Essays on New Keynesian Macroeconomics

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Dissertation submitted for the degree of
Doctor of Philosophy
in Economics

July 2008
Acknowledgements

I would like to express my gratitude to all the people who made this thesis possible. I am deeply indebted to my supervisor Jordi Galí. Throughout my doctoral studies, he provided excellent guidance, sound advice, great teaching, and excellent ideas. It was really a pleasure and an honor for me to have had the possibility of discussing my research with him. I am also very indebted to Thijs van Rens for his encouragement and excellent academic support. I received invaluable comments from him.

The research that I present in this thesis also benefited from many comments and discussions with Filippo Brutti, Andrea Caggese, Fabio Canova, Davide Debortoli, Francisco Grippa, Albert Marcet, Ricardo Nunes, and Michael Reiter.

I am also very grateful to Marta Aragay, Marta Araque, Gemma Burballa, Anna Cano, Carolina Rojas, and Anna Ventura. They kindly assisted me with all the administrative issues related to my doctoral studies.

Finally, I wish to thank my family and friends for their company, support, and encouragement.
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Introduction

The standard New Keynesian (NK) model has become one of the most influential tools in discussions of macroeconomic dynamics, monetary policy and welfare. Moreover, it has emerged as the backbone of the medium scale macroeconomic models that several central banks and policy institutions use for simulation and forecasting purposes. This model integrates the Real Business Cycle (RBC) Paradigm with NK Theory. In fact, the NK model adopts a stochastic dynamic general equilibrium modeling approach from the RBC theory and combines it with two Keynesian ingredients: monopolistic competition and nominal price rigidity. In this sense, this model has much stronger theoretical foundations than traditional Keynesian models.

The purpose of this thesis is to evaluate the accuracy of the following three implications of the standard NK model. First, with full price stability the welfare losses resulting from price stickiness should be zero. Second, inflation is a forward-looking phenomenon. Third, money does not play an independent role in the monetary transmission mechanism.

Traditionally, price stickiness has been studied within the New Keynesian framework as a source of monetary policy non-neutrality or to understand inflation persistence. In contrast, I try to answer the following question in this thesis: how harmful can price stickiness be for society? Theoretically, in the face of exogenous shocks, this rigidity can cause welfare losses by creating relative price distortions that lead to an inefficient sectoral allocation of resources. According to the standard NK model, by attaining zero inflation, relative price distortions are eliminated and the economy reaches the flexible price allocation.\(^1\) Therefore, in the NK setup, monetary policy is able to avoid all the welfare losses that could arise from price stickiness.

In chapter 1, titled "The Welfare Losses of Price Rigidities", I introduce firm-
specific productivity shocks in the standard NK model and compute the welfare losses of price stickiness. Moreover, I also depart from the time-dependent pricing assumption that is used in the NK model. In particular, I compute the welfare losses when price rigidities are incorporated by using state-dependent pricing.\textsuperscript{2} Several interesting results stand out. First, price stickiness may be a source of large welfare losses, even in economies with price stability. Second, state-dependent pricing dampens the size of the welfare losses, but they remain non-negligible. Third, the variance of the firm-specific productivity shock and the frequency of price adjustment are the key determinants of the size of the welfare losses.

Studying inflation dynamics is crucial for monetary policy analysis; in particular, exploring how plausible it is that inflation is mainly determined by forward-looking behavior. This is especially important for understanding the different sources of inflation persistence and the costs of disinflation processes.\textsuperscript{3} The New Keynesian Phillips Curve (NKPC), which describes the aggregate supply block of the NK model, predicts that inflation is determined exclusively by forward-looking behavior of firms. However, several studies have found evidence of backward-looking behavior. The evidence about its quantitative importance is mixed. Galí and Gertler (1999), Galí et al. (2001) and Galí et al. (2005) find a predominant role for forward-looking behavior. In contrast, Fuhrer and Moore (1995) and Rudd and Whelan (2005) find the backward-looking component to be more important.

In chapter 2, titled "Testing for Rule of Thumb Price-Setting", I contribute to the academic debate on inflation dynamics by proposing a novel methodology to test the importance of backward-looking behavior in the form of rule of thumb price-setting.\textsuperscript{4} By using Galí and Gertler’s (1999) hybrid model, I derive a dynamic structural relationship between the cross sectional variance of individual price changes and aggregate inflation. I argue that this relation has several features that make it more attractive than the hybrid NKPC in order to test backward-looking behavior. Finally, I estimate the proposed equation with Spanish data. I find that the fraction of firms that follow

\textsuperscript{2}There exists an important part of the literature on price rigidities that considers pricing policies that are state dependent. See Dotsey, King and Wolman (1999), Gertler and Leahy (2006), Golosov and Lucas (2007), Nakamura and Steinsson (2007) and Caballero and Engel (2007).

\textsuperscript{3}Credible disinflations are relatively costless when inflation is determined by the standard NKPC, but are quite costly when backward-looking behavior in price setting is quantitatively important. See Ball (1994) and Roberts (1998) for a discussion of this topic.

\textsuperscript{4}An alternative way to introduce backward-looking behavior is assuming indexation. See Smets and Wouters (2003) and Christiano et al. (2005).
rule of thumb price-setting is high and quantitatively important.\textsuperscript{5} Therefore, inflation is not only a forward-looking phenomenon in Spain.

Finally, the last topic I explore in this thesis is the importance of money for monetary policy analysis. The basic NK model assigns no role for money. This practice has been justified by Woodford (2003), who concludes that central banks can abstract from money demand if they control interest rates and utility is separable in consumption and real money balances. Moreover, Woodford (2003) and Ireland (2004) provide structural empirical evidence for separability. However these findings are against the reduced form evidence presented by Meltzer (2001), Nelson (2002) and Hafer et al. (2007) showing that money matters for monetary policy analysis.

In chapter 3, titled "Resurrecting the Role of Real Money Balance Effects", I revisit the relevance of money for monetary policy design. I present a structural econometric analysis that suggests that money still plays an independent role in the monetary transmission mechanism in the United States. In particular, it indicates that real money balance effects are quantitatively important but smaller than they used to be in the early postwar period. Therefore, the specification of money demand is necessary in order to determine the evolution of inflation and output, even if the central bank controls the interest rate. The empirical evidence presented in this chapter has three additional implications. First, by including real money balance effects into the standard sticky price model, two stylized facts can be explained: the modestly procyclical real wage response to a monetary policy shock and the supply side effects of monetary policy. Second, much higher volatility of output and much lower volatility of interest rates should arise under the optimal monetary policy when real money balance effects exist in the magnitude estimated in this chapter. Third, the reduction in the size of real money balance effects can account for a significant decline in macroeconomic volatility. This would support financial innovation as a potential source of the Great Moderation.

\textsuperscript{5}This result is consistent with previous studies that estimate the hybrid NKPC for Spain.
Chapter 1

The Welfare Losses of Price Rigidities

1.1 Introduction

The existence of nominal price rigidity seems uncontroversial. The fact that individual goods prices adjust sluggishly has been well documented by different studies for the United States and the Euro Area.\(^1\) This fact naturally raises the following question: What are the welfare consequences of this rigidity in the economy? Theoretically, in the face of exogenous shocks, price stickiness can cause welfare losses by creating relative price distortions that lead to an inefficient sectoral allocation of resources. The general belief in macroeconomics is that these losses would be negligible if monetary policy were to fully stabilize the aggregate price level. This idea is supported by models with price rigidities in which firms face only aggregate shocks.\(^2\) The story behind all these models is that by attaining zero inflation, relative price distortions are eliminated and the economy reaches the flexible price allocation.

Empirical evidence suggests that firms are also hit by idiosyncratic productivity shocks.\(^3\) In this chapter, I consider these shocks in the analysis of the welfare losses of

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\(^1\)Among these studies, Bils and Klenow (2004) point out that the average duration of a price spell is 7 months for US; whereas Dhyne et al.(2006) find that this duration is 13 months for the Euro Area. Both studies used the monthly price records underlying the computation of the CPI.


\(^3\)See Blundell and Bond(2000), Cooper et al.(2004) among others.
price rigidities. In a simple model, in which firms face both aggregate and idiosyncratic productivity shocks, I develop a general framework that allows for the measurement of the welfare losses of price rigidities. These losses are defined as the difference between the households' utility under sticky prices and the one under flexible prices. I then derive a second order approximation of the utility function and obtain the analytical expression for the welfare losses. I show that these losses depend on two different elements, independently of the way the price-setting is modeled. The first is the aggregate output gap, which measures the deviation of total output from the natural output. The second component is the dispersion of output gaps across goods. This component indicates how inefficient the sectoral allocation of goods is, given the aggregate output. Moreover, I show that a direct relationship exists between the dispersion of output gaps across goods and the dispersion of price gaps across goods. The latter measures how distorted relative prices are. Therefore, I confirm the intuition that inefficient output composition is associated with relative price distortions.

Once I find the analytical expression for the welfare losses, I need to assume a price-setting structure in order to compute these. Given the lack of consensus about how price stickiness should be modeled, I use two alternative price-settings to evaluate the magnitude of the welfare losses. The first one is the time-dependent pricing and the second one is the state-dependent pricing. The main difference between these two approaches is that the timing of price changes is exogenous in the time-dependent framework, while it is endogenous in the state-dependent one. In the latter case, the timing depends basically on how far the price of a firm is from its optimal price.

The introduction of idiosyncratic shocks has important consequences regarding the welfare losses associated with price rigidities. Accounting for all the uncertainty that exists on the structure of the economy, I find that these losses are between 0.5 and 4.4 percent of steady state consumption when the time dependent pricing is considered, while they are between 0.1 and 2.3 percent of steady state consumption when the state-dependent pricing is used. In both cases, these losses arise even if price stability is followed. These results suggest that price rigidities are relevant from a welfare point of view; and consequently, that it is important to think more carefully about their determinants in order to investigate if there exist alternative policies that can help

\[4\] The natural output is defined as the equilibrium level of output that would prevail if prices were flexible.

\[5\] The price gap is defined as the difference between the actual price and the one that would be set if prices were flexible.
to reduce the welfare losses arising from price stickiness. Moreover, the results show
that the size of the welfare losses is very sensitive to the price-setting, to the variance
of the idiosyncratic productivity shock and to the frequency of price adjustments.
Regarding the sensitivity to the price-setting, I show that price rigidities in the form
of pricing policies that are state-dependent are always significantly less harmful than
those based on time-dependent rules. The intuition of this result is related to the
existence of the selection effect identified by Golosov and Lucas (2007) in the case of
the state-dependent pricing. In the latter case, firms that are further away from their
optimal price are more likely to change their price, diminishing the distortions that
price rigidities can cause.

The remainder of the chapter proceeds as follows. Section 1.2 presents the model.
Section 1.3 derives an analytical expression for the welfare losses and shows some im-
portant analytical results. Section 1.4 introduces the standard Calvo price-setting in
order to compute the welfare losses when the pricing decisions are time-dependent.
In this case, I show analytically that the dispersion of output gaps across goods de-
pends, in the long run, on aggregate inflation and on the variance of the idiosyncratic
productivity shock. The part of the dispersion that is due to idiosyncratic shocks is
independent of aggregate macroeconomic variables, and consequently, independent of
monetary policy. Therefore, it is concluded that there does not exist any monetary
policy that can reach the flexible price allocation when some firms cannot adjust prices
to their idiosyncratic shocks. The welfare losses are computed under different plau-
sible calibration exercises and assuming that price stability is followed. Section 1.5
presents a modified version of the Generalized Ss model developed by Caballero and
Engel (2007). This model is used in order to compute the welfare losses when the
pricing decisions are state-dependent. Section 1.6 concludes.

1.2 The Model

Most of the structure of the model developed in this section is taken from the one
developed in Galí (2008). The main difference is that firms are hit also by idiosyncratic
productivity shocks.
1.2.1 Households

The representative household seeks to maximize the objective function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, H_t)$$ (1.1)

where $0 < \beta < 1$ is the discount factor, $C_t$ is an index of consumption goods and $H_t$ is the number of hours worked in period $t$. The household purchases differentiated goods and combines them into a composite good using a Dixit-Stiglitz aggregator:

$$C_t = \left( \int_0^1 C_t(i)^{(\epsilon-1)/\epsilon} di \right)^{\epsilon/(\epsilon-1)}$$ (1.2)

where $C_t(i)$ is the differentiated good of type $i$ and $\epsilon > 1$ is the constant elasticity of substitution among goods. The households maximize the index $C_t$, given the total cost of all differentiated goods and their nominal prices $\{P_t(i)\}$. Then, the demand for each good is given by:

$$C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t$$ (1.3)

where $P_t$ is the aggregate price level and is defined as follows:

$$P_t = \left( \int_0^1 P_t(i)^{1-\epsilon} di \right)^{1/(1-\epsilon)}$$ (1.4)

The maximization of the expected utility is subject to an intertemporal budget constraint of the form:

$$\sum_{t=0}^{\infty} E_0 Q_{0,t} P_t C_t \leq B_0 + \sum_{t=0}^{\infty} E_0 Q_{0,t} [(1 + \eta)W_t H_t - T_t]$$ (1.5)

where $B_0$ is the initial level of wealth, $W_t$ is the nominal wage per hour worked, $T_t$ represents a lump sum tax and $\eta$ denotes a constant rate of employment subsidy that is
funded by the lump sum tax. This subsidy is introduced in the model in order to offset the distortion associated with imperfect competition in goods markets. Moreover, \( Q_{0,t} \) is a stochastic discount factor that satisfies \( Q_{0,0} = 1 \) and \( E_0 Q_{0,t} = \prod_{s=0}^{t-1} (1 + i_s)^{-1} \) where \( i_t \) denotes the interest rate at period \( t \). The labor market is perfectly competitive and wages are flexible.

The household’s optimization problem is then to choose processes \( C_t \) and \( H_t \) for all dates \( t \) satisfying (1.5), given its initial wealth \( B_0 \), the goods prices, the nominal wage and the stochastic discount factors that it expects to face, so as to maximize (1.1).

For the purpose of this chapter, the intratemporal first order condition (associated with labor supply) is the only one to be presented. This condition is:

\[
-\frac{U_h}{U_c} = (1 + \eta) \frac{W_t}{P_t} \tag{1.6}
\]

### 1.2.2 Firms

Each firm \( i \) has a production function of the form:

\[
Y_t(i) = \tilde{A}_t A_t(i) H_t(i) \tag{1.7}
\]

where \( Y_t(i) \) is the level of output at period \( t \) of firm \( i \), \( \tilde{A}_t \) is the aggregate level of productivity in period \( t \), \( A_t(i) \) is the firm \( i \)’s idiosyncratic productivity level at period \( t \) and \( H_t(i) \) is the total hours hired by firm \( i \) in period \( t \). The idiosyncratic productivity level is assumed to follow an AR(1) process of the form:

\[
\log A_t(i) = \rho \log A_{t-1}(i) + \varepsilon_t(i) \tag{1.8}
\]

where \( \varepsilon_t(i) \) follows an i.i.d process with zero mean and constant variance \( \sigma^2_{\varepsilon} \). Firms face a rigidity in changing their price. Two ways of modeling this rigidity are explored in the paper: the Calvo pricing (1983) and a modified version of the Generalized Ss model developed by Caballero and Engel (2007). The details on the price settings proposed in these models are left for Sections 4 and 5 respectively.
1.2.3 Equilibrium

Market clearing in the goods market requires that $C_t(i) = Y_t(i)$ for all $i$ and at all times. This implies that the index of aggregate consumption $C_t$ must at all times equal the index of aggregate output $Y_t = \left(\int_0^1 Y_t(i)^{(e-1)/e} di\right)^{e/(e-1)}$. Moreover, labor supply must equal labor demand, which means:

$$H_t = \int_0^1 H_t(i) di$$  \hspace{1cm} (1.9)

By using the market clearing condition in the goods markets, the demand for goods and the production function of the firm, the market clearing condition in the labor market implies:

$$H_t = \left(\frac{Y_t}{A_t}\right)^{1/\alpha} \int_0^1 \left(\frac{Y_t(i)/Y_t}{A_t(i)}\right)^{1/\alpha} di$$  \hspace{1cm} (1.10)

Taking logs in (1.10), I get:

$$\alpha h_t = y_t - \tilde{a}_t + d_t$$  \hspace{1cm} (1.11)

where the lower case letters are used to denote the logs of original variables and $d_t = \alpha \log \int_0^1 \left(\frac{Y_t(i)/Y_t}{A_t(i)}\right)^{1/\alpha}$ is a measure of output dispersion across goods adjusted by the presence of idiosyncratic shocks. Throughout this chapter, I will use the term adjusted output dispersion when I refer to $d_t$. This term captures how the composition of output between firms affects total output. Alternatively, $d_t$ can be written as:

$$d_t = \alpha \log \int_0^1 \left(\frac{1}{A_t(i)}\right)^{1/\alpha} \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon/\alpha} di.$$

1.3 The Welfare Losses of Price Rigidities

The welfare losses of price rigidities are given by the difference between the households’ utility under sticky prices and the one under flexible prices. In this section I derive a second order approximation of the utility function around a zero inflation steady state.
I then evaluate this approximation under sticky prices and flexible prices. Finally, I obtain the analytical expression for the welfare losses.

### 1.3.1 A Second Order Approximation to Utility

The second order Taylor expansion of $U_t$ around a steady state $(C,N)$ with zero inflation yields:

$$U_t - U \simeq U_c C \left( \hat{y}_t + \frac{1 - \sigma \hat{y}_t^2}{2} \right) + U_h H \left( \hat{h}_t + \frac{1 + \chi \hat{h}_t^2}{2} \right)$$  \hspace{1cm} (1.12)

where hat variables represent log deviations from steady state, $\sigma = \frac{U_{cc}}{U_c}$ and $\chi = \frac{U_{hh}}{U_h}$. Moreover, I have made use of the market clearing condition $b_y t = b_c t$. Next, it is convenient to rewrite $\hat{h}_t$ in terms of $\hat{y}_t$ by using (1.11) and the fact that $d_t$ is a term of second order around a zero inflation steady state. Then, we have:

$$U_t - U \simeq U_c C \left( \hat{y}_t + \frac{1 - \sigma \hat{y}_t^2}{2} \right) + \frac{U_h H}{\alpha} (\hat{y}_t - \tilde{a}_t + d_t) + U_h H \frac{1 + \chi}{2\alpha^2} (\hat{y}_t - \tilde{a}_t)^2$$  \hspace{1cm} (1.13)

Efficiency in the zero inflation steady state, which is guaranteed by the government subsidy to labor, implies that $-\frac{U_{hh}}{U_h} = \alpha \frac{Y}{H}$. Therefore, period $t$ utility function can be written as:

$$\frac{U_t - U}{U_c C} \simeq \frac{1 - \sigma \hat{y}_t^2}{2} \tilde{a}_t - d_t - \frac{1 + \chi}{2\alpha} (\hat{y}_t - \tilde{a}_t)^2$$  \hspace{1cm} (1.14)

The latter expression measures the deviation of period utility from its steady state. It is expressed as a fraction of steady state consumption.

### 1.3.2 An Analytical Expression for the Welfare Losses

The welfare losses of price stickiness, expressed as a fraction of steady state consumption, can be defined as follows:

$$L_t = \frac{U_t - U_t^F}{U_c C}$$  \hspace{1cm} (1.15)

where $U_t$ and $U_t^F$ are the utilities under sticky prices and flexible prices respectively. Therefore, in order to compute the welfare losses, it is necessary to obtain utilities
The deviation of utility from the steady state under flexible prices can be expressed as:

\[
\frac{U^F_t - U}{U_t C_t} \approx \frac{1 - \sigma}{2} \hat{y}^n_t + \hat{a}_t - d^n_t - \frac{1 + \chi}{2\alpha} (\hat{y}^n_t - \hat{a}_t)^2
\]  

(1.16)

where \( \hat{y}^n_t \) and \( d^n_t \) denote the natural output and the adjusted output dispersion without price rigidities respectively. By using (1.14), I can define the deviation of utility from the steady state under sticky prices. Then, by taking into account that \( \hat{y}^n_t = \frac{1 + \chi}{\alpha x + 1 - \alpha + \chi} \hat{a}_t \) and substracting (1.16) from (1.14), I get the following expression for the welfare losses of price rigidities:

\[
L_t = - \left[ \frac{\alpha \sigma + 1 - \alpha + \chi}{2\alpha} (\hat{y}_t - \hat{y}^n_t)^2 - (d_t - d^n_t) \right]
\]  

(1.17)

It can be seen that these losses depend on two different components. The first one, known in the literature as the output gap, measures how close total output is from the natural output. The second element has two possible interpretations. One is that it captures how distorted relative prices are. The other one is that it reflects how inefficient the sectoral allocation of goods is.

In order to illustrate the two possible interpretations of the difference between \( d_t \) and \( d^n_t \), it is helpful to define two concepts. The first one is the dispersion of price gaps across goods. It is defined as the variance across goods of the difference between actual prices and the ones that these goods would have if prices were flexible. In Appendix A.2, it is shown that the dispersion of the price gaps across goods is related to the second component in (1.17) in the following way:

\[
d_t - d^n_t = \frac{\epsilon}{2\Theta} \text{Var}_i \left\{ p_t(i) - p_f^i(i) \right\}
\]  

(1.18)

where \( \Theta = \frac{\alpha}{\alpha + (1 - \alpha) \epsilon} \); \( p_t(i) \) is the logarithm of the actual price of good \( i \) and \( p_f^i(i) \) is the logarithm of the price that a good \( i \) would have if price rigidities were permanently removed. The magnitude of the variance in (1.18) measures how distorted relative prices are. From expression (1.18), it is clear that higher relative price distortions due to price stickiness imply more welfare losses. Moreover, by using (1.18), it is obvious that the second element in (1.17) is always non-negative. This implies that

\[\text{Notice that } \text{Var}_i \left\{ p_t(i) - p_f^i(i) \right\} = \text{Var}_i \left\{ p_t(i) - p_t - (p_f^i(i) - p_f^i) \right\} \text{ where } p_t \text{ and } p_f^i \text{ are the price levels under sticky and flexible prices respectively.}\]
there always exist welfare losses in this model, unless the flexible price allocation is
reached.

The second concept that is useful to develop is the dispersion of output gaps across
goods. This is equal to the variance across goods of the difference between actual
output of good $i$ and the natural output of good $i$. The size of this variance measures
how inefficient the sectoral composition of output is, given the aggregate output. By
using the structure of the demand for good $i$, it is straightforward to see that the
dispersion of price gaps across goods and the dispersion of output gaps across goods
are related in the following way:

$$Var_i \left\{ p_i(i) - p^f_i(i) \right\} = \frac{1}{\epsilon^2} Var_i \left\{ y_t(i) - y^n_t(i) \right\}$$  \hspace{1cm} (1.19)

where $y_t(i)$ is the logarithm of the actual output of good $i$ and $y^n_t(i)$ is the logarithm
of the natural output of good $i$. Expression (1.19) confirms the intuition that there is
a direct relationship between relative price distortions and the inefficiency in sectoral
allocation of real resources. Moreover, by using (1.18) and (1.19), it can be concluded
that the second interpretation of the difference between $d_t$ and $d^n_t$ is also right. Given
this interpretation, throughout the rest of this study, I will refer to the gap $d_t - d^n_t$ as
the dispersion of output gaps across goods.

To conclude this section, it is convenient to show the particular form of the welfare
losses when there are no idiosyncratic shocks. In this case, the frictionless price is
the same for every firm $i$. Therefore, the dispersion of price gaps across goods can be
expressed only as a function of the cross sectional variance of actual prices. This means
that expression (1.17) can be written as the standard welfare losses in the literature
on optimal monetary policy with $d_t - d^n_t = -\frac{\epsilon^2}{25} Var_i \left\{ p_t(i) \right\}$.

### 1.4 The Welfare Losses with Calvo Pricing

In this section, first, I present the model with Calvo price-setting. Then, I show how
to use it in order to compute the welfare losses when this price setting takes place.
Finally, I present estimates of the welfare losses under different plausible calibrations
of the parameters of the model.
1.4.1 Calvo Price-Setting

Firms set prices as in the sticky price model of Calvo (1983). In this model, during each period, a randomly chosen fraction of firms \((1 - \theta)\) is allowed to change the prices; whereas the other fraction \(\theta\) do not change. Those firms resetting prices will choose an optimal price \(P_t^*(i)\). Notice that in this case, given the idiosyncratic productivity shock, the optimal price for each firm would not be the same among those firms that change.

1.4.1.1 Optimal Price-Setting

A firm reoptimizing in period \(t\) will choose a price \(P_t^*(i)\) that maximizes the current market value of the profits generated while that price remains effective. This means solving the following problem:

\[
\max_{P_t^*(i)} \sum_{k=0}^{\infty} \theta^k E_t \{ Q_{t,t+k}(P_t^*(i)Y_{t+k}(i) - W_{t+k}H_{t+k}(i)) \} 
\]

subject to the sequence of demand constraints and production functions.

The first order condition associated with this problem, up to a first order approximation around the zero inflation steady state, is:

\[
p_t^*(i) = \Theta \left\{ \mu + (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k E_t x_{t+k} \right\} - \frac{(1 - \beta \theta)}{\alpha} \Theta \sum_{k=0}^{\infty} (\beta \theta)^k E_t a_{t+k}(i) \tag{1.21}
\]

where the lower case letters are used to denote the logs of original variables, \(\mu = \log \frac{\epsilon^*}{\epsilon_{t-1}}\) and \(x_t\) is given by the following expression:

\[
x_t = - \log \alpha + w_t - \frac{1}{\alpha} \tilde{a}_t + \frac{1 - \alpha}{\alpha} (\epsilon p_t + y_t) \tag{1.22}
\]

Notice from (1.21) that the optimal price has two components: the first one is a macro component (common across firms) and the second one is a firm specific component. Then, it is convenient to express this condition as:

\[
p_t^*(i) = p_t^C - \frac{(1 - \beta \theta)}{\alpha} \Theta \sum_{k=0}^{\infty} (\beta \theta)^k E_t a_{t+k}(i) \tag{1.23}
\]
where $p_t^C = \Theta \left\{ \mu + (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k E_t x_{t+k} \right\}$. Finally, by using the fact that the idiosyncratic shock follows an AR(1) process, (1.23) can be written as:

$$p_t^*(i) = p_t^C - \frac{(1 - \beta \theta) \Theta}{\alpha(1 - \beta \theta \rho)} a_t(i)$$

(1.24)

### 1.4.1.2 Aggregate Price Level Dynamics

Using the definition of the aggregate price level, the log of the price level can be written as:

$$p_t = \int_0^1 p_t(i) di$$

(1.25)

Then, by using the Calvo pricing, this relation can be written as:

$$p_t = \theta \int_0^1 p_{t-1}(i) di + (1 - \theta) \int_0^1 p_t^*(i) di$$

(1.26)

Finally, by combining (1.24) and (1.26), I get:

$$p_t^C - p_t = \frac{\theta}{1 - \theta} \pi_t$$

(1.27)

where $\pi_t$ is the inflation rate between periods $t - 1$ and $t$.

### 1.4.1.3 The New Keynesian Phillips Curve with Idiosyncratic Shocks

The first step to derive the aggregate supply curve with idiosyncratic shocks consists in defining the economy’s real average marginal cost ($mc_t$) as the difference between the real wage and the economy’s average product of labor. Then, this definition implies:

$$mc_t = w_t - p_t - \frac{1}{\alpha} \tilde{a}_t + \frac{1}{\alpha} y_t - \log \alpha$$

(1.28)

By combining the previous definition with the one of $x_t$, I get:

$$x_t = mc_t + \frac{1}{\Theta} p_t$$

(1.29)
Plugging the latter relationship into the definition of $p_t^C$ and rearranging some terms, I obtain:

$$p_t^C = (1 - \beta \theta) \Theta + \sum_{k=0}^{\infty} (\beta \theta)^k E_t \hat{m} c_{t+k} + (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k E_t p_{t+k} \quad (1.30)$$

Subtracting $p_{t-1}$ from both sides, I get:

$$p_t^C - p_{t-1} = (1 - \beta \theta) \Theta + \sum_{k=0}^{\infty} (\beta \theta)^k E_t \hat{m} c_{t+k} + (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k E_t \pi_{t+k} \quad (1.31)$$

Notice that the previous expression can be rewritten more compactly as a difference equation in the following way:

$$p_t^C - p_{t-1} = \beta \theta (p_{t-1}^C - p_t) + (1 - \beta \theta) \Theta \hat{m} c_{t+k} + \pi_t \quad (1.32)$$

Finally, by using the fact that $p_t^C - p_{t-1} = \frac{\pi_t}{1 - \theta}$, which is derived from equation (1.27), equation (1.32) yields the following inflation equation:

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1 - \theta)(1 - \beta \theta)}{\theta} \Theta \hat{m} c_{t+k} \quad (1.33)$$

It has been shown that the existence of idiosyncratic shocks does not affect the first order approximation of the standard relationship between inflation and real marginal costs. This is because the mean of the idiosyncratic productivity shocks is zero. Now, for the welfare analysis, it is convenient to obtain a relationship between inflation and the output gap. Galí (2008) shows that the following relationship between the economy’s real average marginal cost and the output gap holds in the model developed in Section 2:

$$\hat{m} c_{t+k} = \left[ \sigma + \frac{\chi + 1 - \alpha^2}{\alpha} \right] (\hat{y}_t - \hat{y}_t^p) \quad (1.34)$$

To conclude the derivation of the relationship between inflation and the output gap, I combine (1.33) and (1.34) to obtain:

Notice that the existence of idiosyncratic shocks does not affect Galí’s result on this relationship because the mean of firm-specific productivity shocks is zero. The latter means that these shocks do not have any impact on the average of aggregate variables.
\[ \pi_t = \beta E_t \pi_{t+1} + \frac{(1 - \theta)(1 - \beta \theta)}{\theta} \left[ \sigma + \frac{\chi + 1 - \alpha}{\alpha} \right] \Theta(\hat{y}_t - \hat{y}_t^n) \] (1.35)

### 1.4.2 Measuring Welfare Losses

In this case, it is convenient to write the welfare losses as in equation (1.17):

\[ L_t = - \left[ \frac{\alpha\sigma + 1 - \alpha + \chi}{2\alpha} \right] (\hat{y}_t - \hat{y}_t^n)^2 - (d_t - d_t^n) \]

Now, it is necessary to find an expression for the dispersion of output gaps across goods that depends on aggregate inflation and on the variance of the idiosyncratic component of productivity. This expression will be useful in order to decompose the welfare losses of price rigidities in two parts: one that is dependent of monetary policy and another one that is not. By using the lemmas developed in Appendix A.3, it can be shown that the dispersion of output gaps across goods, as \( t \to \infty \), is given by:

\[ d_t - d_t^n = \frac{\epsilon}{2\Theta (1 - \theta)} \sum_{j=0}^{\infty} \theta^j \pi_{t-j}^2 + \left\{ \frac{\epsilon}{2\Theta} \phi^2 + \frac{1 + (\epsilon - 1)\Theta}{2\alpha} - \frac{\epsilon(1 - \theta)}{\alpha(1 - \theta \rho)} \phi \right\} \sigma_a^2 \] (1.36)

where \( \phi = \frac{(1 - \beta \theta)\Theta}{(1 - \beta \theta \rho)\alpha} \) and \( \sigma_a^2 = \frac{\sigma^2}{1 - \rho^2} \).

Some comments about the last expression are useful. First, in the long run, the dispersion of output gaps across goods depends on aggregate inflation and on the variance of the idiosyncratic productivity shock. Second, the first component in (1.36) measures the dispersion that is generated due to the fact that some firms cannot adjust prices to aggregate shocks; whereas the second component in (1.36) measures the dispersion that is created because the same firms cannot adjust prices to their idiosyncratic shocks. Under sticky prices (\( 0 < \theta < 1 \)), both components are always non-negative. Third, when \( \theta = 0 \), it can be shown that the dispersion is zero, which implies \( d_t = d_t^n \). Fourth, the part of the dispersion that is due to idiosyncratic shocks is independent of aggregate macroeconomic variables, and consequently, independent of monetary policy. Therefore, it can be concluded that no monetary policy exists that can reach the flexible price composition of output among goods when some firms cannot adjust their prices to their idiosyncratic shocks. Fifth, the dispersion is increasing in the elasticity of substitution among goods, in the degree of price rigidity and in the
variance of the idiosyncratic productivity shock.

By using (1.36), it is clear that we can decompose the welfare losses of price rigidities in two parts: one that is dependent of monetary policy and another one that is not. The losses that depend on monetary policy are given by the following expression:

\[
L_P^t = -\frac{\epsilon}{2\Theta (1-\Theta)} \sum_{j=0}^{\infty} \theta^j \pi^2_{t-j} - \left[ \frac{\alpha \sigma + 1 - \alpha + \chi}{2\alpha} \right] (\hat{y}_t - \hat{y}_t^n)^2
\] (1.37)

whereas the ones that are independent are given by:

\[
L_{IP}^t = - \left\{ \frac{\epsilon}{2\Theta} \phi^2 + \frac{1 + (\epsilon - 1)\Theta}{2\alpha} - \frac{\epsilon (1 - \Theta)}{\alpha (1 - \Theta \rho)} \phi \right\} \frac{\sigma^2_{\epsilon}}{1 - \rho^2}
\] (1.38)

Then, the natural question is: how big are these welfare losses? Clearly, \(L_P^t\) will depend on the monetary policy that is followed. For simplicity, I assume a policy that fully stabilizes the price level. This implies that the output gap is also zero up to a first order approximation, according to the Phillips Curve presented in (1.35). Consequently, under zero inflation, \(L_P^t\) is zero up to a second order approximation.\(^8\) Therefore, the only source of welfare losses is \(L_{IP}^t\), which can be measured in the model without resorting to the monetary policy. The next subsection seeks to quantify that term.

### 1.4.3 Quantifying \(L_{IP}^t\)

In order to measure \(L_{IP}^t\), it is necessary to calibrate the parameters of the model. The frequency chosen to perform this exercise is monthly. The baseline calibration is shown in Table 1.1. Before discussing this calibration, it is worth mentioning that four out of six of the structural parameters are calibrated by using information from the Dominick’s database and some relationships derived from the model.\(^9\) These parameters are \(\epsilon, \alpha, \theta\) and \(\sigma^2_{\epsilon}\). The main advantage of calibrating the majority of

---

\(^8\)When firms face idiosyncratic productivity shocks, a zero inflation policy cannot attain the natural level of output. In fact, the second order approximation of the standard New Keynesian Phillips curve is different from the one derived when firms are hit by idiosyncratic shocks. In the latter, there is a constant term that depends on the variance of the idiosyncratic shocks. Therefore, zero inflation cannot lead to a zero output gap, up to a second or higher order approximation. However, the impact of non zero output gap on the welfare function is of third or higher order with price stability.

\(^9\)The Dominick’s database contains nine years (from 1989 to 1997) of weekly store level data on the prices and quantities of more than 4500 products for 86 stores in the Chicago area. For more details on this database, see Midrigan (2006).
parameters by using the same database is that it provides consistency between the different choices of parameters.

<table>
<thead>
<tr>
<th>Table 1.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
</tr>
<tr>
<td>0.997</td>
</tr>
</tbody>
</table>

It is assumed that $\beta = 0.997$, implying a steady state real return of financial assets of about four percent in annual terms. I set $\epsilon = 3$, based on the evidence provided by Chevalier, Kayshap and Rossi (2003). They estimate price elasticities using the quantity and price data from Dominick’s database. Most of their elasticity estimates range between 2 and 4. I set $\alpha$ so that it equals the average labor income share (0.66 in this calibration) times the markup implied by the choice of $\epsilon$. On price stickiness, it is assumed $\theta = 0.8$ such that the model matches the average price duration of five months estimated by Midrigan (2006) using the Dominick’s database. This price duration is also close to those found in the studies performed by Bils and Klenow (2004) and Altig et al. (2004). The persistence of the idiosyncratic component of productivity is assumed to be very high by setting $\rho = 0.95$. This is the preferred point estimate of $\rho$ in Blundell and Bond (2000). They estimate an AR(1) process for the firm’s idiosyncratic productivity by using a panel data covering 509 U. S. manufacturing companies observed for 8 years. Finally, the calibration of $\sigma^2$ is performed such that I match the observed variance of individual price changes. This is done by using the following expression derived from the model presented above:

$$\sigma^2 = \frac{(1 - \theta \rho)(1 - \rho^2)}{2(1 - \rho)(1 - \theta)} \left\{ \text{Var}_i \{ \pi(i) \} - \frac{2\theta}{1 - \theta} \pi^2 \right\}$$  (1.39)

where $\text{Var}_i \{ \pi(i) \}$ is the variance of monthly individual price changes across goods and $\pi$ is the monthly inflation. Now, it is straightforward how $\sigma^2$ is computed. Given

---

10 Firms’ profits maximization in the steady state implies that $1 = \frac{1}{\epsilon} mc_t$ where $mc_t$ denotes the real marginal cost. Moreover, the assumption about technology implies that the real marginal cost is equal to the labor share $(ls)$ divided by $\alpha$. Therefore, $\alpha = \frac{mc_t}{\epsilon}$.

11 The average price duration is computed by considering regular prices only (no sales). See Midrigan (2006) for the details on this calculation.

12 They provide an estimate of $\rho$ equal to 0.565 in annual frequency. In order to translate this estimate into the monthly frequency, I use $\rho_u = \rho^m$. This approximation assumes that productivity is end of period sampled and interprets it as a stock variable. I use ”$u$” to denote annual frequency and ”$m$” to denote monthly frequency.

13 See the Appendix A.5 for the derivation of this expression.
equation (1.39), the values set above for $\beta, \epsilon, \alpha, \theta$ and $\rho$, a constant monthly inflation of 0.03/12 and a variance of monthly individual price changes across goods equal to 0.002116 (consistent with the observed standard deviation of monthly individual price changes of 4.6 percent found in the Dominick’s database), it yields $\sigma^2_\epsilon = 0.0036$. The latter value is slightly lower than the one set by Golosov and Lucas (2007).\footnote{They choose a variance equal to 0.011 in their baseline calibration in quarterly frequency. Then, in order to translate my estimate into quarterly frequency and compare it with the one of Golosov and Lucas (2007), I apply the following relation: $\sigma^2_{\epsilon, q} = (1 + \rho^2_m + \rho^2_{\epsilon, m})\sigma^2_{\epsilon, m}$. My monthly estimate is equivalent to a quarterly estimate of 0.010. I use “$q$” to denote quarterly frequency and “$m$” to denote monthly frequency.} Under this calibration, the welfare losses of price rigidities are equivalent to 1.7 percent of steady state consumption.

1.4.3.1 Robustness Exercise

Four important sources of uncertainty can affect the baseline estimate. First, even assuming that the Dominick’s database is a representative sample of the economy, there exists uncertainty about the persistence of the idiosyncratic component of productivity and the elasticity of substitution.\footnote{Notice that the uncertainty in $\rho$ and $\epsilon$ leads to uncertainty in $\sigma^2_\epsilon$. Given that the latter is pinned down from all the other parameters and from the standard deviation of price changes, it is not considered that $\sigma^2_\epsilon$ induce uncertainty by itself.} Second, there is uncertainty about the determinants of the observed heterogeneity in the size of individual price changes. In the baseline calibration, it has been assumed that the variance of the idiosyncratic productivity shock can account for almost all the variance of individual price changes. However, it is possible that there exist ex-ante heterogeneity, like different frequencies of price adjustment, that can help to explain this variance. Third, the estimates of $\epsilon$ and $\theta$, obtained by using the Dominick’s database, are significantly lower than others presented in alternative studies. Therefore, there is uncertainty about how well the economy is represented by the information contained in the Dominick’s database. Moreover, given the way I calibrate the variance of the firm specific productivity shock, this third source of uncertainty introduces a fourth one on $\sigma^2_\epsilon$ and its relation with $\epsilon$ and $\theta$. In this subsection, I analyze and discuss how the baseline estimate changes when we consider all these sources of uncertainty separately.

In order to show how much the first source of uncertainty may matter, Table 1.2 presents the welfare losses by allowing the parameters $\rho$ and $\epsilon$ to vary between
reasonable values.\textsuperscript{16} In all these cases, $\alpha$ and $\sigma^2_e$ are also changed appropriately such that the procedure followed to obtain the baseline estimate is the same, except in the choice of $\rho$ and $\epsilon$. From this table, it can be seen that the welfare losses are very sensitive to the elasticity of substitution. This sensitivity is not significantly affected by the values of $\rho$. The degree of autocorrelation of the firm’s productivity is less important in order to determine the welfare losses for low values of $\epsilon$.

Table 1.2: Welfare Losses (in %)

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>0.93</th>
<th>0.95</th>
<th>0.97</th>
<th>0.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon$</td>
<td>2</td>
<td>0.62</td>
<td>0.56</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.85</td>
<td>1.68</td>
<td>1.51</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>3.70</td>
<td>3.36</td>
<td>3.02</td>
</tr>
</tbody>
</table>

The second source of uncertainty is explored by analyzing how the baseline estimate changes when only the variance of the idiosyncratic productivity shock varies. Table 1.3 presents this sensitivity analysis. I consider five different values for $\sigma^2_e$ in the table. The first column corresponds to the baseline estimate. The second row in the table indicates the fraction of the observed variance of individual price changes that is explained by the model. Clearly, in the baseline calibration, this fraction is 1, which means that basically all the observed heterogeneity is due to idiosyncratic shocks. However, it could be argued that there exists some ex-ante heterogeneity that can also account for the variance of the individual price changes. Midrigan (2006) performs an analysis by using the Dominick’s database and concludes that only 20 percent of the variance of price changes could be explained by ex-ante heterogeneity. This case corresponds to the calibration in the second column. Given that ex-ante heterogeneity is not incorporated in the model, I calibrate the variance of the idiosyncratic productivity shock such that only 80 percent of the variance of individual price changes is explained by the model. It can be seen that the estimate of the welfare losses diminishes to 1.34 percent of steady state consumption in this case. This result is not so different from the one obtained with the baseline calibration.

\textsuperscript{16}The two standard error confidence interval for $\rho$, implied by Blundell and Bond’s estimation, is $[0.93,0.97]$. I also consider 0.99 in order to see what happens when the idiosyncratic productivity is very close to a unit root process. In the case of $\epsilon$, the range $[2,4]$ has been chosen based on the evidence provided by Chevallier et al (2003) using the Dominick’s database.
Table 1.3: Sensitivity Analysis for $\sigma^2_e$

<table>
<thead>
<tr>
<th>$\sigma^2_e$</th>
<th>0.0036</th>
<th>0.0029</th>
<th>0.0025</th>
<th>0.0022</th>
<th>0.0018</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Var}^{\text{pi}}{x(i)}$</td>
<td>1.00</td>
<td>0.80</td>
<td>0.70</td>
<td>0.60</td>
<td>0.50</td>
</tr>
<tr>
<td>$\text{Var}^{\text{pi}}{x(i)}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Welfare Losses (in %)</td>
<td>1.68</td>
<td>1.34</td>
<td>1.17</td>
<td>1.00</td>
<td>0.83</td>
</tr>
</tbody>
</table>

The third source of uncertainty is related with the convenience of employing the Dominick’s database to calibrate some parameters. There exist some evidence that can cast doubt on the usefulness of this database. In particular, this evidence suggests that the degree of price rigidity and the elasticity of substitution among goods are much higher than the ones estimated with the Dominick’s database. Nakamura and Steinsson (2007) report that the average price duration is between 11.6 and 13 months; while Klenow and Kristow (2007) find that the average price duration is 8.6 months. Golosov and Lucas (2007) mention that $\epsilon$ typically falls in the range between 6 and 10. This implies different values from for $\alpha$ as well. Therefore, given the conflicting evidence for $\theta$ and $\epsilon$, it is necessary to perform a sensitivity analysis to the baseline calibration by changing only these parameters and $\alpha$ accordingly. Notice that $\sigma^2_e$, in this analysis, corresponds to the one used in the baseline estimate, given that the information on the variance of individual price changes is not available in the alternative studies. I evaluate later how welfare losses change when $\sigma^2_e$ varies for different values of $\theta$ and $\epsilon$.

Figure 1.1 shows the results of this exercise. On the vertical axis, the welfare losses are measured as percentage of steady state consumption. The parameter $\theta$ is allowed to vary between 0.8 and 0.92, which implies that average price duration is between 5 and 13 months. The lines in the graph describe how welfare losses change with the degree of price rigidity for three different levels of the elasticity of substitution among goods. Two interesting results arise from this picture. First, for any degree of price rigidity, the estimation of the welfare losses is very sensitive to variations in $\epsilon$ in the range between 3 and 6; while it is not severely affected when $\epsilon$ moves between 6 and 10. Second, the whole picture reveals that the uncertainty in $\theta$ and $\epsilon$ is translated in a huge uncertainty about the welfare losses, which vary from 1.7 percent ($\theta = 0.8, \epsilon = 3$) to 4.4 percent of steady state consumption ($\theta = 0.92, \epsilon = 10$).

To conclude the robustness exercises, I quantify how movements in $\sigma^2_e$ affect the

---

\[ \text{Notice that the model establishes a relationship between } \epsilon \text{ and } \alpha. \]
estimates of welfare losses for different degrees of $\theta$ and $\epsilon$. Figures 1.2 and 1.3 present the results of these exercises. Figure 1.2 shows how welfare losses change with the degree of price rigidity for three different levels of $\sigma_\epsilon$. It can be seen that the degree of price rigidity does not significantly affect the losses for low levels of volatility of the shock ($\sigma_\epsilon = 0.03$ or less). The picture considers $\epsilon = 3$, but this result also holds if $\epsilon = 10$. Moreover, the uncertainty in $\theta$ and $\sigma_\epsilon$ also implies an enormous uncertainty about the welfare losses, which vary from 0.4 percent to 3.3 percent of steady state consumption. Figure 1.3 presents how the losses vary with the elasticity of substitution for the same levels of $\sigma_\epsilon$. This picture shows that the uncertainty in the elasticity of substitution does not matter much for low levels of $\sigma_\epsilon$. Besides, the impact of the uncertainty in $\epsilon$ is lower than the one of $\theta$. Notice that the size of the range for the welfare losses is lower in figure 1.3 than in figure 1.2.
Figure 1.2: Sensitivity Analysis for $\theta$ and $\sigma_\varepsilon$

Figure 1.3: Sensitivity Analysis for $\varepsilon$ and $\sigma_\varepsilon$
1.5 Welfare Losses with State-Dependent Pricing

In the previous section, the Calvo price-setting was used in order to estimate the welfare losses resulting from price stickiness. One weakness of this approach is that it does not incorporate the fact that it is more likely that those firms that have their prices further away from their target prices have a higher probability of changing their prices.\(^{18}\) In this section, I use a modified version of the Generalized Ss model proposed by Caballero and Engel (2007) in order to let the probability of changing prices be an increasing function of the difference between the actual price and the target price. The section is divided into three parts. First, I present the model. Then, I show how to use it in order to compute the welfare losses. Finally, I present the baseline calibration of the model and some robustness exercises.

1.5.1 The Model

Consider a firm \(i \in [0, 1]\) at time \(t\) that sets its price at \(P_t(i)\) but would choose its price at \(P^*_t(i)\) if price rigidities were momentarily removed. Let the difference between these two prices (the actual and the target prices respectively) be defined, in logarithms, as follows:

\[
x_t(i) = \log(P_t(i)) - \log(P^*_t(i))
\]

For simplicity, in this section I assume that there exists idiosyncratic productivity shocks only, which are independent across firms and across time. All these shocks have zero mean and variance \(\sigma^2\). Moreover, under the assumption that increments in productivity are approximately independent (over time for each \(i\), I can approximate \(p^*_t(i)\) by the following expression: \(^{19}\)

\[
p^*_t(i) \approx \Omega + p^f_t(i)
\]

where \(p^f_t(i)\) is the log of the frictionless price (the price that a firm would choose if price rigidities were permanently removed) and \(\Omega\) is an uninteresting constant. In

\(^{18}\)The cost of deviating from the target price is increasing with respect to the distance from this price. Therefore, adjustment is more likely when this distance is larger.

\(^{19}\)This assumption has been used in other applications by Caballero and Engel (1993a,1993b). It seems very plausible according to the empirical evidence provided by Blundell and Bond (2000). Notice that, in other words, this assumption means that the idiosyncratic productivity should be very persistent. In the limiting case, when \(\rho = 1\), increments in productivity are independent.
general, the target price would be a weighted average of current and expected future frictionless prices. When productivity is very persistent (in the limit it is a unit root), it can be shown that the expectation of the future frictionless prices is approximately the current price.\(^{20}\) Therefore, it holds that the target price is approximately given by the frictionless price.\(^{21}\)

From (1.41), it holds that:

\[
\Delta p_t^*(i) \simeq \Delta p_t^f(i) \tag{1.42}
\]

Notice from Section 2 that the frictionless price is given by:

\[
p_t^f(i) = \Theta \left[ -\log \alpha + w_t + \frac{1 - \alpha}{\alpha} (\varepsilon p_t + c_t) - \frac{1}{\alpha} a_t(i) \right] \tag{1.43}
\]

Considering that there are no aggregate shocks and that inflation is equal to zero, it can be concluded that \(w_t = \bar{w}, p_t = \bar{p}\) and \(c_t = \bar{c}\). The latter implies that the target price follows the process:

\[
\Delta p_t^*(i) \simeq \Delta p_t^f(i) = -\frac{\Theta}{\alpha} \Delta a_t(i) \tag{1.44}
\]

Given that \(\rho\) is very close to 1, \(\Delta a_t(i) \simeq \varepsilon_t(i)\). Therefore:

\[
\Delta p_t^*(i) \simeq -\frac{\Theta}{\alpha} \varepsilon_t(i) \tag{1.45}
\]

The existence of idiosyncratic productivity shocks every period implies that the target price changes every period; and, consequently the price imbalance \(x\) also varies. To complete the model, I need to specify how firms would adjust their prices after being hit by the idiosyncratic shock. I assume that the probability that a firm \(i\) changes its price is equal to \(\Lambda(x_t(i))\), where \(\Lambda(x)\) represents the adjustment hazard. In this way, I capture the most distinguishing feature of state-dependent models: the fact that the disequilibrium variable \(x_t(i)\) influences how likely it is that a firm adjusts its price in a given time period.\(^{22}\) In principle, a hazard function could take any shape. Reasonable hazard functions should be increasing with respect to the absolute value of \(x\), given

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\(^{20}\) Notice that, in the limiting case, when the productivity is a unit root, the frictionless price is a unit root.

\(^{21}\) See Caballero and Engel (1993b) for more details on this issue.

\(^{22}\) The adjustment hazard framework has been used by Caballero and Engel (1993a, 1993b, 2006, 2007). In their 2006 paper, they claim that almost any \(Ss\) model can be approximated by using the adjustment hazard framework.
that it seems unlikely that firms tolerate large deviations as much as they tolerate the small ones.\textsuperscript{23} This feature is known in the literature as the increasing hazard property (Caballero and Engel 1993a).

The timing convention of the model is as follows. At the beginning of period $t$, firm $i$ has a price imbalance of $x_{t-1}(i)$. Then, an idiosyncratic productivity shock hits the firm. This implies that $x$ moves from $x_{t-1}(i)$ to $x_{t-1}(i) + \Delta p_t^*(i)$. Finally, the adjustment hazard is applied on the price deviation after the idiosyncratic shock. With probability $\Lambda(x_{t-1}(i) + \Delta p_t^*(i))$ the firm changes its price and eliminates the price imbalance\textsuperscript{24} ($x_t(i) = 0$) and with probability $1 - \Lambda(x_{t-1}(i) + \Delta p_t^*(i))$ the firm does not change its price and keeps its price deviation in $x_{t-1}(i) + \Delta p_t^*(i)$. Therefore, for each firm $i$, the following process for $x_t(i)$ holds:

$$x_t(i) = I_t(i) \left[ x_{t-1}(i) - \frac{\Theta}{\alpha} \epsilon_t(i) \right]$$

where:

$$I_t(i) = \begin{cases} 
1 & \text{with Probability } 1 - \Lambda(x_{t-1}(i) + \Delta p_t^*(i)) \\
0 & \text{with Probability } \Lambda(x_{t-1}(i) + \Delta p_t^*(i))
\end{cases}$$

\subsection*{1.5.2 Measuring Welfare Losses}

In this case, it is convenient to combine (1.17) with (1.18) in order to write the welfare losses as:

$$L_t = -\frac{\epsilon}{2\Theta} Var_i \left\{ p_t(i) - p_t^*(i) \right\} - \left[ \frac{\alpha \sigma + 1 - \alpha + \chi}{2\alpha} \right] (\bar{y}_t - \bar{y}_t^n)^2$$

Again, these losses have two parts: one that depends on policy and one that does not. Equation (1.45) is consistent with zero inflation, which is assumed. Moreover, I assume that the standard New Keynesian Phillips curve is still a good approximation to relate output gap and inflation.\textsuperscript{25} Under this assumption, a zero inflation policy leads to a

\begin{itemize}
  \item In fact, menu costs models are consistent with increasing hazard functions.
  \item When the price imbalance is positive (negative), eliminating this imbalance implies that the firm has decreased (increased) its price.
  \item Gertler and Leahy (2006) show that the standard New Keynesian Phillips curve is consistent with state-dependent pricing. The main difference with respect to the time dependent pricing is the sensitivity of the output gap to movements in inflation. In the latter case, the sensitivity is much
\end{itemize}
zero output gap, up to a first order approximation. Consequently, the welfare losses are given by:

\[ L_t = -\frac{\epsilon}{2\Theta} \text{Var}_i \{ p_t(i) - p_t'(i) \} \]  

(1.48)

This implies that the only source of welfare losses is the dispersion of price gaps across goods. Given that the model is defined in terms of the price deviation from the desired price (or target price), it is convenient to rewrite the welfare losses as a function of the cross sectional variance of \( x_t \). By using (1.41) in (1.48), the welfare losses can be expressed as:

\[ L_t = -\frac{\epsilon}{2\Theta} \text{Var}_i \{ x_t(i) \} \]  

(1.49)

### 1.5.3 Quantifying \( \text{Var}_i \{ x_t(i) \} \)

The cross sectional variance is estimated by finding the variance of the ergodic distribution of the state variable \( x \) for a given firm \( i \). In order to simulate the process \( x \) I need to assume a functional form for \( \Lambda(x) \). Following Caballero and Engel (2006), I assume the simplest quadratic hazard they present in their paper, which is given by the following expression:

\[ \Lambda(x) = \begin{cases} \delta_p x^2, & x \leq 0 \\ \delta_n x^2, & x \geq 0 \end{cases} \]  

(1.50)

The parameters \( \epsilon \) and \( \alpha \) are the same as those in the baseline calibration in Section 4. The remaining parameters \( \delta_p, \delta_n \) and \( \sigma_t^2 \) are calibrated in two slightly different ways. In the first one, I impose \( \delta_p = \delta_n \) and calibrate the parameters such that I match the fraction of price adjustments and the standard deviation of individual price changes observed in the Dominick’s database.\(^{26}\) In the second one, I remove the restriction \( \delta_p = \delta_n \), such that I can match additionally the fraction of positive price changes.\(^{27}\)

\(^{26}\) Notice that the fraction of price adjustments \( f \) is approximately related to the average price duration \( d \) by the following expression: \( f \approx d^{-1} \)

\(^{27}\) Of course, there exists other dimensions of the data that could be matched. It would be interesting to see how they affect our understanding of the welfare losses of price rigidities.
Table 1.4: Calibration of the Hazard Models

<table>
<thead>
<tr>
<th>Statistics (In %)</th>
<th>Data</th>
<th>Models</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Fraction of Price Adjustments</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Standard Deviation of Price Changes</td>
<td>4.6</td>
<td>4.6</td>
</tr>
<tr>
<td>Fraction of Positive Price Changes</td>
<td>13</td>
<td>10</td>
</tr>
<tr>
<td>Mean of Price Adjustments</td>
<td>7.7</td>
<td>9.8</td>
</tr>
<tr>
<td>Mean of Price Increase</td>
<td>9.8</td>
<td>6.8</td>
</tr>
<tr>
<td>Mean of Price Decrease</td>
<td>9.8</td>
<td>13.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_p$</td>
<td>50</td>
<td>205</td>
<td>-</td>
</tr>
<tr>
<td>$\delta_n$</td>
<td>50</td>
<td>15</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>0.047</td>
<td>0.047</td>
<td>0.047</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Welfare Losses (In %)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.28</td>
</tr>
</tbody>
</table>

Table 1.4 summarizes the results of the simulations. Model 1 reports the symmetric quadratic hazard model. The model fails to match the mean of the absolute value of individual price changes (it overestimates it). This is consistent with the failure of menu costs models to generate many small price changes. By using this model, the welfare losses are 0.3 percent of steady state consumption. Model 2 reports the asymmetric quadratic hazard model. Notice that $\delta_p$ is much higher than $\delta_n$ in order to capture the fact that price increases occur more frequently than price reductions. Moreover, the asymmetric hazard allows for matching the fraction of price increases, which is higher than the one of price decreases. Like Model 1, it predicts an absolute value of price changes that is much higher than the one observed in the data. With this model, the welfare losses are 0.4 percent of steady state consumption. Model 3 reports the constant-hazard model (Calvo 1983). This model has been calibrated so that $\Lambda(x) = 1 - \theta = 0.2$. In contrast to the previous two models, it matches fairly well the mean of the absolute value of price adjustments. However, it does not capture (by construction) the higher probability of a price increase. By using this model, the welfare losses are much higher (1.3 percent of steady state consumption). Finally, notice that the welfare losses estimated by using model 3 are a very good

---

28 As an example, see the menu cost model developed in Golosov and Lucas (2007).
approximation to the ones estimated by using the complete structure of the Calvo model under the assumption that the idiosyncratic productivity is highly persistent. In fact, when using the adjustment hazard approach, the estimated losses are 1.33 percent; whereas when using the model of section 4 with $\rho = 0.99$, these losses are 1.35 percent.

1.5.3.1 Robustness Exercise

In the previous calibration exercises, there exist two important sources of uncertainty. Conditional on the representativity of the Dominick’s database, the first source is the estimation of the elasticity of substitution. In fact, estimates of this parameter based on the use of the Dominick’s database are in the range 2-4. This implies, after following the same type of procedure performed in table 3, that the welfare losses are between 0.1 and 0.6 percent of the steady state consumption if model 1, with $\delta_{p} = \delta_{n}$, is used. When model 2 is considered to perform this robustness analysis, the range for the welfare losses is 0.1-0.7 percent of the steady state consumption.

The second source of uncertainty is related with the convenience of using the Dominick’s database. As mentioned before, other studies present estimates of the elasticity of substitution among goods and the average price duration that are much higher than those obtained by using this database. For this reason, I also perform some additional calibration exercises of the welfare losses that consider: a) lower frequency of price adjustments (average price duration equal to 13 months instead of 5 months) b) three different estimates for the elasticity of substitution among goods and c) two different values for $\sigma_{\varepsilon}$. All these exercises are performed by calibrating model 2 (with $\delta_{p} \neq \delta_{n}$) such that the fraction of price adjustments and the fraction of positive price changes are the same as in the data on individual price changes due to Nakamura and Steinsson (2007). Results are presented in Table 1.5.

Several interesting results emerge from this robustness exercise. First, the impact of the degree of price rigidity on the welfare losses is crucially affected by the size of the standard deviation of the idiosyncratic productivity shock. When $\sigma_{\varepsilon} = 0.03$, these losses are between 0.4 and 0.6 percent of the steady state consumption; while they are between 1.8 and 2.3 percent when the standard deviation of the idiosyncratic productivity shock is doubled. In both cases, the degree of price rigidity is the same. Second, based on the ability of the different calibrations of the model to match the data on the size of individual price changes, it is difficult to take a position on the amount
of the welfare losses. In particular, calibrations 1 and 5 do a great job in matching the absolute value of the median of price adjustments and the median of price increases but yield completely different welfare losses (0.4 versus 2.3 percent). Independent evidence on the variance of the idiosyncratic productivity shocks is necessary in order to obtain a more precise estimate of the welfare losses. Third, given \( \sigma_z \), the impact of varying \( \epsilon \) on welfare is not very important. This result holds because in order to match the fraction of price changes and the fraction of positive adjustments, an increase in \( \epsilon \) implies a reduction in the variance of the price imbalance \( x_t \). Fourth, this exercise shows clearly that a lower frequency of price adjustments would not necessarily imply significantly more welfare losses. If we compare the result obtained by using calibration 1 with the baseline estimate of 0.37 percent, we see that the difference between the two is small. This is because it is plausible that economies with lower frequency of price adjustments are economies with smaller idiosyncratic productivity shocks. Fifth, the model does not fit the disaggregated data on prices when \( \epsilon = 10 \). A higher variance of the idiosyncratic productivity shock will solve this problem. In general, when choosing any value for the elasticity of demand higher than 6, the model would require a higher \( \sigma_z \) in order to match adequately the data on individual price changes.

<table>
<thead>
<tr>
<th>Table 1.5: Robustness Exercise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>STATISTICS (In %)</strong></td>
</tr>
<tr>
<td>Frac. of Price Adj.</td>
</tr>
<tr>
<td>St. Dev. of Price Ch.</td>
</tr>
<tr>
<td>Frac. of Pos. Adj.</td>
</tr>
<tr>
<td>Median of Price Adj.</td>
</tr>
<tr>
<td>Median of Price Inc</td>
</tr>
<tr>
<td>Median of Price Decr</td>
</tr>
<tr>
<td><strong>PARAMETERS</strong></td>
</tr>
<tr>
<td>( \delta_p )</td>
</tr>
<tr>
<td>( \delta_n )</td>
</tr>
<tr>
<td>( \epsilon )</td>
</tr>
<tr>
<td>( \sigma_z )</td>
</tr>
<tr>
<td>WL (In %)</td>
</tr>
</tbody>
</table>
1.6 Concluding Remarks

I have presented a new perspective on the importance of the study of price rigidities. Traditionally, these rigidities have been analyzed in order to understand the real effects of monetary policy, inflation persistence or the design of optimal monetary policy. In this sense, price stickiness has been an important element in monetary policy analysis. Here I provide an additional motivation to pay attention to price rigidities. In particular, I emphasize that price stickiness is relevant because it can cause important welfare losses, even in economies with price stability. This conclusion has been obtained after considering idiosyncratic productivity shocks in the welfare analysis of price rigidities.

The results of this paper also allow for the identification of two aspects of price rigidities that are relevant from a welfare point of view. First, they highlight how crucial it is to understand why firms would decide in favor of state dependent behavior or time dependent behavior. In fact, this study has shown that the welfare losses are significantly higher with time dependent pricing. Secondly, they emphasize the importance of investigating the determinants of the frequency of price adjustments. According to my results, this variable is a key factor in determining the size of welfare losses. Research on these two aspects would also be helpful in order to see if there exist policies that can help to reduce the negative impact of price rigidities.

\footnote{Alvarez (2007) develops an econometric analysis in this line of research. He estimates a multinomial logit model with Spanish Survey data in order to explain the relationship between the use of time dependent pricing strategies and industry characteristics. He finds that time dependent behavior is associated with higher labor intensity in the production, lower degree of competition and large firms.}

\footnote{The other factor is the variance of the idiosyncratic productivity shocks. Clearly, this factor is exogenous.}
Chapter 2

Testing for Rule of Thumb Price-Setting

2.1 Introduction

Much of the recent literature on monetary policy uses the New Keynesian Phillips curve (NKPC) to describe the aggregate supply block of the economy. Many authors, however, have criticized the NKPC on the basis that it cannot explain the degree of inertia observed in inflation. In particular, Fuhrer and Moore (1995) show that the NKPC predicts a degree of persistence in inflation that is much lower than the one detected in the data. To overcome this empirical deficiency, three alternative strategies have been developed. First, the work of Galí and Gertler (1999) incorporates backward-looking behavior by assuming that a fraction of firms follows a simple rule of thumb. Alternatively, Smets and Wouters (2003) and Christiano et. al. (2005) assume that firms partially index prices to past inflation when not re-optimizing. Either through rule of thumb behavior or indexation, the modified inflation equation, known as the hybrid NKPC, can rationalize a lagged inflation term and account for persistence in inflation. The third strategy is the one followed by Mankiw and Reis (2002). They build the sticky information model in which firms are assumed to update their information sets infrequently, due to the presence of costs of collecting and processing information.

All the previous strategies have been evaluated by determining how well they match
macroeconomic data. The main conclusion is that all these models are useful in order to replicate inflation dynamics with plausible parameters. However, how good are these strategies in matching the microeconomic evidence on price-setting? According to Alvarez (2007), all these models fail to match some of this evidence. Models with indexation as well as sticky information models fail in matching infrequent adjustment, a decreasing hazard rate, annual spikes in hazard rate, and heterogeneity in price adjustment. Instead, the model of Galí and Gertler (1999) only fails in capturing a decreasing hazard rate and annual spikes in hazard rate. Therefore, the latter model seems to be much in accord with the microeconomic evidence.

In this chapter, I propose a novel methodology in order to evaluate the quantitative importance of the rule of thumb behavior proposed by Galí and Gertler (1999). By using their hybrid model, I derive a structural relationship over time between the cross sectional variance of individual price changes and aggregate inflation. There are four important features of this relation that make it more attractive than the hybrid NKPC in order to identify rule of thumb behavior. First, the parameters that appear in the equation I derive are only those related with the nature of price-setting: the one that measures the degree of price stickiness and the one that measures the fraction of backward-looking firms. Both of them are identified directly from estimates of that equation. The latter means that the estimates of these parameters are not affected by how real rigidities are modeled and calibrated, as it is the case when they are identified by estimating the hybrid NKPC. Second, the variables that appear in the relationship I propose are predetermined in period $t$. This implies that I do not need an assumption on how expectations about the future are formed. Instead, the estimation of the hybrid NKPC requires to take a position on this issue, given that expected inflation appears in that relation. Third, estimating the equation I propose is not

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1See Galí and Gertler (1999), Galí et al. (2001), Smets and Wouters (2003), Christiano et. al. (2005), and Mankiw and Reis (2002).
2The sticky information model additionally fails in not allowing the presence of non optimal price setters.
3Evaluating the plausibility of rule of thumb firms is not only useful to study inflation persistence but also for monetary policy analysis. Steinsson (2003) shows that optimal monetary policy change in important ways if some firms obey a rule of thumb.
subject to the criticism of Rudd and Whelan (2005), who claim that fitting the hybrid NKPC may be biased in favor of finding a significant role for forward-looking behavior. Fourth, the definition of the variables I use do not change for alternative assumptions on the degree of openness, the form of the production function and the way how real rigidities are introduced. Instead, the construction of measures of real marginal costs in the hybrid NKPC is very sensitive to the previous assumptions.6

I estimate the derived structural relationship with Spanish monthly data covering the period 1993-2001. The estimation technique is the Generalized Method of Moments (GMM). Several interesting results stand out. First, the structural relationship proposed in this paper fits the data well. Second, the backward-looking price setting is statistically significant and quantitatively important. Third, the estimates of the fraction of rule of thumb firms are very close to those found by Galí and Lopez Salido (2001) and Benigno and Lopez Salido (2006) by estimating the Spanish hybrid NKPC. Fourth, the degree of price stickiness implied by the estimates is consistent with the average price duration estimated using only disaggregated data. Fifth, the estimates imply that forward-looking behavior is only slightly dominant in shaping inflation dynamics in Spain.

The rest of the chapter is organized as follows. In Section 2.2, I present the model and the basic assumptions. In Section 2.3, I derive analytically the dynamic relationship between the cross sectional variance of individual price changes and aggregate inflation. In Section 2.4, I expose the methodology and the econometric specification used in order to estimate the degree of backward-lookingness. The estimates and related comments are also presented in this section. In Section 2.5, I present a detailed comparison of the methodology that I propose and the one developed by Galí and Gertler (1999), who estimate the hybrid NKPC. Conclusions are given in Section 2.6.

2.2 The New Keynesian Hybrid Model

In this section I briefly describe the hybrid model developed by Galí and Gertler (1999). This model is used in the next section in order to derive the dynamic structural reduced form of the hybrid NKPC.

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6Galí and Lopez Salido (2001) show that the definition of real marginal cost can change with the assumptions made on the production function and on the degree of openness of the economy. Additionally, Thomas (2008) finds that the measure of real marginal cost differs from the standard one when real rigidities are introduced considering matching frictions.
tionship between the cross sectional variance of individual price changes and aggregate inflation.

2.2.1 Households

The household purchases differentiated goods and combines them into composite goods using a Dixit-Stiglitz aggregator:

$$C_t = \left( \int_0^1 C_t(i)^{(\epsilon-1)/\epsilon} di \right)^{\epsilon/(\epsilon-1)}$$  \hspace{1cm} (2.1)

where $C_t(i)$ is the differentiated good of type $i$ and $\epsilon > 1$ is the constant elasticity of substitution among goods. The households maximize the index (2.1) given the total cost of all differentiated goods and their nominal prices $P_t(i)$. Then, the demand for each good is given by:

$$C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t$$  \hspace{1cm} (2.2)

where $P_t$ is the aggregate price level and is defined as follows:

$$P_t = \left( \int_0^1 P_t(i)^{1-\epsilon} di \right)^{1/(1-\epsilon)}$$  \hspace{1cm} (2.3)

2.2.2 Firms

In the model, it is assumed a continuum of firms indexed by $i \in [0, 1]$. Each firm is a monopolistic competitor and produces a differentiated good $Y_t(i)$ that sells at price $P_t(i)$. Firms set prices as in the sticky price model of Calvo (1983). In this model, during each period, a fraction of firms $(1 - \theta)$ are allowed randomly to change the prices; whereas the other fraction $\theta$ do not change. From those firms resetting prices, only a fraction $(1 - \omega)$ resets price optimally, as in the standard Calvo model. The
remaining fraction $\omega$ chooses the (log) price $p_t^b$ according to the simple rule of thumb:

$$p_t^b = p_{t-1}^* + \pi_{t-1}$$

(2.4)

where $p_{t-1}^*$ is the (log) of the average reset price in $t-1$ (across both backward and forward-looking firms) and $\pi_{t-1}$ is inflation in period $t-1$. Galí and Gertler (1999) point out two appealing features of this rule. First, there are no persistent deviations between the rule and the optimal behavior as long as inflation is stationary. Second, the rule implicitly incorporates information about the future, given that $p_{t-1}^*$ is partly determined by forward-looking firms.

### 2.2.3 Aggregate Price Level Dynamics

After using the law of large numbers and log-linearizing the aggregate price level around a zero inflation steady state, the following expression for the (log) aggregate price level $p_t$ is obtained:

$$p_t = \theta p_{t-1} + (1 - \theta)p_t^b$$

(2.5)

The (log) index for newly set prices is given by the following expression:

$$p_t^* = \omega p_t^b + (1 - \omega)p_t^f$$

(2.6)

where $p_t^f$ is the optimal price chosen by forward-looking firms at period $t$. Notice that all firms that reoptimize in period $t$ choose the same value $p_t^f$, given that there are no firm specific state variables.

### 2.2.4 The Hybrid NKPC

Although the hybrid NKPC is not necessary to derive the relationship between the cross sectional variance of individual price changes and aggregate inflation, I present it in this section for two reasons. First, it will be useful to discuss the implications of my estimates for $\theta$ and $\omega$ on the dynamics of inflation. Second, it will be helpful in explaining the main differences between my estimation strategy of $\theta$ and $\omega$ and the one performed by previous studies in which the hybrid NKPC is estimated.7

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7Galí and Gertler (1999), Galí et. al (2001), Galí and Lopez Salido (2001), and Benigno and Lopez Salido (2006) use the hybrid NKPC derived with rule of thumb behavior to estimate the fraction of
Given that the analysis performed in the empirical section is with monthly data, for simplicity I will focus on the case in which $\beta = 1$.\(^8\) Galí et al. (2001) show that the hybrid NKPC in this case is given by the following expression:

$$\pi_t = \lambda_n \lambda_r \bar{mc}_t + \gamma_b \pi_{t-1} + \gamma_f E_t \{\pi_{t+1}\} + \varepsilon_t$$  \hfill (2.7)

with

$$\lambda_n = \frac{(1 - \omega)(1 - \theta)^2}{\theta + \omega}, \quad \lambda_r = \frac{1 - \alpha}{1 + \alpha(\epsilon - 1)}, \quad \gamma_b = \frac{\omega}{\theta + \omega}, \quad \gamma_f = \frac{\theta}{\theta + \omega}$$

where $\alpha$ measures the curvature of the production function of the firm, which is given by $Y_t(i) = A_t N_t(i)^{1-\alpha}$, $\bar{mc}_t$ is average real marginal cost (in percent deviations from its steady state level) and $\varepsilon_t$ is an error term that may arise from either measurement errors or shocks to the desired markup.

Note that the slope coefficient on real marginal cost depends on two different groups of parameters. The first group, given by $\theta$ and $\omega$, are related to the nature of the price setting. Their impact on real marginal cost is given by $\lambda_n$, which measures the degree of nominal rigidities. The second group, composed by $\alpha$ and $\epsilon$, are associated with real factors of the economy: the structure of the production function and of demand. The effect of these parameters on the real marginal cost is determined by $\lambda_r$, which quantifies the degree of "real rigidities". In this case, these rigidities arise from assuming decreasing returns to scale in labor ($\alpha < 1$).\(^9\)

Finally, the coefficients $\gamma_b$ and $\gamma_f$ capture the influence of backward and forward-looking behavior on inflation dynamics. Notice that these coefficients depend only on $\theta$ and $\omega$.\(^10\) This implies that the expressions for $\gamma_b$ and $\gamma_f$ are not affected by the assumptions made on the production function and on demand.

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\(^8\) A quarterly discount factor of 0.99 is equivalent to a monthly discount factor of 0.997, which is very close to 1.

\(^9\) There are alternative ways to generate real rigidities. See Woodford (2003), Christiano et al. (2004) or Thomas (2008).

\(^10\) When the discount factor is lower than one, this parameter also affects $\gamma_b$ and $\gamma_f$. 

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2.3 Relationship Between the Cross Sectional Variance of Individual Price Changes and Aggregate Inflation

In this section, I show that the previous model implies a dynamic structural relationship between the cross sectional variance of individual price changes and aggregate inflation.

**Proposition:** In the hybrid New Keynesian model, up to a second order approximation around a zero inflation steady state, the cross sectional variance of individual price changes evolves over time according to:

\[ \text{Var}_i \{ \pi_t(i) \} = \theta \text{Var}_i \{ \pi_{t-1}(i) \} + f(\pi_t, \pi_{t-1}) + (1 - 2\theta)f(\pi_{t-1}, \pi_{t-2}) \]  

(2.8)

where \( f(\pi_t, \pi_{t-1}) \) is given by:

\[ f(\pi_t, \pi_{t-1}) = \frac{\theta}{1 - \theta} \pi_t^2 + \frac{\omega}{(1 - \theta)(1 - \omega)} (\pi_t - \pi_{t-1})^2 \]  

(2.9)

**Proof:** First, notice that the cross sectional variance of individual prices evolves according to:

\[ \text{Var}_i \{ p_t(i) \} = \theta \text{Var}_i \{ p_{t-1}(i) \} + f(\pi_t, \pi_{t-1}) \]  

(2.10)

Moreover, we know that the cross sectional variance of individual price changes is given by:

\[ \text{Var}_i \{ \pi_t(i) \} = \text{Var}_i \{ p_t(i) \} - 2\text{Cov}_i \{ p_t(i), p_{t-1}(i) \} + \text{Var}_i \{ p_{t-1}(i) \} \]  

(2.11)

Using the fact that in the hybrid model, \( \text{Cov}_i \{ p_t(i), p_{t-1}(i) \} = \theta \text{Var}_i \{ p_{t-1}(i) \} \), the previous expression can be expressed as:

\[ \text{Var}_i \{ p_{t-1}(i) \} = \frac{\text{Var}_i \{ \pi_t(i) \} - \theta \text{Var}_i \{ p_t(i) \}}{1 - 2\theta} \]  

(2.12)

\(^{11}\)See Steinsson(2003) for a formal proof of (2.10).
By plugging (2.12) into (2.10), the cross sectional variance of individual prices evolves according to:

\[ Var_i \{p_t(i)\} = \theta Var_i \{\pi_t(i)\} + \frac{1 - 2\theta}{1 - \theta} f(\pi_t, \pi_{t-1}) \tag{2.13} \]

Finally, by using (2.13) evaluated in periods \(t\) and \(t-1\); and plugging them into (2.10), we get (2.8).

### 2.4 Empirical Evidence

This part contains two subsections. In the first one, I describe the econometric specification used to estimate equation (2.8) by applying GMM. In the second one, I present the data and estimates of the model.

#### 2.4.1 Econometric Specification

In order to perform the GMM technique, an orthogonality condition should be inferred from the model developed in the previous section. In this particular case, the orthogonality condition comes from equation (2.8) and arises from allowing measurement error term in this equation. There are two reasons to justify this error term. First, there exists an approximation error given that (2.8) holds up to second order. Moreover, an error term can be allowed because there can exist measurement errors in the cross sectional variance of individual price changes or in aggregate inflation. I assume that this measurement error at period \(t\) is not correlated with earlier information. Therefore, the following orthogonality condition can be established:

\[ E_t \{ [Var_i \{\pi_t(i)\} - \theta Var_i \{\pi_t(i)\} - f(\pi_t, \pi_{t-1}) - (1 - 2\theta)f(\pi_{t-1}, \pi_{t-2})] z_{t-1} \} = 0 \tag{2.14} \]

where \(f(\pi_t, \pi_{t-1})\) is given by (2.9) and \(z_{t-1}\) denotes a vector of variables dated at period \(t - 1\) and earlier.
2.4.2 Data and Estimates

The data that I use is Spanish monthly data running from February 1993 through December 2001. In order to measure the evolution over time of the cross sectional variance of individual price changes, a large panel database containing around 1.1 million price records has been used.12 The dataset includes product categories that cover around 70 percent of the expenditures on the CPI basket over the whole sample period.13 Inflation is measured by the percentage change in monthly CPI.

Table 2.1 presents the GMM estimation of the parameters $\theta$ and $\omega$, as well as the average price duration implied by $\theta$ and the coefficients $\gamma_b$ and $\gamma_f$ that help to measure the relative importance of backward versus forward-looking behavior. The last column of the table presents the p-value for the Hansen’s J statistic of overidentifying restrictions. The results are presented for two different set of instruments.14 Standard errors (with a Newey West correction) for all the estimates are reported in brackets.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\omega$</th>
<th>$D$</th>
<th>$\gamma_b$</th>
<th>$\gamma_f$</th>
<th>J test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set 1</td>
<td>0.935</td>
<td>0.835</td>
<td>15.402</td>
<td>0.472</td>
<td>0.528</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.064)</td>
<td>(4.008)</td>
<td>(0.023)</td>
<td>(0.023)</td>
</tr>
<tr>
<td>Set 2</td>
<td>0.936</td>
<td>0.732</td>
<td>15.748</td>
<td>0.439</td>
<td>0.561</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.089)</td>
<td>(3.570)</td>
<td>(0.033)</td>
<td>(0.033)</td>
</tr>
</tbody>
</table>

Note: Standard errors shown in brackets.

Several interesting results arise from these estimations, which are robust to the set of instruments. First, the point estimates of the fraction of rule of thumb firms are high

12The computation of the cross sectional variance has been done by Luis Alvarez and Ignacio Hernando from Bank of Spain.
13More details on this database can be found in Alvarez and Hernando (2004).
14Set 1 includes the cross sectional variance of individual price changes and aggregate inflation from $t - 1$ to $t - 8$. Set 2 includes the same variables but just until $t - 5$. 

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and statistically different from zero. Second, these estimates are very close to those found by Galí and Lopez Salido (2001) and Benigno and Lopez Salido (2006) for the Spanish case.\textsuperscript{15} This is a very interesting result, given that the methodology applied in this study is completely different from the one followed by the previous authors. Third, the point estimates of the average price duration are very close to the ones obtained using the disaggregated data.\textsuperscript{16} Alvarez and Hernando (2004) find that the average price duration for the period 1993-2001 is 15.7 months if censored and uncensored spells are considered; whereas it is 14.7 months when censored spells are discarded. Fourth, the estimates of $\theta$ and $\omega$ implies that forward-looking behavior is only slightly more important than the backward-looking one in order to explain inflation dynamics in Spain. Fifth, the validity of all the regressions is confirmed by the p-value for the Hansen’s J statistic of overidentifying restrictions with a significance level of 5 percent.

2.5 Comparison with Galí and Gertler (1999)

In this section, I compare my methodology to the one proposed by Galí and Gertler (1999).\textsuperscript{17} Basically, these authors propose to estimate the structural parameters $\theta$ and $\omega$ by fitting the hybrid NKPC using GMM. In particular, they use the following orthogonality condition:

$$E_t \left\{ \left[ \pi_t - \lambda_n \lambda_r \bar{m} c_t - \gamma_b \pi_{t-1} - \gamma_f \pi_{t+1} \right] z_t \right\} = 0 \quad (2.15)$$

where $\lambda_n$, $\lambda_r$, $\gamma_b$, $\gamma_f$ are given by the expressions presented in Section 2. Condition (2.15) follows from the fact that the expectational error should be unforecastable with information dated in period $t$ and earlier under rational expectations in inflation.

There are four important differences between the procedure I present and the one developed by Galí and Gertler. First, the way how the orthogonality condition is derived. Notice that Galí and Gertler assume rational expectations in order to infer (2.15). This is the reason why they can use $\pi_{t+1}$ in their estimation. However, if

\textsuperscript{15}The point estimates of this fraction obtained by Gali and Lopez Salido (2001) are between 0.58 and 0.74; whereas those obtained by Benigno and Lopez Salido (2006) are between 0.67 and 0.80. In both cases, they use quarterly data. However, in the first case, they use data from 1980 to 1998; while in the second case the sample goes from 1970 to 1997.

\textsuperscript{16}Notice that the average price duration is measured by computing $1 - \sigma_t^{-1}$.

\textsuperscript{17}Their methodology has been used by different studies like the ones by Gali et al (2001), Gali and Lopez Salido (2001), Benigno and Lopez Salido (2006), among others.

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rational expectations does not hold, then (2.15) would be incorrect. Instead, the condition that is used in this study does not contain expectations about the future. For this reason, it is assumed that there can be measurement errors in the observed variables that are uncorrelated with past information. The latter only means that people elaborating statistics are smart enough such that they do not make systematic mistakes. Therefore, my procedure is consistent with any learning scheme that could be used in order to forecast inflation.

The second difference is related with the parameters that appear in the orthogonality conditions and their identification. Condition (2.15) contains four structural parameters: $\theta$, $\omega$, $\alpha$ and $\epsilon$. Only two of them can be identified. Galí and Gertler calibrate $\alpha$ and $\epsilon$, which means calibrating the degree of real rigidity, in order to identify $\theta$ and $\omega$. Therefore, their results are conditional on their identification assumption on the degree of real rigidity. In general, it can be said that their estimation procedure is sensitive to the mechanism that induces real rigidities. In their proposal, the existence of real rigidities arises from the departure of constant returns to scale in labor. However, there are alternative ways to generate real rigidities. Woodford (2003) proposes to consider segmented labor markets. Christiano et al. (2004) propose firm specific capital. Thomas (2008), in a recent paper, proposes search frictions to induce real rigidities. In all these cases, the slope of the hybrid NKPC is different from the one used by Galí and Gertler. Therefore, how real rigidities arise and how the parameters that determine them are calibrated matter for the identification and estimation of $\theta$ and $\omega$. On the contrary, my procedure does not require any assumption about the nature or importance of real rigidities. In my view, this is a great advantage, given the uncertainty and absence of consensus on how to model and calibrate real rigidities.

The third difference is related with the power of the estimation procedure to detect backward-looking behavior. Rudd and Whelan (2005) criticize the estimation procedure of Galí and Gertler because it is very likely that their estimation is biased in favor of finding a significant role of the forward-looking behavior. The reason for this bias is that it is very plausible that the instrument set contains variables that directly cause inflation but are omitted from the hybrid NKPC specification. If this is the case, the estimation is biased in favor of finding a significant role of the forward-looking behavior.

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18 Adam and Padula (2003) provide some evidence that supports that inflation expectations are not rational. They propose to use survey expectations to measure $E_t \pi_{t+1}$ instead of $\pi_{t+1}$ and to formalize the orthogonality condition by allowing a measurement error.

19 Moreover, in the case of Thomas (2008), the measure of real marginal cost is also different from the one used by Galí and Gertler (1999).
case, it follows that the coefficient next to $\pi_{t+1}$ is going to capture the effect of the omitted variables. On the other hand, the procedure I propose does not suffer this shortcoming. In fact, a priori there is no reason to believe that my estimation can favor forward or backward-looking price-setting.

The fourth difference is related with the data that appear in the orthogonality conditions. The definition of the variables that I employ is robust to alternative assumptions on the degree of openness, the form of the production function and the way how real rigidities are introduced. Instead, the definition of real marginal cost can change with the previous assumptions. Galí and Lopez Salido (2001) show that the definition of real marginal cost is affected by assumptions on the production function and on the degree of openness of the economy. Additionally, Thomas (2008) finds that the measure of real marginal cost is different from the standard one when matching frictions are introduced to generate real rigidities.

### 2.6 Concluding Remarks

The identification of backward-looking behavior to set prices is an important issue in the design of monetary policy because it helps to explain inflation persistence and the costs of disinflation processes. In this chapter, I evaluate the plausibility of the existence of rule of thumb firms to account for backward-lookingness. Previous studies have explored this issue by estimating the hybrid NKPC developed by Galí and Gertler (1999). Instead, by using their model, I derive a dynamic structural relationship between the cross sectional variance of individual price changes and aggregate inflation. I show that the identification of rule of thumb behavior by using this relation does not require assumptions on rationality of expectations, the degree of real rigidities, production functions and openness of the economy. Moreover, it seems that fitting this relation has more power than estimating the hybrid NKPC in order to detect backward-looking behavior.

By using Spanish data, I estimate the structural relationship that I derive in this chapter. I find that the fraction of rule of thumb firms is statistically significant and quantitatively important. The point estimates of this fraction are similar to those found by previous studies estimating the Spanish hybrid NKPC. From these estimates, it is concluded that backward-looking behavior is almost as important as the forward-looking one in describing Spanish inflation.
Finally, it is worth mentioning that there are two interesting extensions of the study presented in this chapter. The first one would incorporate the structural relationship between the cross sectional variance of individual price changes and aggregate inflation in a second order approximation of a DSGE model that uses the hybrid NKPC with rule of thumb firms. This would allow to identify parameters that affect the degree of real rigidities. The second extension consists in evaluating different rules of thumb. In this sense, it would be interesting to evaluate the one proposed by Nunes (2005). He proposes that backward-looking firms use survey expectations instead of past inflation in order to set prices. I plan to explore these extensions in future research.

Chapter 3

Resurrecting the Role of Real Money Balance Effects

3.1 Introduction

The standard New Keynesian model, commonly used in discussions about monetary policy analysis, assigns no role to money in the monetary transmission mechanism. In fact, the standard model is a cashless one. The widespread use of this type of models is justified by Woodford (2003) and Ireland (2004). Woodford argues that money does not play an important role in determining the equilibrium of the economic variables because the central bank controls interest rates (without responding to money) and real money balance effects are not quantitatively important. Woodford evaluates the size of these effects to be very low after calibrating the money in utility function (MIU) model for the U.S economy. In addition, Ireland provides econometric estimates of a bigger structural model, by using Maximum Likelihood (ML), that support Woodford’s position about the negligible size of real money balance effects.

The previous conclusions contrast with some empirical reduced form evidence. Meltzer (2001) shows that money is a significant determinant of consumption growth.

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1Real money balance effects exist when consumption (or aggregate demand) is directly influenced by the level of real money balances held by private sector, for reasons that are independent of movements in the interest rates that ordinarily accompany a change in real money supply. In the money in utility function framework, real money balance effects take place when utility is non-separable in consumption and real money.

2McCallum (2000) performs a different calibration exercise that leads to the same conclusion.
in the US, controlling for the short term real interest rate, its lags and lagged values of consumption growth. Nelson (2002) and Hafer et al. (2007) find the same results as Meltzer for US but using output gap instead of consumption.

Given that the structural analysis and the reduced form evidence point in two different directions, in this chapter, I revisit the importance of real money balance effects by using a structural estimation methodology different from the ones proposed by Woodford and Ireland. I provide empirical evidence showing that these effects are still quantitatively important in United States but lower than they were in the early postwar period. Therefore, real money balances enter directly in the aggregate demand; which implies that the specification of money demand is still relevant in order to determine inflation and output. Moreover, I show and analyze three additional important implications of my empirical evidence on real money balance effects. First, the modestly procyclical real wage response to a monetary policy shock and the supply side effects of monetary policy can be explained by the existence of quantitatively important real money balance effects in a model with sticky prices and flexible wages. Second, the design of the optimal monetary policy should imply much higher volatility of output and much lower volatility of the interest rate when there are real money balance effects. Third, the diminishment in the size of real money balance effects, which occurred in the beginning of the 1980’s, can explain a significant reduction in the volatility of output and inflation. This would support the hypothesis that financial innovation explains part of the Great Moderation in U.S.

The remainder of the chapter is organized as follows. Section 3.2 presents the MIU model and describes the equilibrium conditions that determine the Euler equation and the money demand that are used in the empirical part. The model allows, but does not require, that real money balance effects arise from non-separable utility in consumption and real money. It is shown that the size of these effects is given by the elasticity of marginal utility of consumption with respect to real money divided by the coefficient of risk aversion. Section 3.3 presents the methodology and the econometric specification used in order to estimate the parameters that measure the magnitude of real money balance effects. The estimates, robustness exercises and a comparison between my estimation procedure and those of Woodford and Ireland are also presented in this

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3 There exist other explanations for these stylized facts. The most common explanation for the modestly procyclical real wage response after a monetary policy shock is the existence of sticky prices and sticky wages. The supply side effects of monetary policy are commonly explained with the cost channel of monetary transmission.
section. Section 3.4 contains the analysis of the three additional implications of my empirical evidence. Section 3.5 concludes.

### 3.2 Money in Utility Function Model

In this section, I present briefly a slightly modified version of this model developed by Woodford (2003). The main goal of this part is to show the log-linearized representation of the Euler equation (IS curve or aggregate demand) and money demand (LM curve) that are going to be used in the empirical part.

The representative household seeks to maximize the following expected discounted utility:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t [U(C_t, M_t/P_t; \xi_t) - V(H_t)] \right\} \tag{3.1}$$

where $0 < \beta < 1$ is a discount factor, $C_t$ is the level of consumption of the economy’s single good, $M_t$ is the household’s end-of-period money balances, $P_t$ is the price of the single good in terms of money in period $t$, $\xi_t$ is a disturbance in the liquidity services provided by money and $H_t$ is the quantity of labor supplied (measured in hours). The period indirect utility is composed by the sum of two functions: $U$ and $V$. The function $U$ is concave and strictly increasing in each of the arguments (consumption and real money balances). All these assumptions are consistent with the microfounded transaction cost model and shopping time model. Moreover, utility is allowed to be non separable in consumption and real money balances. However, the sign of $U_{cm}$ is not assumed because the previous microfounded models do not provide it. If $U_{cm} = 0$, then utility is separable in consumption and money; and, consequently, there are no real money balance effects. It is also assumed that disturbances in liquidity services affect both the marginal utility of consumption and money ($U_{c\xi} \neq 0$, $U_{m\xi} \neq 0$). Finally, the function $V$ is an increasing and convex function that represents the disutility of labor.

Notice that it is assumed that the indirect utility function is separable with respect to labor.\footnote{This assumption is consistent with a microfounded transactions costs model but it is not with a shopping time model.} This means that marginal utility of consumption and real money balances do not depend on labor. Therefore, as it is shown later, labor affects neither the Euler
equation nor the money demand equation directly.

The maximization of the expected utility is subject to an intertemporal budget constraint of the form:

\[
\sum_{t=0}^{\infty} E_0 Q_{0,t} [P_t C_t + \Delta_t M_t] \leq A_0 + \sum_{t=0}^{\infty} E_0 Q_{0,t} [W_t H_t - T_t] \tag{3.2}
\]

where \( \Delta_t = \frac{i_t - i^m_t}{1 + i^m_t} \), \( i_t \) is the nominal interest rate paid on a riskless one period bond, \( i^m_t \) is the nominal interest rate paid on money balances held at the end of period \( t \), \( A_0 \) is the initial level of wealth, \( W_t \) is the nominal wage per hour worked and \( T_t \) represents net (nominal) tax collections by the government. Moreover, \( Q_{0,t} \) is a stochastic discount factor that satisfies \( Q_{0,0} = 1 \) and \( E_0 Q_{0,t} = \prod_{s=0}^{t-1} \frac{1}{1 + i^s} \). It is also worth noting that the price of a riskless one period bond is given by:

\[
\frac{1}{1 + i_t} = E_t [Q_{t,t+1}] \tag{3.3}
\]

The household’s optimization problem is to choose processes \( C_t, M_t, H_t \geq 0 \) for all dates \( t \geq 0 \), satisfying (3.2) given its initial wealth \( A_0 \) and the good price, the nominal wage and the stochastic discount factors that it expects to face, so as to maximize (3.1).

The first order conditions associated with the household’s problem are:

\[
\frac{U_c(C_t, M_t/P_t; \xi_t)}{U_c(C_{t+1}, M_{t+1}/P_{t+1}; \xi_{t+1})} = \beta \frac{P_t}{Q_{t,t+1}} \frac{P_t}{P_{t+1}} \tag{3.4}
\]

\[
\frac{U_m(C_t, M_t/P_t; \xi_t)}{U_c(C_t, M_t/P_t; \xi_t)} = \Delta_t \tag{3.5}
\]

\[
\frac{V_H(H_t)}{U_c(C_t, M_t/P_t; \xi_t)} = \frac{W_t}{P_t} \tag{3.6}
\]

Equation (3.4) is a standard intertemporal optimality condition (Euler equation) whereas equations (3.5) and (3.6) are the optimality conditions for money demand and labor supply respectively. Using (3.3) and (3.4), I can rewrite the Euler equation as:
In order to conduct the empirical part of the paper, I just need to approximate conditions (3.7) and (3.5). A log linear approximation to condition (3.7) is then given by:

\[
1 + i_t = \beta^{-1} \left\{ E_t \left[ \frac{U_c(C_{t+1}, M_{t+1}/P_{t+1}; \xi_{t+1})}{U_c(C_t, M_t/P_t; \xi_t)} \frac{P_t}{P_{t+1}} \right] \right\}^{-1}
\]

(3.7)

Equation (3.8) represents the basis for building the aggregate demand block in most of the macroeconomic models that are used for monetary policy analysis. In fact, equation (3.8) combined with a market clearing condition that will be seen later completely defines the aggregate demand in a closed economy without capital. The parameter \( \sigma_c^{-1} \) is the coefficient of relative risk aversion. According to the assumptions I made on the utility function, it is strictly positive. The parameter \( \chi \) is the elasticity of marginal utility of consumption with respect to real money. The importance of real money balance effects is given by the ratio \( \frac{\chi}{\sigma_c} \), which measures the effect of a one percent deviation of money from its steady state on the percentage deviations of consumption from its steady state. If this ratio is significantly different from zero, then real money affects directly aggregate demand; and, consequently, influences the equilibrium evolution of all the macroeconomic variables. Notice that in this model, real money balance effects arise from non-separable utility in consumption and real money balance effects (\( \chi \) different from zero).

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5Condition (6) can also be log linearized and be taken into account when estimating real money balance effects. However, given that using (6) implies imposing the assumption of flexible wages, it is chosen in this study not to do it. The reason is that it is preferable to have an estimate of real money balance effects that is not sensitive to the assumption about wage setting.

6It can also be shown that the components of this ratio are important determinants of the weight of output in the welfare loss function when real money balances effects are allowed in a model. I discuss the impact of these components later.
A corresponding log linear approximation to condition (3.5) is given by:

\[
\hat{m}_t = \eta_c \hat{C}_t - \eta_i \left( \hat{i}_t - \hat{i}_m \right) + \left[ \frac{U_{mg}}{U_m} - \frac{U_{c\xi}}{U_c} \right] \frac{1}{\sigma_m^{-1} + \chi} \xi_t
\]

(3.9)

where \( \eta_c = \frac{\nu}{\sigma_m^{1/\chi}} \), \( \eta_i = \left( \frac{1+\tau_m}{1-\tau_m} \right) \frac{1}{\sigma_m^{1/\chi}}, \hat{i}_m = \log \left( 1 + \tau_m \right), \sigma_m = \frac{-U_m}{\bar{m}U_{mm}}, \bar{v} = \frac{\bar{C}}{\bar{m}} \) and \( \bar{v}, \bar{i}_m, \bar{\Delta} \) are the steady state values of money velocity, the nominal interest rate paid on money and the opportunity cost of holding money respectively. The parameters \( \eta_c \) and \( \eta_i \) are the consumption elasticity and the interest semielasticity of money demand correspondingly. According to the assumptions on the utility function, both of them are strictly positive. The last term represents a money demand shock (given by a linear function of the disturbance on the liquidity services provided by real money balances). Finally, it is assumed that this disturbance has mean equal to zero and follows the autoregressive process:

\[
\xi_t = \rho \xi_{t-1} + \eta_t
\]

(3.10)

where \( \eta_t \) is an innovation with mean zero and serially uncorrelated. These assumptions on this disturbance and its innovation are consistent with the structure of the money demand shock that is assumed in the literature.\(^7\)

In equilibrium all output must be consumed, thus implying a goods market clearing condition given by \( \hat{C}_t = \hat{Y}_t.\)\(^8\) This condition could be used in order to write (3.8) and (3.9) as a function of the percentage deviation of output (instead of consumption) from its steady state. I consider this alternative in the empirical part of the chapter. Moreover, when goods market clearing is combined with (3.8), the aggregate demand arises in its usual form in macroeconomic models.

### 3.3 New Estimates of Real Money Balance Effects

This section contains four parts. In the first one, I describe the econometric specification used to estimate jointly the Euler equation and the money demand by applying the Generalized Method of Moments (GMM). In the second one, I present the data and baseline estimates of the model in order to answer how important real money

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\(^7\)See Ireland (2004) and Bouakez et al. (2005).

\(^8\)Notice that this condition also holds in steady state, which means that \( \bar{C} = \bar{Y}. \)
balance effects are from a quantitative point of view. Two robustness exercises on the estimation process are presented in the third subsection. Finally, I make a comparison of my estimation procedure with those of Woodford (2003) and Ireland (2004) in order to understand why my estimates are different from theirs.

3.3.1 Econometric Specification

In order to perform the GMM technique, two orthogonality conditions can be inferred from the model developed in the previous section. One can be inferred by combining equations (3.8), (3.9) and (3.10) in the following way: first, by using (3.10), we can compute the last expectation term that is present in equation (3.8). Then, we have:

$$E_t (\xi_{t+1} - \xi_t) = E_t (\rho_\xi \xi_t + \eta_{t+1} - \xi_t) = -(1 - \rho_\xi) \xi_t$$  \hspace{1cm} (3.11)

Using (3.11) and (3.9), we can rewrite (3.8) in the following way:

$$E_t (\tilde{C}_{t+1} - \tilde{C}_t) = \frac{\chi}{\sigma_c} E_t (\tilde{m}_{t+1} - \tilde{m}_t) + \frac{(i_t - E_t \tilde{\pi}_{t+1})}{\sigma_c} - \frac{(1 - \rho_\xi) \mu (\sigma_m^{-1} + \chi) v_t}{\sigma_c}$$  \hspace{1cm} (3.12)

where

$$\mu = \frac{U_{c\xi}}{U_{mc}} - \frac{U_{c\xi}}{U_{cc}}$$

and

$$v_t = \tilde{m}_t - \eta_c \tilde{C}_t + \eta_i (i_t - \tilde{\pi}_m^n)$$

The first orthogonality condition comes from equation (12) and arises from the fact that, under rational expectations, the forecast errors in consumption, real money and inflation one period ahead should be uncorrelated with the information set dated at period \(t\) and earlier. Then, this orthogonality condition is given by:

$$E_t \left\{ \left[ (\tilde{C}_{t+1} - \tilde{C}_t) - \frac{\chi}{\sigma_c} E_t (\tilde{m}_{t+1} - \tilde{m}_t) - \frac{(i_t - E_t \tilde{\pi}_{t+1})}{\sigma_c} + \frac{(1 - \rho_\xi) \mu (\sigma_m^{-1} + \chi) v_t}{\sigma_c} \right] z_t \right\} = 0$$  \hspace{1cm} (3.13)

where \(z_t\) denotes a vector of variables dated at period \(t\) and earlier.

The second orthogonality condition comes from equation (3.8) and (3.9). It arises from the properties of the innovation \(\eta_t\). Under rational expectations, this innovation should be uncorrelated with earlier information. Then, the following orthogonality
condition can be established:

\[ E_t \left\{ \left[ \tilde{m}_t - \eta_c \tilde{C}_t + \eta_i (\tilde{i}_t - \tilde{i}_t^m) - \rho_\xi (\tilde{m}_{t-1} - \eta_c \tilde{C}_{t-1} + \eta_i (\tilde{i}_{t-1} - \tilde{i}_{t-1}^m)) \right] z_{t-1} \right\} = 0 \quad (3.14) \]

where \( z_{t-1} \) denotes a vector of variables dated at period \( t - 1 \) and earlier.

The orthogonality conditions given by equations (3.13) and (3.14) constitute the basis for estimating the structural parameters of the model via GMM. Notice that there are eight structural parameters in the system: \( \sigma_c^{-1}, \chi, \tilde{\tau}, \tilde{\tau}^m, \tilde{\tau}, \sigma_m^{-1}, \rho_\xi \) and \( \mu \). All of them are not simultaneously identifiable from the system. For this reason, three of them (\( \tilde{\tau}, \tilde{\tau}^m, \tilde{\tau} \)) are calibrated to perform the estimation, because they are pinned down more directly from first moments of the data.\(^9\) I set them using their averages during the sample period. The rest of parameters are estimated.

Before performing the estimation, one econometric issue should be faced. In small samples, the way the orthogonality conditions are written (or normalized) affects the GMM estimates.\(^{10}\) More specifically, there is no agreement about how to specify the orthogonality condition (3.13) in order to estimate \( \sigma_c^{-1} \) and \( \chi \), the set of parameters that measure the importance of real money balance effects. An alternative normalization for the moment restriction (13) is given by the following expression:\(^{11}\)

\[ E_t \left\{ \left[ \tilde{i}_t - \tilde{\tau}_{t+1} - \sigma_c^{-1}(\tilde{C}_{t+1} - \tilde{C}_t) + \chi(\tilde{m}_{t+1} - \tilde{m}_t) - (1 - \rho_\xi)\mu(\sigma_m^{-1} + \chi)\tilde{v}_t \right] z_t \right\} = 0 \quad (3.15) \]

Hansen and Singleton (1983) and Hall (1988) use normalization (3.13) and (3.15) respectively, without allowing for the existence of real money balance effects (\( \chi = 0 \)) and the presence of money demand shocks (\( \mu = 0 \)) in the Euler Equation. Hansen and Singleton estimate the coefficient of relative risk aversion, whereas Hall estimates its reciprocal (the intertemporal elasticity of substitution). They find very different results, as surveyed in Neely et al. (2001) and confirmed by updated estimates performed by Campbell (2003).\(^{12}\) In particular, the implied coefficient of relative risk aversion

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\(^9\)Given a definition of money, there is agreement about what \( \tilde{\tau}, \tilde{\tau}^m, \tilde{\tau} \) are in the model. In fact, all monetary models used for macroeconomic analysis set them equal to their averages during the sample period. This is not the case for the rest of parameters. For instance, the range of values used in calibration for \( \sigma_c^{-1} \) goes from 0.16 (Woodford (2003)) to 10 (the maximum level considered plausible by Mehra and Prescott (1985)).

\(^{10}\)See Campbell (2003), Hamilton et al. (2005), Neely et al. (2001) and Yogo (2004).

\(^{11}\)Notice that normalization (3.15) arises from multiplying the orthogonality condition (3.13) by \( \sigma_c^{-1} \).

\(^{12}\)Campbell (2003) reports point estimate of 0.71 and 15 for the coefficient of relative risk aver-
estimated by Hall is much higher than the one directly estimated by Hansen and Singleton. Then, these two alternative specifications of the orthogonality conditions are taken into account in order to see how sensitive the results are to the normalization issue. Specification 1 considers equations (3.15) and (3.14) whereas specification 2 considers equations (3.13) and (3.14).

3.3.2 Data and Baseline Estimates

The data that I use is United States quarterly data and runs from the first quarter of 1959 through the fourth quarter of 2004. Consumption is measured by real personal consumption expenditures, real money balances are measured by dividing M2 money stock by the CPI, inflation is measured by changes in the CPI, the interest rate is measured by the three-month Treasury bill rate, expressed in quarterly terms and the interest rate paid on money is measured by M2 money own rate, expressed in quarterly terms. Consumption and real money balances are expressed in per capita terms, by dividing by the civilian noninstitutional population, age16 and over. Prior to estimation, the logarithm of per-capita consumption and per-capita real money balances have been detrended by using a deterministic linear trend in order to get stationary series, given that the application of GMM requires this kind of series.\footnote{Alternative ways to detrend time series have been used in the literature. In this case, I follow the procedure presented in Ireland (2004).} Given this data set, \( \bar{\pi} = 0.29 \) (M2 consumption velocity), \( \bar{i} = 0.0136 \) and \( \bar{i}^m = 0.0091 \).

The election of M2 as the monetary aggregate to be used in this study is related to the fact that it is the one that includes all the assets that provide liquidity services. Given that it is clear that M1 furnishes these services, a way to show that M2 is the correct measure of money is by arguing that (M2-M1) also provides liquidity services. To test the latter, I check if the opportunity cost of (M2-M1) is significantly different from zero. The intuition for this comes from the fact that \( U_{m2-m1} > 0 \) implies that (M2-M1) furnishes liquidity services according to the model and that \( \Delta_t = \frac{u_{0t} - i_t}{1+i_t} > 0 \). So, after computing the average own rate of return of (M2-M1) and compare it with the average rate of return of the short term Treasury bond, I get that the average opportunity cost of holding (M2-M1) is 1 percent annually. Then, (M2-M1) provides...
liquidity services.\footnote{Alternatively, Alvarez et al. (2003) decompose (M2-M1) into saving deposits, time deposits and retail money market funds. They conclude, by doing the same type of analysis I do, that saving deposits and time deposits provide liquidity services whereas retail money market funds do not. So, their proposal of a monetary aggregate that provides liquidity services is M2 minus retail money market funds. However, it is difficult to argue that retail money market funds furnish no liquidity services. In fact, they are extremely liquid (most even checkable), have essentially no default risk and no interest rate risk.}

Table 3.1

Estimates of the Structural Parameters: 1959 – 2004

<table>
<thead>
<tr>
<th>Specification 1</th>
<th>$\sigma_c^{-1}$</th>
<th>$\chi$</th>
<th>$\sigma_m^{-1}$</th>
<th>$\frac{\chi}{\sigma_c}$</th>
<th>$\eta_c$</th>
<th>$\eta_i$</th>
<th>$\rho_\xi$</th>
<th>$\mu$</th>
<th>$J$ test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set 1</td>
<td>0.82</td>
<td>0.48</td>
<td>37.27</td>
<td>0.59</td>
<td>0.85</td>
<td>5.95</td>
<td>0.96</td>
<td>-0.03</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>(0.24)</td>
<td>(0.09)</td>
<td>(6.19)</td>
<td>(0.12)</td>
<td>(0.15)</td>
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<td>(0.11)</td>
<td>(8.56)</td>
<td>(0.16)</td>
<td>(0.18)</td>
<td>(1.45)</td>
<td>(0.01)</td>
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<tr>
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<th>$\chi$</th>
<th>$\sigma_m^{-1}$</th>
<th>$\frac{\chi}{\sigma_c}$</th>
<th>$\eta_c$</th>
<th>$\eta_i$</th>
<th>$\rho_\xi$</th>
<th>$\mu$</th>
<th>$J$ test</th>
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<td>(0.33)</td>
<td>(0.16)</td>
<td>(6.37)</td>
<td>(0.06)</td>
<td>(0.18)</td>
<td>(1.46)</td>
<td>(0.01)</td>
<td>(0.05)</td>
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<tr>
<td>Set 2</td>
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<td>23.60</td>
<td>0.33</td>
<td>1.64</td>
<td>9.29</td>
<td>0.97</td>
<td>0.09</td>
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<td>(0.33)</td>
<td>(0.14)</td>
<td>(5.70)</td>
<td>(0.07)</td>
<td>(0.23)</td>
<td>(2.24)</td>
<td>(0.01)</td>
<td>(0.07)</td>
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Note: Standard errors shown in brackets.

Table 3.1 presents the GMM estimation of the structural parameters $\sigma_c^{-1}$, $\chi$, $\sigma_m^{-1}$, $\rho_\xi$ and $\mu$. It also shows the ratio $\frac{\chi}{\sigma_c}$, which measures the importance of the real
money balance effects, and the consumption elasticity ($\eta_c$) and interest semielasticity
($\eta_i$) of money demand implied by the estimated and calibrated parameters. The last
column of the table reports the p-value for the Hansen’s J statistic of overidentifying
restrictions. The results are presented for the two specifications of the orthogonality
conditions discussed earlier and two sets of instruments.\textsuperscript{15} Standard errors (with a
Newey West correction) for all the parameter estimates are reported in brackets.

Some interesting results arise from these estimations, which are robust to the spec-
cifications and to the set of instruments. First, real money balance effects are quanti-
tatively important. In all the cases, the ratio is significantly different from zero and
its point estimates are higher than 0.32. This result contrasts considerably with those
provided by Woodford (2003) and Ireland (2004), who obtain point estimates of 0.05
and 0.00 respectively for this ratio. Woodford uses a calibration procedure, whereas
Ireland performs Maximum Likelihood estimation. A detailed analysis of the com-
parison of these results with mine is presented in a special subsection later. Second,
the estimates of the coefficient of risk aversion are strictly positive and significantly
different from zero in all the cases. This result is consistent with the restriction I
imposed theoretically on this parameter. Third, all the point estimates of the degree
of risk aversion belong to the 95 percent confidence interval of this parameter provided
by Campbell (2003). Without taking into account real money balance effects, he sug-
gests values for this coefficient between -0.73 and 2.14 when instrumental variables
and normalization 1 are used.

Fourth, the elasticity of marginal utility of consumption with respect to real money
balances (the parameter $\chi$) is significantly different from zero and strictly positive.
This result implies that utility is not separable in consumption and money and discards
the possibility that this parameter could be negative.\textsuperscript{16} Moreover, from the latter result
and the fact that the ratio that measures the importance of real money balance is
significantly different from zero, it can be concluded that money plays an independent
role in the monetary transmission mechanism. Fifth, both parameters of the money
demand are also significantly different from zero. Sixth, the money demand shock is
quite persistent. This result is consistent with those found by Ireland (2004, 2001) and
Bouakez et al. (2005). Finally, the validity of all the regressions is confirmed by the

\textsuperscript{15}Set 1 includes interest rate, inflation, real money balances and consumption from $t - 3$ to $t - 6$.
Set 2 includes the same variables but just until $t - 5$.

\textsuperscript{16}There are some microfounded models where $\chi$ can be negative. See Wang and Yip (1992) for a
detailed discussion.
p-value for the Hansen’s J statistic of overidentifying restrictions with a significance level of 5 percent.

From Table 3.1, it is clear that real money balance effects are quantitatively important but the magnitude is not apparent. Under normalization 1, the ratio $\frac{\chi}{\sigma_c}$ is around 0.6; whereas under normalization 2, it is around 0.3. The main reason behind this result is that the estimate of the degree of risk aversion is very sensitive to the normalization in the GMM estimation. Using normalization 1, the degree of risk aversion is close to 1; while in the second normalization it is around 2.

Finally, there exist minor differences in the estimates of the elasticity of marginal utility of consumption and in those of the interest rate semielasticity. The point estimates of $\chi$ are between 0.5 and 0.7; whereas those of $\eta_i$ go from 6.0 to 9.0.\textsuperscript{17} All the interest rate semielasticity estimates are in line with the money demand estimation performed by Reynard (2004) for the postwar period.\textsuperscript{18}

### 3.3.3 Robustness Exercises

In this subsection, I perform two robustness exercises. First, I use the goods market clearing condition so that $\hat{C}_t = \hat{Y}_t$ and $\overline{C} = \overline{Y}$. Second, I check sub sample stability.

#### 3.3.3.1 Using GDP Data

Clearly, consumption is different from output in the data. However, I will assume that consumption equals output because a lot of macroeconomic studies (e.g. Ireland (2004)) impose this condition in the estimation of macroeconomic models. Then, I specify the orthogonality conditions in the same way as I did when consumption was used. Specification 1 considers equations (3.15) and (3.14) whereas specification 2 considers equations (3.13) and (3.14). Both specifications impose that consumption equals output. The latter means that $\hat{C}_t = \hat{Y}_t$ in (3.13), (3.14) and (3.15). Moreover, some parameters change their definition in the equations as follows:

$$
\sigma_c = \sigma_y = \frac{-U_y}{U_{yy}}, \quad \chi = \frac{mU_{my}}{U_y}, \quad \eta_c = \eta_g = \frac{\chi}{\sigma_m^2 + \chi}, \quad v = \frac{\overline{U}}{\overline{m}}, \quad \mu = \frac{U_g}{U_y} \frac{U_{mg}}{U_m} \frac{U_{gy}}{U_y}
$$

\textsuperscript{17}These point estimates for the interest semielasticity imply that the interest rate elasticity is between 0.085 and 0.121.

\textsuperscript{18}He uses output and M2 minus instead of consumption and M2 respectively in order to perform his money demand study.
The frequency of the data and the sample period are the same as in previous subsection.
Now, output is measured by real GDP, real money balances are measured by dividing M2 money stock by the GDP deflator, inflation is measured by changes in the GDP deflator and the interest rates are the same as before. Real GDP and real money balances are expressed in per capita terms, by dividing by the civilian noninstitutional population, age 16 and over. Prior to estimation, the logarithm of per-capita real GDP and per-capita real money balances have been detrended by using a deterministic linear trend in order to get stationary series, as required by GMM estimation. Again, \((\bar{\bar{i}}, \bar{\bar{\tau}}^m, \bar{\pi})\) are calibrated and the rest of parameters are estimated. Given that I use data on output, \(\bar{\pi} = 0.45\) in this case.

Table 3.2 presents GMM estimates of the structural parameters \(\sigma_y^{-1}, \chi, \sigma_m^{-1}, \rho\) and \(\mu\). It also shows the ratio \(\frac{\chi}{\sigma_y}\), which measures the importance of real money balance effects; and the income elasticity (\(\eta_y\)) and interest semielasticity (\(\eta_i\)) of money demand implied by the estimated and calibrated parameters. The results are presented for both specifications of the orthogonality conditions discussed earlier and two sets of instruments.\(^{19}\) Standard errors (with a Newey West correction) for all the parameter estimates are reported in brackets.

Under both specifications, the estimates of the real money balance effects are statistically significant and much higher than those obtained when consumption is used. When normalization 1 is used, the point estimates are between 1.2 and 1.3; while they are around 0.5 when the second normalization is used. Therefore, all this evidence implies that the result obtained in the baseline case is not driven by using consumption instead of output.

---

\(^{19}\)Set 1 includes interest rate, inflation measured by the percentage change in the GDP deflator, real money balances and output from \(t - 3\) to \(t - 6\). Set 2 includes the same variables but just until \(t - 5\).
Table 3.2  
Estimates of the Structural Parameters: 1959 – 2004  
(Employing the Market Clearing Condition)  

| Specification 1 |  |  |  |  |  |  |  |  |  |  
|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
|                 | $\sigma_y^{-1}$ | $\chi$         | $\sigma_m^{-1}$ | $\frac{\chi}{\sigma_y}$ | $\eta_y$    | $\eta_i$    | $\rho_\xi$ | $\mu$ | $J$ test     |
| Set 1           | 0.28 (0.14)    | 0.36 (0.07)    | 52.23 (10.19)   | 1.27 (0.54)     | 0.70 (0.12)  | 4.27 (0.83)  | 0.96 (0.01)  | -0.02 (0.01) | 0.52          |
|                 | 0.31 (0.16)    | 0.37 (0.07)    | 41.33 (9.52)    | 1.19 (0.37)     | 0.90 (0.18)  | 5.39 (1.24)  | 0.96 (0.02)  | -0.03 (0.02) | 0.24          |

<table>
<thead>
<tr>
<th>Specification 2</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_y^{-1}$</td>
<td>$\chi$</td>
<td>$\sigma_m^{-1}$</td>
<td>$\frac{\chi}{\sigma_y}$</td>
<td>$\eta_y$</td>
<td>$\eta_i$</td>
<td>$\rho_\xi$</td>
<td>$\mu$</td>
<td>$J$ test</td>
</tr>
<tr>
<td>Set 1</td>
<td>1.03 (0.12)</td>
<td>0.56 (0.12)</td>
<td>65.14 (16.05)</td>
<td>0.54 (0.10)</td>
<td>0.88 (0.12)</td>
<td>3.42 (0.84)</td>
<td>0.95 (0.01)</td>
<td>-0.01 (0.01)</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>1.05 (0.17)</td>
<td>0.50 (0.12)</td>
<td>50.12 (15.83)</td>
<td>0.47 (0.16)</td>
<td>1.02 (0.16)</td>
<td>4.44 (1.40)</td>
<td>0.96 (0.02)</td>
<td>-0.01 (0.02)</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Note: Standard errors shown in brackets.

The parameter $\sigma_y^{-1}$ measures the inverse of the interest sensitivity of real expenditure that is exclusively due to the interest rate channel.\(^\text{20}\) The point estimates are strictly positive (as theory predicts) and statistically significant. When normalization

\(^{20}\)When there are real money balance effects, a change in the interest rate affects aggregate demand through two channels: the interest rate channel and the real money balance effect channel. The interest rate channel is the one by which interest rates impact on the desired timing of private expenditures. The other channel is the one by which a movement in the interest rates affects marginal utility of consumption through their impact on real money balances.
1 is used, they are around 0.3. This value is small and very close to what has been found in other macroeconomic papers that estimate this parameter. Rotemberg and Woodford (1997) find that it is equal to 0.16 whereas Amato and Laubach (2003) estimate it equal to 0.26. Under normalization 2, the point estimates are much higher (around 1) and support the standard practice in macroeconomics of calibrating this value equal to one. Moreover, it should be noticed that the values obtained for the interest sensitivity of total output ($\sigma_y$) are higher than those found for the interest sensitivity of real consumption ($\sigma_c$), which makes sense. The intuition is as follows: since the purchases of investment goods (included in output and not in consumption) are likely to be more interest rate-sensitive, it is reasonable that $\sigma_y$ is higher than $\sigma_c$.

When output is used instead of consumption, $\chi$ represents the elasticity of marginal utility of real income with respect to real money. All the point estimates for this parameter are statistically significant and very similar to those found when consumption was used. Therefore, the main conclusions related to this parameter do not change: utility is non-separable and $\chi$ is strictly positive. Moreover, it can be concluded that the increase in the estimates of the real money balance effects when the goods market clearing condition is imposed are associated with the drop in $\sigma_y^{-1}$.

Both parameters of the money demand are also significantly different from zero. According to three out of the four estimates, it cannot be rejected that income elasticity ($\eta_y$) is equal to 1, which is consistent with several empirical studies about money demand. Moreover, the point estimates of the interest rate semielasticity go from 3.4 to 5.4. All these values are plausible under the money demand estimation for the postwar period performed by Reynard (2004). He finds a point estimate of 10.4 for this parameter, with a standard error of 4.4.

Finally, the last column of the table reports the p-value for the Hansen’s J statistic of overidentifying restrictions, which confirms the validity of all the regressions with a significance level of 5 percent.

---

21 Both papers consider cashless sticky price models.
22 He uses M2 minus and reports interest elasticity. The implicit interest semielasticities have been calculated by multiplying the interest elasticity by the inverse of the opportunity cost of the monetary base.
3.3.3.2 Sub-Sample Stability

In this part, I explore if the baseline estimates (those from Table 3.1) are sensitive to the election of the sample. In order to do it, I divide the full sample in two sub-samples: 1959:1-1979:4 and 1980:1-2004:4. The beginning of the second sub-sample is chosen such that it coincides with the beginning of the sample used by Ireland (2001, 2004). This strategy allows a fair comparison of my estimates with his. Results are presented in Table 3.3 for both specifications of the orthogonality conditions and the instrument set 1.\(^{23}\)

The quantitative importance of real money balance effects is also confirmed by this exercise. Under both specifications, the ratio \(\frac{\lambda}{\sigma_z}\) is positive and significantly different from zero across sub-samples. However, the point estimates are not constant across time. Prior to 1980, they are 0.85 and 0.74; while since 1980 they are 0.54 and 0.21. Thus, this result suggests that real money balance effects would have diminished its quantitative importance in the recent period. Nevertheless, the magnitude of the reduction in the size of real money balance effects is not apparent. Notice that the decrease with normalization 2 is much higher than the one with normalization 1.

Other interesting results arise from these estimations, which are robust to the specifications. First, the estimates of the coefficient of relative risk aversion and the elasticity of marginal utility of consumption are lower since 1980. Second, the reduction in the elasticity of marginal utility of consumption is the main determinant of the decrease in the size of real money balance effects. Third, the interest semielasticity has increased considerably since 1980. Fourth, the degree of persistence of the money demand shocks is higher in the recent period.

\(^{23}\)Results are very similar when Set 2 is used.
### Table 3.3

Subsample stability

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<tr>
<th>Specification 1</th>
<th>Period</th>
<th>$\sigma^{-1}$</th>
<th>$\chi$</th>
<th>$\sigma^{-1}_m$</th>
<th>$\frac{\chi}{\sigma_m}$</th>
<th>$\eta_c$</th>
<th>$\eta_i$</th>
<th>$\rho_\xi$</th>
<th>$\mu$</th>
<th>$J$ test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1959-</td>
<td>0.87</td>
<td>0.73</td>
<td>45.55</td>
<td>0.85</td>
<td>1.05</td>
<td>4.85</td>
<td>0.94</td>
<td>-0.05</td>
<td>0.97</td>
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<td></td>
<td>1979</td>
<td>(0.12)</td>
<td>(0.06)</td>
<td>(3.16)</td>
<td>(0.09)</td>
<td>(0.06)</td>
<td>(0.34)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td></td>
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<tr>
<td></td>
<td>1980-</td>
<td>0.54</td>
<td>0.29</td>
<td>19.79</td>
<td>0.54</td>
<td>0.98</td>
<td>11.19</td>
<td>0.98</td>
<td>0.04</td>
<td>0.84</td>
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<td>2004</td>
<td>(0.13)</td>
<td>(0.05)</td>
<td>(2.66)</td>
<td>(0.14)</td>
<td>(0.13)</td>
<td>(1.50)</td>
<td>(0.01)</td>
<td>(0.03)</td>
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<th>Specification 2</th>
<th>Period</th>
<th>$\sigma^{-1}$</th>
<th>$\chi$</th>
<th>$\sigma^{-1}_m$</th>
<th>$\frac{\chi}{\sigma_m}$</th>
<th>$\eta_c$</th>
<th>$\eta_i$</th>
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<th>$J$ test</th>
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<tr>
<td></td>
<td>1959-</td>
<td>2.09</td>
<td>1.53</td>
<td>84.63</td>
<td>0.74</td>
<td>1.19</td>
<td>2.61</td>
<td>0.92</td>
<td>-0.02</td>
<td>0.97</td>
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<tr>
<td></td>
<td>1979</td>
<td>(0.24)</td>
<td>(0.21)</td>
<td>(11.13)</td>
<td>(0.05)</td>
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<td>1980-</td>
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<td>0.88</td>
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<td>2004</td>
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<td>(0.06)</td>
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<td>(1.66)</td>
<td>(0.01)</td>
<td>(0.15)</td>
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*Note: Standard errors shown in brackets.*

### 3.3.4 Previous Studies on Real Money Balance Effects: a comparison

The estimates of the parameter that measures the quantitative importance of real money balance effects differ dramatically from those obtained before in the literature. Therefore, it is interesting, at this point, to understand why this can be so. For this reason, in this subsection, I describe the estimation procedures performed by Woodford (2003) and Ireland (2004); and then, compare them with my procedure.
Woodford, by using a calibration procedure and considering the goods market clearing condition, suggests a value of 0.05 for the size of real money balance effects. This result is obtained by setting $\sigma_y^{-1} = 0.16$ and $\chi = 0.008$. The first parameter is calibrated using an estimate from the study performed by Rotemberg and Woodford (1997). To obtain a value for $\chi$, he uses the following relation implied by the MIU model:

$$\frac{\eta_y}{\eta_i} = \frac{\frac{\bar{v}}{\sigma_y} + \sigma_y^{-1}}{(1 + \tau^m)} \quad (3.16)$$

Notice that $\chi$ can be found, given the income elasticity, the interest semielasticity, the money velocity, the interest rate, the interest rate paid on money and the inverse of the interest sensitivity of real expenditure that is exclusively due to the interest rate channel. He considers $\eta_y = 1$ and $\eta_i = 28$ from a long run money demand study performed by Lucas (2000). He also sets $\bar{v} = 4$ (monetary base velocity), $\tau^m = 0$ and $\bar{t} = 0.01$.

**Table 3.4**

<table>
<thead>
<tr>
<th>Calibration of Real Money Balance Effects</th>
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<tr>
<td>Woodford Calibration1 Calibration2</td>
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<tr>
<td>$\sigma_y^{-1}$</td>
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<tr>
<td>$\eta_y$</td>
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<tr>
<td>$\eta_i$</td>
</tr>
<tr>
<td>$\bar{v}$</td>
</tr>
<tr>
<td>$\frac{\chi}{\sigma_y}$</td>
</tr>
</tbody>
</table>

Clearly, there exist two important differences between Woodford’s procedure and mine. First, the definition of money is different: monetary base versus M2. Second, the methodology of estimation is also different: calibration versus GMM. In order to illustrate how these two differences explain the discrepancy between my estimates and the one presented by Woodford, I perform two different calibration exercises that
are shown in Table 3.4. Calibration 1 follows Woodford (2003) but changes only the definition of the money for M2; and consequently, changes appropriately the money velocity and the interest rate semielasticity. This exercise shows how Woodford conclusion on real money balance effects changes with the definition of money. It can be seen that there exists a huge change. The size of real money balance effect goes from 0.05 to 3.05. Therefore, the definition of money matters considerably. Calibration 2 just changes the degree of risk aversion used in Calibration 1 by assuming an estimate consistent with my empirical evidence in order to see how GMM estimates make a difference. The size of real money balance effects again changes dramatically going from 3.05 to 0.49 (a value that is very close to the average of all my baseline estimates). Therefore, the estimation technique also matters.

Ireland estimates a small macroeconomic model by ML, containing seven relations: an Euler equation, a M2 money demand equation, a Phillips curve, an interest rate rule, a process for a preference shock, a process for a money demand shock and a process for a technology shock. All these relations contain twenty parameters that he estimates by using quarterly data that run from 1980:1 through 2001:3 and imposing the market clearing condition. One of these parameters is $\frac{1}{\sigma_y}$. Ireland estimates it equal to zero, with a standard error of 0.26. The point estimates of the determinants of this ratio were obtained in two different ways: $\sigma_y^{-1}$ was calibrated and set equal to 1, whereas $\chi$ was estimated. Ireland argues that $\sigma_y^{-1}$ was calibrated because preliminary attempts to estimate it, described in Ireland (2001), led to unreasonably high levels of this parameter.

There exist three important differences with respect to Ireland’s procedure. First, in the present study, both parameters that determine the size of real money balance effects are estimated. In fact, I always find reasonable degrees of risk aversion, so I do not need to impose reasonable values on this parameter. Second, the model he estimates is much bigger than the one I estimate (7 equations versus 3 equations I estimate). Third, the econometric procedure is different: ML versus GMM. It is clear that the first difference goes in favor of my approach. The second difference means that he imposes more structure; and therefore, more cross equation restrictions in his estimation. For instance, among other restrictions, he imposes in his estimation that if money enters the IS curve, it should also enter in the Phillips curve. Clearly, the risk of misspecification is much higher in Ireland’s approach. Thus, according to the criteria of minimizing misspecification, the second difference also goes in favor of my
approach. However, it is not clear which econometric procedure is better. For this reason, it is useful to discuss on the convenience of each econometric method.

Cochrane (2001) emphasizes that the issue of which econometric procedure is the best in such circumstances is absolutely open. He points out that there are no theorems or Monte Carlo simulations that suggest which one is preferable. It is known that if the model is correct, ML is more efficient than GMM. However, it is very difficult to argue that an economic model is completely well specified. In particular, in the case of Ireland, there are three reasons why his model could be misspecified. First, the market clearing condition in the way he imposes does not hold: consumption is different from output. Second, his specification of the Phillips curve could be inadequate. He defines it in terms of the detrended output and real money balances; instead of using, as the theory suggests, the real marginal cost. Galí and Gertler (1999) point out that the latter outperforms detrended output in the estimation of the Phillips curve. Third, not all the shocks necessarily satisfy the normality assumption. Then, given that the consistency of the estimates obtained by ML is very sensitive to the misspecification of the model, Ireland estimates could be inconsistent.

GMM allows the researcher to estimate part of the model, limiting the problem of misspecification. In particular, the model I estimate is silent about the market clearing condition, the Phillips Curve, the monetary policy rule and the evolution of two of the shocks considered by Ireland (productivity and preference shocks). In this sense, fewer assumptions on the structure of the model are needed in order to get consistent estimates. However, GMM has also some disadvantages, the main one being the use of irrelevant instruments (or weak identification). Stock et al. (2002) emphasize that estimates may be very sensitive to the choice of instruments when there is weak identification. The estimates I present by using both specifications seem not to have this problem.

3.4 Implications of my Findings

In previous section, I present econometric estimates that suggest that real money balance effects are quantitatively important but lower than they used to be in the beginning of the 1980’s. Given the model used to evaluate the existence of real money balance effects, the results implies that utility is non-separable, and that money plays a direct role in determining the dynamic behavior of inflation and output. More-
over, there are three additional important implications of my evidence on real money
balance effects that I analyze in this section. First, the existence of quantitatively
important real money balance effects in a model with sticky prices and flexible wages
is a possible explanation for two stylized facts: the modestly procyclical real wage
response to a monetary policy shock and the supply side effects of monetary policy.
Second, conditional on productivity and money demand shocks, much higher volatil-
ity of the output and much lower volatility of the interest rate should arise under the
optimal monetary policy when there exist real money balance effects of the magnitude
estimated in this study. Third, the reduction in the size of real money balance effects
can explain an important part of the diminishment in the volatility of inflation and
output. This would support the hypothesis that financial innovation is an important
source of the Great Moderation.

Before analyzing these implications, I need to extend the model I derived in section
2 by allowing for monopolistic competition, sticky prices a la Calvo and a labor market
with flexible wages. This extension allows me to have a Phillips curve and an equation
for the evolution of real wages. As was mentioned in section 2, the Euler equation and
the money demand equation still hold. The derivation of this extension can be found
in Woodford (2003).

Given that I am also interested in analyzing impulse responses of output and real
wages to a monetary policy shock, I also define a monetary policy rule as a part of the
model. This rule is a standard Taylor rule that responds to current inflation and to
current output gap.

3.4.1 MIU Model with Monopolistic Competition, Sticky Prices
and Flexible Wages

In this part, I present all the equations that I need in order to explain all the implica-
tions of my empirical evidence. All of them are log-linearized around a zero inflation
steady state. Moreover, the calibration of the model is presented.

3.4.1.1 Equations

a) IS Curve:
\[ E_t(\hat{Y}_{t+1} - \hat{Y}_t) = \frac{\chi}{\sigma_c} E_t(\hat{m}_{t+1} - \hat{m}_t) + \frac{1}{\sigma_c} (\hat{\pi}_t - E_t\pi_{t+1}) \]  

(3.17)

where \(\pi_{t+1}\) is inflation and the rest of variables and parameters are defined as in section 2.

b) Money Demand:

\[ \hat{m}_t = \eta_c \hat{Y}_t - \eta_i \hat{i}_t + \xi_t \]  

(3.18)

where \(\xi_t\) is a money demand shock that follows an autoregressive process of the form:

\[ \xi_t = \rho \xi_{t-1} + \eta_t \]  

(3.19)

where \(\eta_t\) is an i.i.d. mean zero innovation with variance \(\sigma^2_q\).

c) AS Curve:

\[ \pi_t = \frac{(1 - \alpha)(1 - \alpha \beta)(\omega_w + \omega_p + \sigma^{-1} - \eta_c \chi)}{\alpha(1 + (\omega_w + \omega_p)\theta)} \tilde{m_c}_t + \beta E_t\pi_{t+1} \]  

with

\[ \tilde{m_c}_t = x_t + \frac{\eta_c \chi}{\omega_w + \omega_p + \sigma^{-1} - \eta_c \chi} \hat{i}_t \]  

(3.21)

where \(\tilde{m_c}_t\) is the average real marginal cost and \(x_t\) is the output gap, which is defined as the difference between actual output \((\hat{Y}_t)\), and the natural level of output \((\hat{Y}^n_t)\).\(^{24}\)

Moreover, \(\alpha\) is the fraction of good prices that remain unchanged, \(\beta\) is the discount factor, \(\theta\) is the elasticity of demand, \(\omega_w\) is the elasticity of marginal disutility of work with respect to output and \(\omega_p\) is the negative of the elasticity of marginal product of labor with respect to the level of output. It should also be noticed that in this model the elasticity of real marginal cost with respect to aggregate output is equal to \(\omega_w + \omega_p + \sigma^{-1} - \eta_c \chi\).

d) Natural Output:

\[ \hat{Y}^n_t = \frac{1 + \omega_w + \omega_p}{\omega_w + \omega_p + \sigma^{-1} - \eta_c \chi} \hat{A}_t + \frac{\chi}{\omega_w + \omega_p + \sigma^{-1} - \eta_c \chi} \xi_t \]  

(3.22)

\(^{24}\)The natural output is defined as the equilibrium level of output at each point in time that would be under flexible prices, given a monetary policy that maintains \(i_t = \bar{i}\).
where \(\hat{A}_t\) represents the log deviation of the technology factor with respect to its steady state level. This factor follows an autoregressive process of the form:

\[
\hat{A}_t = \rho_a \hat{A}_{t-1} + \zeta_t
\]  

(3.23)

where \(\zeta_t\) is an i.i.d mean zero technology shock with variance \(\sigma^2_\zeta\).

e) Interest Rate Policy Rule:

\[
\hat{i}_t = \tilde{i}_t + \phi_\pi \pi_t + \phi_x x_t
\]

(3.24)

where \(\phi_\pi > 0, \phi_x > 0\) and \(\tilde{i}_t\) is a monetary policy disturbance that has the following process:

\[
\tilde{i}_t = \rho_\tilde{i} \tilde{i}_{t-1} + \varepsilon_t
\]

(3.25)

where \(\varepsilon_t\) is an i.i.d mean zero shock.

f) Equation for Real Wages:

\[
\hat{w}_t^r = (\omega_w + \sigma^{-1}_e)\hat{Y}_t - \chi \hat{m}_t - \omega_w \hat{A}_t
\]

(3.26)

where \(\hat{w}_t^r\) is the real wage.

### 3.4.1.2 Calibration

The calibrated parameters of the model are:

\[
\begin{align*}
\sigma^{-1}_e &= 1, & \chi &= 0.48, & \eta_c &= 1, & \eta_i &= 7, & \beta &= 0.99, & \theta &= 11, & \omega_w &= 0.09, & \omega_p &= 0.38, & \alpha &= 0.75, \\
\phi_x &= 3.0, & \phi_n &= 0.5, & \rho_\xi &= 0.96, & \rho_a &= 0.95, & \rho_\iota &= 0.7, & \sigma_\eta &= 0.0104, & \sigma_\zeta &= 0.0071, & \sigma_\varepsilon &= 0.0025
\end{align*}
\]

According to the baseline estimates, the size of the ratio \(\frac{\chi}{\sigma_y}\) can take four values: 0.33, 0.38, 0.59 and 0.61. For illustrative purposes, I explore a calibration that sets the size of the real money balance effects equal to the mean of all these possible values. The coefficient of risk aversion is set equal to 1; and consequently, \(\chi = 0.48\).

The values assigned to the parameters of the money demand are consistent with my

\[25\] It is implicit by the solution of the model. It is not assumed ad hoc.
empirical evidence and with other studies as I discussed in section 3. The value for \( \theta \) implies a markup of 10 percent and is taken from Galí et al. (2001). The parameter \( \omega_p \) is obtained by means of the following procedure. I assume a Cobb Douglas aggregate production function of the form \( F(H) = H^\lambda \). Given this production function, \( \omega_p = \lambda^{-1} - 1 \). Then, using the fact that \( \lambda \) is equal to the markup times the labor share (from first order conditions of the firm), \( \omega_p = (1.1 \times 0.66)^{-1} - 1 = 0.38 \). The value for \( \alpha \) is consistent with the macro study performed by Galí and Gertler (1999) and implies that prices are fixed four quarters. This period length is close to the average price duration found with microeconomic evidence\(^{26}\). The coefficients of the interest rate rule are the standard ones of a Taylor rule, except for \( \phi_r \), which is higher than the traditional value of 1.5.\(^{27}\) The parameter \( \omega_w \) is picked by assuming an elasticity of real marginal cost of an individual firm with respect to its output equal to 0.47, which is taken from Rotemberg and Woodford (1997). The calibration of the persistence of the technology factor and the standard deviation of its innovation is the standard one in the literature of Real Business Cycle. The degree of persistence of the money demand shock and the standard deviation of its innovation are calibrated according to my empirical evidence. The persistence of the monetary policy disturbance is calibrated by following Woodford (2003). Finally, the standard deviation of the innovation to the monetary policy disturbance is taken from Ireland (2004).

### 3.4.2 The Modestly Procyclical Real Wage Response to a Monetary Policy Shock

It is a stylized fact that there is a very modest response of real wages relative to the one of output after a monetary policy shock. Studies developed by Altig et al. (2004) and Christiano et al. (2001) support this stylized fact by using an impulse response function derived from a structural VAR. The most common explanation for this is the existence of sticky prices and sticky wages.\(^{28}\) In this section, I show that this stylized fact can also be explained without sticky wages and with real money balance effects.

Figure 3.1 displays the response of real wage and output to a contractionary monetary policy shock in the case when \( \chi = 0.48 \). The solid line represents the response of

---

\(^{26}\)See Nakamura and Steinsson (2007).

\(^{27}\)It is set equal to 3.0 in order to have a determinate equilibrium.

\(^{28}\)See Woodford (2003).
Figure 3.1: Response of Output and Real Wage when $\chi = 0.48$

output whereas the dashed line represents the response of real wage. We can see that the real wage response is much lower than the one of output, as the empirical studies show. Moreover, the difference between these two responses is increased significantly by the addition of real money balance effects. Figure 3.2 displays the responses when there are no such effects and it is clear that the difference between the responses is much lower in this case. In particular, when there are no real money balance effects, the response of real wages is a bit higher than the one of output. The difference between Figures 1 and 2 is explained by two facts: real wage responds more and output responds less when there are no real money balance effects. Then, I can conclude that the existence of quantitatively important real money balance effects can be a way to explain the very modest response of real wages relative to output after a monetary policy shock.

The intuition behind my result is as follows. After a contractionary monetary policy shock, in a model with real money balance effects, both labor demand and labor supply move in the same direction. On the one hand, the monetary contraction reduces the demand for an industry’s output, which means that firms respond by
Figure 3.2: Response of Output and Real Wage when $\chi = 0$

lowering their output and consequently labor demand. On the other hand, it increases the opportunity cost of holding money, and hence, diminishes real money holdings. This diminishment decreases the marginal utility of consumption (given that marginal utility of consumption depends positively on real money balances), and therefore, increases the real wage asked by labor suppliers. The latter means that there is also a reduction in labor supply. Then, the impact of monetary policy is basically on average hours worked (and consequently on output), and not on real wages, given the calibration I propose.

### 3.4.3 The Supply Side Effects of Monetary Policy

Barth and Ramey (2001) show empirically that a monetary policy shock can affect inflation and output also through the supply side. These effects are commonly explained with the cost channel of monetary transmission, which is present when firms’ marginal

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29 Notice that labor supply is given in this model by $W_t P_t = \frac{V_t(H_t)}{V_c(Y_t, m_t)}$. 

72
cost depends directly on the nominal interest rate. In a general equilibrium model, this channel is usually incorporated by assuming that firms must borrow money to pay their wage bill. The need to borrow introduces an additional component to the cost of labor. In this setting, the marginal cost of hiring labor is the real wage multiplied by the gross nominal interest rate. So, when the interest rate increases, the marginal cost of hiring increases because of these effects, and hence, inflation. Notice also that the supply side effects of monetary policy are associated with a shift in the labor demand after a monetary policy shock. In this section, I claim that these effects can be due to the existence of real money balance effects. Moreover, I show that the supply side effects in this case are associated to shifts in labor supply.

The aggregate supply curve, when real money balance effects exist, is given by equations (3.20) and (3.21). By combining these, it can be shown that inflation is not only affected by the output gap but also by the interest rate through the term \( \frac{(1-\alpha)(1-\alpha^3)}{\alpha(1+(\omega_u+\omega_p)\theta)} \eta_i \chi i_t \). In this way, the model with real money balance effects generates the supply side effects. The mechanism by which these effects arise is the following. An increase in the interest rate increases the opportunity cost of holding money, and consequently, diminishes the real money holdings. This diminishment decreases the marginal utility of consumption (given that marginal utility of consumption depends positively on real money balances), and consequently, increases the real wage asked by labor suppliers. Then, this implies that real marginal cost increases, and hence, inflation and the price index also increase.

3.4.4 The Impact of Real Money Balance Effects on the Design of Optimal Monetary Policy

In this subsection, I analyze how real money balance effects affect optimal monetary policy analysis. In particular, I will show how optimal volatility of the economic variables change when real money balance effects are considered in the analysis.

In order to characterize the optimal policy solution, I assume full commitment of the monetary authority and a non distorted steady state. Under these assumptions,
Woodford (2003) shows that the optimal policy problem can be written as:

\[ \text{Min} \sum_{t=0}^{\infty} \beta^t L_t + t.i.p \]

where \( L_t = \pi_t^2 + \lambda_x x_t^2 + \lambda_i \beta_t^2 \) is the quadratic period loss function with the weights (\( \lambda_x \) and \( \lambda_i \) respectively) expressed by the following formulas:\(^{32}\)

\[
\lambda_x = \frac{(1 - \alpha)(1 - \alpha \beta)(\omega_w + \omega_p + \sigma_c^{-1} - \eta_c \chi)}{\alpha \theta (1 + (\omega_w + \omega_p) \theta)}, \quad \lambda_i = \frac{(1 - \alpha)(1 - \alpha \beta) \eta_i}{\alpha \theta (1 + (\omega_w + \omega_p) \theta) \pi}
\]

Before solving this problem, it is convenient to make some comments on these weights. First, when the economy is cashless, \( \pi \) goes to infinity; and, therefore, the weight on the interest rate goes to zero. This is the case in the standard optimal monetary policy analysis. Then, it can be shown in this case that the optimal volatility of inflation and output gap is zero. Second, when money is introduced in the analysis through separable utility in consumption and real money, then \( \lambda_i \) is different from zero. Therefore, there exists a trade-off between stabilizing inflation and the interest rate. Third, by considering non-separable utility (\( \chi \) different from zero), the weight on the output gap diminishes with respect to the case of separable utility.

The optimal monetary policy problem should be solved subject to the constraints imposed by the equations of the model developed in previous subsections. Notice that these equations do not include the interest rate rule because the idea of this part is to derive the optimal policy. After performing a numerical procedure, I find the solution to this problem. Then, by using 100 simulations of 100 years period length, I compute the optimal volatility of the main economic variables under two different scenarios (\( \chi = 0.48 \) and \( \chi = 0 \)).

Table 3.5 presents the simulated standard deviations of the economic variables. The following results emerge. First, when utility is non-separable, the optimal volatility of output and real money balances is much higher. The intuition of this result is as follows. The introduction of non-separability in the utility function decreases the importance of output gap stabilization in favor of inflation and interest rate stabilization. This is also translated in higher volatility of output. Moreover, since output affects

---

\(^{32}\)The abbreviation \( t.i.p. \) in the objective function stands for terms independent of policy.
money through the money demand equation, the volatility of real money increases as well. Second, the optimal volatility of interest rate and real wages is lower. Third, the existence of real money balance effects does not affect significantly the optimal volatility of inflation. Fourth, notice that in the case of separable utility, the optimal volatility of inflation is not zero because money is in the utility function. In this case, as mentioned before, there exists a trade-off between stabilizing inflation and the interest rate. Clearly, this trade-off is solved in favor of inflation stabilization.

<table>
<thead>
<tr>
<th></th>
<th>Separable Utility</th>
<th>Non – Separable Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inflation 1/</strong></td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td><strong>Output</strong></td>
<td>2.36</td>
<td>4.15</td>
</tr>
<tr>
<td><strong>Interest Rate 1/</strong></td>
<td>0.46</td>
<td>0.33</td>
</tr>
<tr>
<td><strong>Real Wage</strong></td>
<td>2.36</td>
<td>1.93</td>
</tr>
<tr>
<td><strong>Real Money</strong></td>
<td>5.10</td>
<td>7.05</td>
</tr>
</tbody>
</table>

1/ Standard deviation expressed in annual terms.

3.4.5 The Diminishment in the Size of Real Money Balance Effects, Greater Macroeconomic Stability and Financial Innovation

Since 1984, the U.S. economy and other industrialized economies have experienced a substantial decline of macroeconomic volatility. This phenomenon is known in the literature as “the Great Moderation”. There exist a lot of potential explanations for this phenomenon. Galí and Gambetti (2007) classify all of them in two groups. The first one suggests that the greater macroeconomic stability is due mainly to smaller shocks hitting the economy (good luck hypothesis). The second group attributes the reduction in macroeconomic volatility to changes in the structure of the economy and/or in the way policy has been performed. Three explanations can be distinguished...
in this group: better monetary policy (Clarida et al. (2000)), improved inventory management (Khan et al. (2002)) and financial innovation (Dynan et al. (2006)).

In this subsection, I show that the diminishment in the size of real money balance effects can explain a significant fraction of the reduction in inflation and output volatility. Moreover, I argue that this result supports financial innovation as source of the Great Moderation. In order to illustrate the first point, I analyze the behavior of the model described in subsection 3.4.1 but assuming two different structures of the economy that only differ in the values for $\chi$ and $\eta_i$. The first one (which I refer to Pre 1984 calibration) assumes that $\chi = 0.80$ and $\eta_i = 4.2$; while the second one (which I refer to Post 1984 calibration) sets $\chi = 0.38$ and $\eta_i = 8.8$. These two different structures are chosen such that the reduction of real money balance effects is present in the model. Notice that the sub-sample stability analysis presented in section 3.3 suggests that the size of real money balance effects has decreased mainly due to a reduction in the elasticity of marginal utility of consumption with respect to real money balances. This is why the risk aversion is kept constant at 1 and the elasticity of marginal utility of consumption with respect to money is changed across the two calibrations (or periods). Moreover, in order to fit the cross equation restriction imposed by the MIU model (equation 3.16), a change in the elasticity of marginal utility of consumption with respect to money requires an adjustment of the interest rate semielasticity, given that the rest of parameters remain constant. This explains why it goes from 4.2 to 8.8.

| Table 3.6 |
| Changes in Volatility 1/|

<table>
<thead>
<tr>
<th></th>
<th>Calibrated Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre 1984</td>
<td>Post 1984</td>
</tr>
<tr>
<td><strong>Inflation</strong></td>
<td>1.00</td>
<td>0.71</td>
</tr>
<tr>
<td><strong>Output</strong></td>
<td>1.00</td>
<td>0.52</td>
</tr>
</tbody>
</table>

1/ Standard deviations in the Pre 1984 period are normalized to 1.

Table 3.6 presents the standard deviation of inflation and output generated by the
model, considering the two different structures of the economy (one before 1984 and the other one after 1984). The Pre 1984 volatilities are normalized to 1 in order to facilitate comparison. It can be seen that the reduction in the size of real money balance effects can account for 89 percent of the decline in output volatility and 50 percent of the decline in inflation volatility. This result suggests that the diminishment of real money balance effects can explain quite well the reduction in output volatility but other explanations, like better monetary policy, are necessary to fully explain the reduction of inflation volatility.

From previous analysis, and by a direct interpretation of the MIU model, it could seem that the decrease in the elasticity of marginal utility of consumption with respect to money is an alternative source of the Great Moderation. I claim that this is not the most appropriate way to understand the results driven by the previous simulation. As Walsh (2003) points out, the MIU approach has to be thought of as a shortcut for a fully specified model of the transactions technology faced by households that give rise to a positive demand for money. Instead, the diminishment in the size of real money balance effects should be interpreted as result of the financial innovation that took place in U.S. in the early 1980’s. In order to support the latter argument, I use the functional equivalence between the transaction cost model developed by Schmitt-Grohé and Uribe (2004) and the MIU model. By using this equivalence, \( \chi \) can be expressed as:

\[
\chi = \frac{\pi(2s'(\pi) + \pi s''(\pi))}{1 + s(\pi) + \pi s'(\pi)}
\]

where \( s(\pi) \) represents a transaction cost that is proportional to consumption purchases, \( s'(\pi) \) denotes the first derivative of the transaction cost function with respect to money velocity and \( s''(\pi) \) represents the second derivative of the same function. Given the previous expression, a plausible story that can explain the decrease in \( \chi \) is a financial innovation that affects the transaction cost function such that a reduction in this parameter takes place. Therefore, this analysis provides formal support to the one developed by Dynan et al (2006), where they conclude that financial innovation is an important source of the greater stability in the economy.
3.5 Concluding Remarks

GMM joint estimation of the Euler equation and money demand, derived from a small structural MIU model, suggests that real money balance effects, arising from non-separable utility in consumption and real money, are still quantitatively important. This finding is consistent with previous reduced form evidence provided by Meltzer (2001), Nelson (2002) and Hafer et al. (2007). However, it contrasts considerably with the results found in previous studies by Woodford (2003) and Ireland (2004). Two important differences with respect to Woodford’s approach explain the discrepancy in results: the definition of money used (money base versus M2) and the estimation procedure (calibration versus GMM). With respect to Ireland’s approach, there are also two important aspects of the procedures that drive the different results: the structure of the model (7 equations versus a subset that include only three of those equations) and the estimation procedure (ML versus GMM).

A sub-sample stability analysis suggests that the size of real money balance effects is still significant but lower than it used to be before the beginning of the 1980’s. The main determinant of this reduction seems to be the diminishment of the elasticity of marginal utility of consumption with respect to money. By using a functional equivalence between the MIU model and the transaction cost model developed by Schmitt-Grohé and Uribe (2004), it has been shown that the decrease in the elasticity of marginal utility of consumption with respect to money can be interpreted as a change in the transaction cost technology that diminishes the importance of real money balances in the determination of consumption or aggregate demand.

There are four important implications of the empirical evidence presented in this chapter. First, money is not redundant in order to determine inflation and output. Second, the existence of quantitatively important real money balance effects in a model with sticky prices and flexible wages can explain two stylized facts: the modestly procyclical real wage response to a monetary policy shock and the supply side effects of monetary policy. Third, the optimal monetary policy changes when there exist real money balance effects of the magnitude estimated in this study. In particular, much higher volatility of output and much lower volatility of interest rate should be attained. Fourth, the reduction in the size of real money balance effects can account for a significant reduction in the volatility of inflation and output. This suggests that financial innovation, through a technological progress in the transaction technology,
can be a source of the Great Moderation.

Finally, this study uses the MIU approach in the estimation process for the following reason. Given that the conclusion that real money balance effects play a minimal role in the monetary business cycle was derived by using this model and two different structural estimation techniques, the use of the same model allows a clear and direct comparison of my analysis with those of previous studies. Nonetheless, the MIU model has to be thought of as shortcut of a fully specified model of transaction technology. In this sense, the evidence provided in this paper supports that it would be worth exploring the development of models that provide plausible and clear stories that generate the MIU model with non-separable utility in consumption and real money balances. So far, there exist microfounded models that provide a framework to show that the evolution of the size of real money balance effects, arising from non-separable utility, is related with the evolution of the transaction technology. However, none of these models provide a clear understanding on how to link the transaction technology (in the form of a transaction cost function, for instance) with the transaction frictions we observe in reality.
Appendix A

Addendum to Chapter 1

A.1 The Adjusted Output Dispersion (d_t)

The adjusted output dispersion, up to a second order approximation, can be expressed as:

\[ d_t = \frac{\epsilon}{2\Theta} \text{Var}_i \{ p_t(i) \} + \frac{1}{2\alpha} \text{Var}_i \{ a_t(i) \} + \frac{\epsilon}{\alpha} \text{Cov}_i \{ p_t(i), a_t(i) \} \]  

(A.1)

Proof: First, notice that the adjusted output dispersion (in logs) can be written as:

\[ d_t = \alpha \log \int_0^1 \left( \frac{P_t(i)}{A_t(i)} \right)^{-\epsilon/\alpha} \left( \frac{1}{A_t(i)} \right)^{1/\alpha} di \]  

(A.2)

Then, a second order approximation of \( d_t \) around a zero inflation steady state is given by\(^1\):

\[ d_t = \alpha \left[ -\frac{\epsilon}{\alpha} \int_0^1 (p_t(i) - p_t) di + \frac{1}{2} \left( \frac{\epsilon}{\alpha} \right)^2 \int_0^1 (p_t(i) - p_t)^2 di + \frac{1}{2\alpha^2} \int_0^1 a_t^2(i) di \right] \]  

(A.3)

\(^1\)It has been taken into account that \( a_t(i) \) has a zero mean, which means that \( \int_0^1 a_t(i) di = 0 \).
Now, by taking into account that 
\[ \int_0^1 (p_t(i) - p_t) \, di = -(1 - \epsilon) \int_0^1 (p_t(i) - p_t)^2 \, di \] (from a second order approximation around a zero inflation of the identity of the price level), the following expression arises:

\[
d_t = \frac{\epsilon}{2\Theta} \int_0^1 (p_t(i) - p_t)^2 \, di + \frac{1}{2\alpha} \int_0^1 \alpha_t^2(i) \, di + \frac{\epsilon}{\alpha} \int_0^1 (p_t(i) - p_t) \alpha_t(i) \, di \quad (A.4)
\]

Finally, by noticing that \( p_t \) is the mean of \( p_t(i) \) and that \( \alpha_t(i) \) has mean zero, then (A.4) can be written as (A.1).
A.2 The Relationship between $d_t - d^n_t$ and the Dispersion of Price Gaps Across Goods

The difference between $d_t$ and $d^n_t$ can be expressed as follows:

$$
\begin{align*}
   d_t - d^n_t &= \frac{\epsilon}{2\Theta} Var_i \left\{ p_i(i) - p_f^t(i) \right\} \\
   &= \frac{\epsilon}{2\Theta} \left[ Var_i \left\{ p_i(i) \right\} + 2Cov_i \left\{ p_i(i), p_f^t(i) \right\} \right]
\end{align*}
$$

(A.5)

Proof: First, up to a second order approximation around a zero inflation, it holds that:

$$
\begin{align*}
   Var_i \left\{ p_i(i) - p_f^t(i) \right\} &= Var_i \left\{ p_i(i) \right\} + Var_i \left\{ p_f^t(i) \right\} - 2Cov_i \left\{ p_i(i), p_f^t(i) \right\} \\
   &= Var_i \left\{ p_i(i) \right\} + Var_i \left\{ p_f^t(i) \right\} - \frac{\Theta}{\alpha} Cov_i \left\{ p_i(i), a_t(i) \right\}
\end{align*}
$$

(A.6)

Now, notice that the frictionless price $p_f^t(i)$ is given by the following expression:

$$
\begin{align*}
   p_f^t(i) = \Theta \left[ -\log \alpha + \gamma - \frac{1}{\alpha} a_t + \frac{1}{\alpha} (\epsilon p_t + c_t) - \frac{1}{\alpha} a_t(i) \right]
\end{align*}
$$

(A.7)

Then, by using (A.7), $Cov_i \left\{ p_i(i), p_f^t(i) \right\} = -\Theta \alpha Cov_i \left\{ p_i(i), a_t(i) \right\}$. This means that if both sides of (A.6) are multiplied by $\frac{\epsilon}{2\Theta}$ and terms are rearranged, the following expression holds:

$$
\begin{align*}
   \frac{\epsilon}{2\Theta} Var_i \left\{ p_i(i) \right\} + \frac{\epsilon}{\alpha} Cov_i \left\{ p_i(i), a_t(i) \right\} &= \frac{\epsilon}{2\Theta} \left[ Var_i \left\{ p_i(i) - p_f^t(i) \right\} - Var_i \left\{ p_f^t(i) \right\} \right]
\end{align*}
$$

(A.8)

The latter expression is useful to find an alternative expression of $d_t$ that relates it with $Var_i \left\{ p_i(i) - p_f^t(i) \right\}$. In fact, by combining (A.1) and (A.8), I have:

$$
\begin{align*}
   d_t &= \frac{\epsilon}{2\Theta} \left[ Var_i \left\{ p_i(i) - p_f^t(i) \right\} - Var_i \left\{ p_f^t(i) \right\} \right] + \frac{1}{2\alpha} Var_i \left\{ a_t(i) \right\}
\end{align*}
$$

(A.9)

Using the latter, the adjusted output dispersion under flexible prices is given by:

$$
\begin{align*}
   d^n_t &= -\frac{\epsilon}{2\Theta} Var_i \left\{ p_f^t(i) \right\} + \frac{1}{2\alpha} Var_i \left\{ a_t(i) \right\}
\end{align*}
$$

(A.10)

Finally, by subtracting (A.10) from (A.9), (A.5) is obtained.
A.3 Proofs of Lemmas

Lemma 1: The adjusted output dispersion \( d_t \), up to a second order approximation, is given by the following expression:

\[
d_t = \frac{\epsilon}{2\Theta} \text{Var}_i \{p_t(i)\} + \frac{1}{2\alpha} \text{Var}_i \{a_t(i)\} + \frac{\epsilon}{\alpha} \text{Cov}_i \{p_t(i), a_t(i)\}
\]  
(A.11)

**Proof:** See appendix A.

Lemma 2: The adjusted output dispersion under flexible prices \( d^n_t \), up to a second order approximation, can be written as:

\[
d^n_t = \frac{1}{2\alpha} \theta \text{Var}_i \{a_t(i)\}
\]  
(A.12)

**Proof:** By using lemma 1, \( d^n_t \) can be expressed as:

\[
d^n_t = \frac{\epsilon}{2\Theta} \text{Var}_i \{p_t(i)\} + \frac{1}{2\alpha} \text{Var}_i \{a_t(i)\} + \frac{\epsilon}{\alpha} \text{Cov}_i \{p_t(i), a_t(i)\}
\]  
(A.13)

Then, by considering (A.7) and the fact that the idiosyncratic shocks and the aggregate variables are uncorrelated, it is straightforward to find \( \text{Var}_i \{p_t(i)\} \) and \( \text{Cov}_i \{p_t(i), a_t(i)\} \) as a function of \( \text{Var}_i \{a_t(i)\} \). More precisely:

\[
\text{Var}_i \{p_t(i)\} = \left( \frac{\Theta}{\alpha} \right)^2 \text{Var}_i \{a_t(i)\}
\]  
(A.14)

\[
\text{Cov}_i \{p_t(i), a_t(i)\} = \frac{-1}{\alpha + (1-\alpha)\epsilon} \text{Var}_i \{a_t(i)\}
\]  
(A.15)

Finally, by plugging (A.14) and (A.15) into (A.13) and adding all the resulting terms, lemma 2 is found.

Lemma 3: The variance of prices across goods, up to a second order approximation, is given by the following expression:

\[
\text{Var}_i \{p_t(i)\} = \frac{\theta}{(1-\theta)} \sum_{j=0}^{\infty} \theta^j \pi_{t-j}^2 + (1-\theta)\phi^2 \sum_{j=0}^{\infty} \theta^j \text{Var}_i \{a_{t-j}(i)\}
\]  
(A.16)

where \( \phi = \frac{(1-\beta\theta)\Theta}{(1-\beta\theta)\alpha} \).
Proof: The proof can be divided into four steps.

Step 1: Define \( E_t = \int_0^1 \left( \frac{P_t(i)}{\lambda} \right)^{-\epsilon/\alpha} di \). Then, up to a second order approximation around zero inflation, this variable (in logs) can be expressed as:

\[
e_t = \frac{\epsilon}{2\Theta\alpha} Var_i \{ p_t(i) \} \tag{A.17}
\]

Step 2: Take into account the Calvo price setting. In this case, \( e_t \) can be written as:

\[
e_t = \frac{\epsilon}{2\Theta\alpha} \left[ \theta \int_0^1 (p_{t-1}(i) - p_t)^2 di + (1 - \theta) \int_0^1 (p_t(i) - p_t)^2 di \right] \tag{A.18}
\]

Now, by plugging \( p_t = p_{t-1} + \pi_t \) and \( p_t(i) = p_t^C - \phi a_t(i) \) into (A.18); and considering that \( p_t^C - p_t = \frac{\theta}{1-\theta} \pi_t \) holds, then after some algebra I get:

\[
e_t = \frac{\epsilon}{2\Theta\alpha} \left[ \theta Var_i \{ p_{t-1}(i) \} + \frac{\theta}{1-\theta} \pi_t^2 + (1 - \theta) \phi^2 Var_i \{ a_t(i) \} \right] \tag{A.19}
\]

Step 3: Combine (A.17) and (A.19) in order to find:

\[
Var_i \{ p_t(i) \} = \theta Var_i \{ p_{t-1}(i) \} + \frac{\theta}{1-\theta} \pi_t^2 + (1 - \theta) \phi^2 Var_i \{ a_t(i) \} \tag{A.20}
\]

Step 4: Considering that \( 0 \leq \theta < 1 \), (A.20) can be solved backward in order to obtain (A.16).

Lemma 4: The covariance between prices and the idiosyncratic productivity shocks can be expressed as:

\[
Cov_i \{ p_t(i), a_t(i) \} = -\phi(1 - \theta) \sum_{j=0}^{\infty} \theta^j Cov_i \{ a_t(i), a_{t-j}(i) \} \tag{A.21}
\]

Proof: First, use the following definition:

\[
Cov_i \{ p_t(i), a_t(i) \} = \int_0^1 (p_t(i) - p_t) a_t(i) di \tag{A.22}
\]
Second, considering the Calvo price setting, the previous expression can be written as:

\[
Cov_i \{p_t(i), a_t(i)\} = \theta \int_0^1 (p_{t-1}(i) - p_t) a_t(i) di + (1 - \theta) \int_0^1 (p_t^C(i) - p_t) a_t(i) di \quad (A.23)
\]

Third, by plugging \(p_t = p_{t-1} + \pi_t\) and \(p_t^C(i) = p_t^C - \phi a_t(i)\) into the latter equation; and considering that the idiosyncratic and the aggregate variables are uncorrelated, I get after some algebra:

\[
Cov_i \{p_t(i), a_t(i)\} = \theta Cov_i \{p_{t-1}(i), a_t(i)\} - (1 - \theta) \phi Var_i \{a_t(i)\} \quad (A.24)
\]

If the previous steps are repeated to find an expression for \(Cov_i \{p_{t-1}(i), a_t(i)\}\), I get:

\[
Cov_i \{p_t(i), a_t(i)\} = \theta^2 Cov_i \{p_{t-2}(i), a_t(i)\} - \phi(1 - \theta) \sum_{j=0}^{1} \theta^j Cov_i \{a_t(i), a_{t-j}(i)\} \quad (A.25)
\]

If this process is repeated infinitely many times, I get (A.21).

**Lemma 5:** As \(t \to \infty\), \(Var_i \{a_t(i)\} = \frac{\sigma_a^2}{1 - \rho} = \sigma_a^2\)

**Proof:** Without loss of generality, assume that the initial level of productivity (in logs) for every firm is zero. Then, at time 0, it holds that \(a_0(i) = \varepsilon_0(i)\). Then, \(Var_i \{a_0(i)\} = \sigma_\varepsilon^2\). Now, at \(t = 1\), \(a_1(i) = \rho a_0(i) + \varepsilon_1(i)\), which implies that \(Var_i \{a_1(i)\} = \sigma_\varepsilon^2 (1 + \rho^2)\). In general, at \(t = n\), \(Var_i \{a_n(i)\} = \sigma_\varepsilon^2 (1 + \rho^2 + ... + \rho^{2n})\).

Therefore, as \(t \to \infty\), I get lemma 5.

**Lemma 6:** \(Cov_i \{a_t(i), a_{t-j}(i)\} = \rho^j \sigma_a^2\)

**Proof:** Notice that \(Cov_i \{a_0(i), a_1(i)\} = \rho Var_i \{a_1(i)\}\);
\(Cov_i \{a_2(i), a_1(i)\} = \rho^2 Var_i \{a_1(i)\}\).

In general, \(Cov_i \{a_t(i), a_{t-j}(i)\} = \rho^j Var_i \{a_{t-j}(i)\}\).

Finally, applying lemma 5 to the previous expression, I get lemma 6.

**Lemma 7:** \(Cov_i \{p_t(i), p_{t-1}(i)\} = \theta Var_i \{p_{t-1}(i)\} - \phi(1 - \theta) Cov_i \{a_t(i), p_{t-1}(i)\}\)

**Proof:** Up to a second order approximation, the following identity holds:

\[
Cov_i \{p_t(i), p_{t-1}(i)\} = \int_0^1 (p_t(i) - p_t)(p_{t-1}(i) - p_{t-1}) \quad (A.26)
\]
Considering the Calvo price setting, the latter expression can be simplified until the lemma is finally proved in the following way:

\[
Cov_i \{p_t(i), p_{t-1}(i)\} = \theta \int_0^1 (p_{t-1}(i) - p_t) (p_{t-1}(i) - p_{t-1}) \, di \\
+ (1 - \theta) \int_0^1 (p_t^*(i) - p_t) (p_{t-1}(i) - p_{t-1}) \, di \\
= \theta \int_0^1 (p_{t-1}(i) - p_{t-1} + \pi_t) (p_{t-1}(i) - p_{t-1}) \, di \\
+ (1 - \theta) \int_0^1 (p_t^C - \phi a_t(i) - p_t) (p_{t-1}(i) - p_{t-1}) \, di \\
= \theta Var_i \{p_{t-1}(i)\} - \phi(1 - \theta) Cov_i \{a_t(i), p_{t-1}(i)\}
\]
### A.4 Decomposing the Variance of Price Changes

The variance of price changes across goods, up to a second order approximation around zero inflation, is given by the following identity:

$$\text{Var}_i \{\pi_t(i)\} = \text{Var}_i \{p_t(i)\} - 2\text{Cov}_i \{p_t(i), p_{t-1}(i)\} + \text{Var}_i \{p_{t-1}(i)\}$$  \hspace{1cm} (A.27)

By using lemma 7, the previous relationship can be expressed as:

$$\text{Var}_i \{\pi_t(i)\} = \text{Var}_i \{p_t(i)\} + \kappa \text{Var}_i \{p_{t-1}(i)\} + \lambda \text{Cov}_i \{a_t(i), p_{t-1}(i)\}$$  \hspace{1cm} (A.28)

where $\kappa = (1 - 2\theta)$ and $\lambda = 2\phi(1 - \theta)$.

From (A.28), $\text{Var}_i \{p_{t-1}(i)\}$ can be expressed as:

$$\text{Var}_i \{p_{t-1}(i)\} = \frac{\text{Var}_i \{\pi_t(i)\} - \text{Var}_i \{p_t(i)\} - \lambda \text{Cov}_i \{a_t(i), p_{t-1}(i)\}}{\kappa}$$  \hspace{1cm} (A.29)

Then, by plugging (A.29) into (A.20) and rearranging terms, I obtain:

$$\text{Var}_i \{p_t(i)\} = \left(\frac{\theta}{1 - \theta}\right) \left[\text{Var}_i \{\pi_t(i)\} + \frac{\kappa}{1 - \theta} \pi_t^2\right]$$

$$- 2\phi \theta \text{Cov}_i \{a_t(i), p_{t-1}(i)\} + \kappa \phi^2 \text{Var}_i a_t(i)$$  \hspace{1cm} (A.30)

Combining the previous expression with (A.16), I get:

$$\text{Var}_i \{\pi_t(i)\} = \sum_{j=0}^{\infty} \theta^j \pi_{t-j}^2 - \left(\frac{1 - 2\theta}{1 - \theta}\right) \pi_t^2 + \text{idio}$$  \hspace{1cm} (A.31)

where $\text{idio}$ is given by:

$$\text{idio} = \frac{(1 - \theta)^2}{\theta} \phi^2 \sum_{j=0}^{\infty} \theta^j \text{Var}_i a_{t-j}(i) + \lambda \text{Cov}_i \{a_t(i), p_{t-1}(i)\}$$

$$- \frac{(1 - \theta)\kappa \phi^2}{\theta} \text{Var}_i a_t(i)$$  \hspace{1cm} (A.32)

Some comments about expression (A.31) are useful. First, notice that the variance
of price changes across goods can be decomposed in two parts: one that is driven by aggregate shocks (summarized by current aggregate inflation and its lags) and another one that is driven by the idiosyncratic productivity shock ($\text{idio}$). Second, considering an annual inflation of 3 percent ($0.03/12$ in monthly frequency), an average price duration of 5 months ($\theta = 0.8$), and the baseline values set in Chapter 1 for $\beta, \epsilon, \alpha$, and $\rho$, the model predicts that the standard deviation of monthly individual price changes across goods is 0.7 percent. However, in the data, the standard deviation is 4.6 percent$^2$. The introduction of idiosyncratic productivity shocks helps the model to fit much better this dimension of the data.

$^2$Midrigan (2006) reports that the standard deviation of price changes, conditional on price adjustment is 10.4 percent. Then, assuming that 80 percent of prices do not change (consistent with the average duration he found), it can be inferred that the standard deviation of all price changes (including zeros) is 4.6 percent.
A.5 Calibration of the Variance of the Idiosyncratic Productivity Shock

In this appendix, I show the way how to derive (1.39) in order to calibrate the variance of the idiosyncratic productivity shock. First, it is convenient to find an expression for \( \text{idio} \) that depends only on the current and past cross sectional variances of the idiosyncratic productivity shock. By looking at (A.32), it is clear that I should obtain an alternative expression for \( \text{Cov}_i \{ a_t(i), p_{t-1}(i) \} \). The way to find it consists in plugging (A.21) into (A.24) to obtain, after rearranging some terms, the following expression:

\[
\text{Cov}_i \{ a_t(i), p_{t-1}(i) \} = -\frac{(1 - \theta)\phi}{\theta} \sum_{j=1}^{\infty} \theta^j \text{Cov}_i \{ a_t(i), a_{t-j}(i) \}
\]  

(A.33)

By replacing the previous expression into (A.32), and considering that a large enough period of time has passed until now (as \( t \to \infty \)), I can apply lemmas 5 and 6 in order to write \( \text{idio} \) as:

\[
\text{idio} = \frac{2(1 - \theta)(1 - \rho)\phi^2}{(1 - \theta\rho)} \frac{\sigma_z^2}{1 - \rho^2}
\]

(A.34)

As \( t \to \infty \), I can also assume that \( \pi_t = \pi \). Therefore, as \( t \to \infty \), the cross sectional variance of price changes is given by:

\[
\text{Var}_i \{ \pi(i) \} = \frac{2\theta}{1 - \theta} \pi^2 + \frac{2(1 - \rho)(1 - \theta)\phi^2}{(1 - \theta\rho)} \frac{\sigma_z^2}{1 - \rho^2}
\]

(A.35)

Finally, rearranging terms, I obtain the following expression for the variance of the idiosyncratic productivity shock:

\[
\sigma_z^2 = \frac{(1 - \theta\rho)(1 - \rho^2)}{2(1 - \rho)(1 - \theta)\phi^2} \left\{ \text{Var}_i \{ \pi(i) \} - \frac{2\theta}{1 - \theta} \pi^2 \right\}
\]

(A.36)
Bibliography


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