^2) - 2(2 + y(3 + 2y))m + (2 + y)^2 m}}{2(1 + y)^2}$$

It is easy to see that the function $h(m, y)$ is such that $h(1, y) = 1$ and $h(m, y) < 1$ for all $m > 1$. This implies that for any value of y, as $r \to 0$ monopolist's profits are *strictly* higher when selling information to only one trader than in any other case. Furthermore, one can readily check that $\Pi'(1) < 0$. The statement in the Proposition then follows from the fact that the profit function as defined by (2.6) and the constraints (2.73)-(2.74) are continuous in κ. $\square$

Proof of Proposition 8.

First we characterize the solution of the monopolist's problem for $m \to \infty$. Consider the problem for fixed y and rewrite (2.73) making explicit the dependence of the endogenous variables on a particular value of m with a subscript, i.e. write $\zeta_m \equiv \zeta(m)$ and $\psi_m \equiv \psi(m)$:

$$y = \frac{(m - \psi_m)(\zeta_m - \psi_m)}{\psi_m(1 + \psi_m(m - 2) - \zeta_m(m - \psi_m))}. \tag{2.79}$$

For (2.79) to hold as an equality in the limit as $m \to \infty$, it is clear that, denoting $\psi_\infty \equiv \lim_{m \uparrow \infty} \psi_m$, we must have $\lim_{m \uparrow \infty} \zeta_m = \psi_\infty$. Moreover, denote $\alpha_m \equiv (m -$

$\psi_m)(\zeta_m - \psi_m)$, and $\alpha_\infty \equiv \lim_{m\uparrow\infty} \alpha_m$. Then, taking limits as $m \to \infty$ in (2.6, one can verify that

$$\lim_{m\uparrow\infty} \Pi(m, y; \kappa) = \frac{y(1 - 2\psi_\infty)}{2r(1 + y\psi_\infty)}, \tag{2.80}$$

and that the constraint (2.21) reduces to

$$\kappa\sqrt{\frac{\psi_\infty}{y(1 - \psi_\infty)}} = 1 - 2\psi_\infty. \tag{2.81}$$

Using (2.81) to eliminate y in (2.80) the monopolist's problem becomes

$$\max_{\psi_\infty} \quad \Pi(\psi_\infty) = \frac{\kappa^2}{2r} \frac{\psi_\infty(1 - 2\psi_\infty)}{\left((1 - \psi_\infty)(1 - 2\psi_\infty)^2 + \kappa^2\psi_\infty^2\right)}$$

Equating to zero the derivative of the above expression yields the optimality condition for $\psi_\infty(k)$ in (2.11). Moreover, from (2.11) we derive $\kappa^2\psi_\infty^2 = (1 - 2\psi)^2(1 - 2\psi^2)$, and using this into (2.81) yields (2.10) and (2.12); using (2.10) into (2.80) yields (2.13).

Finally, notice that in this case profits have a finite limit as risk aversion diverges: using (2.13)-(2.11) we get

$$\lim_{r\uparrow\infty} \Pi_\infty(r, \sigma_z) = \frac{\sigma_z}{4}. \tag{2.82}$$

Next we show that profits are indeed increasing in m for κ large enough. Define $\hat{\kappa} \equiv 1/\kappa$. Let y be given. For (2.21) to hold as $\hat{\kappa} \to 0$, we must have $\psi \to 0$. This implies that, for (2.73) to hold as $\hat{\kappa} \to 0$, we must have both $\zeta \to \psi$ and $(\zeta - \psi)/\psi \to y/m$. As a consequence, from (??),

$$\lim_{r\uparrow\infty} v(\zeta, \psi)q(\zeta) = \frac{y}{m}. \tag{2.83}$$

As $\kappa = r\sigma_z$, we consider separately the two cases. First, let σ_z be fixed. Then as $\hat{\kappa} \to 0$, from (2.6) and (2.83), we have that for all y, $\lim_{r\uparrow\infty} \Pi(m, y; r, \sigma_z) = 0$. Therefore, as risk aversion grows large, profits converge to zero when the monopolist sells to a finite m , and converge to a positive limit when the monopolist sells to an infinite number of traders from.(2.82). Second, let r be fixed. Then as $\sigma_z \to \infty$, from (2.6) and (2.83), we have that

$$\lim_{\sigma_z\uparrow\infty} \Pi(m, y; r, \sigma_z) = \frac{m}{2r} \log\left(1 + \frac{y}{m}\right),$$

which is clearly increasing in m. The statement in the Proposition then follows from the fact that the profit function as defined by (2.6) and the constraints (2.73)-(2.74) is continuous in $\hat{\kappa}$.

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Finally, in order to see that prices are less informative as κ increases, we note that it is sufficient to show that $d\psi_\infty/d\kappa < 0$ for all $\kappa > \bar{\kappa}$. The later statement follows by applying the implicit function theorem to (2.11), and then using (2.11) itself and the restriction $\psi \in [0, 1/2]$ in order to conclude $d\psi_\infty/d\kappa < 0$. $\square$

Proof of Proposition 9.

We show showing that as $m \to \infty$ the monopolist's problem in the model with market orders and with limit orders coincide, as it follows that the solution and the equilibrium properties considered coincide as well. Consider the problem for fixed y and rewrite (2.71) denoting $\zeta_m \equiv \zeta(m)$ and $\psi_m \equiv \psi(m)$:

$$y = \frac{m(\zeta_m - \psi_m)}{\psi_m(1 - \zeta_m)}. \tag{2.84}$$

For the above equation to hold as $m \to \infty$, it is clear that, denoting $\psi_\infty \equiv \lim_{m\uparrow\infty} \psi_m$, we must have $\lim_{m\uparrow\infty} \zeta_m = \psi_\infty$. Moreover, denote $\alpha_m \equiv m(\zeta_m - \psi_m)$, and $\alpha_\infty \equiv \lim_{m\uparrow\infty} \alpha_m$. Then, taking limits as $m \to \infty$ in the profit function where each agent certainty equivalent is given by (2.25), one can verify that

$$\lim_{m\uparrow\infty} \Pi(m, s_\epsilon; \kappa) = \frac{y(1 - 2\psi_\infty)}{2r(1 + y\psi_\infty)}, \tag{2.85}$$

and that the constraint (2.72) reduces to

$$\kappa \sqrt{\frac{\psi_\infty}{y(1 - \psi_\infty)}} = 1 - 2\psi_\infty. \tag{2.86}$$

As the last two equations are identical to (2.80) and (2.81), the problems with limit and market orders coincide. $\square$

Proof of Proposition 10.

Using the constraint (2.26) to eliminate s_ϵ one can verify that, for a fixed m, the monopolist seller of information is solving

$$\max_\phi \hat{\Pi}(m, \phi) = \frac{m}{2r} \log(1 + \Gamma); \tag{2.87}$$

$$\Gamma \equiv \frac{\kappa^2 \phi}{(1 - \phi)\left[m - 1 + \phi + \frac{(\kappa m \phi)^2}{(m-1)(1-\phi)^3}\right]}. \tag{2.88}$$

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The first-order condition for ϕ_m yields :

$$\frac{(\kappa m \phi_m^*)^2}{(m-1)(1-\phi_m^*)^3} = \frac{m - 1 + (\phi_m^*)^2}{1 + \phi_m^*}. \tag{2.89}$$

By the envelope theorem

$$\frac{d\Pi^*(m)}{dm} = \frac{\partial \Pi(m, \phi_m^*)}{\partial m} = \frac{1}{2r}\left[\log\left(1 + \Gamma\right) - \frac{\Gamma}{1 + \Gamma}\alpha\right], \tag{2.90}$$

where

$$\alpha \equiv \frac{m + \frac{m-2}{m-1}\frac{(\kappa m \phi_m^*)^2}{(m-1)(1-\phi_m^*)^3}}{m - 1 + \phi + \frac{(\kappa m \phi)^2}{(m-1)(1-\phi)^3}}. \tag{2.91}$$

From (2.90), a sufficient condition for profits to be increasing in m is that $\alpha \leq 1$. Substituting from (2.89) into (2.91) and rearranging one can verify that indeed $\alpha \leq 1$, so the sufficient condition is satisfied and, as a consequence, the monopolist finds optimal to sell to $m = N$, in the limit as $N \uparrow \infty$ to an infinite amount of traders. The rest of expressions in the Proposition are immediate taking limits in (2.87) and (2.26), using (2.89). $\square$

Proof of Corollary 3. Informational efficiency is measured by the precision conditional on the market price, given by $\tau_u = 1 + y\psi$. Notice that in the limit as $m \to \infty$, we have $\phi \to \psi$. In the competitive case we can express (2.18) as $(\kappa \phi_\infty)^2 = \frac{(1-\phi_\infty)^3}{1+\phi_\infty}$ and substitute into (2.17) to get $y^*\phi_\infty = \frac{1}{1+\phi_\infty}$; as $\phi_\infty \in [0, 1]$, we have $\tau_u \in [1.5, 2]$. In the strategic case, we consider separately the newsletter and the exclusivity equilibria. In the latter case, as informed traders trade on perfectly precise signals, we have the usual result that $\tau_u = 2$. In the newsletter equilibrium, we have that maximizing (2.13) over ϕ yields the optimality condition $\phi_\infty^4 - \phi_\infty^3 + \frac{\kappa^2-2}{8}\phi_\infty^2 + \frac{1}{2}\phi_\infty - \frac{1}{8} = 0$. Solving for $(\kappa \phi_\infty)^2$ yields $(\kappa \phi_\infty)^2 = (1 - 2\phi_\infty^2)(1 - 2\phi_\infty)^2$; substituting into (2.10) we get $y^*\phi_\infty = \frac{1-2\phi_\infty^2}{1-\phi_\infty}$; as in this case $\phi_\infty \in [0, 1/2]$, we have that $\tau_u \geq 2$. As one can verify by applying the implicit function theorem to the polynomials that define them, $\phi_\infty(\kappa)$ are monotonically decreasing functions of κ. In turn, this implies that τ_u is monotonically *increasing* in κ in the competitive equilibrium, while one can verify numerically that τ_u is monotonically decreasing in κ for $\kappa \geq \kappa$. $\square$

Information seller offers signals   Agents observe signal Y_i Agents submit demand schedules   Trading profits π_i are realized

Market clears

$$t=0 \qquad\qquad t=1 \qquad\qquad t=2 \qquad\qquad t=3$$
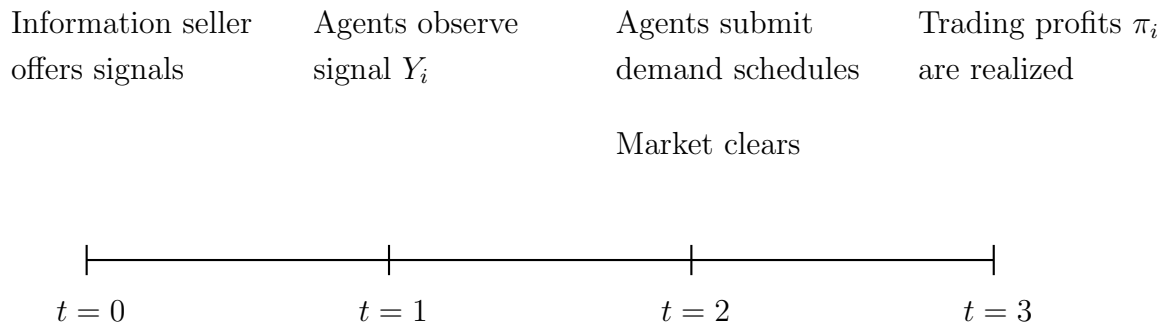
Figure 2.1: Timeline.

Profits as a function of kappa

Figure 2.2: Equilibrium consumer surplus for different values of m and κ, at the optimal noise s_ϵ, in the Kyle (1989) model. The solid lines correspond to the profits when $m = 1$ and $m = N$ (for large N), whereas the dashed lines correspond to values of $m = 2, \ldots, 40$.
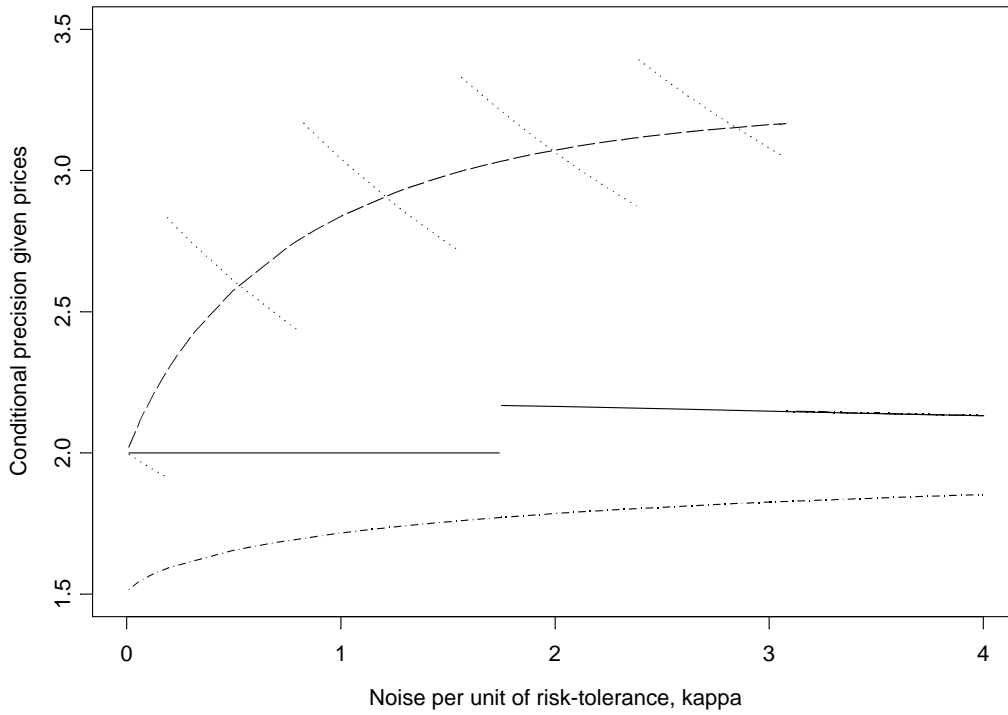
Figure 2.3: Equilibrium values for $\text{var}(X|P_x)^{-1}$ for different values κ. The solid lines corresponds to the model with strategic traders and limit orders. The dotted and long-dash lines correspond to the model with strategic traders and market orders (dotted for the case where m is treated as an integer, dashed when m is treated as a continuous variable). The line with dashes and dots corresponds to the model where traders are price takers.
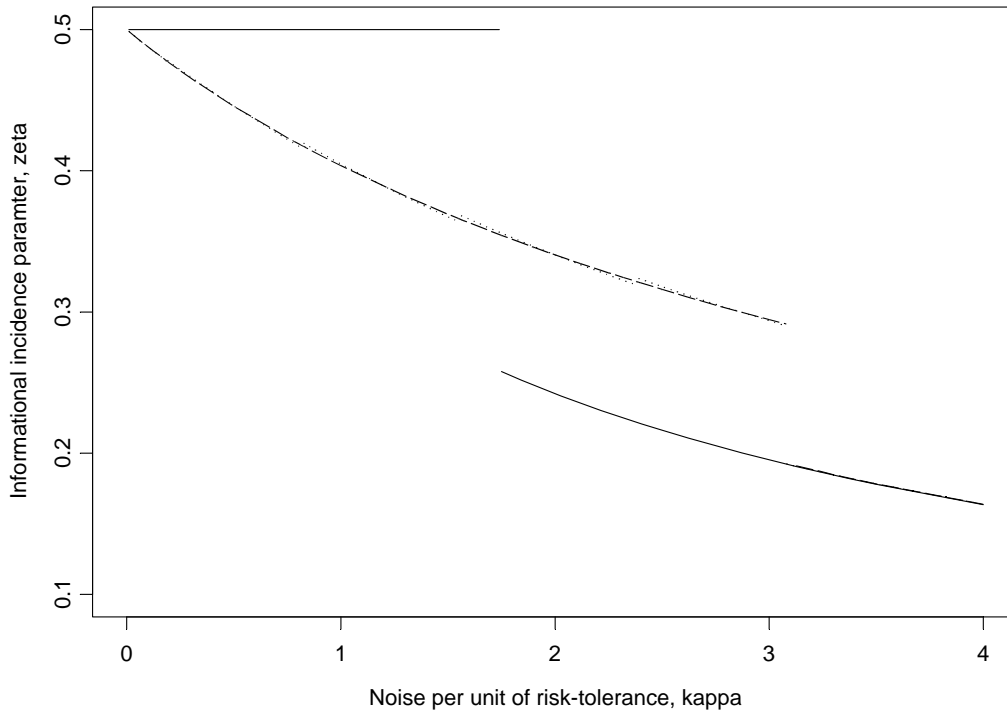
Figure 2.4: Equilibrium values for ζ for different values κ. The solid lines corresponds to the model with strategic traders and limit orders. The dotted and long-dash lines correspond to the model with strategic traders and market orders (dotted for the case where m is treated as an integer, dashed when m is treated as a continuous variable). The line with dashes and dots corresponds to the model where traders are price takers.
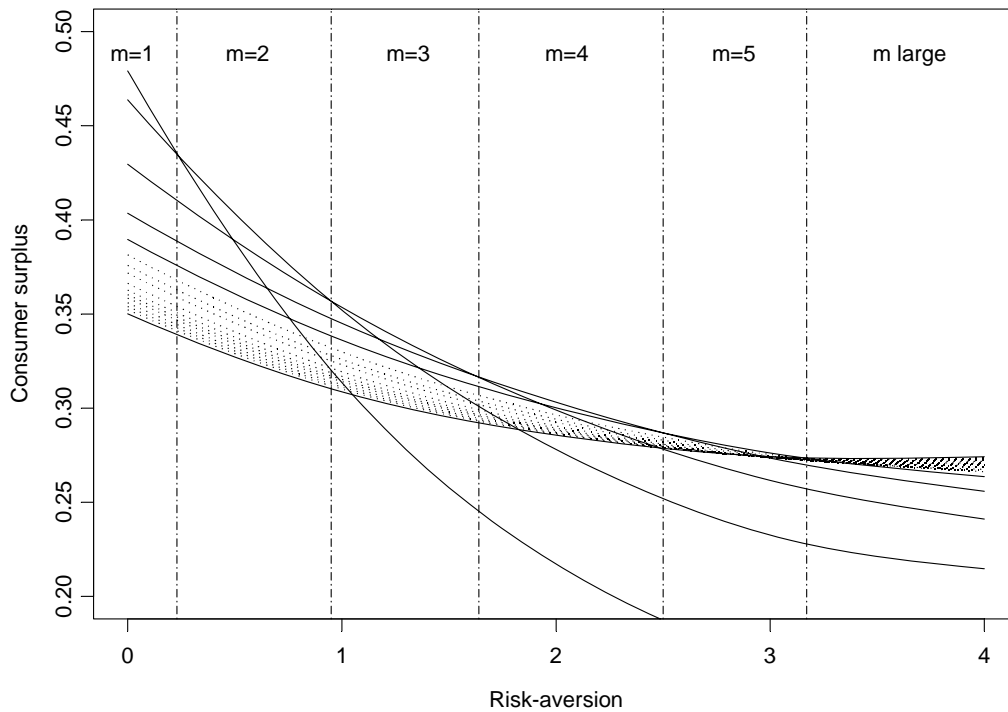
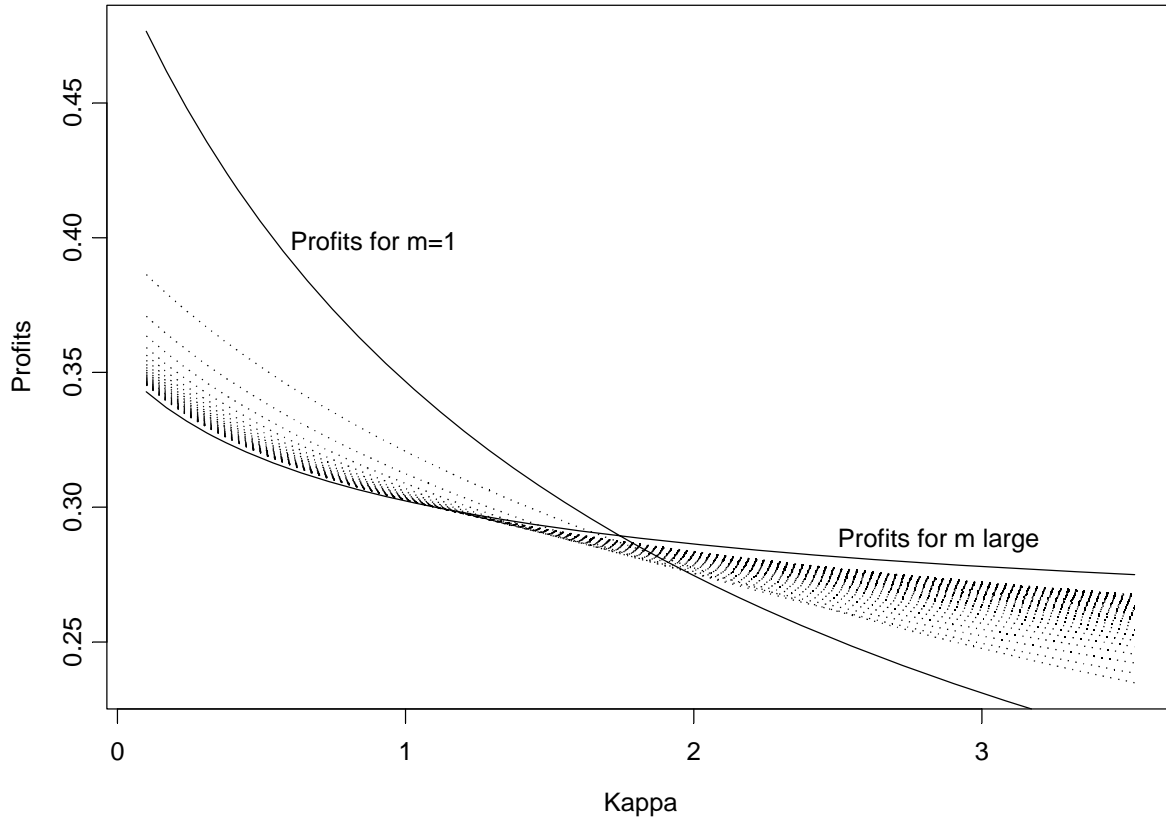Figure 2.5: Equilibrium consumer surplus for different values of m and κ, at the optimal noise s_ϵ, in the Kyle (1985) model. The solid lines correspond to the profits when $m = 1, 2, 3, 4, 5$ and $m = N$ (for large N), whereas the dotted lines correspond to values of $m = 6, \ldots, 40$. The vertical lines give the breakpoints between regions where different m are optimal.
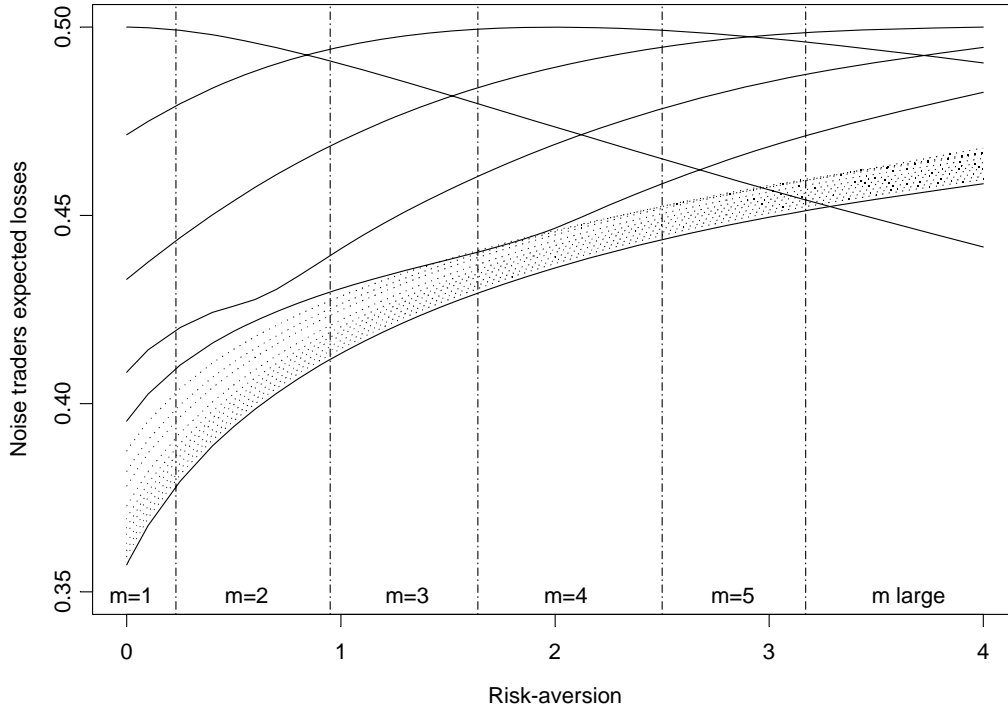
Figure 2.6: Equilibrium expected noise trader losses, or aggregate expected profits for the informed traders, for different values of m and κ, at the optimal noise s_ϵ, in the Kyle (1985) model. The solid lines correspond to the profits when $m = 1, 2, 3, 4, 5$ and $m = N$ (for large N), whereas the dotted lines correspond to values of $m = 6, \ldots, 40$. The functions corresponding to $m = 4$ and $m = 5$ have singularities at $\kappa \approx 0.65$ and $\kappa \approx 1.87$. The vertical lines give the breakpoints between regions where different m are optimal.

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Chapter 3

Information and Expected Returns with Large Informed Traders

3.1 Introduction

The issue of how asset prices form and evolve when agents participating in financial markets are asymmetrically informed has been a concern to financial economists for a long time. Indeed, the core of the market microstructure literature is precisely to understand the mechanics of the process by which the information that is dispersed among market participants prior to the trading stage gets incorporated into trading prices. Understanding whether and how prices become efficient is of major interest, because prices contribute to the efficient allocation of resources in the economy only to the extent that they effectively reflect information about the fundamental value of the underlying asset. In particular, one way in which asset prices transmit to the real economy is by determining firms' cost of capital, which is the required return on equity. Understanding the determinants of expected returns is the major focus of the asset pricing literature. The problems of modeling and comprehending the price formation mechanism one the one hand, and the determinants of expected returns on the other seem therefore naturally related. Despite this apparent connection, the literature on market microstructure and asset pricing typically abstract one from the other. Generally speaking, asset pricing models focus on the pricing of risk assuming that all efficiency issues have been resolved, and that investors have homogenous beliefs; market microstructure models focus on price formation and efficiency when traders are asymmetrically informed but abstracting from the pricing of risk.

This paper provides a framework to investigate how the degree of asymmetric information influences the risk premium. The mechanism we propose relates the endogenous

volume of informed trading with the pricing of risk: when large (i.e. strategic) informed traders are faced by a smaller number of uninformed, a situation which we associate to more asymmetric information, they are forced to trade less on their information not to dissipate their profits. As a result prices convey less information and the risk faced by uninformed traders increases, resulting in higher risk premium. Our model has two key elements: risk-averse pricing and strategic behavior of informed agents. As traders are expected to hold some positive amount of risk in equilibrium, the risk-aversion assumption guarantees that this risk will be priced. In turn, the price of risk depends on the uncertainty perceived by uninformed traders, measured by the volatility of the future cash flows conditional on the information available to them at the time of trading. In a rational expectations framework, uninformed traders will efficiently use the information transmitted by equilibrium prices, as prices convey a noisy signal of what better informed traders know. The precision of this signal is endogenous. On their side, informed traders are aware of the impact that their trades have on equilibrium prices. Effectively, they realize that they face an upward sloping supply schedule, i.e. the price moves against them as they trade, which has a damaging effect on their profits. When the insider is faced by a smaller number of risk averse traders, she has an incentive to reduce her trading aggressiveness as the slope of the residual supply, and hence the price impact, is high. As a result of lower informed trading volume, the precision of the uninformed estimates the fundamental value of the asset is reduced, which in turn increases the price impact of informed traders, reinforcing the initial incentive to reduce their trading volume. As information leakage is reduced, the uninformed side of the market perceives the asset as riskier, and demands a higher compensation to absorb the order flow and hold the asset. The latter effect reflects into higher illiquidity (price impact of the order flow) and higher expected returns.

We remark that in deriving our results the per capita amount of risk that agents have to bear (the per capita supply of the risky asset) is deliberately kept fixed. We therefore provide an additional reason for why an increase in the investor base should result in lower cost of capital, on top of the risk sharing considerations discussed in Merton (1987).

3.1.1   Related literature

A number of empirical studies investigate the correlation between different measures of asymmetric information and risk premia in the cross section of stock returns. One of such measures is the price impact of the order flow, often denoted as Kyle's lambda. In the spirit of Kyle (1985), this should be related to the extent of asymmetric information as uninformed investors who set prices respond to the adverse selection problem that arises

from trading against better informed traders. Brennan and Subrahmanyam (1996) is a leading example. They find a significant relationship between required rates of return and measures of illiquidity, or price impact, in the cross section. Amihud (2002) finds similar results at different frequencies. Easley, Hvidjkaer, and O'hara (2002) derive a different trade based measure of information risk: the probability that a trade is initiated by a trader with private information (PIN). Their main result is that stocks with higher probability of information-based trading have higher rates of return. Despite the empirical evidence of a positive relationship between various proxies of asymmetric information and expected returns, the economic mechanism behind such a relationship is not clear. The reason is that the theoretical microstructure models that inspire those measures of asymmetric information as Kyle's lambda or PIN, are based on the assumption that the price is set by an infinitely liquid risk neutral market maker. This assumption implies that execution prices equal the conditional expected value of the payoff of the asset X. The price P_t therefore satisfies

$$P_t = \mathbb{E}\left[X|\mathcal{F}_t\right];$$

as a direct consequence, the expected (excess) return per share is zero by construction since

$$\mathbb{E}[X - P] = \mathbb{E}[X - \mathbb{E}\left[X|\mathcal{F}_t\right]] = 0.$$

The rationale for a positive relationship between asymmetric information and expected returns is therefore an indirect one: asymmetric information should be responsible for the price impact in the first place, and then, taking illiquidity as given, investors should require a compensation for the trading costs associated with such illiquidity.[1] In our model both the risk premium and illiquidity are endogenously derived and their relationship can be directly analyzed. We show that (absent any specific preference for liquidity that investors might have) asymmetric information induces a positive correlation between the two, but searching for such an indirect causality might lead to underestimate the direct effects of asymmetric information to expected returns that we discuss.

Our work is related to Easley and O'Hara (2004), that develop a model in the spirit of Grossman and Stiglitz (1980) in which traders are competitive and risk averse. Their main result is to show how the composition of information between private and public affects the cost of capital: for a given amount of information investors require higher returns to hold stocks with more private and less public information. Our approach is complementary to theirs since we identify the degree of asymmetric information with the relative size of informed traders over the uninformed, and not with the fraction of

[1]Amihud and Mendelson (1986) show and test the positive relationship between expected returns and the bid-ask spread. A number of other papers since then investigate the same relationship with mixed evidence. See, for instance, Eleswarapu and Reinganum (1993).

information that is private versus public. Moreover informed traders are non competitive in our model and this assumption is necessary for the mechanism that we illustrate to work.[2] Gârleanu and Pedersen (2004) study how the trading *costs* associated with asymmetric information affect the required return. They show that the bid-ask spread generated by adverse selection has no effect on prices with ex-ante identical traders: costs associated with the spread are offset on average by profits from trading on information.[3] Our contribution is complementary because we relate asymmetric information to the *risk* faced by investors.

3.2   The setup

This section describes the basic assumptions and definitions shared by the models described in the rest of the paper. Two assets are traded in the economy: a risk-less asset (cash) in perfectly elastic supply, and a risky asset with final payoff X. Agents in the economy are asymmetrically informed ex-ante. Prior to the trading stage, one class of agents (which we will refer to as the informed agents, or the insider) has access to private information about the payoff in the form of a noisy signal, denoted $Y = X + \varepsilon$. The rest of the market, the uninformed agents, has only access to public information. The supply of the risky asset, denoted Z, is assumed to be random in order to prevent information asymmetries to disappear in equilibrium. All random variables are defined on some probability space $(\Omega, \mathcal{F}, \mathbb{P})$, are normally distributed and uncorrelated. We let σ_x^2, σ_z^2, and σ_ε^2 denote the variances of X, Z and ε respectively. X and ε are assumed to have zero mean[4]. The above assumptions are standard. Next we discuss elements of the model which are more related to the specific problems studied in this paper. As the analysis focuses on risk premia, want expected returns to be positive. For this to be the case, we need: i) agents in the economy to bear a positive amount of risk on average, and ii) agents to demand a compensation in order to hold this risk. In order to obtain i), we assume the average supply, denoted $\bar{Z}$, to be strictly positive. In order to obtain ii), we assume that uninformed agents are risk averse. For tractability, we assume negative exponential utility functions with common constant absolute risk aversion r. Thus, given terminal wealth W_j, each agent j utility is given by $u_j(W_j) = -e^{-rW_j}$.

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[2]In our model, comparative statics with respect to the composition of information would produce the same qualitative results in Easley and O'Hara (2004). The converse is not true: increasing the fraction of informed traders over the uninformed will generically reduce the cost of capital if traders are competitive. The comparison with a competitive model is carried out in section 5.

[3]However, they find that allocation costs associated with inefficient trading decisions do have an effect on prices.

[4]None of the result would change if we assumed that the payoff had a non-zero mean. Prices are going to be linear in equilibrium, and a positive expected payoff would simply scale the intercept of the price function.

Informed agents will be assumed to be risk averse or risk neutral depending on the context. The second key ingredient of the model is that we want informed traders to realize the impact that trading on superior information has in equilibrium. For this reason we assume informed traders to act strategically. As strategic behavior that is *not* related to private information has a negligible impact on equilibrium (see Grinblatt and Ross (1985)), uninformed traders are assumed to be price takers.[5]

Denote θ_i to be the trading strategy of agent i, i.e. the number of shares of the risky asset that agent i acquires. With this notation, the final wealth for agent i is given by $W_i = \theta_i(X - P_x)$, where P_x denotes the price of the risky asset. Agent i chooses a strategy to maximize utility given the available information. An uninformed trader information set $\mathcal{F}_U$ consists of the risky asset price P_x alone, while an informed trader information set $\mathcal{F}_I$ consists of the signal and of the price in the case of limit orders; of the signal alone in the case of market orders.[6]. Equilibrium requires agents to optimize and markets to clear:

Definition 5. *An equilibrium is defined by a set of trading strategies θ_i and a price function $P_x : \Omega \to \mathbb{R}$ such that:*

1. *Each agent i chooses her trading strategy so as to maximize her expected utility given her information set $\mathcal{F}_i$:*

$$\theta_i \in \arg \max_{\theta} \mathbb{E}\left[u\left(W_i\right) | \mathcal{F}_i\right]. \tag{3.1}$$

2. *Market clears:*

$$\Theta_I + \Theta_U = Z; \tag{3.2}$$

where Θ_j is the total demand of class $j = I, U$.

3.3   Model I: single period batch auction

The purpose of this section is to illustrate the mechanism relating strategic behavior, asymmetric information and expected returns in the simplest setup by means of closed form solutions. As a consequence it is going to be very stylized.

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[5]A similar structure is adopted in Bhattacharya and Spiegel (1991).

[6]Formally, $\mathcal{F}_U$ denotes the σ-algebra generated by (P_x); $\mathcal{F}_I$ denotes the σ-algebra generated by (Y, P_x) or by (Y).

3.3.1 The model

The model is based on Kyle (1985), and replaces the assumption of risk neural market makers (which implies risk neutral pricing and therefore zero expected returns) with risk averse pricing. As in Kyle (1985), there is a strategic, risk neutral insider who knows the value of the fundamental X[7]. Once she observes the realization of X, she chooses a market order taking into account the price impact that her strategy will have in equilibrium. Insider's market order θ_I plus noise trading Z constitute the order flow. Once the order flow is realized, a measure μ of atomistic risk averse traders submit a limit order conditional on the order flow, and the price is set so that market clears. As the relative size of the insider is $1/(1+\mu)$, we interpret μ as an inverse measure for the degree of asymmetric information. A smaller value for μ corresponds to a market in which the insider has to be faced by a smaller number of uninformed traders, a situation that we associate with more asymmetric information.

As the solution of the model is simple and instructive, we provide it in detail. We proceed backwards. Each uninformed trader chooses θ_U to maximize expected utility $\mathbb{E}\left[-e^{-r\theta_U(X-P)}|\mathcal{F}^U\right]$. Standard arguments imply that the optimal demand is linear in the expected excess return per share:

$$\theta_U(P) = \frac{\mathbb{E}\left[X|\mathcal{F}^U\right] - P}{r\mathbb{V}\left[X|\mathcal{F}^U\right]}. \tag{3.3}$$

As market clearing requires

$$\theta_I(X) + \mu\theta_U(P) = Z, \tag{3.4}$$

an uninformed agent at the trading stage can infer a noisy signal of insider's market order $\xi \equiv \theta_I(X) - Z$. As a consequence, her information set can be equivalently expressed in terms of the signal ξ. At this point, in order to compute the conditional moments in (3.3), uninformed traders need to guess how the trading strategy of the informed trader is related to her private information. Not surprisingly, the guess takes a linear form: $\theta_I(X) = \alpha + \beta X$, with $\beta > 0$. Under the conjectured strategy, uninformed posterior beliefs can be readily verified to be:

$$\mathbb{E}\left[X|\xi\right] = \frac{\beta\sigma_x^2}{\beta^2\sigma_x^2 + \sigma_z^2}\left(\xi - \mathbb{E}\left[\xi\right]\right); \qquad \mathbb{V}\left[X|\xi\right] = \frac{\sigma_x^2\sigma_z^2}{\beta^2\sigma_x^2 + \sigma_z^2}. \tag{3.5}$$

We can now turn to the insider's problem. As she is strategic, she takes into account

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[7]In terms of the setup presented in the previous section, we are considering the simplifying assumption that $\sigma_\varepsilon^2 \to 0$. None of the results would change if $\sigma_\varepsilon^2 > 0$.

the impact that her trading strategy has on the price. Substituting the demand (3.3) into (3.4) the insider anticipates that equilibrium price will be

$$P = \mathbb{E}\left[X|\xi\right] + \frac{r}{\mu}\mathbb{V}\left[X|\xi\right]\xi, \tag{3.6}$$

where the beliefs in (3.6) are given from (3.5). As she is risk neutral, she chooses θ_I in order to maximize expected profits $\mathbb{E}\left[\theta_I(X - P)|\mathcal{F}^I\right]$. The first-order condition for her problem is[8]:

$$X - \mathbb{E}\left[P|X\right] - \theta_I \frac{d\mathbb{E}\left[P|X\right]}{d\theta_I} = 0, \tag{3.7}$$

$$X - \mathbb{E}\left[P|X\right] - \theta_I \underbrace{\frac{d\left(\mathbb{E}\left[\mathbb{E}\left[X|\xi\right] + \frac{r}{\mu}\mathbb{V}\left[X|\xi\right]\xi|X\right]\right)}{d\theta_I}}_{price\ impact} = 0, \tag{3.8}$$

$$X - \mathbb{E}\left[P|X\right] - \theta_I \left(\underbrace{\frac{\beta\sigma_x^2}{\beta^2\sigma_x^2 + \sigma_z^2}}_{information\ (Kyle\ '85)} + \underbrace{\frac{r}{\mu}\mathbb{V}\left[X|\xi\right]}_{market\ size,\ risk} \right) = 0. \tag{3.9}$$

The insider realizes that she effectively faces an upward-sloping residual supply curve, i.e. the price will move against her. The marginal increase in the price as she demands an additional unit is clearly illustrated by the last two terms in (3.9). The first term is standard, and is independent of risk aversion. As with risk neutral market makers in Kyle (1985), it reflects the fact that a marginal increase in the order flow drives up uninformed conditional expectation, that in turn increases the price. The second term instead, is related to the risk-aversion assumption. It reflects the compensation required by the uninformed fringe to hold an additional amount of the risky asset. Intuitively, all else equal, it is increasing in the conditional amount of risk faced by the uninformed fringe $\mathbb{V}\left[X|\xi\right]$ and in risk aversion r. The role played by μ is the interesting one, as it is at the heart of the relationship between asymmetric information and the risk premium as it is studied here. To gain intuition, consider the effect of increasing the degree of asymmetric information. A marginal decrease in μ has a direct effect evident from (3.9): the slope of the residual supply curve faced by the insider increases because she is trading against a smaller group of uninformed traders. With a bigger price impact, the insider has an incentive to restrict the quantity she trades, producing an indirect effect via information leakage. The expression for the conditional variance in (3.5) shows that $\mathbb{V}\left[X|\xi\right]$ decreases in the insider's trading aggressiveness β. Therefore, the less she trades

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[8]The second-order condition is satisfied if $\beta > 0$, that is going to be the case in equilibrium.

on information because of a lower value of μ, the less information is going to be revealed by the price (at the market clearing stage), the higher the conditional variance $\mathbb{V}[X|\xi]$. In turn, the higher the risk faced by the uninformed fringe, the bigger the price impact. Solving the model in closed form confirms this intuition.

Lemma 7. *The unique linear equilibrium satisfies*

$$\alpha = \frac{r}{\mu}\bar{Z}\sigma_x^2\beta; \qquad \beta = \sqrt{\frac{\sigma_z^2}{\sigma_x^2} + \left(\frac{r}{\mu}\sigma_z^2\right)^2} - \frac{r}{\mu}\sigma_z^2. \tag{3.10}$$

Proof: using (3.6) and (3.5) into (3.9) and matching coefficients in with the conjectured insider's strategy yields the result in the Lemma. $\square$

3.3.2 Comparative statics: asymmetric information, depth and the risk premium

Simple inspection of (3.10) and (3.5) implies

$$\frac{d\beta}{d\mu} > 0, \qquad \frac{d\mathbb{V}[X|\xi]}{d\mu} < 0. \tag{3.11}$$

The above comparative statics confirm the intuition presented in the previous discussion: when the insider trades against a smaller number of uninformed risk-averse traders, she is forced to reduce the quantity she trades. In turn, this reduction in informed trading volume implies lower information leakage and higher conditional risk faced by the uninformed side of he market. We now turn to the implications for the risk premium and market liquidity.

First we define the following two market statistics: the risk premium (RP), defined as the unconditional expectation of the excess return per share: $RP \equiv \mathbb{E}[X - P]$; and market depth ($D$), or liquidity, defined as the inverse of the price impact of trade: $D \equiv \lambda^{-1} \equiv \left(\frac{dP}{d\xi}\right)^{-1}$. We will equivalently refer to λ as an "illiquidity" measure. As we are interested in comparative statics with respect to μ, we assume the average *per capita* supply of the risky asset to be constant and equal to $\bar{S}$, i.e. total average supply as a function of μ is $\bar{Z} = \bar{S}(1 + \mu)$. The reason to do so it that we want to isolate the effect of asymmetric information from pure risk sharing considerations that would affect the risk premium if the per capita supply were to change with μ.[9] Next Proposition provides

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[9]The risk sharing effects of an increase in the investor base are describe by Merton (1987) in an incomplete but homogenous information economy. Controlling for risk sharing, we therefore provide an additional channel by which an increased investor base might result in lower cost of capital.

the expressions for these statistics and the relationship with the degree of asymmetric information.

Proposition 11. *i) Risk premium and depth satisfy:*

$$RP = r\mathbb{V}\left[X|\mathcal{F}^U\right]\frac{(\bar{Z}-\alpha)}{\mu}, \tag{3.12}$$

$$(D)^{-1} = \lambda_k + \frac{r}{\mu}\mathbb{V}\left[X|\mathcal{F}^U\right], \tag{3.13}$$

where

$$\lambda_k = \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2 + \sigma_\theta^2}.$$

ii) Both the risk premium and illiquidity are increasing in the degree of asymmetric information (i.e. decreasing in μ)

Proof: expression (3.12) follows by taking unconditional expectations of (3.3) and of the market clearing condition; (3.13) follows by (3.6) and (3.5). Proof of part ii) follows by substituting $\mathbb{V}\left[X|\mathcal{F}^U\right]$ from (3.5) and the expressions for α, β in Lemma 1 and taking the partial derivatives of (3.12) and (3.13) with respect to μ. $\square$

The risk premium in (3.12) is the product of two terms: the effective risk aversion of an uninformed $r\mathbb{V}\left[X|\mathcal{F}^U\right]$, and the amount of risk they expect to hold. The first term we know from (3.11) that is decreasing in μ because of more information leakage. As for the second term, it can be shown (combining equations (3.10) and (3.12)) that expected holdings per capita of the uninformed are positively related to μ. To gain intuition on the behavior of expected holdings, consider the effects of a marginal increase in μ if the risk premium was held fixed. Form (3.3), an uninformed trader would hold more of the asset because more informative prices lower her effective risk-aversion. On the other hand, from (3.9), also the insider would increase her order because of a lower price impact, but the incentive to do so are reduced by strategic considerations: she anticipates that the more she trades the more the price moves against her. This heterogeneity in behavior and incentives results in uninformed (per capita) expected holdings to be increasing in μ. To aspects are worth noting at this point. The first one is that this result is special to this microstructure setup: as shown in the section 5, it is not going to hold if we assume that the insider can submit limit orders. The second one is that it reinforces our statement: even if the quantity of risk hold by the uninformed fringe *decreases* with asymmetric information, the effect on the conditional variance via reduced information leakage is strong enough that the risk premium increases.

Expression (3.13) shows that the inverse of depth, which is the price impact of the order flow, is the sum of two components. The first one, λ_k, reads exactly as Kyle's

lambda: is the price impact that obtains with risk neutral market makers and is related to the sensitivity of the uninformed conditional expectation. We notice that, even if the functional form is the same, it's equilibrium value is lower than in Kyle (1985) because the insider's trading aggressiveness β is dampened by the risk aversion of the uninformed. The second term in (3.13) is related to the risk aversion of the uninformed. Intuitively, one can verify that λ_k is *increasing* in μ, as the insider trades more and the order flow is more informative, while the second term $\frac{r}{\mu}\mathbb{V}\left[X|\mathcal{F}^U\right]$ is *decreasing* in μ as the uninformed require a lower compensation to hold additional units of the risky asset. As already discussed, the marginal impact of μ on this term is reinforced by a feedback effect via information leakage. As a result the net effect of increase in information asymmetry as measured by a lower value of μ, is to decrease market depth.

3.3.3   Expected Return-Illiquidity relationship

In various empirical studies, (see Brennan and Subrahmanyam,1996), the relationship between asymmetric information and the cost of capital is investigated by regressing risk premia on a measure of illiquidity (the price impact of trades), to which authors typically refer as Kyle's lambda. Since in Kyle (1985) the risk premium is zero by construction, a direct theoretical support for why risk premia and price impact should be positively correlated is missing. The motivation that is typically provided is an indirect one: asymmetric information should be responsible for the price impact λ in the first place, and then, taking λ as given, investors should require a compensation for the trading costs associated with the price impact. In this model, both illiquidity and the risk premium are derived endogenously, and as the results in Proposition 1 suggest, are both directly affected by the degree of asymmetric information: lower values of μ imply both higher risk premia and lower liquidity, inducing a positive correlation in the cross section.

The results in Proposition 1 are obtained without assuming that investors value liquidity *per se*, nor that they take into account the trading costs associated with future illiquidity[10], so despite the positive relationship, there is no direct causality from illiquidity to expected returns. The relationship has a mechanical nature as both are related to the demand slope of the uninformed risk-averse traders who set the price: the risk premium in (3.12) is proportional to the uninformed required compensation for the perceived risk of future cash flows, measured by $\frac{r}{\mu}\mathbb{V}\left[X|\mathcal{F}^U\right]$, that is one of the two components of the price impact (3.13). Moreover, this relationship is not special to the asymmetric information assumption: absent any information asymmetry the order

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[10]This model is obviously static, but the same argument can be made in the dynamic extension developed in next section.

flow would be uninformative and the term λ_k would be zero, so that the risk premium would be simply proportional to λ.[11] In a cross sectional regression of returns onto price impact measures, a positive λ_k term would result in a negative intercept, that might be interpreted as a pricing error. Evaluating the effect of asymmetric information on risk premia in an indirect way according to this methodology would therefore underestimate the direct effect through information leakage that we discuss.

3.4   Model II: dynamics

Among the many simplifying assumptions made in the setup of the previous section, one is clearly time. The model is static in the sense that, even if with a different timing, agents get to trade only once and after that the world ends. An obvious concern is whether the results presented there are robust to a dynamic framework in which agents interact repeatedly. The purpose of this section is to show that this is in fact the case.

In developing a dynamic model, an obvious choice could be to extend the multi-period model in Kyle (1985) to allow for risk-averse pricing. In that model the insider would initially receive some information regarding the realization of the fundamental at some future time T, and would trade against the market maker (or an uninformed fringe in our case) in each period until T. By construction, the model would have a non-stationary nature, and so would any endogenous variable such as the risk premium or liquidity. As the question we are after is how the unconditional risk premium is related to the degree of asymmetric information, it's more natural to frame the analysis in a stationary environment.

We build on the stationary model of Chau and Vayanos (2006). The model has an infinite-horizon and is in discrete time.[12] The assumptions are the natural dynamic extension of the setup presented in Section 2. A riskless asset and a risky asset are traded in each period. The riskless asset is in perfectly elastic supply and pays an exogenous gross return R; the risky asset pays a dividend D_t each period according to the process

$$D_t = g_{t-1} + \varepsilon_{D,t},$$

where the fundamental process g_t determines the conditional expected dividend and evolves according to:

$$g_t = \rho g_{t-1} + \varepsilon_{g,t}.$$

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[11]See Novy-Marx (2006) for a related point on illiquidity premia.

[12]Chau and Vayanos (2006) develop the continuous time limit.

Each period there is a random per capita supply of the asset given by

$$Z_t = \bar{Z} + \varepsilon_{z,t}.$$

The innovations to the three processes above are i.i.d. normally distributed with zero mean and variances σ_D^2, σ_g^2 and σ_z^2. There is an infinitely lived strategic risk-neutral insider, who privately observes the process g. Each period, the insider's market order plus the noisy supply constitute the order flow. In Chau and Vayanos (2006), the price is set each period by a competitive market maker, implying weak form efficiency. As we did in Model 1 with respect to Kyle (1985), here is where we depart form Chau and Vayanos (2006) by assuming that the rest of the market is populated by a measure μ of risk averse (CARA) uninformed competitive traders who submit limit orders. For tractability, we assume that each period an overlapping generation of such traders is born, and the old one leaves the market after consumption takes place. We denote with G_t the generation of uninformed traders that enters the market at time t. The timing of events is as follows: at each time t:

1. G_t enters the market; the insider observes the realization of g_{t-1} and submits a market order θ_t^I.

2. D_t is announced to the market.

3. The noisy supply $\varepsilon_{z,t}$ realizes, each trader in G_t submits a limit order θ_t^U and the price P_t is set to clear the market.

4. G_{t-1} consumes and leaves the market.

5. D_t is paid to asset holders.

As the correspondence of this dynamic setup to the static one of the previous section might not seem immediate at a first look, we provide some further details to ease the interpretation. We start considering G_t optimization problem. Uninformed traders have access to public information only, which is given by the time series of current an past dividends and prices. Hence $\mathcal{F}_t^U = \{D_s, P_s | s \leq t\}$. Given what she knows, a G_t investor cares about next period consumption, so what she wishes to maximize is

$$U_t = \mathbb{E}\left[-e^{-rc_{t+1}} | \mathcal{F}_t^U\right], \tag{3.14}$$

subject to the dynamic budget constraint[13]

$$c_{t+1} = \theta_t^U (P_{t+1} + RD_t - RP_t) + W_{Gt}R. \tag{3.15}$$

An uninformed trader that purchases θ_t^U units of the stock today is given θ_t^U units of the announced dividend D_t. As her consumption takes place tomorrow, she carries this payment to the next period invested at the risk free rate. If the investor did not have to re-trade she would face no uncertainty, but as she leaves the market next period she will have to clear her position at tomorrow's price. Here is where the dynamic aspect of the problem enters into the picture. Tomorrow's price will depend on D_{t+1}, and therefore on g_t. In deciding how much to trade today the uninformed investor has to form an estimate of the current value of the unobserved fundamental process g. As the insider observes g each period, she has an informational advantage over the uninformed.

Even if the solution of the model is clearly more involved, the economics of the problem are the same described in Model 1. We will therefore only provide a brief intuition of the qualitative aspects of it in the text, and relegate the details of the solution to Appendix A. In the same spirit of Model 1, the uninformed will make use of the market clearing condition to learn a noisy signal of the insider's market order. The uninformed conjecture that the insider's order is linear in the error they make in estimating the fundamental process, meaning that they guess $\theta_t^I = \alpha + \beta(g_{t-1} - \mathbb{E}\left[g_{t-1}|\mathcal{F}_{t-1}^U\right])$. Given this conjecture, the uninformed have to solve a Kalman Filtering problem in order to compute their estimates of g. As the risk they face is given by the conditional variance of next period price, and next period price will depend on the value of the fundamental process, the precision of their estimates regarding the process g will affect directly the perceived risk of holding the stock. In turn, this determines the risk premium and liquidity in the market. Clearly, the insider takes all this into account when optimizing over her strategy. The same incentives to restrict her trading for lower values of μ and feedback effects via information leakage onto conditional risk described in Model 1 apply to this context as well. Here we provide a numerical example to illustrate that the qualitative results described by means of closed form solutions in Model 1 are robust to this dynamic environment. The particular values of the parameters chosen are meant to be illustrative and should not be considered an attempt to calibrate the model to real world data. Following our interpretation, define $w = \frac{1}{1+\mu}$ as the degree of asymmetric information. Let the risk premium be the unconditional expectation of the excess return per share $RP = \mathbb{E}\left[P_{t+1} + RD_t - RP_t\right]$, and $Depth$ be the inverse of the price impact of the order flow. Set the following parameter values: $\bar{Z} = \sigma_D^2 = \sigma_g^2 = \sigma_z^2 = 1$; $r = 4$, $\rho = 0.5$; $R = 1.05$.

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[13]The term W_{Gt} in (3.15) is the wealth that each trader in G_t is endowed with when she enters the market. It plays no role because of the CARA preference specification.

w	β	$\mathbb{V}\left[g_t|\mathcal{F}_t^U\right]$	$\mathbb{V}\left[P_{t+1}|\mathcal{F}_t^U\right]$	$Depth$	RP
3%	0.26	1.128	6.05	1.20	1.37
4%	0.19	1.130	6.75	0.86	1.41
5%	0.12	1.132	8.74	0.55	1.74

The table shows what already anticipated: when the insider is faced by a smaller number of risk averse traders, she has an incentive to reduce her trading aggressiveness. As a result of lower informed trading volume, the precision of the unformed estimates the fundamental value of the asset is reduced. This increases the risk faced by the uninformed by increasing the conditional volatility of future prices. As a consequence, market depth decreases and the risk premium increases.

3.5 Model III: limit orders, risk aversion and competitive behavior

The fact that risk-aversion is a necessary condition for risks to be priced is obvious. Perhaps less obvious is the fact that informed traders need to be strategic for the mechanism described in the previous sections to obtain. For this reason we provide a comparison with a competitive model in order to highlight the differences. To make such a comparison, and to relate it to existing literature on the subject (e.g., Easley and O'Hara (2004)), we will assume that informed traders are risk averse[14] and submit limit orders. Risk aversion among informed and uninformed are allowed to be different. The rest of the setup is as described in section 2. Informed traders condition their (linear) strategies not only on their private information but also on the market price; β denotes the coefficient on the private signal in an informed trader's strategy, and w denotes the fraction of informed traders. Under the strategic assumption, the interpretation of the model is similar to the previous sections: the informed trader has measure one,[15] and a the uninformed have measure μ; noise is expressed in per capita terms and $w = 1/(1 + \mu)$.

In order to provide the comparison, we denote with a superscript $k = c, s$ the endogenous parameters that obtain when the model is solved assuming competitive behavior of all traders (c) and when informed traders are assumed to behave strategically (s). For expositional purposes, we make a slight change of notation: for each random variable, let us denote τ the inverse of the variance, and refer to it as the precision. In particular denote $\tau_h \equiv \mathbb{V}\left[X|\mathcal{F}^h\right]^{-1}$ for $h = I, U$ as the precision of the payoff conditional on a

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[14]If an informed competitive trader was risk neutral, his information would be immediately (and trivially) revealed by the equilibrium price.

[15]We can interpret the speculator as a coalition of informed traders who manage to solve competition problems and act a as a single trader.

trader's information set. Next Lemma provides the expressions for the risk premium in this setup. Proofs for this section are contained in Appendix B.

Lemma 8. *The risk premium satisfies*

$$RP^k = \bar{Z} \left(w\beta^k \frac{\tau_I}{\tau_\varepsilon} + (1-w)\frac{\tau_U^k}{r^u} \right)^{-1}, \tag{3.16}$$

where

$$\beta^c = \frac{\tau_\varepsilon}{r^I} > \beta^s; \qquad \tau_U^k = \tau_x + \psi^k \tau_\varepsilon; \qquad \psi^k \equiv \frac{\left(w\beta^k\right)^2}{\left(w\beta^k\right)^2 + \tau_\varepsilon/\tau_z}. \tag{3.17}$$

From the expressions in the Lemma a first consideration is immediate: the risk premium is strictly higher if informed traders are strategic than if they are competitive for any value of the primitives. This result is expected as existing literature already pointed out the informational properties of strategic and competitive equilibria (see Kyle, 1989). What is more interesting, and perhaps less immediate, is the relationship between in the fraction of informed traders and the risk premium in the two models, which is what we focus on. From (3.16) we have:

$$Sign\left(\frac{\partial RP^k}{\partial w}\right) = -Sign\left(\underbrace{\frac{\partial\left(w\beta^k\right)}{\partial w} - \frac{\tau_U^k}{r^u}}_{\text{holdings effect}} + (1-w)\frac{\tau_\varepsilon}{r^u} \underbrace{\frac{\partial\psi}{\partial\left(w\beta\right)}\frac{\partial\left(w\beta^k\right)}{\partial w}}_{\text{information leakage effect}} \right). \tag{3.18}$$

An increase in w affects the risk premium in two ways: by changing the relative equilibrium holdings of each class, and by affecting the amount of information leakage. We begin our analysis with the competitive case as it provides a useful benchmark. In this case (as can easily be verified from the expressions in the Lemma) the risk premium can be simply interpreted as a weighted average of investors' conditional uncertainty about future cash flows, where the weights depend on the aggregate risk tolerance of each class.[16] Crucially, (3.17) shows that an informed competitive trader's strategy is independent of w, making the two effects in (3.18) straightforward to describe. For the holdings effect, a marginal increase in w has to be interpreted as replacing uninformed traders with informed ones: informed traders face less uncertainty and require lower compensation to hold the stock than uninformed do[17]. For the information leakage

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[16]In the special case $w = 1/2$, the risk premium is the harmonic average of traders' effective risk-aversions.

[17]We are implicitly assuming $r_I \leq r_U$. It is a very natural assumption to make since $r_I > r_U$ would never hold if the allocation of information was endogenous: more risk tolerant traders value information more and are the first to buy it in equilibrium.

effect, a marginal increase in w mechanically increases informed trading volume $w\beta$. More informed trading volume increases the fraction of the precision of the private information that is revealed through the market price (measured by ψ), reducing the uninformed perception of risk. Both effects tend to reduce the risk premium, that monotonically decreases in w. On the other hand, from the discussion in the previous sections we should expect strategic informed traders to modify their strategies depending on the market structure. In fact, we have the following result:

Corollary 4. $\dfrac{\partial \beta^s}{\partial w} < 0$. *Moreover, if* $\dfrac{r^U}{r^I}\dfrac{\tau_x + \tau_\varepsilon}{\tau_x} > \left(\dfrac{1-w}{w}\right)^2$, *then* $\dfrac{\partial (w\beta^s)}{\partial w} < 0.$

An informed strategic trader scales down his aggressiveness for higher values of w, in line with our previous results. Wether in aggregate informed trading volume $w\beta^s$ increases or decreases with w is parameter dependent. In particular, $\partial (w\beta^s)/\partial w$ is positive for w small, and it changes sign as w increases, as the Corollary clearly implies. The fact that aggregate informed trading volume increases for low w is an obvious consequence of this setup. In Model 1 and it's dynamic extension in fact, we were considering comparative statics keeping the mass of speculators fixed, while here as w becomes small, speculators effectively disappear from the market. As a consequence, informed trading can only increase at that point. Nevertheless, as informed traders' relative size increases, eventually individual trading aggressiveness reduces so much that in aggregate informed trading volume becomes a decreasing function of w. From (3.18), this is clearly a sufficient condition for the risk premium to increase: both the holdings and the information leakage effects tend to increase the risk premium in this case. The latter effect is in common with the previous sections: lower trading volume increases the uncertainty faced by uninformed traders, increasing their required return per unit of holdings of the stock. The former effect is new to this setup: by trading less, informed per-capita holdings *decrease*, implying that the amount of risk per capita to be borne by the uninformed *increases* in w. A direct implication is the that the risk premium might exceed the value that would obtain if all traders were uninformed, something that is impossible in the competitive case. As this discussion makes clear, the strategic behavior assumption on the part of informed traders modifies substantially the qualitative aspects of the equilibrium. In particular, the pricing effects of asymmetric information discussed in this paper do not hold if traders behave competitively.

Appendix A

In this Appendix we solve for the equilibrium in the dynamic model.

I. Market clearing and the price function

The first-order condition for the uninformed problem in (3.14) and (3.15) yields

$$\theta_t^U = \frac{\mathbb{E}\left[P_{t+1}|\mathcal{F}_t^U\right] + RD_t - RP_t}{r\mathbb{V}\left[P_{t+1}|\mathcal{F}_t^U\right]}; \tag{3.19}$$

market clearing requires

$$\theta_t^I + \mu\theta_t^U = Z_t; \tag{3.20}$$

plugging (3.19) into (3.20) and solving for the price yields

$$P_t = D_t - \frac{r\mathbb{V}\left[P_{t+1}|\mathcal{F}_t^U\right]}{\mu R}\left(\bar{Z} - \xi_t\right) + \frac{\mathbb{E}\left[P_{t+1}|\mathcal{F}_t^U\right]}{R}. \tag{3.21}$$

We conjecture that the insider's strategy takes the form

$$\theta_t^I = \alpha + \beta\Delta g_{t-1}, \tag{3.22}$$

where $\Delta g_{t-1} = g_{t-1} - \mathbb{E}\left[g_{t-1}|\mathcal{F}_{t-1}^U\right]$. Notice that the market clearing condition can be equivalently stated as

$$\mu\theta_t^U = \bar{Z} - \underbrace{\left(\theta_t^I - \varepsilon_{z,t}\right)}_{\equiv\xi_t},$$

so that the signal $\xi_t \in \mathcal{F}_t^U$.

As we are looking for a stationary solution we impose a stationarity condition:

$$\mathbb{V}\left[P_{t+1}|\mathcal{F}_t^U\right] = \Sigma_p. \tag{3.23}$$

At this point we can use the conjectured strategy for the insider (3.22) and (3.23) and solve forward the equation for the price in (3.21). Ruling out bubbles yields

$$P_t = D_t + \frac{\hat{g}_t}{R - \rho} + \frac{r\Sigma_p}{\mu R}\xi_t - p0, \tag{3.24}$$

where

$$p0 = r\frac{\Sigma_p}{\mu(R - 1)}\left(\bar{Z} - \alpha\right). \tag{3.25}$$

II. The uninformed inference problem

In order to model the uninformed learning process, we conjecture that the conditional expectation $\hat{g}_t \equiv \mathbb{E}\left[g_{t-1}|\mathcal{F}^U_{t-1}\right]$ evolves according to

$$\hat{g}_t = \rho\hat{g}_{t-1} + \lambda_\xi\left(\xi_t - \mathbb{E}\left[\xi_t|\mathcal{F}^U_{t-1}\right]\right) + \lambda_D\left(D_t - \mathbb{E}\left[D_t|\mathcal{F}^U_{t-1}\right]\right), \tag{3.26}$$

where λ_ξ and λ_D are constants to be determined. Again we impose a stationarity condition

$$\mathbb{V}\left[g_t|\mathcal{F}^U_t\right] = \Sigma_g. \tag{3.27}$$

Using (3.26) and (3.27) we can compute the variance of the price form (3.24). Some algebra yields

$$\Sigma_p = \Sigma_g\left(1 + \frac{\lambda_D + \beta\lambda_\xi}{R - \rho} + \frac{\beta r\Sigma_p}{\mu R}\right)^2 + \sigma_D^2\left(1 + \frac{\lambda_D}{R - \rho}\right)^2 + \sigma_z^2\left(\frac{r\Sigma_p}{\mu R} + \frac{\lambda_\xi}{R - \rho}\right)^2. \tag{3.28}$$

Next we solve for the uninformed recursive (Kalman) filtering problem. Suppose that conditional on $\mathcal{F}^U_{t-1}$, the uninformed believe $g_{t-1} \sim \mathcal{N}\left(\hat{g}_{t-1}, \Sigma_g\right)$. Then at time t, once the uninformed observes the two signals D_t and ξ_t, the posterior belief about g_{t-1} is going to be

$$g_{t-1} = \mathbb{E}\left[g_{t-1}|\mathcal{F}^U_t\right] + \eta_t, \tag{3.29}$$

where η_t is orthogonal to all elements in $\mathcal{F}^U_t$. By joint normality, the conditional mean $\mathbb{E}\left[g_{t-1}|\mathcal{F}^U_t\right]$ and variance $\mathbb{V}\left[g_{t-1}|\mathcal{F}^U_t\right]$ are going to be

$$\mathbb{E}\left[g_{t-1}|\mathcal{F}^U_t\right] = \mathbb{E}\left[g_{t-1}|\mathcal{F}^U_{t-1}\right] + \tag{3.30}$$

$$+\Sigma\left[g,(D,\xi)|\mathcal{F}^U_{t-1}\right]\Sigma\left[D,\xi|\mathcal{F}^U_{t-1}\right]^{-1}\left(\begin{array}{c} D_t - \mathbb{E}\left[D_t|\mathcal{F}^U_{t-1}\right] \\ \xi_t - \mathbb{E}\left[\xi_t|\mathcal{F}^U_{t-1}\right] \end{array}\right); \tag{3.31}$$

$$\mathbb{V}\left[g_{t-1}|\mathcal{F}^U_t\right] = \Sigma_g - \Sigma\left[g,(D,\xi)|\mathcal{F}^U_{t-1}\right]\Sigma\left[D,\xi|\mathcal{F}^U_{t-1}\right]^{-1}\Sigma\left[g,(D,\xi)|\mathcal{F}^U_{t-1}\right]' \tag{3.32}$$

where

$$\Sigma\left[g,(D,\xi)|\mathcal{F}^U_{t-1}\right] \equiv \left(\ \mathbb{C}\left[g_{t-1},D_t|\mathcal{F}^U_{t-1}\right],\ \ \mathbb{C}\left[g_{t-1},\xi_t|\mathcal{F}^U_{t-1}\right]\ \right), \tag{3.33}$$

$$\Sigma\left[D,\xi|\mathcal{F}^U_{t-1}\right] \equiv \left(\begin{array}{cc} \mathbb{V}\left[D_t|\mathcal{F}^U_{t-1}\right], & \mathbb{C}\left[D_t,\xi_t|\mathcal{F}^U_{t-1}\right] \\ \mathbb{C}\left[D_t,\xi_t|\mathcal{F}^U_{t-1}\right], & \mathbb{V}\left[\xi_t|\mathcal{F}^U_{t-1}\right] \end{array}\right). \tag{3.34}$$

The terms in (3.33) and (3.34) are easily computed using the fact that $g_{t-1}|\mathcal{F}^U_{t-1} \sim \mathcal{N}\left(\hat{g}_{t-1}, \Sigma_g\right)$ and the definitions of D_t, g_t and ξ_t. Notice that we started assuming $g_{t-1}|\mathcal{F}^U_{t-1}$ to be normally distributed; now we can verify that $g_t|\mathcal{F}^U_t$ is normally distributed as well: the time t conditional mean is

$$\hat{g}_t \equiv \mathbb{E}\left[g_t|\mathcal{F}^U_t\right] = \mathbb{E}\left[\rho g_{t-1} + \varepsilon_{gt}|\mathcal{F}^U_t\right] = \rho\mathbb{E}\left[g_{t-1}|\mathcal{F}^U_t\right],$$

and substituting (3.30) in the last expression verifies the conjecture in (3.26). Matching the coefficients in the resulting expression with the conjecture yields

$$\lambda_\xi = \frac{\rho \Sigma_g \sigma_D^2 \beta}{\sigma_D^2 \sigma_z^2 + \Sigma_g \left(\sigma_z^2 + \sigma_D^2 \beta^2\right)}; \qquad \lambda_D = \frac{\rho \Sigma_g \sigma_z^2}{\sigma_D^2 \sigma_\theta^2 + \Sigma_g \left(\sigma_z^2 + \sigma_D^2 \beta^2\right)}. \tag{3.35}$$

Finally, the time t conditional variance is given by

$$\begin{aligned} \Sigma_g &= \rho^2 \mathbb{V}\left[g_{t-1}|\mathcal{F}_t^U\right] + \sigma_g^2, \\ &= \frac{\Sigma_g \rho^2 \sigma_D^2 \sigma_\theta^2}{\sigma_D^2 \sigma_\theta^2 + \Sigma_g \left(\sigma_\theta^2 + \sigma_D^2 \beta^2\right)} + \sigma_g^2. \end{aligned} \tag{3.36}$$

III. The insider optimization problem: sketch.

At the moment of trading at time t, the insider's information set consists of all private and public information up to the previous period: $\mathcal{F}_t^I = \{D_{s-1}, P_{s-1}, g_{s-1}|s \leq t\}$. As she is risk neutral and lives forever, she maximizes

$$J_t = \underset{\{\theta_s^I\}_{s=t}^\infty}{Max} \mathbb{E}\left[\sum_{s=t}^\infty \frac{\theta_s^I \left(v_s - P_s\right)}{R^{s-t}}|\mathcal{F}_t^I\right], \tag{3.37}$$

where

$$v_t = \mathbb{E}\left[\sum_{s=t}^\infty \frac{D_s}{R^{s-t}}|\mathcal{F}_t^I\right].$$

Consider the following quadratic trial solution to (3.37)

$$J_t = A\Delta g_{t-1}^2 + B\Delta g_t + C.$$

The problem becomes

$$J_t = \underset{\theta_t^I}{Max}\mathbb{E}\left[\theta_t^I \left(v_t - P_t\right) + \frac{A\Delta g_t^2 + B\Delta g_t + C}{R}|\mathcal{F}_t^I\right]. \tag{3.38}$$

Using (3.24) and (3.26) one can easily compute:

$$\begin{aligned} \mathbb{E}\left[\left(v_t - P_t\right)|\mathcal{F}_t^I\right] &= \mathbb{E}\left[\frac{\Delta g_t}{R - \rho}|\mathcal{F}_t^I\right] - \frac{r\Sigma_p}{\mu R}\theta_t^I + p_0, \\ \mathbb{E}\left[\Delta g_t|\mathcal{F}_t^I\right] &= \Delta g_{t-1}(\rho - \lambda_D) + \lambda_\xi(\alpha - \theta_t^I), \\ \mathbb{V}\left[\Delta g_t|\mathcal{F}_t^I\right] &= \sigma_g^2 + \lambda_\xi^2 \sigma_z^2 + \lambda_D^2 \sigma_D^2. \end{aligned}$$

The last three expressions are sufficient to compute the conditional expectation in (3.38). Then matching the coefficients of the first-order condition with respect to θ_t^I with the conjecture in (3.22) provides a system of two equations for the unknowns α, β. Substituting the optimal trading strategy into the Bellman Equation and matching coefficients with the trial solution provides 3 equations for the unknowns A, B and C. These five equations with (3.35), (3.28) and (3.36) provide the system of 9 equations in the 9 unknowns $A, B, C, \alpha, \beta, \lambda_\xi, \lambda_D, \Sigma_g, \Sigma_p$ that completely characterize the equilibrium. Moreover in equilibrium the insider second-order condition has to be satisfied. The system has 2 solutions, one for each root of Σ_p. One solution is unstable: minor expectation errors (Δg) significantly impact prices and destabilize the economy (Σ_p is orders of magnitude bigger that Σ_g). In the numerical example we provide the values for the stable solution.

Appendix B

In this appendix we solve for the equilibrium in the model with limit orders. Up to the informed traders' problem, the solution of competitive and strategic models coincide, and we omit the superscripts used in the text to differentiate the models.

Strategies are conjectured to be linear:

$$\theta^I(P, S) = \alpha_I + \beta Y - \gamma_I P; \qquad \theta^U(P) = \alpha_U - \gamma_U P, \tag{3.39}$$

and market clearing implies

$$w\theta^I(P, Y) + (1 - w)\theta^U(P) = Z. \tag{3.40}$$

Making use of (3.39), the market clearing condition (3.40), can be rearranged into

$$\tilde{Y} - \frac{Z - \bar{Z}}{w\beta} = \frac{\bar{Z} - w\alpha_I - (1 - w)\alpha_U + P(w\gamma_I + (1 - w)\gamma_U)}{w\beta}. \tag{3.41}$$

It follows that the uninformed information set contains the public signal $\xi = \tilde{Y} - \frac{Z - \bar{Z}}{w\beta}$. Standard results are used to derive the posterior beliefs:

$$\mathbb{E}[X|\xi] = \frac{\psi\tau_\varepsilon}{\tau_x + \psi\tau_\varepsilon}\xi; \qquad \tau_U = \tau_x + \psi\tau_\varepsilon, \tag{3.42}$$

where

$$\psi \equiv \frac{(w\beta)^2}{(w\beta)^2 + \tau_\varepsilon/\tau_z}. \tag{3.43}$$

Uninformed first-order condition yields

$$\theta_U = \frac{\tau_U}{r^U}\left(\mathbb{E}\left[X|\xi\right] - P\right);$$ (3.44)

using (3.42) and the right hand side of (3.41) into the last expression and matching coefficients with (3.39) yields

$$\alpha_U = \frac{\psi\tau_\varepsilon}{w\beta r^U + \psi\tau_\varepsilon(1 - w)}; \qquad \gamma_U = \frac{w\left(\beta\tau_U - \psi\tau_\varepsilon\gamma_I\right)}{w\beta r^U + \psi\tau_\varepsilon(1 - w)}.$$ (3.45)

Informed traders conditional moments are

$$\mathbb{E}\left[X|Y\right] = \frac{\tau_\varepsilon}{\tau_x + \tau_\varepsilon}Y; \qquad \tau_I = \tau_x + \tau_\varepsilon.$$ (3.46)

Under the competitive assumption, the first-order condition implies

$$\theta_I^c = \frac{\tau_I}{r^I}\left(\mathbb{E}\left[X|Y\right] - P\right).$$

Making use of (3.46) and the last equation and matching coefficients with (3.39) yields

$$\alpha_I^c = 0; \qquad \beta^c = \frac{\tau_\varepsilon}{r^I}; \qquad \gamma_I^c = \frac{\tau_I}{r^I}.$$ (3.47)

The system of equations (3.43), (3.45) and (3.47) can be solved explicitly in terms of the primitives and fully characterizes the competitive equilibrium.

Under the strategic assumption, notice that the market clearing equation can be rearranged into

$$P = \frac{(1 - w)\alpha_U - Z}{1 - w} + \frac{w}{1 - w}\theta_I;$$ (3.48)

taking into account (3.48), informed traders' first-order condition yields

$$\theta_I^s = \frac{\tau_I}{r^I + \lambda_I\tau_I}\left(\mathbb{E}\left[X|Y\right] - P\right),$$

where

$$\lambda_I = \frac{w}{(1 - w)\gamma_U}.$$ (3.49)

Using (3.46) and the last two equations and matching coefficients with (3.39) yields

$$\alpha_I^s = 0; \qquad \beta^s = \frac{\tau_\varepsilon}{r^I + \lambda_I\tau_I}; \qquad \gamma_I^s = \frac{\tau_I}{r^I + \lambda_I\tau_I}.$$ (3.50)

The system of equations (3.43), (3.45), (3.49) and (3.50) can be reduced to a single equation in β that implicitly characterizes the equilibrium, given by

$$\beta = \frac{\tau_z^{-1}(1-w)\tau_\varepsilon^2\tau_x}{(1-w)\tau_\varepsilon(r^I\tau_z^{-1}\tau_x + w^2\beta\tau_I) + r^U w(w^2\beta^2 + \tau_z^{-1}\tau_\varepsilon)\tau_I}. \tag{3.51}$$

Informed second-order condition is satisfied in equilibrium as $\beta > 0$. Existence and uniqueness can be easily shown using (3.51) following Kyle (1989), and the proof is omitted.

Proof of Lemma 8

The market clearing equation in the competitive and strategic case can be written as

$$w\frac{\tau_I}{r^I}\left(\mathbb{E}\left[X|Y\right] - P\right) + \frac{\tau_U^c}{r^U}\left(\mathbb{E}\left[X|\xi\right] - P\right) = Z,$$

$$w\frac{\tau_I}{r^I + \lambda_I\tau_I}\left(\mathbb{E}\left[X|Y\right] - P\right) + \frac{\tau_U^s}{r^U}\left(\mathbb{E}\left[X|\xi\right] - P\right) = Z.$$

The equation for the risk premium follows by taking unconditional expectations of both sides of the market clearing equation, solving for $\mathbb{E}\left[X - P\right]$ and using the definitions of β^c, β^s. Also, from the definitions clearly follows that $\beta^c > \beta^s$.

Proof of Corollary 4

The proof follows directly by applying the implicit function theorem to (3.51).

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