Household Heterogeneity and Incomplete Financial Markets: Asset Return Implications in a Real Business Cycle Setup

Eva Cárceles Poveda

DEPARTMENT OF ECONOMICS

UNIVERSITAT POMPEU FABRA

Doctoral Thesis

Barcelona, July 2001

Dipòsit legal: B.23236-2008
ISBN: 978-84-692-0962-2

To my parents

Uncertainty is an intimate dimension of our daily lives. For some, it is the zest of life. Without uncertainty, the distinction between the present and the past is blurred; there are no surprises and no anticipations, hence no thrills; there is no scope for achievement, hence no rewards; and love, which always entails risks as well as the joy of discovery, looses its sharp edge. Yet, for others, uncertainty is the course of life. It is so for those who feel threatened with loss of life or individual freedom, who have no assured shelter or subsistence, who lack job security and fear unemployment. Uncertainty is thus an intimate dimension of economics as well.

Jaques H. Drèze

Acknowledgements

It is difficult to name all the persons who have contributed to the realization of the present work, so, I will just mention the names of some of them here.

First, I would like to thank Daniel Gross. He encouraged me to do a Ph.D. and gave me the first insights into economics.

During my years at Pompeu Fabra and my visiting year at the London Business School, I have met exceptional graduate students, who have accompanied and supported me along the whole dissertation process. I would specially like to mention Evi Pappa . Without her company, these five years would have been much harder. I also want to thank Manuel Fernandez for his always helpful attitude, Irina Yakadina for her encouragement, Arpad Abraham, Alessandro Secchi and Chryssi Giannitsarou for many good moments and many intellectually exciting conversations, and Antoni Calvó Armengol for his help at the final thesis stage and for many interesting talks.

There are two persons, with whom I have an enormous debt. The first is Jose M. Marín. I met him during my first Ph.D year and, from the very first moment, he motivated and encouraged me. Although he has been involved in the thesis at a later stage, I have benefited a lot from very fruitful discussions with him. I am very grateful for his always friendly attitude and support. The second is Christian Haefke. He is an exceptional person and I have learned a lot from him.

I also benefited from fruitful discussions with many other people, impossible to name them all. In particular, I would like to thank James Costain and Chris Telmer.

Finally, I am extremely grateful to both, my supervisor Morten Ravn and my tutor Albert Marcet. I want to thank Morten for his guidance and dedication to the development of this thesis, and for his encouragement and support during the whole process. I want to thank Albert for his very useful suggestions and for, without knowing it, making me get passioned about economics.

Contents

List of figures

List of tables

Chapter 1

Introduction

One of the main problems with the modern real business cycle (RBC) literature is its inability to replicate the empirical behavior of the main asset returns in the data. Several authors[1] have incorporated financial markets into the basic model showing that, regardless of the parameterization or the incorporation of other frictions, like capital adjustment costs, these models are unable to replicate the key financial statistics in the data, predicting an equity premium which is essentially zero, and asset return volatilities that are also far from reality.

One of the main reasons for the lack of success of the previous models may be the fact that they are using a representative agent environment. In this case, financial markets are effectively complete, independently of the existing asset structure. In the present thesis, this assumption is relaxed by incorporating both, idiosyncratic labor income risk and imperfect risk sharing, leading to ex-post household heterogeneity and to an incomplete financial market structure. One of the main objectives is therefore to see, if these extensions can help to improve the asset pricing implications of the standard model.

We have to mention that, since the original statement of the asset pricing puzzles by Mehra and Prescott (85), there has been a large strand of literature trying to analyze the asset pricing implications of a context with household heterogeneity and incomplete financial markets. Among others, Aiyagari and Gertler (91), Heaton and Lucas (96), Lucas (94), Marcet and Singleton (99), and Telmer (93) have studied such a framework under the assumption of exogenously determined asset returns and consumption processes. Note, however, that our analysis goes one step further in the sense that it incorporates a production technology, offering a better foundation

[1] See, for example, Rouwenhorst (95), Lettau (96), Lettau and Uhlig (97), Jermann (98) or Danthine, Donalson and Mehra (92).

of asset prices than the standard exchange economy. In particular, consumption is derived from explicit utility maximization instead of being specified exogenously. In addition, the value of price of equity is determined endogenously via the optimization problem of the firm, which also breaks the identity between dividends and consumption processes in exchange economies. We indeed believe, that a detailed and rigorous analysis of asset pricing requires a general equilibrium model of this type.

Note also that the presence of a non-trivial production sector involves addressing an important issue, which has not been given very much attention in the previous asset pricing literature. Under incomplete financial markets and household (shareholder) heterogeneity, the usual profit maximization of the firm is no longer well defined. Thus, unless one assumes that the firm is myopic, in the sense that it solves a static optimization problem by maximizing period by period profits[2], one has to incorporate non-standard firm objectives into the model. A second important objective or contribution of the present thesis is therefore to illustrate how to get around the problem of the firm by incorporating a firm objective which is adequate for the case in which financial markets are incomplete.

Concerning the firm objective when markets are incomplete, there exists a branch of literature, which started already in the late sixties, analyzing the formulation of objectives for the firm that are suited for an incomplete markets environment. The contributions in this direction can be classified into three different groups. The first group tries to define a firm discount factor which reflects the preferences of the firm's shareholders. The second group simply assumes that the firm maximizes the expected utility of profits without taking into account the shareholders' interests. Finally, a third group studies the possibility of introducing a control mechanism based on voting to decide among alternative production plans. In the present work, we consider the first two approaches, extending the proposed firm objectives to a multi-period setup to incorporate them into our model.

[2] This is done by Krussel and Smith (97), who also study equilibrium asset returns in the presence of an endogenous production sector. In contrast to our work, however, the authors assume that households are the owners of the capital stock. Under this assumption, the firm rents the capital period by period, and no difficulty arises when defining its static optimization problem. It is not in the scope of the present work to compare out setup with theirs. As we will see, however, an important advantage of our approach is the fact that it allows for different dividend specifications, which we study in the second chapter.

Using the previous firm objectives, we analyze the asset pricieng implications of the model from two different perspectives in chapters 2 and 3. Each chapter provides its own introduction and conclusions, raising the questions addressed and highlighting the main results obtained.

In chapter 2, the standard model RBC model is compared to the case in which two classes of households, distinguished by their realizations of an idiosyncratic labor income shock, can insure against uncertainty through restricted trade in an equity or a bond market. In order to assess the effects of the two different firm objectives independently of the financial market structure, the model is first solved under the assumption that households are identical in all respects. In this case, it is possible to find conditions under which the two different firm objectives have locally the same behavior. Household heterogeneity is then introduced under two different assumptions: the existence of either a bond or a stock market for firm shares, providing a measure of the degree of risk sharing achieved with the two different asset markets. In both cases, it is assumed that investment is entirely financed by retained earnings, while the residual of profits and investment is paid out as dividends. Finally, the effect of dividend behavior is analyzed by comparing the previous setup to the case in which dividends are defined as a stochastic fraction of profits, while the firm can finance its investment through retained earnings or equity issue.

In chapter 3, the risk sharing opportunities of the two households are completely shut down by imposing no-trade constraints in the two asset markets. This assumption allows for the derivation of approximated closed-form solutions for the different asset moments, giving a much deeper insight of the channels through which idiosyncratic uncertainty may affect asset returns in the RBC model. The different asset moments are decomposed into an aggregate and an idiosyncratic component, which accounts for most of the moments. Apart from the risk aversion parameter and the consumption elasticity with respect to the idiosyncratic shock, it is shown that the last component crucially depends from the idiosyncratic innovation variance, the idiosyncratic shock persistence, and the probability of each household of being unconstrained in the asset markets. The effects of each parameter are highlighted using different parameterizations. Further, it is shown that some persistence in the household that is unconstrained is needed for idiosyncratic uncertainty to have an impact on aggregate asset returns.

Chapter 2

Asset prices and business cycles under market incompleteness

1 Introduction

One of the main problems with the modern real business cycle (RBC) literature is its inability to replicate the empirical behavior of US asset returns. As reported by several authors, the standard model with a single source of risk, the aggregate technology shock of the firm, has very counterfactual asset pricing implications. While the reason for the lack of success of the previous models may be the fact that they are using a representative agent environment, implying that markets are effectively complete, departing from this setup by introducing household heterogeneity and incomplete financial markets involves addresing an important issue: as long as there are several shareholders in the firm, its usual profit maximization objective is no longer well defined.

The main objective of the present chapter is twofold. First, we want to illustrate how to get around the problem of the firm by incorporating a firm objective which is adequate for the case in which financial markets are incomplete. Second, we want to see if the presence of household heterogeneity and incomplete financial markets can help to improve the performance of the standard RBC model concerning the empirical asset pricing observations. Apart from looking at the size and volatility of the different asset returns, we also analyze the effects of dividend behavior, an important matter which has not been given very much attention in the literature until the present.

The two firm objectives assumed are a variant of the usual market value maximization, proposed by Grossman and Hart, and the expected utility of the firm's profits, an objective originally proposed by Radner, implying that the firm is risk averse. In order to assess the effects of the two objectives independently of the financial market structure, the model is first solved under the assumption that households are identical in all respects. In this case, we are able to find conditions under which the two objectives have locally the same behavior. In particular, we show that the presence of a risk averse firm is locally equivalent to the usual market value maximization in the presence of capital installation costs. In addition, this distortion alone is not sufficient to account for the asset return moments in the data.

We then introduce household heterogeneity under two different assumptions: the existence of either a bond or a stock market for firm shares, providing a measure of the degree of risk sharing achieved with the two different asset markets. While the presence of incomplete financial markets considerably alters the behavior of asset returns, we find that the improvement is much lower if households can insure by trading in equity shares of the firm. In this case, the behavior of the equity price, linked to the firm's technology, leads to a much higher variability and to a higher correlation with labor income of the trading volume value, making it much easier for the households to smooth consumption by trading in this asset. We also show that the usual specification of dividend payments as a residual payment leads to a counterfactual dividend behavior under both firm objectives. On the other hand, modelling dividends as an exogenous process to fit their behavior in the data hardly affects the asset moments, suggesting that dividend behavior is not one of the key determinants of the size and variability of risk premia.

The chapter is structured as follows. In the following section, we briefly illustrate the firm objective problem under market incompleteness and review the literature on the subject, focusing on the two approaches that we will use in the main part. The model and the calibration are presented in sections three and four respectively. Section five presents the simulation results, and section six summarizes and concludes.

2 The Firm Objective under Incomplete Markets

It is well known that, if financial markets are complete, the stock market assigns a well defined value to the firm's profits, and that all the shareholders of a firm agree on discounted profit maximization. If markets are incomplete, however, this firm objective is no longer well defined, and shareholder disagreement may occur in equilibrium. A systematic discussion of this problem started already in the late sixties with the influential paper of Diamond (67). In the present section we briefly illustrate the problem and review the literature on the subject, focusing on two important contributions, from which we will derive the two firm objectives used in the main part of the chapter.

Assume, for simplicity, that there are two periods ($t = 0, 1$) and S alternative possible outcomes or states of nature in period 1. There is a single good, a finite number of consumers ($i = 1, ...I$) and one firm, selling its ownership shares on the stock market. Consumers are characterized by their exogenous initial endowment of the good $w_i = (w_{i0}, w_{i1s})_{s=1}^{S}$, their initial ownership in the firm, $\overline{\theta}_i \geq 0$, their consumption set $X_i = R_{+}^{S+1}$, and their chosen consumption stream $c_i = (c_{i0}, c_{i1s})_{s=1}^{S}$, over which they have a preference ordering that can be represented by $u_i : X_i \rightarrow R$. The single firm in the economy is characterized by its production set $Y \epsilon R^{S+1}$ and its production plan $y = (y_0, y_{1s})_{s=1}^{S}$, where y_0 can be interpreted as an input (or investment cost) at date 0, generating the vector of output y_{1s} (or revenue) in state s at date 1. In other words, the firm has a production function $y_{1s} = \psi_s(y_0)$, summarizing the relationship between output at date 1 and input at date 0, where ψ_s is assumed to be concave and differentiable.

In this context, it can be shown that a production plan is optimal for consumer (shareholder) i if it maximizes[3]:

$$\max \left\{ \pi_i y = \sum_{s=0}^{S} \pi_{is} y_{1s} \right\} \tag{1}$$

where π_{is} represents the marginal rate of substitution or present value factor of shareholder i. It is clear that, if markets are complete, the marginal rates of substitution will be equalized in equilibrium, so that all the shareholders of the firm will agree on maximizing the discounted present value of profits, πy, where $\pi_i = \pi$ for all i. On the other hand, due to the fact that trading in the financial markets no longer forces to have a unique present value factor π under market incompleteness, the previous objective is no longer well defined in this case, and shareholder disagreement may result in equilibrium.

[3] Appendix 1 provides a brief derivation of this result.

I. The Value Maximizing Approach

One of the approaches that has been pursued in the literature to address this problem is to define an adequate discount factor for the firm's profits, which appropriately reflects the preferences of the shareholders. The most general choice for this factor was proposed by DeMarzo (88). The author assumes that, in order to maximize its current share value, the firm should simply conjecture an arbitrary discount factor which is consistent with current market prices. This leads to the following objective:

$$\max\left\{\pi^f y : \pi^f \in Q(v,y) \equiv \left(q \in R^S_{++} : v = qy\right)\right\} \tag{2}$$

This objective is open to the criticism that the production choice of the firm does not take into account the preferences of the shareholders. Therefore, it may be rejected by the controllers of the firm. Recognizing this, two further criteria have been proposed in the literature, reflecting the fact that, in a two period economy, there are two groups of shareholders, the new (θ_i) and the original ($\overline{\theta}_i$) shareholders. Dreze (74) suggests that the firm should maximize according to a weighted average of the present value coefficient of the *new* shareholders, while Grossman and Hart (79) propose to choose the weighted present value factor of the *original* shareholders. This leads to the following two objective functions:

$$\max\left\{\sum_{i=1}^{I}\theta_i\pi_i y : y \in Y\right\} \tag{3}$$

$$\max\left\{\sum_{i=1}^{I}\overline{\theta}_i\pi_i y : y \in Y\right\} \tag{4}$$

As pointed out by Grossman and Hart, the first objective becomes very implausible as soon as ones extends the model to more than two periods. In a T period model, for example, the final shareholders are those consumers who hold shares after trading at $T-1$, while many of the firm decisions have been already made at this stage[4]. Therefore, we will use the objective in (4), which we denote by Value Maximizing (VM) firm objective in what follows. As shown by the authors, the objective can easily be extended to a multi-period setup, where uncertainty is described by date-events s^t, with s^t specifying both date t and a particular history of the environment $s_1,...s_t$ until date t. Using this notation, equation (4) can be rewritten as:

[4] For a more detailed discussion of this problem see Grossmann and Hart (79). For a brief derivation of the Grossmann and Hart firm objective from shareholders' preferences see appendix 1.

$$VM = \max \sum_{i=1}^{I} \overline{\theta}_i \sum_{s^t} \pi_i(s^t) y(s^t) \tag{5}$$

II. The Utility Maximizing Approach

The previous framework implicitly assumes that there exists a firm manager who is able to deduce or make the shareholders reveal information about their present value factors, which he will have to aggregate into a single index. Recognizing that consultation of shareholders may be time consuming and costly, a second group of authors simply assume the existence of a utility function for the firm, defined exogenously over state distribution of profits. Important contributions here are due to Radner (72), Sandmo (72), Sondermann (74), and Leland (72) among others. The authors assume that the firm faces the following maximization problem:

$$\max \; EU_f(p) \tag{6}$$

where p represents profits, defined as revenues R minus costs C. If we denote by MR and MC the marginal revenues and marginal costs respectively, the output decision of the firm in this case will be determined by the following first order condition:

$$E\left\{ \frac{\partial U_f(p)}{\partial p} \left[MR - MC \right] \right\} = 0 \tag{7}$$

As pointed out by Leland, if profits are linear in output, a risk averse firm is sufficient for the second order conditions of this problem to be satisfied. Formally, a firm is risk averse if its utility function satisfies the following condition: $\partial U_f(p)/\partial p \geq 0$ and $\partial^2 U_f(p)/\partial p^2 \leq 0$. Finally, using the notation above, one can also extend this second objective, denoted by Utility Maximizing (UM) firm objective, to a multi-period setup. If we denote by π_f the firm's subjective probability measure, equation (6) can be rewritten as:

$$UM = \max \sum_{s_t} \pi_f(s^t) U_f(p(s^t)) \tag{8}$$

It is important to note that the UM objective has been mainly criticized by the fact that firms have no physical identity on which preferences could be rooted. In other words, one has to assume that there is a manager who decides on the utility for the firm. However, his identity is exogenously given, independently on the ownership structure, and it may be true that his preferences are at variance with those of the other shareholders. On the other hand, the difficulties in consulting the shareholders, and in implementing a transfer scheme across them, a necessary assumption to derive the VM objective, are evident under the Grossman and Hart criterium. Thus, none of the two objectives is completely satisfactory from a conceptual point of view. While giving a resolution to the previous discussion is outside the scope of the present work, we still think that we can contribute to the debate by analyzing, in a dynamic context, the implications of the two firm objectives[5].

3 The Model

The Environment

The economy is populated by a firm f and by two (classes of) households, $i \in I \equiv \{1,2\}$. Households are only distinguished by their realizations of an idiosyncratic labor income shock ϵ_{it}, which follows the stationary Markov process[6]:

$$\log(\epsilon_{1t}) = c + \psi_\epsilon \log(\epsilon_{1t-1}) + \varepsilon_{\epsilon t},\ \ \varepsilon_{\epsilon t} \sim N(0, \sigma_\epsilon^2) \tag{1}$$

$$\epsilon_{2t} = 1 - \epsilon_{1t} \tag{2}$$

Each period t, household i is endowed with one unit of labor, which he can transform into ϵ_{it} productivity or efficiency labor units that will be supplied to the firm. Therefore, his period labor income is equal to $\epsilon_{it}w_t$, where w_t is the aggregate wage rate paid by the firm, while the total labor supply in the economy is always equal to one, since, $L_t = \epsilon_{1t} + \epsilon_{2t} = 1$. Note that, with this normalization, the idiosyncratic shock has a redistributive character, since it determines the period share of each household in total labor income, $\frac{\epsilon_{it}w_t}{w_t L_t}$. In addition, the symmetric nature of the shock implies that it will lead to no effects on the aggregate macroeconomic variables.

By assumption, households are not allowed to trade in contingent claim contracts to fully insure against uncertainty. Instead, they can only trade in two assets: a perfectly divisible equity share of the firm, and a risk free one-period bond, assumed

[5]As stated in the introduction of the thesis, a further contribution on the subject is basically due to Dréze (85) and DeMarzo (93), who study the possibility of introducing a control mechanism based on voting to decide among alternative production plans. In the present work, however, we concentrate on the two approaches described above.

[6]The shock is bounded below and above at 0.1 and 0.9 to avoid individual consumption from being too low when a household is constrained for several periods.

to be in zero net supply. At time t, the equity share, with price v_t, provides a claim to a flow of firm dividends d_t from time $t+1$ onwards. The bond, with price p_t^b, provides a claim to one unit of consumption at time $t+1$. Trade is also limited by short selling and borrowing constraints, implying that the resulting market structure is incomplete[7].

At date t each agent's preferences over sequences of consumption, $\{c_{it}\}_{t=0}^{\infty}$ are given by:

$$U_{it} = E_t \left\{ \sum_{j=0}^{\infty} \beta^j \frac{c_{it+j}^{1-\gamma}}{1-\gamma} \right\},\ i = 1, 2 \tag{3}$$

where $0 < \beta < 1$ is the subjective time discount factor, and γ is the relative risk aversion, assumed to be equal for both households. Each period, household i maximizes (3) subject to the following constraints:

$$c_{it} + v_t\theta_{it} + p_t^b b_{it} = (v_t + d_t)\,\theta_{it-1} + b_{it-1} + w_t\epsilon_{it},\ i = 1, 2 \tag{4}$$

$$\theta_{it} \geq K^e,\ i = 1, 2 \tag{5}$$

$$b_{it} \geq K^b,\ i = 1, 2 \tag{6}$$

The first equation represents the sequence of budget constraints faced by each household, while equations (5) and (6) are the no short-selling and borrowing constraints. Market prices (v_t, p_t^b, w_t), dividends, d_t, and initial asset holdings $(\overline{b}_{i0}, \overline{\theta}_{i0})$ are taken as given. Further, it is assumed that households hold initially zero debt and one half of the equity share of the firm.

As stated before, there is a single firm in the economy which combines its owned capital with the fixed labor supply from the households to produce a single good, according to the constant returns to scale production function:

$$y_t = z_t F(k_{t-1}, 1) = z_t k_{t-1}^{\alpha},\ k_0 \text{ given} \tag{7}$$

The variable z_t represents an aggregate technology shock, following the stationary Markov process:

$$\log(z_t) = \psi_z \log(z_{t-1}) + \varepsilon_{zt},\ \varepsilon_{zt} \sim N(0, \sigma_z^2) \tag{8}$$

[7]Note that the imposition of asset constraints is necessary to rule out Ponzi schemes or equilibria with unbounded asset holdings.

The previous shock is assumed to be independent from the idiosyncratic shock of the households. Thus, households will be subject to both aggregate and idiosyncratic uncertainty.

At each date t, the firm has to decide on the amount of investment i_t and on the way of financing it. Concerning the last, it is assumed that the firm has no access to any source of external financing, implying that investment will be solely financed by retained earnings[8]. Under these assumptions, we have:

$$i_t = k_t - (1-\delta)k_{t-1} = R_{et} \tag{9}$$

$$y_t - w_t = R_{et} + d_t \tag{10}$$

where R_{et} and d_t denote retained earnings and dividends respectively. Combining both equations, one obtains that dividends are equal to the residual of the value of output after wages have been paid and investment has been financed. In other words, dividends are equal to the net cash flow of the firm every period, i.e.,

$$N_t = y_t - w_t - i_t = d_t \tag{11}$$

Given that the market structure is incomplete the firm will maximize either:

$$VM = E_t \left\{ \sum_{j=0}^{\infty} \sum_{i=1}^{2} \overline{\theta}_{i0} M_{i,t+j} N_{t+j} \right\} \text{ where } M_{i,t+j} = \beta^j \frac{c_{it+j}^{-\gamma}}{c_{it}^{-\gamma}} \tag{12}$$

or

$$UM = E_t \left\{ \sum_{j=0}^{\infty} \beta_f^j U_f(N_{t+j}) \right\} = E_t \left\{ \sum_{j=0}^{\infty} \beta^j \frac{N_{t+j}^{1-\gamma_f}}{1-\gamma_f} \right\} \tag{13}$$

As to the second objective, we assume that the firm has the same discount factor β_f, and utility functional form U_f as the households, while we allow for a different risk aversion value. While the first assumption leads to the same deterministic steady state under both firm objectives, the concavity of U_f ensures that the second order conditions for the firm's problem are satisfied.

---

[8]Note that, under identical households (or effectively complete financial markets), internal and external finance are perfect substitutes, implying that the problem of the firm is indeterminate. In the present setup, this is solved by fixing the amount of outstanding equity and by ruling out any other source of external finance. Since the identical household case is the benchmark case to evaluate the incomplete market economies, the same assumption concerning the capital structure of the firm is preserved in this second case.

The Equilibrium

At period t, market clearing requires:

$$\sum_i c_{it} + i_t = y_t \tag{14}$$

$$\sum_i \theta_{i,t} = 1 \tag{15}$$

$$\sum_i b_{i,t} = 0 \tag{16}$$

Note that equations (15) and (16) imply that there is a single outstanding firm equity share, while bonds are in zero net supply. Concerning the maximization problem of the firm, its first order conditions imply that, for all t:

$$w_t = z_t F_2'(k_{t-1}, 1) = (1-\alpha) y_t \tag{17}$$

$$1 = E_t \left\{ \sum_{i=1}^{2} \overline{\theta}_{i0} M_{i,t+1} \left[z_{t+1} \alpha k_t^{\alpha-1} + (1-\delta) \right] \right\} \tag{18}$$

or

$$1 = E_t \left\{ \beta \frac{N_{t+1}^{-\gamma_f}}{N_t^{-\gamma_f}} [z_{t+1} \alpha k_t^{\alpha-1} + (1-\delta)] \right\} \tag{19}$$

Equation (17) implies that labor is paid its marginal product, while the capital stock is determined by equations (18) or (19), depending on the firm objective. As reflected in the Euler equations, the difference between the two firm objectives lies in the firm discount factor, which affects the law of motion of the capital stock, and thus, the behavior of the aggregate macroeconomic variables. This difference is discussed in more detail below. Finally, the first order conditions from the optimization problem of the households lead to the usual Euler equations determining asset prices:

$$v_t \geq E_t \left[M_{i,t+1} (v_{t+1} + d_{t+1}) \right] \text{ and } \theta_{it} \geq K^e,\ i = 1, 2 \tag{20}$$

$$p_t^b \geq E_t \left[M_{i,t+1} \right] \text{ and } b_{it} \geq K^b,\ i = 1, 2 \tag{21}$$

As reflected in the equations, if the asset constraints are binding, the associated Euler equations will only be satisfied as an inequality. In addition, the structure of our two household economy implies that, at period t, the constraint can only be binding for one of the two households, while the unconstrained household, which is the one with the highest price valuation, will determine the asset price that period. Given this, we can write the two Euler inequalities as follows:

$$v_t = \max_i E_t[M_{i,t+1}(v_{t+1} + d_{t+1})] \tag{22}$$

$$p_t^b = \max_i E_t[M_{i,t+1}] \tag{23}$$

The Representative Household Economy

Before turning to the parameterization of the previous model, it will be useful to consider the identical household case, i.e., the case in which they are not subject to any idiosyncratic uncertainty. In this case, ϵ_{it} is set to its expected value, which is assumed to be 0.5, implying that each household receives one half of the total labor income w_t. Since there is no room for asset trading under identical households, the equilibrium asset holdings will be equal to $b_{it} = 0$ and $\theta_{it} = 0.5$, while individual consumptions and asset prices will be given by:

$$c_{it} = \frac{1}{2} c_t, \;\; i = 1, 2 \tag{24}$$

$$v_t = E_t \left[M_{t+1}(v_{t+1} + d_{t+1}) \right] \tag{25}$$

$$p_t^b = E_t \left[M_{t+1} \right] \tag{26}$$

where M_{t+1} is the marginal rate of substitution of aggregate consumption. Finally, depending on the firm objective, the capital stock will be determined by the following two Euler equations:

$$1 = E_t \left\{ M_{t+1}[z_{t+1} \alpha k_t^{\alpha - 1} + (1 - \delta)] \right\} \tag{27}$$

or

$$1 = E_t \left\{ \beta \frac{N_{t+1}^{-\gamma_f}}{N_t^{-\gamma_f}} [z_{t+1} \alpha k_t^{\alpha - 1} + (1 - \delta)] \right\} \tag{28}$$

Note that, if the firm has a VM objective, the equilibrium outcome of this setup is the same as under complete markets. In this case, idiosyncratic uncertainty would be completely diversified away, and households would consume one half of the aggregate resources in the economy. In addition, the capital Euler equation derived from the usual market value maximization firm objective would be given by equation (27). Given this, the economy with a VM firm objective and identical households can be used as a benchmark case to evaluate the incomplete market economies. On the other hand, in spite of the fact that markets are complete, the first best frictionless outcome will not result any more if the firm is risk averse, given that this introduces a distortion in the economy. This is discussed in what follows.

The Implications of a Risk Averse Firm

If households are identical, an interesting implication of the presence of a risk averse firm is the fact that, around the steady state, the economy behaves as if there were capital adjustment costs. This is summarized by the following proposition:

Proposition 1 *Assume that the economy is populated by a representative household (or many identical households) and a risk averse firm, with the same discount factor* β_f *and CRRA utility* U_f *as the households. Under this assumption, the economy is locally equivalent to an economy in which the firm maximizes the discounted present value of profits, but is subject to capital installation costs of the form* $g(\frac{i_t}{k_{t-1}})$*, with the property that* $g(\frac{i_{ss}}{k_{ss}}) = \frac{i_{ss}}{k_{ss}}$*, and* $g'(\frac{i_{ss}}{k_{ss}}) = 1$*, where* $\frac{i_{ss}}{k_{ss}}$ *is the steady state investment to capital ratio*[9].

Proof: In the appendix. To understand the proposition better, define

$$\zeta = -\frac{g''(\frac{i^{ss}}{k^{ss}})\frac{i^{ss}}{k^{ss}}}{g'(\frac{i^{ss}}{k^{ss}})} \tag{29}$$

Note that the steady state elasticity ζ, which depends on the curvature of the adjustment cost function, is the parameter determining the degree of capital adjustment costs in the economy. In particular, if ζ is equal to zero, there are no installation costs. Proposition one states that, given a level of firm risk aversion γ_f in an economy with a utility maximizing firm, there exists a level of ζ, such that the same economy

[9]The property on the adjustment cost function required by proposition one ensures that the economy has the same steady state as the economy with a risk averse firm and no adjustment costs. An example of a function satisfying this property is the following: $g(\frac{i_t}{k_{t-1}}) = \frac{b}{1-a}(\frac{i_t}{k_{t-1}})^{1-a} + c$, with $b = (\frac{i_{ss}}{k_{ss}})^a$ and $c = -\frac{i_{ss}a}{k_{ss}(1-a)}$. Note that a is the parameter governing the degree of adjustment costs in this case.

with a value maximizing firm and capital installation costs, whose degree is determined by ζ, has the same behavior around the steady state[10]. Intuitively, if the firm is risk averse, it will try to smooth its net cash flow over time, implying that it will choose a lower (higher) investment level after a good (bad) shock, as compared to the investment level it would choose under the VM objective. Thus, investment will be smoother, and consumption more volatile, if the firm is risk averse, as happens in the presence of capital installation costs.

4 Calibration

When choosing the parameter values of the previous model, it is important to note that the economy is essentially a real business cycle model, which has been extended to incorporate incomplete markets due to restricted asset trade. While the main purpose of the present work is to evaluate the model concerning the impact of the different firm objectives, as well as to asses the effects of market incompleteness on the different asset return moments, it is important that the model still replicates the salient business cycle statistics. For this reason, a first set of parameters is chosen following the RBC literature.

As usual in RBC models, the time period is one quarter. The constant capital/output share α of the Cobb-Douglas production function is set to 0.36, the approximate percentage of GNP of capital owners for the postwar period. The capital depreciation rate δ is chosen to be 0.025, a value that matches the steady state investment to capital ratio for the same period, and the subjective discount rate of the households β is set to 0.99, leading to an annual interest rate of 4%, as usual in the literature simulating quarterly data.

Concerning the two parameters governing the law of motion of the aggregate technology shock, on the basis of the estimated autocorrelation of the output equation Solow residuals, several authors have found values for the shock persistence in the range $\psi_z \in [0.95 : 1]$. Further, some data analysis suggests that the standard deviation of the innovation σ_z can be expected to lie in the interval $\sigma_z \in [0.007 : 0.01]$.

[10] Although it has only been proved that this equivalence holds locally (by log linearizing the two economies around the steady state), we have obtained the same equivalence by numerically solving the two models with a method that does not rely on aproximations around the steady state, suggesting that the equivalence also holds globally.

Given this, the parameters chosen are $(\psi_z, \sigma_z) = (0.95, 0.00712)$.

Household risk aversion is assumed to be equal to 1, a value that lies in the acceptable interval [1 : 5] suggested in the literature[11]. Concerning the two asset constraints, it is assumed that households cannot short sell equity, i.e., $K_i^e = 0$, while they can only borrow up to a certain level, which is set at approximately 1/3 of the steady state labor income, corresponding to a borrowing constraint of $K_i^b = -0.4$.

As to the idiosyncratic shock, it has been noted earlier that it represents the period share of each household in total labor income. The mean of this share for the first household is set to 0.5, leading to the same expected value for household two[12]. Further, the parameters ψ_ϵ and σ_ϵ are chosen to match the persistence and innovation standard deviation of the household labor income share in the data.

In the existing literature, idiosyncratic labor income uncertainty has been usually modelled in exchange economies, with annual estimates for the lognormal labor income share in the following ranges: $\psi_\epsilon^a \in [0.529 : 0.9]$ and $\sigma_\epsilon^a \in [0.19 : 0.28]$. For example, using data from the Panel Study on Income Dynamics (PSID), Heaton and Lucas (96) estimate a lognormal process for individual labor income as a fraction of aggregate labor income, obtaining averaged estimates of $(\psi_\epsilon^a, \sigma_\epsilon^a) = (0.529, 0.25)$. Using the same data, a more recent estimate of these two parameters is due to Telmer, with values of $(\psi_\epsilon^a, \sigma_\epsilon^a) = (0.9, 0.2828)$, while Aiyagari and Mcgrattan have used the estimates $(\psi_\epsilon^a, \sigma_\epsilon^a) = (0.6, 0.19)$. To convert these numbers into quarterly values, one can use the following time series properties. Assume the process for quarterly data:

$$y_t = \rho_q y_{t-1} + u_{qt}, \text{ where } u_{qt} \sim N(0, \sigma_{u_q}^2) \tag{1}$$

Substituting iteratively for y_{t-1}, one can obtain the dependence of the variable y_t on annual data (four quarters ahead). This is given by the following equation:

$$y_t = \rho_q^4 y_{t-4} + u_{qt} + \rho_q u_{qt-1} + \rho_q^2 u_{qt-2} + \rho_q^3 u_{qt-3} \tag{2}$$

which can be rewritten as:

[11] Following the finding that the main asset market facts can only be replicated in representative agent models with extremelly high risk aversion parameters, the appropriateness of such large values has been widely discussed in the literature. In general, values greater than 10 have been found highly unrealistic, and several authors have suggested as a reasonable range the interval [1:5].

[12] As noted by Telmer (93), one could introduce further heterogeneity into the model by varying the expected value of the labor income share for the first household.

$$y_t = \rho_a y_{t-1} + \varepsilon_t \text{ where } \varepsilon_t \sim (0, \sigma_\varepsilon^2) \tag{3}$$

where ρ_a and σ_ε would be the persistence and the innovation standard deviation for annual data. Looking at the equations, one can infer the two quarterly estimates from the annual values, noting that $\rho_q = \rho_a{}^{1/4}$ and that the annual innovation variance is given by $\sigma_\varepsilon^2 = \sigma_{u_q}^2(1 + \rho_q^2 + \rho_q^4 + \rho_q^6)$. Using the previous procedure, and annual estimates for the two parameters at the lower end, we have obtained the following quarterly numbers, which we will use in our model $(\psi_\epsilon, \sigma_\epsilon) = (0.8528, 0.1169)$.

The only remaining parameter to be calibrated is the risk aversion of the firm, which is chosen to match the dividend/net cash flow cyclicality in the US data. To analyze the behavior of the these two variables in the US, we have used two different data sets, the NIPA and the CRSP, described in more detail in appendix 3. The following table reports the basic statistics for the two variables.

Table 2.1: Dividend behavior in the US economy

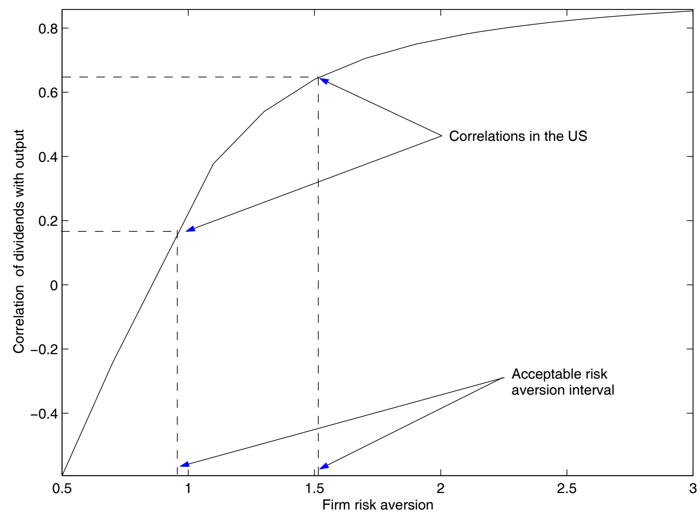
	std in %	std(x)/std(y)	corr(x,y)
Dividends (CRSP)	6.64	3.96	0.39
Dividends (NIPA)	6.53	3.89	0.16
Net cash flow (NIPA)	5.27	3.14	0.65

All data have been detrended with the HP filter

As reflected in the table, dividend payments have been subject to substantial variability during the postwar period, with a standard deviation of around 6.6 percent, and a relative standard deviation with respect to output of around 3.9 percent[13]. In addition, we can see that dividends are procyclical, with a correlation with output of 0.39 and 0.16 for CRSP and NIPA data respectively. Finally, the last row of the table reflects a slightly less variable and a more procyclical firm net cash flow, with an output correlation of around 0.65. Figure 1 below displays the relationship between the correlation of dividends (or firm net cash flow) with output and the firm risk aversion in the UM model.

[13]In the literature, several authors using CRSP data have reported a higher dividend variability. See, for example, Campbell (98). As the author points out, part of this variance is due to a strong seasonal effect in the quarterly series. Since there is no seasonality in our model, we have deseasonalized the CRSP series before computing the statistics in the table.

F



As reflected in the figure, the dividend correlation is increasing with the firm risk aversion, while the risk aversion value that better matches the correlations in the data lies approximately between 1 and 1.5. Given this, we have chosen a firm risk aversion of 1.5. Using the previous parameter values, the solution to the model has been computed numerically with the Parametrized Expectations Approach (PEA), originally proposed by Den Haan and Marcet (90), and described in appendix 4.

5 Simulation Results

The present section reports the results obtained for several economies with differing assumptions concerning the uncertainty and trading opportunities of the households. The model is first solved under the assumption that both households are identical in all respects. As already seen in section 3, the solution obtained with a VM firm objective and identical households is the same as under complete markets, and will thus serve as a benchmark case for comparison with the incomplete market economies. In addition, the solution with identical households and a UM firm will allow us to compare the effects of the two firm objectives independently of the existence of market incompleteness. After doing this, household heterogeneity is introduced under two different assumptions: the existence of either a bond, or a stock market for firm shares, providing a measure of the degree of risk sharing achieved with the two different asset markets.

5.1 Macroeconomic and Financial Stylized Facts

The performance of the model will be evaluated along two different dimensions. Concerning the macroeconomic features of the business cycle, the following table summarizes the basic statistics for the US economy that the model should be able to replicate.

Table 2.2: Macroeconomic statistics in the US economy

Data from 59:1-96:4	std in%	std(x)/std(y)	corr(x,y)
Output	1.70	1.00	1.00
Consumption	1.33	0.782	0.871
Investment	7.73	4.551	0.905
Capital Stock	0.762	0.448	0.448

We have used GNP, Personal Consumption Expenditures and Gross Private Domestic Investment. The capital stock is constructed from the investment series, using a depreciation of 0.025 and a starting capital stock to match the postwar investment-to-capital ratio. All data are HP filtered. Source: NIPA.

As reflected in the table, investment is more variable than output, while consumption is less variable and the capital stock much less so. We also see that all variables are highly procyclical except the stock of capital, whose contemporaneous correlation with output is around 0.45. The key financial observations of US asset returns are reported in table 3 below.

Table 2.3: Financial statistics in the US economy

Data from 48:1-95:4	Mean	Std in%
Equity Return (SP500)	2.17	7.60
Risk Free Rate (T-Bills)	0.23	0.78
Equity Premium	1.94	7.49
ρ(SP500,T-Bills)	0.195	

Returns are at quarterly frequency and in %. The equity premium is calculated as one plus the total return on stocks divided by one plus the T-Bill rate. The numbers are arithmetic averages. Source: Ibbotson Associates.

Looking at the table, we note that the average real return on stocks has been much higher than the average risk free rate. Over the period from 1948:1 until 1995:4, stocks have carried an equity premium of about 2 percent per quarter, while the average return on T-Bills has been around 0.2 percent. In addition, the risk free rate exhibits a much lower variability as compared to stocks, with a contemporaneous correlation of the two asset returns of about 0.2 during the postwar period.

5.2 The Representative Household Economy

The following table summarizes the performance of the representative household economy with regard to the basic macroeconomic aggregates with a Value Maximizing (VM) and a Utility Maximizing (UM) firm objective.

Table 2.4: Macroeconomic statistics, Identical households

Series	VM std(x)/std(y)	corr(x,y)	UM std(x)/std(y)	corr(x,y)
Output	1.00	1.00	1.00	1.00
Consumption	0.35	0.93	0.87	0.99
Investment	2.99	0.99	1.38	0.99
Capital Stock	0.27	0.35	0.12	0.25
Dividends	4.02	-0.97	0.11	0.64

The simulated data have been detrended with the HP filter.

Given that the economy with a VM firm objective is a standard RBC model, it is not surprising that it reproduces the macroeconomic regularities relatively well. As reflected in the first column of the table, investment is more variable than output, while consumption and capital are less variable, with the capital stock exhibiting the lowest variability. As in the data, the correlation with output is high for consumption and investment, while the capital stock is less procyclical. We also see, however, that the VM model predicts a very counterfactual fact regarding dividend payments. While the relative variability of this variable is close to the data, the model predicts a correlation of dividends with output of -0.97, caused by the fact that dividends are defined as the residual payment of profits and investment, which increases, in this case, by relatively more than profits after a good shock.

The last two columns of the table display the same statistics for the utility maximizing firm. Although the basic pattern of the macro aggregates is maintained, i.e., investment is more volatile than output, while consumption and capital are less volatile, the relative variability of aggregate consumption is now higher, while the opposite happens to the relative variability of the capital stock, the investment level, and the dividend payments. As we see, dividends are now procyclical but have a too low volatility as compared to the data. These differences, confirming the finding of proposition one, are also illustrated by the impulse response functions, depicted for investment, consumption and dividends in figure 2 below.

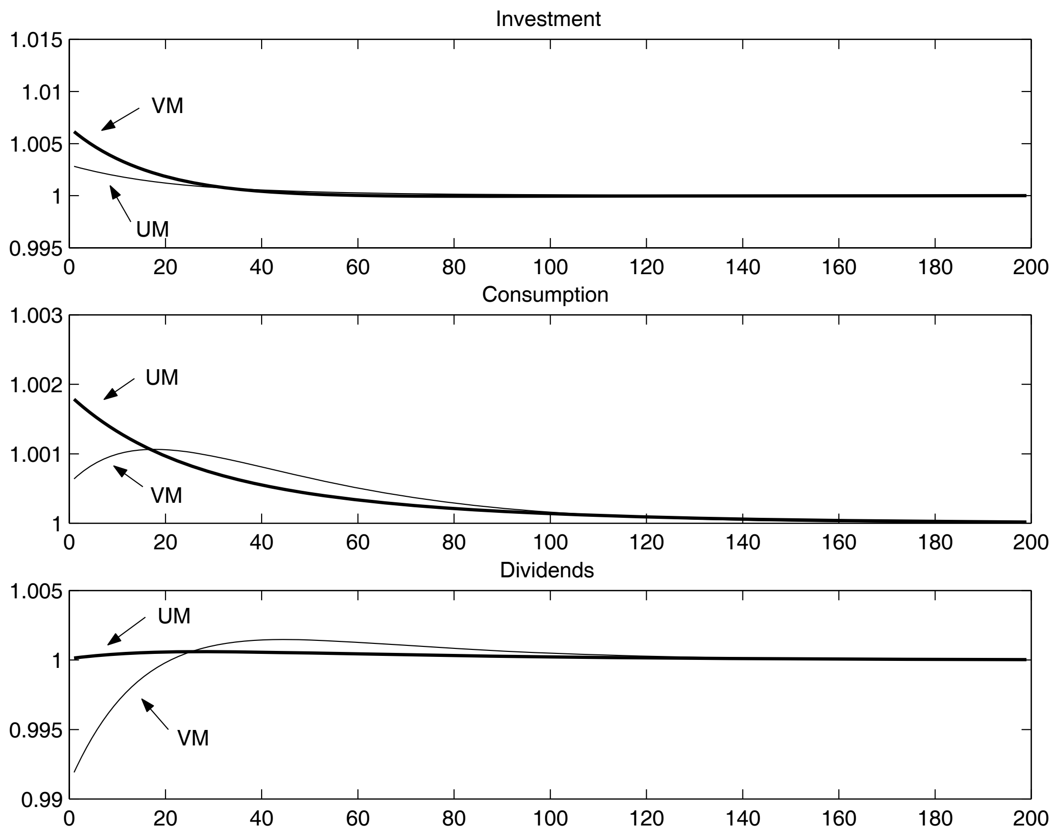


Figure 2.2: Impulse responses under the VM and UM firm objectives

The figure shows that the UM firm smooths its net cash flow (or dividends) over time through a less elastic investment, leading, in turn, to slightly procyclical dividend payments and to a more variable consumption pattern, as in the presence of capital installation costs. The basic financial statistics predicted by the two models are displayed in table 5 below.

Table 2.5: Financial statistics, Identical households

Series	**VM**	Mean	Std	**UM**	Mean	Std
Equity Return (r_t^e)		1.0101	0.0694		1.0119	0.5909
Risk Free Rate (r_t^f)		1.0100	0.0647		1.0082	0.0775
Equity Premium		0.0001	0.0250		0.0037	0.5757
$\rho(r_t^e, r_t^f)$		0.93			0.13	

Returns are computed as: $(1 + r_t^e) = (d_{t+1} + v_{t+1})/v_t$ and $(1 + r_t^f) = 1/p_t^b$. The premium is computed as the difference of average returns. All numbers are at quarterly frequency and in percent.

As noted by other authors[14], while the standard RBC model performs relatively well concerning the basic business cycle features, it is unable to replicate the key financial statistics in the US data. The first two columns of the table show that the VM model leads to an almost zero equity premium, and to asset return volatilities that are very far from reality. Concerning the UM firm, the results are slightly better, with a more volatile equity return and a slightly higher premium, which is, however, still too small.

To better understand the reasons for this failure, we can use the log linear closed-form solutions for risk premia derived in Lettau (98) for the standard RBC model. As usual, the equity premium is equal to the negative of the covariance between the logarithm of the pricing kernel, equal to the marginal rate of substitution of aggregate consumption, and the logarithm of the gross equity return. As the author shows, this covariance is given in the model by the following expression:

$$r_{t+1}^{rp} = -Cov_t(m_{t+1}, r_{t+1}^e) = \gamma \eta_{cz} \eta_{r^e z} \sigma_z^2 \qquad (1)$$

where η_{xy} is the elasticity of variable x with respect to variable y. Apart from the aggregate shock standard deviation and the household risk aversion, the equation shows that the covariance is determined by both, the elasticity of consumption with respect to the shock (η_{cz}) and the equity return elasticity with respect to the shock ($\eta_{r^e z}$). In addition, this last elasticity can be decomposed into a component due to revisions in expectations of future dividend changes ($\eta_{r^e z}^d$) and a component due to

[14]See, for example, Rouwenhorst (95), Lettau (96), Lettau and Uhlig (97) or Jermann (98).

revisions in expectations of the future risk free rate $(\eta^{rf}_{r^e z})$[15]. Using our parameterization and the same procedure as the author, we have calculated the elasticities and the risk premium generated by the two firm objectives. The results are displayed in table 6 below.

Table 2.6: Premium and elasticities from the closed-form solution

$\gamma = 1$	r^{rp}	η_{cz}	$\eta_{r^e z}$	$\eta^{d}_{r^e z}$	$\eta^{rf}_{r^e z}$
VM	0.0001	0.305	0.035	-0.045	0.079
UM	0.0035	0.871	0.799	0.154	0.645
$\gamma = 26$	r^{rp}	η_{cz}	$\eta_{r^e z}$	$\eta^{d}_{r^e z}$	$\eta^{rf}_{r^e z}$
VM	0.0013	0.282	0.035	-1.42	1.4
UM	1.9433	0.871	16.93	0.154	16.8

As reflected in the upper part of the table, displaying the results for the benchmark risk aversion of 1, the consumption and equity return elasticities are smaller under the VM firm objective. In particular, the relatively small return elasticity in this case, due to the negative dividend component, leads to the almost zero premium already reported above. As we see, the two components of the return elasticity are positive and higher under the UM firm objective, a fact that, together with the higher consumption elasticity, leads to a slight improvement in the premium, which is, however, still too small. Thus, the failure of the previous models lies in the relatively low elasticities, which are not able to generate sizesable asset moments.

The results for a higher household risk aversion are displayed in the second part of the table, illustrating an important difference between the two firm objectives which has not been noted before. Under the VM firm objective, the premium is extremely insensitive to an increase in risk aversion, whose positive effect is mitigated by the behavior of the two elasticities. While the two return elasticity components almost cancel out, the higher risk aversion leads to a smoother consumption profile, decreasing the consumption elasticity, and leaving the moments almost unaffected. Given this, it becomes clear that the representative household VM model has little chance to generate the asset moments in the data, even with an unrealistically high risk aversion parameter. In contrast to this, while the consumption smoothing channel of

[15] A brief derivation of the elasticities and of the expression for the premium is given in appendix 5. See Lettau (98) for details on the derivation, and chapter 3 for a more comprehensive discussion of the results below.

a higher household risk aversion is eliminated under the UM firm objective, the equity return elasticity increases considerably, leading to a much higher risk premium[16]. As reflected by the table, however, a risk aversion value of 26 would still be needed to generate the premium in the data, confirming the fact that, with a single source of uncertainty, reasonable values for γ and σ_z are not able to generate good results under this second firm objective either.

5.3 The Model with Heterogeneous Households

In what follows, we present the results of extending the model with an idiosyncratic labor income shock, leading to ex-post household heterogeneity and to an incomplete financial market structure. As stated earlier, the implications of the different economies are studied under two different assumptions. First, it is assumed that households can insure against uncertainty by trading only in the stock market, while a second experiment analyzes the case in which households are allowed to trade in bonds, while there is no market for firm shares. In this case, they hold the equity share to receive dividend payments, while they can only insure by trading in the riskless asset.

The following tables summarize the macroeconomic statistics generated by the VM and UM models in the cases in which there is only an equity market (VME and UME), and the cases in which there is only a bond market (VMB and UMB)[17].

[16]Note that, under the UM firm objective, household risk aversion does not enter the expectational Euler equation governing the behavior of the system. This eliminates the consumption smoothing channel, leaving η_{cz} unaffected after an increase in risk aversion. In addition, since the two return elasticity components directly depend on η_{cz}, this implies that the dividend component, not affected γ, will remain unaltered, while the component due to the future risk free rate, directly affected by γ, will considerably increase.

[17]In the presence of asset trade, asset prices are determined by the household that is unconstrained, which is the one with the higuest price valuation. In the absence of trade, this still holds, since the no trade environment can be interpreted as the limiting case in which the asset constraints are equal to $K^e = 0.5$ and $K^b = 0$ respectively. In this case, the household with the highest valuation will still be the one determining the underlying asset price, since he is the one who would like to take long positions in the non-traded asset.

Table 2.7: Macroeconomic statistics, VME and VMB

Series	VME	(a)	(b)	VMB	(a)	(b)
Output		1.00	1.00		1.00	1.00
Consumption		0.36	0.93		0.43	0.76
Investment		2.95	0.99		3.02	0.96
Capital Stock		0.26	0.35		0.28	0.31
Dividends		3.99	-0.97		8.73	-0.87

Table 2.8: Macroeconomic statistics, UME and UMB

Series	UME	(a)	(b)	UMB	(a)	(b)
Output		1.00	1.00		1.00	1.00
Consumption		0.87	0.99		0.87	0.99
Investment		1.38	0.99		1.38	0.99
Capital Stock		0.13	0.25		0.13	0.25
Dividends		0.11	0.62		0.11	0.63

All data have been detrended with the HP filter. (a)std(x)/std(y), (b)corr(x,y)

As reflected in the table, the fact that idiosyncratic uncertainty has no aggregate effects leads to relative variabilities and correlations for the aggregate macroeconomic variables that are very similar to the ones obtained under identical households. Only under the VMB case, we observe a somehow higher variance and a lower output correlation of consumption and dividends, caused by the different behavior of the firm discount factor in this case[18]. Looking at the overall results, however, we see that the behavior of the macroeconomic aggregates is essentially unaltered by the presence of idiosyncratic risk. The relevant asset return statistics are displayed below.

[18]Recall that, under the VM objective, the firm discount factor is equal to a weighted average of the individual marginal rates of substitution, and is therefore determined by individual consumptions. As we will see later, the low consumption smoothing achieved under the bond trade environment leads to a high individual consumption variability, affecting the behaviour of the firm discount factor, and thus, of the macroeconomic aggregates. In particular, the VMB case results in higher and more variable levels for output, capital, investment and consumption, while dividends, defined as a residual payment, are lower and therefore more volatile, as reflected in the table. Given that individual consumption variability is much lower with equity trade (VME), the mentioned distortion does not arise, and this last case leads to the same moments as the identical household economy.

Table 2.9: Financial statistics, VME and VMB

Series	**VME**	**Mean**	**Std**	**VMB**	**Mean**	**Std**
Equity Return		1.001	0.335		0.980	13.9
Risk Free Rate		0.897	0.449		0.328	1.99
Equity Premium		0.104	0.525		0.652	13.7
$\rho(r_t^e, r_t^f)$		0.21			0.15	

Table 2.10: Financial statistics, UME and UMB

Series	**UME**	**Mean**	**Std**	**UMB**	**Mean**	**Std**
Equity Return		0.997	0.377		1.099	13.8
Risk Free Rate		0.892	0.445		0.362	1.76
Equity Premium		0.105	0.558		0.738	13.7
$\rho(r_t^e, r_t^f)$		0.19			0.2	

All units in %. Data at quarterly frequency

In contrast to the previous results, the behavior of the different asset moments is considerably altered by the presence of idiosyncratic risk and incomplete financial markets. The general pattern across the two firm objectives and financial market structures is an increase in the mean and volatility of the premium, as compared to the identical household economies, while, in the VM cases, we also observe a considerable decrease in the correlation of the two asset returns, which is now much closer to the US data. Apart from the general improvement, we note, however, a substantial difference between the two financial market structures: while the resulting equity premium is around 0.1 percent when households can insure against uncertainty through trade in the equity share, it increases to approximately 0.7 percent in the two cases in which households can only trade in the risk free bond, leading also to asset return variabilities that are closer to the US data. Thus, according to the numbers, the bond market structure leads to a higher degree of market incompleteness. This is also confirmed by the behavior of individual consumptions, reported in table 11.

Table 2.11: Moments for Consumptions and Labor Incomes

	σ_{w_1}	σ_{w_2}	σ_c	μ_{c_1}	σ_{c_1}	μ_{c_2}	σ_{c_2}
VM	0.032	0.032	0.025	1.38	0.025	1.38	0.025
UM	0.028	0.028	0.026	1.38	0.026	1.38	0.026
VME	0.220	0.214	0.025	1.38	0.091	1.38	0.094
UME	0.219	0.213	0.026	1.38	0.091	1.38	0.094
VMB	0.220	0.214	0.023	1.40	0.185	1.42	0.173
UMB	0.219	0.213	0.026	1.37	0.173	1.39	0.167

Results directly computed from the simulations (σ represents the volatility).

As illustrated earlier, under complete markets (or identical households), individual consumptions are equated, and fluctuate therefore as much as per capita consumption. This case is displayed in the first two rows of the table. Given this, the difference between the individual and the per capita consumption variabilities can be taken as a measure of the degree of market incompleteness in the different models. As shown by the table, idiosyncratic uncertainty considerably increases household risk, since it leads to a much higher variability of the individual labor incomes w_i. We observe, however, that this risk is considerably diversified away when households can trade in the equity share. In this case, individual consumptions are more variable than per capita consumption, while this variability is still much lower than the variability of labor income, reflecting that there is a relatively high consumption smoothing in this case. In contrast to this, individual consumption is highly variable when households can only trade in the risk free bond, which mitigates labor income risk much less. Thus, consumption smoothing is much lower, and the degree of market incompleteness much higher under the bond market structure, as already reflected by the asset moments.

Before further discussing the differences between the equity and bond market structures, we can first analyze, using the analytical framework of the previous section, the way in which the presence of idiosyncratic risk may affect the risk premium in the model. As shown by equation (1), the covariance between the pricing kernel and the equity return with a single source of uncertainty (or identical households) is equal to a single term, whose magnitude crucially depends on the equity return and consumption elasticities with respect to the aggregate shock. With idiosyncratic uncertainty, however, the premium is likely to increase, given that this additional source of risk will enter the expressions of both, the pricing kernel and the equity return. In particular, under the extreme form of market incompleteness in which households cannot insure against uncertainty through any asset trade, one can show that the premium is given by the following expression:

$$r_{t+1}^{rp} = -Cov_t(m_{i,t+1}^{u_t}, r_{t+1}^{e}) = \gamma\eta_{cz}\eta_{r^e z}\sigma_z^2 + \gamma\eta_{c_i\epsilon}\eta_{r^e\epsilon}\sigma_\epsilon^2(2P_t - 1) \qquad (2)$$

where u_t denotes that household i is unconstrained at period t, and P_t denotes the probability that the first household is unconstrained[19]. As reflected by the equation, idiosyncratic uncertainty affects the premium through a new term, determined by the household risk aversion, the idiosyncratic innovation variability, and the equity return and consumption elasticities with respect to the idiosyncratic shock, $\eta_{c_i\epsilon}$ and $\eta_{r^e\epsilon}$. Under the no trade assumption, chapter 3 shows that, due to the presence of this new term, the model is able to generate the mean equity premium in the data with reasonable parameter values. Note, however, that the fact that households can insure against uncertainty through asset trade in the present setup is likely to mitigate the impact of idiosyncratic risk through the effects of consumption smoothing on the two idiosyncratic elasticities[20]. In particular, a high degree of consumption smoothing, as under equity trade, is likely to decrease both, the consumption elasticity $\eta_{c_i\epsilon}$ and the equity return elasticity $\eta_{r^e\epsilon}$, since the last depends positively on the first, considerably mitigating labor income risk. In addition, since $\eta_{r^e\epsilon}$ is directly determining the standard deviation of the equity return, a high degree of consumption smoothing (or a lower $\eta_{r^e\epsilon}$) will also lead to a low premium variability. This is reflected in tables 9 and 10, which show that the lower risk sharing achieved under the bond market structure leads, not only to a higher mean, but also to a much higher premium variability. Finally, the equation also suggests that, under poor consumption smoothing, an increase in household risk aversion is likely to have a considerable impact on the premium, since it also enters multiplying the idiosyncratic term. This is confirmed

[19]Note that, if households can insure against uncertainty through asset trade, the non-differentiable asset constraints do not allow for a closed-form solution. See chapter 3 for a derivation of this expression and for further discussion.

[20]As stated before, we can interpret the no trade solution as the limiting case in which the two asset constraints are equal to $K^e = 0.5$, and $K^b = 0$. Thus, although we cannot directly use the previous equation to evaluate the economies with asset trade, it still can give important insights about the solution obtained in these cases.

by the following table, reporting the different asset return moments for the UMB case after a slight increase in risk aversion.

Table 2.12: Asset return moments for a higher risk aversion

	$\gamma = \mathbf{1.1}$		$\gamma = \mathbf{1.2}$		$\gamma = \mathbf{1.3}$	
UMB	**Mean**	**Std**	**Mean**	**Std**	**Mean**	**Std**
Equity Return	1.451	16.6	1.841	19.21	2.151	20.85
Risk Free Rate	0.367	2.07	0.399	2.43	0.441	2.82
Equity Premium	1.083	16.4	1.442	18.98	1.709	20.58

In contrast to the results under identical households, a small increase in household risk aversion leads now to a considerable impact on the mean and variability of the equity return and the premium. In particular, the model is able to approximately generate the mean equity return and equity premium in the data with a risk aversion of 1.3[21]. Unfortunately, the increase in the asset return variabilities is more than proportional, implying that the model is able to generate the mean returns in the data, but not the right risk-return trade-off. In other words, the Sharpe ratio generated by the model, probably due to the preference specification, is still too low.

In what follows, we investigate on the reasons leading to the lower insurance provided by the bond market structure. To do this, we can first look at the household trading pattern under both financial market structures. This is depicted, for a sample period and the two UM cases, in figure 3 below[22].

[21]Increasing risk aversion when households are allowed to trade in equity has also a much higher impact than the identical households, but the effect is much lower than under bond trade. In particular, increasing risk aversion from 1 to 1.5 increases the premium from 0.1 to 0.3 percent. We have also investigated the effects of relaxing the borrowing constraint. In these cases, the model is also able to generate the mean returns in the data if we use higher, but still reasonable risk aversion values.

[22]We obtain am almost identical picture for the VM firm, reflecting the fact that the firm objective does not affect the pattern of trade.

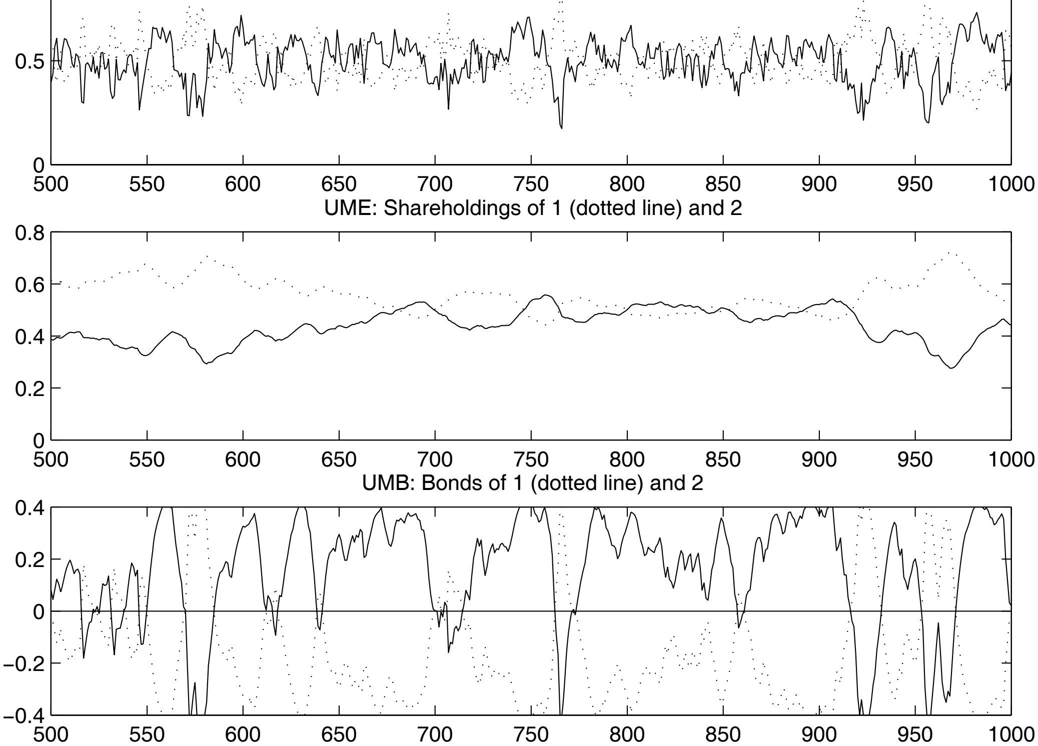


Figure 2.3: Productivity and asset holdings under the UM objective

As reflected in the figure, the pattern of trade in the two asset markets is very similar. To smooth their consumption over time, households invest (disinvest) in the equity share or the risk free bonds when they are hit by a positive (negative) productivity shock. Further, while the no short-selling constraint is almost never binding, the borrowing constraint does not bind for too many periods. In spite of the trading pattern similarity, however, there is an important aspect that could help to explain the different insurance provided by the two asset markets: the different behavior of the two asset prices, v_t and p_t^b, leading to a different variability and a different correlation with labor income of the trading volume value tv_i, which we define as $v_t(\theta_{it} - \theta_{it-1})$ and $p_t^b b_{it} - b_{it-1}$ for equity and bond trade respectively. Recall that, for the two trading environments, the households' budget constraints are given by the following equations:

$$c_{it} + v_t(\theta_{it} - \theta_{it-1}) = d_t\theta_{it-1} + w_t\epsilon_{it},\ \text{VME and UME} \tag{3}$$

$$c_{it} + p_t^b b_{it} - b_{it-1} = d_t/2 + w_t\epsilon_{it},\ \text{VMB and UMB} \tag{4}$$

Looking at the equations, we see that a higher variability and correlation with labor income of the trading volume value will make it easier for the households to mitigate the labor income risk imbedded in $w_t\epsilon_{it}$, leading to a smoother consumption profile. These two statistics are displayed for the first household and the two asset market structures in table 2.13 below.

Table 2.13: Moments for prices, asset holdings and trading volume

	σ_{w_1}	σ_{tv_1}	σ_v or σ_{p^b}	σ_{s_1} or σ_{b_1}	ρ_{w_1,tv_1}	ρ_{w_1,in_1}
VME	0.259	0.223	1.631	0.161	0.96	0.173
UME	0.259	0.222	1.486	0.162	0.96	0.174
VMB	0.280	0.052	0.019	0.213	0.48	0.577
UMB	0.259	0.051	0.017	0.217	0.52	0.626

σ represents the standard deviation

As reflected in the second column of the table, the variability of the trading volume value σ_{tv} is substantially higher, and much closer to the variability of labor income, in the two cases with equity trade, while the third and fourth columns show that this is caused by a considerably higher standard deviation of the equity price, which increases the variability of tv, in spite of the fact that bond holdings are subject to a higher fluctuation, as also shown by figure 3 above. In addition, the last two columns of the table show that the correlation between the trading volume value and labor income is much higher, while the correlation between the household's asset income in and his labor income is lower, with equity trade, making it much easier to insure against labor income risk by trading in this asset. Note that these differences are probably caused by the fact that the equity price is linked to the technology of the firm. Indeed, under a VM firm objective and identical households, one can show that the equity price is equal to the capital stock. Although this exact correspondence breaks down under the UM firm objective or with household heterogeneity, the equity price is still linked to the firm's underlying assets, and is therefore directly affected by the technology shock. This leads, not only to a higher price variability, but also to the different correlations observed above. Finally, it is important to note that the previous statistics have not taken into account dividend income. If we do this, however, the statistics are hardly altered, suggesting that dividend behavior is not one of the key determinants of the degree of risk sharing provided by the different asset markets. This is further investigated below.

Does dividend behavior really matter?
Until the present, we have mainly considered the differences between the two trading environments, without paying too much attention to dividend behavior, which is, as we have seen, counterfactual under both firm objectives. Note that an endogenous determination of dividend payments which exhibit a reasonable behavior could be probably achieved by allowing for external finance in the presence of other market frictions for the firm. While investigating this in detail is outside the scope of the present paper, we can still study the effects of dividend behavior by specifying them as an exogenous process that fits the relative variability and the correlation with output in the data.

To do this we assume that, each period, the firm pays out a stochastic fraction ϕ_t of its profits as dividends, while it keeps the rest as retained earnings to finance part of its investment. In addition, the firm can also finance its investment by issuing equity. Thus, we have that:

$$D_t = \phi_t(y_t - w_t) \tag{1}$$

$$R_{et} = y_t - w_t - D_t \tag{2}$$

$$i_t = R_{et} + v_t\left[e_t - e_{t-1}\right] \tag{3}$$

where D_t represents total dividend payments, and e_t denotes the number of outstanding equity shares. Further, the fraction of profits paid out as dividends, ϕ_t, is specified as:

$$\log\phi_t = c_\phi + \psi_\phi \log\phi_{t-1} + \varepsilon_{\phi t},\ \varepsilon_{\phi t} \sim N(0, \sigma_\phi^2) \tag{4}$$

Note that this specification of the dividend policy of the firm is not that unrealistic. In a study of corporate dividend behavior for US firms, Lintner (56) pointed out the following aspects. First, most of the companies considered had a rather definite policy regarding the target ratio of dividends to current earnings. Second, most shareholders preferred a reasonably stable payout rate, and this was normally achieved by a practice of changing dividends in any given year by only part of the amounts indicated by financial figures. In other words, most corporations had a fixed target payout ratio, which they adjusted every period to reflect changes in earnings, with this adjustment being only partial to achieve relatively stable dividend payments.

Using quarterly NIPA data from corporate US businesses on profits after taxes (PAT), dividend payments (DIV) and undistributed profits (UDP), which satisfy the following relationship:

$$PAT_t = DIV_t + UDP_t \tag{5}$$

we have constructed the quarterly payout ratio $\phi_t = \frac{DIV_t}{PAT_t}$ for US corporations. The ratio looks very similar to a persistent AR(1) process whose mean could be interpreted as the fixed target payout ratio of US corporations. In addition, detrending it with the Hodrick Prescott filter, one also observes a negative correlation with the HP filtered series of logged profits after taxes, implying that its behavior also reflects the partial adjustment of dividend payments discussed above. Using the constructed ratio, we have estimated a persistence of 0.9565 for the process in (4). With this persistence, and a standard deviation for the innovation that generates a dividend variability of around 4% in the model, the resulting output correlation of dividend payments is around 0.2, as in the data. In this case, when the firm is hit by a positive shock, it increases both, dividend payments and retained earnings, while it also issues equity, with a smaller issuing amount if the firm is risk averse, given that investment is less elastic. The following table displays the asset moments for the UM cases resulting from the new dividend specification.

Table 2.14: Financial statistics, UM with exogenous dividends

Series	**UME**	**Mean**	**Std**	**UMB**	**Mean**	**Std**
Equity Return		1.002	0.53		1.034	13.2
Risk Free Rate		0.887	0.44		0.367	1.78
Equity Premium		0.115	0.67		0.666	12.9
$\rho(r_t^e, r_t^f)$		0.1			0.16	

All units in %. Data at quarterly frequency

Comparing the results with tables 2.9 and 2.10, we see that the return moments are very similar, suggesting again that the cyclicality and variability of dividend payments does not seem to be a key determinant of the different premium moments[23]. Note that this has been implicitly pointed out already by the closed-form solution under identical households displayed in table 2.6. As we have seen, the dividend component of the equity return elasticity was relatively low under both firm objectives, while the whole increase in the premium caused by a higher household risk aversion under the UM firm objective was accounted for by the return elasticity component due to the future risk free rate. Altogether, these results suggest that dividend behavior does not matter in the present context. Instead, other factors, like the variability of the underlying asset price, seem to be quantitatively more important for the determination of the premium.

6 Summary and Concluding Remarks

The present work has analyzed the asset pricing implications of a real business cycle model, extended with household heterogeneity and incomplete financial markets. Apart from evaluating the effects of this extension, we have also been able to address two important issues, which have not been given very much attention in the previous macroeconomic literature. First, in order to integrate an incomplete markets environment with the presence of a dynamic firm, one has to incorporate non-standard firm objectives. We have looked at two objectives, a variant of the usual market value maximization, the VM objective, and the expected utility of the firm's profits, the UM objective, which implies that the firm is risk averse. In addition to this, the presence of a non trivial production sector in the model has also allowed us to investigate the ways in which production affects asset behavior.

If households are identical, the VM objective reduces to the usual profit maximization under complete markets, while the UM firm is locally equivalent to the presence of capital adjustments costs. In the latter case, the distortion introduced by the firm objective alone does not alter the asset moments substantially. In both cases, the main reason for the failure lies in the low consumption and equity return elasticities with respect to the aggregate shock.

[23]The results are also very similar for the two VM cases.

When the model is extended with idiosyncratic risk, the behavior of the macroeconomic aggregates is essentially unaltered, while the mean and variability of the equity premium increases. If households can insure against uncertainty through equity trade, however, the high degree of consumption smoothing achieved leads to a smaller improvement in the different asset moments. In this case, the relatively variable equity price considerably increases the volatility of the equity turnover, while the correlation of this variable with labor income is very high, making it much easier for the households to mitigate labor income risk by trading in the asset. In contrast to this, when households can only trade in the risk free bond, the turnover has a much lower variability and a lower correlation with labor income, leading to a lower degree of consumption smoothing and to asset return moments that are much closer to the US data.

As stated by Telmer (93), a general finding of the exchange economy literature with household heterogeneity and incomplete financial markets is the fact that: *a very limited set of securities have the potential to go a long way in terms of providing a vehicle for the sharing of endowment risks. Specifically, almost all of an agent's idiosyncratic income risk can be diversified through trade in a single discount bond market with constraints on borrowing.* This is also confirmed by other authors, pointing out that trade in risk free bonds, as opposed to equity trade, is the main way of insurance. As we have seen, this conclusion seems to be reversed when production is endogenous. In this case, the equity price, closely linked to the firm's underlying assets, is directly affected by the technology shock of the firm, leading to the differences observed above.

Further, a natural implication of the presence of a production sector is the endogenous determination of dividend payments. Lettau (96) has pointed out that: *there are many ways to model dividends...In models with a production sector, dividends are usually regarded as payments to the production factor capital. As an alternative, Rouwenhorst (95) defines dividends as firm revenues minus wages and investments. Prices of assets with these different types of dividends are potentially very different, resulting in fragile conclusions regarding the equity premium.* Our analysis suggests that dividend behavior does not matter. First, when dividends are defined as a residual of firm profits minus investment, their behavior is counterfactual under both firm objectives. The VM objective leads to strongly countercyclical payments, while they are procyclical, but too smooth, if the firm is risk averse. In this case, while the two firm objectives have different financial implications, these differences are not mainly caused by dividend behavior. Second, when the specification of dividends is modified, asset returns are hardly altered.

Finally, we should also point out that the model is able to generate the right mean returns, but not the right return variabilities, leading to a Sharpe ratio that is smaller than in the data. One of the reasons for this failure may be the CRRA preference specification. Thus, a natural extension of the present work would be to investigate the risk-return trade-off implications of alternative preferences.

APPENDIX 1

(i) The Optimal Production Plan

In the two period economy of section two, every shareholder maximizes his expected utility subject to the following budget constraint:

$$B_i = \left\{ (c_i, \theta_i) \in X_i \times R : \begin{array}{c} c_{i0} + v\theta_i = w_{i0} + \overline{\theta}_i(y_0 + v) \\ c_{i1s} = w_{i1s} + \theta_i y_{1s} \ \forall s \end{array} \right\} \tag{1}$$

where v represents the equity price. Suppose that the economy is initially in an equilibrium relative to a production plan, and that all shareholders meet to decide on a change in this plan. If u_i is the maximized value of the utility of shareholder i (relative to the old plan), differentiating this utility with respect to y_0, one can see that a change in the production plan will lead to the following utility change[24]:

$$\frac{\partial u_i}{\partial y_0} = \left\{ \overline{\theta}_i \frac{\partial u_i}{\partial c_{i0}} \left[\frac{dv_i}{\partial y_0} + \frac{\partial y_0}{\partial y_0} \right] + \theta_i \left(\sum_{s=1}^{S} \frac{\partial u_i}{\partial c_{i1s}} \frac{\partial y_{1s}}{\partial y_0} - \frac{\partial u_i}{\partial c_{i0}} \frac{\partial v_i}{\partial y_0} \right) \right\} \tag{2}$$

which can be rewritten as:

$$b_i = \frac{\partial u_i}{\partial y_0} / \frac{\partial u_i}{\partial c_{i0}} = \left\{ \overline{\theta}_i \left[\frac{dv_i}{\partial y_0} + \frac{\partial y_0}{\partial y_0} \right] + \theta_i \left(\sum_{s=1}^{S} \pi_{is} \frac{\partial y_{1s}}{\partial y_0} - \frac{\partial v_i}{\partial y_0} \right) \right\} \tag{3}$$

where b_i can be interpreted as the maximum amount of date 1 income that shareholder i will give up to change the firm's production plan. The first term on the right hand side of the equation represents the perceived change in the shareholder's wealth at period 0 due to the new production plan (effect as initial shareholder), while the second term represents the effect as a final shareholder. As reflected in the equation, the final shareholder will be affected in two ways: first, he will receive a different dividend from his investment strategy, second, he will face a different price or cost of purchasing new firm shares. If one assumes that shareholders' perceptions are competitive, in the sense that the cost of the security price is perceived to adjust to the changed dividend stream, i.e.,

---

[24]Note that a change in the production plan can be implemented through a change in the initial investment, y_0, leading to a new random revenue y_{1s} at date 1.

$$\frac{\partial v_i}{\partial y_0} = \sum_s \pi_{is} \frac{\partial y_{1s}}{\partial y_0} \tag{4}$$

the last term of the equation cancels out, and equation (3) becomes[25]:

$$b_i = \left\{ \overline{\theta}_i \left[\frac{dv_i}{\partial y_0} + \frac{\partial y_0}{\partial y_0} \right] \right\} = \left\{ \overline{\theta}_i \left[\sum_s \pi_{is} \frac{\partial y_{1s}}{\partial y_0} + \frac{\partial y_0}{\partial y_0} \right] \right\} \tag{5}$$

Looking at the equation, it becomes clear that every shareholder will want to maximize the present value of the firm's profits, discounted by his own marginal rate of substitution. In other words, a production plan will be optimal for shareholder i if it maximizes:

$$\pi_i y = \sum_{s=0}^{S} \pi_{is} y_{1s} \tag{6}$$

The Grossman and Hart Firm Objective

Using the previous equations, one can easily show how to derive the Grossman and Hart firm objective from shareholders' preferences, as long as it is possible to implement a transfer mechanism among them. As pointed out by the authors, the term b_i on the left hand side of equation (5), will have the same sign for all the shareholders if markets are complete, while its sign may differ across them under market incompleteness. The authors assume that, at date 0, it is possible to transfer income from the shareholders who favor a change in the production plan (those with $b_i \in R_+$) and those who do not favor it (those with $b_i \in R_-$), such that all of them are made better off, i.e., such that $\sum_{i=1}^{I} b_i \in R_{++}$. Thus, a change in the production plan is assumed to take place if and only if this sum is strictly positive. From equation (5), it follows that:

[25] Equation (4) always holds under complete markets, while it is a reasonable assumption under market incompleteness. Note that it just models the fact that, if the firm produces better output (or pays higher dividends), its market value will bid up to reflect it, meaning that the shareholder will not be better off by the extra consumption benefits. As the authors point out, this assumption should not be made in the presence of no short selling constraints, since the net benefits from a marginal share would not be appropriately measured by $\sum_s \pi_{is} y_{1s}$ when $\theta_i = 0$. Using the same reasoning, note, however, that the presence of a such a constraint will not cause any problems as long as it is never or almost never binding. In addition, note that this assumption is not necessary to derive equation (5) in the particular case in which households cannot trade in the equity share. In this case, substituting for $\theta_i = \overline{\theta}_i$ in (4), equation (5) also obtains.

$$\sum_i b_i = \sum_{i=1}^{I} \overline{\theta}_i \left[\sum_{s=1}^{S} \pi_{is} \frac{\partial y_{1s}}{\partial y_0} \right] + 1 \tag{7}$$

Finally, since the expression is a concave function of y_0, for a plan not to be blocked by any shareholder, the firm should maximize:

$$V = \sum_{i=1}^{I} \overline{\theta}_i \sum_{s=1}^{S} \pi_{is} y_{1s}(y_0) + y_0 \tag{8}$$

which exactly coincides with the proposed firm objective.

APPENDIX 2

Proof of Proposition 1

To prove proposition 1, denote by economy 1 the economy with a UM objective and by economy 2 the economy with the VM objective and capital installation costs. Note that the difference between the two economies lies in the firm Euler equation and in the transitional equation for the capital stock. These equations are given by:

Economy 1:

$$k_t - (1-\delta)k_{t-1} = i_t$$

$$1 = \beta E_t \frac{{N_{t+1}}^{-\gamma_f}}{N_t^{-\gamma_f}} \left\{ \alpha z_{t+1} k_t^{\alpha-1} + (1-\delta) \right\}$$

Economy 2:

$$k_t - (1-\delta)k_{t-1} = k_{t-1} g(\frac{i_t}{k_{t-1}})$$

$$\frac{1}{g'(\frac{i_t}{k_{t-1}})} = \beta E_t \frac{c_{t+1}^{-\gamma}}{c_t^{-\gamma}} \left\{ \alpha z_{t+1} k_t^{\alpha-1} + \frac{(1-\delta) + g(\frac{i_{t+1}}{k_t})}{g'(\frac{i_{t+1}}{k_t})} - \frac{i_{t+1}}{k_t} \right\}$$

Apart from these two equations, determining the investment level and the capital stock, the level of output, aggregate consumption and dividends (or net cash flow) are determined by the following equations in both economies:

Economies 1 and 2:

$$y_t = z_t k_{t-1}^{\alpha}$$

$$c_t = y_t - i_t$$

$$N_t = \alpha y_t - i_t = d_t$$

(i) It is easy to show that the assumptions on β_f , U_f and g required by proposition 1 lead to the same steady state in the two economies.

(ii) Following Campbell (94), one can find an approximate analytical solution to the previous two economies by replacing the true constraints and Euler equations with log linear approximations. The models become then a system of expectational difference equations which can be solved using the method of undetermined coefficients. Combining the log linearized constraints, and denoting by $\widehat{x}$ the variables in logs, we obtain, for the two economies, the following equations:

Economies 1 and 2:

$$\widehat{y}_t = \widehat{z}_t + \alpha \widehat{k}_{t-1}$$

$$\widehat{i}_t = \lambda_{01} \widehat{y}_t - \lambda_{02} \widehat{c}_t$$

$$\widehat{d}_t = \lambda_{03} \widehat{y}_t - \lambda_{04} \widehat{i}_t$$

$$\widehat{k}_t = \lambda_1 \widehat{k}_{t-1} + \lambda_2 \widehat{z}_t + \lambda_3 \widehat{c}_t$$

where the coefficients denoted by λ are constants depending on the model parameters α, δ, and β. In addition, guessing that log consumption takes the form:

$$\widehat{c}_t = \eta_{ck} \widehat{k}_{t-1} + \eta_{cz} \widehat{z}_t$$

where η_{xy} is the elasticity of variable x with respect to variable y, it is easy to show that the elasticities of output, capital, investment, and dividends with respect to the two state variables are given, in the two economies, by:

Economies 1 and 2

$$\eta_{yk} = \alpha \text{ and } \eta_{yz} = 1$$

$$\eta_{kk} = \lambda_1 + \lambda_3 \eta_{ck} \text{ and } \eta_{kz} = \lambda_2 + \lambda_3 \eta_{cz}$$

$$\eta_{ik} = \lambda_{01} \alpha - \lambda_{02} \eta_{ck}, \text{ and } \eta_{iz} = \lambda_{01} - \lambda_{02} \eta_{cz}$$

$$\eta_{dk} = \lambda_{03} \alpha - \lambda_{04} \eta_{ik} \text{ and } \eta_{dz} = \lambda_{03} - \lambda_{04} \eta_{iz}$$

Given that the log linearized constraints and the elasticities of the previous variables are the same in both economies, the two cases will only be equivalent if the two consumption elasticities, η_{ck} and η_{cz}, are also the same.

(iii) To determine the consumption elasticities, one needs to make use of the log linearized Euler equations. In economy 2, this equation is given by:

$$\gamma E_t(\widehat{c}_{t+1} - \widehat{c}_t) + \zeta(\widehat{i}_t - \widehat{k}_{t-1}) = E_t(\lambda_4 \widehat{z}_{t+1} + \lambda_5 \widehat{i}_{t+1} + \lambda_6 \widehat{k}_t)$$

where the coefficients denoted by λ are constants that now also depend on the steady state elasticity ζ, given by:

$$\zeta = -\frac{g''(\frac{i^{ss}}{k^{ss}})\frac{i^{ss}}{k^{ss}}}{g'(\frac{i^{ss}}{k^{ss}})}$$

Substituting the log linearized constraints in the previous Euler equation, and using the method of undetermined coefficients, one can show that the two consumption elasticities for this economy are the solutions to:

$$\eta_{ck,2} = \frac{1}{Q_{2,2}}\left\{-Q_{1,2} - \sqrt{Q_{1,2}^2 - 4Q_{0,2}Q_{2,2}}\right\}$$

$$\eta_{cz,2} = f_2(\phi, \gamma, \zeta, \eta_{ck,2})$$

where $Q_{0,2}$, $Q_{1,2}$ and $Q_{2,2}$ are constants depending of the model parameters $\phi = (\alpha, \beta, \delta, \rho_z)$, the risk aversion of the consumer γ and ζ, and f_2 is a function depending on the previous things and the consumption elasticity with respect to the capital stock $\eta_{ck,2}$. Note that, for a given parameter vector ϕ, and a given pair (γ, ζ), the solution of economy 2 is entirely determined by the previous two equations.

As to economy 1, the log linearized Euler equation is given by:

$$\gamma_f E_t(\widehat{d}_{t+1} - \widehat{d}_t) = E_t(\lambda_7 \widehat{z}_{t+1} + \lambda_8 \widehat{k}_t)$$

while the two consumption elasticities are given by:

$$\eta_{ck,1} = \frac{1}{Q_{2,1}}\left\{-Q_{1,1} - \sqrt{Q_{1,1}^2 - 4Q_{0,1} \times Q_{2,1}}\right\}$$

$$\eta_{cz,1} = f_1(\phi, \gamma_f, \eta_{ck,1})$$

where the constants denoted by Q depend on ϕ and γ_f, and f_1 is a function of ϕ, γ_f, and $\eta_{ck,1}$. Again, for a given parameter vector ϕ and a given level of firm risk aversion γ_f, the solution of economy 1 is entirely determined by the previous two equations.

(iv) To show the local equivalence of the two economies, one just needs to find a vector $(\gamma, \zeta, \gamma_f)$ that solves the following equation system:

$$\eta_{ck,2}(\phi, \gamma, \zeta) - \eta_{ck,1}(\phi, \gamma_f) = 0$$

$$\eta_{cz,2}(\phi, \gamma, \zeta) - \eta_{cz,1}(\phi, \gamma_f) = 0$$

Note that the previous system consists of two equations and three unknowns (or free parameters). Thus, we can fix one of the parameters at the desired level, and find the pair for the other two that solves the equation system. In particular, we can fix the level of household risk aversion γ in the two economies, and find then the pair (ζ, γ_f) that leads to the same consumption elasticities and, thus, to the same local behavior in the two economies.

APPENDIX 3

In order to construct the statistics reported in table 2.1, we have used two different data sources. The first one is the National Income and Product Accounts database (NIPA), from which we have directly obtained the two series of interest, given by:

-Quarterly real dividends paid by corporate businesses, 1959:1-1996:4

-Quarterly real net cash flow of corporate businesses, 1959:1-1996:4

Further, we have also used the Center for Research in Security Prices database (CRSP) at the University of Chicago, to obtain the following series:

-Monthly nominal value weighted return index on the NYSE, AMEX and NASDAQ markets (including all distributions), I_t^d, 1947:1-1996:12

-Monthly nominal value weighted return index on the NYSE, AMEX and NASDAQ markets (excluding dividends), I_t, 1947:1-1996:12

-Monthly Consumer Price Index, 1947:1-1996:12

To construct the dividend series from the CRSP data, we have used the fact that total return indices are reported with and without dividend payments. This implies that one can extract the total dividends paid on the index using the following equation:

$$\frac{I_t^d}{I_{t-1}^d} = \frac{I_t + D_t}{I_{t-1}}$$

After doing this, we have constructed the real dividend series by deflating the nominal dividends with the CPI, while the quarterly dividend series has been computed by adding up the monthly dividends during the quarter.

APPENDIX 4

Solving the model with the PEA Algorithm

The essence of the PEA approach is to parametrize the expectations in the system as functions of the state variables, which are known at period t, in order to be able to solve for the endogenous variables. The exogenous state vector in the model is described by the process for the technology shock, governing aggregate income, and by the process for the idiosyncratic shock of one of the households. In addition, the entire state vector includes last period capital stock and the endogenous distribution of asset holdings. Given the symmetry of the two households, once we know the asset holdings of one of them, the holdings of the second carry no additional information. Further, the relevant information about asset holdings that matters for a household's decision is his total asset wealth, given by $w_{it} = (v_t + d_t)\theta_{it-1} + b_{it-1}$. Since, in the present model, households just trade in one of the assets, we can include as a state the household's asset holdings. Therefore, the chosen state vector is $s_t = \{\log z_t, k_{t-1}, \log \epsilon_{1t}, x_{1t-1}\}$, where $x_{1t-1} = s_{1t-1}$ or b_{1t-1}.

Once the state variables have been identified, the conditional expectations in the Euler equations of the system are parametrized as exponentiated polynomials, for which one could increase the degree to obtain a higher approximation accuracy. This implies that the parametrized expectations look as follows:

$$Pea_t = f_t(\beta, s_t) \equiv \exp(\beta' s_t) = \exp(\beta_1 + \beta_2 \log z_t + \beta_3 \log k_{t-1} + \beta_4 \log \epsilon_{1t} + \beta_5 x_{1t-1})$$

As reflected in the equation, each expectation is a function of the states and of a vector of parameters β, which determine the explanatory power of each state in the expectation term. To find a vector β that is consistent with the true expectations, the following iterative procedure is repeated until convergence:

(i) In the first step, the model is solved using the parameters obtained in the previous iteration. One then obtains a time series of length T for all endogenous variables. To do this, initial values for the equity and bond holdings, $\overline{\theta}_{i0}$ and $\overline{b}_{i0}$, and the capital k_0, as well as T random shocks z_t and ϵ_{1t} are needed. These shocks are only generated once and kept the same for every iteration.

(ii) The time series for the variables are then used to estimate the parameters of the power functions Pea_{it}. To ensure consistency with the true expectations, the following nonlinear least square minimization is used:

$$Min_{\beta}T^{-1}\sum_{t=1}^{T}\{E_{it}-Pea_{it}(\beta,s_t)\}^2$$

where E_{it} are the true expectations.

(iii) If one denotes the parameter estimates of this minimization by $\overline{\beta}$, one can then use a weighted average of the old parameters β and these new estimates for the next iteration, i.e., the parameters for the new iteration will be: $\beta = \lambda\beta + (1-\lambda)\overline{\beta}$, where $\lambda \in (0,1)$.

Chapter 3

Idiosyncratic shocks and asset returns in the RBC model: an analytical approach

1 Introduction

In the present chapter, we investigate the effects on the different asset moments of extending the standard Real Business Cycle (RBC) model, where the only source of risk is the aggregate technology shock of the firm, with a second source of uncertainty, an idiosyncratic labor income shock, which leads to household heterogeneity. While previous studies of a setup with household heterogeneity have relied on numerical solutions, we propose to use an approximated analytical approach in the present work, giving a much deeper insight of the channels through which idiosyncratic uncertainty may affect asset returns in the model. In particular, we extend the analytical methodology used by Lettau (98), who derives closed form solutions for the different asset moments under the assumption of a representative household. In a first step, the relevant equations are approximated in log linear form, and the elasticities of the macroeconomic aggregates with respect to the state variables are calculated as functions of the model parameters using the method of undetermined coefficients. After doing this, Campbell's decomposition of unexpected returns is used to obtain closed form solutions for the different asset moments as functions of the elasticities of the macroeconomic variables.

One of the problems of extending Lettau's methodology to our case is the fact that households will want to smooth their consumption through trade in the existing asset markets. If one allows for asset trade, however, the model cannot be approximated in log-linear form. Given that we want to preserve analytical tractability, we shut down all the trading opportunities in the present setup. Although this implies that our analysis is done under an extreme form of market incompleteness, we believe that the present setup can help to evaluate the potential of idiosyncratic shocks to explain the different asset moments. One just needs to take into account that the values of the asset moments obtained from a calibrated model have to be considered as an upper bound for the given parameterization. On the other hand, the no trade outcome is obtained by imposing no-trade constraints, i.e., constraints such that, in equilibrium, households do not trade with each other. Since only one of the households can be constrained each period, the unconstrained household will uniquely determine asset prices for that period. Note, however, that this implies that the asset moments are determined by the two households, depending from who is constrained or unconstrained every period. In the present work, this is taken into account by re-defining the asset moments as functions of the probability that a household is unconstrained at a given period.

Finally, we also have to consider that, under shareholder heterogeneity and market incompleteness, the usual value maximization objective for the firm used by Lettau is not longer well defined. In the present work, we deal with this problem by assuming that the firm is risk averse, and that it maximizes the expected utility of its net cash flow. As discussed in the previous chapter, this is one of the approaches that have been proposed in the literature to get around the firm objective problem when markets are incomplete.

The chapter is organized as follows. The following section presents the model, while section 3 explains the solution method and extends it for the case in which households are heterogeneous. The results are discussed in section 4. In order to have a benchmark case to evaluate the impact of idiosyncratic uncertainty, the solution to the model is first computed under the assumption that households are identical in all respects. In this case, the setup with a utility maximizing firm is compared to the case in which the firm has the usual market value maximization objective, while we also study the impact of different dividend specifications. After doing this, labor income risk is added to the model in order to evaluate the potential of the incomplete markets economy to explain the different asset return moments. Section 5 summarizes and concludes.

2 The Model

Households

The economy is populated by a firm f and by two (classes of) households $i = \{1, 2\}$, only distinguished by the realization of an idiosyncratic labor productivity shock ϵ_{it}. Each household solves the following problem:

$$Max\ E_t \sum_{j=0}^{\infty} \beta^j \frac{C_{i,t+j}^{1-\gamma}}{1-\gamma} \tag{1}$$

s.t.

$$C_{it} + V_t\theta_{it} + P_t^b b_{it} = (V_t + D_t)\,\theta_{it-1} + b_{it-1} + W_t\epsilon_{it} \tag{2}$$

$$b_{it} \geq K^b \text{ and } \theta_{it} \geq K^e \tag{3}$$

Equation (2) represents the households' budget constraint, and it implies that they can potentially invest in equity shares of the firm (θ), providing a claim to the firm's dividends (D) from period $t+1$ onwards, or in risk free one period bonds (b), providing a claim to one unit of consumption at period $t+1$. It is assumed that there is a single outstanding equity share, while bonds are in zero net supply. Further, initial asset holdings are assumed to be symmetric, i.e., households hold initially zero debt and one half of the equity share of the firm. As shown by equation (3), households are subject to trading constraints in the two assets, leading to incomplete financial markets.

Apart from their asset income, households receive labor income, equal to the aggregate wage rate paid by the firm (W) times the idiosyncratic productivity shock. If households are identical, $\epsilon_{it} = 0.5$, otherwise,

$$\log \epsilon_{1t} = c_\epsilon + \psi_\epsilon \log \epsilon_{1t-1} + \varepsilon_t^1,\ \varepsilon_t^1 \sim N(0, \sigma_\epsilon^2), \quad \text{with} \quad 0 \leq \epsilon_{1t} \leq 1 \ \text{ and } \ \psi_\epsilon < 1 \tag{4}$$

$$\epsilon_{2t} = 1 - \epsilon_{1t} \tag{5}$$

One can think about the previous setup as households having one fixed unit of labor, which they can transform into ϵ_{it} efficiency labor units that will be supplied to the firm. Under this assumption, aggregate labor supply will always be equal to one, since $L_t = \epsilon_{1t} + \epsilon_{2t} = 1$, and idiosyncratic uncertainty will have no effects on the aggregate macroeconomic variables. If we define $M_{t+1}^i = \beta(C_{it+1}/C_{it})^{-\gamma}$, the first order conditions for the problem above lead to the following Euler equations determining the two asset prices:

$$V_t \geq E_t[M_{t+1}^i(V_{t+1} + D_{t+1})] \text{ and } \theta_{it} \geq K^e \tag{6}$$

$$P_t^b \geq E_t[M_{t+1}^i] \text{ and } b_{it} \geq K^b \tag{7}$$

As stated before, it is only possible to obtain a closed form solution under the assumption that households cannot use asset markets to insure against uncertainty, a limiting outcome which obtains by setting the two asset constraints in the previous setup at 0 and 0.5 respectively, i.e.,

$$K^b = 0 \text{ and } K^e = 0.5 \tag{8}$$

Under this assumption, there will be a constrained household every period, while the equilibrium asset holdings will always be equal to the initial ones, i.e., $b_{it} = 0$ and $\theta_{it} = 0.5$, leading to the following equilibrium consumption:

$$C_{it} = D_t 0.5 + W_t \epsilon_{it} \tag{9}$$

Concerning the two asset prices, note that the structure of the model implies that the asset constraints can only be binding for one of the two households each period, while the unconstrained household, for whom the asset pricing equations hold with equality, will be the one determining asset prices that period. Given this, we can define the pricing kernel as the marginal rate of substitution of the household that is unconstrained, and write the two pricing equations as equalities as follows[26]:

$$PK_t = \{M_{t+1}^{u_t}\} \tag{10}$$

$$V_t = E_t[PK_t(V_{t+1} + D_{t+1})] \qquad \text{or} \qquad 1 = E_t[PK_t R_{t+1}^e] \tag{11}$$

$$P_t^b = E_t[PK_t] \qquad \text{or} \qquad 1 = E_t[PK_t] R_{t+1}^f \tag{12}$$

where R_{t+1}^j is the gross return of asset j, and the superscript u_t corresponds to the household that is unconstrained at period t, i.e., if the first (second) household is unconstrained at period t, $u_t = 1$ ($u_t = 2$).

[26] Note that, if a household wants to go short in one of the two asset markets, since he is not allowed to do so by the no-trade assumption, he will want to short sell the remaining asset, given that it is his only way to smooth consumption. Consequently, one of the two households will determine the two asset prices each period. Note also that the Kuhn Tucker conditions imply that this will be the household with the highest expected marginal rate of substitution. In other words, the pricing kernel can also be expressed as: $PK_t = \{M_{i,t+1} : E_t[M_{i,t+1}] = \max_i E_t[M_{i,t+1}]\}$.

The Firm

Apart from the two households, there is a firm owning the entire capital stock (K) in the economy, which it combines with the fixed labor supply from the households to produce output (Y). Investment (I) is entirely financed by retained earnings or profits (output net of wage payments), and the residual of profits and investment is paid out as dividends. Each period, the firm maximizes the expected discounted utility of its nets cash flow (N), by solving the following problem:

$$Max\ E_t \sum_{j=0}^{\infty} \beta_f^j \frac{N_{t+j}^{1-\gamma_f}}{1-\gamma_f} \tag{13}$$

s.t.

$$L_t = 1 \tag{14}$$

$$Y_t = Z_t K_{t-1}^{\alpha} L_t^{1-\alpha} \tag{15}$$

$$I_t = K_t - (1-\delta) K_{t-1} \tag{16}$$

$$N_t = D_t = Y_t - W_t L_t - I_t \tag{17}$$

$$\log Z_t = \psi \log Z_{t-1} + \varepsilon_t^z,\ \ \varepsilon_t^z \sim N(0, \sigma_z^2) \tag{18}$$

As reflected in the equations, the firm is subject to an aggregate technology shock (Z), which is assumed to be independent from the idiosyncratic shock of the households. In addition, its presence implies that households will be subject to both, aggregate and idiosyncratic uncertainty. The first order conditions for the problem above are given by:

$$W_t = (1-\alpha) Y_t \tag{19}$$

$$1 = \beta E_t \frac{{N_{t+1}}^{-\gamma_f}}{N_t^{-\gamma_f}} \left\{ \alpha Z_{t+1} K_t^{\alpha-1} + (1-\delta) \right\} \tag{20}$$

The first equation determines the aggregate wage rate, and it implies that $N_t = D_t = \alpha Y_t - I_t$, while the second determines the law of motion of the capital stock. Note that, under identical households (or complete markets) the marginal rates of substitution, M^i_{t+1}, would be equalized across them, and the usual market value maximization objective for the firm would be given by:

$$Max\ E_t \sum_{j=0}^{\infty} M_{t+j} N_{t+j}, \text{ where } M_{t+j} = \beta^j (\frac{C_{t+j}}{C_t})^{-\gamma} \tag{21}$$

leading to the following first order condition with respect to the capital stock:

$$1 = E_t M_{t+1} \left\{ \alpha Z_{t+1} K_t^{\alpha-1} + (1-\delta) \right\} \tag{22}$$

where M_{t+1} is the marginal rate of substitution of aggregate consumption. Looking at the two capital Euler equations in (20) and (22), it becomes clear that the difference between the two firm objectives lies in the firm discount factor, which will affect the law of motion of the capital stock, and, thus, the behavior of the aggregate macroeconomic variables. Finally, the goods market clearing condition closing the model is given by:

$$C_t + I_t = Y_t \tag{23}$$

while the two asset market clearing conditions, given by:

$$\sum_i \theta_{it} = 1 \ \text{ and } \ \sum_i b_{it} = 0 \tag{24}$$

hold trivially under the no trade assumption. The equilibrium solution of the economy above is characterized by a system formed by the constraints and first order conditions of the firm in equations (14) to (20), by the household's budget constraint and the process for the idiosyncratic shock in equations (4), (5) and (9), by the market clearing condition in (23), and by the asset pricing equations in (10) to (12).

3 An Analytical Solution

In what follows, we present an analytical solution to the previous equation system, following Campbell [1] and Lettau (98) . We denote by lower case letters the variables in logs, while the log of ϵ_{it} is denoted by $\widehat{\epsilon}_{it}$. When linearizing, we always suppress the constants and approximate the logarithm of sums with a first order Taylor expansion around the steady state.

We first present the solution under identical households assuming two different objectives for the firm, the utility maximizing objective (UM) in equation (13) and the usual value maximizing objective (VM) in equation (21). Except for the dividend specification, this last case corresponds to the model solved in Lettau (98)[27]. Further, the second section extends the solution method for the case in which the firm has a UM objective, households are heterogenous, and markets are incomplete due to the presence of idiosyncratic risk and no insurance channels.

3.1 Identical Households

If households are identical, the vector of model parameters is equal to

$$\Phi = \{\alpha, \delta, \beta, \psi, \sigma_z, \gamma \text{ or } \gamma_f\}$$

while the vector of states includes the last period capital stock and the aggregate technology shock, i.e., $s_t = (k_{t-1}, z_t)$. Log linearizing the equation system, the endogenous macroeconomic variables in logs can be expressed as $x_t = \eta_{xk} k_{t-1} + \eta_{xz} z_t$, where η_{xs} is the elasticity of variable $x = \{k_t, y_t, i_t, c_t, d_t\}$ with respect to the corresponding state variable in logs[28]. In addition, using lag operators, one can see that the AR(1) process for the technology shock leads to an AR(2) process for the capital stock and to an ARMA(2,1) process for output, consumption, investment and dividends. As shown by Lettau, these processes are given by the following equations:

$$z_t = \frac{1}{1 - \psi L} \varepsilon_t^z \tag{25}$$

$$k_t = \frac{\eta_{kz}}{(1 - \eta_{kk} L)(1 - \psi L)} \varepsilon_t^z \tag{26}$$

[27] Note that we cannot directly use the results of Lettau as a benchmark case to evaluate our incomplete market economies, given that we have a different dividend specification and a different objective for the firm. On the other hand, if households are identical, these differences only affect the system used to solve for the elasticities of the macroeconomic aggregates, while they do not modify the equations used by Lettau to compute the different asset moments. Since a detailed derivation of these equations is already provided by the author, we just state his main results in the following subsection, providing the exact expresions for the terms with the superscript* in appendix 2.

[28] Appendix one illustrates how to solve for these elasticities. See also Campbell [1] for details about the solution method.

$$x_t = \frac{1+\Theta_x^* L}{(1-\eta_{kk}L)(1-\psi L)} w_{xt} \tag{27}$$

where $x = \{y, i, c, d\}$ and $w_{xt} = \eta_{xz}\varepsilon_t^z$. To compute the asset return moments, one can log linearize the two asset pricing equations in (11) and (12), obtaining:

$$E_t r_{t+1}^e + \frac{1}{2}Var_t(r_{t+1}^e) = -E_t(pk_t) - \frac{1}{2}Var_t(pk_t) - Cov_t(pk_t, r_{t+1}^e) \tag{28}$$

$$r_{t+1}^f = -E_t(pk_t) - \frac{1}{2}Var_t(pk_t) \tag{29}$$

The Risk Free Rate

As reflected in the previous equation, the risk free rate is just a function of the pricing kernel, which only depends on aggregate consumption. Substituting into the equation the ARMA(2,1) consumption process, and suppressing the constant variance term, Lettau shows that the risk free rate also follows an ARMA(2,1) process, given by:

$$r_{t+1}^f = \frac{1+\Theta_{r^f}^* L}{(1-\eta_{kk}L)(1-\psi L)} w_{r^f t} \tag{30}$$

where $w_{r^f t} = \gamma(\eta_{ck}\eta_{kz} + \eta_{cz}(\psi - 1))\varepsilon_t^z$. In addition, transforming the previous process into its $MA(\infty)$ representation, one can obtain its unconditional standard deviation, given by:

$$\sigma_{r^f} = A^*(\frac{(\psi+\Theta_{r^f})^2}{1-\psi^2} - 2\frac{(\psi+\Theta_{r^f})(\eta_{kk}+\Theta_{r^f})}{1-\psi\eta_{kk}} + \frac{(\psi+\Theta_{r^f})^2}{1-\eta_{kk^2}})^{\frac{1}{2}} \tag{31}$$

The Equity Premium

The expected risk premium, adjusted for the effect of Jensen's inequality, is given by the difference of the two log linearized asset pricing equations in (28) and (29). This difference is equal to the negative of the covariance between the pricing kernel and the equity return, i.e.,

$$E_t r_{t+1}^{rp} = E_t r_{t+1}^e + \frac{1}{2}Var_t(r_{t+1}^e) - r_{t+1}^f = -Cov_t(pk_t, r_{t+1}^e) = \gamma Cov_t(\Delta c_{t+1}, r_{t+1}^e) \tag{32}$$

To compute this covariance, one can make use of the fact that the log of consumption growth and of the equity return can be decomposed into their expected value and an innovation term, i.e.,

$$\Delta c_{t+1} = E_t \Delta c_{t+1} + \eta_{cz} \varepsilon^z_{t+1} \tag{33}$$

$$r^e_{t+1} = E_t r^e_{t+1} + \eta_{r^e z} \varepsilon^z_{t+1} \tag{34}$$

implying that:

$$E_t r^{rp}_{t+1} = \gamma E_t[(\Delta c_{t+1} - E_t \Delta c_{t+1})(r^e_{t+1} - E_t r^e_{t+1})] = \gamma \eta_{cz} \eta_{r^e z} \sigma^2_z \tag{35}$$

The consumption elasticity η_{cz} has been obtained before. On the other hand, an expression for the equity return elasticity $\eta_{r^e z}$ can be obtained, as in Lettau [1], using Campbell's decomposition of unexpected returns into revisions in expectations of future dividend changes and revisions in expectations of future returns. Further, given that the equity premium is constant in the model, only news of the expected future risk free rate can lead to an unexpected equity return. Therefore,

$$\begin{aligned} r^e_{t+1} - E_t r^e_{t+1} &= (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} - (E_{t+1} - E_t) \sum_{j=2}^{\infty} \rho^{j-1} r^f_{t+j} \\ &= (\eta^{d*}_{r^e z} + \eta^{rf*}_{r^e z}) \varepsilon^z_{t+1} = \eta_{r^e z} \varepsilon^z_{t+1} \end{aligned} \tag{36}$$

where ρ is equal to the discount factor β in the present model. As we see, the equity return elasticity with respect to the aggregate shock is the sum of two components, a first component reflecting the effect of dividends, $\eta^d_{r^e z}$, and a second component capturing the effect of the future risk free rate, $\eta^{rf}_{r^e z}$. Both components can be obtained by substituting in the expression the $MA(\infty)$ and $ARMA(2,1)$ processes for the risk free rate and the dividends respectively, and by computing the infinite sums. Finally, using the previous equation, one can also calculate the equity return standard deviation, given by:

$$\sigma_{r^e} = [E_t(\eta_{r^e z} \varepsilon^z_{t+1})^2]^{\frac{1}{2}} = |\eta_{r^e z}| \, \sigma_z \tag{37}$$

The Sharpe Ratio

An expression for the Sharpe ratio can be directly obtained by expanding the excess return pricing equation, which implies that $E_t[PK_t R^{rp}_{t+1}] = 0$. If we do that, we obtain:

$$SR_t^{rp} = \frac{E_t(R_{t+1}^{rp})}{\sigma_t(R_{t+1}^{rp})} = -\rho_t(PK_t, R_{t+1}^e)\frac{E_t(PK_t)}{\sigma_t(PK_t)} \tag{38}$$

where ρ_t denotes a conditional correlation. Further, in a lognormal environment, the two terms appearing on the right hand side of the expression can be approximated as:

$$\frac{E_t(PK_t)}{\sigma_t(PK_t)} = (e^{\sigma_{pk}^2} - 1)^{\frac{1}{2}} \simeq \sigma_{pk} \tag{39}$$

$$-\rho_t(PK_t, R_{t+1}^e) = \frac{-Cov_t(PK_t, R_{t+1}^e)}{\sigma_t(PK_t)\sigma_t(R_{t+1}^e)} = \frac{-(e^{\sigma_{pk,r^e}} - 1)}{(e^{\sigma_{pk}^2} - 1)^{\frac{1}{2}}(e^{\sigma_{r^e}^2} - 1)^{\frac{1}{2}}} \simeq \frac{-\sigma_{pk,r^e}}{\sigma_{pk}\sigma_{r^e}} \tag{40}$$

Therefore,

$$SR_t^{rp} = \frac{-\sigma_{pk,r^e}}{\sigma_{r^e}} = \frac{E_t(r_{t+1}^{rp})}{\sigma_{r^e}} = \gamma\,|\eta_{cz}|\,\sigma_z \tag{41}$$

3.2 Heterogeneous Households

If households are heterogenous, the vector of model parameters has to be extended by the persistence and standard deviation of the idiosyncratic shock, while the new state vector also includes the shock itself, i.e., $\Phi = \{\alpha, \delta, \beta, \psi, \sigma_z, \gamma, \gamma_f, \psi_\epsilon, \sigma_\epsilon\}$ and $s_t = \{k_{t-1}, z_t, \widehat{\epsilon}_{1t}\}$[29].

As pointed out earlier, the first thing we can observe is that the presence of this new source of uncertainty has no effect on the behavior of the macroeconomic aggregates, i.e., the elasticities of output, aggregate consumption, investment, capital and dividends with respect to k_{t-1} and z_t are the same as before, while the elasticity of these variables with respect to the idiosyncratic shock is equal to zero. In other words, the equation system determining the elasticities of the macroeconomic aggregates is the same as in appendix one. On the other hand, with the presence of the new shock, individual consumptions are given by:

$$c_{it} = c_t + \eta_{c_i\epsilon}\widehat{\epsilon}_{it} = \eta_{ck}k_{t-1} + \eta_{cz}z_t + \eta_{c_i\epsilon}\widehat{\epsilon}_{it} \tag{42}$$

[29] Given the symmetry of the idiosyncratic shock, we only need to include the process of the first household in the state vector.

where[30]

$$\widehat{\epsilon}_{1t} = c_{\epsilon} + \psi_{\epsilon}\widehat{\epsilon}_{1t-1} + \varepsilon_t^1 = \frac{c_{\epsilon}}{1-\psi_{\epsilon}} + \frac{1}{1-\psi_{\epsilon}L}\varepsilon_t^1 \tag{43}$$

$$\widehat{\epsilon}_{2t} = -\widehat{\epsilon}_{1t} \tag{44}$$

As we can see, the elasticities of individual consumption with respect to k_{t-1} and z_t are the same as for aggregate consumption, while there is now a new term capturing the effect of idiosyncratic uncertainty, with the same elasticity $\eta_{c_i\epsilon}$ for both households with respect to their individual shock. We denote this elasticity by $\eta_{c\epsilon}$ in what follows. Using the process for the idiosyncratic shock, we can now compute the expressions for expected and unexpected individual consumption growth, which will be used to compute the different asset moments. These are given by:

$$E_t\Delta c_{it+1} = E_t\Delta c_{t+1} + \frac{\eta_{c\epsilon}(\psi_{\epsilon}-1)}{1-\psi_{\epsilon}L}\varepsilon_t^i \tag{45}$$

$$\Delta c_{it+1} - E_t\Delta c_{it+1} = \eta_{cz}\varepsilon_{t+1}^z + \eta_{c\epsilon}\varepsilon_{t+1}^i \tag{46}$$

Note that expected and unexpected individual consumption growth have both, an aggregate and an idiosyncratic component. As we will see, this will allow for a similar decomposition of the different asset moments. Further, since the asset moments are determined by the two households, depending on who is unconstrained each period, we will have to take into account the fact that: $E_t(\varepsilon_{1t}\varepsilon_{1t}) = E_t(\varepsilon_{2t}\varepsilon_{2t}) = \sigma_{\epsilon}^2$, while, the perfectly negative correlation between the two idiosyncratic innovations implies that $E_t(\varepsilon_{1t}\varepsilon_{2t}) = -\sigma_{\epsilon}^2$.

The risk free rate

As before, the risk free rate is equal to the negative of the expected value of the pricing kernel, given by:

$$r_{t+1}^f = -E_t(pk_t) = \gamma E_t(\Delta c_{it+1}^{u_t}) \tag{47}$$

[30]The expression for $\widehat{\epsilon}_{2t}$ is obtained by doing a first order Taylor expansion of equation (5) around the mean of $\widehat{\epsilon}_{1t}$, where $E_t[\log(\epsilon_{1t})] = \log(0.5)$. Note that the shock of the second household can also be written as: $\widehat{\epsilon}_{2t} = c_{\epsilon_2} + \psi_{\epsilon}\widehat{\epsilon}_{2t-1} + \varepsilon_t^2$, where $c_{\epsilon_2} = -c_{\epsilon}$ and $\varepsilon_t^2 = -\varepsilon_t^1$.

where u_t denotes as usual that household i is unconstrained at period t. Substituting into the previous equation the expression for individual expected consumption growth, we obtain:

$$r^f_{t+1} = \frac{1 + \Theta^*_{rf} L}{(1 - \eta_{kk} L)(1 - \psi L)} w_{r^f_t} + \frac{1}{1 - \psi_\epsilon L} \upsilon_{rf\,t} \tag{48}$$

where $w_{rf\,t} = \gamma(\eta_{ck}\eta_{kz} + \eta_{cz}(\psi - 1))\varepsilon^z_t$ and $\upsilon_{rf\,t} = \gamma\eta_{c\epsilon}(\psi_\epsilon - 1)\varepsilon^{u_t}_t$. As we see, the first term is equal to the risk free rate obtained under identical households, while there is now a new AR(1) term coming from the presence of idiosyncratic risk. To compute its standard deviation, the process can be rewritten as follows:

$$r^f_{t+1} = \frac{1}{\psi - \eta_{kk}} \sum_{j=0}^{\infty} (\psi^j(\psi + \Theta_{r^f}) - \eta^j_{kk}(\eta_{kk} + \Theta_{r^f})) w_{r^f_{t-j}} + \sum_{j=0}^{\infty} \psi^j_\epsilon \upsilon_{rf\,t-j} \tag{49}$$

Finally, using the independence of the aggregate and the idiosyncratic shocks, we obtain:

$$\sigma(r^f_{t+1}) = [\sigma^2(r^{fz}_{t+1}) + \frac{(\gamma\eta_{c\epsilon}(\psi_\epsilon - 1)\sigma_\epsilon)^2}{1 - \psi^2_\epsilon}]^{\frac{1}{2}} \tag{50}$$

where the first term in the brackets, only affected by the aggregate shock, is equal to the risk free rate variance under identical households, while the second term is the contribution to the total variance of the idiosyncratic shock. Note that the variance is independent of the household that is unconstrained at period t, since, as we have seen, $\sigma^2(\varepsilon^i_t) = \sigma^2_\epsilon$ for $i = 1, 2$.

The Equity Premium

As shown previously, the expected risk premium is given by the negative of the covariance of the pricing kernel with the equity return, which is now equal to:

$$E_t r^{rp}_{t+1} = \gamma E_t[(\Delta c^{u_t}_{it+1} - E_t \Delta c^{u_t}_{it+1})(r^e_{t+1} - E_t r^e_{t+1})] \tag{51}$$

As shown by equation (46), unexpected individual consumption growth can be decomposed into its expected value and its innovation terms with respect to the two shocks. On the other hand, to calculate the unexpected equity return, we can again use its decomposition into revisions in expectations of future dividend growth and revisions in expectations of the future risk free rate due to Campbell:

$$r^e_{t+1} - E_t r^e_{t+1} = (E_{t+1} - E_t)\sum_{j=1}^{\infty} \rho^{j-1}\Delta d_{t+j} - (E_{t+1} - E_t)\sum_{j=2}^{\infty}\rho^{j-1} r^f_{t+j} \tag{52}$$

When calculating the two infinite sums, it is important to note that dividends are only affected by the aggregate technology shock, thus, the first sum on the right hand side is the same as before, i.e.,

$$(E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^{j-1}\Delta d_{t+j} = \eta^{d*}_{r^e z}\varepsilon^z_{t+1} \tag{53}$$

On the other hand, if we substitute for the risk free rate process in the second term of the expression, we obtain:

$$-(E_{t+1} - E_t)\sum_{j=2}^{\infty}\rho^{j-1} r^f_{t+j} = \eta^{r^f *}_{r^e z}\varepsilon^z_{t+1} + \rho\gamma\eta_{c\epsilon}(1-\psi_\epsilon)\sum_{j=0}^{\infty}(\rho\psi_\epsilon)^j \varepsilon^{u_{t+j+1}}_{t+1} \tag{54}$$

where $\varepsilon^{u_{t+j+1}}_{t+1}$ is the innovation of the household, $i = 1$ or 2, that is unconstrained at period $t + j + 1$. As reflected in the equation, the presence of the idiosyncratic shock in the expression for the risk free rate implies that the unexpected return elasticity coming from revisions in expectations of future returns has both, an aggregate and an idiosyncratic component. In addition, using the decomposition of the risk free rate into the process it follows under identical households and the new idiosyncratic term, it is easy to show that the aggregate component, $\eta^{r^f *}_{r^e z}$, is the same as before. Given this, the unexpected equity return can now be written as:

$$r^e_{t+1} - E_t r^e_{t+1} = \eta_{r^e z}\varepsilon^z_{t+1} + \rho\gamma\eta_{c\epsilon}(1-\psi_\epsilon)\sum_{j=0}^{\infty}(\rho\psi_\epsilon)^j\varepsilon^{u_{t+j+1}}_{t+1} \tag{55}$$

Define $B = \rho\gamma\eta_{c\epsilon}(1 - \psi_\epsilon)$ and $P_{t+j+1} = prob(u_{t+j+1} = 1)$, i.e., P_{t+j+1} is the probability that the first household is unconstrained at period $t + j + 1$. Since $\varepsilon^2_{t+1} = -\varepsilon^1_{t+1}$, each of the terms in the previous infinite sum can be expressed as $P_{t+j+1}\varepsilon^1_{t+1} + (1 - P_{t+j+1})\varepsilon^2_{t+1} = P_{t+j+1}\varepsilon^1_{t+1} - (1 - P_{t+j+1})\varepsilon^1_{t+1} = (2P_{t+j+1} - 1)\varepsilon^1_{t+1}$. Given this, the unexpected equity return can be finally rewritten as:

$$r^e_{t+1} - E_t r^e_{t+1} = \eta_{r^e z}\varepsilon^z_{t+1} + \eta_{r^e\epsilon}\varepsilon^1_{t+1} \tag{56}$$

where

$$\eta_{r^e\epsilon} = B\sum_{j=0}^{\infty}(\rho\psi_\epsilon)^j(2P_{t+j+1}-1) \tag{57}$$

As we see, the effect of idiosyncratic uncertainty on the unexpected equity return, and, thus, on the two risky asset moments is a function of the probability that the first household is unconstrained from period $t+1$ onwards. This effect will be discussed later. Using this expression, the expected risk premium is then given by:

$$E_t r_{t+1}^{rp} = \gamma\eta_{cz}\eta_{r^e z}\sigma_z^2 + \gamma\eta_{c\epsilon}\eta_{r^e\epsilon}E_t(\varepsilon_{t+1}^{u_t}\varepsilon_{t+1}^1)] \tag{58}$$

As with all other asset moments, the expected risk premium is now equal to the sum of two terms. The first term is the premium one would obtain under identical households, while the second term reflects the presence of a second source of uninsurable uncertainty. Note that the size of this second term will also depend from the household that is unconstrained at period t. Since $E_t(\varepsilon_{t+1}^1\varepsilon_{t+1}^1) = \sigma_\epsilon^2$ and $E_t(\varepsilon_{t+1}^2\varepsilon_{t+1}^1) = -\sigma_\epsilon^2$, the previous expectation can be written as $P_t\sigma_\epsilon^2 - (1-P_t)\sigma_\epsilon^2 = (2P_t-1)\sigma_\epsilon^2$, where $P_t = prob(u_t = 1)$. Thus,

$$E_t r_{t+1}^{rp} = \gamma\eta_{cz}\eta_{r^e z}\sigma_z^2 + \gamma\eta_{c\epsilon}\eta_{r^e\epsilon}\sigma_\epsilon^2(2P_t-1) \tag{59}$$

Using equations (56), we can also compute the standard deviation of the equity return, given by:

$$\sigma(r_{t+1}^e) = (\eta_{r^e z}^2\sigma_z^2 + \eta_{r^e\epsilon}^2\sigma_\epsilon^2)^{\frac{1}{2}} \tag{60}$$

As with the risk free rate, the first term in the brackets represents the variance of the equity return under identical households, while the second is the contribution to the total variance of the idiosyncratic shock.

The Sharpe Ratio

As before, the Sharpe ratio will be given by:

$$SR_t^{rp} = \frac{E_t r_{t+1}^{rp}}{\sigma(r_{t+1}^e)} = \frac{\gamma\left|\eta_{cz}\eta_{r^e z}\right|\sigma_z^2 + \gamma\left|\eta_{c\epsilon}\eta_{r^e\epsilon}\right|\sigma_\epsilon^2(2P_t-1)}{(\eta_{r^e z}^2\sigma_z^2 + \eta_{r^e\epsilon}^2\sigma_\epsilon^2)^{\frac{1}{2}}} \tag{61}$$

Constrained Persistence

As we have seen, the different asset moments can be decomposed into an aggregate and an idiosyncratic component. Further, the idiosyncratic component entering into the expression of the risky asset moments crucially depends from the probability that the first household is unconstrained. Recall the two components are given by:

$$E_t r_{t+1}^{rp(idio)} = \gamma \eta_{c\epsilon} \sigma_\epsilon^2 (2P_t - 1) B \sum_{j=0}^{\infty} (\rho \psi_\epsilon)^j (2P_{t+j+1} - 1) \tag{62}$$

$$\sigma^2_{r^e(idio)} = B^2 \sigma_\epsilon^2 [\sum_{j=0}^{\infty} (\rho \psi_\epsilon)^j (2P_{t+j+1} - 1)]^2 \tag{63}$$

Consider the case in which P_t is constant over time. In this case, it is possible to obtain an exact closed form solution, and the two components are equal to:

$$E_t r_{t+1}^{rp(idio)} = \frac{\gamma \eta_{c\epsilon} \sigma_\epsilon^2 B (2P - 1)^2}{1 - \rho \psi_\epsilon} \tag{64}$$

$$\sigma^2_{r^e(idio)} = \frac{B^2 \sigma_\epsilon^2}{(1 - \rho \psi_\epsilon)^2} (2P - 1)^2 \tag{65}$$

As we see, the effect of idiosyncratic uncertainty on the two asset moments disappears if $P = 0.5$. This is the case in which, if a household is unconstrained at period t, he has the same probability of being unconstrained next period, i.e., there is no persistence in who is constrained. On the other hand, the effect will be maximized if $P = 1$ or $P = 0$. In these cases, there is full constrained persistence, i.e., if a household is unconstrained at period t, he will be unconstrained forever. An important conclusion which we can draw from the previous expressions is therefore that, for idiosyncratic uncertainty to have an effect on the different asset moments, one needs persistence in who is constrained.

Apart from this, note that, while the assumption of full constrained persistence is clearly unrealistic, we now argue that the outcome of setting $P = 1$ or $P = 0$ would approximately obtain in the case in which there is some constrained persistence, i.e., to obtain this outcome, one would only need to assume that the same household is (un)constrained only for a certain number of periods. To see this note that the infinite sum appearing in the two asset moments is discounted with $\rho \psi_\epsilon$, where $\rho = \beta < 1$ and $\psi_\epsilon < 1$. Thus, the sum can be approximated arbitrarily well with the first k periods, since, for $P_t \in [0, 1]$, there exists a period k such that:

$$\gamma\eta_{c\epsilon}B\sigma_{\epsilon}^{2}(2P_t-1)\sum_{j=k}^{\infty}(\rho\psi_{\epsilon})^{j}(2P_{t+1+j}-1)<m \tag{66}$$

$$(\rho B)^{2}\sigma_{\epsilon}^{2}[\sum_{j=k}^{\infty}(\rho\psi_{\epsilon})^{j}(2P_{t+1+j}-1)]^{2}<m \tag{67}$$

where m is an arbitrarily small number. Since, after period k it would not really matter if household one or two is unconstrained, the case in which the same household is (un)constrained for the first k periods, or the case in which there is only some persistence in who is constrained, can be well approximated with the solution obtained by setting $P_t = 1$ or $P_t = 0$, as stated earlier[31].

Finally, note that, in the present model, households are constrained or unconstrained depending on their idiosyncratic shock, since they are only distinguished by the shock realization. In particular, if their shock is lower (higher) than the average, they will want to smooth their consumption by short selling (investing in) the assets and will therefore be constrained. On the other hand, given the autoregressive and persistent nature of the process, if a household receives a lower (higher) than average shock, it is likely that he gets a lower (higher) than average shock for some periods after. In other words, the model is likely to generate persistence in who is constrained. On the other hand, since the number of periods a household is unconstrained may be lower then k, the results presented in the following section, corresponding to the case with $P_t = 1$ or $P_t = 0$, should be interpreted as the best possible outcome, i.e., the moments obtained have to be considered as an upper bound[32].

4 Results

4.1 Identical Households

In the present section, we present the results obtained for the identical household economy. As stated earlier, this economy has been extensively studied by Lettau

[31] In the present model, and depending from the parameterization, a k between 50 and 150 periods leads to a difference between the true and the approximated asset moments (calculated using only the first k periods) of less than 0.0002

[32] Note also that, if the probability is not constant, in which case the sum would have to be approximated with the first k periods, it would be decreasing over time, and we would obtain similar results as long as it stays sufficiently above 0.5 for a certain number of periods.

(98), using a different dividend specification and a different objective for the firm. Thus, the main purpose of the section is both, to illustrate the differences of these specifications, and to obtain a benchmark for comparison with the heterogenous household economies. Unless otherwise specified, we will use the following parameter values:

$$\Phi = \{\alpha, \delta, \beta, \psi, \sigma_z, \gamma, \gamma_f\} = \{0.36, 0.025, 0.99, 0.95, 0.00712, 1, 1.5\}$$

The value of the first five parameters is standard in the real business cycle literature. The capital/output share α is set to approximately replicate the percentage of GNP to capital owners in the postwar period. The capital depreciation δ is chosen to match the steady state investment to capital ratio for the same period, and the subjective discount rate $\beta = 0.99$ leads to an annual interest rate of 4%, as usual in the literature simulating quarterly data. Further, the two parameters for the technology shock are chosen to replicate the US postwar output behavior. As to the risk aversion parameters, the benchmark value for the household risk aversion has been set to one, while the risk aversion of the firm of 1.5 has been chosen to generate a dividend cyclicality that is close to the data. The following table displays the elasticities of the main macroeconomic aggregates for the two different firm objectives and the benchmark parametrization.

Table 3.1: Macroeconomic elasticities

$\gamma = 1$	η_{kk}	η_{kz}	η_{ik}	η_{iz}	η_{ck}	η_{cz}	η_{dk}	η_{dz}
VM	0.965	0.075	-0.389	3.015	0.618	0.305	2.214	-3.987
UM	0.980	0.034	0.209	1.374	0.412	0.871	0.735	0.075

If we look at the elasticities of the different variables with respect to the aggregate technology shock, we can clearly see the difference between the two firm objectives. The capital stock and investment level are less elastic, while consumption reacts more to the shock if the firm is risk averse. In addition, the risk aversion of the firm leads to a relatively small dividend elasticity, or a high dividend smoothness, while this elasticity is much higher and negative under the VM objective, leading to a higher variability and a countercyclical behavior of this variable.

Clearly, if the firm is risk averse, it will want to smooth its dividends over time, and will therefore, invest less (more) after a relatively good (bad) shock, leading to a less elastic investment and a less smooth consumption pattern. On the other hand, the higher elasticity of investment under the VM objective will lead to more variable and countercyclical dividends, given that they are defined as the residual of profits and investment. Note that these findings just confirm the fact that a risk averse firm will behave as if there were capital adjustment costs in the economy, as illustrated in the second chapter. The relevant asset return moments for the benchmark parameterization and the two firm objectives are displayed in the following table.

Table 3.2: Asset moments under identical households

$\gamma = 1$	r^{rp}	SR	$\sigma(r^e)$	$\sigma(r^f)$	$\eta_{r^e z}$	$\eta^d_{r^e z}$	$\eta^{rf}_{r^e z}$
VM	0.0001	0.0022	0.025	0.061	0.035	-0.045	0.079
UM	0.0035	0.0062	0.569	0.077	0.799	0.154	0.645

US data: r^{rp}=1.94, SR=0.27, $\sigma(r^e)$= 7.6, $\sigma(r^f)$= 0.78

As reflected in the first row of the table, the standard RBC model is unable to replicate the main asset return moments in the data. The benchmark parametrization leads to an almost zero equity premium, to a Sharpe ratio that is approximately 100 times lower than in the data, and to asset return volatilities that are very far from reality. Concerning the UM firm objective, the results are slightly better, with a more volatile equity return and a slightly higher premium and Sharpe ratio. The improvement, however, is rather small.

The reasons for this failure can be clearly seen by looking at the different factors affecting asset return moments. As illustrated earlier, apart from the aggregate shock standard deviation and the household risk aversion, the equity premium is determined by both, the elasticity of consumption with respect to the shock (η_{cz}) and the equity return elasticity with respect to the shock ($\eta_{r^e z}$). The last three columns of the table display this last elasticity, followed by its two components, coming from revisions in expectations of future dividend changes ($\eta^d_{r^e z}$) and from negative revisions in expectations of the future risk free rate ($\eta^{rf}_{r^e z}$). As we see, the dividend component is negative under the VM objective, while the second component is relatively small, leading to a very small return elasticity and a low mean and volatility of the premium. On the other hand, the two components are positive and a bit higher under the UM objective, a fact that, together with the higher consumption elasticity reported earlier, leads to slightly better moments. As already stated, however, the distortion caused by the firm risk aversion alone is not able to provide reasonable asset return moments either.

In spite of this result, there is still an important difference between the two firm objectives, which is not evident from the previous analysis. If the firm is risk averse, an increase in household risk aversion has a considerably higher impact on the different asset moments. This is reflected in the following table.

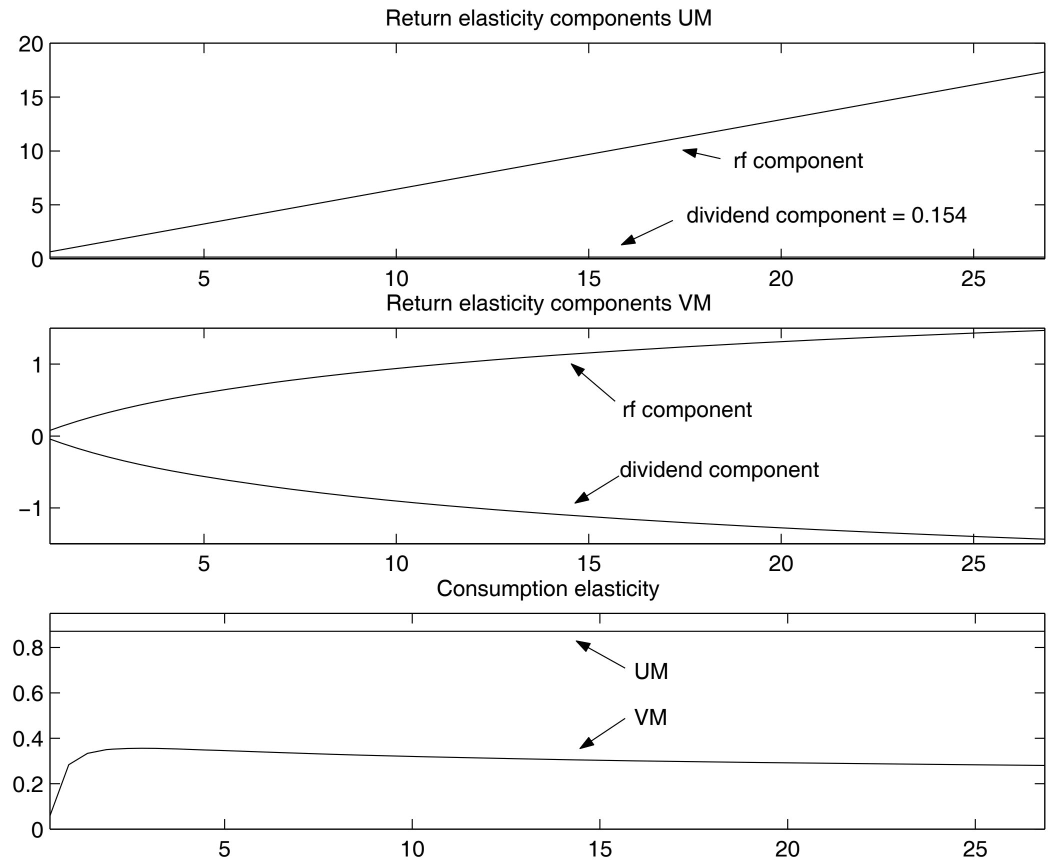
Table 3.3: Asset moments and risk aversion under identical households

	r^{rp}	SR	$\sigma(r^e)$	$\sigma(r^f)$	η_{cz}	$\eta_{r^e z}$	$\eta^d_{r^e z}$	$\eta^{r^f}_{r^e z}$
VM $\gamma = 15$	0.0008	0.032	0.025	0.19	0.303	0.035	-1.12	1.15
UM $\gamma = 15$	0.6511	0.093	6.99	1.15	0.871	9.830	0.154	9.68
VM $\gamma = 26$	0.0013	0.052	0.025	0.24	0.282	0.035	-1.42	1.45
UM $\gamma = 26$	1.9433	0.161	12.05	2.01	0.871	16.93	0.154	16.8

US data: r^{rp}=1.94, SR=0.27, $\sigma(r^e)$= 7.6, $\sigma(r^f)$= 0.78

The table displays the different asset moments generated by the two firm objectives with household risk aversion parameters of 15 and 26. As reflected in the first and third rows, the different asset moments are highly insensitive to an increase risk aversion under the VM objective. Increasing this parameter from 1 to 26 leads to an increase in the premium from 0.0001 to 0.0013, while the Sharpe ratio increases from 0.002 to 0.05. In contrast to this, a risk aversion of 26 matches the mean premium in the data if the firm is risk averse, while it increases the Sharpe ratio from 0.006 to 0.16. Clearly, the main reason for this difference lies in the behavior of the two elasticities directly affecting the asset return moments. These are displayed in the last columns of the table and are also depicted in figure 1 below.

Figure 3.1: Elasticities under the VM and UM objectives



As we see in the table, there are two effects mitigating the positive effect of a higher risk aversion under the VM objective. First, a higher risk aversion leads to a more smooth consumption pattern or a lower consumption elasticity with respect to the shock, having a negative impact on the premium. In contract to this, the consumption elasticity is unaffected if the firm is risk averse, given that the household risk aversion does not enter the expectational equation determining the law of motion of the system. This difference is reflected in the third panel of figure 1. Second, we can also observe that a higher risk aversion under the VM objective increases the return elasticity component due to revisions in the future risk free rate, while it decreases the component due to revisions in future dividend changes, leading to a negligible change in the total elasticity and, thus, in the asset return moments. Again, while the dividend component is unaffected under the UM objective, the risk free rate component experiences a substantial increase with a higher household risk aversion, leading to a much higher return elasticity in this case. These effects are reflected in the first two panels of figure 1.

Given the previous results, it becomes clear that the two opposite effects of a higher risk aversion on the return elasticity components under the VM firm objective, give the model no chance to generate reasonable results, even with an unrealistically high household risk aversion parameter. Note, however, that an important aspect generating this behavior may be the residual dividend specification, which drives the strong negative effect on the return elasticity, as we saw before. To evaluate the importance of this effect, we have calculated the same moments defining dividends as the marginal product of capital, as in Lettau (98). The results are displayed in the following table.

Table 3.4: Asset moments under a different dividend specification

		r^{rp}	SR	$\sigma(r^e)$	$\sigma(r^f)$	η_{cz}	$\eta_{r^e z}$	$\eta^d_{r^e z}$	$\eta^{r^f}_{r^e z}$
VM	$\gamma = 15$	0.0008	0.032	0.025	0.19	0.303	0.035	-1.120	1.15
VM^L	$\gamma = 15$	0.0264	0.032	0.815	0.19	0.303	1.145	-0.011	1.15
VM	$\gamma = 26$	0.0013	0.052	0.025	0.24	0.282	0.035	-1.420	1.45
VM^L	$\gamma = 26$	0.0534	0.052	1.024	0.24	0.282	1.438	-0.014	1.45

US data: r^{rp}=1.94, SR=0.27, $\sigma(r^e)$= 7.6, $\sigma(r^f)$= 0.78

The first and third columns reproduce the results in table 3.3, while the second and fourth columns display the moments with the dividend specification used by Lettau. As reflected in the table, an increase in risk aversion leads now to a higher equity return elasticity, due to the smaller negative effect of the dividend component, increasing the equity premium and equity return standard deviation, while leaving the other moments unaffected. In spite of the improvement, however, the sensitivity of the moments is still very small. As we see, with a risk aversion of 26, the model generates a premium of 0.05, much smaller than the premium generated by the UM objective with the same risk aversion parameter. Indeed, with the new dividend specification and the VM objective, a risk aversion of approximately 710 would be needed to generate the mean equity premium in the data.

Apart from this, we can also observe that the better asset moments under the second firm objective are only driven by the increase in the return elasticity component due to future returns, since the dividend component remains unchanged. Altogether, these facts suggest that dividends play a minor role, as compared to future returns, in determining the size of the unexpected equity return, and thus, of the different asset moments. Finally, we should also note that the UM model is able to generate the right mean, although it does it with a too high risk aversion, but not the right return volatilities. As we see in table 3.3, the standard deviations of the two asset returns are too high, leading to a Sharpe ratio that is smaller than in the data.

4.2 Heterogenous Households

In the present section, we analyze the impact on the different asset return moments of the presence of a second source of uninsurable uncertainty. The benchmark parametrization is the same as before. In addition, the two parameters of the idiosyncratic shock, $(\psi_\epsilon, \sigma_\epsilon)$, are quarterly adjusted estimates following the literature on asset pricing with idiosyncratic shocks in exchange economies. In this literature, the annual estimates for the two parameters are in the range $\psi_\epsilon^a \in (0.6, 0.9)$ and $\sigma_\epsilon^a \in (0.19, 0.28)$. Since we want to make sure that the shock is below without having to truncate the distribution, we study a quarterly standard deviation in the annual range $\sigma_\epsilon^a \in (0.15, 0.19)$. Using these ranges, the two annual estimates at the lower and upper ends lead to the following quarterly numbers $(\psi_\epsilon^L, \sigma_\epsilon^L) = (0.8801, 0.0890)$ and $(\psi_\epsilon^U, \sigma_\epsilon^U) = (0.974, 0.1128)$.

As explained before, the nature of the idiosyncratic shock has no effect on the aggregate macroeconomic variables. Therefore, the elasticities displayed in table 3.1 remain unchanged. The following table displays the asset moments generated by the model for different values of the household risk aversion and the two idiosyncratic parameter values at the lower end. The moments generated by the identical household economy are also displayed in the first row of the table for $(\psi_\epsilon^a, \sigma_\epsilon^a) = (0.6, 0.15)$.

Table 3.5: Asset moments and risk aversion under heterogenous agents

	r^{rp}	$\sigma(r^e)$	$\sigma(r^f)$	SR	η_{cz}	$\eta_{r^e z}$	$\eta_{c\epsilon}$	$\eta_{r^e \epsilon}$
UM $\gamma = 1$	0.003	0.57	0.08	0.006	0.87	0.79	0	0
UM^h $\gamma = 1$	0.545	7.09	1.94	0.077	0.87	0.79	0.86	0.79
UM^h $\gamma = 1.4$	1.067	9.92	2.71	0.108	0.87	1.06	0.86	1.11
UM^h $\gamma = 1.8$	1.764	12.8	3.48	0.138	0.87	1.32	0.86	1.43
UM^h $\gamma = 2.0$	2.178	14.2	3.87	0.154	0.87	1.44	0.86	1.59

US data: r^{rp}=1.94, SR=0.27, $\sigma(r^e)$= 7.6, $\sigma(r^f)$= 0.78

As we see, the presence of idiosyncratic uncertainty leads to a dramatic improvement in the different asset moments. With the benchmark household risk aversion of 1, the model generates a premium of 0.55 percent, while only a risk aversion of 2 is needed to generate its value in the data. In this case, the Sharpe ratio is equal to 0.154, much higher than under identical households, while the asset return variabilities are also closer to reality.

To understand the mechanisms leading to this results, recall that idiosyncratic uncertainty affects the mean and variability of the different asset moments through a new idiosyncratic component in the expressions of the unexpected equity return and the unexpected individual consumption growth. As we have seen, this leads in turn to a new term in the expressions for the premium and asset return variabilities, which we can now decompose into their value under identical households plus the value of a new term, only affected by idiosyncratic risk. Given the relatively small size of the premium and asset variabilities under identical households, it becomes clear that the new idiosyncratic term is now the key determinant of the different asset moments. Indeed, it accounts for approximately 99.6 percent of the premium and for more than 99.5 percent of the total variabilities. The following equations display this new component for the three asset moments:

$$r_{id}^{rp} = \gamma \eta_{c\epsilon} \eta_{r\epsilon} \sigma_{\epsilon}^2 \tag{68}$$

$$\sigma_{re,id}^2 = \eta_{r^e \epsilon}^2 \sigma_{\epsilon}^2 \tag{69}$$

$$\sigma_{r^f,id}^2 = \eta_{r^f \epsilon}^2 \sigma_{\epsilon}^2 \tag{70}$$

where

$$\eta_{r^e \epsilon} = \frac{\gamma \rho \eta_{c\epsilon}(1 - \psi_{\epsilon})}{1 - \rho \psi_{\epsilon}} \tag{71}$$

$$\eta_{r^f \epsilon} = \frac{\gamma \eta_{c\epsilon}(\psi_{\epsilon} - 1)}{(1 - \psi_{\epsilon}^2)^{\frac{1}{2}}} \tag{72}$$

As reflected in the equations, the risk aversion parameter appears in the three terms, multiplying both, the idiosyncratic consumption elasticity, $\eta_{c\epsilon}$, and the idiosyncratic innovation variance, σ_ϵ^2. Since these variables are independent from γ, an increase in risk aversion will clearly lead to a considerable increase in the different asset moments. This is reflected by the previous table, showing that the premium and return variances are indeed much more sensitive to an increase in risk aversion in the presence of idiosyncratic uncertainty. Using the same reasoning, it is clear that a similar effect will obtain if we increase the idiosyncratic innovation variance σ_ϵ^2, while leaving constant the risk aversion parameter. This is reflected by the following table, displaying the moments for $(\gamma, \psi_\epsilon^a) = (1.5, 0.6)$ and for different values of the standard deviation along the estimated range.

Table 3.6: Asset moments and shock variance under heterogenous agents

	r^{rp}	$\sigma(r^e)$	$\sigma(r^f)$	SR	η_{cz}	$\eta_{r^e z}$	$\eta_{c\epsilon}$	$\eta_{r^e \epsilon}$
UM	0.007	0.799	0.116	0.009	0.87	1.12	0	0
UM^h $\sigma_\epsilon^a = 0.15$	1.225	10.62	2.904	0.115	0.87	1.12	0.86	1.19
UM^h $\sigma_\epsilon^a = 0.17$	1.575	12.04	3.292	0.131	0.87	1.12	0.86	1.19
UM^h $\sigma_\epsilon^a = 0.19$	1.963	13.45	3.679	0.146	0.87	1.12	0.86	1.19

US data: r^{rp}=1.94, SR=0.27, $\sigma(r^e)$= 7.6, $\sigma(r^f)$= 0.78

As with the risk aversion, the sensitivity of the different moments to an increase in the shock innovation variance is very high. With a risk aversion of 1.5 and the idiosyncratic standard deviation at the upper end, the model is also able to approximately generate the mean premium in the data. In this case, the Sharpe ratio is again around 0.15, while the two return variabilities are similar to the ones obtained above. Thus, both the household risk aversion and the idiosyncratic innovation variance, entering the new idiosyncratic term in a multiplicative way, are key determinants of the size of the asset moments. In addition, we see that the model is able to approximately generate the mean equity premium in the data for reasonable values of the two parameters. In spite of the considerable improvement, note, however, that the results are not completely satisfactory, given that the Sharpe ratio is still around one half of its value in the data. In other words, the equity return variability generated by the model is too high, while the same happens with the variability of the risk free rate. As shown by the tables, this happens when varying both γ and σ_ϵ. In what follows, we investigate the robustness of this result to a change in the idiosyncratic shock persistence ψ_ϵ, which is the third parameter directly affecting the new components. The following table displays the results of increasing the risk aversion parameter for the two idiosyncratic persistence levels at the lower and upper ends.

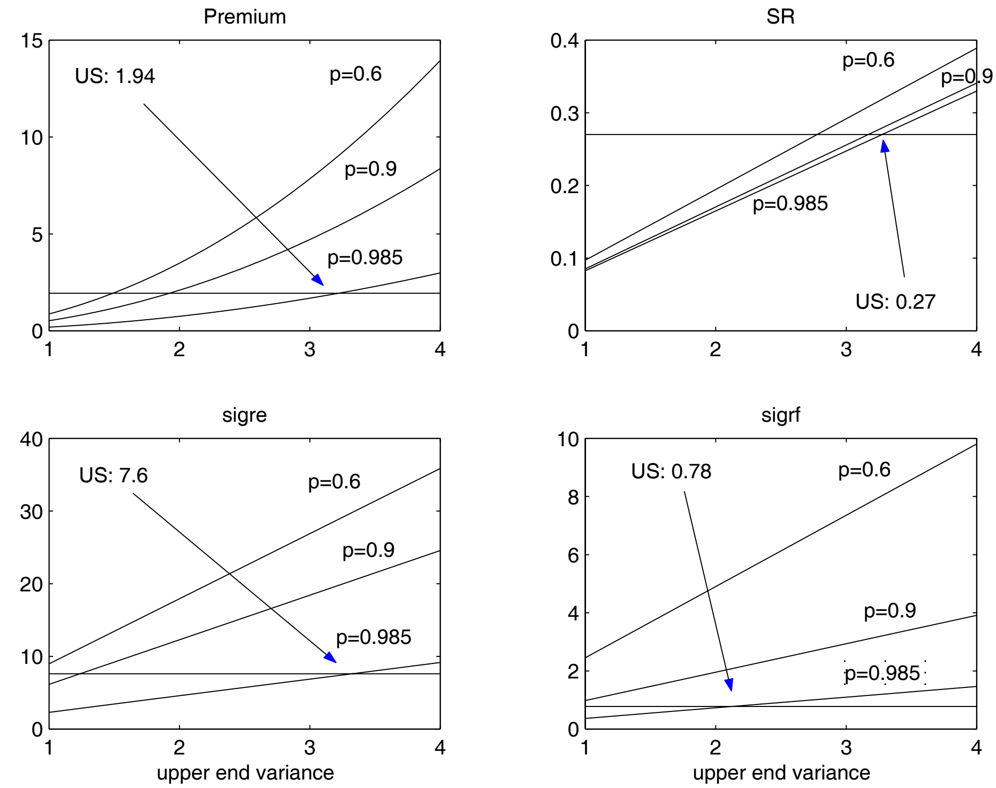
Table 3.7: Asset moments and shock persistence under heterogenous agents

$\sigma^a_\epsilon = 0.19$	r^{rp}	σ_{re}	σ_{rf}	SR	r^{rp}_{id}	$\sigma^2_{re,id}$	$\sigma^2_{rf,id}$
UM^h $(\gamma, \psi^a_\epsilon) = (1, 0.6)$	0.87	8.97	2.45	0.09	.0087	.0080	.0006
UM^h $(\gamma, \psi^a_\epsilon) = (1.3, 0.6)$	1.48	11.66	3.18	0.13	.0147	.0135	.0010
UM^h $(\gamma, \psi^a_\epsilon) = (1.5, 0.6)$	1.96	13.45	3.68	0.15	.0196	.0180	.0014
$\sigma^a_\epsilon = 0.19$	r^{rp}	$\sigma(r^e)$	$\sigma(r^f)$	SR	r^{rp}_{id}	$\sigma^2_{re,id}$	$\sigma^2_{rf,id}$
UM^h $(\gamma, \psi^a_\epsilon) = (1.5, 0.9)$	1.18	9.22	1.47	0.13	.0117	.0084	.0002
UM^h $(\gamma, \psi^a_\epsilon) = (1.7, 0.9)$	1.51	10.44	1.66	0.15	.0156	.0108	.0003
UM^h $(\gamma, \psi^a_\epsilon) = (1.93, 0.9)$	1.95	11.85	1.89	0.17	.0194	.0139	.0004

US data: r^{rp}=1.94, SR=0.27, $\sigma(r^e)$= 7.6, $\sigma(r^f)$= 0.78

As we see, a higher persistence level leads to an improvement in the results concerning the risk-return trade-off predicted by the model. While a risk aversion of 1.5 generates the mean premium in the data with the lower persistence level, a risk aversion of 1.93 generates the same mean premium with $\psi^a_\epsilon = 0.9$, with a lower variability of the equity return in this case, leading also to a higher Sharpe ratio. In addition, the standard deviation of the risk free rate is approximately one half of the one obtained with $\psi^a_\epsilon = 0.6$.

Clearly, the improvement in the variabilities are caused by the effect of the shock persistence on the new components entering the three asset moments, displayed in the last three columns of the table. As reflected in the equations above, apart from the innovation variance σ^2_ϵ, the two return variances crucially depend on the squared return elasticities $\eta^2_{r^e\epsilon}$ and $\eta^2_{r^f\epsilon}$, while the premium is directly affected by the term $\gamma\eta_{c\epsilon}\eta_{r\epsilon}$. One can show that, for a given risk aversion parameter, these three terms decrease with a higher shock persistence, leading to a lower premium and to lower return variabilities. In addition, it can also be shown that the sensitivity of the idiosyncratic components of the two return variabilities is lower than the premium sensitivity with a higher persistence. This is also reflected in the table. While a higher risk aversion increases the risk premium component to 1.9% in both cases, the variance components are equal to 1.8% and 0.14% with the lower persistence level, while they only increase to 1.4% and 0.04% with $\psi^a_\epsilon = 0.9$. In other words, the more than proportional effect on the equity return variance caused by an increase in risk aversion is partly mitigated by the higher persistence level. These effects are also illustrated below.



The figure shows the effects of a higher risk aversion, displayed on the x-axis, on the different asset moments for the lower and upper end persistence levels, as well as for $\psi_\epsilon^a = 0.985$, which is the persistence level needed to generate the right risk-return trade-off. The straight line represents the value of the different moments in the data. As discussed above, the slope of the lines determining the asset moments decreases with a higher shock persistence. In addition, while a persistence level inside the estimated range has no chance of generating the right return variabilities, we see that the third line, corresponding to $\psi_\epsilon^a = 0.985$, intersects the US line in the same risk aversion region for the premium, the equity return variance and the Sharpe ratio. With the mentioned persistence, a risk aversion of around 3.3 generates the three moments in the data, while the risk free rate variability is around 1.2 percent, slightly higher than in the US level. Note, however, that this is only possible with an annual idiosyncratic persistence outside the estimated range of $\psi_{\epsilon}^{a} = (0.6, 0.9)$. In this sense, we can say that, in spite of the considerable improvement in the asset moments, the success of this type of idiosyncratic uncertainty is only partial. As we have seen, while the model has the potential to generate the right risk premium with a reasonable parametrization, it generates too high return variabilities.

5 Summary and Conclusions

The present paper extends the approximated analytical approach used by Lettau (98) to study the asset pricing implications of extending the standard real business cycle model with an idiosyncratic labor income shock. To be able to obtain closed form solutions for the asset moments, it is assumed that households cannot use asset markets to insure against uncertainty, resulting in an incomplete financial market structure.

The firm maximizes the expected utility of its net cash flow (UM objective), given that the usual value maximization firm objective (VM objective) is no longer well defined under shareholder heterogeneity and market incompleteness. In addition, while Lettau defines dividends as the marginal product of capital, we assume that the firm owns the entire capital stock, decides on the investment level, and pays the residual of profits (output net of wage payments) and investment out as dividends.

The model is first studied under the assumption that households are identical in all respects. In this case, the different asset moments are essentially determined by the household risk aversion, the aggregate innovation standard deviation, and by two elasticities, the elasticity of consumption and the elasticity of the equity return with respect to the aggregate shock, which can be decomposed into a component due to future dividend growth and a component due to future returns.

As already shown by Lettau, when the firm has the usual VM objective, both elasticities are relatively small, implying that the model has no chance of generating reasonable asset moments, even with an unrealistically high risk aversion parameter. Interestingly, we find that this is independent of the dividend specification. When dividends are defined as a residual payment, an increase in risk aversion hardly alters the moments. On the other hand, when they are equal to the marginal product of capital, as in Lettau, a risk aversion parameter of around 700 would be needed to generate the mean premium in the data.

When the model is studied under the assumption of a risk averse firm (UM objective), the results are slightly better, but the improvement is still relatively small when using reasonable parameter values. In this case, however, the sensitivity of the asset moments with respect to an increase in risk aversion considerably increases. In particular, a risk aversion of 26, while being still too high, is able to yield the mean equity premium in the data. As before, we find that dividends play a minor role, since all the improvement is driven by the equity return component due to future returns. Thus, independently of the firm objective and the dividend specification, the identical household economy has no chance of generating reasonable results.

The model is then studied under the assumption that households are subject to a symmetric idiosyncratic labor income shock. In this case, the new source of risk leads to a new idiosyncratic component in the expressions of the pricing kernel and of the unexpected equity return. Concerning the last, the idiosyncratic component arises from the fact that news about future returns can only be due to news about future risk free rates in the model, given the constant risk premium over time. Since the risk free rate is directly affected by idiosyncratic risk, its presence in the expression of the unexpected equity return implies that this return will also contain an idiosyncratic component. We show that this component crucially depends from the degree of constrained persistence, i.e., from the degree to which the same household is constrained for a certain number of periods. In particular, idiosyncratic uncertainty has an affect as long as there is constrained persistence. We study the results under the assumption that the same household will be unconstrained for the first k periods. Since the number of periods a household is usually unconstrained may be lower than k, the results have to be interpreted as an upper bound.

Using the closed form solutions for unexpected individual consumption and the unexpected equity return, we are able to decompose the equity premium and the two asset return variabilities into their value under identical households and a new term, only affected by idiosyncratic risk. With the presence of the new term, which accounts for approximately 99.5 % of the moments, the model is able to generate the right equity premium in the data with a reasonable parameterization.

We show that the idiosyncratic components are essentially driven by three parameters, the household risk aversion, the idiosyncratic innovation variance and the idiosyncratic shock persistence. Since the first two enter the new terms in a multiplicative way, all the asset moments are found to be highly sensitive to an increase in these two parameter values. In addition, we find that an increase in the premium, due to a higher risk aversion or a higher innovation variance, is accompanied by a more than proportional effect on the two asset return variabilities, which are too high as compared to the data. This result can be improved by increasing the idiosyncratic shock persistence, which dampens the effect of an increase in risk aversion on the two return variabilities. To generate the right risk return trade-off, however, a too high persistence outside the estimated range would be needed. Therefore, while the model with idiosyncratic risk leads to a considerable improvement, we can say that its success is only partial.

APPENDIX 1

The following log linearized system of equations is used to solve for the elasticities of the macroeconomic variables:

$$z_t = \psi z_{t-1} + \varepsilon_{zt} \tag{1}$$

$$y_t = z_t + \alpha k_{t-1} \tag{2}$$

$$i_t = \lambda_{01} y_t - \lambda_{02} c_t \tag{3}$$

$$k_t = \lambda_1 k_{t-1} + \lambda_2 z_t + \lambda_{12} c_t \tag{4}$$

$$d_t = \lambda_{03} y_t - \lambda_{04} i_t \tag{5}$$

$$\gamma_f E_t \Delta d_{t+1} = \lambda_3 E_t (z_{t+1} + (\alpha - 1) k_t) \tag{6}$$

or, in the firm has the usual VM objective,

$$\gamma E_t \Delta c_{t+1} = \lambda_3 E_t (z_{t+1} + (\alpha - 1) k_t) \tag{7}$$

where the coefficients denoted by λ are constants depending on the parameters of the model Φ. Guessing that logged aggregate consumption takes the form:

$$c_t = \eta_{ck} k_{t-1} + \eta_{cz} z_t \tag{8}$$

where η_{xy} is the elasticity of variable x with respect to variable y, it is easy to show that the elasticities of output, capital, investment, and dividends with respect to the two state variables are given by:

$$\eta_{yk} = \alpha \tag{9}$$

$$\eta_{yz} = 1 \tag{10}$$

$$\eta_{kk} = \lambda_1 + \lambda_{12}\eta_{ck} \tag{11}$$

$$\eta_{kz} = \lambda_2 + \lambda_{12}\eta_{cz} \tag{12}$$

$$\eta_{ik} = \lambda_{01}\eta_{yk} - \lambda_{02}\eta_{ck} \tag{13}$$

$$\eta_{iz} = \lambda_{01}\eta_{yz} - \lambda_{02}\eta_{cz} \tag{14}$$

$$\eta_{dk} = \lambda_{03}\eta_{yk} - \lambda_{04}\eta_{ik} \tag{15}$$

$$\eta_{dz} = \lambda_{03}\eta_{yz} - \lambda_{04}\eta_{iz} \tag{16}$$

Further, substituting for the corresponding variables in the capital stock expectational equations, setting $E_t z_{t+1} = \psi z_t$, and using the method of undetermined coefficients, we can obtain the consumption elasticities with respect to the two state variables, by solving the two following equations:

$$\eta_{ck} = \frac{1}{Q_2}\left\{-Q_1 - \sqrt{Q_1^2 - 4Q_0Q_2}\right\} \tag{17}$$

$$\eta_{cz} = f(\Phi, \eta_{ck}) \tag{18}$$

where Q_0, Q_1 and Q_2 are constants that differ for the two firm objectives and depend on the model parameters Φ, and f is a function depending on Φ and on the consumption elasticity with respect to the capital stock η_{ck}. From the system above it becomes clear that, once we have solved for the two consumption elasticities, we can easily obtain the elasticities for the other endogenous variables.

APPENDIX 2

The present appendix provides the exact expressions for the terms of subsection 3.1 with the superscript*:

$$\Theta_x^* = \frac{(\eta_{xk}\eta_{kz} - \eta_{xz}\eta_{kk})}{\eta_{xz}} \tag{1}$$

$$\Theta_{rf}^* = \frac{-(\eta_{ck}\eta_{kz} + \eta_{cz}\eta_{kk}(\psi - 1))}{(\eta_{ck}\eta_{kz} + \eta_{cz}(\psi - 1))} \tag{2}$$

$$A^* = \frac{\gamma(\eta_{ck}\eta_{kz} + \eta_{cz}(\psi - 1))\sigma_z}{\psi - \eta_{kk}} \tag{3}$$

$$\eta_{r^e z}^{d*}\varepsilon_{t+1}^z = (E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^{j-1}\Delta d_{t+j} = \frac{1-\rho}{1-\rho\psi}(\eta_{dz} + \rho\frac{\eta_{dk}\eta_{kz}}{1-\rho\eta_{kk}})\varepsilon_{t+1}^z \tag{4}$$

$$\eta_{r^e z}^{r^f *}\varepsilon_{t+1}^z = -(E_{t+1} - E_t)\sum_{j=2}^{\infty}\rho^{j-1}r_{t+j}^f = (\frac{\gamma\rho}{(1-\psi\rho)}((1-\psi)\eta_{cz} - \frac{(1-\rho)\eta_{ck}\eta_{kz}}{(1-\eta_{kk}\rho)})\varepsilon_{t+1}^z \tag{5}$$

Bibliography

Aiyagary, S. R. and Gertler, M., *Asset Returns with Transaction Costs and Uninsured Individual Risk*, Journal of Monetary Economics 27, 1991, 311-331

Altug S., Labadie P., *Models with Production and Capital Accumulation,* in Dynamic Choice and Asset Markets

Campbell J. Y., *Asset Prices, Consumption, and the Business Cycle*, 1998, forthcoming in Handbook of Macroeconomics, edited by Taylor, J., B. and Woodford, M.

Campbell J. Y., *Inspecting the Mechanism, An Analytical Approach to the stochastic Growth Model*, Journal of Monetary Economics 33, 1994, 463-506

Danthine, J. P., Donaldson, J. B. and Mehra, R., *The Equity Premium and the Allocation of Income Risk*, Journal of Economic Dynamics and Control 16, 1992, 509-532

Danthine, J. P. and Donaldson, J. B., *Labor Relations and Asset Returns*, 1999.

DeMarzo, P.M., *Majority Voting and Corporate Control: The rule of the dominant shareholder*, RES, 1993, 60, 713-734

DeMarzo, P.M., *An Extension of the Modigliani-Miller Theorem to Stochastic Economies with Incomplete Markets and Interdependent Securities*, JET, 45, 353-369, 1988

Den Haan, W. J. and Marcet, A., *Solving the Stochastic Growth Model by Parametrized Expectations*, Journal of Business and Economic Statistics, 1990

Diamond, P.A., T*he role of a Stock Market in a General Equilibrium Model with Technological Uncertainty*, American Economic Review, vol. 57, 759-76, 1967

Dréze, J. H., *Investment under Private Ownership: Optimality, Equilibrium and Stability, in Allocation under Uncertainty: Equilibrium and Optimality*, ed. by Drèze, J., NY, Macmillan, 1974

Drèze, J. H., *(Uncertainty and) the Firm in GE Theory*, Economic Journal, 95, 1985, 1-20

Grossman, S. J. and Hart, O. D., *A Theory of Competitive Equilibrium in Stock Market Economies*, Econometrica, vol. 47, No. 2, March 1979

Heaton, J. and Lucas, D., *Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Pricing*, Journal of Political Economy, 1996, vol. 104, No. 3

Jermann U. J., *Asset Pricing in Production Economies*, Journal of Monetary Economics 41, 1998, 257-275

Krussel P. and Smith A. A., Income and Wealth Heterogeneity, Portfolio Choice, and Equilibrium Asset Returns, *Macroeconomic Dynamics, 1997, vol. 1, Nr. 2, 387-422*

Leland, H.,E., *Theory of the Firm Facing Uncertain Demand*, The American Economic Review,1972, 279-291

Lettau M., *Inspecting the Mechanism: The Determination of Asset Prices in the Real Business Cycle Model,* CEPR Discussion Paper No. 1884, May 1998

Lettau, M., and Uhlig, H., *Asset Prices and Business Cycles: Successes and Pitfalls of the General Equilibrium Approach*, November 1996, prepared for the Special Conference Issue of Finance and Stochastics

Lettau, M., *Idiosyncratic Risk and Volatility Bounds or, Can models with idiosyncratic risk solve the equity premium puzzle?*, April 1999

Lettau, M., and Uhlig, H., *Preferences, Consumption Smoothing, and Risk Premia*, March 1997

Lucas, J. D., *Asset Pricing with Undiversifiable income risk and short sale constraints, Deepening the Equity Premium Puzzle*, Journal of Monetary Economics 34, 1994, 325-341

Lintner, John, Distribution of Incomes of Corporations Among Dividends, Retained Earnings, and Taxes, The American Economic Review, Vol. 46, Issue 2, 1956

Magill, ,M. and Quinzii, M.,*Theory of Incomplete Markets*, vol. 1, The MIT Press, Cambridge, Massachussets

Marcet A. and Singleton K., *Equilibrium Asset Prices and Savings of Heterogeneous Agents in the Presence of Incomplete Markets and Portfolio Constraints*, Macroeconomic Dynamics, 3, 1999, 243-277

Marcet A. and Lorenzoni, G., *Parametrized Expectations Approach: Some Practical Issues*, in Marimon, R. and Scott, A., Computational Methods for the Study of Dynamic Economies, Oxford University Press, forthcoming 1998

Mehra R., Prescott E. C., *The Equity Premium: A puzzle*, Journal of Monetary Economics 15, 1985, 145-161

Radner, R., *New ideas in pure theory, Problems in the Theory of Markets under Uncertainty*, American Economic Review, 1972, 454-460

Radner R., *Existence of Equilibrium of Plans, Prices, and Price Expectations in a Sequence of Markets*, Econométrica, 40(2), March 1972, 289-303

Sandmo, A., *On the Theory of the Competitive Firm Under Price Uncertainty*, The American Economic Review, 1972, 65-73

Sondermann, D., *Temporary Competitive Equilibrium Under Uncertainty,in Allocation under Uncertainty: Equilibrium and Optimality*, ed. by Drèze, J., NY, Macmillan, 1974

Telmer, C. I., Asset Pricing Puzzles and Incomplete Markets, The Journal of Finance, vol. XLVIII, No. 5, December 1993

Rouwenhorst K.G., *Asset Prices Implications of Equilibrium Business Cycle Models*, in Cooley, T., New Frontiers of Modern Business Cycle Research 1995