Sticky Information and Non-pricing Policies in DSGE Models

by

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B.A. (Università di Roma "La Sapienza") 2001
M.S. (Universitat Pompeu Fabra) 2004

A dissertation submitted in partial satisfaction of the
requirements for the degree of
Doctor of Philosophy

in

Macroeconomics

in the

DEPARTMENT OF ECONOMICS and BUSINESS
of the
UNIVERSITAT POMPEU FABRA

Thesis Advisor:
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(IAE and UPF)

Summer 2008

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Abstract

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This thesis is organized in two parts. In the first one, I seek to understand the relationship between frictions in information flows among firms and inflation persistence. To this end, I present a novel estimator for the Sticky Information Phillips Curve (Mankiw and Reis, 2002), and I use it to estimate this model with U.S. postwar data. The second part presents new evidence about aggregate advertising expenditures in U.S., and analyzes the effect of advertising in the aggregate economy by the mean of a dynamic stochastic general equilibrium model. Chapter 2 focuses on the short run impact of advertising on the aggregate dynamics, and chapter 3 on its long run effects.

Ai miei genitori,

Enrico and Fabrizia,

che mi hanno sempre accudito

e lasciato libero di cercare

la mia strada

Contents

Acknowledgments

This thesis was written during my stay in Barcelona. Many people contributed to make this stay an amazing experience in my life, and all of them contributed in various ways to this thesis through discussions, exchange of ideas, and nice talks. To them go my best acknowledgements. In particular, some people deserve a special mention. The first one is my advisor Albert Marcet that supported and encouraged me throughout my Ph.D. course. His trust in my work has been the best incentive to complete this thesis. Another one is my friend and co-author Francesco Turino. In the last 5 years we shared invaluable talks about economics and life, we worked jointly on a nice piece of economic writing, and we spent together many hours of study that I'll never forget.

Some other people worth a special mention, in particular Giacometta Zucconi, Guidotto Colleoni, William Russel, Giampiero Daví, Marcello Dececco, Pierluigi and Nicoletta Ciocca, our great secretary staff Marta Araque and Marta Aragay, Davide Debortoli, Luca Gambetti, Aniol Llorente, and Paolo Ghinetti.

Finally, a special acknowledgement goes to Carolina Maria Sales Gomes da Silva and to my parents, whom this thesis is dedicated to.

Part I

Introduction

This thesis is organized in two parts, three chapters and an introduction. In the first part I analyze the role of sticky information in explaining inflation persistence. The first section of chapter 2 explains in details what sticky information is, and introduces the theory of pricing with sticky information, i.e. the Sticky Information Phillips Curve originally proposed by Markiw and Reis (2002). The contribution of this part is both econometric and economic. First, I show how to bring this model to the data, providing an estimator that overcomes the problem of infinite dimensions usually associated with estimations of the Sticky Information Phillips Curve. In particular, I propose to estimate this model matching the covariance between current inflation and current and lagged exogenous shocks that enters in firms' decisions about pricing. The main result is that the Sticky Information Phillips Curve cannot jointly match all the selected covariances from the data. This result hinges on the fact that the Sticky Information Phillips Curve can eventually match inflation persistence only at the cost of mispredicting inflation variance. I conclude that the Sticky Information Phillips Curve is a valid model to explain inflation persistence, but not an overall valid theory of inflation.

The second part of the thesis is about the macroeconomics of advertising. The first chapter seeks to understand the observed behavior of advertising over the Business Cycle, and its effect on the aggregate dynamics. To this end, firstly we showed that actual data of U.S. aggregate advertising expenditures have a well-behaved pattern over the Business Cycle. Secondly, we build a dynamic stochastic general equilibrium model that can rationalize this pattern within the neoclassical growth models theory. Thirdly, we show that a loglinearized version of the model can fit very well actual data of U.S. aggregate advertising. Finally, we make use of the estimated model to assess the importance of aggregate advertising in explaining the level of aggregate consumption. Our main finding is that advertising affects aggregate consumption and, through this channel, the entire aggregate dynamics. This result hinges on the significativeness of aggregate advertising as explanatory variable of the volatility of the data on aggregate consumption.

The third chapter is devoted to analyze the long run effect of advertising on the supply of labor of the agents. Using the model presented in chapter 2, we show that advertising substantially affects the steady state of labor. The presence of advertising results in a higher level of worked hours, output, and consumption. Also, our model predicts that advertising can be potentially important in explaining the observed differences in worked hours between U.S. and the European countries. We propose a counterfactual exercise with a calibrated version of the model to bring evidence in support of this intuition. Finally, by the mean of a welfare analysis we show that despite the higher consumption generated by advertising, consumers are always worse off with advertising. Welfare losses are driven in the model by the "overworking" effect induced by advertising.

Part II

Sticky Information

Chapter 1

Sticky Information and Inflation Persistence: Evidence from U.S. data

1.1 Introduction

This chapter estimates the degree of information stickiness in U.S. post-war economy. The model I refer to is the Sticky Information Phillips Curve (henceforth SIPC), proposed by Mankiw and Reis (2002, henceforth MR) as structural theory of inflation.

MR goal was to understand why actual inflation responds gradually to a large number of different shocks, as observed in post-war U.S. data. As a matter of fact, several theories of prices, such as the neoclassical model with no nominal rigidities and the New Keynesian Phillips Curve, predict an inflation dynamics far less persistent than what we observe in actual data.[1] In general, if firms maximize profits and have

[1] Actually, MR presented the SIPC as an alternative theory to the New Keynesian Phillips Curve, which was criticized because it lacks of persistence. The criticisms pointed out that: (i) actual inflation responds gradually to monetary policy shocks, while NKPC implies an immediate adjustment (Mankiw 2001); (ii) output losses typically accompany a reduction in inflation, while this is not true

rational expectations, they will react to any exogenous shock adjusting their prices to the new target level as soon as they acquire information about the shock. Hence, the effect of a shock on price changes disappears rapidly, and the only source of persistence in inflation dynamics can be the one of the exogenous shocks (e.g. cost push shocks, monetary policy shocks, demand shocks).

To overcome the lack of intrinsic persistence in inflation dynamics, MR model is based on the idea that firms absorb only sporadically the information they need to choose their price plans. In those periods when information inflows are limited or absent, firms set prices based on outdated information. When a shock occurs only a fraction of firms adjust the price contemporaneously, while the other firms delay some periods in adjusting their prices to the "new" shock, meanwhile relying on outdated price plans. Thus, the overall effect of shocks on changes of prices lasts in time, and inflation turns out to be a persistent process as real data suggest.

The key parameter that controls for inflation persistence in the model is the frequency of firms' information updating λ. In fact, holding equal the persistence of the exogenous shocks, for high (low) values of λ the SIPC model predicts low (high) persistence of inflation. This point can be shown with an easy example. Following Reis (2004),[2] I simulate the SIPC assuming a simple univariate $AR(1)$ model for the exogenous shocks. In figure (**??**), for all the values of $\lambda \in (0,1]$ I plot the ratio between the first autocovariance function, $acf(1)$, of fitted inflation from the SIPC model over the $acf(1)$ of actual inflation. The $acf(1)$ can be used in this example as a measure of persistence. From figure (A.1) is apparent that the higher is λ, the smaller is the percent of actual persistence explained by the model.

In the literature there isn't consensus on the estimates of λ. Reis (2004) suggested

with NKPC (Able and Bernanke, 1998); (iii) NKPC implies that announced disinflation causes a boom, while in real economy it is the opposite (Ball 1994).

[2] I thank Ricardo Reis for making his codes freely available on the web.

that $\lambda = 0.25$[3] is the best parameter for the SIPC to match the persistence of U.S. postwar inflation. Later on, Khan and Zhu (2002, 2006), Kiley (2006) and Korenok (2005) estimated $\lambda_T \in [0.15, 0.4]$ using limited information estimators, while Laforte (2006), Mankiw and Reis (2006) estimated $\lambda_T \in [0.7, 0.85]$ using full information estimators.[4] As we can see from figure (**??**), there is a relevant difference in the degree of actual inflation persistence explained by the model for λ_T belonging to one or the other range.[5]

In this chapter I propose to estimate the model exploiting the information contained in the covariance functions between current inflation and current and lagged exogenous shocks that are relevant in firms' pricing decisions. These covariances seem the appropriate moments to estimate the SIPC because the more firms' are inattentive to new shocks (low λ), the longer a shock today will affect future price plans, and so the longer it will be correlated with inflation.

To pursue this estimation strategy, I write the expectations terms that appear in the SIPC as functions of forecast errors, and then the forecast errors in terms of exogenous shocks.[6] Once the model is transformed in this way, it is easy to derive a set of orthogonality conditions that are based on the covariances between inflation and the exogenous shocks. I show them in section 3.1. These orthogonality conditions are estimated pursuing a two steps approach: first, I fit a vector autoregression (VAR) model for the exogenous shocks, which is used to calculate the covariances we need. Second, I use the simulated moments to estimate λ with the GMM.

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[3]This is the calibration originally used in MR (2002) paper. In a model where a period is equal to one quarter, $\lambda = 0.25$ implies that firms update information on average once a year.

[4]These are "full information" estimators in the sense that the SIPC is estimated – as aggregate supply equation – joint with an equation for aggregate demand, and an equation for nominal interest rate adjustment (e.g. the Taylor rule) in a fully fledge model of the aggregate economy.

[5]Note that the SIPC with $\lambda = 1$ encompasses the RE model with monopolistic competitive firms and flexible prices.

[6]The idea of writing the model as function of exogenous errors is used also in Mankiw and Reis (2006), and in Wang and Wen (2006). However, the results presented in this paper are contemporaneous and independent from those of the cited papers.

With respect to the other papers that estimated the SIPC using limited information estimators, e.g. Khan and Zhu (2002, 2006), Kiley (2006) and Korenok (2005), this econometric strategy has two advantages: (i) since my orthogonality conditions have a finite number of terms, I avoid the infinite dimensions problem usually associated with the SIPC without using any truncation or approximation of the model. (ii) I use more information about the inflation process to pin down λ_T. This last point is not straightforward to see because the other papers use different methodologies from mine. I make such comparison in section 2.2.

The main finding of this chapter is that the model cannot explain jointly inflation persistence and inflation variance. This happens because the estimates of model parameters are significantly different depending on the moment we are matching. In particular, if we use the model to match the covariances between current inflation and lagged shocks, then it will match inflation persistence by construction, and firms are predicted to update information every 6 to 9 months, i.e. $\lambda_T \in [0.31, 0.58]$ in line with previous estimates. On the contrary, if we use the model to match the conditional variance of inflation, then it predicts that firms update information every 4 months. The discrepancy is not only statistically significant, but also economically relevant because in the second case the SIPC predicts inflation dynamics with little persistence, at odds with the data.

The rest of the chapter is organized as follows: in section 2 I review the SIPC model and the literature about sticky information and inflation persistence. In section 3 I present econometric strategy and results. Section 4 analyzes whether the degree of information stickiness changed during the sample. Some conclusions are given in section 5.

1.2 The Model

1.2.1 The Sticky Information Phillips Curve

Suppose a continuum $(0,1)$ of monopolistic competitive rational firms. They are rational in the sense that in every period use all the information available to set a price p_t^* in order to maximize profits. The main assumption in the SIPC model is that in every period there is only a fraction λ that observe new information about the conditions of the market (e.g. exogenous demand shocks, changes in the nominal marginal cost, etc.).

Accordingly, a fraction λ of firms maximize profits conditional on the newly updated information, while the rest of firms maximize profits conditional on outdated information. In other words, they set a price according to old price plans. Notice that each firm has the same probability to update information, regardless of how long has been since its last update.

Since all firms are ex-ante identical in what they share the same technology and they face demands with the same price elasticity, the optimal price will be the same for all those firms that updated information j periods ago. In particular, these firms adjust today price according to:

$$x_t^j = E\left[p_t^* \mid \Omega_{t-j}\right] \tag{1.1}$$

where x_t^j is price adjustment at period t. All the variables are expressed in logs, and Ω_{t-j} is the information set at period $t-j$.

MR derived the optimal pricing rule in this model, and they showed that inflation evolves according to:

$$\pi_t = \frac{\alpha\lambda}{1-\lambda} y_t + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E\left[\pi_t + \alpha \Delta y_t \mid \Omega_{t-1-j}\right] \tag{1.2}$$

where $\Delta y_t = y_t - y_{t-1}$ is the growth rate of output gap, and λ is the probability that the agent updates his information in period t.[7]

As we can see from equation (1.2), in the SIPC model inflation is persistent because current inflation depends on past periods expectations about the current inflation and output growth, where past expectations are weighted with a weight that fades out at the rate $(1 - \lambda)$. The mechanism is the following: suppose that in period t occurs a shock ε_t that increases the output gap. The information about this shock is included in period t information, i.e. $\varepsilon_t \subset \Omega_t$. According to (1.2), inflation raises contemporaneously because of the trade off term $\left(\frac{\alpha\lambda}{1-\lambda} y_t\right)$. In period $t+1$, when (1.2) holds for π_{t+1}, a fraction λ of agents gets aware of the shock occurred in t, so inflation raises again because $E\left[\Delta y_{t+1} \mid \Omega_{(t+1)-1-j}\right]$ is positive for $j = 0$. Same happens in $t + 2$, when a fraction $\lambda(1 - \lambda)$ gets aware of the shock, and then in all the following periods $t + j \ for \ j > 1$, when the effect of the shock on inflation fades out at rate $(1 - \lambda)^j$. Hence, in this model a shock today affects future inflation level for infinite periods. This implies that the inflation process is serially correlated for many periods, as real data suggest.

1.2.2 Sticky Information and Inflation Persistence

The model of sticky information belongs to the literature about *Rational Inattention*.[8] This conjecture has been proposed as explanation of the stickiness observed in macroeconomic variables, and it is related with the early papers on limited information of Lucas (1973), Fischer (1977), Taylor (1980), Sims (1998), and Woodford (2001). MR combined some elements of Fisher's and Lucas's contributions and proposed the Sticky Information Phillips Curve to model inflation dynamics.

In their original paper, MR assumed that producers get new information in every

[7] For the proof and the details see Mankiw and Reis (2002).

[8] The name *Rational Inattention* first appeared in Sims (2003).

period with an exogenous probability λ, and they calibrated firms' average information duration of 1 year (in a model where each period is a quarter this corresponds to $\lambda = 0.25$). In that paper MR achieved their goal: fitted inflation responded gradually to several exogenous shocks like demand shocks and monetary policy shocks.

After the original MR work, the SIPC model has been estimated in different ways. Reis (2004) proposed a validation test based on the simulation of the model. First, he provided a rigorous microfoundation of SIPC based on cost-benefit analysis: firms gather new information only if the expected benefit of changing the price is higher than the cost of acquiring the information.[9] Then, he simulated the SIPC calibrating model parameters. Parameter α, which he found to be function of intertemporal elasticity of substitution, of Fisher elasticity of labor, and of the elasticity of demand of single-variety goods, is calibrated within the interval $\alpha \in (0.1, 0.2)$, following the RBC literature about these deep parameters. For λ Reis used the calibration $\lambda = 0.25$ originally proposed by MR. Using these parameters values he showed that the model did a good job in matching some selected moments of the aggregate distribution of prices, including the first autocovariance function of inflation, which he used as measure of persistence. Reis (2004) gave an important contribution to support the SIPC, showing that the model can generate an inflation similar to the actual one. However, there are some reasons of concern with his results. When his simulations he assumed an arbitrary process for the driving force of prices which is highly persistent itself. Therefore, we don't know how much of the persistence of fitted inflation came from the intrinsic dynamics of inflation with sticky information, and how much from the exogenous process.

A different approach is followed in Mankiw Reis and Wolfers (2003) that showed

[9] Reis (2004) is not the only paper that provide a microfoundation of inattentive agents. Branch (2004) explains individual inattentiveness as function of the increase in forecasts accuracy once new information is processed. Hence, in Branch's model the more new information improves the Thail index of forecasts (with respect to outdated information), the more agents are attentive.

how the sticky information conjecture explains well the main features of expectations about inflation, as they are observed in the Michigan Consumers Survey and in the Survey of Professional Forecasters. Moreover, they showed that $\lambda = 0.25$ was the best value for the SIPC to match the moments of the distribution of inflation expectations. Although Mankiw Reis and Wolfers surely provided an evidence that the SIPC model can explain micro data, it is less clear whether their work can be used also to support MR calibration of λ. In fact, there is a good number of examples in the literature where macro models calibrated with micro data does not match the moments of aggregate variables distributions.[10] I don't want to get into this issue here. My point is just to underline that a proper estimation of the SIPC using macro data seems of necessity to draw conclusions about the relationship between sticky information and inflation persistence. In the last two years this has been done by several papers. Among the others, Khan and Zhu (2002, 2006), Kiley (2006) Korenok (2005), Laforte (2006), Mankiw and Reis (2006). The reason why I propose another estimation of the SIPC is mainly because the estimates of λ_T vary a lot among those papers, as I showed in the introduction.

Also, the estimator I pursue here improves the ones used by the other papers because it exploits more information about inflation. To show that I focus on those papers that use limited information estimators (e.g. Khan and Zhu, Coibion, Kiley) as I do in this chapter. In general, the estimation strategies of the other papers have a common first step. They truncate the infinite sum of expectations in equation (1.2) at $t - j_{\max}$, and then they substitute the remaining expectations terms with the predictions of a VAR model set ad-hoc to forecast inflation and output gap. For instance, today expectations conditional on information dated $t - 5$ are replaced by

[10] One example is the Frish labor supply elasticity in the standard RBC model. The micro evidences about this elasticity point a value around 1/6, whereas to fit aggregate labor volatility the RBC model need a value of close to 1, so much higher.

$proj_{t-5}(\pi_t + \alpha\Delta y_t)$. Thus, the specification they estimate is:

$$\pi_t = \frac{\alpha\lambda}{1-\lambda} y_t + \lambda proj_{t-1}(\pi_t + \alpha\Delta y_t) + ... + \lambda(1-\lambda)^{j_{\max}} proj_{t-1-j_{\max}}(\pi_t + \alpha\Delta y_t)$$

Now, since the $proj_{t-j}(\pi_t + \alpha\Delta y_t)$ is a linear combination of lagged inflation and output gap,[11] what they do is a nonlinear regression of inflation at time t on $t - j_{\max}$ lags of inflation and output gap. In turns, this means that they exploit the covariances between inflation and lagged inflation and output gap terms.

Hence, they don't exploit any information regarding the covariance between current inflation and contemporaneous shocks. On the contrary, the simulated moments estimator used in this chapter allows us to estimate λ matching jointly the moments that measure inflation persistence, i.e. the covariances between inflation and lagged shocks, and the moments that measure the conditional variance of inflation, i.e. the covariance between current inflation and contemporaneous shocks. If the SIPC model is the true DGP, then it should match all the moments jointly or just a subset of them for same values of parameters. The bulk of my results points out that with the SIPC this is not the case. In section 3.3 we see that estimates of λ changes substantially depending on the selected set of moments.

1.3 The Estimation

1.3.1 Econometric strategy

I use here the standard assumption that the dynamics of inflation and output gap result from the interaction of n macroeconomic variables, which I define as elements of

[11] The sentence should read: "since the $proj_{t-j}(\pi_t + \alpha\Delta y_t)$ is a linear combination of lagged inflation and output gap, plus past values of other variables possibly included in the VAR." The specification of the VAR model differs in all the papers cited above, therefore it is not possible to make general statement about the information that come from other variables. However, the bulk of the argument remains true, since that information regards lagged variables.

a covariance-stationary vector process Z_t. This assumption poses very few structure on inflation and output gap processes, nonetheless it allows to find a useful result:

Lemma 1 *Let* $\{Z_t\}_{t=0}^{\infty}$ *be a covariance stationary* $(n \times 1)$ *vector process s.t.* $\{\pi_t, \Delta y_t\} \subset Z_t$*. Then SIPC (1.2) implies:*

$$\frac{\alpha\lambda}{1-\lambda} y_t + \alpha \Delta y_t = \sum_{i=0}^{\infty} (1-\lambda)^i \, \delta A_i \varepsilon_{t-i} \tag{1.3}$$

where the $(n \times n)$ *matrices* A_i *are the dynamic multipliers of the* Z_t *process, and* ε_t *is a* $(n \times 1)$ *vector of exogenous shocks.* δ *is a* $(1 \times n)$ *row vector that picks up* $(\pi_t + \alpha \Delta y_t)$ *within* Z_t*.*

Proof. See Appendix B.[12] ■

Equation (1.3) is useful to derive a set of orthogonality conditions. Multiplying (1.3) by a vector of lagged shocks ε_{t-i} for $i = 0, ..., l$ and taking the expectations, I obtain:

$$\begin{aligned} E\left[\left(\frac{\alpha\lambda}{1-\lambda} y_t + \alpha \Delta y_t\right) (\delta \varepsilon_{t-i})'\right] &= (1-\lambda)^i \, \delta A_i \Sigma \delta' \\ for \quad i &= 0, \ldots, l \end{aligned} \tag{1.4}$$

where $\Sigma \equiv E\left[\varepsilon_t \varepsilon_t'\right]$ is the VCV matrix of the shocks.[13]

Is useful to see that each orthogonality condition in (1.4) matches a lag of the covariance between Z_t and a linear combination of ε_t. In other words, the RHS of (1.4) is a linear combination of the impulse response functions of Z_t, weighted by the frequency of firms' that don't update information. Intuitively, in the SIPC model this linear combination is function of the output gap process, which is the driving force

[12] This result is similar to those found by Mankiw and Reis (2006) and Wang and Wen (2006). However, I obtained it contemporaneously and independently from those papers.

[13] Equation (1.4) follows multiplying (1.3) by $(\delta \varepsilon_{t-j})'$ and taking the expectations conditional on information at time t. It uses the fact that $E\left[\varepsilon_t \varepsilon_{t-j}\right] = 0$, $j = 1, \ldots$

of prices, and of λ, which measure how many firms are attentive to the shocks, i.e. how rapidly the effect of the shocks on prices fades out.

The moments (1.4) depend on the unknown regressors $\{\varepsilon_t, A_i, \Sigma\}$.[14] In order to estimate them, I pursue a two steps approach. First, I estimate a vector autoregression model of Z_t to obtain consistent estimates of $\{\varepsilon_t, A_i, \Sigma\}$. Second, I estimate the orthogonality conditions (1.4) with the GMM using

$\left\{\widehat{\varepsilon}_t(\beta), \widehat{A}_i(\beta), \Sigma_T(\beta)\right\}|_{\beta=\beta_T^{VAR}}$ as regressors.

This econometric strategy implies that some variables in the second step are generated regressors from the first step. Therefore, to make statistical inference the asymptotic standard errors calculated from the GMM estimator should be corrected.

To do it, we can compute the asymptotic standard errors of λ in a model that estimates jointly the parameters of the VAR(p) and the SIPC. The problem is described formally in the Appendix. The corrected variance of λ_T is then:

$$V(\lambda_T) = \left[(TV_{na}(\lambda_T))^{-1} - E\frac{\partial g_{1,t}'}{\partial \lambda}\Sigma_{g_1}^{-1}E\frac{\partial g_{1,t}}{\partial \beta'}\cdot \left(E\frac{\partial g_{1,t}'}{\partial \beta}\Sigma_{g_1}^{-1}E\frac{\partial g_{1,t}}{\partial \beta'} + E\frac{\partial g_{2,t}'}{\partial \beta}\Sigma_{g_2}^{-1}E\frac{\partial g_{2,t}}{\partial \beta'}\right)^{-1}\cdot E\frac{\partial g_{1,t}'}{\partial \beta}\Sigma_{g_1}^{-1}E\frac{\partial g_{1,t}}{\partial \lambda}\right]^{-1}/T \quad (1.5)$$

where $g_{1,t}$ is the vector of orthogonality conditions (1.4), $g_{2,t}$ is the vector the orthogonality conditions used to estimate the VAR(p) in the first step,[15] β is the vec of the VAR(p) matrices of parameters, and Σ_x is the variance of moments x.

[14]It can be shown that if $\left\{\widehat{\varepsilon}_t, \widehat{A}_i, \Sigma_T\right\}$ are consistent estimators of $\{\varepsilon_t, A_i, \Sigma\}$, then the sample analog

$$\frac{1}{T}\sum_{t=1}^{T}\left[\left(\frac{\alpha\lambda}{1-\lambda}y_t + \alpha\Delta y_t\right)(\delta\widehat{\varepsilon}_{t-i})' - (1-\lambda)^i\,\delta\widehat{A}_i\Sigma_T\delta'\right]$$

converges almost surely to the population moment (1.4).

[15]The VAR(p) is estimated LS equation by equation.

I write the correct variance (1.5) as function of the not-adjusted one, $V_{na}(\lambda_T)$. It is useful to see that $V(\lambda_T) \geq V_{na}(\lambda_T)$.

Finally, note that if the residuals from the first step estimation are uncorrelated with the ones from the second step, then the two steps estimator is also the most efficient among the GMM estimators of (1.4).

1.3.2 VAR estimation

I provide estimates for two main specifications of the $VAR(p)$ model. (i) the baseline, where Z_t includes inflation, output gap and interest rate; (ii) a second one that I named *min RMSE,* where Z_t includes the most relevant variables to forecast inflation and output gap according to Stock and Watson (2003a).[16]

In details, I estimate

$$Z_t = \sum_{j=1}^{p} B_j \cdot Z_{t-j} + \varepsilon_t \tag{1.6}$$

where

$$\underbrace{Z_t}_{n\times 1} = \begin{bmatrix} \Delta y_t & \pi_t & X_t' \end{bmatrix}'$$

and X_t can be either $X_t = i_t$ (the baseline specification), or a $(n-2\times 1)$ vector that includes: short term interest rate (the Fed Fund Rate), the term spread (10 years Government bond minus short term interest rate), the real Stocks Price Index (S&P500, deflated by CPI); IMF price index of commodities; real money (real M2 minus small time deposits); unemployment rate; total capacity utilization rate (TCU).

I estimate both the specifications for inflation measured either with CPI, or with the implicit GDP deflator. Output gap is detrended with the HP filter. All the variables are taken in logs except for unemployment, TCU, and interest rates.

[16]Stock and Watson (2003a) analyzed the contribution of several variables in forecasting inflation and output gap.

The variables have been detrended or taken in first difference when necessary, so all the series used in the VAR(p) are stationary. Also, the VAR(p) model has the minimum number of lags in order for the residuals to be not serially correlated. These two conditions assure that the VAR(p) estimator is consistent.

The sample goes from 1957q1 to 2005q4; The database come from FRED II database of U.S. economy.[17]

1.3.3 GMM Estimation

To control for the small sample bias problem that affect nonlinear GMM estimators I estimate two alternative specifications of (1.4). The first one is (1.4) multiplied by $(1-\lambda)$, the second one is (1.4) multiplied by $\left(\frac{1-\lambda}{\alpha\lambda}\right)$. They are referred to as (1) and (2) in next tables. The sample goes from 1958q4 to 2005q4 (189 observation).[18]

I provide estimations only for parameter λ, while I calibrate α. In the original model α depends on the intertemporal elasticity of substitution of consumers, on Fisher elasticity of labor, and on the elasticity of demand of single-variety goods. Since I don't use data on neither about consumption or about labor or about firms' markup, then I don't attempt to estimate α.

Results

My first attempt has been to estimate (1.4) using all the orthogonality conditions, i.e. for $i = 0, ..., l$.

The results are not encouraging. We never accept the null hypothesis of overidentifying restrictions in Hansen's J-test, no matter the order of lags l I choose (2, 4, 6, 8, 12), the inflation index I choose (either CPI or GDP deflator), the VAR(p)

[17]Available at Federal Reserve Bank of St. Louis.

[18]The GMM sample is shorter than the VAR one because I loose 7 observations to obtain the VAR(6) estimates.

specification I use to generate the regressors (either the baseline, or the *min RMSE*). Apparently, the model can't match the selected moments all together.

This evidence points out that there are some sources of mispecification in the SIPC model. However, since the main call of the SIPC was to explain inflation persistence, I check whether the model can match the lagged covariances alone. To do this, I discard the first equation in (1.4), and I focus on the other orthogonality conditions, i.e. equations (1.4) for $i = 1, ..., l$.

The following table 1 summarizes the results.

Restricted $\alpha = .2$	Specif.	λ_T^{2s}	Adjusted std.err.	Null MR cal.	t-stat (p-val)	Null RE	t-stat (p-val)	J-stat (p-val)
O.C. (1.4)	$i = 1, ..., 6$							
defl; VAR	(1)	0.35	0.149	0.25	0.71	1	-4.30	2.22
$\{\Delta y_t, \pi_t, i_t\}$					(0.47)*		(0.00)	(0.81)*
	(2)	0.36	0.055	0.25	2.17	1	-11.41	1.95
					(0.03)		(0.00)	(0.85)*
defl; VAR	(1)	0.38	0.106	0.25	1.35	1	-5.70	2.18
minRMSE					(0.17)*		(0.00)	(0.82)*
	(2)	0.41	0.075	0.25	2.15	1	-7.81	2.79
					(0.03)		(0.00)	(0.87)*
cpi; VAR	(1)	0.47	0.061	0.25	3.67	1	-8.44	2.70
$\{\Delta y_t, \pi_t, i_t\}$					(0.00)		(0.00)	(0.74)*
	(2)	0.49	0.057	0.25	4.21	1	-8.85	2.24
					(0.00)		(0.00)	(0.81)*
cpi; VAR	(1)	0.54	0.101	0.25	2.88	1	-4.48	3.38
minRMSE					(0.00)		(0.00)	(0.64)*
	(2)	0.57	0.094	0.25	3.48	1	-4.47	2.74
					(0.00)		(0.00)	(0.73)*

Table 1. 2-step GMM with optimal weighting matrix. U.S. data, sample 1958q4 – 2005q4. Output gap filtered with HP filter. Newey-West HAC standard errors adjusted for stochastic regressors. p-values in parenthesis. J-statistics is Hansen test of overidentifying restrictions (5 d.o.f.).

The estimates are reasonable and quite precise. The model fits very well all the moments according to the J-test. We can never reject the null hypothesis of overidentifying restrictions.

The estimates of the frequency of information updating λ are our main concern. In all specifications λ_T^{2s} is in the range assumed by the theory, i.e. within the $(0, 1]$ interval. More precisely, it ranges between $[0.35, 0.57]$. In column 4 of table 1 I report the p-value for the null hypothesis $\lambda = 0.25$, which is the calibration used by Reis (2004) to match actual inflation persistence with the SIPC model. λ_T^{2s} does not significantly differs from the original MR calibration, at least when we use the GDP deflator as inflation index.

Finally, it seems that the estimates of λ are sensitive to the magnitude of the shocks. Specifically, the smaller are forecast errors $\{\widehat{\varepsilon}_t\}_{t=1}^{T}$, the bigger is λ_T^{2s}, as we can see comparing the estimates of the *min RMSE* model against those of the baseline.[19]

Using this restricted set of moments, the results are substantially different from the ones found before. Now the model fit well the data, and the estimates of firms' average information updating comfort MR calibration. We could have expected this result: Reis (2004) showed that $\lambda = 0.25$ is the correct value for the SIPC model to reproduce the persistence of actual U.S. inflation. Therefore, if we force the model to match the moments that measure persistence, then it is likely that the best parameter to do it is indeed the 0.25 proposed by Reis.

It is worth noticing that the estimates in table 1 are in line with the ones of the other empirical papers that estimate the SIPC. For the sake of comparison, I estimate λ using the same information those papers used, which turns out to be the information contained in the first $acf(i)$ of δZ_t for $i = 1, ..., l$.

[19] The *min RMSE* VAR(p) predicts better Z_t with respect to the baseline specification because it uses more information. Therefore, the residuals $\widehat{\varepsilon}_t$ in the *min RMSE* are smaller than the ones in the baseline VAR(p).

Using lagged δZ_t as instruments, I derive and estimate the following simulated orthogonality conditions:

$$E\left[\left(\frac{\alpha\lambda}{1-\lambda}y_t + \alpha\Delta y_t\right)(\delta Z_{t-i})'\right] = \sum_{j=0}^{\infty}(1-\lambda)^{i+j}\,\delta A_{i+j}\Sigma A_j'\delta'$$
$$for \quad i = 1,...,l$$

In this case the estimates of λ_T^{2s} ranges between $[0.30, 0.41]$. This result is close to the one obtained using lagged ε_t as instruments, and resembles much the ones found by Khan and Zhu (2002, 2006), Kiley (2006) Korenok (2005). In conclusion, it seems we can reproduce the same results they found once we use the same information they used, although the estimators are different. Aside, this works as a double check on the correctness of the methodology that I use in this chapter.

At this point, the question that naturally follows is whether the SIPC model can match the conditional variance of inflation alone. To do it, I estimate the first orthogonality condition in (1.4). In order to obtain more precise estimates, that orthogonality condition is multiplied by a vector of instruments x_t, which contains all variables dated $t-1$ and before.[20] Using the additional assumption that the errors $\varepsilon_t' s$ are i.i.d. I obtain

$$E\left[\left(\left(\frac{\alpha\lambda}{1-\lambda}y_t + \alpha\Delta y_t\right)(\delta\widehat{\varepsilon}_t)' - \delta\Sigma_T\delta'\right)\cdot x_t\right] = 0 \tag{1.7}$$

which is a new vector of orthogonality conditions with dimensions 19×1 (the number of instruments used).

(1.7) is estimated using the same procedure as before. In this case the GMM estimator turns out to be the Non-linear IV estimator, but with smaller variance. To minimize the standard errors of the estimates, the weighting matrix is chosen to be the inverse of the variance of moments. The results are summarized in table 2.

[20] I use a total of 19 instruments, namely: a constant, 4 lags of inflation, 4 lags of output gap, two lags of unemployment rate, interest rate, marginal cost, money growth, and the term spread.

Restricted $\alpha = .2$ O.C. (1.7)	Specif.	λ_T^{2s}	Adjusted std.err.	Null MR cal.	t-stat (p-val)	Null RE	t-stat (p-val)	J-stat (p-val)
defl; VAR	(1)	0.75	0.089	0.25	5.58	1	-2.76	22.21
$\{\Delta y_t, \pi_t, \mathrm{i}_t\}$					(0.00)		(0.00)	(0.22)*
	(2)	0.86	0.084	0.25	7.24	1	-1.59	15.29
					(0.00)		(0.11)*	(0.64)*
defl; VAR	(1)	0.71	0.099	0.25	4.64	1	-2.90	22.62
minRMSE					(0.00)		(0.00)	(0.205)*
	(2)	0.84	0.103	0.25	5.70	1	-1.55	15.55
					(0.00)		(0.12)*	(0.623)*
cpi; VAR	(1)	0.82	0.073	0.25	7.83	1	-2.43	20.26
$\{\Delta y_t, \pi_t, \mathrm{i}_t\}$					(0.00)		(0.01)	(0.318)*
	(2)	0.85	0.067	0.25	9.03	1	-2.11	13.97
					(0.00)		(0.03)	(0.731)*
cpi; VAR	(1)	0.80	0.083	0.25	6.61	1	-2.39	22.93
minRMSE					(0.00)		(0.01)	(0.197)*
	(2)	0.85	0.079	0.25	7.70	1	-1.79	16.39
					(0.00)		(0.07)*	(0.565)*

Table 2. 2-step GMM with optimal weighting matrix. U.S. data, sample 1958q4 – 2005q4. HP filter for output gap. Newey-West HAC standard errors adjusted for stochastic regressors. p-values in parenthesis. J-statistics is Hansen test of overidentifying restrictions (18 d.o.f.).

Again, the estimates are quite precise and lie in the range assumed by the theory. The model fits quite well the moments according to the J-test. We can never reject the null hypothesis of overidentifying restrictions.[21]

λ_T^{2s} now ranges in the interval $[0.71, 0.86]$, which is significantly higher than the one found in table 2. This value implies that average information duration ranges from 3.5 to around 4 months. As before, I report the p-value for the null hypothesis

[21] A yellow flag should be lied here. Standard distributions for hypothesis testing with IV estimators are reliable only if the instruments are not weak.

Unfortunately, is still unclear how to check for weak instruments in nonlinear estimators with possibly nonspherical residuals.

$\lambda = 0.25$: it is rejected in all the specifications, suggesting that MR calibration should be dismissed if we want the SIPC to match the conditional variance of inflation.

In some specification, however, λ_T^{2s} is close to 1. Since the SIPC with $\lambda = 1$ encompasses RE equilibrium with flexible prices, then I test whether the hypothesis of RE is accepted or rejected by the data. In column 6 of table 2 I report the t-statistics and the p-value for the null hypothesis $\lambda = 1$. The evidences are not overwhelming, the null is rejected at 5% level in most of the specifications, but it is accepted in more than half of them at 1%. We can't say anything about the RE equilibrium, at least from this estimation.

All in all, this second estimation suggests that the SIPC can fit also the conditional variance of inflation, although the model needs an higher frequency λ of firms' information updating to match it.

Interpretation of the results and the Hybrid SIPC

Basically the previous results show that if we use the SIPC model to match inflation persistence, then it predicts an inflation volatility higher than the one we observe in the data. While if we use the model to match the conditional variance of inflation, then it predicts a lower inflation persistence with respect to the actual one.

One possible explanation is the following. The SIPC has been often criticized because it predicts that all producers change the price in every period, while there are robust (across countries and sample periods) evidences that a sizeable fraction of firms does not change price for many periods.

According with this evidence, the SIPC model would be mispecified because it does not take into account that in every period a fraction of producers keep last period price. Would this argument be useful to understand the results above? In principle yes. In a model with both adaptive and inattentive producers the effect of a shock on inflation lasts in time for two reasons. First, since inattentive agents use past information to set prices in the future, then a current shock will affect price changes, i.e. inflation, in the future. Second, since adaptive producers use lagged prices to set their current price, then a shock that shift inflation today will affect also prices tomorrow. Thus, the covariance between current inflation and lagged shocks in such model depends both on the frequency of information updating and on the size of the fraction of adaptive producers. In particular, that covariance may be high even with a low degree of information stickiness insofar as there is a big fraction of adaptive producers. In this latter case, if we estimated the (mispecified model) SIPC matching the orthogonality conditions (1.4) with $i = 1, ..., l$ we may find (downward) biased estimates of λ_T.[22]

Given that, it would be useful to derive and test a model with heterogeneous agents, where some of them are inattentive and some others use adaptive pricing rules. Dupor, Kitamura and Tsuruga (2006) goes in this direction. They proposed a model of "dual stickiness" where producers change prices only sporadically, plus they absorb the relevant information in random periods, as in the SIPC.

Basically, what DKT do, is to nest together Calvo's sticky price framework with the sticky information of Mankiw and Reis. In DKT model inflation in period t is function of all past periods expectations of a stream of future variables from $t+1$ onwards. It is not immediate to accommodate such inflation dynamics in the framework used in this chapter. I leave this for future research, and I pursue here a simpler approach. I derive and test a model where a fraction of agents are inattentive and the rest is purely adaptive firms.

As result, in this model economy there are two types of producers. The first type

[22]It is easy to see that. If the RHS of (1.4) increases because there are more adaptive producers and inflation is more persistent, helding fixed $y_t, \Delta y_t$ and ε_t, then λ must decrease in order for the equality (1.4) to hold.

is a fraction φ of adaptive firms who set the price equal to last period price, either accounting for previous period inflation (indexation of prices), or not. The other $(1-\varphi)$ producers are inattentive as in the SIPC model.

In this economy the aggregate price index is given by:

$$p_t = (1-\varphi)\, p_t^{SI} + \varphi p_t^b$$

where

$$\begin{aligned} p_t^{SI} &= \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-j} (p_t + \alpha y_t) \\ p_t^b &= \begin{cases} p_{t-1} & \text{case (a)} \\ p_{t-1} + \pi_{t-1} & \text{case (b)} \end{cases} \end{aligned}$$

Inflation in this "hybrid" model evolves according to:

$$\pi_t = \begin{cases} (1-\varphi)\left[\frac{\alpha\lambda}{1-\lambda} y_t + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-j-1} (\pi_t + \alpha \Delta y_t)\right] & \\ \qquad +\varphi \pi_{t-1} & \text{case (a)} \\ (1-\varphi)\left[\frac{\alpha\lambda}{1-\lambda} y_t + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-j-1} (\pi_t + \alpha \Delta y_t)\right] & \\ \qquad +\varphi (2\pi_{t-1} - \pi_{t-2}) & \text{case (b)} \end{cases} \tag{1.8a}$$

Applying to the models in (1.8a) the econometric procedure presented in section 3.1, I obtain the following sets of orthogonality conditions:

$$\begin{aligned} E\left[\left(\frac{\alpha\lambda}{1-\lambda} y_t + \alpha \Delta y_t - \frac{\varphi}{1-\varphi} (\pi_t - \pi_{t-1})\right) (\delta \varepsilon_{t-i})'\right] &= \qquad (1.9) \\ &= (1-\lambda)^i \delta A_i \Sigma \delta' \\ &\quad \text{case (a)} \end{aligned}$$

$$\begin{aligned} E\left[\left(\frac{\alpha\lambda}{1-\lambda} y_t + \alpha \Delta y_t - \frac{\varphi}{1-\varphi} (\pi_t - 2\pi_{t-1} + \pi_{t-2})\right) (\delta \varepsilon_{t-i})'\right] &= \qquad (1.10) \\ &= (1-\lambda)^i \delta A_i \Sigma \delta' \\ &\quad \text{case (b)} \\ for \quad i &= 0, ..., l \end{aligned}$$

Orthogonality conditions (1.9) and (1.10) are estimated with GMM. Analogously with previous estimations, I substitute regressors $\{\varepsilon_t, A_i, \Sigma\}$ with

$\left\{\widehat{\varepsilon}_t(\beta), \widehat{A}_i(\beta), \Sigma_T(\beta)\right\}|_{\beta=\beta_T^{VAR}}$.[23]

Hybrid SIPC		(a) O.C. (1.9)			(b) O.C. (1.10)		
$i = 0, ..., l$		λ_T^{2s}	φ_T^{2s}	J-stat	λ_T^{2s}	φ_T^{2s}	J-stat
Restr. $\alpha = .2$	Specif.	(s.e.)	(s.e.)	(p-val)	(s.e.)	(s.e.)	(p-val)
defl; VAR	(1)	0.41	-1.23*	3.56	0.62	-0.66*	3.08
$\{\Delta y_t, \pi_t, \text{i}_t\}$		(0.055)	(0.775)	(0.61)*	(0.101)	(0.476)	(0.68)*
	(2)	0.44	-1.33*	3.18	0.67	-0.81*	2.05
		(0.060)	(0.885)	(0.67)*	(0.101)	(0.664)	(0.84)*
defl; VAR	(1)	0.45	-1.08*	1.85	0.64	-0.54*	1.37
minRMSE		(0.054)	(0.813)	(0.86)*	(0.104)	(0.479)	(0.92)*
	(2)	0.46	-1.14*	1.76	0.66	-0.60*	1.03
		(0.056)	(0.884)	(0.88)*	(0.107)	(0.568)	(0.95)*
cpi; VAR	(1)	0.55	-1.37*	8.05	0.75	-0.66*	4.65
$\{\Delta y_t, \pi_t, \text{i}_t\}$		(0.065)	(0.986)	(0.15)*	(0.064)	(0.416)	(0.45)*
	(2)	0.64	-2.40*	6.35	0.77	-0.79*	3.39
		(0.080)	(2.628)	(0.27)*	(0.065)	(0.553)	(0.64)*
cpi; VAR	(1)	0.69	-0.83*	6.18	0.78	-0.34*	3.88
minRMSE		(0.087)	(0.671)	(0.28)*	(0.070)	(0.265)	(0.56)*
	(2)	0.78	-0.97*	4.46	0.80	-0.39*	3.21
		(0.101)	(1.09)	(0.48)*	(0.072)	(0.330)	(0.66)*

Table 3. 2-step GMM with optimal weighting matrix. U.S. data, sample 1958q4 – 2005q4. HP filter for output gap. Newey-West HAC standard errors not adjusted for stochastic regressors. * means that the coefficient is not significatively different from zero. J-statistics is Hansen test of overidentifying restrictions (18 d.o.f.).

As we can see in table 3 the good result is that now all the specification are accepted by the data: according to the J-statistics in all specifications the selected moment are "close enough" to zero at parameter estimates. Also, λ_T^{2s} lies in between the estimates found before, implying an average information duration of 6 months, or

[23]Notice that the VAR(p) model I used in previous section to obtain $\left\{\widehat{\varepsilon}_t(\beta), \widehat{A}_i(\beta), \Sigma_T(\beta)\right\}$ encompasses both the SIPC model and this hybrid version, since the reduced form AR representations of the two models are observationally equivalent.

twice a year, which seems reasonable. On the contrary, the estimates of the fraction of adaptive producers are a reason of concern. The coefficient φ_T^{2s} has the wrong sign in all the specifications, and it is never significantly different from zero.

All in all, the results with the hybrid model don't seem satisfying. The hypothesis of non-maximizers adaptive producers seems not the key issue to improve the sticky information model, and eventually a different explanation should be find.

1.3.4 Robustness analysis

I check the robustness of previous results along three dimensions.

1. Whether the results are sensitive to the calibration of α (Table 4)

2. Whether they change using a different filter to get the output gap, i.e. Quadratic Detrend instead of the Hodrick-Prescott (Table 5)

3. Whether a different assumption about the dynamics of Z_t changes the estimates of λ. In particular, I assume that $S_t \equiv (\pi_t + \alpha \Delta y_t) \sim AR(2)$, and I estimate jointly λ and the parameters of the AR(2) model (see the appendix) (Tables 6-7)

The evidences about λ are broadly confirmed. Again we find that the model cannot match all the moments (1.4) together, but it does a good job in matching a subset of them once we separate the conditional variance from the lagged covariances. The null hypothesis of $\lambda = 0.25$ is always rejected when we match the variance, and accepted in most of the specifications when we match the lagged covariances. Finally, the estimations with $\alpha = 0.1$ point out a a stable *inverse* relationship between α and the estimates of λ.

1.4 Structural Breaks

During the 1990's the U.S. economy experienced a disinflation accompanied by a fall of inflation persistence.[24] In the SIPC model inflation persistence is inversely related to firms' knowledge about the economy: the more that knowledge is outdated, the more persistent inflation is. Therefore, we might expect that average information duration decreased (λ increased) during the sample, and this would explain the reduction in inflation persistence.

To test the hypothesis of an increase in λ, I perform Andrews's supLM test of structural breaks.[25] This test cuts the tails of the sample and computes the most likely point in time where a break might have occurred in the middle subsample.

The test is applied to both the estimations reported above, and the results are summarized in table 8.

Structural breaks $H_0 : no\ S.B.$		sup LM	Asymptotic critical values 10%	5%	1%
O.C. (1.4) with $i = 1, ..., 6$	$\pi_0 = .2$	1.18***	6.80	8.45	11.69
	$\pi_0 = .1$	1.18***	7.63	9.31	12.69
O.C. (1.7)	$\pi_0 = .2$	2.08***	6.80	8.45	11.69
	$\pi_0 = .1$	1.89***	7.63	9.31	12.69

Table 8. π_0 indicates the percent of each tail cut. SupLM test has non-standard distribution. The asymptotic critical values are given in Andrews (1993). * , ** , *** means significance respectively at 1%, 5%, and 10% level.

According to Andrews's test there is an overwhelming evidence that no structural break to λ occurred during the sample. However, there is one reason of concern with

[24] See Bayoumi and Sgherri (2004) for references.

[25] I choose Andrews's (1993) supLM because it is the most powerful test when timing of (possible) breaks is unknown.

this result. During late 1970's inflation volatility increased sharply because of the oil shock, which surely was an exogenous event uncorrelated with the degree of sticky information in the economy, but it could possibly biases the results of Andrews test. Figure (2), which display the residuals from the estimation of (1.4), show that this is indeed the case.[26]

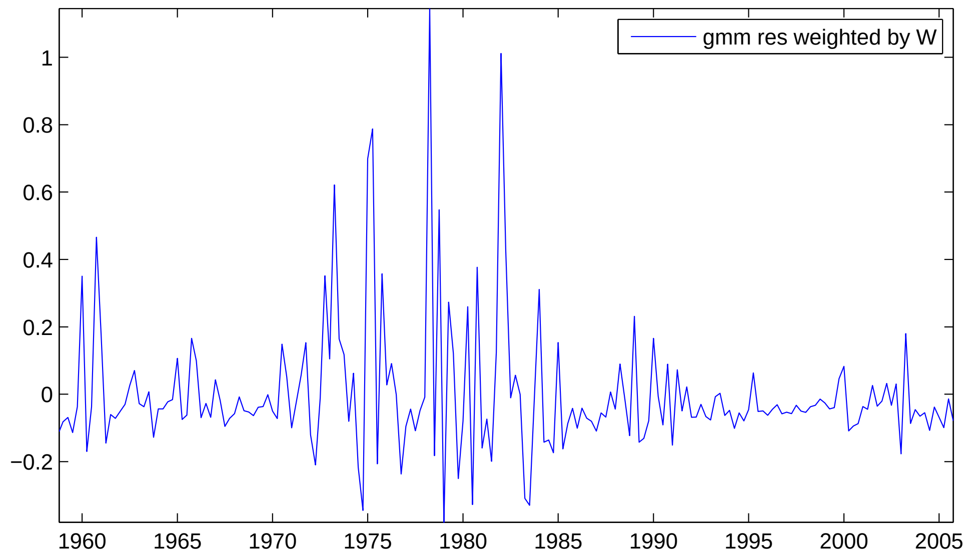

Figure 1.1: **Weighted average of residuals from GMM estimation of (4).**

Therefore, I perform a second test controlling for the effect of the oil shock in the 1970's. Under the null hypothesis that the same model holds throughout the sample, I test whether λ is equal in two subsamples: one that goes from 1959q1 to 1970q1, and the second from 1990q4 to 2005q4. I compare the coefficients using LM and Wald tests. Results are in table 9.

[26]The figure refers to the estimation of (1.4) with i=0. The figure for the estimation of (1.4) with i=1,...,l is similar and it is not reported here.

Structural breaks $H_0 : no\ S.B.$	λ_{60} (s.e.)	λ_{90} (s.e.)	$Wald$ (p-val)	LM (p-val)
O.C. (1.4) with $i = 1, ..., 6$	0.44 (0.069)	0.74 (0.140)	3.83 (0.05)**	5.25 (0.03)*
O.C. (1.7)	0.61 (0.049)	0.95 (0.025)	44.51 (0.00)	1871.1 (0.00)

Table 9. Wald and LM tests both have standard χ^2 distribution with 1 d.o.f. * , ** , *** means significance respectively at 1%, 5%, and 10% level.

The null hypothesis $\lambda_{60}^{Gmm} = \lambda_{90}^{Gmm}$ is now rejected almost always at 5% level. According to this test, it is likely that a structural break have occurred between the first and the last years of the sample. Now, it make sense that in recent years firms have had better knowledge of the economy and the markets they operate in than 40 years ago. In the past information is likely to have been stickier: less media to channel macroeconomic news, less accurate forecasts about the markets, less experienced authorities, less data gathering, etc. It is not surprising that firms acquired the relevant information slower, therefore taking longer time to react to new events.

1.5 Conclusions

In this chapter I find that SIPC is not a valid model to explain U.S. post-war inflation dynamics. The main reason is that the model cannot explain at the same time inflation persistence and inflation variance. In particular, if we use the model to match inflation persistence, then it predicts an inflation volatility higher than what we observe in the data; while if we use the model to match the conditional variance of inflation, then it predicts a lower inflation persistence with respect to the actual one.

This finding is also robust to some theoretical deviation from the original model. I derive and estimate inflation dynamics once we introduce in the model economy a fraction of adaptive producers who use past period price to set their current prices. This attempt is useful to overcome the widespread critique that the SIPC model counterfactually predicts a zero mass in the zero of the distribution of firms for price changes. However, also this "hybrid" model fails to match jointly inflation persistence and inflation variance.

In details, when I estimate the SIPC model matching the covariances between current inflation and only the lagged shocks, then the estimates of frequency of firms' information updating are in accordance with that of the other papers that estimated the SIPC using single equation estimators. I show that this is due to the fact that we are exploiting the same information about the inflation process. In this case I find $\lambda_T^{2s} \in [0.35, 0.57]$. This value implies an average information duration from 6 to 9 months.

Differently, once we use the model to match the conditional variance of inflation I find $\lambda_T^{2s} \in [0.71, 0.86]$. This value implies an average information duration from 3.5 to 4 months, which is just slightly higher than the average information duration in the neoclassical model with RE and flexible prices.

Finally, using different subsamples I find evidence that firms' average information duration was significantly higher in the first years of the sample (1960's) than in the last ones (1990's), as our intuition suggest. This finding suggests that sticky information might have been an important source of inflation persistence in past times, while today is not anymore.

All in all, my analysis suggests that sticky information theory cannot be the main explanation of inflation persistence, at least as it is modeled in the SIPC model.

Part III

Advertising

Chapter 2

Advertising and Business Cycle Fluctuations

> *"... as a matter of fact, the scale of expenditures on advertising varies positively with the general level of economic activity, so that, insofar as the effect of marginal expenditures is positive, advertising itself tends to accentuate the amplitude of economic fluctuations..."*
>
> *Nicholas Kaldor, 1950*

Joint with Francesco Turino

2.1 Introduction

2.1.1 Motivations

Every year firms spend 230 billions of dollars to advertise their products in U.S. media, around 1000 dollars per U.S. citizen. Advertising as an industry is worth 2.2% of GDP, absorbs around 20% of firms' budget for new investments,[1] and 13% of their corporate profits. Beyond any further consideration, these numbers show that

[1] We refer to B.E.A. fixed non-residential investment.

advertising is a matter of crucial importance for firms.[2]

In the economic literature the rationale behind spending in advertising has been identified with its positive effect on the demand. As a matter of fact, consumers' demand is not exogenously given as usually assumed in the textbook theory of prices, but firms can tilt it toward their own products through advertisements. The effectiveness of advertising in enhancing the demand is not only revealed by firm's attitude in spending money on it, but it is also supported by a large number of empirical studies.[3] Overall, the positive relationship between firm's advertising and its own demand is widely accepted as robust evidence.

Now, since the reason of advertising is to increase consumers' demand, targeted advertising increases the demand of a single good, we ask in this chapter whether the relationship between advertising and demand holds also in the aggregate. In other words, would aggregate advertising enhance the aggregate demand? And, if yes, which will be its overall impact on the aggregate economy?

In the literature there isn't consensus about an answer to this question. A widespread opinion among macroeconomists is that the answer should be negative. Building on Solow (1968) and Simon (1970), they argue that it is incorrect to assume aggregate advertising and aggregate consumption to have the same causality relationship that holds between targeted advertising and the sales of the advertised good because advertising raises firm's demand by stealing costumers from competitors, and not by increasing the overall size of the markets. Because of this "competition" effect, advertising would just affect the composition, but not the size of aggregate consumption.

[2]In the economic literature advertising is commonly viewed as a selling cost that does not affect production. In fact, advertising neither enters as factor in the production function, like e.g. investment in equipment and machinery, nor affects production technology, like R&D. Perhaps this is the reason why advertising has been always considered a matter of firms' management, receiving scarce attention in the macroeconomic theory of firms.

[3]A survey of this studies can be found in Bagwell (2005) and Schmalensee (1972).

The relationship between aggregate advertising and aggregate consumption is crucial to assess the impact of advertising on the aggregate dynamics. As we will show in section 3, aggregate consumption is the main channel through which the effect of advertising propagates in the economy. If we shut down this channel then its effect in the aggregate becomes negligible. As a matter of fact, the macroeconomic literature about advertising has mainly looked at empirical evidence to support the relationship between aggregate advertising and aggregate consumption.[4] In this direction went Ashley, Granger, and Schmalensee (1980), or more recently, Jung and Seldom (1995), Fraser and Paton (2003).[5]

However, despite the large amount of evidence provided, none of these studies was conclusive, and after the conjectures of the classical economists (e.g. Marshall, Chamberlain, Kaldor, Galbraith, and Solow) the literature still lacks of a theory to analyze whether and how advertising affects the aggregate economy.[6]

In this chapter we attempt to fill this gap building a general equilibrium model that accounts for advertising. In particular, we assume a specification of preferences that allow to nest in the model two competitive theories about aggregate advertising, i.e. advertising as market enhancing vs. spread-the demand-around advertising. Then, we use this model to analyze the effect of advertising on the aggregate dynamics. The main result of the chapter is to show that, under general conditions, advertising can have a quantitatively relevant impact on the aggregate dynamics by generating a *work and spend* cycle: a consumer who wants to spend more on consumption because of the advertising incentive, but faces the intertemporal budget constraint, will end up working more hours.

[4]Benhabib and Bisin (2002) is one exception.

[5]See Jacobson and Nicosia (1981) for more details on this empirical approach.

[6]There are however some exceptions. In particular, Benhabib and Bisin (2002) analyze what are the conditions such that a neoclassical general equilibrium model can replicate the major predictions of the Postmodernist critique. Grossmann (2007) studies the link between advertising and in-house R&D expenditures in a quality-ladder model of endogenous Growth.

Also, we show that after any of the shocks considered in the model, fluctuations of consumption and labor are bigger when firms advertise more. Apparently, advertising tends to accentuate the amplitude of economic fluctuations, as suggested by Kaldor (1950). We assess the quantitative impact of advertising on the aggregate dynamics by measuring the representative consumer welfare costs of fluctuations when firms are allowed to advertise against the welfare costs when advertising is banned.

Thirdly, by the mean of a Bayesian estimation of our model we test which of the two theories of advertising nested in the model is more likely to be true. One main result is to show that aggregate advertising do affect aggregate consumption, as originally suggested by Galbraith (1967).

In this chapter advertising is intended as a form of manipulation of consumer's preferences and tastes.[7] As with Dixit and Norman (1978) and Benhabib and Bisin (2002), we model advertising to increase the marginal utility of the advertised good through a modification of parameters in the utility function.[8] Notice, however, that this assumption is not itself a sufficient condition to conclude that aggregate advertising enhances aggregate demand. If consumers used savings to pay the extra consumption generated by advertisements, then advertising would at the same time increase consumption and crowd out investments, and the net effect on the demand would be unclear. Also, if advertising shifted purchases toward more expensive goods, then an increase in advertising could imply a reduction in real consumption, and therefore in

[7]The way to integrate advertising in consumer's choice theory is controversial. In general, there are three opposite views in the literature about what advertising does: the Persuasive, the Informative, and the Complementary view. See Bagwell (2005) for an excellent survey. A tastes shifter advertising as the one we model here fits in with the Persuasive view about advertising, as originally proposed by Marshall (1890,1919), Chamberlain (1933), Robinson (1933), Kaldor (1950), and later on used by Dixit and Norman (1978) and Benhabib and Bisin (2002).

[8]Robinson (1933, pp.5) wrote: "...the consumer will be influenced by advertisement, which plays upon his mind with studied skill, and makes him prefer the goods of one producer to those of another because they are brought to his notice in a more pleasing and forceful manner." A way to accommodate this idea of "fundamental" versus "altered" preferences within the neoclassical rational-agent-based theory, is to model advertising as a parameter in the utility function.

the aggregate demand. Moreover, advertising is not just a matter of demand, it can affect the economic activity in various ways. For instance increasing the substitutability among goods, and therefore the market power of firms (price effects). Or, in a dynamic framework, reducing consumers savings, and therefore the future demand. Overall, it seems that its effects on the economy are not trivially predictable. In order to cope with all these issues, our strategy is to embed the modified utility function with advertising into a dynamic stochastic growth model with monopolistic competition, and to analyze the effect of advertising in the general equilibrium solution of this model.

The way we model advertising in the utility function is akin to that used in macroeconomic literature for consumption habits.[9] Likewise external habits, advertising creates consumer dissatisfaction about his actual level of consumption of good i, and pushes him to buy more. Where advertising differs from habits is that its effect on the demand is generated endogenously in the equilibrium by the interaction between firms decisions and consumers reactions, while in the case of habits it is exogenously assumed. We stress this point because one result in our framework is that the conditional demand of single variety goods turns out to be function of past advertising, which itself is function of past sales. Hence, the demand function of good i can be written as function of past consumption of good i, as in models of costumer markets or models with "deep habits" preferences.

2.1.2 The aggregate dynamics results

We show that advertising has several effects in equilibrium. It absorbs resources and increases inefficiency due to monopolistic power. It raises consumption, labor, and the gross production of the economy.

[9]See Abel (1990) or Ravn Schmitt-Grohe and Uribe (2006).

Basically, we identify 3 channels through which advertising affects aggregate dynamics. First, it generates the *work and spend cycle*: in the presence of advertising, people work more in order to afford an higher consumption path. The perceived need for higher consumption is due to the advertising signals that the agent is exposed to, as suggested by Benhabib and Bisin (2002). In fact, consumer's tastes turn out to be endogenous, socially determined, and possibly variable over time, depending on firms' optimal advertising policy.

The second mechanism operates through prices. Advertising increases markup, reduces consumer's wage and therefore reduces the supply of labor. The last mechanism operates through the resource constraint. Since advertising absorbs resources, it puts a wedge between gross production and net GDP, which is defined as consumption plus investment. We show that for a reasonable set of parameter calibrations, the first mechanism prevails over the other two. Equilibrium labor, and therefore gross production increases. Part of the extra production is used to produce advertising, and the rest is sold as consumption goods.

It is worth noticing that we are challenging advertising to affect the aggregate dynamics using the worst scenario to obtain these results. In fact, in a model with nominal rigidity where wages vary less, the consumer would increase further the supply of labor in response to new advertising, thus strengthening the work and spend cycle mentioned above.

Also, in a model with fully flexible prices as the one we use here, any increase of markup due to advertising would make investment goods more expensive, thus reducing the real interest rate and, in turns, household savings. Therefore, advertising in our model has a direct negative effect on investment, which offset the positive effect coming through the work and spend mechanism.[10]

[10]In an early draft of this chapter we showed that in a two sectors model where consumption and investment are composed by different goods, advertising not only increases consumption, but also

This chapter follows the usual structure of papers in the Real Business Cycle literature. In Section 2 we build a database of advertising at business cycle frequency. Aggregate advertising is defined here as the total money spent by all firms to advertise products in U.S. media. Quarterly data of aggregate advertising expenditures are not available among the standard business cycle statistics, so we had to go through various sources to put together the database. In section 2 we explain the empirical work we carried on, together with a description of the data we achieved to collect, and the empirical facts we have found.

In section 3 we set up the model. First, we will derive the demand of a single variety good as function of advertising. Then, we set up firms profits maximization problem, where the producer is called to decide both production quantity and advertising intensity. The optimal advertising policy predicts that firms will spend a strictly positive share of revenues in advertising, and that advertising is optimally used as a complementary tool to pricing. In fact, a side contribution of the chapter is to provide the dynamic version of the Dorfam-Steiner (1954) theorem with monopolistic competitive markets.

Results are reported in section 4. In the model economy advertising affects consumption and labor fluctuations. In general, the model features a stronger internal propagation mechanism with respect to the standard RBC model. Both technology and preferences shocks are amplified and further propagated when firms are allowed to advertise. We assess the increase in aggregate fluctuations by the mean of a welfare costs analysis of fluctuations in a model with advertising compared to the welfare costs of the same model economy where advertising is banned.

Finally, we estimate a loglinearized version of the model to test for the effect of advertising on aggregate consumption. Conditional on our data, the market enhancing

investment, since it lowers the relative price of investment goods.

hypothesis is preferred versus the spread-it-around, meaning that advertising do affect aggregate consumption, and through consumption, the labor supply, the markup and the aggregate production. Section 5 concludes.

2.2 Stylized Facts

In what follows we define aggregate advertising, or advertising expenditures, the total U.S. dollars spent by all firms (domestic and foreigners) to advertise their products in U.S. medias.

Quarterly data on aggregate advertising are not included among the standard business cycle indicators, and we had to go through various sources to collect them.[11] As result, we've built the only up-to-date free-of-charge quarterly series of U.S. aggregate advertising that we are aware of.[12] Our data report firms' expenditures for advertising in 7 media, namely: cable and network televisions, radio, newspapers, magazines and Sunday magazines, billboards, direct mails, and outdoor advertising. The sample goes from the first quarter of 1976 to the second quarter of 2006, (122 quarters).

In order to check whether the series provided in this chapter is actually representative of total advertising expenditures in U.S., we compute total cumulated expenditures by year from our data set, and we compare them with annual data of total advertising spending issued by Robert Coen of Universal McCann, which is considered by advertising experts the most reliable source of aggregate advertising data.[13] During the considered sample our series accounts on average for nearly 30% of Coen's

[11] Details about the data and the sources we used are given in Appendix A.

[12] U.S. Federal administration used to collect quarterly data of aggregate advertising, but it stopped after 1968, when advertising was dismissed from the list of relevant variables used by the FED to analyze the cycle.

[13] See Appendix A.

total advertising, with a minimum of 25%, and an in-sample standard deviation of 2.95 percent points.

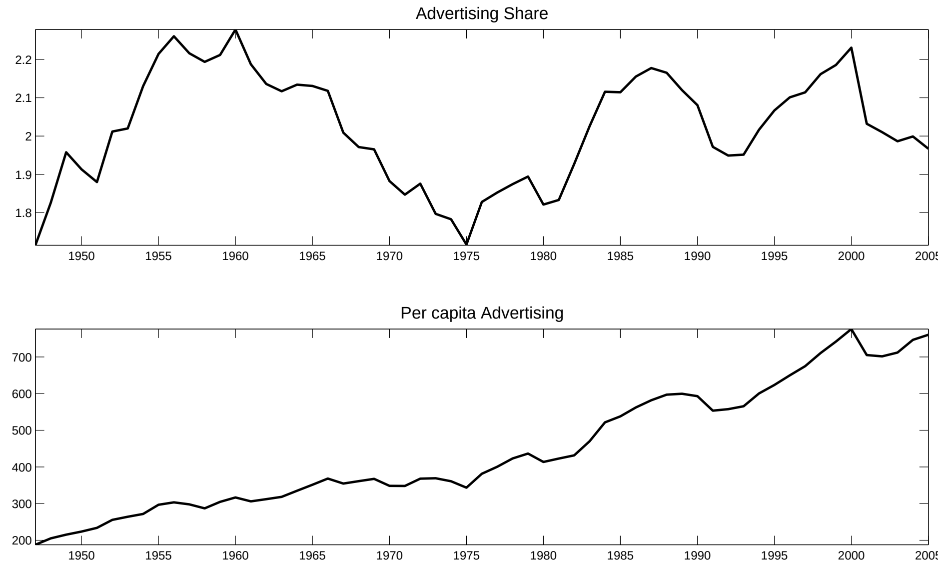

Figure 2.1: **Advertising in Postwar U.S. economy.** Plot 1. Advertising as share of GDP. Plot 2. Per-capita real advertising. Annual data, sample from 1948q1 to 2005q4.

Coen's annual data are useful to assess the magnitude of the phenomenon at issue. As we can see from panel 1 of figure (2.1), the ratio of advertising over GDP fluctuates around 2% throughout the sample, with peaks in late 50's and in 2000.

Also, panel 2 of figure (2.1) plots per-capita real advertising expenditures. This statistics is commonly used in the literature as measure of the number of advertising messages that reaches the consumer, i.e. a proxy for the intensity of advertising in the economy. As we can see from panel 2, there is a strong upward trend of per-capita advertising, which suggests that advertising, whatever might be its effect in the aggregate, is not a temporary characteristics of a newly industrialized economy, but remains a matter of firms' concern also in post-industrialized highly developed economies.

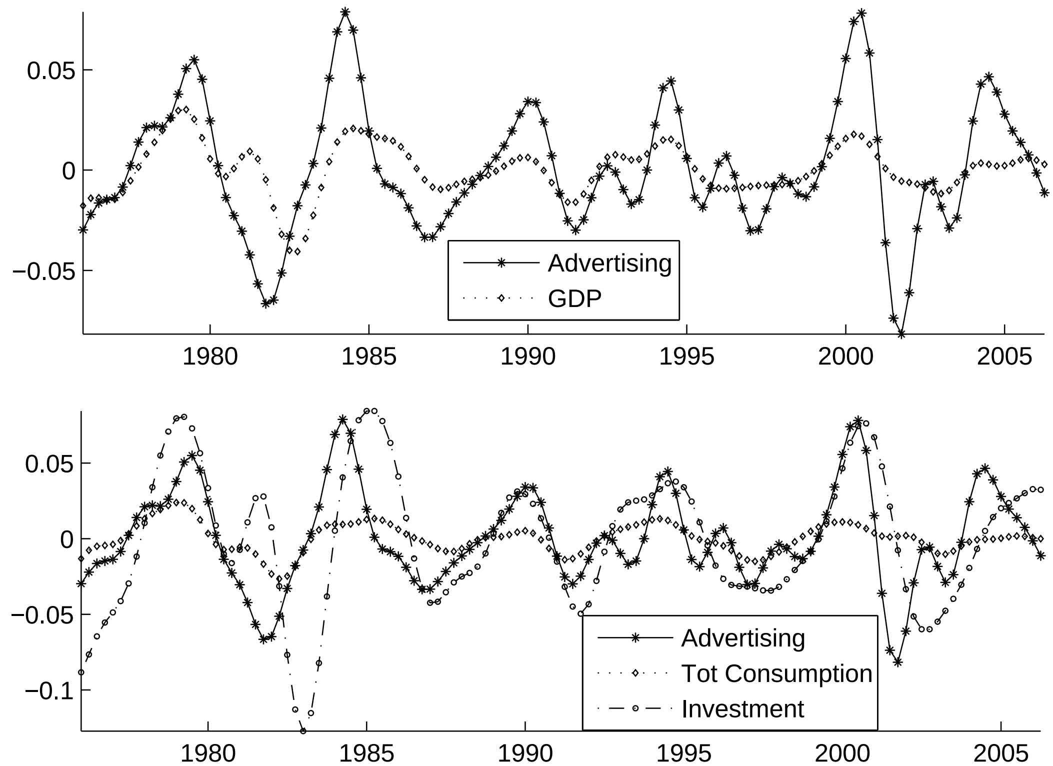

Figure 2.2: **Advertising and the main Business Cycle Indicators.** Quarterly data, sample from 1976q1 to 2006q2.

More interestingly for the purpose of this chapter, figure (2.2) plots quarterly real advertising expenditures along with real GDP, real total consumption, and real fixed non-residential investment. As usual, the series are taken in logs and filtered using band pass filter to isolate cyclical components.[14]

Basically, this picture shows that: (i) advertising is procyclical; (ii) it is more volatile than GDP and consumption, but less than investment.

[14]Before filtering we took away from advertising the seasonal component. Also, to control for spurious facts we calculated all the statistics presented in this section both with BP and Hodrick-Prescott filters. The main empirical evidence we present in this section do not change using one filter or the other.

X_t	$\frac{\sigma(X_t)}{\sigma(GDP_t)}$	$corr(X_t, Adv_t)$	$corr(X_t, X_{t-1})$
Advertising	2.50	1	0.89
GDP	1	0.63	0.92
Consumption	0.75	0.66	0.94
Non-durables	0.82	0.65	0.93
Durables	3.02	0.62	0.94
Investment	3.70	0.50	0.93
$\frac{Adv}{GDP}$	2.49	0.25$^{(1)}$	0.78

Table 2.1: **Second order moments (Quarterly data).** Note: All variables are filtered using the Band Pass (6,32) filter. $\sigma(.)$ is sample standard deviation. (1) This is $corr(X_t, GDP_t)$.

Table (2.1) provides the accompanying business cycle statistics. Advertising displays an high and positive correlation coefficient with GDP, and it is 2.50 times more volatile than output. In addition, it appears to be very persistent over the cycle, with a point estimate of first order autocorrelation of 0.89. Also, the positive correlation (0.25) between advertising-GDP ratio and GDP itself suggests that advertising can't be simply assumed as a constant proportion of the output.The correlation coefficient between advertising and consumption is 0.66, and the relative standard deviations is 3.30, i.e. advertising is more than twice volatile than consumption, and it is half volatile than investment: $\sigma\left(Adv_t\right)/\sigma(I_t) = 0.67$. In details, advertising is 4 times more volatile than Services, 3 times more volatile than non-durable goods, and something less volatile than durable goods (the relative standard deviation is equal to 0.88).

Since we have only a partial series of advertising expenditures we checked the robustness of previous facts by computing the same moments using Coen's data, which report the annual total expenditures in advertising. Results are in table (2.2).

Annual figures confirm the quarterly evidence. Total advertising is procyclical –

X_t	$\sigma(X_t)$	$\frac{\sigma(X_t)}{\sigma(Gdp_t)}$	ρ	$\sigma(X_t, Gdp_t)$
Gdp	1.40	1	0.07	1
Adv	2.40	1.70	0.16	0.69
$\frac{\textbf{Adv}}{\textbf{Gdp}}$	1.77	1.23	0.14	0.10
Newspapers	2.90	2.02	0.16	0.63
Magazines	3.60	2.53	0.19	0.76
Radio	2.40	1.68	0.12	0.57
Television	7.70	5.40	-0.03	0.54
Outdoor	3.80	2.65	0.01	0.51

Note: all variables are in logs, and are detrended with the Band Pass (2,8). Data sample goes from 1947 to 2005.

$cov\,(Adv_t, GDP_t) \simeq 0.7$ –, and more volatile than output – $\sigma(Adv_t)/\sigma(GDP_t) = 1.70$, in accordance with the quarterly evidence.

As last issue we analyze dynamic cross correlations between advertising, consumption, and investment. Dynamic correlations are useful to provide empirical evidence to support or dismiss the idea that advertising can be a leading indicator of the cycle.

As we can see from table (2.2), advertising and consumption move contemporaneously (i.e. the stronger correlation occurs at k=0), and the same is true for advertising and investment, even though in this case the evidence is weaker: the correlation coefficients at k=0 and k=1 are almost the same. Such time path of advertising is in contrast with the one found in Blank (1962). He reported evidence that advertising tends to lag output, and similar results ware found in Yang (1964). The difference between our results and theirs may be due to the different data samples, or to different filters used in those papers.[15] However, the results from the dynamic cross-correlation analysis seem to dismiss the idea that advertising is a leading indicator of the cycle. The fact that advertising slightly leads GDP is possibly due to the

[15] Both Blank and Yang used first differences, as usually done at that time to isolate the business cycle components.

$\sigma\left(\mathbf{X}_t, \mathbf{Gdp_{t+k}}\right)$									
k	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**
X_t									
Adv.	0.00	0.25	0.50	0.69	0.80	0.82	0.72	0.58	0.41
Cons	0.04	0.29	0.54	0.76	0.90	0.93	0.86	0.70	0.47
Inv	0.12	0.42	0.70	0.89	0.95	0.88	0.71	0.49	0.26
$\sigma\left(\mathbf{X}_t, \mathbf{Adv}_{t+k}\right)$									
Cons	0.25	0.47	0.67	0.79	0.83	0.78	0.65	0.48	0.30
Inv	0.06	0.30	0.52	0.71	0.83	0.82	0.72	0.54	0.33

Table 2.2: **Dynamic cross correlations, quarterly data.** All variables are filtered with BP(6,32) filter. Data sample from 1971q1 to 2005q4.

fact that it co-moves with consumption, which itself has been shown to lead GDP in the data.[16]

All in all, we can summarize our main findings as follows:

- The amount of resources invested in advertising in U.S. accounts for roughly 2% of GDP.

- Advertising is strongly procyclical, and positively correlated with both consumption and investment.

- Advertising is highly volatile: more volatile than GDP, consumption, and non durable consumption, but less than investment, and durable consumption. Also, it is persistent over the cycle.

- Advertising and total consumption are contemporaneously correlated, and both slightly lead GDP over the cycle.

[16]See Wen (2001) and Wen and Benhabib (2004).

2.3 A DSGE model with Advertising

In this section we describe our model economy and display the problems of households and firms. The market consists of a continuum of imperfect substitute consumption goods produced by monopolistically competitive producers that posses the technology to advertise their products. Advertising is assumed to generate *urge to consume* the advertised good. We obtain this effect by introducing advertising as a complement of consumption in the utility function. We will support this modeling strategy extensively in section 3.5.

Then, we embed this assumption in an otherwise standard dynamic stochastic growth model with no nominal or real frictions. The sources of uncertainty in this model are: (i) a shock on the production technology; (ii) a shock on preferences; (iii) a shock on the exogenous government spending; (iv) an idiosyncratic shock on the production of advertising; (v) a shock on the markup.

2.3.1 The household and the role of advertising

We assume that exists a representative consumer with preferences defined over consumption and hours worked described by the utility function

$$U(\tilde{C}_t, H_t) = \sum_{t=0}^{\infty} \beta^t \left[\frac{\tilde{C}_t^{(1-\sigma)} - 1}{1-\sigma} - \xi_t \frac{H_t^{1+\phi}}{1+\phi} \right] \tag{2.1}$$

where $\tilde{C}_t$ is consumption aggregator, H_t is the time devoted to work, ξ_t is a preferences shock that varies the disutility of labor.

The consumer draws utility from a "composite consumption" aggregator $\tilde{C}_t$, which is defined as follows:

$$\tilde{C}_t = \left(\int_0^1 \left(c_{i,t} + B\left(g_{i,t} \right) \right)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.2}$$

where $\varepsilon > 1$ is the pseudo-elasticity of substitution across varieties; $g_{i,t}$ is the goodwill associated with good i – the goodwill is meant to represent the stock of firm's advertising accumulated over time –, and $B(\cdot)$ is a decreasing and convex function controlling for the effectiveness of the goodwill on consumer's preferences.

Building on Arrow and Nerlove (1962), we model the dynamic effect of advertising assuming that current and past advertising add up to create the reputation of a good, the producer's *goodwill*, which is defined as the intangible stock of advertising that affects consumer's utility at time t, as shown in (2.2). As usual, the stock of goodwill evolves according to the law of motion:

$$g_{i,t} = z_{i,t} + (1 - \delta_g)\, g_{i,t-1} \tag{2.3}$$

where $z_{i,t}$ is firm's investment in new advertising at time t, $\delta_g \in (0, 1)$ is the depreciation rate of the goodwill.

We introduced the concept of goodwill because several empirical studies argued that advertising campaigns affect sales for longer than one period, and this evidence seems robust across goods, countries, and sample periods.[17] Hence, today marginal utility, which is linked to sales in a way we will make clear below, is likely to be affected not only by current advertising expenditures, but also by past advertising, with an intensity that fades out as time goes by.

According to (2.2), consumer's utility depends positively on the level of consumption and on the level of goodwill. For each good i, the goodwill i is a complementary argument in the utility function. Hence, the marginal utility of good i increases in the level of the goodwill i. Notice, indeed, that

$$\frac{\partial^2 U\left(\tilde{C}_t, H_t\right)}{\partial c_{i,t} \partial gi, t} \propto -\frac{1}{\varepsilon}\left(c_{i,t} + g_{i,t}\right)^{\frac{-(1+\varepsilon)}{\varepsilon}} B'(g_{i,t}) \geq 0 \tag{2.4}$$

[17] In particular, see Clarke (1976) for an empirical study of the dynamic effects of advertising in U.S., and Bagwell (2005) for a survey.

where the last disequality comes from the assumption that $B(\cdot)$ is decreasing in $g_{i,t}$.

This setup reflects what in literature is known as the persuasive role of advertising: advertisements create some added value to the good that wouldn't otherwise exist. Accordingly, the advertised good worths more for consumers, as if it were a newly different good. The way we capture this effect is to assume that an increase of advertising raises the marginal utility of consumed goods.

From a theoretical point of view, the way advertising affects the consumers' decision is a controversial issue. As pointed out in Bagwell (2005), *"no single view of advertising is valid in all circumstances"*. Also the empirical evidence is controversial. Some empirical studies support the informative, others the persuasive nature of advertising.[18] Although a framework that considers only the persuasive role of advertising could appear as a serious limitation of our analysis, there are several reasons to support such choice. Firstly, it has been often emphasized[19] that advertising, since pursued by an interested party, largely tries to persuade rather than to inform consumers. This implies that advertising can be considered as persuasive in nature. Secondly, recent studies of behavioral economists provided evidence on how consumers' tastes are distorted by advertising. For instance, Gabaix and Laibson (2004) have shown that, also in context of informative advertising, it is theoretically possible that information revelation may break down in the presence of consumers that fail to foresee shrouded attributes, such as hidden fees or maintenance costs. Finally, from a modeling point of view, disregarding the information asymmetries implied by the informative advertising, allows us to maintain a certain analytical tractability when solving a general equilibrium model of the aggregate economy.

Recently, non-homothetic preferences as the one in (2.2) have received an increasing attention in macroeconomics. Ravn et al. (2006) have modeled habits formation

[18]See Bagwell (2005, chapter 2) for a very detailed survey.
[19]See for instance Kaldor (1950)

over single variety goods using a non-homothetic consumption aggregator that resemble the one we have assumed here. There are indeed several similarities between our framework and the recent "deep habits" literature.[20] As the habits, the goodwill is a negative externality. Thus, when increases it creates dissatisfaction in the agent about his current level of consumption, causing the demand to increase. In this perspective, advertising can be interpret as an endogenous taste shock which is able to generate dynamic effects on consumption that mimic the ones due to habits formation.

There is however a fundamental difference between these two frameworks. Under the "deep habits" hypothesis, the firm can affect future demand by lowering the price today and, therefore, increasing the number of customers that has habits to consume its good. In this case strategic pricing policy is a form of indirect preferences manipulation. In our framework, oppositely, advertising is a non-price policy that increases the number of customers by directly influencing their preferences. In this perspective, advertising and price are different but complementary policies for the firms.

The rest of the model is standard. We assume that the representative consumer holds one asset, the capital stock K_t, which is assumed to evolve over time according to the following law of motion:

$$K_{t+1} = I_t + (1 - \delta_k) K_t \tag{2.5}$$

where I_t denotes the investment, which is a composite good produced with the fol-

[20]It is also interesting to note that the formation of consumption habits is an important goal of the advertised firms. As pointed out by Arens (1993), advertisers have three habit-related goals: i)Habits breaking-To get consumer to break an existing habit ii) Habits acquisition- To get consumers acquire the habit of buying their brand iii) Habit reinforcement- To convince current users to remain habitual purchasers. Overall, inducing habits it's an important strategy that reduces the firms' risk of demand fluctuations.

lowing technology:

$$I_t = \left(\int_0^1 (i_{i,t})^{\frac{\varepsilon-1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.6}$$

The representative consumer rents whatever capital he owns to firms, and supplies labor services per unit of time. Labor and capital markets are perfectly competitive, with a wage W_t paid per unit of labor services, and a rental rate R_t paid for unit of capital. In addition, the consumer receives net profits Π_t from firms and pays lump sum taxes T_t to finance the exogenous government spending. Under these assumption, the representative agent's nominal budget constraint is defined as:

$$\int_0^1 p_{i,t} \left(c_{i,t} + i_{i,t} \right) di \leq W_t H_t + R_t K_t + \Pi_t - T_t \tag{2.7}$$

The utility maximization problem for the representative consumer can be stated as choosing the processes $\tilde{C}_t$, H_t in order to maximize the utility function (2.1) subject to (2.5) and (2.7).[21] Notice that the consumer does not choose the desired level of goodwill. Following the Persuasive view of advertising, we assume here that the agent passively receives the advertising signaling, which eventually modifies his preferences, without being aware of the effects of advertising on his utility. This feature disentangles our framework from Becker's (1993) *complementary* theory of advertising. In Becker (1993) the agent actively shows a demand of advertising every time he consumes a good, since the information brought by advertisements increases the utility of consuming that good.

[21]To solve the maximization problem is useful to write the budget constraint in the Lagrangian as function of $\tilde{C}_t$, I_t. Notice that in the optimum holds $\int_0^1 p_{i,t} i_{i,t} di = P_t I_t$ and $\int_0^1 p_{i,t} c_{i,t} di = P_t \tilde{C}_t - \int_0^1 p_{i,t} g_{i,t} di$.

The first order conditions for a maximum are:

$$\frac{\tilde{C}_t^{-\sigma}}{P_t} = \lambda_t \tag{2.8}$$

$$\lambda_t = \beta E \left\{ \lambda_{t+1} \left[R_t + (1 - \delta_k) \right] \right\} \tag{2.9}$$

$$\xi_t H^{\phi} = W_t \lambda_t \tag{2.10}$$

where λ_t is the lagrange multiplier associated with the budget constraint. Equation (2.9) is the familiar Euler equation that gives the intertemporal optimality condition, while equation (2.10) describes the labor supply schedule.

Optimality conditions (2.8), (2.9), (2.10) are standard in this type of models. Where our model differs from the standard one is that now the shadow price of consumption and leisure depends not only on aggregate consumption but also on aggregate goodwill. Consequently, consumer's decisions about labor and investment are affected by aggregate advertising. In particular, insofar as $\widetilde{C}_t$ has negative first derivative with respect to the goodwill, then advertising will increase both the level of aggregate consumption and the supply of labor.

This mechanism plays a pivotal role to determine the general equilibrium results that we will see in next section. A Partial Equilibrium analysis of the model is useful to see it.

2.3.2 Partial Equilibrium Analysis

Suppose that $\int B(g_{i,t})\, di$ exogenously increases. Given our assumptions, $\widetilde{C}_t$ decreases and marginal utility λ_t increases.

Consider the labor supply schedule (2.9). When λ_t increases agents evaluate more consumption relative to leisure, i.e. the marginal rate of substitution between consumption and leisure increases. Hence, agents are willing to work more in order to consume more, and an increase in aggregate advertising shifts the labor supply schedule to the right.

Consider now the euler equation (2.10). Since the goodwill is an AR(1) process, an increase in $\int B(g_{i,t})\, di$ raises both the marginal utility of consumption today λ_t, and tomorrow λ_{t+1}. Thus, the overall impact on C_t will then depend on the dynamic response of the goodwill. Besides, notice that the dynamics of the goodwill now influences the intertemporal elasticity of substitution. Intuitively, whenever the growth rate of the goodwill is positive, the consumer will postpone his consumption, since he foresees a higher marginal utility in future periods. Conversely, when the growth rate of the goodwill is negative, then the consumer suffers the *urge in consumption*, and increases his level of consumption today.

2.3.3 Firms

There is a continuum of firms indexed $i \in [0, 1]$, each producing a differentiated product, which is sold as consumption, investment, or government good.

The optimal demand function of consumption goods is the solution to consumer's problem of minimizing consumption expenditures subject to the aggregate constraint (2.2). It can be shown that this demand is equal to:

$$c_{i,t} = \max\left\{ \left(\frac{p_{i,t}}{P_t}\right)^{-\varepsilon} \tilde{C}_t - B(g_{i,t}) \ ; \ 0 \right\} \tag{2.11}$$

where

$$P_t = \left[\int_0^1 p_{i,t}^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}} \tag{2.12}$$

is the nominal price index.

The demand function (2.11) is a key relationship in our model. As we anticipated in previous section, it is increasing in the level of advertising: a positive investment in $z_{i,t}$ increases the stock of goodwill $g_{i,t}$, which in turns decreases $B(g_{i,t})$. The marginal utility of $c_{i,t}$ increases according to (2.4), making the consumer willing to pay more for the same amount of good, or equivalently, willing to buy more for any given price. Accordingly, (2.11) increases.

Thus, advertising acts in this model as a demand shifter. This prediction is in line with a large number of empirical studies about advertising at firm level.[22] In our setup such positive relationship crucially depends on the assumption that advertising is an argument of the utility function. It is worth noticing, however, that this assumption is not arbitrary once we restrict our attention to models with walrasian demand functions and perfect information, since in this models an effect of advertising on the demand function is equivalent to the assumption that advertising affects the preferences relation.[23]

As shown in the bottom panel of figure 2.3, the effect of advertising on the demand depends on the parameter θ, which can be interpreted as a measure of the effectiveness of advertising in shifting the demand.

The price elasticity of the demand is now time varying and decreasing in the level

[22] A positive relationship between advertising and sales one of the few non-controversial empirical evidence about advertising. See Bagwell (2005), section 3.2, for more references.

[23] The argument goes by contradiction. Suppose that advertising shifts the demand, that is the consumer chooses two different bundles pre and post advertising, and the preferences relation remains unchanged. Recall that advertising is assumed to be not a productive factor, nor to affect the production technology. Under these assumption, advertising does not change the marginal cost or the quality of the good. Hence, pre and post advertising the consumer is choosing his preferred bundle among the same basket of goods. Also, since we are assuming perfect information, any direct effect of advertising on prices is ruled out here. In this case the system of prices is unchanged before or after advertising, implying that the bundle chosen post advertising must have been affordable also pre advertising. This in turn implies that such bundle yielded a lower utility than the pre advertising one. As result, post advertising the agent is choosing a bundle which is not preferred to the pre advertising one, violating the Weak Axiom of Revealed Preferences. Therefore, if the bundle chosen by the agent before and post advertising changed, then also the preferences relation must have changed. However, this argument is not true anymore if we consider models with walrasian demand correspondences, instead of functions. Therefore, preferences relation in the model must always be *strictly* convex.

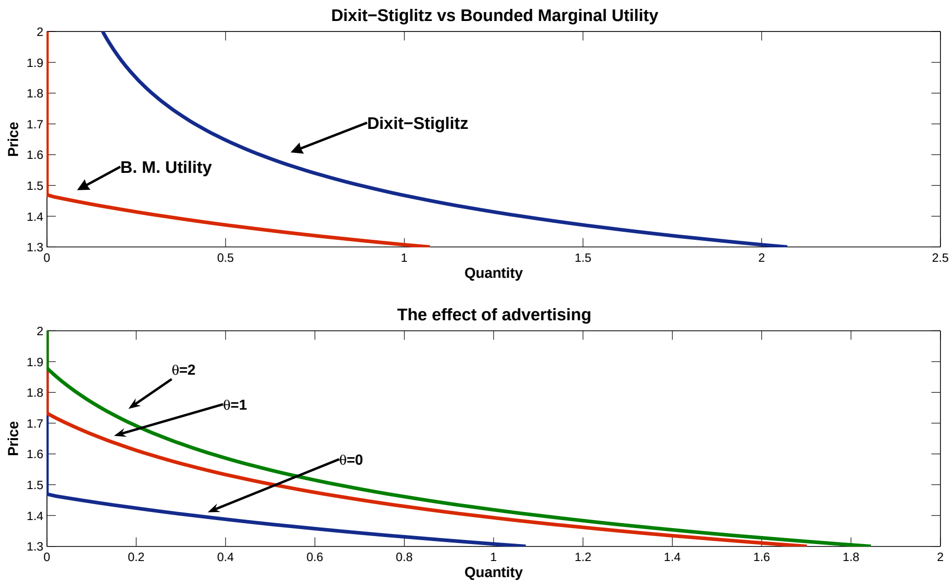

Figure 2.3: **Conditional Demand.**Top panel: Dixit-Stiglitz vs Bounded Marginal Utility. Bottom panel: the effect of alternative values for the intensity of advertising in the preferences, θ.

of goodwill. According to (2.11), the demand function is composed by two terms: the first one, $(P_{i,t}/P_t)^{-\varepsilon}\,\widetilde{C}_t$, with elasticity ε, and the second one, $B\left(g_{i,t}\right)$, inelastic. As consequence, the price elasticity is a weighted average between the elasticity of the two, and its value will depend on the relative importance of the goodwill over the total demand, i.e.

$$\eta\left(y_{i,t}, g_{i,t}\right) = \left|\frac{\partial y_{i,t}}{\partial p_{i,t}}\frac{p_{i,t}}{y_{i,t}}\right| = \varepsilon\left(1 + \frac{B(g_{i,t})}{y_{i,t}}\right) \tag{2.13}$$

In particular, notice that the elasticity of the demand (2.13) is smaller than the elasticity of the demand without advertising, i.e. $g_{i,t} = 0$.[24] The finding of a steeper demand schedule for advertised goods is a well know effect in the literature, called the *fidelization of the consumer*. The intuition behind this mechanism is that advertising, although without modifying the quality of the advertised good, increases the differ-

[24]This occurs because $B(g_{i,t})$ is decreasing in $g_{i,t}$.

entiation among goods perceived by the consumer. Thus, the firm can manipulate the elasticity of the demand for its product using advertisements. The link between advertising and price elasticity reveals another important mechanism inherent to our model: advertising increases the monopolistic power of the firm, thus affecting the level of prices through the markup.

Finally, it is interesting to note that the demand (2.11) implicitly embeds the combative nature of advertising. By differentiating equation (2.11) with respect to the goodwill and assuming that a sufficiently large fraction λ of firms changes together their advertising levels, we get:

$$\frac{\partial c_{i,t}}{\partial g} = \int_0^{\lambda} \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon} B'(g_{i,t}) di \ < \ 0 \, \forall \, i \ \in [\lambda, 1]$$

Therefore, the demand for a good reduces when other firms increase their advertising levels.

Contrarily to consumption, the demand function for good i sold as investment or government good is standard,[25] since we assume that advertising does not affect either the technology to combine single variety goods to form a unit of capital, nor the purchases of government. While the first assumption is hardly be questionable, the one about government expenditures may appear arbitrary. Notice, however, that the introduction of a positive effect of advertising on government expenditures would strengthen the results that we will find in next section. Consequently, we use it as a conservative assumption about the effect of advertising in the aggregate economy.

All together, the demand of good $i_{i,t}$ for consumption, investment, and government goods, forms the total demand by firm i at time t, i.e.:

$$y_{i,t} \equiv c_{i,t} + i_{i,t} + gex_{i,t} = \left(\frac{p_{i,t}}{P_t}\right)^{-\varepsilon} \left(\tilde{C}_t + I_t + Gex_t\right) - B(g_{i,t})$$

[25]The demands are derived in Appendix B.

Firm i will choose a price for sales and a level of advertising in order to maximize the discounted flow of future profits subject to the constraint given by the total demand. The optimal policy rules are derived formally in the Appendix B. We report them here:

$$p_{i,t} = \frac{\varepsilon \left(1 + \frac{B(g_{i,t})}{y_{i,t}}\right)}{\varepsilon \left(1 + \frac{B(g_{i,t})}{y_{i,t}}\right) - 1} \phi_t \equiv \mu_{i,t}\, \varphi_t \tag{2.14}$$

$$-\left(p_{i,t} - \varphi_t\right) B'(g_{i,t}) + E_t \left[(1 - \delta_g) \left(\nu_{t+1} r_{t,t+1}\right) \right] = \nu_{i,t} \tag{2.15}$$

where ϕ_t is the marginal cost of production, and $\nu_{i,t}$ is the marginal cost of producing new advertising $z_{i,t}$.

Equation (2.14) describes the familiar pricing policy in monopolistic competition models. The firm exploits its monopolistic power by charging a positive markup ($\mu_{i,t}$) over the marginal cost. However, differently from that standard monopolistic competition model, markup in (2.14) is time varying. In particular, it increases in the level of goodwill, due to the effect that the goodwill has on the elasticity of the demand, as we showed in (2.13).

Equation (2.15) is firm's optimal advertising policy. It states that the firm invests in advertising until the marginal benefit from an extra dollar of advertising equals the marginal costs of producing it. Given the dynamic nature of the goodwill, the marginal benefit in the LHS of (2.15) has now two components: the increase in current revenues associated with a marginal increase in advertising, and the discounted opportunity cost of not producing tomorrow the surviving goodwill produced today. Equation (2.15) turns out to be the dynamic version with monopolistic competitive markets of the well known Dorfam-Steiner (1954) theorem, which states that firms budget for advertising expenditures must be equal in the optimum to the ratio between the elasticity of the demand w.r.t. advertising over the elasticity of the demand w.r.t. price.

The optimal advertising policy is useful to understand the determinants of advertising. According to (2.15), advertising is sensitive both to costs and demand variations. On the one side, reductions in the marginal cost of advertising leads to higher investments in advertising. On the other side, the marginal benefit of advertising positively depends on the markup, which in turns is positively affected by the aggregate demand (see equation (2.14)). Therefore, any exogenous increase of the demand increases the markup and therefore raises the optimal level of advertising.

??Besides, notice that the markup itself is increasing in the level of goodwill. So, when advertising increases also markup increases. In fact, in this framework advertising and price are complementary policies, in accordance with the theory of optimal advertising as the outcome of firms playing a supermodular game, as showed in Tremblay (2005). This evidence should support our assumptions about advertising in utility function as key feature to develop a well behaved model of advertising.??

Also, notice that in the extreme case where advertising has no effect on the demand, i.e. $B'(\cdot) = 0$, then equation (2.15) implies that optimal advertising is equal to zero. Therefore, in this framework the only incentive for firms to advertise is to manipulate the demand. In particular, no strategic reason, as for instance entry deterrence is taken into account here.

An interesting result that comes from the optimal advertising policy is that advertising implies persistence in the dynamics of consumption. The reason is that, if equation (2.15) holds, then the conditional demand functions (2.11) will depend on past sales, as in the case of models with habits in consumption, or in models of costumers markets.[26] Notice, indeed, that equation (2.15) can be written as:

[26] About this issue, see Ravn, Schmitt-Grohé and Uribe (2006), and the "habit persistence" entry of Palgrave Economic Dictionary, written by Schmitt-Grohé and Uribe (2006).

$$g_{i,t} = \Phi_t y_{i,t} \tag{2.16}$$

where Φ_t is a time varying coefficient that depends only on the marginal costs of advertising and production, which are common to all firms.

Thus, in this model each firm invests in advertising a proportion Φ_t of sales, where the ratio of advertising over sales is bigger when the marginal cost of advertising ν_t is smaller, i.e. when advertising is cheaper. Now, working out $g_{i,t-1}$ from the law of motion of goodwill (2.3) using (2.16) lagged one period, and using the resulting equation to work out $g_{i,t}$ in the conditional demand (2.11), we obtain:

$$y_{i,t} = \left(\frac{p_{i,t}}{P_t}\right)^{-\varepsilon} \left(\widetilde{C}_t + I_t + Gex_t\right) + \psi\left(\Phi_t, y_{i,t-1}, z_{i,t}\right) \tag{2.17}$$

As we anticipated the demand (2.17) depends on past sales $y_{i,t-1}$. This result derives from two properties of our setup: (i) the assumption that consumer's preferences are affected by advertising; (ii) the dynamic effect of advertising, whose signals lasts in agents' memory for several periods, implying that the effect of goodwill fades out gradually in the utility function. Compared with other models that explained consumption persistence in the literature, the relevance of our result hinges on the fact that consumption persistence with advertising is endogenously derived in equilibrium, and not exogenously assumed as, for instance, with habits persistence.

Equation (2.17) reveals another interesting aspect of advertising. Since $z_{i,t}$ is now a positive component of the demand, an increase in advertising shifts the demand upward with an effect similar to the one of an exogenous demand shock. Now, given that a positive technology shock will increase firms desired level of goodwill, then when a technology shock occurs we also observe that the aggregate demand shifts upward. Therefore, after a technology shock the level of demand will increase not only because prices are lower (in this framework a technology shock decreases the marginal cost) but also because for any given price the consumers are now willing to buy more goods. In other words, advertising channel of transmission of technology shocks to "endogenous demand shocks", possibly explaining some percentage of the demand volatility that we would otherwise account as exogenous demand shocks. In section 4 we will use the variance decomposition from model estimation to test for this prediction of the model.

2.3.4 The Symmetric Equilibrium

In this model firms are price takers on factor markets, and they share all the same production technology. Thus, they all face the same marginal cost.[27] Moreover, all firms face the same gross elasticity of substitution among goods ε. These two conditions jointly imply that there exists a symmetric equilibrium where all firms set the same price, produce the same quantities, and invest the same amount of resources in advertising. In addition, in each period the equilibrium (common) price of goods is normalized to unity, i.e. $p_t = 1 \quad \forall\, t$. So, all the other prices (e.g. wage, rental rate) are expressed in terms of contemporaneous consumption.

Let X_t to be the vector of all the endogenous variables in equilibrium,[28] then the symmetric equilibrium for this model is a process $\{X_t\}_{t=0}^{\infty}$ that satisfies: (2.8)-(2.10), (2.14)-(2.15), plus the production function of consumption goods and advertising, the optimal factors demand for these productions,[29] the law of motion of capital (2.5) and the one of goodwill (2.3), the clearing market condition on the goods market, $Y_t = C_t + I_t + Gex_t$, and the clearing market condition on the labor market, $H_t = H_{p,t} + H_{a,t}$.

[27]The reader can check that inspecting the RHS of equation (B.3).
[28]Specifically, $X_t = (\lambda_t, G_t, \mu_t, Z_t, H_t, Gex_t, H_{a,t}, H_{p,t}, C_t, K_t, I_t, Y_t, R_t, W_t, Q_{t,t+1})$.
[29]See Appendix B for details.

2.3.5 Advertising in Utility Function: Functional Forms Assumptions

In order to characterize the utility function we need to assume a functional form for $B(\cdot)$. As we said in section 3.1 the requirements are, (i) $B^{'}(\cdot) < 0$ s.t. $\frac{U^{II}\left(\tilde{C}_t, H_t\right)}{\partial c_{i,t} \partial gi,t} \geq 0$, and (ii) $B^{''}(\cdot) \leq 0$ i.e. that advertising has decreasing returns in enhancing the conditional demand (2.11).

Moreover, by assuming an appropriate specification for $B\left(\cdot\right)$ we want to nest in this model the two competitive theories of advertising that we mention before: i.e. market enhancing and spread-it-around hypothesis. The advantage of this strategy is that it allows us not to take a stand on these different views, but to leave the data to choose which theory is more likely to be true (conditional on macroeconomic data). It worths stressing the fact that this point is crucial to assess all the effects of advertising in the aggregate economy. Indeed, as we said in the introduction, if advertising enhances the single-good demands because it steals costumers to competitors, then it has no effect on aggregate consumption, which in turns implies that in the aggregate advertising only absorbs resources without affecting any other macroeconomic variable. On the contrary, if it increases the market size for a good regardless the effects on other goods, then it affects the aggregate consumption and, through this channel, all the other aggregate variables, like labor, output and prices.

We assume:

$$B(g_{i,t}) \equiv F(g_{i,t}) - \gamma \int_{o}^{1} 1 - F\left(g_{i,t}\right) di \tag{2.18}$$

where

$$F(g_{i,t}) \equiv \frac{1}{1 + \theta g_t} \tag{2.19}$$

In (2.18) the goodwill enters in quasi-difference from its aggregate mean value. Therefore, advertising enhances firms demand only if they advertise more than their competitors. In the symmetric equilibrium solution of the model this implies:

$$B(G_t) \equiv \frac{1 + \gamma\theta G_t}{1 + \theta G_t}$$

Now, when $\gamma = 1$, the aggregate goodwill does not enter in equilibrium in the marginal utility of consumption (see equations (2.8) and (2.2)). So, it does not affect the representative consumer's decisions about labor supply (2.9) and consumption (2.10). As a consequence, advertising does not affect aggregate consumption level, i.e. the spread-it-around hypothesis holds.

In this case the effects of the aggregate advertising on the economic activity are easy to predict. It absorbs resources without enhancing the demand, and it has no price effects (notice that the markup (2.14) will be the same that in a model without advertising). Thus, it is a deadweight loss both for firms and consumers. However, notice that firms will still be employing factors to advertise - in the optimal advertising policy γ does not enter in $B'(\cdot)$. The reason is that in the non cooperative solution firms don't internalize the effect of their decisions on the aggregate level of advertising in the market. As a consequence, they keep wasting money in an unproductive factor, while the effects of their advertisements on the demand functions is offset by other firms' advertising.

Differently, when $\gamma = 0$ the goodwill enters in level in the utility function. Accordingly, firm's i advertising affects the marginal utility of $c_{i,t}$, and in turns shifts upward the demand function of good i, no matter what the other firms do. In this case advertising affects aggregate consumption, labor, and firms' markup. Finally, any value of $\gamma \in (0, 1)$ implies a model where the effect of advertising is a linear combination between the two extreme cases (complete spread-it-around vs. market enhancing).

A consideration apart deserve the choice of $F(\cdot)$. Equation (2.19) implies that the marginal utility of consumption is bounded (for this reason we might refer to this model as "bounded marginal utility"). In fact, in the demand function of good i there exists a maximum price above which the demand is zero − this is clearly due to the bound we impose on the marginal utility of consumption −. When the price is too high the marginal benefit of consuming that good is smaller than its cost, and the consumer drops it from his basket of purchases.

In absence of advertising the bound is 1 in every period, whereas in presence of advertising the value of the bound depends on the level of goodwill. In particular, the incentive for firms to advertise is the effect of the goodwill in reducing the bound, which implies that the consumer is willing to pay an higher price for the advertised good. Finally notice that the effect of the goodwill is bigger the higher is θ, and the higher is the level of goodwill of firm i with respect to the mean goodwill of other firms. θ can be interpreted as a measure of the effectiveness of advertising in manipulation consumer's tastes.

2.4 Results

In what follows we refer to a log-linear approximation of model's policy functions in the neighborhood of the non-stochastic steady state. Rational expectations are solved to obtain an AR representation of the model. This loglinearized version of the model is used to accomplish a number of tasks.

1. Impulse Response Functions (IRFs). We characterize the response of model's variables to several exogenous shocks, namely: a technology shock (figure 2.4), a preferences shock[30] (figure 2.5), a shock on the exogenous government spending (figure 2.6), and an idiosyncratic shock to the advertising production function (figure).

[30]We model preferences shock as a shock on the disutility of labor.

2. Model estimation. We use Bayesian technique based on likelihood maximization to estimate the set of equation (B.17)-(B.37).[31]

3. Finally, we compute some selected statistics using parameters estimates to address an answer to the research question that we posed at the beginning of the chapter: Does advertising effectively influence the level of aggregate consumption?

2.4.1 The aggregate dynamics

To compute the IRFs we calibrate parameter values such that model steady states match selected long run moments of U.S. postwar data. In particular, the ratio of consumption over GDP is around 75%, the representative consumer works 25% of his time, and labor share is around 2/3. Time period in the model is a quarter. The discount factor β is calibrated such that interest rate in steady state is 4% on annual basis. The depreciation rate of capital δ_k is equal to 2.5% per quarter, and the gross elasticity of substitution across varieties equal to 6. The following table (2.3) summarizes the whole set of calibrated parameters.

We plot the IRFs for different values of the spread-it-around parameter γ, and we use the associated model economy where advertising is banned as benchmark to evaluate the impact of advertising.

A number of results are worth emphasizing about the dynamics of this model. First, advertising responds positively to any of the shocks considered (see figure 2.7). Whenever the aggregate demand increases, firms want an higher level of goodwill. This occurs because the marginal benefit of goodwill increases with the demand, as apparent from equation (2.15). As result advertising in the model is procyclical as in real data.

[31]See Appendix C.

Parameter	Value	Description
β	.9902	Subjective discount factor s.t. $R = 1.04^{1/4}$
ε	6	Elasticity of substitution across varieties s.t.
		Steady State of markup is ca. 1.07
$\delta_{\mathbf{k}}$	0.025	Depreciation rate of Capital, 10% annual
$\boldsymbol{\Xi}$	3.10	Steady State of the Preference shock s.t.
		$H = 25\%$ total endowment of hours
$\delta_{\mathbf{g}}$	0.3	Depreciation rate of goodwill s.t.
		half life of goodwill is ca. 2 quarters (Clarke, 1976)
$1/\phi$	1.3	Frisch elasticity of labor supply
θ	2.70	Intensity of advertising in Utility s.t. $\frac{ADV}{GDP}$ is 2%
α	0.75	s.t. aggregate labor share is around 2/3
σ	2	Inverse of intertemporal elasticity of substitution
ρ_a, ρ_h, ρ_z	0.95	Persistence of exogenous shocks

Table 2.3: Parameters calibration

Out of all the shocks considered, the effect of advertising on consumption is strongest in case of a technology shock, as we can see comparing figures (2.4) (2.5) and (2.6). This occurs because technology shocks not only increase the aggregate demand, thus increasing marginal benefit of advertising, but also diminishes firm's marginal cost, thus making advertising cheaper. The two effects, higher marginal benefit and lower marginal cost, jointly push upward the level of advertising.

Second, advertising amplifies the response of total worked hours. An increase in advertising raises marginal utility of consumption, thus diminishing the marginal rate of substitution between leisure and consumption. So, in presence of advertising the consumer is less reluctant to work, and therefore works more hours to afford more consumption goods. This mechanism is known in the literature as the *work and spend*

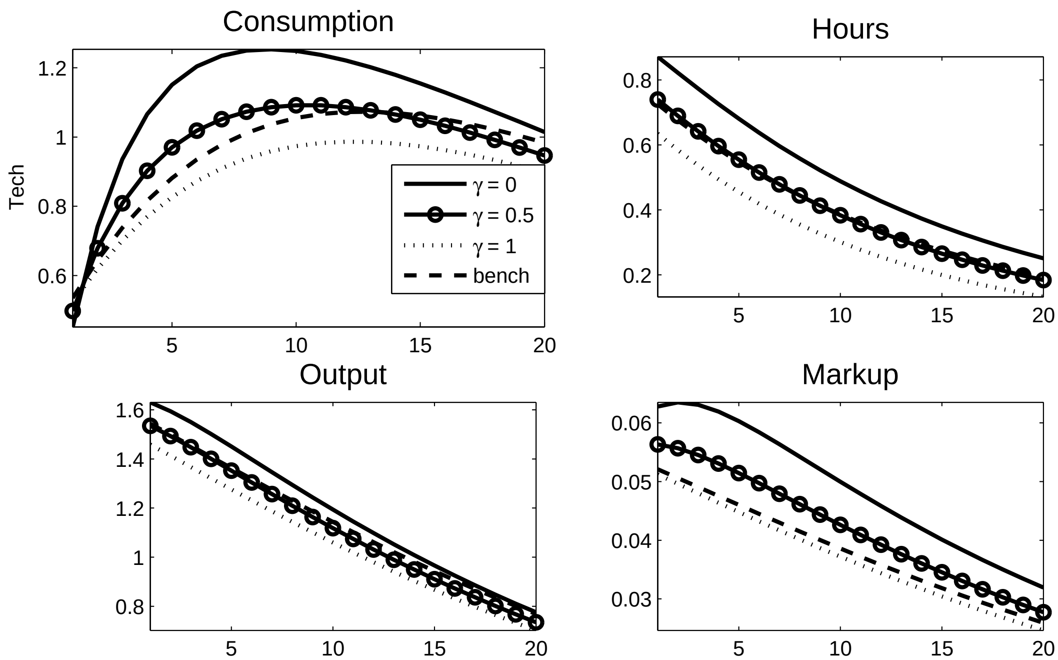

Figure 2.4: **Impulse Response Functions to technology shock.** Each plot displays percent deviation from steady state of corresponding variable in response to a shock of magnitude 1.

cycle, and has been supported by various empirical works, like Brack and Cowling (1983) for US economy, and Fraser and Paton (2003) for UK.[32]

Third, advertising increases the volatility of the markup. Note that in a model with bounded marginal utility the markup is procyclical, since the elasticity of the demand is inversely related to the output (i.e. higher output steeper demand). Now, according to equation (2.15) during booms firms advertise more because each unit of advertising yields higher marginal benefit. Thus, after a positive shock the higher goodwill further reduce the elasticity of the demand (see equation 2.14), amplifying the response of the markup. The intuition is that with an higher level of advertising the consumer values more his consumption, and firms exploit the consumer's higher willingness to pay by raising the markup.

[32]See Molinari and Turino (2007) for details.

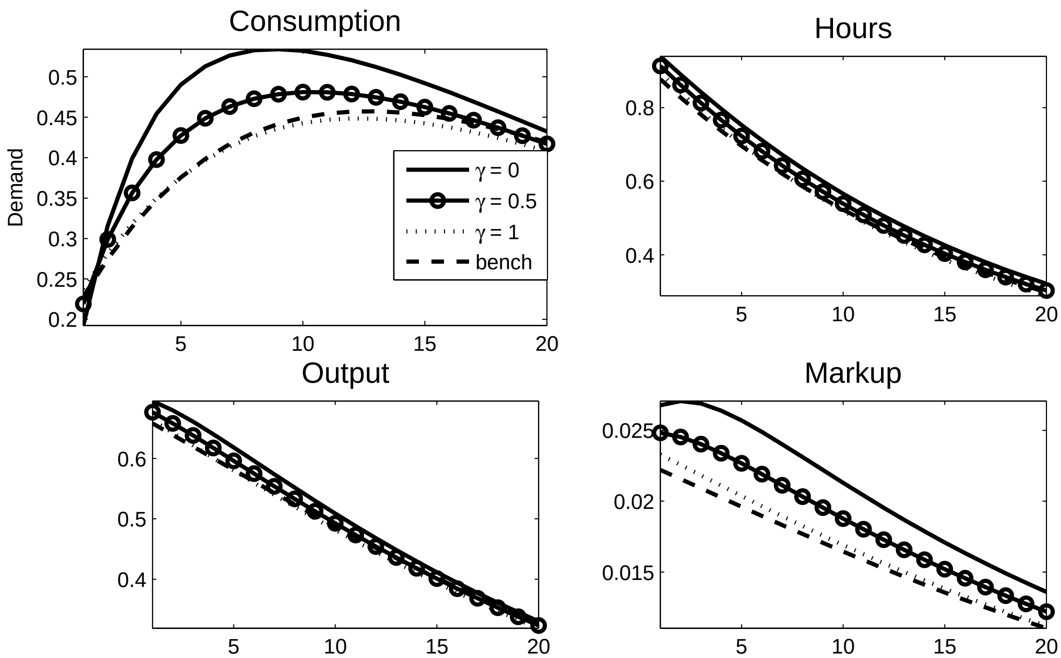

Figure 2.5: **Impulse Response Functions to preferences shock.** Each plot displays percent deviation from steady state of corresponding variable in response to a shock of magnitude 1.

Welfare Cost of fluctuations

Previous results can be quantified computing the the welfare cost of fluctuations, and comparing them with the ones in the benchmark economy. As usual in the literature on welfare cost of fluctuations,[33] these costs are defined as the units of Steady State consumption that consumer would be willing to pay to eliminate variability in his consumption stream. Such costs are computed as the second order Taylor approximation around the steady state difference between the welfare of an agent endowed with optimal consumption and labor bundle from the model equilibrium solution and, and the welfare of an agent that gets permanently a constant consumption in the same amount of the steady state consumption from the model solution, and works

[33] See Erceg Henderson and Levin (2000), or Otrok (2001).

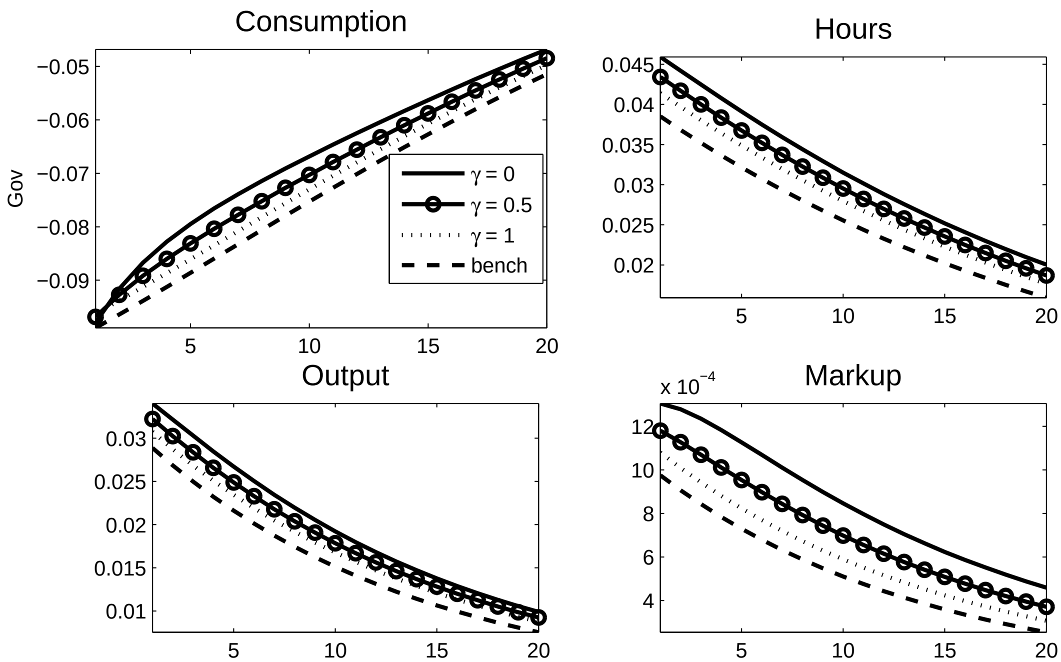

Figure 2.6: **Impulse Response Functions to government spending shock.** Each plot displays percent deviation from steady state of corresponding variable in response to a shock of magnitude 1.

the steady state amount of hours.

In case of welfare costs comparison between two different economies endowed with different preferences relations, the literature suggests to make a double comparison between the welfare costs by computing them with any of the two preferences relations at issue.[34]

In this case, however, it seems a more appropriate choice to consider only the ex-ante preferences[35] for the welfare comparison, since "ex-post" preferences account for the volatility of the goodwill, biasing the costs of fluctuations of consumption and labor.

[34] See Benhabib and Bisin (2002). This point is explained in details in Molinari and Turino (2007).

[35] In the literature about the welfare cost of advertising, the preferences relation where advertising does not enter in utility function is typically called *ex-ante* preferences relation. In our case this is the preferences relation that holds in the benchmark economy

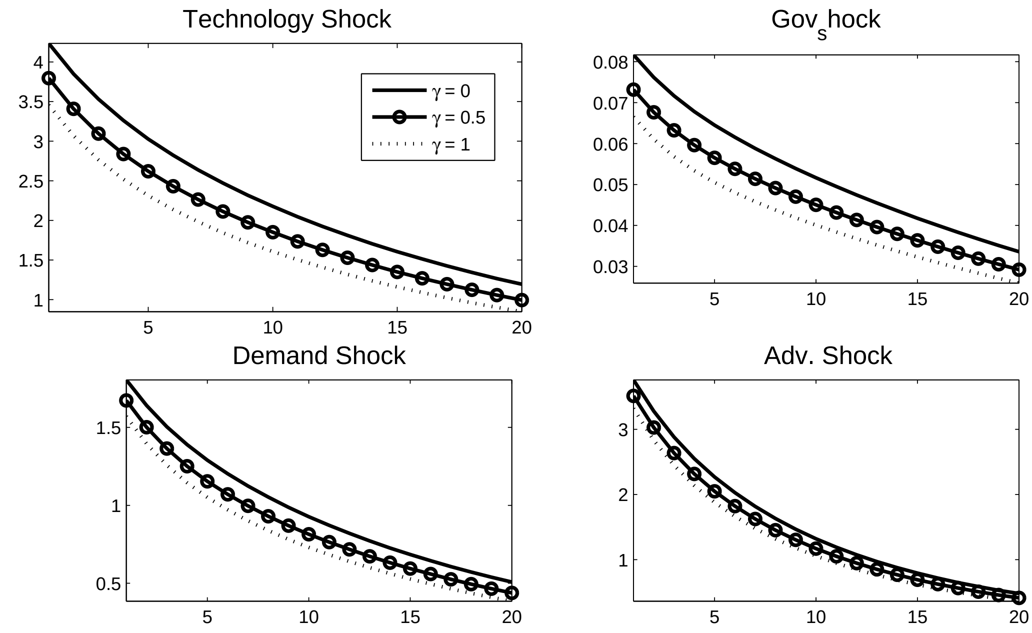

Figure 2.7: **Impulse Response Functions of advertising to ALL the shocks considered.**Each plot displays percent deviation from steady state of corresponding variable in response to a shock of magnitude 1.

Table (5) reports the welfare cost of fluctuations in the model with advertising and $\gamma = 0$ (strongest effect of advertising in the aggregate). Costs are measured in percentage of Steady State consumption. We assess the effect of advertising by comparing the percent difference between the welfare costs in the economy with advertising against the ones in the benchmark model. Also, we report different costs for different calibrations of the two parameters that enter in the utility function, i.e. the risk aversion parameter σ and the Frisch elasticity of labor $\frac{1}{\phi}$.

As we can see, the consumer is always willing to pay an higher percentage of his consumption to get rid of fluctuations when consumption goods are advertised.

Ex-ante preferences relation, $\gamma = 0$			
	Costs in percentage of SS consumption		
σ	$\phi = 0$	$\phi = 0.77$	$\phi = 3$
1	0.044 (53.8%)	0.008 (25.3%)	0.001 (17.7%)
2	0.171 (63.4%)	0.047 (23.6%)	0.021 (16.4%)
5	0.247 (145%)	0.11 (75.8%)	0.079 (42.3%)

Table 2.4: **Welfare costs of Fluctuations.** In parentheses the percent increase in welfare costs from the benchmark model without advertising to the model with advertising and $\gamma = 0$.

2.4.2 Model Estimation

We estimate the model using a Bayesian approach based on maximization of the likelihood of observable data. Bayesian estimation is preferred among other estimation techniques, e.g. GMM, for several reasons. First, in our model a crucial variable to estimate advertising related parameters is the unobservable goodwill, and the maximum likelihood approach with the Kalman filter used to evaluate the likelihood is the more natural way to cope with unobservable variables. Second, the effects of advertising typically spreads in the economy through various transmission channels, which can entirely be assessed only once we compute the general equilibrium solution of the model. Therefore, any estimation that exploits only the information contained in partial equilibrium relationships, e.g. a GMM estimation of the Euler Equation, would neglect some important information from the model.

For the estimation we add to the system of equations that characterize the equilibrium solution of the model, 4 measurement equations that link the observable variables to the variables in the model. Our data set consists in log-differences data of consumption, output net of exports, total hours worked and aggregate advertising as defined in section 2. Details about the data are given in Appendix A, and the full set of equations that we estimate is given in Appendix C. To avoid the potential problem of singularity in the covariance matrix of forecast errors due to model misspecification, we estimate a model in which the number of structural shocks equals the number of observables to which the model is fitted. A good strategy to use joint information from all the observables to identify all the parameters is to shut down the idiosyncratic shock in advertising production function and to introduce a shock on the elasticity of the demand. This shock is usually interpreted in the literature a as a cost push shock on firms markup.

We keep some parameters fixed, namely the discount rate β, the gross elasticity of demand ϵ, the depreciation rate of capital δ_k, the depreciation rate of the goodwill δ_g, and the steady state value of the preference shock Ξ. The first three parameters are typically of difficult identification in RBC models, while δ_g turns out to be not identifiable separately from θ. Finally, we fixed Ξ because using data on worked hours in difference leaves undetermined the mean level of hours worked. Fixed parameters are calibrated according to the values in table (2.3).

All the other parameters, $\{\sigma, \phi, \alpha, \alpha_z, \theta, tg_q, \gamma, \rho_a, \rho_h, \rho_i, \rho_{mk}, \sigma_a, \sigma_h, \sigma_i, \sigma_{mk}\}$, are estimated combining the information from data likelihood with the one contained in the priors. Distributions for the priors are chosen according to what used in the literature, while priors means are chosen such that steady states of model variables match selected long run moments of U.S. postwar data at priors means. Details are given in the tables (2.5) and (2.6):

In general our priors for the structural parameters are quite flat. The prior on θ is a gamma distribution (i.e. θ is bounded in zero) with mean 2.50 and variance 0.4. Given $\delta_g = 0.3$, this value of θ implies a ratio of advertising over GDP equal to 0.02, in line with the empirical evidence presented in section 2. The prior for γ is an uniform (0,1) distribution, which reflects our neutral stand between the spread-it-around and

Parameter	Density	Domain	Mean	Std. Dev.
σ	Gamma	$\mathbb{R}^+$	2.00	0.40
ϕ	Gamma	$\mathbb{R}^+$	0.77	0.40
θ	Gamma	$\mathbb{R}^+$	2.50	0.40
γ	Uniform	$[0,1]$	0.5	0.08
α	Beta	$[0,1)$	0.75	0.04
$\alpha_{\mathbf{z}}$	Beta	$[0,1)$	0.75	0.04
tg_q	Normal	$\mathbb{R}_+$	.005	.002

Table 2.5: Prior Distributions of Structural Parameters

the market enhancing hypotheses of advertising. For the shocks processes we use very standard priors, following Smets and Wouters (2007), Chang, Doh and Schorfheide (2006), An and Schorfheide (2007).

The estimation is performed in three steps. First, we maximize the posterior kernel in order to find the mode of the posterior distribution. Second, starting from a random perturbation around the mode, we sample from the posterior distribution using a random walk Metropolis-Hastings algorithm.[36] We run this algorithm 5 times from 5 different perturbations. This strategy assures relatively fast convergence of the Markov Chains generated from the algorithm, at least compared with what reported in related literature. Convergence diagnostic indicates that around 30,000 drawings are enough to attain convergence. We check for convergence computing the 80% interval around the cumulated mean (both within and between chains) of accepted candidates over total drawings, and then checking whether the cumulated means stabilize for an increasing number of drawings and whether they converge to the same value.

As last step, we report selected statistics for the posterior distributions by comput-

[36]The variance of the jumping distribution is the inverse of the Hessian from the maximization of the mode, multiplied by 0.35. Acceptance rate is around 35%.

Parameter	Density	Domain	Mean	Std. Dev.
$\sigma_{\mathbf{y}}$	InvGamma	$\mathbb{R}_+$	.008	Inf
$\sigma_{\mathbf{h}}$	InvGamma	$\mathbb{R}_+$	.034	Inf
$\sigma_{\mathbf{i}}$	InvGamma	$\mathbb{R}_+$	.099	Inf
$\sigma_{\mathbf{mk}}$	InvGamma	$\mathbb{R}_+$	.039	Inf
$\rho_{\mathbf{y}}$	Beta	$[0, 1)$	0.6	0.20
$\rho_{\mathbf{h}}$	Beta	$[0, 1)$	0.6	0.20
$\rho_{\mathbf{i}}$	Beta	$[0, 1)$	0.6	0.20
$\rho_{\mathbf{mk}}$	Beta	$[0, 1)$	0.6	0.20

Table 2.6: Prior Distributions of Shock Processes

ing the correspondent moments from the Markov Chains where we randomly discard 40% of observations from each chain in order to reduce the autocorrelation of the drawings.

In tables (2.7) and (2.8) we report mean and the 90% interval from the posterior distributions. In Appendix C we provide also a set of figures that plot posterior against prior distributions for each parameter.

Our first concern is for the results of θ and γ, the two parameters related with advertising. The estimates seems quite informative, both posteriors have less variance than corresponding priors. The posterior mean of θ suggests that a value of 3 is the appropriate choice for this parameter. This result supports the calibration used both in previous section and in the following chapter, and implies a ratio of advertising over GDP of $1,6\%$ in the estimated model. This results is quite surprising because it is obtained using data in differences, therefore without any information about levels of advertising or output from the data.

The posterior mean of γ is close to 0.02, and the upper bound of 1 is significatively

	Prior		*Posterior*		
Parameter	Mean	Std.Dev.	Mean	Std.Dev.	90% interval
σ	2.00	0.40	2.98	0.397	[2.28 3.59]
ϕ	0.77	0.40	2.59	0.504	[1.76 3.42]
θ	2.50	0.40	3.05	0.440	[2.34 3.79]
γ	0.50	0.08	0.02	0.001	[0.017 0.022]
α	0.75	0.04	0.58	0.030	[0.53 0.63]
$\alpha_{\mathbf{z}}$	0.75	0.04	0.69	0.030	[0.64 0.74]
tg_q	.005	.002	.0033	.0007	[.0020 .0044]

Table 2.7: Posterior Distributions of Structural Parameters

rejected from the data.[37] This suggests that aggregate advertising is a significant explanatory variable of aggregate consumption, as conjectured by the market enhancing hypothesis. As a consequence, this estimation comfort the view that advertising affects aggregate consumption and through this channel, the whole aggregate economy.

About the other parameters, the estimates of σ and ϕ deserve some attention. Posterior means are relatively high compared to their estimations in similar models, e.g. Smets and Wouters (2007), or to the calibrations typically used in the literature. In order to check this fact, we estimate a standard RBC model without advertising using the same data set used for the advertising model. In particular, we estimate two verison of the model, one using the standard utility function with the Dixit-Stiglitz consumption aggregate, and the other using the bounded marginal utility. Estimates of (σ, ϕ) are reported in the following table:

While the evidence on ϕ are mixed, the ones on σ clearly indicates higher estimates in the model with advertising. Our interpretation of this result relies on the

[37] We estimated several specifications of the model, in all of them estimates of $\widehat{\gamma}$ where significantly different from 1, ranging in $\widehat{\gamma} \in (0.00, 0.39)$.

	Prior		*Posterior*		
Parameter	Mean	Std.Dev.	Mean	Std.Dev.	[90% interval]
$\sigma_{\mathbf{y}}$	.008	Inf	.007	.0005	[.0063 .0080]
$\sigma_{\mathbf{h}}$	.034	Inf	.022	.0031	[.0169 .0271]
$\sigma_{\mathbf{i}}$	.099	Inf	.080	.0186	[.0475 .1087]
$\sigma_{\mathbf{mk}}$	.004	Inf	.022	.0079	[.0164 .0280]
$\rho_{\mathbf{y}}$	0.6	0.20	0.96	0.008	[0.950 0.979]
$\rho_{\mathbf{h}}$	0.6	0.20	0.99	0.002	[0.982 0.999]
$\rho_{\mathbf{i}}$	0.6	0.20	0.90	0.045	[0.857 0.959]
$\rho_{\mathbf{mk}}$	0.6	0.20	0.97	0.026	[0.948 0.990]

Table 2.8: Posterior Distributions of Shock Processes

effect of advertising on the volatility of the marginal utility of consumption. Typically, in standard RBC model the estimation of σ turns out to be low because the model predict an excess of consumption smoothing with respect to the data. In this model with advertising, instead, the goodwill is an argument of the utility function that co-moves with consumption, but has an opposite effect on the marginal utility.[38] Therefore variations in advertising offset the ones of consumption, maintaining the marginal utility relatively stable and its volatility relatively low, and therefore reconciling the evidence of constant marginal utility from the model, with the volatile series of consumption from the actual data.

Finally, the estimates of the parameters of the shocks processes are in line with the ones found in the empirical literature for similar DSGE models, e.g. Smets and Wouters (2007), suggesting that our model is able to treat the uncertainty present in the data with the same degree of accuracy than the one reported from similar model

[38]Notice that the marginal utility has negative derivative w.r.t. C_t, and positive derivative w.r.t. G_t.

Model	σ	ϕ	$LogMarginal$ $DataDensity$
Dixit-Stiglitz	1.53 (0.261)	2.16 (0.497)	1,312
Bounded Marginal Utility	1.88 (0.370)	2.04 (0.468)	1,314
B.U. with Advertising	2.98 (0.397)	2.58 (0.504)	1,530

Table 2.9: Posterior Distributions for different models. Standard deviations in parentheses.

estimations in the literature.

In section 4.1 we made a point about the behavior of advertising as a build-in mechanism of transmission of technology shocks to the aggregate demand. A variance decomposition analysis using model estimates is useful check our intuition. We plot variance decomposition from the estimated model with advertising against the variance decomposition from an estimated version of the benchmark economy where advertising in banned.

Results from table (2.8) confirm our previous intuition. In the model with advertising the technology shock accounts for a bigger proportion of Consumption, Output, labor and Investment volatility. In particular, with advertising technology shock accounts for roughly 10% more of the volatility of consumption, and 17% more of the volatility of output than in the model without advertising, whereas the exogenous preferences shock accounts for 15.3% less of the volatility of consumption, and around 15% less of the volatility of output.

Moment	Estimated model, log-diff data $\mathfrak{M}_1$	$\mathfrak{B}_1$	Filtered data
σ_Z/σ_Y	2.44	–	2.50
σ_Z/σ_C	1.99	–	3.30
σ_Z/σ_I	1.16	–	0.70
σ_H/σ_Y	0.66	0.53	0.32
σ_C/σ_Y	1.22	1.20	0.74
$\rho(Z_t, Y_t)$	0.47	–	0.63
$\rho(Z_t, C_t)$	0.30	–	0.67
$\rho(Z_t, H_t)$	0.41	–	0.58
$\rho(H_t, Y_t)$	0.59	0.62	0.73
$\rho(C_t, Y_t)$	0.71	0.67	0.88
σ_{μ_t}/σ_Y	0.041	.0012	–
$\rho(\mu_t, Y_t)$	0.41	1.00	–
$\rho(\mu_t, Z_t)$	0.96	–	–
$\rho(\mu_t, H_t)$	0.27	0.62	–

Table 2.10: Estimated Models with log difference data vs. actual BP(6,32) filtered data

2.5 Conclusions

In this chapter we seek to understand the observed behavior of advertising over the Business Cycle, and its effect on the aggregate dynamics. To this end, firstly we showed that actual data of U.S. aggregate advertising expenditures have a well-behaved pattern over the Business Cycle. Secondly, we build a model that can rationalize this pattern within the neoclassical growth models theory. Thirdly, we show that a loglinearized version of the model can fit well the actual data. Finally, we use an estimated version of the model to test Solow's spread-it-around hypothesis against Galbraith's market enhancing hypothesis.

Our main finding is that the second hypothesis is preferred by the data. This result hinges on the significativeness of aggregate advertising as explanatory variable of the volatility of aggregate consumption.

The previous result implies that advertising, entering in the representative agent's

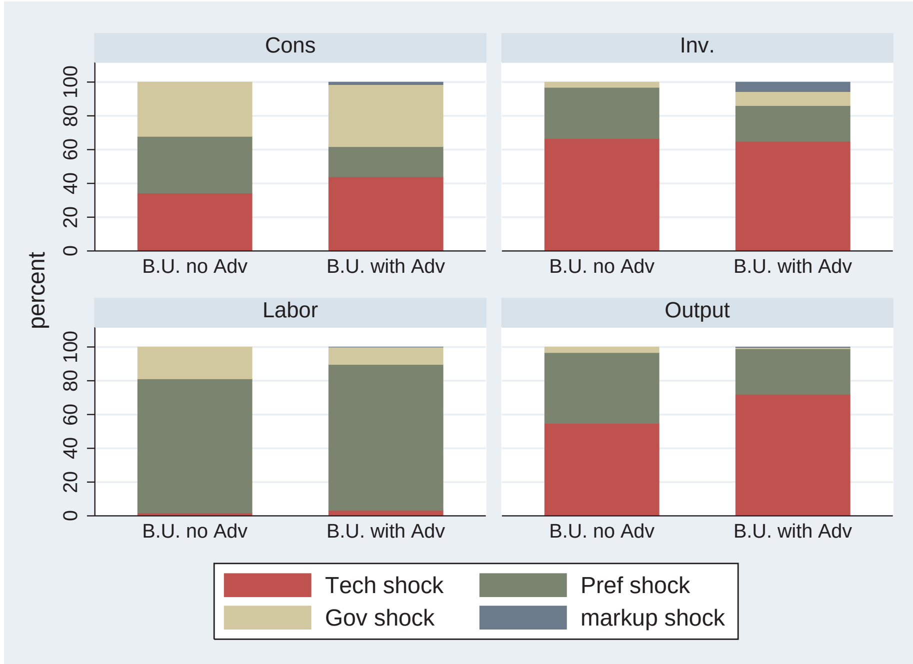

Figure 2.8: **Variance decomposition for selected variables.** ...

choice about the desired level of consumption in any period, has a non-negligible impact over the whole aggregate dynamics. Interestingly, in our framework advertising increases optimal labor supply, i.e. it pushes the consumer to work more hours, as conjectured by Benhabib and Bisin (2002). Despite the modest size of advertising industry over total production, 2.2% of GDP in U.S., the short run impact of advertising fluctuations turns out to be quantitatively important, exacerbating the welfare costs of fluctuations of 23% when going from an economy where advertising is banned to one economy where firms are free to advertise their products. Overall, the model goes in the direction suggested by Kaldor (1950): advertising amplifies fluctuations of the aggregate dynamics.

Also, we show that advertising have important effect on prices, affecting the behavior of the markup over the cycle. Since in our framework the optimal advertising policy is a positive function of the sales, and the price elasticity of the demand is a negative function of advertising, then the markup turns out to increase over the cycle along with the aggregate demand. Moreover, its volatility is 20% higher when firms advertise than in the benchmark economy with no advertising.

As last issue, we show that advertising in this model behaves as an endogenous taste shock, with an intensity which is controlled by firms, and varies in the optimum whenever a technology shock occurs. This feature of the model leads to the observation that a shock to the productivity directly affects the aggregate demand through advertising. In fact, we find evidence that disregarding the advertising channel in a RBC model may lead to underestimate the effect of technology shocks in generating the cycle.

Chapter 3

Advertising and Labor Supply: a Long Run Analysis

Joint with Francesco Turino

3.1 Introduction

In previous chapter we showed that advertising absorbs a substantial amount of resources in U.S. economy, and that it has an important impact on macroeconomic outcomes. In this chapter we provide evidence that the case of U.S. is not unique. Similar magnitudes, but with a substantial degree of variation, characterize also the advertising sectors in Europe and Japan. Moreover, we document a novel stylized fact: advertising expenditures are in general positive correlated with output, consumption and hours worked across OECD countries.

In chapter 2 we introduced advertising into a standard neoclassical growth model with monopolistically competitive structure of goods market. Following Dixit and Norman (1978), we treated advertising as affecting consumers' tastes in a way that generates a positive link between advertisements and firms' sales. As a result, firms decide not only the optimal price and the production plan, but also how much to invest in advertising by using a specific technology that requires labor. Also, consumers' preferences are no longer given, as in the neoclassical tradition, but are endogenously determined by the interaction between firms and consumers through advertising. In this perspective, advertising can be interpreted in a wider sense: it represents a force that is able to generate a culture of "consumerism", distorting consumers' choice and eventually affecting the aggregate variables.[1]

We show that the presence of advertising considerably modifies the steady state equilibrium. Three different channels can be identified: advertising affects preferences by increasing the marginal utility of consumption, absorbs resources, and increases firms' market power. In a model with advertising calibrated to match U.S. data, we find that in equilibrium worked hours, output and consumption are unambiguously higher than in the associated economy where advertising is banned, despite the fact that firms' market power is higher with advertising. This occurs because advertising shifts the marginal rate of substitution between consumption and leisure in a way that unambiguously increases the labor supply, a result that is consistent with the available evidence documented in Brack and Cowiling (1983) and, more recently, in Fraser and Paton (2003).

The link between advertising and labor supply is also a crucial mechanism to generate quantitatively relevant effects. While the impact of advertising in a model with exogenous labor supply is modest, when the labor is endogenously decided by the representative agent, the effect of advertising on the steady state equilibrium can

[1] In this respect, our framework embeds the spirit of early studies of Duesenberry (1949): the individual is the product of the society. This theme is also a central element of other social sciences, like sociology or psychology, in studying the influence of advertising in the society. As emphasized by Benhabib and Bisin (2002), "social constructionism", the view that individual self is socially constituted, has largely influenced the Postmodernist theory, so that words like "consumerism", "commodification" of culture and "manipulation of preferences" have become its central core.

be large. For example, in the baseline case the model predicts that in the United States, without advertising, the equilibrium level of hours worked would be about 10% lower.

Another important prediction of our framework is that for a sufficiently high Frisch elasticity, hours, GDP, consumption are monotonically increasing function of the amount of resources invested in advertising. All other things equal, our framework predicts a pattern of cross-country correlations among advertising expenditures, consumption and hours consistent with the ones we find in the data. In particular, our model can explain about $1/5-1/3$ of the observed differences in worked hours among several European countries and the U.S. This is true even when controlling for other sources of cross-country heterogeneity like total factor productivity or taxation.

Finally, by the mean of a welfare analysis we show that in the long run consumers are always worse off with advertising. Despite the higher level of consumption, the welfare loses are driven by an "overworking" effect induced by advertising. Also, the welfare gain associated with a policy that bans advertisements is shown to be a strictly increasing function of the advertising share.

The chapter is organized as follows. Section 2 documents the empirical evidence, Section 3 describes the model economy, Section 4 summarizes the principal long run effect of advertising, Section 5 quantifies the results, Section 6 evaluates the predictions of the model comparing different countries, Section 7 derives a welfare criterium and analyze the results, Section 8 concludes.

3.2 Empirical Evidence

In this section we analyze the behavior of aggregate advertising in several OECD countries. We make use of two indicators: advertising share (i.e. the ratio of advertising expenditures to GDP) and per capita real advertising.[2] The advertising share,

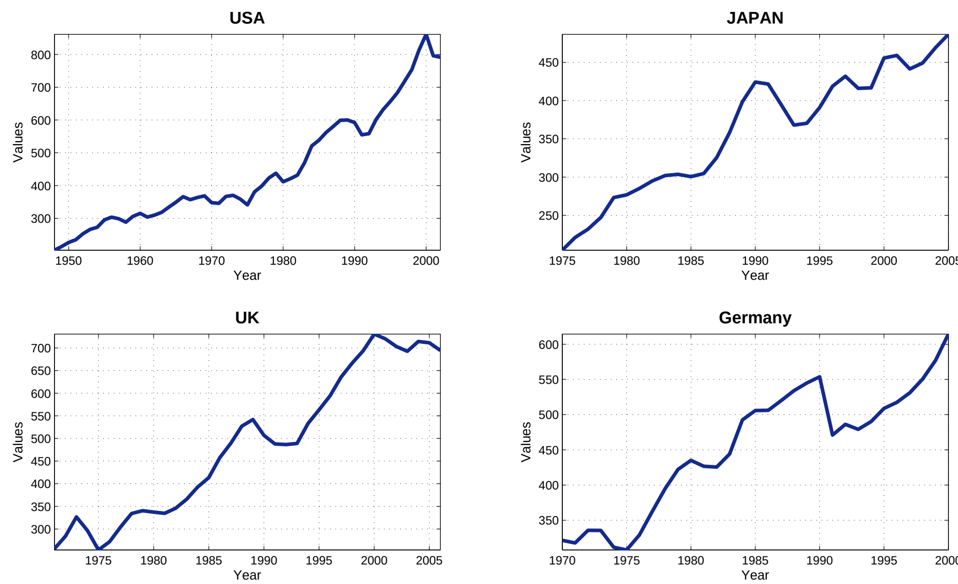

Figure 3.1: **Real Advertising per capita in US, Japan, UK and Germany**. Advertising is expressed in terms of Us dollars with constant PPP and constant prices (2000=100). Population is the total number of person aged 15-64.

the amount equivalent of resources invested each year, is the natural indicator to highlight the relative importance of advertising industry in the aggregate economy. Per capita real advertising can be interpreted as a measure of the advertising intensity in the economy.

We start analyzing the path of advertising intensity in the most industrialized countries. Figure 3.1 graphs per capita real advertising for United States, Japan, Germany, and the UK. To render the figures comparable, advertising has been expressed in terms of constant purchasing power parity. As evident from the graph, all series show a strong upward trend: the number of messages per individual, or

[2] Our data set contains information on macro aggregates as hours worked, population and advertising expenditures that derive from various sources (see data appendix). In particular, depending on the source, aggregate advertising is defined in different ways. By using advertising share and per capita advertising we try to minimize the source of heterogeneity in these two indicators.

equivalently the intensity of advertising, has constantly increased over time. The US is the economy that has experienced the biggest increase in the advertising intensity. According to the graph, the same individual living in the eighties, today would be exposed to double the amount of advertising. Japan is instead the economy with the lowest growth rate. Compared with the US, between 1980 and 2000, the Japan's growth rate was less than half of that of the US. Beyond the quantitative aspects already discussed, the graphs show a clear pattern which is very familiar to macroeconomists, that is like consumption, investment and GDP, advertising has constantly grown in the second half of the last century.

But what about the relative importance of advertising in the aggregate economy? As mentioned before, the natural indicator to explore this question is the advertising-GDP ratio. Figure 3.2 graphs the advertising share, again for the most industrialized countries.

Two main features have to be pointed out. First off all the graph clearly indicates that advertising is a sizeable sector in the countries we have considered. In all the cases, advertising share is well above 1% of GDP. There are, however, quantitative differences among countries. Germany and the UK are very similar with an average advertising share slightly below 1.4%. The US is the economy in which a bigger amount of resources are invested in advertising: on average, its advertising share accounts for more than 2% of GDP. Japan is the country with the smallest advertising share. The total resources invested in this country account, on average, almost 1.15% of GDP.

Secondly, there is not a clear trend in all the figures we have considered, rather it seems that advertising share has fluctuated around a constant mean, which implies that the average growth rate of advertising and GDP should be approximately the same. The observed fluctuations are probably due to cyclical episodes. Indeed, at

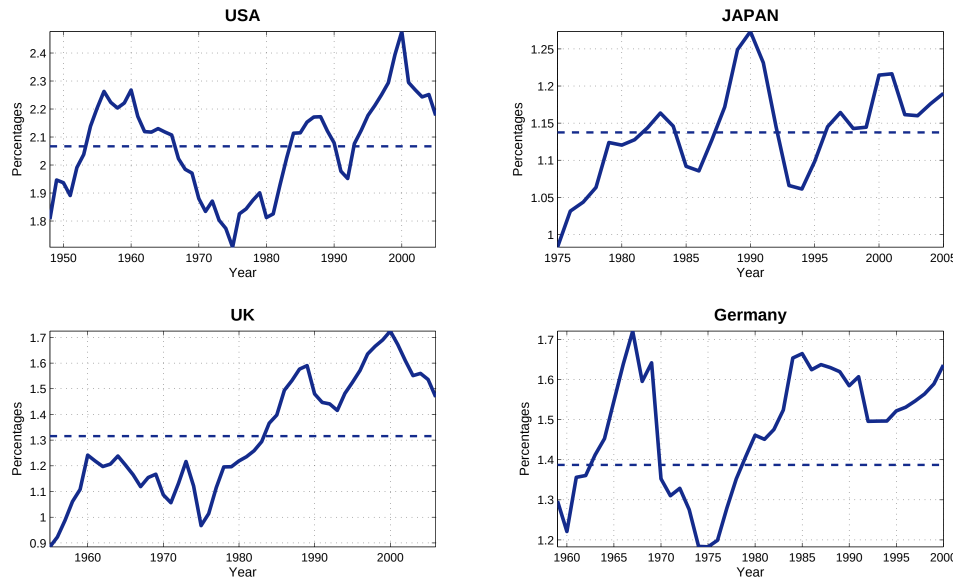

Figure 3.2: **Advertising share in the US, Japan, the UK and Germany**. Advertising share is calculated as the ratio of total advertising expenditures (all the media) to GDP. The horizontal line indicates the sample average mean.

least for the US economy, advertising is a procyclical variable that is more volatile than GDP.[3]

To emphasize the relative importance of advertising in the aggregate economy, another useful indicator is the ratio of advertising to total fixed non residential investments. This ratio proxies the relative weight of advertising to the total productive investment of the economy.[4] It can then be considered as a raw indicator of the relative importance of advertising in firms' investments. Table 3.1 shows, among other indicators, the average mean for the ratio of advertising to non residential fixed investment for the G7 countries. As we can see, in all the countries considered, advertising accounts for a relevant part of total productive investment: the advertising-productive investment ratio varies from a minimum of 4.17% for Italy to a maximum of 16.23%

[3]The cyclical properties of Us advertising expenditures are documented in chapter 2.

[4]In fact, total fixed non residential investment is defined as the total investment net of housing and can be then considered as a proxy for the total productive investment.

for the US. Accordingly, the data for the US is remarkable. The relative weight of advertising on productive investment is, indeed, very high. This is particularly true once we consider that advertising in not a productive investment in *stricto sensu*. Indeed, advertising cannot be considered as a necessary factor of production [5] like, for instance, fixed capital, it is rather an immaterial investment that firms bear in order to increase their profits by influencing the market demand. In this sense, the fact that advertising accounts to almost 17% of total productive investment for the United States is quite surprising. This essentially reveals that in this country, advertising is a very important component of firms' "competition" policies.

In analyzing advertising from an aggregate perspective, the natural issue to be explored is its potential link with GDP. In table 3.1 we have also reported the average of per capita real GDP. It seems that there exists a positive correlation between per capita real GDP and advertising. With the exception of Germany, each block of the table shows that in general, countries with the highest level of advertising are also characterized by the highest level of per capita GDP. To shed more light to the issue, figure 3.3 (panel A) graphs the logs of per capita GDP against the logs of advertising for 18 OECD countries. Again, there exists a strong positive correlation between the two variables considered. As reported in table 3.2 (column A), the estimated elasticity of real per capita GDP to advertising equal to 0.425 and is statistically significant at a five percent level of significance. Hence, our data suggests that, in general, countries with a high level of per capita GDP are also characterized by a high level of per capita real advertising.

However, it has been often asserted that the natural locus to study the macroeco-

[5] In a world without perfect information, the claim that advertising is not a productive factor could be questioned. This is trivially true whenever advertising contains an informative component. However, it is also true that advertising contains a persuasive component perhaps more important than the informative one. In this sense, interpreting advertising as non productive factor is not that problematic.

Country	Period	$\frac{Adv}{GDP}$	$\frac{Cons}{GDP}$	$\frac{Adv}{Inv}^a$	$\frac{Adv}{Pop}^b$	$\frac{GDP}{Pop}^c$	$\frac{Cons}{Pop}^d$	$\frac{Hours}{Pop}$	$\frac{Hours}{Empl.}$
USA	1983 – 2000	2.26	66.79	16.23	1.03	45.34	30.81	1309	1832
GBR		1.51	60.73	10.52	0.52	33.60	22.49	1172	1668
DEU		1.48	57.6	9.76	0.52	34.95	20.12	994	1546
JPN		1.15	53.43	4.90	0.38	32.79	18.04	1351	1832
CAN	1996 – 2006	0.91	55.41	6.35	0.37	41.70	23.04	1239	1793
FRA		0.69	54.86	4.74	0.28	40.71	22.63	970	1586
ITA		0.67	58.66	4.17	0.25	37.63	22.42	1004	1621

Table 3.1: **Summary statistics for selected countries**. a: The ratio has been calculated using total fixed investment without housing. b,c,d: the variables have been expressed in terms of dollars with constant ppp and constant prices. The population refers to the total person aged 15-64. Notice that the two blocks in the table are not comparable since they refer to different aggregate for advertising expenditures. See the data appendix for details.

nomic effects of advertising is the link between consumption and advertising expenditures, rather than the link with GDP.[6] The idea seems to be reasonable and lies in the own nature of advertising. If indeed advertising is a marketing activity devoted to influencing the firms' final sales and considering that aggregate consumption expenditures is a raw measure of total sales, then it seems reasonable to think that there could exist a link between aggregate advertising and consumption expenditures. Perhaps, we would add, if such a link exists, the quantitative dimension of advertising is relatively unimportant. In such a case, the effects of advertising on GDP would be indeed amplified through consumption so that the relative weight of advertising as a component of GDP would be only marginally important.

[6]In this respect, the vision of Galbraith (1967) was surely influential. Galbraith, in describing American society, was among the first to suggest that advertising can be an important factor of the consumption pattern. This link has been studied for instance in Ashley et al. (1980) or, more recently, in Jung and Seldom (1994).

To explore the issue in table 3.1 we have also reported the average mean of consumption share and per capita consumption for the G7 countries. The country order has been chosen according to the advertising share. As evident from the table, at least for the first block, the order is maintained for all the consumption indicators we have reported. For instance, the United States which is the leading country in terms of advertising share is also the country with the highest consumption share, per capita real advertising and per capita real consumption. The relationship, however, is less clear in the second block of the table: Canada shows the biggest value for advertising share but is not the leading country in terms of consumption share. However, there still exists a positive relationship between advertising and consumption per capita.

Our data suggests that in the G7 countries, at least in the period of time we have considered, there exists a positive correlation between consumption and advertising. This is true not only in terms of share of GDP (as the Galbrathian hypothesis suggests) but also in per capita units. In panel B of figure 3.3 the apparent relationship between consumption and advertising is investigated further. The picture shows consumption against advertising defined in per capita units. As in the case of GDP, the picture clearly shows that there exists a positive cross-country correlation between consumption and advertising. The estimated elasticity is positive and, again, statistically significant. It is interesting to note that even when we consider only European countries, the sample correlation is lower (0.56) but still positive. Of course, the reported statistics are only descriptive, in the sense that nothing can be inferred about the causal relationship between consumption and advertising. Still, there remains the observation that on average, countries characterized by an higher level of consumption are also characterized by a higher level of advertising. Remarkable is the case of the US economy that seems to be very peculiar with respect to all the other countries considered: the economy is characterized by the highest level of consumption share,

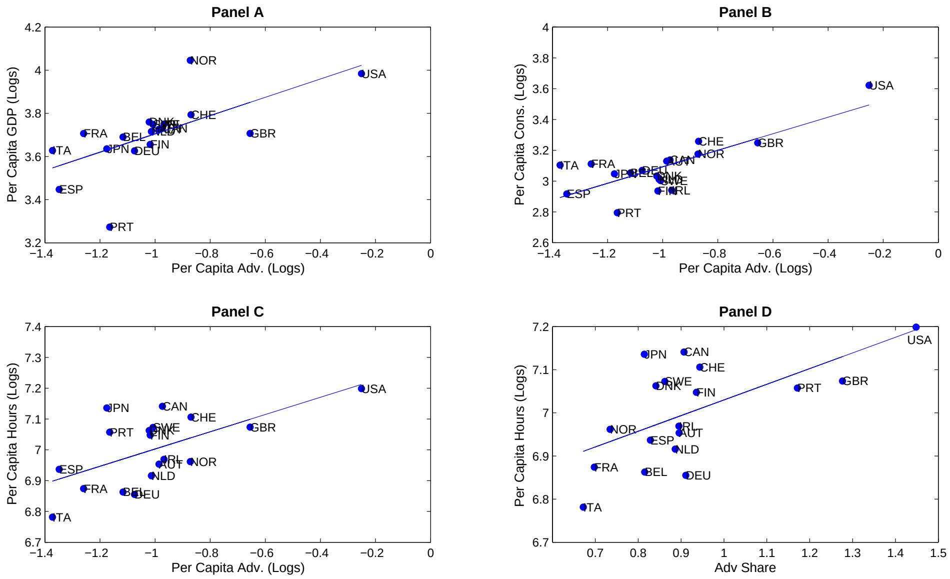

Figure 3.3: **Scatter plots.**.Panels A, B, and C graph respectively the logs of per capita GDP, Consumption, and Hours against per capita advertising. Panel D graphs the logs of per capita hours versus advertising share (in percentage). Period 1996-2005. Source for aggregate advertising expenditures: WARC. See the data appendix for details.

per capita consumption and advertising.

In comparing the United States with the European countries, it has been often noted that Americans work more than Europeans do. The last two columns of table 3.1 show indeed that, with the exception of Japan, United States is the country with the highest level of both per capita and per employees hours worked. There are several different explanations for this empirical regularity. Prescott (1994) suggested that the observed differences in hours worked between Europe and USA can be totally explained by differences in marginal tax rates on labor income. Alesina et all (2005), indicated that the major differences between Europe and USA are largely due to the European labor market regulations advocated by politically powerful unions. Blanchard (2004) argues that Europeans like leisure more than Americans do.

Cowling and Poolsomnute (2007) instead argued *"that the intensity of creation*

Regressors	Dependent Variables (A) $\frac{GDP}{Pop}$	(B) $\frac{Cons}{Pop}$	(C) $\frac{Hours}{Pop}$	(D) $\frac{Hours}{Pop}$	(E) $\frac{Hours}{Pop}$
Per capita Adv.	0.425 (0.018)	0.536 (0.013)	0.278 (0.008)	-	0.3061 (0.006)
Advertising Share	-	-	-	0.363 (0.008)	-
Constant	11.48 (0.003)	3.629 (0.000)	7.282 (0.006)	6.668 (0.006)	7.289 (0.001)
R^2	0.418	0.605	0.384	0.380	0.551

Table 3.2: **Simple Regressions**. Columns A-D refer to the estimated relationship reported in the figure 3.3. Column E refers to the estimated relationship excluding Japan and Canada from the sample. All the variables are expressed in logs with the exception of advertising share. P-values are reported in parenthesis. 18 OECD countries. Average mean 1996-2006. Source of advertising expenditures data: WARC.

of wants through advertising and marketing might be an influence on decisions made by Americans about how much time they should devote to paid work, and how much time to leisure". The argument of the authors is based on the vision that advertising *"creates a continuing dissatisfaction with current levels of consumption, that may encourage people to offer a larger fraction of their time for the generation of income in order to satisfy their increased demands for material consumption".* As a consequence, advertising inducing pressure to consume will affect the labor supply. Evidence for such a phenomena are documented in Brack and Cowling (1983) for the US labor supply and, more recently, in Fraser and Paton (2003) for the UK economy.

In figure 3.3 we explore the potential link between advertising and hours worked for several countries. As in the previous cases, in our sample there exists a positive cross-country correlation between advertising and per capita hours worked: countries with high level of hours worked are in general associated with high level of advertising. The sample correlation is 0.62 when advertising is expressed as share of GDP, and 0.60 in per capita terms. Either the cases, the estimated elasticity is positive and statistically significant. In the case of per capita variables, the estimated relationship predicts that a one percent increase in per capita advertising would lead to 0.27 % increase in the per capita hours worked. Finally, as summarized in the last column of table 3.2, repeating the experiment by excluding from the sample Japan and Canada revels that the result remains valid even if we compare only the US with Europe.

To summarize, the main stylized facts that we found analyzing the data so far are the following:

- In selected OECD countries per capita advertising has constantly grown over time. The U.S. is the economy that experienced this growth mostly.
- Advertising share of GDP varies substantially across countries.
- There exists a positive and statistically significant correlation across countries among advertising, GDP, consumption and hours worked.

Of particular interest from our perspective is the positive correlation between advertising and hours worked. The evidence is interesting and leads to a natural question: can advertising be considered an important variable to explain the observed differences in hours worked between the US and Europe? As we have mentioned before, the economic explanation of such a differences has generated a lively debate. With the exception of Crowling (2007), advertising has been ignored in this debate, and in general in macroeconomics. Advertising may redistribute demand across firms of the same industry but unlikely can affect the total demand of such industry, never mind hours worked. However, advertising could also be an important factor in influencing the consumers' decisions, and so the aggregate variables like hours worked. Moreover, the positive correlation founded in the data, with the obvious cautions required, can be considered as an encouraging evidence in favor of this research direction.

3.3 The Model and The Symmetric Equilibrium

The model we use here is the one described in chapter 2, section 3. We report here for the convenience of the reader the equations that define the symmetric equilibrium solution.

Let $X_t = (G_t,\ \mu_t,\ Z_t,\ H_t,\ H_{a,t},\ H_{p,t},\ \tilde{C}_t,\ K_t,\ I_t,\ Y_t,\ R_t,\ W_t, r_{t,t+1})$ be the vector of the all the endogenous variables, we say that, given the exogenous process A_t and the initial conditions K_0, G_0, the symmetric equilibrium for the model economy is a process $\{X_t\}_{t=0}^{\infty}$ that satisfies the following system of equations:

$$W_t = \alpha A_t^{\alpha} \mu_t^{-1} \left(\frac{K_t}{H_{p,t}} \right)^{1-\alpha} \tag{3.1}$$

$$R_t = (1-\alpha) A_t^{\alpha} \mu_t^{-1} \left(\frac{H_{p,t}}{K_t} \right)^{\alpha} \tag{3.2}$$

$$\tilde{C}_t^{-\sigma} = \beta \left\{ \tilde{C}_{t+1}^{-\sigma} \left[R_t + (1-\delta_k) \right] \right\} \tag{3.3}$$

$$\xi H_t^{\phi} = W_t \tilde{C}_t^{-\sigma} \tag{3.4}$$

$$\tilde{C}_t = C_t + B(G_t) \tag{3.5}$$

$$\mu_t = \frac{\varepsilon \left(1 + \frac{B(G_t)}{Y_t} \right)}{\varepsilon \left(1 + \frac{B(G_t)}{Y_t} \right) - 1} \tag{3.6}$$

$$\frac{W_t}{\alpha\gamma A_t} H_{a,t}^{1-\alpha} = -\sum_{j=0}^{\infty} (1-\delta_g)^j \, r_{t,t+j} \, B'(G_{t+j}) \, (1-\mu_{t+j}^{-1}) \tag{3.7}$$

$$r_{t,t+1} = \beta \left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-\sigma} \tag{3.8}$$

$$H_t = H_{a,t} + H_{p,t} \tag{3.9}$$

together with the goods market clearing condition, the production function of advertising, the definition of output, and the aggregate version of the law of motion for both capital and goodwill.[7]

3.4 The Steady State

In this section we analyze the effect of advertising on the balanced growth path, i.e. an equilibrium in which all the variables growth at a constant rate, with the exception of labor, markup, and interest rate which have to be constant. In our setup, the most problematic aspect is related with the preferences we have chosen. In particular, the additive term in equation (3.5) makes the requirement of a constant interest rate more complicated to be satisfied. However, as pointed out in the next proposition, a slightly modification of the function $B(G)$ guaranties that a balance growth equilibrium exists.

Proposition 2 *A sufficient condition for the existence of a balance growth path equilibrium is:*

$$B(G_t) = \frac{A_t}{1 + \theta \frac{G_t}{A_t}}$$

[7]See chapter 2 section 3 and Appendix B.

Proof. See appendix C.3. ■

Under the previous condition the major aggregates growth at the same rate of technology so that a steady state can be defined as an equilibrium in which all the variables expressed in terms of units of efficient labor are constant. For the sake of simplicity, from now on we will assume that the requirement of proposition 2 is satisfied and we will concentrate in a steady state equilibrium in which the technology A_t is normalized to be one. That assumption does not affect in any respects the validity of our results.

In our setup the equilibrium level of advertising depends, in particular, on the intensity of advertising in the preferences θ, and on the rate of transformation of advertising in goodwill γ. We will concentrate in analyzing the effects due to parameter γ. This is because while it is probable true that cultural differences across countries affect also the entrepreneurs' beliefs about advertising effectiveness (and so the amount of resources invested in marketing activities), at the same time such differences are less identifiable than differences in institutional aspect of advertising regulation.[8]

In what follows we will distinguish between two alternative equilibria depending on whether the labor choice is endogenous or not. As will become evident later, the assumption we make on labor supply dramatically affects the way advertising influences the steady state. From a quantitative point of view, we will see that advertising affects substantially the equilibrium only when the labor supply is allowed to be endogenous.

Before going any further, it is worth pointing out that, in all this section, to isolate the effects of advertising we will compare the equilibrium with positive advertising expenditures with the standard case (without advertising). However, this standard

[8] A typical example could be the product placement, a practice for a long time banned in Europe but totally legitim in USA.

equilibrium is exactly the same as the one we obtain by setting the parameter γ to zero in our model. In this case, any firm optimally chooses to not advertise its products so that in the symmetrical equilibrium the amount of resource invested in advertising is zero. Therefore, any differences between the two equilibria when the parameter γ is different from zero can be defined as the direct effect of advertising.

Independently on our assumption on the labor supply, we can state a first result that holds in general.

Proposition 3 *The consumption share increases with advertising.*

Proof. See appendix C.5. ■

Proposition 3 states that no matter if the labor choice is endogenous or not, advertising alters the steady state in a way that makes the economy more consumption based. As in Galbrath's (1967), the share of consumption to GDP increases whenever the equilibrium is characterized by a positive amount of advertising expenditures. We turn to this point later.

3.4.1 Exogenous labor supply

In this section we assume that the representative consumer supplies inelastically $\bar{H}$ unit of labor services per unit of time, and has preferences over consumption represented by the following intertemporal utility function:

$$U(\tilde{C}_t) = \sum_{t=0}^{\infty} \beta^t \left[\frac{\tilde{C}_t^{(1-\sigma)} - 1}{1-\sigma} \right]$$

where $\tilde{C}_t$ is defined as in equation (2.2). The corresponding symmetric equilibrium is exactly the same as in the case with endogenous labor supply except for intratemporal condition (C.1) which disappears.

As shown in appendix C.5, all the endogenous variables of the model economy can be expressed as functions of advertising-related labor, and the ratio of production-related labor to capital. Thus, let V_t denotes the vector collecting the remaining endogenous variables, all the equilibrium relationships can be conveniently summarized by introducing a map $V_t = V\left(H_{a,t}, \frac{H_{p,t}}{K_t}\right)$. The following proposition characterizes precisely the steady state equilibrium.

Proposition 4 *A steady state with exogenous labor supply is defined as a sequence* $\{V_t\}_{t=0}^{\infty}$ *such that* $V_t = V(H_a, \frac{H_p}{K})$ $\forall$ t *satisfies*

$$-\gamma \frac{B'(G)}{H_a^{1-\alpha}} = c_1 \frac{\left(\frac{H_p}{K}\right)^{\alpha}}{\left(\frac{1-\alpha}{r}\right)\left(\frac{H_p}{K}\right)^{\alpha} - 1}\left(\frac{1}{\frac{H_p}{K}}\right)$$

$$\bar{H} = (1-\alpha) H_a + \varepsilon B(G) \left\{ \frac{\left(\frac{H_p}{K}\right)\left[\left(\frac{1-\alpha}{R}\right)\left(\frac{H_p}{K}\right)^{\alpha} - 1\right]}{(1-\varepsilon)\frac{H_p}{K}^{\alpha} + \varepsilon\left(\frac{R}{1-\alpha}\right)} \right\}$$

Moreover, if an equilibrium exists, it is unique.

Proof. See appendix C.5. ■

We are now in the position to characterize the principal effect of advertising when the labor supply is exogenous.

Proposition 5 *With exogenous labor supply and advertising, the steady state equilibrium results in a higher level of average markup, and a lower level of output, consumption and investment.*

Hence, when the labor supply is assumed to be exogenous advertising essentially exacerbates the negative effects associated with monopolistic competition: the resulting equilibrium is characterized by a lower level of output, consumption and investment. Two channels can be identified: advertising increases the market power and absorbs resources. As we have seen before, the price elasticity decreases with advertising. The monopolistic power of the firm then increases, and so the equilibrium markup does. Clearly, the increases in average markup negatively affects output, and consequently consumption and investment. Such an effect is equivalent to one we would obtain by exogenously decreases the elasticity of substitution across varieties, ε, in the benchmark model without advertising. However, advertising absorbs labor, which reduces the amount of hours available for producing goods. At the same time, the labor supply is fixed so that the former effect eventually decreases total output and its components. As a consequence, the negative effects associated with the increase in market power are further amplified by the resource absorption due to advertising.

The actual impact of advertising on the aggregate equilibrium is well captured by the equilibrium level of output that, as proved in appendix, can be expressed as follows:

$$Y = \left(\frac{R}{1-\alpha}\right)\left(\frac{K}{H_p}\right)\left[\bar{H} - (1-\alpha)\,H_a\right] \tag{3.10}$$

Accordingly, the equilibrium of output is proportional to the product of two terms: the ratio of capital to production related labor and the quasi differences between total hours and advertising related labor. The first term, from equation (3.2), is inversely related with the equilibrium markup and thus captures the negative effect due to the increase in monopolistic power. The second decreases with the amount of time devoted to produce advertising, and thus captures its resources absorption.

3.4.2 Endogenous labor supply

We now turn back to the model in which the labor supply is allowed to be endogenous. As in the previous case, the steady state can be defined in terms of a map collecting all the the equilibrium relationships.

Proposition 6 *A steady state with endogenous labor supply is defined as a sequence* $\{V_t\}_{t=0}^{\infty}$ *such that* $V_t = V(H_a, \frac{H_p}{K}, H)\ \forall\ t$ *satisfy*

$$-\gamma \frac{B'(G)}{H_a^{1-\alpha}} = c_1 \frac{\left(\frac{H_p}{K}\right)^{\alpha}}{\left(\frac{1-\alpha}{r}\right)\left(\frac{H_p}{K}\right)^{\alpha} - 1} \left(\frac{1}{\frac{H_p}{K}}\right)$$

$$H = (1-\alpha) H_a + \varepsilon B(G) \left\{ \frac{\left(\frac{H_p}{K}\right)\left[\left(\frac{1-\alpha}{R}\right)\left(\frac{H_p}{K}\right)^{\alpha} - 1\right]}{(1-\varepsilon)\frac{H_p}{K}^{\alpha} + \varepsilon\left(\frac{R}{1-\alpha}\right)} \right\}$$

$$\xi H^{\phi} = R\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{K}{H_p}\right)(C + B(G))^{-\sigma}$$

where C *is defined as equation (C.11) in the appendix.*

When the labor supply is allowed to be endogenous, advertising affects the aggregate economy through an extra channel: all the other things equal, the marginal utility of consumption increases, leading to a stronger substitutional effect the causes the labor supply schedule to shift downwards. The former mechanism has a natural interpretation. In fact, aggregate advertising is a negative externality for the consumers: the total utility of a given consumption bundle decreases as the aggregate advertising increases. This generates a disaffection that makes the consumers to feel unhappy with their consumption levels, promoting then a larger needs of material consumption. Obviously, all the consumers' choices are affected and in particular the labor one in a way that causes the labor supply to move downwards. In other words, the consumers are willing to work more in order to consume more.

Despite the implied movent in the labor supply, the general equilibrium effect of advertising on the level of hours worked is ambiguous. From the demand side, there are two competitive effects. The demand of advertising related labor increases while, given the higher markup, the demand of production related labor decreases so that the demand schedule can shift upwards, downwards or remain unchanged. In the last two cases, the equilibrium level of hours worked can increases only if the shift in the supply schedule is big enough. Also, the former condition does not guaranty that with advertising the equilibrium level of output is higher. As equation (3.10) clearly indicates, output can increase only if the effect on hours worked is strong enough to overcompensate the negative one due to the increase in the market power. On the contrary, an equilibrium in which a positive amount of advertising expenditures leads to a higher level of hours worked but a lower level of output is theoretically possible.

These arguments suggest that if the effect of advertising on the labor supply is particularly strong, the equilibrium results in a higher level of both hours worked and output, in spite of the fact that advertising increases at the same time the firms' market power. This theoretical possibility is particularly interesting since implies that increase in the market power is not necessarily associated with a lower level of hours worked and output (see, for instance, Blanchard and Kyotaky (1987)). Moreover, it is interesting to understand to what extent such an effect can mitigate the inefficiency associated with the monopolistic competition or whether it induces an even higher welfare loses due to an "overworking" effect. We turn to this point in section 3.7.

3.5 Quantitative Analysis

We have seen that theoretically advertising affects the steady state equilibrium through different channels. However, while with exogenous labor supply it is clear that it essentially exacerbates the inefficiency related with the monopolistic competitive structure of the good markets, in the case of endogenous labor supply there exists instead room for interesting effects that potentially affect the economy in a way that contrasts with the conventional theory.

Thus, to evaluate precisely how the general equilibrium is affected by advertising we perform a quantitative analysis. To this end, first we calibrate the parameters of the model to match several characteristics of the US economy and, then, we check, counterfactually, how the steady state changes with advertising by comparing the level of the main endogenous variables with the one we obtain in the benchmark model without advertising. In this way, we are also able to asses whether the impact of advertising on the aggregate economy can be considered quantitatively important. Also, we compare the equilibrium with exogenous and endogenous labor supply in order to understand not only what are the major differences but also in which circumstance, if any, advertising exerts quantitative relevant effects.

3.5.1 Calibration

To perform the quantitative exercise, we calibrate the model to match the data on the US economy. Table 3.3 summarizes the baseline calibration for the model economy. Following Ravn et al (2006), we set the labor share in GDP to be approximately 75 percent, the annual real interest rate to 4 percent, the Frisch labor supply elasticity equal to 1.3, the elasticity of substitution across varieties, ε, equal to 5.3, and the inverse of the intertemporal elasticity of substitution, σ, to 2. These restrictions imply that the labor elasticity of output in production, α , is 0.75, the subjective discount factor, β, is 0.99, and the inverse of the Frish labor supply elasticity, φ, is 0.77. The depreciation rate, δ_k, is 0.025 so that the ratio of consumption to GDP is about 0.7.

To calibrate the advertising related parameters, δ_g, θ, and γ we used the following strategy. According to Clark (1976), we set the goodwill depreciation rate, δ_g to 0.3, so that the effect of a specific advertising campaign vanishes between 3 and 4 quarters. With the information on the advertising share, we calibrate the intensity

Symbol	Value	Description
δ_k	0.025	Capital depreciation rate
β	0.9902	Subjective discount factor (R = $1.04^{0.25}$)
α	0.75	Labor intensity (Labor Share $\approx$ 0.75)
ξ	3.11	Preferences Parameter
δ_g	0.3	Goodwill depreciation rate (Clark 1972)
ε	6	Elasticity of substitution across varieties
θ	2.76	Intensity of advertising (adv share = 0.0224)
σ	2	Inverse of I. elasticity of substitution
φ	0.7692	Inverse of Frisch labor supply elasticity (1.3)

Table 3.3: Baseline Calibration

of advertising in preferences, θ. Since the rate of transformation of advertising in goodwill γ cannot be calibrated separately from θ we normalize γ to be one in the U.S. In the other countries we will keep θ fixed and we will calibrate γ in order to match the advertising share. Finally, the preference parameter ξ is calibrated such that $H = 0.2$ in the steady state, the average work week as a fraction of total weekly hours in postwar U.S. data.[9]

3.5.2 Quantitative Results

In this section we quantify the macroeconomic effects of advertising expenditures. To address the issue, we numerically evaluate the predictions of our framework regarding key endogenous variables as advertising efficiency changes. Figure 3.4 and 3.5 respectively illustrate the results in the model with endogenous and exogenous labor supply. All the variables, with exception of advertising share and the ratio

[9]See for instance King et al. (1988)

of advertising-related hours to total hours (which we have labeled as resources absorption), are expressed as percentage deviations from the benchmark value without advertising.

First, we note that the advertising share is an increasing function of γ. Thus, independently on the labor supply elasticity, the amount of resources invested in advertising increases with its efficiency. All the others thing equal, such a result indicates that the observed differences in advertising share across countries can be related to differences in advertising efficiency.

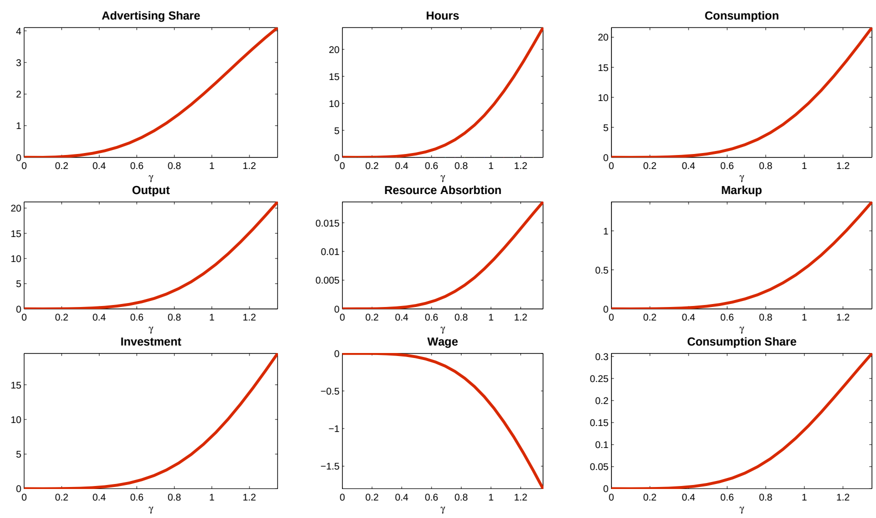

Figure 3.4: **Steady state allocations as a function of advertising productivity γ. Endogenous labor supply**.All the variables are expressed as percentage deviation with respect to the benchmark model without advertising (i.e. $\gamma = 0$). Resource absorption refers to the ratio of advertising-related hours to total hours worked.

Turning back on the macroeconomic consequences of advertising, we note that in the case of endogenous labor supply the results are quite surprisingly. Compared with the benchmark equilibrium, hours worked, output and its components are increasing functions of advertising productivity and, depending from the value of γ, their percentage deviation can be significantly high. To take an example, our framework predicts that without advertising the equilibrium level of hours worked in the

US economy ($\gamma = 1$) would have been of about 10% lower. Of the same magnitudes are also the variations of output and its components. However, part of the increase in hours worked goes to advertising production, so that the increase in output is lower than the one in hours. Not surprisingly, the average markup is an increasing function of advertising efficiency. The resulting movent in the labor demand together with the shift in the labor supply schedule implies that the real wage monotonically decreases with advertising efficiency.

As we have already mentioned, our framework predicts that an increase in the market power is not necessarily associated with a lower level of both hours worked and output. Our analysis instead suggests that such a results strictly depend on the sources that causes the market power to rise. In the standard Dixit-Stiglitz framework with monopolistic competition in the goods market, the market power results in a wedge that affects the consumers' intratemporal condition in a way that makes the consumers less willing to substitute from leisure into consumption. This effect results in a suboptimal level hours and, consequently, output. In our framework instead, advertising by modifying the consumers' tastes increase the willingness to pay for the goods and, at the same time, promotes a culture of consumerism that increases the willingness to work. As a result, firms' gain market power that exploit by charging a higher markup over the marginal cost, while the consumers feeling a disaffection with the current level of consumption that leads to a higher supply of hours. . The negative effect related to the increase of firms' market power is then partly offset by the stronger substitutional effect induced by advertising The numerical results indicate, moreover, that the latter effect is so strong that the resulting equilibrium is characterized by a higher level of hours worked and output.

Another important prediction of our framework is that the equilibrium level of hours worked, consumption, and GDP are monotonic increasing functions of adver-

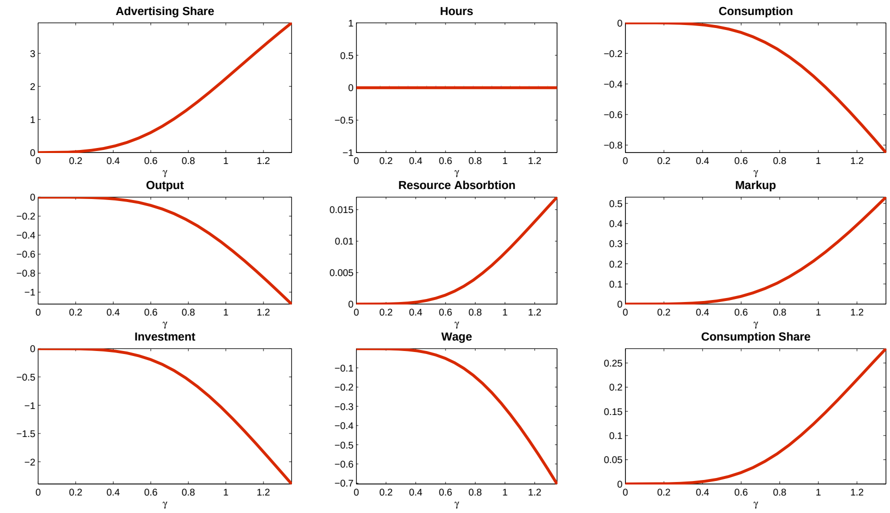

Figure 3.5: **Steady state allocations as a function of advertising productivity γ. Exogenous labor supply.**All the variables are expressed as percentage deviation with respect to the benchmark model without advertising (i.e. $\gamma = 0$). Resource absorption refers to the ratio of advertising related labor to total hours worked.

tising efficiency. All the other parameters equal, the model predicts that if we compare two economies in which one of them is characterized by a higher advertising share, then we should observe that the same economy is also characterized by a higher level of hours worked, consumption and output. This is consistent with the positive cross-country correlations observed in the data.

As a comparison, figure 3.5 plots the steady state level of the main aggregates as a function of advertising productivity in the case of exogenous labor supply. The numerical results confirm the analytical ones: output, consumption and investments now decrease when the equilibrium is characterized by a positive amount of advertising expenditures. From a quantitative point of view, however, the effect of changing γ is considerably different in the two cases. With the exception of advertising share, resource absorption, and consumption share which are essentially the same, we see that the quantitative effects of advertising are considerably higher in the case of endogenous labor supply. To take an example, the consumption maximal variation is about 20 times bigger (in absolute value) when the labor choice is endogenous rather than exogenous. This is also the case of output and investment while the differences in the long run markup are instead more contained.

The effect of advertising on the steady state consumption share is almost trivial and independent from the labor choice. Either the cases, compared with its benchmark value, the consumption share increases only up to a maximum of 0.3% which requires empirically unobserved advertising share of more than 4%.

The latter result has a straightforward intuition: in this model the consumption share, reflecting an intertemporal decision, is essentially determined by the long run interest rate which in turn is not affected by advertising. As shown in the appendix, conditional on $\gamma = 0$ the equilibrium consumption share can be written as follows:

$$\frac{C}{Y} = \left(\frac{1}{R}\right)\left(\frac{1-\beta\left(1-\alpha\delta_k\right)}{\beta}\right)$$

When we allow for positive advertising expenditures instead, the previous equation is modified by adding an extra term which reflects the resource absorption due to advertising but which is almost constant with respect to the productivity of advertising. As a consequence, the consumption share moves little with γ. Thus, conditional to the baseline calibration the model predicts that advertising is not an important variable in explaining the observed cross-country differences in the consumption share.

Overall, beyond the differences related with the labor choice, the analysis clarifies that the steady state is significantly affected by advertising only when the labor choice is endogenous. In this case the results are remarkable. It seems thus the effect of advertising on hours worked is not only an important prediction of the model but it is also the main mechanism that is able to generate important macroeconomic effects. Perhaps, this is the main result of all our analysis.

It also clear, however, that the actual impact of advertising in general equilibrium depends, eventually, from how strong is the labor supply channel compared

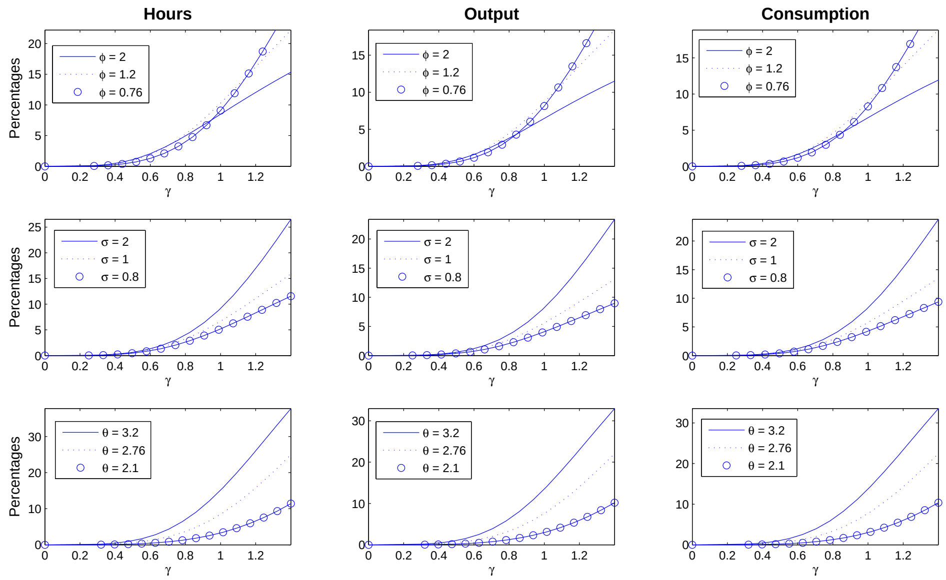

Figure 3.6: **Steady State Hours Worked, Output, and Consumption as function of advertising productivity for various value of ϕ, σ and θ.** All the variables are expressed as percentages deviation from the benchmark value in the model without advertising expenditures ($\gamma = 0$)

with the other ones. This is true not only for what concern the equilibrium level of hours worked, but also the one of output and its components. In this perspective, it is interesting to check whether alternative calibrations can lead to different predictions. To this end, we check how the steady state levels of hours worked, output and consumption move with advertising by calibrating the model using alternative parameters. We will concentrate in particular on the ones affecting the labor supply schedule, that is, the inverse of intertemporal elasticity σ, the intensity of advertising in the preferences γ, and the inverse of the Frisch elasticity ϕ. Figure 3.6 illustrates the main results for hours, output and consumption.

Not surprisingly, the quantitative impact of advertising varies with alternative parametrization. Regarding the effect of changing the intertemporal elasticity of substitution, we note that decreasing σ causes the effect of advertising efficiency on the main aggregates to decline. This is because, for any given level of consumption and goodwill, the marginal evaluation of consumption decreases, so that the substitution effect becomes smaller. As a result, the consumers' willingness to work declines so that the labor supply schedule shifts upwards, causing the equilibrium level of hours worked to be lower than the in the baseline calibration. The lower availability of hours causes then output and consumption to decline. However, when σ is extremely low we see that the main results are reverted. Now hours worked and the main aggregates monotonically decrease with advertising efficiency. In such a case, the effect of advertising on the labor supply is trivial so that the equilibrium value of hours worked is totally driven by the demand side. The implied decrease in the level of hours worked exacerbates the negative effect associated to the markup so that output and consumption further decrease.

The effects of changing the intensity of advertising in the preference, θ, are similar: as the parameter is fixed to a lower value, the effect of advertising on the main steady state aggregates declines. Again, the increase in the substitutional effect induced by advertising decrease, leading then to a lower level of hours worked and output.

Inspecting the first panel of figure 3.6 reveals, not surprisingly, that the effect of advertising is stronger the lower is parameter ϕ. In a competitive labor market without human capital accumulation, the disutility of working ϕ is equal to the inverse of the Frisch elasticity of labor supply. As ϕ increases the Frisch elasticity declines and, at the same time, the utility cost of working becomes higher. For this reason, the labor supply becomes steeper and the equilibrium level of hours worked reacts less to variations in wage and marginal utility of consumption. In the limiting case in which the labor supply is totally inelastic ($\phi = \infty$) we would have find exactly the same results as in the case of exogenous labor supply. Notice, moreover, that when ϕ is fixed to a particular high value the equilibrium level of output and consumption decrease even thought hours worked increase. This is because of the resource absorption due to advertising expenditures.

To summarize, comparing our framework with the benchmark one, we have found that the presence of advertising can result in the steady state equilibrium in which the level of hours worked and the main aggregates is higher. However, this is true under the condition that the intertemporal elasticity of substitution and the Frisch elasticity of the labor supply are sufficiently low, otherwise it is still, theoretically, possible an equilibrium in which hours and the main aggregates decrease.

3.6 Cross-Country comparisons

One of major prediction of our framework is that, conditional to the baseline calibration, the steady state advertising share and the level of hours worked, output and consumption are monotonic increasing functions of advertising efficiency. All the other parameters equal, the model predicts a pattern of cross-country correlations among advertising expenditures and the major aggregates that is consistent with the evidence we observe in the data. This is particularly important since, in analyzing the data, we have argued that the positive cross-country correlation between advertising expenditures and per capita hours worked can be read as suggesting a possible role for advertising in explaining the differences between the US and Europe. As such, it is important to know what are the theoretical predictions of our framework in comparing different countries.

The issue is addressed by comparing the predicted cross-country differences in hours worked with the observed ones, assuming that the principal source of cross-country heterogeneity is the advertising efficiency. For any countries, the parameters are set to their baseline values with the exception of γ that is instead fixed at the value consistent with the observed adverting share.

Panel A of table 3.4 illustrates the results. As we can see, differences in advertising efficiency can explain part of the observed differences in hours worked among countries. For what concern the baseline calibration, we note that the results are surprisingly large: differences in advertising efficiency can account between 20% and more than 34% of the observed differences in hours worked. Not surprisingly, reducing the Frisch elasticity or rising the intertemporal elasticity of substitution causes the quantitative differences to decline. This is particularly true in the former case: the reaction of hours worked to changes in advertising efficiency is about two times lower than in the baseline calibration.

Our model predicts that the huge differences in hours worked between the US and Europe can then be partly explained by advocating differences in advertising expenditures. The mechanism behind our results comes from the effect that advertising exerts on preferences. As explained before, the marginal evaluation of consumption increases with advertising leading hours worked to rise. Equivalently, all the other things equal, the marginal evaluation of leisure (relative to consumption) declines as the amount of resources invested in advertising increases.

Such a mechanism is consistent with the idea, suggested by Blanchard (2004), that the differences in hours worked might be related to the evaluation of leisure. It could be, for instance, that Europeans like leisure more than Americans do in such a case for any equal increase of wage the income effect would be stronger in Europe than in the US. In our model differences in hours worked are related to differences in marginal evaluation to leisure not only because the European could exogenously have a biggest culture of leisure, but also because advertising endogenously induces a culture of consumerism that distorts the consumers' optimal allocation. Indeed, the model predicts that even if Americans and European have the same ex ante preferences toward consumption and leisure, the former would work more because of the larger needs of consumption induced by advertising.

So far we have assumed that the only difference across countries is given by advertising efficiency. In our model, however, other factors can affect the advertising expenditures decisions. For instance, any exogenous variation in total factor productivity, A, or in factors affecting the labor market, such as taxes on labor income, result in different advertising decisions. As such, it is important to evaluate what are the prediction of our model by assuming that the countries differ with respect to those factors rather than to advertising efficiency.

We start analyzing the effect of assuming that the only source of cross-country heterogeneity is given by total factor productivity, A. To address the issue, we calibrate this parameter by matching, for any countries, the GDP per hours worked expressed in relative terms with respect the US economy. All the other parameters are fixed to their baseline values. Panel B of table 3.4 illustrates the results.

Two main should be pointed out. First, total labor productivity is not an appropriate dimension to explain differences in advertising shares. With the exception of the UK, the model over-predicts the amount of resources invested in advertising. This is because the degree of variation across countries in labor productivity is not enough to generate substantial differences in the amount of resources invested in advertising when its efficiency is instead constant. Second, the same argument applies for the case of predicted hours worked. Comparing the predictions for the UK with the others countries, we note that the model can explain a large part of the differences in hours worked only when the former are large enough. This is conditional to a sufficiently large Frisch elasticity otherwise, as shown in the last collum of panel B, the predictions dramatically decline.

Next, we analyze what are the predictions of our framework assuming that the only source of cross-country heterogeneity is given by differences on consumption and labor income taxes. As in Prescott (2004), this assumption results in a wedge, τ, that affects the consumers' intratemporal condition as follows:

$$\xi H^{\phi} = (1-\tau) W \left(C + B(G)\right)^{-\sigma} \tag{3.11}$$

Accordingly, all the other things equal, the marginal tax rate shifts upwards the labor supply, causing a different equilibrium level of both hours worked and real wage. Given equation (**??**), the different availability of labor together with variation in its cost affect also the advertising expenditures decision.

Panel C of table 3.4 illustrates the results. All the parameters are set to their baseline values with the exception of marginal tax rates which are fixed according to the one reported in Prescott (2004, pag.7). Again, we note that differences in marginal taxes is not an appropriate dimension to explain the observed differences in advertising investment. In this case, with the exception of Italy and France, the model tends to under-predict the country's advertising share. This is particularly true in the case of the US. Increasing the intertemporal elasticity of substitution, or decreasing the Frisch elasticity of labor supply do not improve the results. For what concern the predicted differences in hours worked, we note that in general the model under-predicts the equilibrium level of hours so that the differences among the US and the European countries are over-estimated. This is because the induced differences in advertising amplify, through the marginal utility of consumption, the negative effects due to taxes. For this reason our results substantially differ from the ones reported in Prescott. However, reducing the Frisch elasticity of labor supply considerably improves the results.

Overall, the analysis indicates that to properly match the observed differences in advertising share our model requires differences in advertising efficiency. In this particular case, the mechanisms inherent to our model generate a substantial cross-country variation in hours worked that is consistent with the observed differences among the US and European countries. This implies that theoretically adverting could have played a role in this particular issue.

It is worth noting, however, that the model we used is an extremely stylized representation of the reality that does not take into account several elements that are surely important in determine the equilibrium level of hours worked but still, we believe that our results can be read as indicating that advertising may affect the aggregate economy in a non trivial way. This is true, however, up to the point that advertising generates a culture of consumerism that is able to distort the optimal allocation between consumption and leisure. Provide empirical evidence of such a mechanism is surely an important task in analyzing the macroeconomic effects of advertising.

3.7 Welfare Analysis

The effects of advertising on welfare has been one of the main focuses of research on advertising. However, as pointed out in the survey of Bagwell (2005), this point is rather controversial. Indeed dependently whether advertising is intended as informative or persuasive, its welfare effects could be dramatically different. According with Dixit and Norman (1978), if advertising is meant as informative, it can increase the social welfare since *"it primarily conveys price information, and so it can induced improved market search, thus rising demand elasticity and lowering price-cost margins"*. When advertising is persuasive, the effect on social welfare can instead be negative since *"it makes a product to appear a poorer substitute for the other, and thus lowers its elasticity of demand and rises the price-cost margin and therefore the price"*. In a such a case, private profitability of advertising is only necessary but not sufficient to increases the social welfare.

In section 3.5, we have seen that in our framework advertising leads to different equilibrium outcomes. Depending on the calibration chosen, it can either exacerbates the effect associated with the monopolistic competitive structure of the goods market or leading to a shift in the labor supply that results in a steady state equilibrium in which hours, output and consumption are higher compared with the benchmark case. While in the former case it is obvious that advertising generates negative welfare effects, in the latter, instead, it can improve welfare by bringing the economy closer to the competitive equilibrium in spite of the fact that it increases, at the same time, firms' market power. However, in this case it is still possible that the aggregate effects of advertising result in an equilibrium outcome characterized by an excessive level of working, causing then a negative effect on the social welfare, even though the consumers experience a bigger amount of consumption. To understand then which effect prevails in the equilibrium, we concentrate only in studying the welfare effect of advertising when it leads to a higher level of both hours and consumption. In all the other cases, the results are trivially obvious.

However, exploring the welfare consequences of such effects is complicated by the fact that preferences are endogenous and socially determined. The model is essentially characterized by two set of different tastes: pre-advertising and post-advertising preferences, i.e, the respective utility functions affected or not by advertising. Obviously, there is not a special reason to analyze the welfare effect of advertising refereing to a particular representation of the preferences rather than the other. One could believe, for instance, that since the pre-advertising tastes represent the consumers' fundamental will (exogenously given), the appropriate yardstick for welfare comparisons is the utility function not affected by advertising. At the same time, the consumers' choices eventually depend on preferences manipulated by advertising so that the idea that the welfare comparisons have to be done with the post-advertising preferences seems to be also reasonable. For this reason we follow Benhabib and Bisin (2002) in defining a welfare criterion that takes into account both the pre and post-advertising preferences. Thus, given the parameter γ, we indicate with $(C(\gamma), H(\gamma))$ the vector of the consumer's equilibrium allocations and with $U(C(\gamma), H(\gamma), \gamma)$ his equilibrium utility function. The pre-advertising allocations and taste are the equilibrium values corresponding to $\gamma = 0$. The next definition summarizes the criterion used in the welfare comparisons.

Definition 1 *Given the advertising productivity, γ, we say that the consumer's welfare increases due to advertising if and only if it increases with respect to post-advertising preferences so that*

$$U(C(\gamma), H(\gamma), \gamma) \geq U\left(C\left(0\right), H(0), \gamma\right)$$

and it also increases with respect to pre-advertising preferences

$$U(C(\gamma), H(\gamma), 0) \geq U(C(0), H(0), 0)$$

Accordingly, the consumer is better off with advertising only when he/she prefers the post advertising allocations independently whether his/her tastes are manipulated by advertising. Before going any further, it is worth noting that the welfare effects of advertising are less obvious than it could appear. One could argue, for instance, that the results are totally driven by the fact that by construction advertising is not welfare enhancing. However, according with the previous proposition, it is important to realize that in comparing the equilibrium allocations we are using the same utility functions so that differences in welfare are totally due to differences in the equilibrium allocations rather than by the negative effect that advertising exerts on preferences.

Figure 3.7 graphs the welfare comparison as a function of the advertising efficiency, γ. All the remain parameters are calibrated according to their baseline values, so that

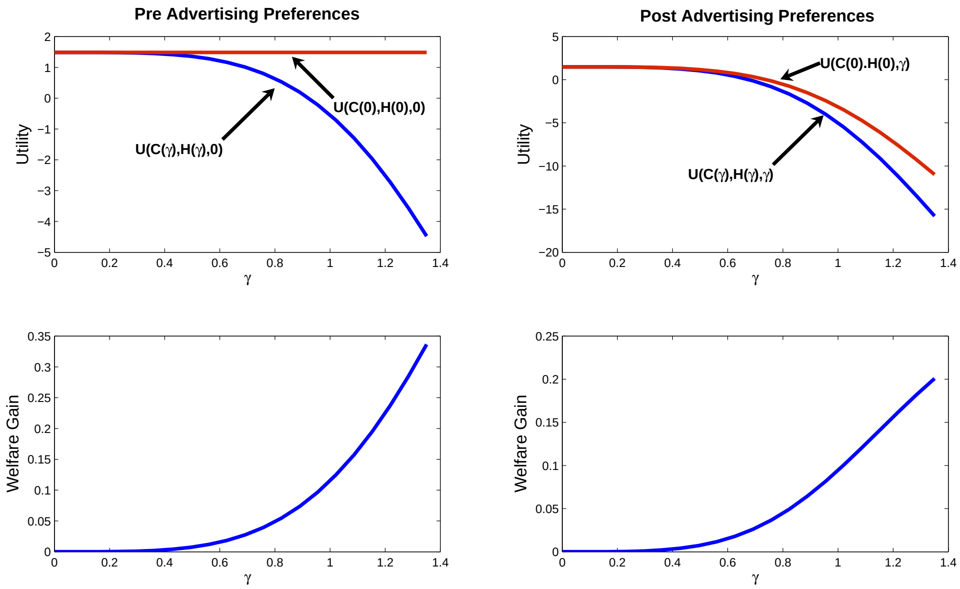

Figure 3.7: **Welfare Analysis**.Steady state utility functions. Left panel: pre-advertising preferences. Right panel: post-advertising preferences. The bottom panel illustrates the welfare gain, in terms of consumption equivalence, associated with a policy that completely bans advertisements.

the equilibrium outcome results in a higher level of hours worked and consumption. As evident from the picture, no matter which types of preferences we use as welfare yardstick, the consumer always prefers the equilibrium allocations without advertising. Based on definition 1, we can conclude then that the consumer is always worse off with advertising. It generates then an "overworking" that results in a welfare loses for the society, even though the equilibrium level of consumption increases. As a result, in this case the effect of advertising on the social welfare is even worse than the one induced by the monopolistic structure of the goods markets.

The bottom panel of figure 3.7 quantifies the welfare gain, in terms of consumption equivalence, associated with a policy in which advertising is totally banned.[10] As

[10]More precisely, for any given γ the welfare gain is defined as the value of λ that solve the equation: $U(\lambda C(\gamma), H(\gamma), \gamma) = U(C(0), H(0), \gamma)$. The pre-advertising gain is calculated considering the ex ante utility function $U(.,.,0)$

we can see, the welfare gain is strictly increasing function of γ or, equivalently, it increases with the amount of resources invested in the advertising sector. Based on this prediction, the USA would be the countries in which the consumers are worst of, a result that contrasts with the conventional view. In this case, our frameworks predicts that reducing the amount of resource invested in advertising in this country would lead to a large welfare gain.

Also, it is interesting to note that our model predicts that consumers' satisfaction and per capita income are not necessarily correlated. As we have seen in section 3.5, conditional to the baseline calibration total output is indeed an increasing function of advertising efficiency while the utility associated with equilibrium allocations is decreasing. This is true even if we use pre-advertising preferences as a welfare yardstick.

This result is consistent with the fact that, across countries, no clear relationship between average income and average happiness can be found (see Graham (2005)). The literature on the economics of happiness suggests that other factors, like aspirations or relative income considerations, are at play. For instance, Laynards (2005) highlights the extent to which people happiness is affected by status. This results in a rat race approach to work and income gains, which eventually reduces well-being. Also, the increasing flow of information about the living standards of others can increases frustration with relative income differences. This can be true even if the individual experiences an increasing level of income. In our framework, the negative welfare effect of advertising are driven by consumption aspirations that induce consumers to work more. If in the western societies individuals care about relative income differences, than it seems reasonable to believe that material (and conspicuous) consumption can be an indirect signal of the individual relative position in the income distribution. In this respect advertisements can play an important role by attaching to consumption goods particular status.

Finally, it is worth stressing out that the generality of our welfare implications is somehow limited for several reasons. Firstly, we have deliberately excluded any informational aspect of advertising that could have opposite welfare effects. Secondly, by fixed the number of goods we have ruled out any possible effects related with the love for varieties property that characterizes CES consumption aggregator. As shown in Grossmann (2007), it is theoretically possible to have positive welfare effects even in the context of purely combative advertising whenever the former stimulates research in development expenditures that can extend the set of available varieties. Finally, a welfare cross-country comparisons, from our point of view, would require a more complicated analysis in which it could be possible to extend the sources of possible heterogeneity rather then considering only the advertising dimension.

3.8 Conclusions

This chapter shows that advertising can have important macroeconomic consequences in the long run. The main mechanism operates through the effect that advertising exerts on the consumers' labor supply. If aggregate advertising can increase the level of consumption, then the individual is induced to work more in order to consume more. This can result in an equilibrium in which output, consumption and hours worked increase with the amount of resources invested in advertising.

This result opens other interesting questions. We show that, theoretically, the observed differences in the advertising spending between the US and Europe can be considered partly responsible for the ones in hours worked. This issue deserves more attention, in particular because the welfare implications we derive can lead to non-trivial policy implications.

Overall, we think that the potential link between advertising and the labors supply needs to be empirically documented. Despite the evidence reported in Brack and Cowling (1983) and Fraser and Paton (2002), it is still missing an empirical analysis involving a cross-countries comparisons. This issue is particularly relevant to properly understand the way advertising affects the aggregate economy, in general, and the labor choice in particular. Exploring this aspect is the next priority of our research agenda.

Panel A			Baseline		$\sigma = 1$		$\phi = 2$	
Country	$\frac{Adv}{GDP}\%$	Hours	H	Diff.%	H	Diff.%	H	Diff.%
USA	2.24	100	100	–	–	–	–	–
GBR	1.51	89.5	96.36	34.71	97.86	20.38	98.17	17.45
DEU	1.40	75.9	95.05	20.55	97.08	12.12	97.50	10.38
FRA	0.69	74.1	93.21	26.20	95.98	15.52	96.55	13.31
ITA	0.67	76.7	93.15	29.70	95.94	17.60	96.52	15.09
Panel B								
Country	$\frac{Adv}{GDP}\%$	$Labor P.$	z/y	Diff.%	z/y	Diff.%	z/y	Diff.%
USA	2.24	100	–	–	–	–	–	–
GBR	1.51	91	1.50	67.01	1.71	85.94	1.83	31.16
DEU	1.40	96	1.89	12.79	2.01	21.65	2.06	5.91
FRA	0.69	100	2.24	0	2.24	0	2.24	0
ITA	0.67	95	1.80	16.75	1.95	16.56	2.02	7.76
Panel C								
Country	$\frac{Adv}{GDP}\%$	Taxes	z/y	Diff.%	z/y	Diff.%	z/y	Diff.%
USA	2.24	0.40	1.31	–	1.15	–	1.71	–
GBR	1.51	0.44	1.2	63.13	1.03	70.46	1.65	28.54
DEU	1.40	0.59	0.71	132	0.6	144.9	1.4	59.92
FRA	0.69	0.59	0.71	124.6	0.60	134.9	1.4	64.40
ITA	0.67	0.64	0.61	175.9	0.47	190.8	1.30	87.94

Table 3.4: **Cross-Countries comparisons.** Panel A: Heterogeneity in advertising efficiency. Panel B: Heterogeneity in labor productivity. Panel C: Heterogeneity in marginal taxes on labor income. In all the panels, Diff% $= \frac{\Delta H}{(100-Hours)}$ refers to percentages of hours differential between the US and European countries explained by the predicted difference in hours worked (Δ H). In panel B and C, z/y refers to the predicted advertising share. Hours and labor productivity (Labor P.) are expressed in relative terms with respect to the US economy (USA=100) Sources: Groningen Growth and Development Centre and the Conference Board, Total Economy Database. Taxes are taken from Prescott (2004).

Bibliography

[1] Abel Andrew and Bernanke Ben (1998): "Macroeconomics", 3rd Ed. MA: Addison-Wesley

[2] Alesina A., Glaeser E. and Sacerdote B. (2005): Work and Leisure in the U.S. and Europe: why so different?, CEPR No. 5140.

[3] An, Sungbae and Schorfheide, Frank (2007): "Bayesian Analysis of DSGE Models" Econometric Reviews vol. 26 n^o 2-4 pp. 113-172

[4] Andrews, Donald (1993): "Test for Parameter Instability and Structural Change With Unknown Change Point", Econometrica vol. 61 n^o 4, pp. 821-856

[5] Arens W. (1993): Contemporaney Advertising Irwin, Chicago

[6] Ashley, R. and Granger, C.W.J. and Schmalensee R. (1980): "Advertising and Aggregate Consumption: An Analysis of Causality"

[7] Econometrica vol. 48 n^o 5 pp.1149-1168.

[8] Bagwell, Kyle (2005): "The Economic Analysis of Advertising", Handbook of Industrial Organization, forthcoming

[9] Ball, Laurence (1994): "Credible Disinflation with Staggered Price Setting", American Economic Review 84, pp. 282-289

[10] Bayoumi Tamim and Sgherri Silvia (2004): "Monetary Magic? How the Fed Improved the Flexibility of the U.S. Economy", IMF wp 04-24

[11] Becker, Gary S. and Murphy, Kevin M. (1993): "A Simple theory of Advertising as a Good or a Bad", Quarterly Journal of Economics, vol. 108 n^o 4 pp. 941-964

[12] Benhabib, Jess and Bisin, Alberto (2002): "Advertising, Mass Consumption and Capitalism"

[13] manuscript, Department of Economics NYU

[14] Benhabib, Jess and Wen, Yi (2004): "Indeterminacy, Aggregate Demand, and the Real Business Cycle"

[15] Journal of Monetary Economics vol. 51 n^o 3, pp. 503-530.

[16] Blanchard O. and Kyotaky N. (1987): "Monopolistic Competition and the Effects of Aggregate Demand" The American Economic Review, Vol. 77, No. 4. $(Sep., 1987)$, pp. $647 - 666$.

[17] Blanchard O. (2004): The Economic Future of Europe NBER Working Paper 10310, February. Forthcoming, Journal of Economic Perspectives.

[18] Blank, David (1961): "Cyclical Behavior of National Advertising"

[19] Journal of Business vol. 35 pp. 14-27.

[20] Brack, J and Cowling, K. (1983): "Advertising and Labor Supply: Workweek and Workyear in U.S. Manufacturing Industries, 1919-76"

[21] Kyklos Vol. 36:2 pp.285-303

[22] Branch, A. William (2004): "Sticky Information and Model Uncertainty in Survey Data on Inflation Expectations", Unpublished Manuscript

[23] Calvo, Guillermo (1983): 'Staggered prices in a utility-maximizing framework', Journal of Monetary Economics n^o 12, pp. 383-398

[24] Chamberlain, E.H. (1933): "The Theory of Monopolistic Competition"

[25] Cambridge MA, Harvard University Press.

[26] Chari, V. V. and Kehoe, Patrick J. and McGrattan, Ellen R. (2000): "Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?", Econometrica, vol. 68, pp. 1151-1180

[27] Christiano J. Lawrence, and Eichenbaum Martin, and Evans Charles (2001): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy", NBER wp 8403

[28] Clarke, D. G. (1976): "Econometric Measurement of the Duration of Advertising Effect on Sales"

[29] Journal of Marketing Research vol 13:4 pp 345-357.

[30] Coibion, Olivier (2006): "Testing the Sticky Information Phillips Curve", manuscript University of Michigan

[31] Cowling K. and Poolsombat R. (2007):"Advertising and Labour Supply: Why Do Americans Work Such Long Hours?" The Warwick Economics Research Paper Series n 789.

[32] Datamonitor (2004): "Advertising in The United States"

[33] www.datamonitor.com

[34] Dixit, Avinash K. and Norman, Victor (1978): "Advertising and Welfare"

[35] Bell Journal of Economics vol.9 n^o 1 pp.1-17

[36] Dorfman, R. and Steiner, P.O. (1954). Optimal Advertising and Optimal Quality

[37] American Economic Review vol. 44, No. 5, pp. 826-836

[38] Dupor Bill and Kitamura Tomiyuki and Tsuruga Takayuki (2005): "Do sticky Prices Need to Be Replaced with Sticky Information?", IMES Discussion Paper Series n^o 2006-E-23, Bank of Japan

[39] Erceg, Christopher J. and Anderson, Dale W. and Levin, Andrew T. (2000): "Optimal monetary policy with staggered wage and price contracts", Journal of Monetary Economics vol.46 pp. 281-313.

[40] Erceg Christopher and Levin Andrew (2003) "Imperfect Credibility and Inflation Persistence", Journal of Monetary Economics vol. 50, pp. 915-44

[41] Fischer, Stanley (1977): "Long-term Contracts, Rational Expectations, and the Optimal Money Supply Rule", Journal of Political Economy vol. 85 pp. 191-205

[42] Fraser, J. and Paton, D. (2003): "Does advertising increases the labor supply? Time series evidences from the UK"

[43] Applied Economics vol. 35, pp.1357-1368.

[44] Galbraith, J.K. (1958), "The Affluent Society", Boston, Houghton Mifflin.

[45] Galbright J.K. (1967): "The New Industrial State" Boston: Houghton Mifflin.

[46] Galí Jordi and Gertler Mark (1999): "Inflation dynamics: A structural econometric analysis", Journal of Monetary Economics 44, pp. 195-222

[47] Graham C. (2005): "The Economics of Happiness. Insights on globalization from a novel approach"

[48] World Economics vol 6, No 3.

[49] Grossmann V. (2007). Advertising, in-house R&D, and growth, Oxford Economic Papers, forthcoming.

[50] Jacobson, R. and Nicosia, F. M. (1981), 'Advertising and public policy: The macroeconomic effects of advertising', Journal of Marketing Research 17, 29-38.

[51] Jung C. and Seldom B. (1995), " The macro-economic relationship between advertising and consumption", Southern Economic Journal 62, 577-587.

[52] Kaldor, N.V. (1950): "The Economic Aspects of Advertising"

[53] Review of Economic Studies vol.18 pp.1-27.

[54] Khan Hashmat and Zhu Zhenhua (2002): "Estimates of the Sticky-Information Phillips Curve for the United States, Canada and the United Kingdom", Banque du Canada wp 2002-19

[55] ——————————— (2006): "Estimates of the Sticky-Information Phillips Curve for the United States", Journal of Money, Credit, and Banking vol. 38 n^o 1, pp. 195-207

[56] Kiley, Michael T. (2006): "A Quantitative Comparison of Sticky-Price and Sticky-Information models of price setting", Finance and Economics Discussion series n^o 2006-45, Federal Reserve Board

[57] Laforte, Jean-Philippe (2005): "Pricing Models: A Bayesian DSGE approach for the US", manuscript Federal Reserve Board

[58] Mankiw, N. Gregory (2001): "The Inexorable and Mysterious Tradeoff Between Inflation and Unemployment", Economic Journal vol. 111

[59] Mankiw Gregory N. and Reis Ricardo (2002): "Sticky Information Versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve", Quarterly Journal of Economics vol.117 (4), pg. 1295-1328

[60] Mankiw Gregory N. and Reis Ricardo and Wolfers Justin (2003): "Disagreement about Inflation Expectations", NBER wp 9796

[61] Mankiw Gregory N. and Reis Ricardo (2006): "Sticky Information in General Equilibrium", manuscript Princeton University

[62] Marshall, A. (1918): "Industry and Trade: A Study of Industrial Technique and Business Organization", and of Their Influences on the Conditions of Various

Classes and Nations", MacMillan and Co. (London).

[63] Nerlove, M. and Arrow, Kennet J. (1962): "Optimal Advertising Policy under Dynamic Conditions", Econometrica vol. 29 pp. 129-142.

[64] Reis, Ricardo (2004): "Inattentive Producers", Manuscript Harvard University

[65] Robinson, J. (1933): "Economics of Imperfect Competition", MacMillan and Co. (London).

[66] Otrok, Christofer. (2001): "On measuring the welfare cost of business cycles", Journal of Monetary Economics vol.47 pp.61-92.

[67] Prescott E. C., (2004): Why Do Americans Work So Much More than Europeans? Federal Reserve Bank of Minneapolis Quarterly Review, July 2004, 28, 213.

[68] Ravn, Morten and Schmitt-Grohe, Stephanie and Uribe, Martin (2006): "Deep Habits", Review of Economic Studies vol. 73 pp. 195-218

[69] Schmalensee, Richard (1972), The Economics of Advertising, Amsterdam: North Holland.

[70] Schumpeter, J.A. (1939): "Business Cycles

[71] A Theoretical Historical and Statistical Analisis of the Capitalist Process", McGraw-Hill, New York

[72] ———————— (2003): "Implications of rational inattention", Journal of Monetary Economics n^o 50, pp. 665-690

[73] Smets, Frank and Wouters, Raf (2007): "Shocks and frictions in US Business Cycles: A Bayesian DSGE Approach", American Economic Review, vol. 97 n. 3, pp. 586-606

[74] Solow, Robert (1968): "The truth further refined: a comment on Marris", The Public Interest vol. 11, pp. 47-52.

[75] Stock, James and Watson, Mark (2003a): "Forecasting Output and Inflation: The Role of Asset Prices", Journal of Economic Literature vol. XLI, pp. 788-829

[76] ————————————————- (2003b): "Has the Business Cycle Changed? Evidence and Explanations", forthcoming FRB Kansas City symposium, Jackson Hole, Wyoming, August 2003

[77] Sutherland M. (1996): Advertising and the Mind of Consumer Allen & Unwin, St Leonards, NSW.

[78] Taylor, B. John (1980): "Aggregate dynamics and staggered contracts", Journal of Political Economy n^{o}88, pp. 1-23

[79] Trabandt, Mathias (2003): "Sticky Information vs. Sticky Prices: A Horse Race in a DSGE Framework", Manuscript Humboldt University Berlin

[80] Tremblay, Victor (2005): "Advertising, Price, and Supermodularity", mimeo Oregon State University

[81] Yang, Y. C. (1964): "Variations in the cyclical behavior of advertising", Journal of Marketing 28:2 pp. 25-30.

[82] Wang, Pengfei and Wen, Yi (2006): "Solving Linear Difference Systems with Lagged Expectations by a Method of Undetermined Coefficients", Federal Reserve Bank of St. Louis, Working Paper 2006-003C.

[83] Woodford, Michael (2001): "Imperfect Common Knowledge and the Effects of Monetary Policy", NBER wp 8673

Appendix A

Sticky Information and Inflation Persistence: Evidence from U.S. data

A.1 Inflation persistence in the SIPC model

Figure (A.1) plots the percent of actual inflation persistence explained by the data for differnet values of the degree of information stickiness in the economy. An arbitrary AR(1) process is assumed for the exogenous shock that enters in firms expectations over future marginal cost.

A.2 Proof of Lemma 1

In order to write the Sticky Information Phillips Curve,

$$\pi_t = \frac{\alpha\lambda}{1-\lambda} y_t + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-1-j} [\pi_t + \alpha \Delta y_t] \tag{A.1}$$

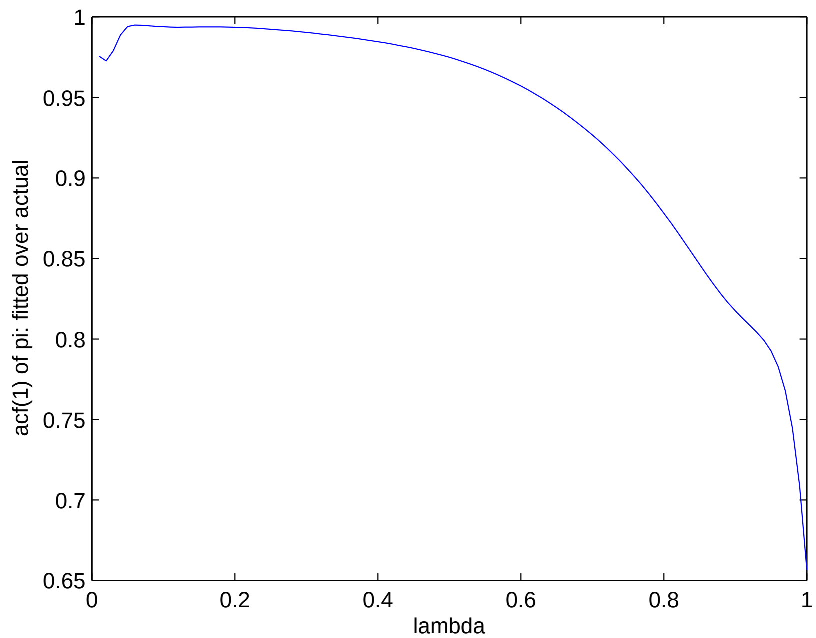

Figure A.1: **Inflation persistence in SIPC model for different values of lambda.** Y-axis measures the ratio of actual over fitted inflation persistence. On X-axis different values of λ used to simulate the SIPC model.

as function of the exogenous shocks, first we define the j-periods-ahead forecast error as:

$$\varepsilon^F_{t|t-j} = Z_t - E\left[Z_t \mid \Omega_{t-j}\right] \tag{A.2}$$

Then, using (A.2) to substitute out the expectations in (A.1) we obtain:

$$\pi_t = \frac{\alpha\lambda}{1-\lambda} y_t + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j \, \delta \left(Z_t - \varepsilon^F_{t|t-j-1}\right) \tag{A.3}$$

where δ is a $(1 \times n)$ row vector that picks $(\pi_t + \alpha \Delta y_t)$ within Z_t. Equation (A.3) can be written as:

$$\frac{\alpha\lambda}{1-\lambda} y_t + \alpha \Delta y_t = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j \, \delta \varepsilon^F_{t|t-j-1} \tag{A.4}$$

Let's focus now on the RHS of (A.4). Using the Wold decomposition of Z_t,

$$Z_t = c + \sum_{i=0}^{\infty} A_i \varepsilon_{t-i} \tag{A.5}$$

we have:

$$\varepsilon^F_{t|t-j-1} = \sum_{i=0}^{j} A_i \varepsilon_{t-i} \tag{A.6}$$

Thus, using (A.5) to substitute out $\varepsilon^F_{t|t-j-1}$ in the RHS of (A.4) we find:

$$\lambda \sum_{j=0}^{\infty} (1-\lambda)^j \, \delta \varepsilon^F_{t|t-1-j} = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j \, \delta \sum_{i=0}^{j} A_i \varepsilon_{t-i}$$

$$= \left(\delta\varepsilon_t + (1-\lambda)\,\delta\varepsilon_t + (1-\lambda)^2\,\delta\varepsilon_t + \ldots\right) + \left((1-\lambda)\,\delta A_1 \varepsilon_{t-1} + (1-\lambda)^2\,\delta A_1 \varepsilon_{t-1} + \ldots\right) + \ldots$$

$$= \frac{\lambda}{1-(1-\lambda)} \sum_{i=0}^{\infty} (1-\lambda)^i \, \delta A_i \varepsilon_{t-i} \tag{A.7}$$

Finally, plugging (A.7) into (A.4) we obtain:

$$\frac{\alpha\lambda}{1-\lambda} y_t + \alpha \Delta y_t = \sum_{i=0}^{\infty} (1-\lambda)^i \, \delta A_i \varepsilon_{t-i}$$

which proves the Lemma.

A.3 Adjusted $V\left(\lambda_T^{2s}\right)$

The problem is defined as follows: let

$$E\left[g_1\left(\lambda, \beta, y_t\right)\right] = 0 \tag{A.8}$$

be the set of orthogonality conditions (1.4), where β is the vec of the matrices of parameters in (1.6).

λ_T^{2s} in section 3.3 is obtained estimating the sample analog

$$\frac{1}{T} \sum_{t=1}^{T} \left[g_1\left(\lambda, \beta_T^{Var}, y_t\right)\right] = 0$$

where β_T^{Var} are estimated coefficients of a pth-order vector autoregression model (henceforth VAR(p)) with errors $\varepsilon_t \sim i.i.d.\ N(0, \Sigma)$. This VAR(p) model has m endogenous variables and it is estimated LS equation by equation. The vector β_T^{Var} has $(m(mp+1) \times 1)$ elements.

Also, let

$$E\left[\underset{km\times 1}{g_2(\beta, y_t)}\right] = 0 \tag{A.9}$$

be the orthogonality conditions that we use to estimate β_T^{Var} in the first step, where $k = mp + 1$.

Now, to estimate jointly $\{\lambda, \beta\}$ we could stack (A.8) and (A.9) in a $(km+n) \times 1$ vector of moments and estimate it by GMM, i.e.

$$E\begin{bmatrix} \underset{n\times 1}{g_1(\lambda, \beta, y_t)} \\ \underset{km\times 1}{g_2(\beta, y_t)} \end{bmatrix} = 0 \tag{A.10}$$

In this model there are no stochastic regressors, and the (correct) VCV matrix of coefficients if optimal weighting matrix is used is

$$V\begin{pmatrix} \lambda_T \\ \beta_T \end{pmatrix} = \left[\underbrace{TE\left(\frac{\partial \begin{pmatrix} g_{1,t} \\ g_{2,t} \end{pmatrix}}{\partial \begin{pmatrix} \lambda & \beta' \end{pmatrix}} \right)}_{\equiv G}{}' \cdot \underbrace{\left(E \begin{pmatrix} g_{1,t} \\ g_{2,t} \end{pmatrix} \begin{pmatrix} g'_{1,t} & g'_{2,t} \end{pmatrix} \right)}_{\equiv \Omega}{}^{-1} E\left(\frac{\partial \begin{pmatrix} g_{1,t} \\ g_{2,t} \end{pmatrix}}{\partial \begin{pmatrix} \lambda & \beta' \end{pmatrix}} \right) \right]^{-1}$$

where G is a $(km+n) \times (km+1)$ matrix, and Ω is the $(km+n) \times (km+n)$ VCV matrix of moments.

By construction $\frac{\partial g_{2,t}}{\partial \lambda} = 0$, so matrix G can be written as

$$G = E \begin{pmatrix} \frac{\partial g_{1,t}}{\partial \lambda} & \frac{\partial g_{1,t}}{\partial \beta'} \\ 0 & \frac{\partial g_{2,t}}{\partial \beta'} \end{pmatrix}$$

In addition, if we assume no covariance between SIPC and VAR residuals,[1] then the inverse of the variance of moments is

$$\Omega^{-1} = \begin{pmatrix} \Sigma_{g_1}^{-1} & 0 \\ 0 & \Sigma_{g_2}^{-1} \end{pmatrix} \tag{A.11}$$

Using these two facts (**??**) is:

$$V \begin{pmatrix} \lambda_T \\ \beta_T \end{pmatrix} = \left[E \begin{pmatrix} \frac{\partial g_1'}{\partial \lambda} & 0' \\ \frac{\partial g_1'}{\partial \beta} & \frac{\partial g_2'}{\partial \beta} \end{pmatrix} \begin{pmatrix} \Sigma_{Gmm}^{-1} & 0 \\ 0 & \Sigma_{Var}^{-1} \end{pmatrix} E \begin{pmatrix} \frac{\partial g_1}{\partial \lambda} & \frac{\partial g_1}{\partial \beta'} \\ 0 & \frac{\partial g_2}{\partial \beta'} \end{pmatrix} \right]^{-1} /T \tag{A.12}$$

or, after some algebra manipulation,

$$V \begin{pmatrix} \lambda_T \\ \beta_T \end{pmatrix} = \begin{pmatrix} E\frac{\partial g_{1,t}'}{\partial \lambda}\Sigma_{g_1}^{-1}E\frac{\partial g_{1,t}}{\partial \lambda} & E\frac{\partial g_{1,t}'}{\partial \lambda}\Sigma_{g_1}^{-1}E\frac{\partial g_{1,t}}{\partial \beta'} \\ E\frac{\partial g_{1,t}'}{\partial \beta}\Sigma_{g_1}^{-1}E\frac{\partial g_{1,t}}{\partial \lambda} & E\frac{\partial g_{1,t}'}{\partial \beta}\Sigma_{g_1}^{-1}E\frac{\partial g_{1,t}}{\partial \beta'} + E\frac{\partial g_{2,t}'}{\partial \beta}\Sigma_{g_2}^{-1}E\frac{\partial g_{2,t}}{\partial \beta'} \end{pmatrix}^{-1} /T \tag{A.13}$$

Now, the not-adjusted variance of the two steps estimator λ_T^{2s} obtained from the estimation of the orthogonality conditions (A.8) alone is:

$$V_{na}(\lambda_T^{2s}) = \left(TE\frac{\partial g_{1,t}'}{\partial \lambda}\Sigma_{g_1}^{-1}E\frac{\partial g_{1,t}}{\partial \lambda} \right)^{-1} \tag{A.14}$$

So, using the definition (A.14) and the formula for the inverse of partitioned matrices, the variance of λ_T in (A.13) can be written as

$$V(\lambda_T) = \left(\left(TV_{na}(\lambda_T^{2s})\right)^{-1} - E\frac{\partial g_{1,t}'}{\partial \lambda}\Sigma_{g_1}^{-1}E\frac{\partial g_{1,t}}{\partial \beta'} \left(E\frac{\partial g_{1,t}'}{\partial \beta}\Sigma_{g_1}^{-1}E\frac{\partial g_{1,t}}{\partial \beta'} + E\frac{\partial g_{2,t}'}{\partial \beta}\Sigma_{g_2}^{-1}E\frac{\partial g_{2,t}}{\partial \beta'} \right)^{-1} E\frac{\partial g_{1,t}'}{\partial \beta}\Sigma_{g_1}^{-1}E\frac{\partial g_{1,t}}{\partial \lambda} \right)^{-1} /T \tag{A.15}$$

[1]It can be shown that this is indeed the case when the errors ε_t are normally distributed.

Thus, (A.15) is the variance of the estimator of λ in this model, and it turns out to be also the variance of λ_T^{2s} once we adjust for the stochastic regressors.

Actually, we don't need to estimate (A.10) to get (A.15), since we can compute it with the information we have from the two steps estimator. In particular, $\frac{\partial g_{1,t}}{\partial \lambda}$ and $\Sigma_{g_1}^{-1}$ are respectively the Jacobian and the weighting matrix of the second step. $\left(E\frac{\partial g_{2,t}'}{\partial \beta}\Sigma_{g_2}^{-1}E\frac{\partial g_{2,t}}{\partial \beta'}\right)$ is the VCV matrix of the VAR(p) parameters in the first step, and $\frac{\partial g_{1,t}}{\partial \beta'}$ is the vector of derivatives of (1.4) with respect to β evaluated at $\beta_T^{Var}, \lambda_T^{2s}$.

It is worth noticing that (A.15) is the correct variance of λ_T only if the covariance between $g_{1,t}$ and $g_{2,t}$ is zero. Otherwise, it would be optimal to estimate jointly λ, β because we could exploit the information in $g_{1,t}$ to pin down β_T. But this means a non linear optimization over a large set of parameters, while in this chapter I do simpler: I estimate β by OLS, then I estimate λ by GMM, using $\Sigma_{g_1}^{-1}$ as weighting matrix. This is akin to estimate (A.10) with the GMM using

$$W = \begin{pmatrix} \Sigma_{g_1}^{-1} & O \\ O' & I_{km} \end{pmatrix} \tag{A.16}$$

as weighting matrix.

Now, if $E\left[g_{2,t}g_{1,t}\right] \neq 0$ then Ω^{-1} in (A.11) is not diagonal, and (A.16) is not the optimal weighting matrix to estimate (A.10). In this case, the correct variance of λ_T is the upper left cell of:

$$V\begin{pmatrix} \lambda_T \\ \beta_T \end{pmatrix} = \left(G'WG\right)^{-1} G'W\Omega WG \left(G'WG\right)^{-1} / T \tag{A.17}$$

and the two-steps estimator is no longer the most efficient estimator of λ_T among GMM estimators of (A.10).

A.4 Robustness Analysis: Tables

- Calibration of α.

Restricted $\alpha = .1$	Specif.	λ_T^{2s}	Adjusted std.err.	Null MR cal.	t-stat (p-val)	Null RE	t-stat (p-val)	J-stat (p-val)
O.C. (1.4)	$i = 1, ..., 6$							
defl; VAR	(1)	0.51	0.071	0.25	3.74	1	-5.18	1.91
$\{\Delta y_t, \pi_t, i_t\}$					(0.00)		(0.00)	(0.86)*
	(2)	0.53	0.073	0.25	3.82	1	-6.36	1.62
					(0.00)		(0.00)	(0.89)*
defl; VAR	(1)	0.54	0.099	0.25	2.93	1	-4.64	1.76
minRMSE					(0.00)		(0.00)	(0.88)*
	(2)	0.55	0.104	0.25	2.90	1	-4.28	1.54
					(0.00)		(0.00)	(0.90)*
O.C. (1.7)								
defl; VAR	(1)	0.93	0.047	0.25	14.42	1	-1.38	17.47
$\{\Delta y_t, \pi_t, i_t\}$					(0.00)		(0.16)*	(0.49)*
	(2)	0.94	0.047	0.25	14.54	1	-1.12	13.39
					(0.00)		(0.26)*	(0.76)*
defl; VAR	(1)	0.92	0.06	0.25	11.14	1	-1.23	20.61
minRMSE					(0.00)		(0.21)*	(0.29)*
	(2)	0.94	0.058	0.25	11.75	1	-1.02	14.11
					(0.00)		(0.30)*	(0.72)*

Table 3. Calibration of α. 2-step GMM estimator with optimal weighting matrix. Orthogonality conditions (1.4). U.S. data, sample 1958q4 – 2005q4. HP filter for output gap. Newey-West HAC standard errors adjusted for stochastic regressors. p-values in parenthesis. J-statistics is Hansen test of overidentifying restrictions.

- Quadratic detrend (QD) filter.

Restricted $\alpha = .2$	Specif.	λ_T^{2s}	Adjusted std.err.	Null MR cal.	t-stat (p-val)	Null RE	t-stat (p-val)	J-stat (p-val)
O.C. (1.4)	$i = 1, ..., 6$							
defl; VAR	(1)	0.29	0.055	0.25	0.83	1	-12.61	1.35
$\{\Delta y_t, \pi_t, i_t\}$					(0.40)*		(0.00)	(0.92)*
	(2)	0.30	0.040	0.25	1.31	1	-17.37	1.17
					(0.18)*		(0.00)	(0.94)*
defl; VAR	(1)	0.33	0.099	0.25	0.84	1	10.73	1.02
minRMSE					(0.40)*		(0.00)	(0.96)*
	(2)	0.34	0.060	0.25	1.56	1	10.73	1.29
					(0.11)*		(0.00)	(0.93)*
O.C. (1.7)								
defl; VAR	(1)	0.64	0.081	0.25	4.82	1	-4.38	29.99
$\{\Delta y_t, \pi_t, i_t\}$					(0.00)		(0.00)	(0.03)
	(2)	0.78	0.090	0.25	5.89	1	-2.37	10.88
					(0.00)		(0.00)	(0.89)*
defl; VAR	(1)	0.58	0.11	0.25	3.01	1	-3.77	35.44
minRMSE					(0.00)		(0.00)	(0.00)
	(2)	0.90	0.15	0.25	4.22	1	-0.58	11.87
					(0.00)		(0.56)*	(0.85)*

Table 4. QD 2-step GMM estimator with optimal weighting matrix. Orthogonality conditions (1.4). U.S. data, sample 1958q4 – 2005q4. Quadratic Detrend filter for output gap. Newey-West HAC standard errors adjusted for stochastic regressors. p-values in parenthesis. J-statistics is Hansen test of overidentifying restrictions.

- Univariate process for the exogenous shocks, i.e. $(\pi_t + \alpha\Delta y_t) \sim AR(2)$

Let's define $S_t \equiv (\pi_t + \alpha\Delta y_t)$. I assume here that demeaned inflation and output gap follow an univariate second order autoregressive process, i.e.

$$S_t = \phi_1 S_{t-1} + \phi_2 S_{t-2} + \varepsilon_t$$

This assumption simplifies the model at issue, and we can estimate jointly the parameters of the $AR(2)$ process and the firms' frequency of information updating parameter λ.

To do that, I estimated jointly the following orthogonality conditions:

$$\begin{aligned} E\left[\begin{pmatrix} S_{t-1} \\ S_{t-2} \end{pmatrix} \cdot (S_t - \phi_1 S_{t-1} - \phi_2 S_{t-2})\right] &= 0 \\ E\left[(S_t - \phi_1 S_{t-1} - \phi_2 S_{t-2})^2 - \sigma_\varepsilon^2\right] &= 0 \\ E\left[\left(\frac{\alpha\lambda}{1-\lambda} y_t + \alpha \Delta y_t\right) \cdot \varepsilon_{t-i} - (1-\lambda)^i A_i \sigma_\varepsilon^2\right] &= 0 \end{aligned} \quad \text{(A.18)}$$

In Table 5 I estimate all the moments (A.18) together. The results are similar to that found before, when I estimate the orthogonality conditions (1.4) all together. The moments are rejected by the data. In other words this model cannot fit inflation variance and perisistence together.

Restricted $\alpha = .2$	Specif.	λ_T^{Gmm} (s.e.)	ϕ_1 (s.e.)	ϕ_2 (s.e.)	σ_ε^2 (s.e.)	H_0: $\lambda = .25$ (p-val)	J-stat (p-val)
O.C. (1.4)	$i = 0, ..., 6$						
GDP deflator	(1)	0.90	0.67	0.28	1.2e-5	7.00	11.24
$\pi_t + \alpha\Delta y_t \sim$AR(2)		(.093)	(.062)	(.065)	(0.2e-5)	(0.00)	(0.08)*
	(2)	0.24	0.69	0.27	0.2e-5	-0.07	23.88
		(.030)	(.063)	(.067)	(0.1e-5)	(0.94)*	(0.00)
CPI	(1)	0.70	0.62	0.32	1.3e-5	5.84	17.72
$\pi_t + \alpha\Delta y_t \sim$AR(2)		(.077)	(.047)	(.044)	(0.3e-5)	(0.00)	(0.00)
	(2)	0.83	0.60	0.32	2.0e-5	9.97	9.04
		(.058)	(.046)	(.046)	(0.3e-5)	(0.00)	(0.17)*

Table 5. Forecast technology. 1-step GMM with optimal weighting matrix. U.S. data, sample 1958q4 – 2005q4. HP filter for output gap. Newey-West HAC standard errors (no stochastic regressors). p-values in parenthesis. J-statistics is Hansen test of overidentifying restrictions.

However, following the same strategy as before, I discard first the conditional variance and I estimate the other moments, and then I discard the moments related with the lagged covariances and I estimate the contemporaneous covariance alone. Results are in Table 6.

Restricted $\alpha = .2$	Specif.	λ_T^{Gmm} (s.e.)	ϕ_1 (s.e.)	ϕ_2 (s.e.)	σ_ε^2 (s.e.)	H_0: $\lambda = .25$ (p-val)	J-stat (p-val)
O.C. (1.4)	$i = 1, ..., 6$						
GDP deflator	(1)	0.51	0.61	0.33	1.4e-5	3.92	0.97
$\pi_t + \alpha\Delta y_t$ ~AR(2)		(.067)	(.061)	(.064)	(0.2e-5)	(0.00)	(0.96)*
	(2)	0.52	0.62	0.32	1.4e-5	4.02	0.83
		(.068)	(.060)	(.063)	(0.2e-5)	(0.00)	(0.97)*
CPI	(1)	0.54	0.66	0.28	1.9e-5	6.74	2.27
$\pi_t + \alpha\Delta y_t$ ~AR(2)		(.043)	(.051)	(.048)	(0.3e-5)	(0.00)	(0.80)*
	(2)	0.55	0.64	0.29	1.9e-5	6.35	1.86
		(.058)	(.058)	(.048)	(0.3e-5)	(0.00)	(0.86)*
O.C. (1.7)							
GDP deflator	(1)	0.91	0.61	0.34	1.4e-5	7.66	exacly
$\pi_t + \alpha\Delta y_t$ ~AR(2)		(.086)	(.066)	(.074)	(0.2e-5)	(0.00)	identif.
	with	0.96	0.58	0.38	1.0e-5	9.75	24.72
	instr.	(.072)	(.044)	(.048)	(0.2e-5)	(0.00)	(0.13)*
CPI	(1)	0.81	0.66	0.28	2.1e-5	9.59	exacly
$\pi_t + \alpha\Delta y_t$ ~AR(2)		(.058)	(.060)	(.056)	(0.4e-5)	(0.00)	identif.
	with	0.98	0.64	0.31	0.9e-5	10.08	24.69
	instr.	(.072)	(.038)	(.037)	(0.2e-5)	(0.00)	(0.13)*

Table 6. Forecast technology. 1-step GMM with optimal weighting matrix. U.S. data, sample 1958q4 – 2005q4. HP filter for output gap. Newey-West HAC standard errors (no stochastic regressors). p-values in parenthesis. J-statistics is Hansen test of overidentifying restrictions.

Appendix B

Advertising and Business Cycle Fluctuations

B.1 Sources of Data

B.1.1 Data on Advertising

Advertising expenditures in TV, Cable, Radio, Magazines, and Outdoor:

Ad$Summary, quarterly issues from 1975:3 to 2006:2, issued by Media Market, N.Y.C.

Advertising expenditures in newspaper:

Newspaper Association of America. Data available on the official website of the Association: http://www.naa.org/

Annual advertising expenditures and its components:

Universal McCann, Robert Coen's Annual Report, Estimated Annual U.S. Advertising Expenditures from 1958 to 2006.

B.1.2 Macroeconomic Data

Source: Database "FRED II" of Federal Reserve Bank of St. Luise

Available at the website: http://research.stlouisfed.org/fred2

Real Gross Domestic Product (GDPC96)

Real Exports of Goods & Services (EXPGSC96)

Real Personal Consumption Expenditures and components (PCEC96)

Real Private Fixed Investment (FPICA)

GDP Implicit Price Deflator (GDPDEF)

Civilian Employment-Population Ratio (EMRATIO)

Civilian Non-Institutional Population (CNP160V)

Source: Bureau of Labor Statistics

Available at the website: http://www.bls.gov/data/home.htm

Total Private Average Weekly Hours of Production Workers (CES050007)

Total Non-farm Employment (CES050001)

Note: The series of worked hours used in the estimation is

$$H = \frac{CES050007 * EMRATIO}{168}$$

where 168 normalizes weekly hours to the total endowment of hours in a week. Alternatively we use the series:

$$H = \frac{CES050001}{CNP160V * 168}$$

B.2 Technical Appendix

B.2.1 Firm's costs minimization problem

To produce its good each firms employs two inputs, labor and capital, combined according to the following production function:

$$y_{i,t} = A_t k_{i,t}^{1-\alpha} \left(\Gamma_t H_{p,t}(i)\right)^{\alpha} - F \tag{B.1}$$

where $y_{i,t}, k_{i,t}, H_{p,t}(i)$, denote respectively firm's output, capital stock, and production-related labor. A_t measures the stochastic technological progress of the Total Factor Productivity, and F is a fixed cost.

Firm's demand of production-related inputs is the solution to the dual problem of total cost minimization, given by $W_t hp_{i,t} + R_t k_{i,t}$, and subject to the production function constraint (B.1).

As result, firm's total cost function, and marginal cost are given respectively by:

$$CT(y_{i,t}) = \frac{D}{A_t} W_t^{\alpha} R_t^{1-\alpha} \left(y_{i,t} + F\right) \tag{B.2}$$

and

$$\varphi_{i,t} = \frac{D}{A_t} W_t^{\alpha} R_t^{1-\alpha} \tag{B.3}$$

where $D = \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} \frac{1}{(1-\alpha)}$ is a positive constant.

Also, each firm promotes its sales by spending money in advertising. As with Grossmann (2007), we assume that advertising is produced by the marketing department of the firm using the following technology:

$$z_{i,t} = U_t^z A_t \left(H_{a,t}(i)\right)^{\alpha} \tag{B.4}$$

where $z_{i,t}$, $H_{a,t}(i)$ denote respectively the new investment in advertising and the marketing-related labor. As apparent from equation (B.1), advertising is an unproductive factor for the firm.

B.2.2 Profits maximization problem

Each producer face three demands for its product. One for consumption, i.e. (2.11), one for investment, and one for government purchases.

The demand function for investment goods derives from the solution to consumer's dual problem of expenditures minimization, subject to the technology constraint (2.6), i.e.

$$i_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon} I_t \tag{B.5}$$

About government's demand of goods, recalling the assumption that advertising does not affect government's choices, we can write it goods as the solution of the consumer's problem of expenditures minimization, subject to the constraint

$$Gex_t \geq \left(\int (gex_{i,t})^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{B.6}$$

where preferences for simplicity we set the bound in utility to zero.

Thus, we can derive the total demand for good i as

$$y_{i,t} \equiv c_{i,t} + i_{i,t} + gex_{i,t} = \left(\frac{p_{i,t}}{P_t}\right)^{-\varepsilon} \left(\tilde{C}_t + I_t + Gex_t\right) - B(g_{i,t}) \tag{B.7}$$

where (B.7) uses (2.11) (B.6) and (B.5).

Then, firm's problem of profits maximization can be stated as choosing a sequences of prices $p_{i,t}$ and advertising-related labor $H_{a,t}(i)$ in order to maximize:

$$\max_{\{H_{a,t}(i),p_{i,t}\}} E\sum_{t=0}^{\infty} Q_{0,t}\left(p_{i,t}y_{i,t} - CT(y_{i,t}) - W_t H_{a,t}(i)\right) \tag{B.8}$$

subject to

$$g_{i,t} = z_{i,t} + (1-\delta_g)\, g_{i,t-1}$$

$$z_i = U_t^z A_t \left(H_{a,t}(i)\right)^{\alpha}$$

$$y_{i,t} = \left(\frac{p_{i,t}}{P_t}\right)^{-\varepsilon} \left(\tilde{C}_t + I_t\right) - B(g_{i,t})$$

where $Q_{0,t}$ is the discount factor. $CT(y_{i,t})$ is defined as in equation (B.2).

It is useful to notice that profits maximization is dynamic problem due to the dynamic structure of the goodwill, and not to the presence of advertising. Notice, indeed, that with $\delta_g = 1$, this problem is again static, as in the standard monopolistic competition case with no nominal rigidities.

The first order conditions for an interior maximum of (B.8) are:

$$P_{i,t} = \frac{\varepsilon\left(1 + \frac{B(g_{i,t})}{y_{i,t}}\right)}{\varepsilon\left(1 + \frac{B(g_{i,t})}{y_{i,t}}\right) - 1}\varphi_t \equiv \mu_{i,t}\varphi_t \tag{B.9}$$

$$\nu_t = \frac{W_t}{\alpha\gamma U_t^z A_t} h a_t^{1-\alpha} S_z'\left(z_{i,t}, g_{i,t-1}\right) \tag{B.10}$$

$$-\left(p_{i,t} - \varphi_t\right) B'(g_{i,t}) + E\left(1 - \delta_g + S_g'(z_{i,t+1}, g_{i,t})\right)\left(\phi_{t+1} Q_{t,t+1}\right) = \nu_t \tag{B.11}$$

B.2.3 On the profit function

This section focuses on the properties of the firms' profit function. Our framework is not standard so that in the maximization problem the sufficiency of the first order conditions have to be checked.

In what follows, we assume for simplicity that the stock of goodwill fully depreciates, i.e $\delta g = 1$. This assumption greatly simplifies the next analysis by making the optimization problem static. The dynamic problem is rather complicated but the existence of the optimal solution should be follows the same line as the one we present in the next analysis.

Before going any further, it should be noted that the admissible space for the optimal price has to be restricted. In fact, our framework implies the existence of a maximal price, say $\bar{p}_t$, above which the demand is equal to zero. As a consequence, the optimal price belongs in the bounded space $(\varphi_t, \bar{p}_{i,t})$, where φ_t is the marginal cost of producing goods.

To study the behavior of the profit function, we will divide the optimization problem in an equivalent to steps maximization. Given the level of advertising related labor, we will maximize the profit with respect to the price and, then, given the optimal price we will maximize with respect to advertising related labor. Either the case, we will show that the resulting function is single peaked so that the maximum is interior. The procedure is useful since from the second order conditions we cannot conclude that the function is globally concave, so that the first order conditions are not sufficient for a maximum.[1]

By combining together the cost function (B.2), the total demand (B.7) and neglecting, without loss of generality, the fixed cost, the profit function for firm i can be expressed as the following:

$$\Pi_{i,t} = (p_{i,t} - \varphi_t)\left[p_{i,t}^{-\varepsilon}\left(\tilde{C}_t + I_t\right) - B(g_{i,t})\right] - W_t H_{a,t}(i) \tag{B.12}$$

For the moment suppose that the amount of advertising related labor is fixed, so that the previous function depends only from the price. We want to study the shape of such a function. To do it, notice that differentiating (B.12) with respect to the price, yields:

$$\frac{\partial \Pi_{i,t}}{\partial p_{i,t}} = (1-\varepsilon)(\tilde{C}_t + I_t)\, p_{i,t}^{-\varepsilon-1}\left(p_{i,t} - \frac{\varepsilon}{\varepsilon-1}\varphi_t\right) - B(g_{i,t}) \tag{B.13}$$

By setting the above expression equal to zero, we see that the necessary condition for an interior maximum requires:

$$(1-\varepsilon)(\tilde{C}_t + I_t)\, p_{i,t}^{-\varepsilon-1}\left(p_{i,t} - \frac{\varepsilon}{\varepsilon-1}\varphi_t\right) = B(g_{i,t}) \tag{B.14}$$

[1]Directed calculation of the Hessian reveals that the matrix is negative semi defined only in a portion of the admissible space.

Let $f(p_{i,t})$ denotes the l.h.s of the above expression. It is easy to verify that such a function is positive only if $p_{i,t} \in (\varphi_t, \frac{\varepsilon}{\varepsilon-1}\varphi_t)$. Moreover, differentiating it with respect to the price yields:

$$f'(p_{i,t}) = \varepsilon(\varepsilon-1)\,(\tilde{C}_t + I_t)\,p_{i,t}^{-\varepsilon-2}\,\left(p_{i,t} - \frac{\varepsilon+1}{\varepsilon-1}\varphi_t\right)$$

which reveals that the function is strictly decreasing for $p_{i,t} \in (\varphi_t, \frac{\varepsilon+1}{\varepsilon-1}\varphi_t)$. Consequently, taking into account that the r.h.s is a positive constant, there exists a unique price, say $p_{i,t}^*$, such that equation (B.14) is satisfied. Moreover, $p_{i,t}^* \in (\varphi_t, \frac{\varepsilon}{\varepsilon-1}\varphi_t)$.[2]

On the other hand, knowing the behavior of $f(p_{i,t})$ enables us to characterized the sign of (B.13). It is straightforward to check that the following relation holds:

$$sign\left(\frac{\partial \Pi_{i,t}}{\partial p_{i,t}}\right) = \begin{cases} > 0 & \text{if } p_{i,t} \in (\varphi_t, p_{i,t}^*) \\ = 0 & \text{if } p_{i,t} = p_{i,t}^* \\ < 0 & \text{if } p_i \in (p_{i,t}^*, \bar{p}_{i,t}) \end{cases}$$

so that, the profit is a single peaked function of the price and, all other things constant, $p_{i,t}^*$ is the price that maximizes the profit and, at the same time, the unique solution of equation (B.14). Moreover, in the optimal price it is true:

$$\frac{\partial^2 \Pi_{i,t}}{\partial^2 p_{i,t}}(p_{i,t}^*) = f'(p_{i,t}^*) < 0$$

as it is required. Finally, using the implicit function theorem it easy to show that following holds:

$$\frac{\partial p_{i,t}^*}{\partial g_{i,t}} = \frac{B'(g_{i,t})}{\frac{\partial^2 \Pi_{i,t}}{\partial^2 p_{i,t}}} > 0 \tag{B.15}$$

so that the optimal price increases with advertising.

We want to show now that once we take into account the optimal price, the resulting profit is also single-peaked function of the advertising related labor, so that

[2] Because of the fact that the function is positive only for this range of the price

there is necessarily an interior maximum for this function. To do it, first substitute the optimal price $p^*_{i,t}$ into (B.12) and then, differentiate the resulting function with respect to $h_{a,t}(i)$. Using the envelope theorem, it easy to show that the derivative of the profit with respect to the advertising related labor is of the form:[3]

$$\frac{\partial \pi_{i,t}(h_{a,t}, p^*_{i,t})}{\partial h_{a,t}} = -B'(g_{t,i})\alpha\gamma A_t h_{a,t}^{\alpha-1}(p^*_{i,t} - \varphi_t) - W_t$$

Setting the above expression equal to zero and using the goodwill low of motion yields:

$$-B'(g_{t,i})(p^*_{i,t} - \varphi_t) = \frac{W_t}{\alpha\gamma} g_{t,i}^{\frac{1-\alpha}{\alpha}} \tag{B.16}$$

This is essentially the static counterpart of equation (2.15). Let $f_1(g_{i,t})$ and $f_2(g_{i,t})$ respectively denote the l.h.s and r.h.s of the above expression. A sufficient condition for a unique solution is that the two functions cross each other just once in all the admissible range of $g_{i,t}$. Moreover, by studying the behavior of such a function we can also infer how the profit changes with advertising.

Studying the function $f_2(g_{i,t})$ is not problematic. It can be readily checked that it strictly increasing, and takes value zero when the goodwill stock is equal to zero. The other one is more complicated. It is composed of the product of two terms, the first of them strictly decreasing while the second one strictly increasing. Differentiating the function with respect to the goodwill we get:

$$f_1'(g_{i,t}) = -B''(g_{i,t})(p^*_{i,t} - \varphi_t) - B'(g_{i,t})\frac{\partial p^*_{i,t}}{\partial g_{i,t}}$$

Now notice that, by definition, the following equalities hold:

$$B'(g_{i,t}) = -\theta B(g_{i,t})^2$$

$$B''(g_{i,t}) = -2\theta B'(g_{i,t})B(g_{i,t})$$

[3]For the sake of brevity, from now on we suppress the index i to the advertising related labor.

so that, by using equation (B.15) we get:

$$f_1'(g_{i,t}) = \theta B'(g_{i,t}) \left[2\, B(g_{i,t})\, (p^*_{i,t} - \varphi_t) + \frac{B(g_{i,t})^2}{\frac{\partial^2 \pi_{i,t}}{\partial^2 p_{i,t}}} \right]$$

The first term in the RHS of the above expression is always negative, thus $f_1(g_{i,t})$ is increasing (decreasing) whenever the second term in the is negative (positive). Inspecting the term and considering it as a function of $B(g_{i,t})$, we see that its a parabola without constant and with the quadratic term multiplied by a negative coefficient. Hence, it is negative for all the values of $B(g_{i,t})$ outside the range between its roots. Clearly, one of them is given by $B(g_{i,t}) = 0$, which is not admissible since it requires $g_{i,t} = \infty$. The second one is given by the value of $g_{i,t}$ that solves the following equality:

$$B(g_{i,t}) = -(p^*_{i,t} - \varphi_t) \frac{\partial^2 \pi_{i,t}}{\partial^2 p_{i,t}}$$

Let $g^*_{i,t}$ be the unique[4] solution of the previous equality. Given the fact that $B(g_{i,t})$ is a decreasing function we can then conclude:

$$sign\left(f_1'(g_{i,t})\right) = \begin{cases} > 0 & \text{if } g_{i,t} \in (0, g^*_{i,t}) \\ = 0 & \text{if } g_{i,t} = g^*_{i,t} \\ < 0 & \text{if } g_{i,t} > g^*_{i,t} \end{cases}$$

This fact, together with the observation that $f_1(0) > 0$, implies that if $f_1(g_{i,t})$ and $f_2(g_{i,t})$ intersect, they must cross once. This also implies that there exists a unique value of advertising related labor, say of $h^*_{a,t}$, that satisfies equation (B.16). Moreover:

$$sign\left(\frac{\partial \pi_{i,t}(h_{a,t}, p^*_{i,t})}{\partial h_{a,t}}\right) = \begin{cases} > 0 & \text{if } h_{a,t} \in (0, h^*_{a,t}) \\ = 0 & \text{if } h_{a,t} = h^*_{a,t} \\ < 0 & \text{if } h_{a,t} > h^*_{a,t} \end{cases}$$

[4]The uniqueness of the solution is guaranties from the fact that $B(g_{i,t})$ is decreasing function of $g_{i,t}$ while $-(p^*_{i,t} - \varphi_t)\frac{\partial^2 \pi_{i,t}}{\partial^2 p_{i,t}}$ is increasing.

so that the $\pi(p^*_{i,t}, h_{a,t})$ is a single peaked function of $h_{a,t}$. It proves that a interior maximum exists.

B.3 Estimation

B.3.1 The estimated model

The model we estimate is the one defined in chapter 2, section 3, and in related Appendix B.2, but for 4 small modifications:

1. The fixed cost is set to zero, i.e. $F = 0$.

2. There is a labor augmenting technology shock that follows a deterministic growth trend in the production function. We introduce this trend to match the one observed in our data. The shock is modeled as: $\Gamma_t = exp(tg_q)\Gamma_{t-1}$.

3. We shut down the shock on U_t^z and we introduce a shock on the elasticity of the demand ε.

4. We allow for two different labor shares in the production function of advertising and of the goods. So, $Z_t = A_t H_{a,t}^{\alpha_z}$

Overall the estimated version of the model is defined by the system of equations:

$$\tilde{C}_t = C_t + \frac{1 + \gamma\theta G_t}{1 + \theta G_t} \tag{B.17}$$

$$\tilde{C}_t^{-\sigma} = \frac{\beta}{exp(tg_q)} E_t \left[\tilde{C}_{t+1}^{-\sigma} \left(R_t + 1 - \delta_k \right) \right] \tag{B.18}$$

$$\xi_t H_t^{\phi} = W_t \tilde{C}_t^{-\sigma} \tag{B.19}$$

$$W_t = \alpha \mu_t^{-1} \left(\frac{K_t}{H_{p,t}} \right)^{1-\alpha} \tag{B.20}$$

$$R_t = (1-\alpha)\, \mu_t^{-1} \left(\frac{H_{p,t}}{K_t} \right)^{\alpha} \tag{B.21}$$

$$\mu_t = \frac{\varepsilon_t \left(1 + \frac{1}{(1+\theta G_t) Y_t}\right)}{\varepsilon_t \left(1 + \frac{1}{(1+\theta G_t) Y_t}\right) - 1} \tag{B.22}$$

$$\left(1 - \mu_t^{-1}\right) \frac{\theta}{\left(1+\theta G_t\right)^2} + E_t \left[\frac{(1-\delta_g)}{exp(tg_q)} Q_{t,t+1} \nu_{t+1} \right] = \nu_t \tag{B.23}$$

$$Q_{t,t+1} = \beta \left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-\sigma} \tag{B.24}$$

$$H_t = H_{a,t} + H_{p,t} \tag{B.25}$$

$$\nu_t = \frac{W_t H_{a,t}}{\alpha_z Z_t} \tag{B.26}$$

$$Z_t = A_t H_{a,t}^{\alpha_z} \tag{B.27}$$

$$G_t = \frac{(1-\delta_g)}{exp(tg_q)} G_{t-1} + Z_t \tag{B.28}$$

$$K_t = \frac{(1-\delta_k)}{exp(tg_q)} K_{t-1} + I_t \tag{B.29}$$

$$Y_t = C_t + I_t + gex_t \tag{B.30}$$

$$Y_t = A_t K_t^{1-\alpha} H_{p,t}^{\alpha} \tag{B.31}$$

$$log(A_t) = \rho_a log(A_{t-1}) + \epsilon_t^a \tag{B.32}$$

$$log(\xi_t) = (1-\rho_h)\, log(\Xi) + \rho_h log(\xi_{t-1}) + \epsilon_t^h \tag{B.33}$$

$$log(gex_t) = (1-\rho_g)\, log(Gex) + \rho_g log(gex_{t-1}) + \epsilon_t^g \tag{B.34}$$

$$log(\varepsilon_t) = (1-\rho_p)\, log(\varepsilon) + \rho_p log(\varepsilon_{t-1}) + \epsilon_t^p \tag{B.35}$$

where the exogenous shocks processes are assumed to satisfy: $\rho_a, \rho_h, \rho_g, \rho_{mk} \in [0,1)$ and $\begin{pmatrix} \varepsilon_t^a \\ \varepsilon_t^\xi \\ \varepsilon_t^g \\ \varepsilon_t^p \end{pmatrix} \sim N \left[\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} ; \begin{pmatrix} \sigma_a^2 & 0 & 0 & 0 \\ 0 & \sigma_\xi^2 & 0 & 0 \\ 0 & 0 & \sigma_g^2 & 0 \\ 0 & 0 & 0 & \sigma_p^2 \end{pmatrix} \right]$

Plus the 4 measurement equations, i.e.

$$gx_t^{obs} = exp(tc_q)\frac{X_t}{X_{t-1}} \tag{B.36}$$

for $X_t = \{C_t, Z_t, Y_t\}$, and

$$gh_t^{obs} = \frac{H_t}{H_{t-1}} \tag{B.37}$$

B.3.2 Convergence diagnostics

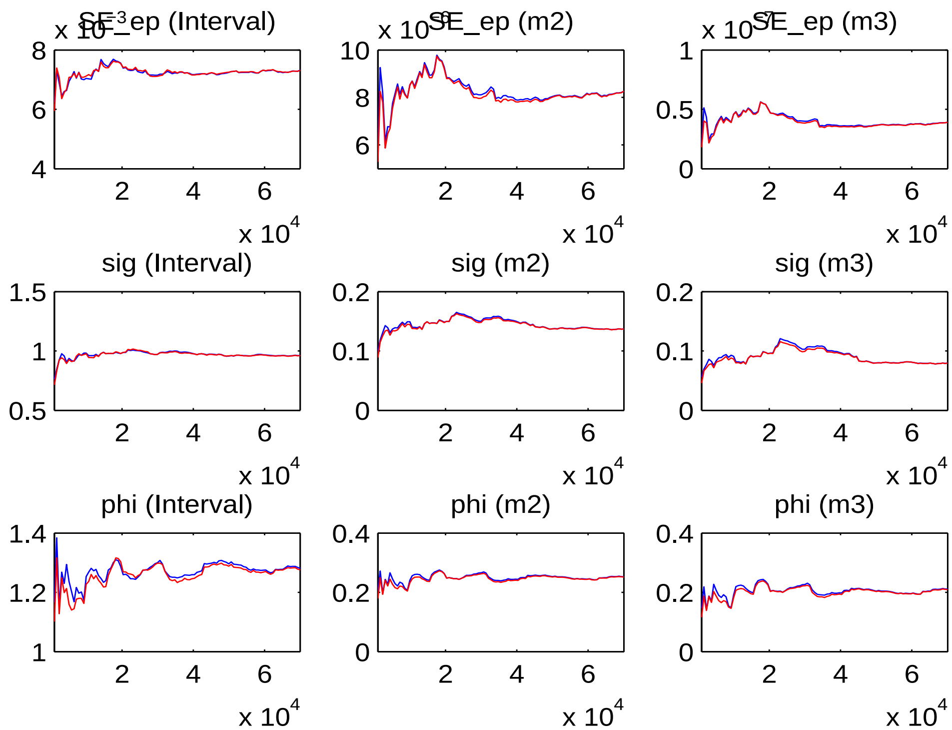

Figure B.1: **Convergence Diagnostic for selected parameters.** Cumulated means: Blue line is between chains, red is within chains.

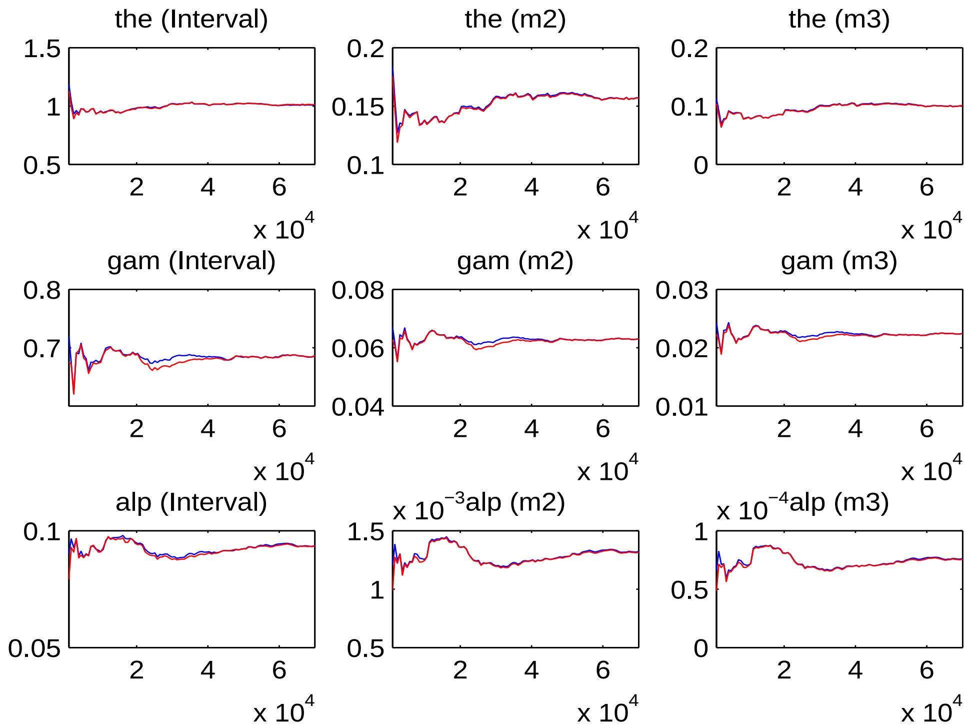

Figure B.2: **Convergence Diagnostic for selected parameters.** Cumulated means: Blue line is between chains, red is within chains.

B.3.3 Prior and Posteriors

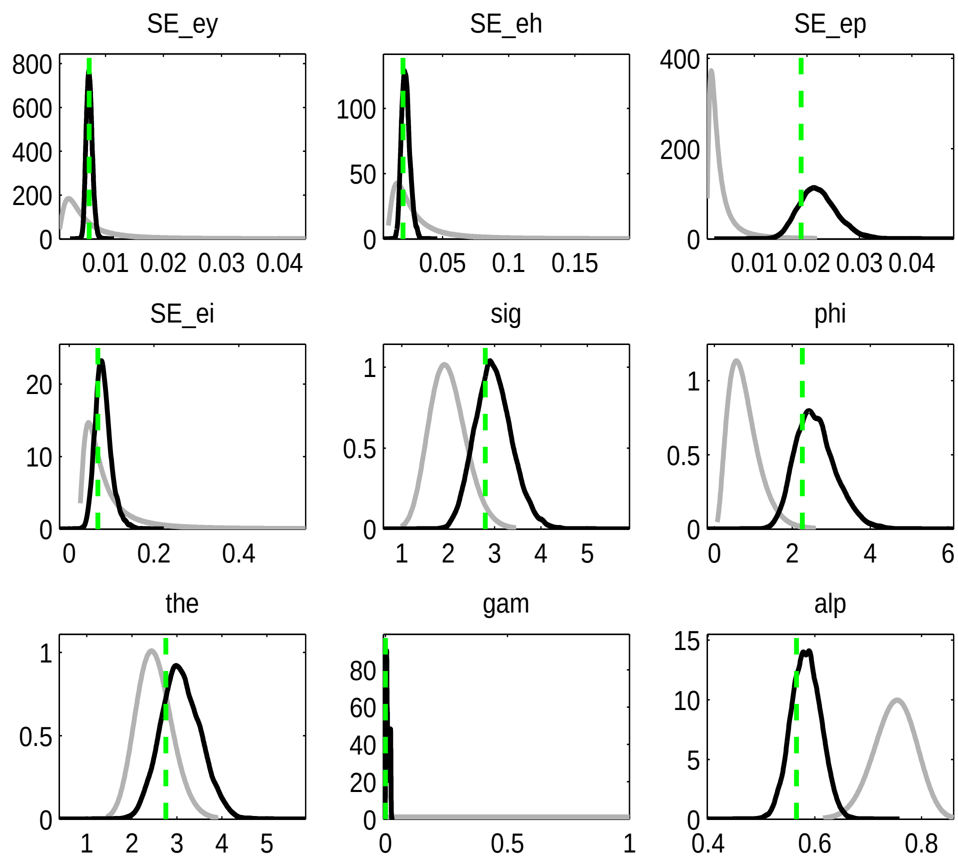

Figure B.3: **Priors And Posteriors distributions.** Priors are plotted in gray, Posteriors in black, and the mode computed at first step in dashed green.

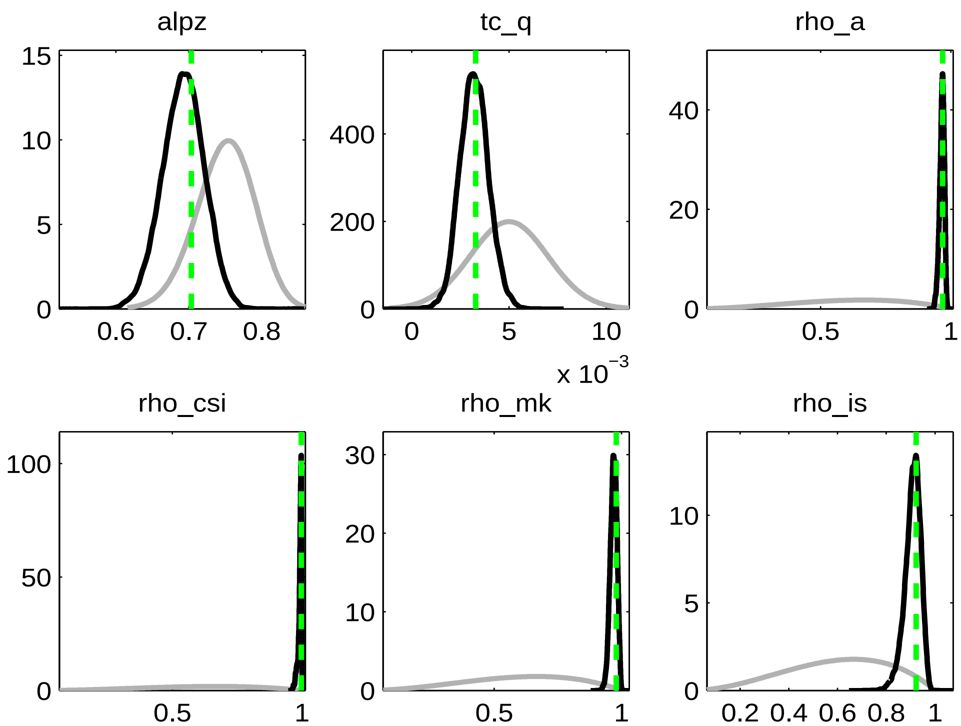

Figure B.4: **Priors And Posteriors distributions.** Priors are plotted in gray, Posteriors in black, and the mode computed at first step in dashed green.

Appendix C

Advertising and Labor Supply: a Long Run Analysis

C.1 Appendix: Data

Advertising expenditures data

- **Germany**: Investment in advertising including expenditures on salaries, media and the production of means of publicity for the period 1950-2000. Sources Rehme G., and Weiser S. (2007) table 7.

- **United Kingdom**: Annual advertising expenditures all the media. The data for 1950 to 1991 are provided to us by courtesy of Stuart Fraser. The data from 1991 to 2005 are taken from IPA (www.ipa.co.uk).

- **USA**: The data for 1948 to 1999 are obtained from an updated version of Robert J. Coens (McCann-Erikson, Inc.) original data published in Historical Statistics of the United States, Colonial Times to 1970. The data from 2000 to 2005 are obtained from the Newspaper Association of America (http://www.naa.org).

The aggregate data include spending for advertising in newspapers, magazines, radio, broadcast television, cable television, direct mail, billboards and displays, Internet, and other forms.

- **Japan**: Data from 1975 to 2005. Source DENTSU (www.dentsu.com)
- **Others OECD countries** data from 1996 to 2006. Source World Advertising Research Center (WARC). Advertising Media and Forecasts.

Macro aggregates.

- Output, Consumption and Investment are from the OECD dataset.
- Per capita hours worked and Labor productivity are from Groningen Growth and Development Centre and the Conference Board, Total Economy Database, January 2007, http://www.ggdc.net
- Tax Wedges are taken from Prescott (2004)

C.2 Appendix: Social Planner Solution

$$\max_{C_t,H_t} U(C_t, H_t) = \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t + B(G_t))^{(1-\sigma)} - 1}{1-\sigma} - \xi \frac{H_t^{1+\phi}}{1+\phi} \right]$$

s.t.

$$\begin{aligned} K_{t+1} &= K_t^{1-\alpha} (H_{p,t} A_t)^{\alpha} - F - C_t - (1-\delta_k) K_t \\ G_t &= (1-\delta_g) G_{t-1} + A_t H_{a,t}^{\alpha} \\ H_t &= H_{a,t} + H_{p,t} \end{aligned}$$

The first order conditions for an interior maximum are the followings:

$$(C_t + B(g_t))^{-\sigma} = \eta_t \tag{C.1}$$

$$\xi H_t^{\phi} = -\vartheta_t \tag{C.2}$$

$$\kappa_t \alpha \gamma A_t H_{a,t}^{\alpha-1} = -\vartheta_t \tag{C.3}$$

$$\eta_t \alpha A_t^{\alpha} \left(\frac{K_t}{H_{p,t}} \right)^{1-\alpha} = -\vartheta_t \tag{C.4}$$

$$B'(G_t)\left(C_t + B(G_t)\right)^{-\sigma} + \beta\left(1-\delta_g\right)\kappa_{t+1} = \kappa_t \tag{C.5}$$

$$\eta_t = \beta \left[(1-\alpha) A_t^{\alpha} \left(\frac{H_t}{K_t} \right)^{\alpha} + (1-\delta_k) \right] \eta_{t+1} \tag{C.6}$$

where η_t, ϑ_t and κ_t denote, respectively, the Lagrange multipliers associated with the resource constraint, the goodwill stock and the labor constraint. By solving equation (C.5) forward we find:

$$\kappa_t = \sum_{i=0}^{\infty} \left[\beta\left(1-\delta_g\right)\right]^i B'(G_{t+i})\left(C_{t+i} + B(G_{t+i})\right)^{-\sigma} < 0$$

since it is the discounted sum of negative terms. On the other hand, equations (C.3) and (C.4) imply that:

$$\frac{\kappa_t}{\eta_t} = \frac{A_t^{\alpha-1}}{\gamma} \left(\frac{K_t}{H_{a,t}} \right)^{1-\alpha} H_{a,t}^{1-\alpha}$$

is strictly positive, as well as, from equation (C.6), η_t. Therefore, the system of first order conditions does not admit a solution which is compatible with the requirement that all the variables have to be positive. It implies that the optimal solution cannot be interior.

Now suppose for the moment that the planner choices that all the hours go to the advertising-related labor, i.e $H_t = H_{a,t}$. In such circumstance, the first order condition associated with the optimal level of hours becomes:

$$\xi H_t^{\phi} = \kappa_t \alpha \gamma A_t H_t^{\alpha-1} \tag{C.7}$$

while the one refereing to optimal goodwill remains the same as (C.5). Again, the sign of κ_t is negative so that (C.7) has no solutions. Hence, we conclude that the optimal level of advertising related labor is equal to zero, or, equivalently, that the social planner choices to set advertising equal to zero each point in time.

C.3 Appendix: Balanced Growth Path

We want an equilibrium in which all the variable growth at a constant rate, with the exception of labor, markup, and interest rate which have to be constant. In this section we will show that a slight modification of the function B, together with all the assumptions we have made on the technology and utility function will guaranty that such an equilibrium exists. By rewriting the consumer's intratemporal condition in the following way:

$$\frac{W_t}{\tilde{C}_t} = \tilde{C}_t^{\sigma-1} H_t^{\phi}$$

we see that the requirement of hours worked constant in steady state implies that two conditions have to be satisfies: i) W_t and $\tilde{C}_t$ has to growth at the same rate; ii) the parameter σ has to be equal to one.

In what follows, we will check what are the conditions such that the first requirement is satisfied. In particular, we will concentrate in finding a functional form for B which assures that a balanced growth path equilibrium exists. With this aim in mind, note that the assumption of Cobb-Douglas (gross) production function implies that in the balanced growth equilibrium the wage and technology have to growth at the same rate γ_a. Therefore, condition i) is satisfies if also $\tilde{C}_t$ growths at the same rate. Writing its growth rate as the following:

$$\frac{\tilde{C}_t}{\tilde{C}_{t-1}} = \left(\frac{C_t}{C_{t-1}} - \frac{B(G_t)}{B(G_{t-1})} \right) \frac{C_{t-1}}{\tilde{C}_{t-1}} + \frac{B(G_t)}{B(G_{t-1})}$$

reveals that it is the case if both C_t and $B(G_t)$ growth at the rate γ_a. While the former is guaranteed from the supply side of the model, the latter strictly depends on the functional form of B, and eventually from the goodwill stock. However, over the balance growth path and again for the property of the production function, one can show that both advertising, and goodwill growth at the rate γ_a. Thus, rewriting the function B as:

$$B(G_t) = \frac{A_t}{1 + \theta \frac{G_t}{A_t}}$$

it is enough to guaranty that it growths at the rate γ_a.

It is worth noting that the same assumption guaranties that also markup is constant in steady state. In fact, from equation (3.6) follows that its value depends on the ratio $\left(\frac{B(G_t)}{y_t}\right)$, which in turn is constant in steady state. Moreover, it is easy to show that a constant markup satisfies the firms' optimal advertising policy, so that all the optimality condition hold over the balanced growth equilibrium. All the other requirement, such as a constant interest rate, follows from previous results together with the assumption we have made on instantaneous utility function.

C.4 Appendix: Steady State

In this section we derive the main relationships characterizing the steady state equilibrium. Using equation (3.3), we see that the steady state value for the rental rate is defined as the following:

$$R = \frac{1 - \beta(1 - \delta_k)}{\beta}$$

Accordingly, in the long run advertising does not affect the interest rate. On the other hand, equation (3.2) reveals that long run markup and the wage can be, respectively, expressed as:

$$\mu = \frac{1 - \alpha}{R} \left(\frac{H_p}{K}\right)^{\alpha}$$

$$W = R\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{K}{H_p}\right) \tag{C.8}$$

Consequently, understanding what is the effect of advertising on both wage and markup is equivalent to study how advertising affects the ratio of production related labor to capital. Indeed, the previous expression clearly indicate that the equilibrium markup increases with that ratio, while the wage rate decreases.

To find the equilibrium value of output, recall that in equilibrium aggregate output is defined as the following:

$$Y = K^{1-\alpha}H_p^{\alpha} - F$$

while the no entry condition implies that the fixed cost is of the form:

$$F = \left(1 - \frac{1}{\mu}\right)H_p^{\alpha}K^{1-\alpha} - WH_a$$

Combining the previous results together and using the definition of wage (C.8), the market clearing condition in the labor market, and the interest rate yields

$$Y = \left(\frac{R}{1-\alpha}\right)\left[H - (1-\alpha)H_a\right]\frac{K}{H_p} \tag{C.9}$$

An expression for the equilibrium level of investment can be find using the low of motion for the capital stock. Accordingly:

$$I = \delta_k\left(\frac{K}{H_p}\right)(H - H_a) \tag{C.10}$$

Finally, according with the goods market clearing condition, the equilibrium level of consumption can be expressed as the following:

$$C = \left(\frac{K}{H_p}\right)\left\{\left[\frac{1-\beta(1-\alpha\delta_k)}{\beta(1-\alpha)}\right]H - \frac{1-\beta}{\beta}H_a\right\} \tag{C.11}$$

The long run version of the intratemporal condition is obtained using equation (3.5) together with the definition of wage and equation (3.6) as:

$$\xi H^{\phi} = R\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{K}{H_p}\right)(C + B(G))^{-\sigma} \tag{C.12}$$

where C is defined as in equation (C.11).

Combining together the goodwill law of motion with the advertising production function we get the long run goodwill stock as a function of related advertising labor:

$$G = \gamma \frac{H_a^{\alpha}}{\delta_g} \tag{C.13}$$

The clearing in the labor market requires:

$$H = H_a + H_p \tag{C.14}$$

The equilibrium relationship between advertising and the ratio $\frac{H_p}{K}$ can be derived using equation (3.7). After bit of algebra, the equation can be equivalently written as the following

$$-\gamma \frac{B'(g)}{H_a^{1-\alpha}} = c_1 \frac{\mu}{\mu - 1} \frac{W}{\alpha} \tag{C.15}$$

where $c_1 = 1 - \beta \left(1 - \delta_g\right)$ is a positive constant. Using the long run markup that we have derived before, it easy to show that the following expression holds:

$$\frac{\mu}{\mu - 1} = \frac{\left(\frac{1-\alpha}{r}\right)\left(\frac{H_p}{K}\right)^{\alpha}}{\left(\frac{1-\alpha}{r}\right)\left(\frac{H_p}{K}\right)^{\alpha} - 1}$$

Finally, substituting the above result in equation (C.15), and using the definition of wage in the resulting equation yields:

$$-\gamma \frac{B'(g)}{H_a^{1-\alpha}} = c_1 \frac{\left(\frac{H_p}{K}\right)^{\alpha}}{\left(\frac{1-\alpha}{r}\right)\left(\frac{H_p}{K}\right)^{\alpha} - 1} \left(\frac{1}{\frac{H_p}{K}}\right) \tag{C.16}$$

The optimal price policy implies that the price elasticity can be rewritten as the following:

$$\varepsilon \left(1 + \frac{B(G)}{Y}\right) = \frac{\mu}{\mu - 1}$$

or equivalently,

$$\varepsilon \left(1 + \frac{B(G)}{Y}\right) = \frac{\left(\frac{1-\alpha}{r}\right)\left(\frac{H_p}{K}\right)^{\alpha}}{\left(\frac{1-\alpha}{r}\right)\left(\frac{H_p}{K}\right)^{\alpha} - 1}$$

Using the previous equation to solve for output yields:

$$Y = \varepsilon B(G) \left[\frac{\frac{H_p}{K}^{\alpha} - \frac{R}{1-\alpha}}{(1-\varepsilon)\frac{H_p}{K}^{\alpha} + \varepsilon\left(\frac{R}{1-\alpha}\right)} \right]$$

Finally, to find an expression for the equilibrium level of hours worked, substitute equation (C.9) into the previous one, and solve for H:

$$H = (1-\alpha) H_a + \varepsilon B(G) \left\{ \frac{\left(\frac{H_p}{K}\right) \left[\left(\frac{1-\alpha}{R}\right) \left(\frac{H_p}{K}\right)^{\alpha} - 1 \right]}{(1-\varepsilon)\frac{H_p}{K}^{\alpha} + \varepsilon\left(\frac{R}{1-\alpha}\right)} \right\} \tag{C.17}$$

Equations (C.8)-(C.17) define a system of non linear equations that fully describes the steady state of the model economy.

C.4.1 Proof of proposition 3

Using (C.9) and (C.10), it is easy to check that in the long run equilibrium the investment is of the form:

$$\frac{I}{Y} = \frac{\delta_k(1-\alpha)}{R} \left[\frac{H - H_a}{H - (1-\alpha)H_a} \right]$$

Accordingly, it is composed of the product of two terms. Not incidentally, the first of them is equal to the investment share in the absence of advertising expenditures, while the second is a positive term always lower than one. As a consequence, no matter whether labor supply is endogenous or not, the investment share always decreases with advertising. Equivalently, the consumption share increases with advertising.

C.5 Appendix: Exogenous Labor Supply

When it is assumed that the consumer supplies inelastically $\bar{H}$ of labor services per unit of time, the steady state is still fully described by equation (C.8)-(C.14) with exception of the infratemporal condition (C.12) which vanishes, and of the variable H which has to be substituted with $\bar{H}$.

C.5.1 Proof of proposition 4

Let $V_t = \left[C_t,\ Y_t,\ I_t,\ W_t, H_{p,t}\ G_t, H_{a,t},\ \frac{H_{a,t}}{K}\right]$ denotes the endogenous variables, we can conveniently summarize equations (C.8)-(C.14) by introducing a map $V : R^2 \rightarrow R^8$ such that $V_t = V\left(H_{a,t}, \frac{H_{p,t}}{K_t}\right)$. This is true because all the endogenous variables have been expressed as a function of H_a and $\frac{H_p}{K}$. Now, the only equilibrium not considered in the map V are equations (C.16) and (C.12). This two expressions determine the steady state values of H_a and $\frac{H_p}{K}$. Consequently, a steady state can be defined as any sequences $\{V_t\}$ such that $V_t = V(H_a, \frac{H_p}{K})\ \forall\ t$ satisfy

$$-\gamma\frac{B'(G)}{H_a^{1-\alpha}} = c_1\frac{\left(\frac{H_p}{K}\right)^{\alpha}}{\left(\frac{1-\alpha}{r}\right)\left(\frac{H_p}{K}\right)^{\alpha} - 1}\left(\frac{1}{\frac{H_p}{K}}\right)$$

$$\bar{H} = (1-\alpha)\, H_a + \varepsilon B(G)\left\{\frac{\left(\frac{H_p}{K}\right)\left[\left(\frac{1-\alpha}{R}\right)\left(\frac{H_p}{K}\right)^{\alpha} - 1\right]}{(1-\varepsilon)\,\frac{H_p}{K}^{\alpha} + \varepsilon\left(\frac{R}{1-\alpha}\right)}\right\}$$

as stated in the first part of the proposition.

To prove the unicity of the equilibrium consider first equation (C.16). Such equation uniquely determines the ratio of production related labor to capital as a function of advertising. To see it, it is enough to note that for any given γ both sides of the equation are monotonically decreasing function of their own arguments. Unfortunately, it is not possible to find a explicit function relating the ratio with adverting, but the equilibrium linkage between them can be easily studied from the previous expression. Our goal, in particular, is to understand how $\frac{H_p}{K}$ moves with advertising for any given γ.

Let $f_1(H_a)$ and $f_2(\frac{H_p}{K})$ respectively denote the l.h.s and r.h.s of the equation (C.16). As already noted, it is easy to see that both functions are decreasing in its own argument. Then, applied the implicit function theorem to equation (C.16) we get:

$$\frac{d\frac{H_p}{K}}{dH_a} = \frac{f_1'(H_a)}{f_2'(\frac{H_p}{K})} \; > \; 0$$

Thus, the ratio of production related labor to capital can be implicitly expressed as an increasing function of advertising. Using equation (C.17) to solve for H_a yields:

$$H_a = \frac{1}{1-\alpha}\left\{\bar{H} - \varepsilon B(G)\left\{\frac{\left(\frac{H_p}{K}\right)\left[\left(\frac{1-\alpha}{R}\right)\left(\frac{H_p}{K}\right)^{\alpha} - 1\right]}{(1-\varepsilon)\frac{H_p}{K}^{\alpha} + \varepsilon\left(\frac{R}{1-\alpha}\right)}\right\}\right\} \equiv \frac{1}{1-\alpha}\left(\bar{H} - F(H_a)\right) \tag{C.18}$$

Thus, given the exogeneity of hours worked, a sufficient condition for uniqueness of equilibrium, whenever it exists, is that $F(H_a)$ is a strictly increasing function. This is not obvious since the function is composed by the product of two terms that potentially move with advertising in opposite directions. However, it is possible to rearrange the expression in a more useful way. To do it, notice that, by definition, equation (**??**) implies

$$B^{'}(g) = -\frac{\theta}{(1+\theta g)^2} \equiv -\theta B(g)^2$$

Combining the previous result with equation (C.16) yields

$$B(g) = \frac{1}{\gamma\theta\left(\frac{B(g)}{c_1 H_a^{1-\alpha}}\right)}\left[\frac{\left(\frac{H_p}{K}\right)^{\alpha}}{\left(\frac{1-\alpha}{r}\right)\left(\frac{H_p}{K}\right)^{\alpha} - 1}\left(\frac{1}{\frac{H_p}{K}}\right)\right]$$

Now, using the previous equations, the function $F(H_a)$ can be rewritten as the following:

$$F(H_a) = \frac{1}{\gamma\theta\left(\frac{B(g)}{c_1 H_a^{1-\alpha}}\right)}\left\{\frac{\varepsilon\left(\frac{H_p}{K}\right)^{\alpha}}{\left[(1-\varepsilon)\left(\frac{H_p}{K}\right)^{\alpha} + \varepsilon\left(\frac{r}{1-\alpha}\right)\right]}\right\}$$

Accordingly, in order to prove that $F(H_a)$ is an increasing function is enough to show that the term $(1-\varepsilon)\left(\frac{H_p}{K}\right)^{\alpha} + \varepsilon\left(\frac{r}{1-\alpha}\right)$ is always positive. Indeed, in such a circumstance $F(H_a)$ is increasing since it is the product of two positive and increasing functions. Thus, by notice that

$$(1-\varepsilon)\left(\frac{H_p}{K}\right)^{\alpha}+\varepsilon\left(\frac{r}{1-\alpha}\right)>0 \Leftrightarrow \frac{\varepsilon}{\varepsilon-1} > \mu$$

it is always guaranty in equilibrium, we conclude that $F(H_a)$ is an increasing function of advertising related labor, or, equivalently, that the equilibrium, whenever it exists, is unique.