Universitat Pompeu Fabra

Department of Economics and Business

Doctoral Thesis

Quantitative Effects of Heterogeneity in Dynamic Macroeconomics

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June 1999

Contents

Acknowlegdements

I am grateful first and foremost to Serguei Maliar. Every-day work with him was both productive and enjoyable. His uncompromising support was indispensable.

I also have enormous debt to my supervisor, Morten Ravn, for the attention he paid to my research. His helpful comments as well as his friendly attitude were invaluable.

I wish to thank to Fabio Canova, Rosa M. Fernandez, Jordi Gali, Timothy Kehoe, Antonio Ladron, Albert Marcet, Jane Marrinan, Nicola Pavoni, Vincenzo Quadrini, Victor Rios-Rull, Xavier Sala-i-Martin, whose suggestions led to substantial improvements in several places.

Chapter 1

Introduction

The thesis studies quantitative implications of the real business cycle models with heterogeneous agents. The questions I ask are: First, Can the studied models not only replicate the aggregate time series facts but also the distributions of individual quantities such as consumption, hours worked, income, wealth observed in the micro data? Second, Does incorporating heterogeneity enhance the aggregate performance of the representative consumer models? To simplify the characterization of the equilibria in the models, I use results from aggregation theory.

The thesis is composed of three chapters, each of which can be read independently. All chapters are joint with Serguei Maliar.

The first chapter analyzes the predictions of a heterogeneous agents version of the model by Kydland and Prescott (1982). I calibrate and solve the model with eight heterogeneous agents to match the aggregate quantities and the distributions of productivity and wealth in the U.S. economy. I find that the model can generate the distributions of consumption and working hours which are consistent with patters observed in the data. I show that incorporating the heterogeneity helps to improve on the model's predictions with respect to labor markets. In particular, unlike a similar representative agent setup, the heterogeneous model can account for the Dunlop-Tarshis observation of weak correlation between productivity and hours worked.

The second chapter constructs a heterogeneous agents version of the indivisible labor model by Hansen (1985) with search and home production. I calibrate and solve the model with five agents to replicate aggregate quantities and differences in productivity across agents in the U.S. economy. The model does reasonably well at reproducing cyclical behavior of the macroeconomic aggregates. At the individual level, it can account for the stylized facts that more productive individuals (i) enjoy a higher employment rate, (ii) have a lower volatility of employment, (iii) spend less time working at home. It is important to emphasize that none of the heterogeneous models with home production existing in the literature has been able to explain fact (ii).

The third chapter studies distributive dynamics of wealth and income in a heterogeneous agents version of Kydland and Prescott's (1982) model. I show that under the assumptions of complete markets and Cobb-Douglas preferences, the evolution of wealth and income distributions in the model can be described in terms of aggregate variables and time-invariant agent-specific parameters. This allows me to characterize explicitly the behavior of such inequality measures as the coefficient of variation of wealth and income over the business cycle. I test the model's implications by using the time series on the U.K. economy. The predictions are in agreement with the data.

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Chapter 2

Heterogeneity in Capital and Skills in a Neoclassical Stochastic Growth Model

Joint with Serguei Maliar

2.1 Introduction

This paper studies a complete market heterogeneous agents version of a standard neoclassical model by Kydland and Prescott (1982). We assume that instead of many identical agents the economy is populated by a number of agents who differ along two dimensions: initial endowments and non-acquired skills. The first question we ask is: Can the heterogeneous model replicate the distributions of individual quantities observed in the cross-sectional data such as consumption and hours worked?

The representative agent version of Kydland and Prescott's (1982) model has been extensively studied in the literature. Although it is successful at matching most of the features of aggregate fluctuations of the real economies, this model has serious drawbacks regarding the labor markets. In particular, it predicts that the correlation between hours worked and labor productivity is in excess of 0.9, while the actual correlation is close to zero. The latter empirical fact is often referred to in the literature as the "Dunlop-Tarshis observation" after J. Dunlop (1938) and L. Tarshis (1939). The capability to account for this observation "continues to play a central role in assessing the empirical plausibility of different business-cycle models" (Christiano and Eichenbaum, 1992). One can reasonably hope that neglecting heterogeneity contributes to the above shortcoming, as there exists an empirical evidence that aggregate measures of fluctuations are affected by cyclical variation in the average skill level, e.g., Hansen (1993), Kydland and Prescott (1993). Therefore, the second question we address is: Does incorporating heterogeneity help to improve the aggregate predictions of the model?

The two questions we ask have been addressed in previous studies. Kydland (1984) analyzes a real business cycle model with two types of agents who are heterogeneous in skills; he finds that allowing for the heterogeneity has a minor positive effect on the aggregate predictions. Garcia-Milá, Marcet and Ventura (1995) (further, GMV) consider a similar two-agents setup and reach a different conclusion, namely, that heterogeneity deteriorates the aggregate performance dramatically: the heterogeneous agents version not only fails to account for the correlation between productivity and hours worked but it also counterfactually predicts extremely low volatility of hours worked, a negative correlation between hours worked and output, etc. They also find that the model generates the failure at the individual level: it counterfactually predicts that rich (skilled) individuals spend less time working in the market compared to the poor (unskilled). The inability of the heterogeneous model to generate the appropriate predictions is referred to in GMV (1995) as a puzzle.

The studies by Kydland (1984) and GMV (1995) differ in several dimensions. In particular, the first paper assumes the preferences of the Cobb-Douglas type, constructs two heterogeneous agents by splitting the panel data into two educational groups and calibrates the model to reproduce the groups' average hours worked. On the other hand, the second considers the preferences of the addilog type, distinguishes two agents according to their level of wealth (wealth to wage ratio) and calibrates the parameters to match the groups' wealth holdings. A significant discrepancy in the findings of Kydland (1984) and GMV (1995) indicates that some of the above assumptions play a determinant role for the model's implications. Also, GMV (1995) argues that the model's predictions are likely to be sensitive to the number of the heterogeneous agents introduced in the model. Given these differences in results and the implied uncertainty about the effects of heterogeneous agents, further study of the model is of interest.

Unlike the previous literature, we characterize the equilibrium in the heterogeneous economy by using aggregation theory. Aggregation allows us to achieve two objectives. First, it simplifies qualitative analysis and makes it possible to describe in a simple way the relation between distributions and aggregate dynamics. In particular, aggregation results allow us to gain intuition into the effect of heterogeneity on labor markets, to understand the origin of the puzzle in GMV (1995) and to elaborate a modification to the calibration procedure which resolves the problems encountered by these authors. Further, aggregation simplifies substantially the numerical analysis and enables us to extend the model to include any number of heterogeneous agents without having the corresponding increase in computational cost. It is worth noting that the concept of aggregation used in the paper is different from the standard one by Gorman (1953). The latter requires that the preferences are quasi-homothetic. We consider two examples of preferences, the Cobb-Douglas and the addilog. The addilog preferences are not quasi-homothetic; they lead to demand which is not linear in wealth and imply aggregate dynamics which depend on the joint distribution of capital and skills.

In the paper, we present several analytical results of interest. We show that the model's time-series performance is directly linked to its distributive implications. In such a way, we establish that in order to generate the appropriate time series predictions on labor markets, it is necessary that the model is able to account for the empirical observation that hours worked by the agents increase in the level of skills (wealth). The numerical results reported in GMV (1995) suggest that such regularity is difficult to generate in the model. We provide an analytical argument in support of this finding: we show that assumptions about preferences which are standard for macro literature are inconsistent with cross-sectional observations. Specifically, under the Cobb-Douglas utility, the heterogeneous agents version is never able to reproduce the joint distribution of capital, skills, consumption and hours worked observed in the data. Under the addilog utility, the model's implications are primarily determined by the value of the intertemporal elasticity of substitution for consumption; however, under the standard range of values for this parameter (smaller than or equal to one) the model fails to account for the distributions as well. Our analysis has also a positive implication, namely, we show that if the intertemporal elasticity of substitution for consumption in the addilog utility is higher than one, then it is likely that the heterogeneous agents version is successful in matching both time series and distributions.

The results from simulations confirm this conjecture. We calibrate and solve the model with eight heterogeneous consumers to match the aggregate quantities and the distributions of productivity and endowments in the U.S. economy. We find that if the utility parameters are chosen so that the distributions of consumption and hours worked in the model are consistent with patterns observed in the data, heterogeneity improves substantially the model's performance at the aggregate level. In particular, the heterogeneous agents version can account for several time-series facts which cannot be reconciled within a similar representative agent setup; for example, the weak correlation between productivity and hours worked. Our results are in contrast with conclusion of most papers on heterogeneity, which do not find a significant difference in the predictions of the heterogeneous and the representative agent versions of the studied models, e.g., Cho (1995), Krusell and Smith (1995), Rios-Rull (1996), etc.

The paper is organized as follows. Section 2 contains the description of the model economy and derives the results from aggregation. Section 3 analyzes qualitative implications of the model. Section 4 describes the calibration procedure. Section 5 discusses the numerical predictions. Finally, Section 6 concludes.

2.2 The model

We start by analyzing a competitive equilibrium in an economy populated by a set of utility-maximizing heterogeneous consumers and a single profit-maximizing firm. Subsequently, we construct a planner's problem generating the optimal allocation which is identical to the competitive equilibrium in the decentralized economy. Finally, we show how to simplify the characterization of the equilibrium in the model by using aggregation theory.

2.2.1 The economy

The consumer side of the economy consists of a set of agents S. The measure of agent s in the set S is denoted by $d\omega^s$, where $\int_S d\omega^s = 1$. The agents are heterogeneous in skills and initial endowments. The skills are intrinsic, permanent characteristics of the agents. We denote the skills of an agent

$s \in S$ by e^s, assume $e^s > 0$ for $\forall s \in S$ and normalize the aggregate skills to one, $\int_S e^s d\omega^s = 1$. The initial endowment of the individual $s \in S$ is denoted by κ_0^s. The timing is discrete, $t \in T$, where $T = 0, 1, ...$

An infinitely-lived agent $s \in S$ seeks to maximize the expected sum of momentary utilities $u(c_t^s, l_t^s)$, discounted at the rate $\delta \in (0,1)$, by choosing a path for consumption, c_t^s, and leisure, l_t^s. The utility function $u(\cdot)$ is continuously differentiable, strictly increasing in both arguments, and strictly concave. In period t the agent owns capital stock k_t^s and rents it to the firm at the rental price r_t. Also, he supplies to the firm n_t^s units of labor in exchange on income $n_t^s e^s w_t$, where w_t is the wage paid for one unit of efficiency labor. The total time endowment of the agent is normalized to one, $n_t^s + l_t^s = 1$. Capital depreciates at the rate $d \in (0, 1]$. When making the investment decision, the agent faces uncertainty about the future returns on capital. We assume that markets are complete: the agent can insure himself against uncertainty by trading state contingent claims, $\{m_t^s(\theta)\}_{\theta \in \Theta}$, where Θ denotes the set of all possible realizations of productivity shocks. The claim of type $\theta \in \Theta$ costs $p_t(\theta)$ in period t and pays one unit of consumption good in period $t+1$ if the state $\theta \in \Theta$ occurs and zero otherwise.

Consequently, the problem solved by agent $s \in S$

$$\max_{\left\{c_t^s, n_t^s, k_{t+1}^s, m_{t+1}^s(\theta)\right\}_{\theta \in \Theta, t \in T}} E_0 \sum_{t=0}^{\infty} \delta^t u(c_t^s, 1 - n_t^s) \tag{2.1}$$

$$\text{s.t. } c_t^s + k_{t+1}^s + \int_{\Theta} p_t(\theta)\, m_{t+1}^s(\theta)\, d\theta = (1 - d + r_t)\, k_t^s + n_t^s e^s w_t + m_t^s(\theta_t). \tag{2.2}$$

Initial holdings of capital and contingent claims, k_0^s and m_0^s, are given.

The production side of the economy consists of a single representative firm. Given the prices, r_t and w_t, the firm rents capital $\{k_t^s\}^{s \in S}$ and hires labor $\{n_t^s\}^{s \in S}$ to maximize period-by-period profits. Capital and labor inputs are given by aggregate capital in the economy and efficiency hours worked by all consumers, $k_t = \int_S k_t^s d\omega^s$ and $h_t = \int_S n_t^s e^s d\omega^s$.

Therefore, the problem of the firm is the following

$$\max_{k_t,\, h_t} \pi_t = \theta_t f(k_t, h_t) - r_t k_t - w_t h_t. \tag{2.3}$$

The aggregate technology shock θ_t follows a first order Markov process with transitional probabilities $\Pr\{\theta_{t+1} = \theta \mid \theta_t = \theta'\}_{\theta,\theta' \in \Theta}$. The value θ_0 is given. The production function $f(\cdot)$ has constant returns to scale, is strictly concave, continuously differentiable, strictly increasing with respect to both arguments and satisfies the appropriate Inada conditions.

We define initial endowment of agent $s \in S$ as the value of initial capital and security payment measured in terms of output in period $t = 0$

$$\kappa_0^s = (1 - d + r_0) k_0^s + m_0^s(\theta_0).$$

The function $\{\kappa_0^s\}^{s \in S}$ will be referred to as (initial) wealth distribution. The rest of the economy's characteristics such as the distribution of skills, the initial condition (k_0, θ_0), etc. will be summarized by the set $\Im$.

A competitive equilibrium in the economy $(2.1) - (2.3)$ is defined as a set $\Im$, a distribution of wealth $\{\kappa_0^s\}^{s \in S}$ and a sequence of contingency plans for the consumers' allocation $\left\{c_t^s, n_t^s, k_{t+1}^s, m_{t+1}^s(\theta)\right\}_{\theta \in \Theta, t \in T}^{s \in S}$, for the firm's allocation $\{k_t, h_t\}_{t \in T}$ and for the prices $\{r_t, w_t, p_t(\theta)\}_{\theta \in \Theta, t \in T}$ such that given the prices, the sequence of plans for the consumers' allocation solve the utility maximization problem $(2.1), (2.2)$ of each agent $s \in S$, the sequence of plans for the firm's allocation leads to zero profit solution to (2.3) for $\forall t \in T$ and all markets clear. Moreover, the equilibrium plans are to be such that $c_t^s \geq 0$, and $1 \geq n_t^s \geq 0$ for $\forall s \in S$, $t \in T$ and $w_t, r_t, k_t \geq 0$ for $\forall t \in T$. We assume that the equilibrium exists, is interior and is unique. The assumption of the uniqueness refers only to the equilibrium plans for prices and for allocations other than the individual holdings of capital and state contingent claims.

The last two are not uniquely defined because the number of assets traded exceeds the number of the economy's states.

2.2.2 The planner's problem

To simplify the analysis of the equilibrium, we exploit two fundamental theorems of welfare economics. Specifically, consider an otherwise identical economy except that it is ruled by a planner who maximizes the weighted sum of the agents' preferences subject to the economy's resource constraint

$$\max_{\{c_t^s, n_t^s, k_{t+1}\}_{t\in T}^{s\in S}} E_0 \sum_{t=0}^{\infty} \delta^t \int_S \lambda^s u(c_t^s, 1 - n_t^s) d\omega^s \tag{2.4}$$

$$\text{s.t.} \quad c_t + k_{t+1} = (1-d)\, k_t + \theta_t f\,(k_t, h_t)\,. \tag{2.5}$$

where $c_t = \int_S c_t^s d\omega^s$ and $h_t = \int_S n_t^s e^s d\omega^s$ are aggregate consumption and aggregate efficiency hours and $\{\lambda^s\}^{s\in S}$ is a set of welfare weights.

In addition, the planner's choice is restricted to satisfy the expected lifetime budget constraint of each agent $s \in S$

$$E_0 \left[\sum_{\tau=0}^{\infty} \delta^\tau \frac{u_1(c_\tau^s, n_\tau^s)}{u_1(c_0^s, n_0^s)} \left(c_\tau^s - n_\tau^s e^s w_\tau \right) \right] = \kappa_0^s. \tag{2.6}$$

where $\{\kappa_0^s\}^{s\in S}$ is the distribution of wealth and $w_\tau = \theta_\tau \partial f\,(k_\tau, h_\tau) / \partial h_\tau$ is the equilibrium wage in decentralized economy $(2.1) - (2.3)$.

A solution to problem $(2.4) - (2.6)$ is defined as a set $\Im$, a distribution of wealth $\{\kappa_0^s\}^{s\in S}$, a set of welfare weights $\{\lambda^s\}^{s\in S}$ and a sequence of contingency plans for the consumers' allocation $\{c_t^s, n_t^s\}_{t\in T}^{s\in S}$ and for aggregate allocation $\{c_t, h_t, k_t\}_{t\in T}$ such that, given the welfare weights, the sequence of plans for the allocations solves problem $(2.4), (2.5)$ and satisfies the constraint (2.6) of each agent $s \in S$. Moreover, the weights are strictly positive, $\lambda^s > 0$ for $\forall s \in S$, and the allocations are such that $c_t^s \geq 0$, and $1 \geq n_t^s \geq 0$ for $\forall s \in S$, $t \in T$ and $k_t \geq 0$ for $\forall t \in T$.

Proposition 1 *If a competitive equilibrium in the decentralized economy* $(2.1) - (2.3)$ *exists and is interior and if the equilibrium sequence of contingency plans for* $\{c_t^s, n_t^s\}_{t \in T}^{s \in S}$ *and for* $\{c_t, h_t, k_{t+1}\}_{t \in T}$ *is unique, then such a sequence is uniquely determined by* $(2.4) - (2.6)$.

Proof. The fact that the equilibrium allocation in the decentralized economy is a solution to $(2.4), (2.5)$ and satisfies the constraint (2.6) for each $s \in S$ follows by the first welfare theorem and the results of appendix A respectively. Therefore, $(2.4) - (2.6)$ are necessary for the equilibrium. The sufficiency follows by the second welfare theorem and by the fact that for any set of weights for which a solution to $(2.4), (2.5)$ exists, is interior and is unique, the distribution of wealth $\{\kappa_0^s\}^{s \in S}$ is uniquely defined by the constraints (2.6). ||

Let us comment on this result. According to the first welfare theorem, under complete markets and in the absence of externalities and other distortions, a competitive equilibrium is Pareto optimal and therefore, can be calculated as a solution to planner's problem $(2.4), (2.5)$. However, to make use of this result, it is necessary to find a set of weights which corresponds to a given distribution of wealth. To identify such weights, we exploit the transversality conditions or, equivalently, the expected life-time budget constraints of the agents in the decentralized economy. Constraints (2.6) give us S restrictions which are sufficient to identify S unknown welfare weights.

In the decentralized economy, the agents's life-time budget constraint allows for a simple economic interpretation. Specifically, it restricts the value of commodities consumed by the agent over the life time to be equal to his endowment of wealth and the value of his life-time labor income.

The fact that welfare weights are endogenous to the model complicates substantially the analysis of the equilibrium. An example of numerical algorithm which solves for weights is described by GMV (1995). It is roughly as follows: fix the weights to some values, find a solution to planner's problem (2.4), (2.5), calculate the left side of the constraints (2.6) and compare the result to the given distribution of wealth; subsequently, iterate on weights until a solution to problem (2.4), (2.5), which is consistent with the constraints (2.6), is found. In fact, this algorithm is costly as each iteration on weights requires finding a solution to the planner's problem and evaluating the expectations in the life-time budget constraint. Moreover, the cost depends on the number of agents and increases significantly if there are more than two agents. Therefore, we will not resort to simulations from the outset but will begin with exploring the possibility to simplify the description of the equilibrium.

2.2.3 Aggregation

In this section, we show how to employ aggregation theory in order to simplify characterization of the equilibrium under the assumption of the Cobb-Douglas and addilog preferences. Given that both of these preferences are commonly used in macroeconomics, it is of interest to study their implications for the economy with heterogeneous consumers. Another reason for our choice is that these two types of preference are assumed in Kydland (1984) and GMV (1995) respectively. Considering both Cobb-Douglas and addilog preferences will enable us to evaluate whether the discrepancy in findings of these studies is explained by the preference choice.

Quasi-homothetic preferences

Gorman's (1953) theorem implies that if the consumers differ only in wealth and have identical quasi-homothetic preferences, then demand is linear in wealth and, as a result, at the aggregate level, the economy behaves as if there was a single representative consumer. Examples of Gorman's (1953) aggregation in neoclassical economies with a single consumption commodity are discussed in Chatterjee (1994) and Caselli and Ventura (1996).

It turns out that with heterogeneity in both wealth and skills, the aggregation result changes. Precisely, demand for physical hours worked n_t^s is not linear in wealth any longer; however, demand for efficiency hours worked $n_t^s e_s^s$ is. Consequently, the preferences of a representative consumer depend on aggregate efficiency hours worked, h_t, and not on aggregate physical hours worked, $n_t = \int_S n_t^s d\omega^s$. Rather than elaborate a strict proof of this fact, we illustrate the aggregation results by using a particular example of quasi-homothetic preferences.

Assume that the agent's momentary utility is of the Cobb-Douglas type

$$u\left(c_t^s, n_t^s\right) = \frac{\left((c_t^s)^\mu \left(1 - n_t^s\right)^{1-\mu}\right)^{1-\eta} - 1}{1-\eta}, \qquad 1 > \mu > 0, \quad \eta > 0. \tag{2.7}$$

Then, the equilibrium in the model can be described by a single-agent utility maximization problem

$$\max_{\{c_t, h_t, k_{t+1}\}_{t\in T}} E_0 \sum_{t=0}^{\infty} \delta^t \frac{\left(c_t^\mu (1-h_t)^{1-\mu}\right)^{1-\eta} - 1}{1-\eta} \quad \text{s.t.} \quad RC, \tag{2.8}$$

a set of equations which relates individual and aggregate allocations

$$c_t^s = c_t f^s, \qquad n_t^s = 1 - (1-h_t)\frac{f^s}{e^s}, \tag{2.9}$$

and a set of equations which determines the agent-specific parameters $\{f^s\}^{s\in S}$

$$f^s = \frac{\kappa_0^s + e^s E_0 \sum_{\tau=0}^{\infty} \delta^\tau \frac{u_1(c_\tau, h_\tau)}{u_1(c_0, h_0)} w_\tau}{E_0 \sum_{\tau=0}^{\infty} \delta^\tau \frac{u_1(c_\tau, h_\tau)}{u_1(c_0, h_0)} \left(c_\tau + w_\tau \left(1 - h_\tau\right)\right)}, \tag{2.10}$$

where notation RC is used to denote the economy's resource constraint (2.5) and f_s is the share of consumption of agent $s \in S$ in aggregate consumption, $f_s = (\lambda^s)^{1/\eta} (e^s)^{-\frac{(1-\mu)(1-\eta)}{\eta}} / \int_S (\lambda^s)^{1/\eta} (e^s)^{-\frac{(1-\mu)(1-\eta)}{\eta}} d\omega^s$.

Proposition 2 *Under utility* (2.7), (2.4) – (2.6) *and* (2.8) – (2.10) *are equivalent.*

Proof. See appendix B. ||

The above characterization of the equilibrium has two important advantages comparing to (2.4) – (2.6). First, the equilibrium relations between individual and aggregate variables are defined explicitly. This fact will make it possible to deduce some qualitative properties of the equilibrium without calculating the exact numerical solution. Second, the quantitative analysis can be carried out without computationally costly iteration on weights. Precisely, the equilibrium can be computed in three steps: solve model (2.8), compute the shares from (2.10) and restore the agents' consumption and hours worked by using (2.9). The cost of calculating a numerical solution by using this algorithm does not depend on the number of heterogeneous agents and is comparable to that of finding the equilibrium in the associated representative agent model.

Addilog preferences

In fact, a representative consumer can be constructed under some preferences which are not quasi-homothetic, though in this case demand will not be linear in wealth. An example of preferences which lead to such "imperfect" aggregation is the addilog utility. This type of preferences is introduced in the literature by Houthakker (1960). The fact that addilog preferences are consistent with aggregation is pointed out by Shafer (1977). Atkeson and Ogaki (1996) consider a neoclassical economy with two consumption commodities and exogenous production and exploit the property of aggregation for estimating the parameters in the addilog utility function.

Assume that the agents have momentary utility of the addilog type

$$u\left(c_t^s, n_t^s\right) = \frac{\left(c_t^s\right)^{1-\gamma} - 1}{1-\gamma} + B \cdot \frac{\left(1-n_t^s\right)^{1-\sigma} - 1}{1-\sigma}, \qquad \gamma, \sigma, B > 0. \tag{2.11}$$

Then, the equilibrium in the model can be characterized by a utility maximization problem of a single-agent

$$\max_{\{c_t, h_t, k_{t+1}\}_{t \in T}} \quad E_0 \sum_{t=0}^{\infty} \delta^t \left\{ \frac{c_t^{1-\gamma} - 1}{1-\gamma} + X \cdot B \frac{\left(1-h_t\right)^{1-\sigma} - 1}{1-\sigma} \right\} \quad \text{s.t.} \quad RC, \tag{2.12}$$

a condition which identifies the parameter X

$$X = \left(\int_S \left(e^s\right)^{1-1/\sigma} \left(f^s\right)^{\gamma/\sigma} d\omega^s \right)^{\sigma}, \tag{2.13}$$

a set of equations which relates individual and aggregate allocations

$$c_t^s = c_t f^s, \qquad n_t^s = 1 - \left(1 - h_t\right) X^{-1/\sigma} \left(e^s\right)^{-1/\sigma} \left(f^s\right)^{\gamma/\sigma}, \tag{2.14}$$

and conditions which determine the agents' shares of consumption $\{f^s\}^{s \in S}$

$$E_0 \sum_{t=0}^{\infty} \delta^{\tau} \frac{u_1\left(c_\tau, h_\tau\right)}{u_1\left(c_0, h_0\right)} \left[c_\tau f^s - w_\tau e^s \left(1 - \left(1 - h_\tau\right) X^{-1/\sigma} \left(e^s\right)^{-1/\sigma} \left(f^s\right)^{\gamma/\sigma}\right) \right] = \kappa_0^s, \tag{2.15}$$

where $f^s = \left(\lambda^s\right)^{1/\gamma} / \int_S \left(\lambda^s\right)^{1/\gamma} d\omega^s$.

Proposition 3 *Under utility* (2.11), (2.4) – (2.6) *and* (2.12) – (2.15) *are equivalent.*

Proof. See appendix B. ||

Here, as in the case of quasi-homothetic preferences, the preferences of the representative consumer depend on aggregate efficiency hours worked and the equilibrium relation between individual and aggregate quantities is explicit. There are two differences, however. First, the utility of the representative consumer depends on the joint distributions of wealth and skills through the parameter X and, second, constraint (2.15) does not allow for a closed form solution with respect to the consumption share, unless $\gamma = \sigma$.

As the utility (2.12) depends on the endogenous parameter X, a numerical solution cannot be computed without iterations. An example of algorithm which calculates the equilibrium is as follows: fix X to some value, solve model (2.12), compute the shares from (2.15) and restore X according to (2.13); iterate on X until a fixed-point solution is found. Note that, unlike the algorithm iterating on welfare weights, the above procedure iterates on a single parameter X independently of the number of heterogeneous agents. As a result, with any number of agents, the cost of calculating a numerical solution by using this algorithm is roughly equal to that of finding the equilibrium in the two-agent model by using iterating on welfare weights.

2.3 Model's implications

This section is dedicated to qualitative analysis of the model's predictions. Specifically, we study the relation between the implications of the model at the individual and aggregate levels and identify the factors which play a determinant role for the properties of the equilibrium.

2.3.1 Aggregate dynamics

Let us first discuss a potential effect of the parameter X which appears in the utility of the representative consumer under the assumption of the addilog utility. According to (2.12) this parameter premultiplies the utility parameter B and, therefore, has the same effect on the properties of the model as a variation in B. Normally, the parameter B is chosen to match average hours worked in the real economy. The estimate of average hours worked, however, largely depends on whether the time endowment is measured in terms of "real" or "discretionary" time (discretionary time is total time minus time spent on personal care such as sleeping, eating, etc.). This implies a substantial degree of freedom in the choice of the parameter B. For instance, when calibrating identical models, Hansen (1985) assumes "discretionary" time and obtains $B = 2$, while Christiano and Eichenbaum (1992) match "real" time and get $B = 2.99$, a value which is about 50% higher. This example suggests that the effect associated with X is of little interest (at least, for the second moments of aggregate variables) unless it differs from one by a very large order of magnitude. Therefore, the parameter X will be disregarded in this section.

Given the supposition above, under both Cobb-Douglas and addilog types of preferences, we would expect the following. First, the time series properties of variables $\{c_t, k_t, w_t, r_t\}$ in the model are not affected by the assumed types of heterogeneity. Second, the variable efficiency hours worked $\{h_t\}$ in the heterogeneous model behaves as physical hours worked in the representative agent case. Therefore, the only implication of heterogeneity which is of potential interest is its effect on time series properties of physical hours worked $\{n_t\}$. Consequently, in the rest of the section, we concentrate on the model's implications regarding labor markets.

Integrating individual hours worked, we get that under both the Cobb-Douglas and addilog utilities, efficiency hours and "simple" hours are related as

$$h_t = 1 - (1 - n_t) \cdot \xi, \tag{2.16}$$

where under Cobb-Douglas utility the parameter ξ is computed from (2.9)

$$\xi = \int_S \frac{f^s}{e^s} d\omega^s, \tag{2.17}$$

while in the case of addilog utility it is obtained from (2.14)

$$\xi = \frac{\int_S (e^s)^{1-1/\sigma} (f^s)^{\gamma/\sigma} d\omega^s}{\int_S (e^s)^{-1/\sigma} (f^s)^{\gamma/\sigma} d\omega^s}. \tag{2.18}$$

We call the parameter ξ *labor input bias.* We will denote by σ_x and $corr\,(x, y)$ the volatility of a variable x and the correlation of variables x and y.

Consider two empirical regularities on labor market behavior:

(*i*) physical hours worked fluctuate more than efficiency hours worked;

(*ii*) average productivity and hours worked are weakly correlated.

The first fact is reported, for example, by Kydland and Prescott (1993). Using the PSID data, they construct aggregate labor-input series by summing individual hours worked weighted by average hourly wage and find that the resulting series fluctuate less than the unadjusted hours worked. Hansen (1993) obtains a similar result using monthly data on age-sex groups from the U.S. Bureau of Labor Statistics' household survey. Note that in order for the model to be consistent with this empirical observation, it is necessary that $\xi < 1$. Indeed, according to (2.16), we have $\sigma_h = \xi^2 \sigma_n$.

The second fact is well-known to the literature and is often referred to as the Dunlop-Tarshis observation. Christiano and Eichenbaum (1992) document this regularity for the U.S. economy. Also, they show that the representative agent version of the model insistently predicts that this correlation is close to one and, therefore, does not reproduce fact (*ii*). Below we demonstrate that by taking into account the effect of heterogeneity on labor markets, we can improve the model's predictions in this dimension.

It follows from (2.16) that physical and efficiency hours worked are perfectly correlated in the model, $corr\,(n, h) = 1$, and, thus,

$$corr\,(y/n, n) = corr\,(y/h \cdot h/n,\; h)\,,$$

where y/n is average labor productivity. Given that in the heterogeneous model, h_t behaves as n_t in the representative agent model and also, that the representative agent model generates almost perfect positive correlation between productivity and hours worked, we have $corr\,(y/h, h) \simeq 1$. Therefore, $corr\,(y/n, n)$ in the heterogeneous case depends on how the variable h_t/n_t behaves over the business cycle. Equation (2.16) implies that whether this variable is pro- or countercyclical depends on the value of the parameter ξ

$$\frac{d\left(h_t/n_t\right)}{dh_t} = \frac{1}{n_t} \cdot \left(1 - \frac{dn_t/n_t}{dh_t/h_t}\right) = -\frac{1-\xi}{\xi n_t^2}. \tag{2.19}$$

It follows by the last result that if $\xi < 1$, then the correlation between productivity and hours worked in the heterogeneous model will be smaller than this statistic in the representative agent setup.

2.3.2 Individual dynamics

In this section, we focus on the relation between aggregate and distributive predictions of the model. For the purpose of subsequent analysis, we assume that there is a continuum of agents in the economy so that decisions of a single individual have no effect on aggregate allocation.

Kydland (1984) and Ríos-Rull (1993) document two empirical regularities of the individual labor choice:

(*iii*) individual hours worked are increasing in skills;

(*iv*) the volatility of individual hours worked is decreasing in skills.

In terms of our model, empirical facts (*iii*) and (*iv*) imply $dn_t^s/de^s > 0$ and $d\varepsilon_t^s/de^s < 0$ respectively, where $\varepsilon_t^s = \left(\frac{dn_t^s}{dw_t} \cdot \frac{w_t}{n_t^s}\right)$ is the period elasticity of agent's labor supply with respect to wage.

First, we demonstrate that if regularity (*iii*) is satisfied in our model, then (*iv*) will also be satisfied. Using the formulas for individual working hours given in (2.9) and (2.14) for the Cobb-Douglas and addilog utilities, one can show that

$$\frac{d\varepsilon_t^s}{de^s} = -\frac{\varepsilon_t h_t}{n_t^s (1-h_t)} \cdot \frac{dn_t^s}{de^s}, \tag{2.20}$$

where $\varepsilon_t = \left(\frac{dh_t}{dw_t} \cdot \frac{w_t}{h_t}\right) > 0$. The latter follows because ε_t is the same as the elasticity of labor with respect wage in the representative agent model.

Let us analyze how distributive facts (iii), (iv) are related to previously discussed aggregate regularities (i), (ii). Using formula (2.19), the change in supply of physical and efficiency hours worked in response to wage increase can be written as

$$dn_t = \frac{dw_t}{w_t} \int_S n_t^s \varepsilon_t^s d\omega^s, \qquad dh_t = \frac{dw_t}{w_t} \int_S n_t^s \varepsilon_t^s e^s d\omega^s.$$

Together with (2.19), these formulas imply

$$\frac{d\left(h_t/n_t\right)}{dh_t} = \frac{\int_{S\times S} n_t^s n_t^{s'} \left(e^s - e^{s'}\right)\left(\varepsilon_t^s - \varepsilon_t^{s'}\right) d\omega^s d\omega^{s'}}{n_t^2 \cdot \int_S n_t^s \varepsilon_t^s e^s d\omega^s}. \tag{2.21}$$

Observe that if individual elasticities decrease in skills, $d\varepsilon_t^s/de^s < 0$, then each term in the numerator of the above expression is negative and, therefore, $d\left(h_t/n_t\right)/dh_t < 0$. As shown in the previous section, the last result corresponds to $\xi < 1$ which is consistent with aggregate facts (i) and (ii).

The elasticity considerations allows us to gain some intuition on why the correlation between productivity and hours worked in the heterogeneous model might be lower than in the representative agent model. In response to a positive shock, all individuals supply more hours on the market. However, if the elasticity of labor supply decreases in skills, the increase in hours of low skilled workers is larger than that of high skilled workers. As a result, a fraction of low skilled hours in total hours worked is larger after the shock than it was before the shock and the average productivity of labor, y_t/n_t, goes down. If the difference in the elasticities across agents is high enough, then it could be the case that $corr\left(y/n, n\right) < 0$.

2.3.3 Working hours

The analysis of the previous section suggests that the model can produce the appropriate aggregate and distributive predictions only if it is capable of generating hours worked which increase in the level of skills (fact (*iii*)). To evaluate the model's ability to account for this observation, we will employ an additional empirical regularity at the individual level, namely:

(*v*) wage differentials across agents do not exceed wealth differentials.

In terms of our model, this implies $d\log\kappa_0^s/d\log e^s \geq 1$, since this ratio gives us the percentage change in wealth relative to one percent change in skills. GMV (1995) divide the PSID sample into two groups according to two alternative criteria, the wage to wealth ratio and the level of wealth. Depending on the criterion, they obtain that the difference in the wages of two groups amount to 2 while the differences in wealth range from 5 to 30. We find that a quantitative expression of fact (*v*) is highly sensitive to a criterion which is used for dividing the population into groups. For example, in the next section, we will show that if the PSID data are divided according to the educational level, then the distributions of wealth and wages across group do not differ significantly.

Let us analyze the relation between facts (*iii*) and (*v*). Formulas (2.10), (2.15) imply that under both Cobb-Douglas and addilog utility functions, the life-time budget constraint can be written as follows

$$E_0\left[\sum_{\tau=0}^{\infty}\delta^{\tau}\frac{u_1(c_\tau,n_\tau)}{u_1(c_0,n_0)}\left(c_\tau f^s-n_\tau^s e^s w_\tau\right)\right]=\kappa_0^s. \tag{2.22}$$

Differentiating (2.22) with respect to skills and subtracting from the resulting condition equation (2.22), previously divided by skills, we get

$$1 - \frac{d \log f^s}{d \log e^s} = \frac{\frac{\kappa_0^s}{e^s}\left(1 - \frac{d \log \kappa_0^s}{d \log e^s}\right) - E_0 \sum_{\tau=0}^{\infty} \delta^{\tau} \frac{u_1(c_\tau, n_\tau)}{u_1(c_0, n_0)} e^s w_\tau \frac{dn_\tau^s}{de^s}}{\frac{f^s}{e^s} E_0 \sum_{\tau=0}^{\infty} \delta^{\tau} \frac{u_1(c_\tau, n_\tau)}{u_1(c_0, n_0)} c_\tau}. \tag{2.23}$$

Under the assumption of Cobb-Douglas utility function, the condition for individual hours provided in (2.9) yields

$$1 - \frac{d \log f^s}{d \log e^s} = \frac{dn_t^s}{de^s} \cdot \frac{1}{1 - n_t^s}. \tag{2.24}$$

Observe that if the model is to reproduce fact *(iii)* and fact *(v)*, then the right-hand-side of (2.23) must be negative. However, the condition (2.24) demonstrates that the model can account for fact *(iii)* only if the left-hand-side of (2.23) is positive. Thus, under the assumption of Cobb-Douglas utility, the model cannot explain facts *(iii)* and *(v)* simultaneously. In other words, if such model is calibrated to reproduce the distribution of wealth in the data, then it counterfactually predicts that rich (skilled) agents supply less labor on the market than poor (unskilled). Equivalently, if the model is calibrated to match the difference in hours worked across productivity groups, then the implied distribution of wealth is such that regularity *(v)* is violated.[1]

For the addilog type of preferences, using (2.14), we obtain

$$1 - \frac{d \log f^s}{d \log e^s} = \frac{dn_t^s}{de^s} \cdot \frac{e^s \sigma}{\gamma (1 - n_t^s)} - \frac{1}{\gamma} + 1. \tag{2.25}$$

Note that equation (2.25) together with (2.23) implies that, similar to the Cobb-Douglas case, the model will also fail with respect to facts *(iii)* and *(v)*, if the agents' preferences are of the addilog type with the standard elasticity of intertemporal substitution for consumption, $\frac{1}{\gamma} \leq 1$.[2] The above

[1] Kydland (1984) does not analyze the model's implications with respect to the distribution of wealth and, therefore, does not pin down this failure of the model.

[2] Precisely this feature of the model accounts for GMV's (1995) puzzle discussed in the

results suggest that the assumptions about preferences which are standard for macroeconomic literature are inconsistent with basic cross-sectional observations.

There is another implication that follows from (2.25). Specifically, if the utility is of the addilog type and if the intertemporal elasticities of substitution for consumption and leisure, $\frac{1}{\gamma}$ and $\frac{1}{\sigma}$, are large enough, then the model might be able to reproduce facts (iii) and (v) simultaneously. Our previous analysis suggests that in this case, the model's predictions will also be consistent with the remaining empirical regularities (i), (ii) and (iv). Given that the model can account for all empirical facts of interest only under the assumption of the addilog preferences, we will study the quantitative implications of the model only under this type of preferences.

2.4 Calibration procedure

This section discusses the calibration procedure. To compute numerical predictions, we use the model which is adjusted to growth as shown in appendix C. The solution algorithm is described in appendix D.

For studying quantitative implications of the model, specific values need to be assigned to the model parameters. We calibrate the model to reproduce the capital to output ratio, π_k, the consumption to output ratio, π_c, and aggregate hours worked, n, in the U.S. economy. The parameter choice is summarized in *Table* 1.

Here, $\widetilde{\delta}$ is the discount rate in the model with growth, and g is the rate of technological progress. The parameters $\widetilde{\delta}$, d, B are computed from the

introduction. This paper assumes the addilog utility with $\frac{1}{\gamma} = 1$, calibrates the model to reproduce the distribution of wealth (fact (v)), and finds that such model generates undesirable predictions at both individual and aggregate levels. Indeed, our analysis implies that in this case regularities $(i) - (iv)$ are violated.

optimality conditions evaluated in the steady state as it is shown in appendix C. We assume that initially the economy is in the steady state, i.e. we set $\theta_0 = 1$ and choose aggregate capital, k_0, to be equal to the steady state value. The remaining parameters are set to the values standard in the RBC literature. The ratios π_k and π_c and the values of the parameters g and α are adopted from Christiano and Eichenbaum (1992); following their paper, we define consumption in π_c as a sum of private and government consumption. The value of n is taken from the micro study by Juster and Stafford (1991). Finally, the aggregate technology shock θ_t has the time series representation $\theta_t = \theta_{t-1}^{\rho} \exp(\varepsilon_t), \rho \in [0,1]$, where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ for $\forall t \in T$. The parameters ρ and σ_ε are calibrated as in Hansen (1985).

To calibrate the heterogeneity parameters, we use the PSID sample (1989). This data set contains the observations on 7114 U.S. households. Following Kydland (1984), we split the population into the groups by the level of education of the head of the household. We remove from the PSID sample 96 households for which the level of education of the head is not available. As a proxy for skills, we use average hourly earnings of the head of the household. Initial endowment is calibrated according to the total wealth of the household. To compute the averages, we adjust the households' observations to PSID sample weights. Exact labels of the cross sections used are $V17545$, $V17536$, $V17389$, $V17612$. *Table* 2 summarizes the estimates.

The groups $1-8$ distinguished in the table correspond to subsamples of the households, whose heads completed: 1) grades $0-5$; 2) grades $6-8$; 3) grades $9-11$; 4) grade 12 grade (high school); 5) grade 12 plus non-academic training; 6) college but no degree; 7) college BA but no advanced degree; 8) college and advanced or professional degree.

2.5 Results

The quantitative implications of the model to a large extent depend on the choice of the parameters γ and σ. To illustrate the tendencies, we report the results from simulations for several pairs of γ and σ.

2.5.1 Distributive predictions

This section reports the model's predictions on consumption and working hours of eight heterogeneous groups distinguished in section 2.4 and compares them with the corresponding quantities in the U.S. economy. As a proxy for consumption, we use monetary income of households. Working hours in the U.S. economy are calibrated according to those worked by the head of the household. The groups' quantities in the U.S. economy are the averages of individual variables in the subsamples. To compute the averages, we adjust the households' observations to PSID sample weights. Exact labels of the cross sections used are $V16335$, $V17533$. The predictions of the model are computed using (2.14), (2.16) which are evaluated in the steady state. *Table* 3 presents the results.

In section 2.3.3, we show qualitatively that the values of the parameters γ and σ play a decisive role in distributive predictions of the model. The results in *Table* 3 allow us to evaluate their effect quantitatively. As we can see, the tendency is that, if $\gamma > 1$, the model fits relatively well the empirical distribution of consumption; however, it counterfactually predicts that working hours decrease with the level of skills. If $\gamma < 1$, the model generates the appropriate predictions for hours worked; however, it fails to produce the appropriate variability of consumption across groups.

The effect of the parameter σ on the groups' consumption is determined by the value of γ. In such a way, the variability of consumption across groups increases with σ if $\gamma > 1$, and it decreases with σ if $\gamma < 1$. The variability of hours worked decreases with σ under any value of γ; this implies that if $\gamma > 1$, an increment in σ induces the low skilled agents to work less and high skilled agents to work more, while if $\gamma < 1$, the opposite is true. If $\gamma \simeq 1$, the effect of the parameter σ on the groups' quantities is ambiguous and small.

Only in two cases from all those reported in *Table* 3, namely, $\gamma = 0.6$, $\sigma = 1.0$ and $\gamma = 0.6$, $\sigma = 0.2$, does the model generate the distributional patterns for both consumption and working hours as in the U.S. economy. The fit of the model is somewhat better under $\sigma = 1.0$ than under $\sigma = 0.2$, although in both cases the variability of consumption across groups is too large. In particular, the model severely underpredicts the value of consumption for two bottom-skill groups.

In fact, the model's failure with respect to individual consumption of unskilled individuals is not surprising. In the real economies, a large portion of the expenditures of such individuals comes from government transfers and public services which are not included in the model. For example, Díaz-Giménez, Quadrini and Ríos-Rull (1996) divide the U.S. population into three groups according to the level of education; they find that the share of transfers in the total income of the bottom group (28 percent) is about six times as high as that of the top group (4.7 percent).

2.5.2 Aggregate predictions

Table 4 contains the estimates of the parameters X and ξ, and selected second moments computed using the series for efficiency hours, h_t, and physical hours worked, n_t. With slight abuse of notation, in this section, σ_x, $corr(x, y)$ will represent the volatility of a logged variable x and correlation between logged variables x and y.

As we see from *Table* 4, in all of the cases considered, the values of X differ from one by less than 15%. In section 2.3.1, we have argued that if the parameter X is not significantly different from one, then efficiency hours in the heterogeneous model behave in the same way as physical hours worked in the representative agent case. To verify this conjecture, we computed the solutions under $X = 1$ and compared the resulting statistics to those reported in the table. We find that the effect of this restriction on the model's predictions is fairly small (a few percent, at most). Therefore, the statistics σ_h, $\sigma_{y/h}$, $corr\,(y/h, h)$ and $corr\,(y/h, y)$ can be viewed as the volatilities of hours worked and productivity, and the correlations between productivity and hours worked and between productivity and output in the associated representative agent model.

Hansen (1985) and Christiano and Eichenbaum (1992) consider the representative agent version of the model under $\gamma = 1$ and $\sigma = 1$. Hansen (1985) shows that such a model underpredicts the volatility of hours worked and fails to account for the large fluctuations in hours compared to the relatively small fluctuations in productivity. Christiano and Eichenbaum (1992) point out another drawback of the model, precisely, its failure to produce the appropriate correlation between productivity and hours worked. Also, as follows from the table, the correlation between productivity and output in such a model is far from the one observed in the data. The results reported in *Table* 4 indicate that a variation in the values of γ and σ improves on statistics σ_h and $\sigma_h/\sigma_{y/h}$; however, it has only a minor effect on $corr\,(y/h, h)$ and $corr\,(y/h, y)$. In particular, our findings suggest that the representative agent version of the model cannot account for the weak correlation between productivity and hours worked under any reasonable values of γ and σ.

As is argued in section 2.3.1, accounting for labor input bias, ξ, may bring the model into closer conformity with the data provided that $\xi < 1$. From *Table* 4, we can observe the following tendency. If $\gamma \simeq 1$, then $\xi \simeq 1$ and the effect of σ on ξ is ambiguous and small. If $\gamma > 1$, then $\xi > 1$ and it decreases with σ; finally, if $\gamma < 1$, then $\xi < 1$ and it increases with σ.

In particular, under the elasticities $\frac{1}{\gamma}$ and $\frac{1}{\sigma}$, which are large enough, the value of ξ is substantially smaller than one, and, consequently, the effect of heterogeneity on aggregate dynamics is quantitatively significant.

Table 4 shows that under $\gamma = 0.6$ and $\sigma \in \{0.3,\ 0.2,\ 0.15\}$, the model is capable of accounting for the weak correlation between productivity and hours worked; it also generates the correlation between productivity and output which is close to that observed in the data. Finally, it reproduces the empirical observation discussed in section 2.3 that the volatility of physical hours worked, σ_n, is larger than the volatility of efficiency hours, σ_h. Unfortunately, under these γ and σ, the model also has undesirable features. Specifically, the volatility of productivity is too low and the model's predictions are not robust to small changes in the parameters.

To understand the origin of the shortcomings, consider the correlation between productivity and hours worked

$$corr\,(y/n, n) = \frac{corr\,(y,n)\,\sigma_y - \sigma_n}{\sigma_{y/n}} = \frac{corr\,(y,n)\,\sigma_y - \sigma_n}{\sqrt{\sigma_y^2 + \sigma_n^2 - 2corr\,(y,n)\,\sigma_y \sigma_n}}. \quad (2.26)$$

If the model is to generate zero correlation between productivity and hours worked, it is necessary that $\sigma_y corr\,(y,n) \simeq \sigma_n$. Given that in our model $corr\,(y,n)$ is close to one (see *Table* 5), the preceding condition together with (2.26) implies both that $\sigma_{y/n}$ is small and that $corr\,(y/n, n)$ is highly sensitive to small changes in statistics σ_n and σ_y. It can be reasonably expected that any modification which reduces $corr\,(y,n)$ will improve the model's performance. Two examples of such modifications are incorporating home production and allowing for variation in labor input along both the hours-per-worker margin and the employment margin.[3]

[3]Introducing home production and the two margins in the representative agent model reduces $corr\,(y,n)$ from 0.99 to 0.91 and 0.75 respectively (source: Kydland, 1995, table 5.2).

In *Table* 5, we report the remaining statistics for the U.S. and artificial economies. For comparison, we also include the predictions of the representative agent version of the model under $\gamma = 1$, $\sigma = 1$. As we see from the table, in our model all the statistics are similar to those in the representative agent model. Observe that under γ and σ which are lower than one, the volatilities of consumption, capital, investment and output are closer to the corresponding volatilities in the data.

Summarizing, on the one hand, the heterogeneous agents model improves on some labor market statistics which the representative agent model seriously fails to predict. On the other hand, the model with heterogeneity preserves the positive features of the representative agent setup.

2.6 Conclusion

This paper incorporates into the neoclassical model with labor-leisure choice heterogeneous agents who differ with respect to endowments and non-acquired skills. We show that the qualitative and quantitative analysis of the equilibrium in the model can be simplified substantially by using the results from aggregation. We demonstrate that under the standard for the macroeconomic literature assumptions on preferences, the model cannot account for cross-sectional observations. Specifically, we find that if agents' preferences are of the Cobb-Douglas type, the model fails to reproduce the simple empirical regularity that high-productivity agents work more compared to the low-productivity. We reach the same conclusion for the case of addilog utility with the standard (smaller than one) intertemporal elasticity of substitution for consumption. We show, however, that if such elasticity in the addilog utility is set to a value which is large enough, then the model is capable of generating the appropriate pattern for hours worked. Moreover, such model can account for several time-series labor market facts which cannot be reconciled within a similar representative agent setup.

2.7 Appendices

This section derives the expected life-time budget constraint, proves the propositions in section 2.2.3, develops a version of the model with growth and outlines the solution algorithm.

2.7.1 Appendix A

With an interior solution, the first order conditions (FOCs) of agent's $s \in S$ utility maximization problem (2.1), (2.2) with respect to insurance holdings, capital holdings and the transversality conditions are as follows

$$u_1(c_t^s, n_t^s)p_t(\theta) = \delta u_1(c_{t+1}^s(\theta), n_{t+1}^s(\theta)) \Pr\{\theta_{t+1} = \theta \mid \theta_t = \theta'\}_{\theta,\theta' \in \Theta}, \quad (2.27)$$

$$u_1(c_t^s, n_t^s) = \delta E_t\left[u_1(c_{t+1}^s, n_{t+1}^s)(1 - d + r_{t+1})\right], \quad (2.28)$$

$$\lim_{t\to\infty} E_0\left[\delta^t u_1(c_t^s, n_t^s)\left(k_{t+1}^s + \int_\Theta p_t(\theta)\, m_{t+1}^s(\theta)\, d\theta\right)\right] = 0, \quad (2.29)$$

where u_1, u_2 are the first order partial derivatives of the utility u with respect to consumption and labor and $c_{t+1}^s(\theta)$, $n_{t+1}^s(\theta)$ are equilibrium consumption and working hours as functions of the realization of the aggregate shock.

FOC (2.27) implies

$$E_{t-1}\left[\delta \frac{u_1(c_t^s, n_t^s)}{u_1(c_{t-1}^s, n_{t-1}^s)} m_t^s(\theta_t)\right] = \int_\Theta m_t^s(\theta)\, p_{t-1}(\theta)\, d\theta. \quad (2.30)$$

Further, FOC (2.28) together with the fact that k_t^s is known at $t-1$ yields

$$E_{t-1}\left[\delta\frac{u_1\left(c_t^s,n_t^s\right)}{u_1\left(c_{t-1}^s,n_{t-1}^s\right)}k_t^s\left(1-d+r_t\right)\right]=k_t^s. \qquad (2.31)$$

Multiplying each term of (2.2) by $\frac{\delta u_1\left(c_t^s,n_t^s\right)}{u_1\left(c_{t-1}^s,n_{t-1}^s\right)}$, taking the expectation E_{t-1} on both sides and using (2.31), (2.30), one can show that for all $t>0$ the following condition holds

$$k_t^s+\int_\Theta m_t^s\left(\theta\right)p_{t-1}\left(\theta\right)d\theta=E_{t-1}\left[\delta\frac{u_1\left(c_t^s,n_t^s\right)}{u_1\left(c_{t-1}^s,n_{t-1}^s\right)}\left(c_t^s-n_t^se^sw_t\right)\right]+$$

$$E_{t-1}\left[\delta\frac{u_1\left(c_t^s,n_t^s\right)}{u_1\left(c_{t-1}^s,n_{t-1}^s\right)}\left(k_{t+1}^s+\int_\Theta m_{t+1}^s\left(\theta\right)p_t\left(\theta\right)d\theta\right)\right].$$

Applying forward recursion, using the law of iterative expectations and imposing transversality condition (2.29), we get

$$\left(1-d+r_0\right)k_0^s+m_0^s\left(\theta_0\right)=c_0^s-n_0^se^sw_0+k_1^s+\int_\Theta m_1^s\left(\theta\right)p_0\left(\theta\right)d\theta=$$

$$E_0\left[\sum_{\tau=0}^{1}\delta^t\frac{u_1\left(c_\tau^s,n_\tau^s\right)}{u_1\left(c_0^s,n_0^s\right)}\left(c_\tau^s-n_\tau^se^sw_\tau\right)\right]+$$

$$E_0\left[\delta\frac{u_1\left(c_1^s,n_1^s\right)}{u_1\left(c_0^s,n_0^s\right)}\left(k_2^s+\int_\Theta m_2^s\left(\theta\right)p_1\left(\theta\right)d\theta\right)\right]=\ldots=$$

$$E_0\left[\sum_{\tau=0}^{\infty}\delta^\tau\frac{u_1\left(c_\tau^s,n_\tau^s\right)}{u_1\left(c_0^s,n_0^s\right)}\left(c_\tau^s-n_\tau^se^sw_\tau\right)\right]+$$

$$\lim_{t\to\infty}E_0\left[\delta^t\frac{u_1\left(c_t^s,n_t^s\right)}{u_1\left(c_0^s,n_0^s\right)}\left(k_{t+1}^s+\int_\Theta m_{t+1}^s\left(\theta\right)p_t\left(\theta\right)d\theta\right)\right]=$$

$$E_0\left[\sum_{\tau=0}^{\infty}\delta^\tau\frac{u_1\left(c_\tau^s,n_\tau^s\right)}{u_1\left(c_0^s,n_0^s\right)}\left(c_\tau^s-n_\tau^se^sw_\tau\right)\right].$$

The last result corresponds to expected life-time budget constraint (2.6) used in the main text.

2.7.2 Appendix B

The *FOC*s of planner's problem (2.4) , (2.5) with respect to c_t^s, n_t^s, k_t are

$$\lambda_s u_1 \left(c_t^s, 1-n_t^s\right) = \zeta_t, \tag{2.32}$$

$$\lambda_s u_2 \left(c_t^s, 1-n_t^s\right) = \zeta_t e^s w_t, \tag{2.33}$$

$$\zeta_t = \delta E_t \left[\zeta_{t+1} \left(1-d+r_{t+1}\right)\right], \tag{2.34}$$

where ζ_t is the Lagrange multiplier associated with the economy's resource constraint, $w_t \equiv \theta_t f_2 \left(k_t, h_t\right)$ is marginal product of labor input.

Proof of proposition 2. Under the Cobb-Douglas utility, solving for c_t^s and $(1-n_t^s)$ from *FOC*s (2.32) , (2.33), we get

$$c_t^s = \left[\frac{\mu}{\zeta_t} \cdot \left(\frac{1-\mu}{\mu w_t}\right)^{(1-\mu)(1-\eta)}\right]^{1/\eta} \cdot \left(\lambda^s\right)_s^{1/\eta} \left(e^s\right)^{-\frac{(1-\mu)(1-\eta)}{\eta}}, \tag{2.35}$$

$$1-n_t^s = \left[\frac{1-\mu}{\zeta_t} \cdot \left(\frac{1-\mu}{\mu w_t}\right)^{-\mu(1-\eta)}\right]^{1/\eta} \left(\lambda^s\right)^{1/\eta} \left(e^s\right)^{\frac{\mu(1-\eta)-1}{\eta}}. \tag{2.36}$$

Integration of (2.35) and (2.36) over the set of agents (both sides of the latter are previously premiltiplied by e_s) yields

$$c_t = \left[\frac{\mu}{\zeta_t} \cdot \left(\frac{1-\mu}{\mu w_t}\right)^{(1-\mu)(1-\eta)}\right]^{1/\eta} \cdot \int_S \left(\lambda^s\right)^{1/\eta} \left(e^s\right)^{-\frac{(1-\mu)(1-\eta)}{\eta}} d\omega^s, \tag{2.37}$$

$$1-h_t = \left[\frac{1-\mu}{\zeta_t} \cdot \left(\frac{1-\mu}{\mu w_t}\right)^{-\mu(1-\eta)}\right]^{1/\eta} \cdot \int_S \left(\lambda^s\right)^{1/\eta} \left(e^s\right)^{-\frac{(1-\mu)(1-\eta)}{\eta}} d\omega^s. \tag{2.38}$$

Dividing (2.35) by (2.37) and (2.36) by (2.38) and introducing f^s, we obtain (2.9) in the main text.

Rearranging the terms of equations (2.37) and (2.38), we get

$$u_1(c_t, 1-h_t) = \zeta_t \cdot \left(\int_S (\lambda^s)^{1/\eta} (e^s)^{-\frac{(1-\mu)(1-\eta)}{\eta}} d\omega^s\right)^{-\eta}, \tag{2.39}$$

$$u_2(c_t, 1-h_t) = \zeta_t w_t \cdot \left(\int_S (\lambda^s)^{1/\eta} (e^s)^{-\frac{(1-\mu)(1-\eta)}{\eta}} d\omega^s\right)^{-\eta}, \tag{2.40}$$

where $u(c_t, 1-h_t) = \frac{\left(c_t^\mu (1-h_t)^{1-\mu}\right)^{1-\eta} - 1}{1-\eta}$. Combining (2.39), (2.40) yields he FOC of (2.8) with respect to labor; substituting (2.39) into (2.34) gives us the FOC with respect to capital; these are respectively

$$u_2(c_t, 1-h_t) = w_t u_1(c_t, 1-h_t),$$

$$u_1(c_t, 1-h_t) = \delta E_t \left[u_1(c_{t+1}, 1-h_{t+1})(1-d+r_{t+1})\right].$$

This proves that aggregate dynamics of the heterogeneous economy is described by single-agent utility maximization problem (2.8). Finally, condition (2.10) follows after substituting into expected life-time budget constraint (2.6) both conditions given in (2.9). ||

Proof of proposition 3. Under the addilog utility, conditions (2.32), (2.33) become

$$c_t^s = \zeta_t^{-1/\gamma} \cdot (\lambda^s)^{1/\gamma}, \tag{2.41}$$

$$1 - n_t^s = (\zeta_t w_t)^{-1/\sigma} B^{1/\sigma} \cdot (\lambda^s)^{1/\sigma} (e^s)^{-1/\sigma}. \tag{2.42}$$

Integrating them over the set of agents (first, premultiplying the latter by e^s) yields

$$c_t = \zeta_t^{-1/\gamma} \cdot \int_S (\lambda^s)^{1/\gamma} d\omega^s, \tag{2.43}$$

$$1 - h_t = (\zeta_t w_t)^{-1/\sigma} B^{1/\sigma} \cdot \int_S (\lambda^s)^{1/\sigma} (e^s)^{1-1/\sigma} d\omega^s. \tag{2.44}$$

Taking the ratios of (2.41) to (2.43) and (2.42) to (2.44), introducing f^s and defining the parameter X as it is in (2.13) of the main text, we get the two conditions in (2.14).

Rearranging the terms in (2.43) and (2.44), we have

$$u_1 (c_t, 1 - h_t) = \zeta_t \cdot \left(\int_S (\lambda^s)^{1/\gamma} d\omega^s \right)^{-\gamma}, \tag{2.45}$$

$$u_2 (c_t, 1 - h_t) = \zeta_t w_t \cdot \left(\int_S (\lambda^s)^{1/\sigma} (e^s)^{1-1/\sigma} d\omega^s \right)^{-\sigma}, \tag{2.46}$$

where $u (c_t, 1 - h_t) = \left(c_t^{1-\gamma} - 1 \right) / (1 - \gamma) + B \cdot \left((1 - h_t)^{1-\sigma} - 1 \right) / (1 - \sigma)$. Equations (2.45), (2.46) combined together yield the FOC of single-agent problem (2.12) with respect to h_t. Further, substituting (2.45) in (2.34), we get the corresponding intertemporal condition of (2.12). They are respectively

$$X \cdot u_2 (c_t, 1 - h_t) = w_t u_1 (c_t, 1 - h_t),$$

$$u_1 (c_t, 1 - h_t) = \delta E_t \left[u_1 (c_{t+1}, 1 - h_{t+1}) (1 - d + r_{t+1}) \right].$$

This verifies that at the aggregate level the heterogeneous economy behaves as single-agent economy (2.12). Finally, after substituting (2.14) into expected life-time budget constraint (2.6), we get (2.15). ||

2.7.3 Appendix C

We introduce growth as it is usually done in the RBC literature. In the economy with growth, the problem of an individual $s \in S$ is

$$\max_{\left\{c_t^s, n_t^s, k_{t+1}^s, m_{t+1}^s(\theta)\right\}_{\theta \in \Theta, t \in T}} E_0 \sum_{t=0}^{\infty} \delta^t \left\{ \frac{\left(c_t^s\right)^{1-\gamma} - 1}{1-\gamma} + B \cdot \frac{\left(1 - n_t^s\right)^{1-\sigma} - 1}{1-\sigma} \cdot g^{t(1-\gamma)} \right\}$$

$$\text{s.t.} \quad c_t^s + k_{t+1}^s + \int_{\Theta} p_t(\theta)\, m_{t+1}^s(\theta)\, d\theta = (1 - d + r_t)\, k_t^s + n_t^s e^s w_t g^t + m_t^s(\theta_t),$$

where the parameter g is the rate of labor-augmenting technological progress. Finding the FOCs of the above problem, introducing new variables $\widetilde{c}_t^s = c_t^s g^{-t}$, $\widetilde{k}_t^s = k_t^s g^{-t}$ and $\widetilde{m}_t^s = m_t^s g^{-t}$ and following the procedure described in appendix B, we obtain the conditions that identify aggregate dynamics

$$g \widetilde{c}_t^{-\gamma} = \widetilde{\delta} E_t \left[\widetilde{c}_{t+1}^{-\gamma} \left(1 - d + \widetilde{r}_{t+1}\right) \right], \tag{2.47}$$

$$\widetilde{c}_t^{-\gamma} \widetilde{w}_t = B \cdot X \cdot \left(1 - h_t\right)^{-\sigma}, \tag{2.48}$$

$$\widetilde{c}_t + \widetilde{k}_{t+1} g = \widetilde{k}_t (1 - d) + \theta_t f\left(\widetilde{k}_t, h_t\right), \tag{2.49}$$

where $\widetilde{r}_t \equiv \theta_t \partial f\left(\widetilde{k}_t, h_t\right) / \partial \widetilde{k}_t$, $\widetilde{w}_t \equiv \theta_t \partial f\left(\widetilde{k}_t, h_t\right) / \partial h_t$, the parameter X is defined by (2.53), and $\widetilde{\delta} \equiv \delta g^{1-\gamma}$ is the discount rate adjusted to growth. The expected life-time budget constraint takes the form

$$E_0 \sum_{\tau=0}^{\infty} \widetilde{\delta}^{\tau} \frac{\widetilde{c}_\tau^{-\gamma}}{\widetilde{c}_0^{-\gamma}} \left[\widetilde{c}_\tau f^s - \widetilde{w}_\tau e^s \left(1 - \left(1 - h_\tau\right) X^{-1/\sigma} \left(e^s\right)^{-1/\sigma} \left(f^s\right)^{\gamma/\sigma}\right)\right] = \kappa_0^s. \tag{2.50}$$

Equations (2.14), (2.16), (2.18) do not change after introducing growth except that the appropriate variables in (2.14) are $\widetilde{c}_t^s$ and $\widetilde{c}_t$.

Optimality conditions (2.47) – (2.49) provide a basis for calibrating the parameters $\widetilde{\delta}$, d and B. Evaluating (2.47) and (2.49) in the steady state yields

$$\widetilde{\delta} = \frac{\pi_k g}{\pi_k g + \pi_c + \alpha - 1}, \qquad d = \frac{1 - \pi_c}{\pi_k} + 1 - g.$$

Condition (2.48) in the steady state can be written as

$$B \cdot X = (1 - \alpha)\, \pi_k^{(1-\gamma)\alpha/(1-\alpha)} \pi_c^{-\gamma} \cdot (1 - h)^{\sigma} h^{-\gamma}, \tag{2.51}$$

where h denotes steady-state efficiency labor. The parameter B is calibrated to the same value as in the representative agent case. Setting $X = 1$ and $\xi = 1$, we get

$$B = (1 - \alpha)\, \pi_k^{(1-\gamma)\alpha/(1-\alpha)} \pi_c^{-\gamma} \cdot (1 - n)^{\sigma} n^{-\gamma}. \tag{2.52}$$

Dividing (2.51) by (2.52) and substituting the resulting condition into (2.16) evaluated in the steady state, we obtain

$$X = \xi^{\sigma} \cdot n^{\gamma} \cdot (1 - (1 - n) \cdot \xi)^{-\gamma}. \tag{2.53}$$

This formula relates the parameters X and ξ.

2.7.4 Appendix D

To compute the solution, we use the following iterative algorithm:

Step 1. Fix ξ to some level and compute X according to (2.53).

Step 2. Use (2.47) – (2.49) to solve for aggregate equilibrium quantities.

Step 3. Recover f^s's from (2.50) and recompute ξ according to (2.18).

Iterate on these steps until the fixed point value of ξ is found.

Once the equilibrium law of motion for the aggregate quantities and the corresponding set of the agent-specific parameters $\{f^s\}^{s \in S}$ is known, the remaining variables can be restored by direct calculations.

To complete *Step 2* of the solution algorithm, we use parametrized expectations algorithm, see, e.g., Marcet (1989). Under this algorithm, the conditional expectation in (2.47) is parameterized by a function of the state variables and, subsequently, iterations on the parameters of this function are performed until the equilibrium law of motion for the marginal utility is found. As a function parametrizing (2.47), we use second order degree exponentiated polynomial; the length of simulations is 10000.

To compute the expectations in expected life-time budget constraints (2.50), we follow the approach described in GMV (1995). Under this approach, the expected infinite sum is approximated by the average of N simulations of length T

$$E_0 \sum_{\tau=0}^{\infty} \left(\frac{\widetilde{c}_\tau^{-\gamma}}{\widetilde{c}_0^{-\gamma}} \right) \widetilde{\delta}^{\tau} z_\tau \simeq \frac{1}{N} \sum_{n=1}^{N} \sum_{\tau=0}^{T} \left(\frac{\widetilde{c}_\tau^{-\gamma}}{\widetilde{c}_0^{-\gamma}} \right) \widetilde{\delta}^{\tau} z_\tau, \tag{2.54}$$

where $z_\tau \in \{\widetilde{c}_\tau, \widetilde{w}_\tau, \widetilde{w}_\tau (1 - h_\tau)\}$. To obtain a precise estimate, both N and T have to be large. Since it is computationally costly to set both N and T large, the right-hand-side of (2.54) is subdivided in two parts, the head and the tail. Subsequently, the head is computed as average of N short draws of length T' and the tail is approximated by a single long draw of the length T''

$$\frac{1}{N} \sum_{n=1}^{N} \sum_{\tau=0}^{T} \left(\frac{\widetilde{c}_\tau^{-\gamma}}{\widetilde{c}_0^{-\gamma}} \right) \widetilde{\delta}^{\tau} z_\tau \simeq \frac{1}{N} \sum_{n=1}^{N} \sum_{\tau=0}^{T'} \left(\frac{\widetilde{c}_\tau^{-\gamma}}{\widetilde{c}_0^{-\gamma}} \right) \widetilde{\delta}^{\tau} z_\tau + \widetilde{\delta}^{T'} \sum_{\tau=0}^{T''} \left(\frac{\widetilde{c}_\tau^{-\gamma}}{\widetilde{c}_0^{-\gamma}} \right) \widetilde{\delta}^{\tau} z_\tau .$$

We choose $N = 400$, $T' = 115$, $T'' = 10000$. As is argued in GMV (1995), a similar choice guarantees substantial accuracy of simulated solutions.

Finally, we show that a solution $\{f^s\}^{s \in S}$ to expected life-time budget constraints (2.50) computed on *Step 3* exists, is unique and such that $f^s > 0$ for $\forall s \in S$. Indeed, let $\{\widetilde{c}_\tau, \widetilde{w}_\tau, h_\tau\}_{\tau \in T}$ be such that $0 < \widetilde{c}_\tau, \widetilde{w}_\tau < \infty$ and $0 < h_\tau < 1$ for any $\tau \in T$. Consider functions $\psi^s (f^s)$ and $\phi^s (f^s)$ such that

$$\phi^s (f^s) \equiv X^{-1/\sigma} (e^s)^{1-1/\sigma} (f^s)^{\gamma/\sigma} \cdot E_0 \sum_{\tau=0}^{\infty} \widetilde{\delta}^{\tau} \frac{\widetilde{c}_\tau^{-\gamma}}{\widetilde{c}_0^{-\gamma}} \widetilde{w}_\tau (1 - h_\tau),$$

$$\psi^s(f^s) \equiv \kappa_0^s + e^s \cdot E_0 \sum_{\tau=0}^{\infty} \widetilde{\delta}^{\tau} \frac{\widetilde{c}_\tau^{-\gamma}}{\widetilde{c}_0^{-\gamma}} \widetilde{w}_\tau - f^s \cdot E_0 \sum_{\tau=0}^{\infty} \widetilde{\delta}^{\tau} \frac{\widetilde{c}_\tau^{-\gamma}}{\widetilde{c}_0^{-\gamma}} \widetilde{c}_\tau.$$

In terms of the above functions, the agent's expected life-time budget constraint (2.50) can be expressed as $\phi^s(f^s) = \psi^s(f^s)$. For any $X \in R_+$, we have $\phi^s(0) = 0$ and $(\phi^s)' > 0$. Further, given that $\kappa_0^s > 0$ for $\forall s \in S$, we have $\psi^s(0) > 0$ and $(\psi^s)' < 0$. Finally, the functions $\phi^s(f^s)$ and $\psi^s(f^s)$ are continuous on R_+. Thus, there exists a unique value f^s which satisfy the expected life-time budget contraint of each agent $s \in S$. A solution $\{f^s\}^{s\in S}$ and formula (2.18) determine uniquely the corresponding value of the parameter ξ. Therefore, if the equilibrium exists, is interior and unique, then the value of ξ which is consistent with the equilibrium is also unique.

General results about the existance of the equilibrium are hard to achieve. Whether the equilibrium in our economy exists will depend on a particular choice of the model's parameters. To see the point, consider, for example, the case when all consumers have the preferences of Cobb-Douglas type and assume that there are some consumers whose endowment to skills ratio is very high. According to formulas (2.9), (2.10) such consumers will choose to work a negative amount of hours, which implies that there is no an interior equilibrium in the model. However, in all the numerical experiments which we reported, the equilibrium exists and is interior; also, the iterative procedure had no difficulties to converge to a fixed point value of the parameter ξ.

Table 1. Model parameters

Parameter	π_k	π_c	n	g	δ	α	d	ρ	σ_ε
Value	10.62	0.727	0.31	1.004	0.993	0.339	0.0217	0.95	0.00712

Table 2. Heterogeneity parameters generated by the household data

	Groups							
Parameter	1	2	3	4	5	6	7	8
κ_s	0.268	0.730	0.419	0.600	0.796	1.270	1.853	1.904
β_s	0.123	0.272	0.624	0.771	0.946	1.129	1.643	2.120
$d\omega_s$	0.173	0.640	1.274	1.781	0.806	1.624	1.188	0.513

Notes: Each heterogeneity parameter is the average of corresponding individual variable in the subsample. The share, $d\omega_s$, is the relative weight of the subsample in the total population. The shares and the heterogeneity parameters weighted by the shares are normalized to 8.

Table 3. Groups' consumption and working hours for U.S. and artificial economies

	Heterogeneous model												U.S.	
	γ=1.5, σ=10.0		γ=1.5, σ=1.0		γ=1.0, σ=1.0		γ=0.6, σ=1.0		γ=0.6, σ=0.2		γ=0.6, σ=0.15		economy	
#	c_s	n_s	c_s	n_s	c_s	n_s	c_s	n_s	c_s	n_s	c_s	n_s	c_s	n_s
1	0.15	0.35	0.21	0.48	0.13	0.29	0.05	0.08	0.03	0.03	0.03	0.02	0.35	0.05
2	0.34	0.33	0.39	0.38	0.29	0.28	0.16	0.14	0.12	0.07	0.11	0.12	0.51	0.12
3	0.64	0.32	0.71	0.34	0.62	0.32	0.49	0.28	0.43	0.25	0.43	0.25	0.64	0.26
4	0.78	0.32	0.84	0.32	0.77	0.32	0.66	0.30	0.60	0.28	0.60	0.28	0.78	0.30
5	0.95	0.31	0.98	0.31	0.94	0.31	0.88	0.32	0.83	0.32	0.83	0.32	0.95	0.34
6	1.14	0.30	1.12	0.28	1.13	0.31	1.13	0.34	1.11	0.35	1.10	0.35	1.10	0.36
7	1.61	0.29	1.48	0.25	1.65	0.31	1.86	0.39	1.97	0.44	1.98	0.44	1.55	0.39
8	2.00	0.29	1.78	0.24	2.11	0.31	2.56	0.43	2.88	0.51	2.92	0.52	2.00	0.39

Notes: Consumption, c_s , and hours worked, n_s , weighted by the shares are normalized to 8 and to 8*0.31 respectively.

Table 4. Endogeneous parameters and selected labor statistics for U.S. and artificial economies

	Heterogeneous model[c]							U.S. economy
	γ=1.5 σ=1.0	γ=1.0 σ=1.0	γ=1.0 σ=0.2	γ=0.6 σ=1.0	γ=0.6 σ=0.3	γ=0.6 σ=0.2	γ=0.6 σ=0.15	
X	1.145	1.000	0.992	0.894	0.897	0.898	0.898	-
ξ	1.031	0.998	0.997	0.951	0.930	0.926	0.923	-
σ_h	0.57 (0.07)	0.70 (0.09)	1.15 (0.14)	0.83 (0.11)	1.33 (0.16)	1.47 (0.18)	1.58 (0.20)	-
σ_n	0.51 (0.06)	0.70 (0.09)	1.16 (0.14)	0.97 (0.13)	1.65 (0.20)	1.85 (0.23)	2.01 (0.26)	1.66[b]
$\sigma_{y/h}$	0.73 (0.09)	0.69 (0.09)	0.55 (0.08)	0.64 (0.09)	0.50 (0.07)	0.45 (0.07)	0.43 (0.07)	-
$\sigma_{y/n}$	0.78 (0.10)	0.69 (0.09)	0.54 (0.08)	0.51 (0.07)	0.25 (0.05)	0.23 (0.05)	0.25 (0.05)	1.18[b]
corr(y/h,h)	0.91 (0.02)	0.94 (0.02)	0.86 (0.03)	0.96 (0.01)	0.90 (0.02)	0.87 (0.03)	0.83 (0.03)	-
corr(y/n,n)	0.92 (0.02)	0.94 (0.02)	0.86 (0.03)	0.93 (0.02)	0.48 (0.05)	0.05 (0.05)	-0.29 (0.09)	-0.20[a]
corr(y/h,y)	0.98 (0.00)	0.98 (0.00)	0.94 (0.01)	0.99 (0.00)	0.94 (0.01)	0.92 (0.01)	0.89 (0.02)	-
corr(y/n,y)	0.99 (0.00)	0.98 (0.00)	0.99 (0.00)	0.97 (0.01)	0.59 (0.04)	0.17 (0.06)	-0.17 (0.11)	0.42[b]

Notes: [a] Source: Christiano and Eichenbaum (1992 table 4).
[b] Source: Hansen (1985 table 1).
[c] The standard deviations and correlations are sample averages of statistics computed for each of 400 simulations; each simulation consists of 115 periods. Numbers in parentheses are sample standard deviations of these statistics. All statistics are computed after first logging and then detrending the simulated time series using the Hodrick-Prescott filter.

Table 5. Selected statistics for U.S. and artificial economies

	RA model[a]	Heterogeneous model[b]						U.S. economy[a]
	γ=1.0 σ=1.0	γ=1.5 σ=1.0	γ=1.0 σ=1.0	γ=1.0 σ=0.2	γ=0.6 σ=1.0	γ=0.6 σ=0.2	γ=0.6 σ=0.15	
n_t	-	0.31 (0.00)	0.31 (0.00)	0.31 (0.01)	0.31 (0.01)	0.31 (0.01)	0.31 (0.01)	0.31[c]
h_t	-	0.29 (0.00)	0.31 (0.00)	0.31 (0.01)	0.34 (0.01)	0.36 (0.01)	0.36 (0.01)	-
σ_c	0.42 (0.06)	0.35 (0.05)	0.41 (0.06)	0.47 (0.07)	0.43 (0.08)	0.53 (0.09)	0.55 (0.10)	1.29
σ_k	0.36 (0.07)	0.33 (0.07)	0.36 (0.08)	0.43 (0.09)	0.39 (0.09)	0.50 (0.10)	0.53 (0.12)	0.63
σ_i	4.24 (0.51)	3.78 (0.45)	4.07 (0.53)	4.99 (0.61)	4.51 (0.59)	5.84 (0.72)	6.18 (0.80)	8.60
σ_y	1.35 (0.16)	1.27 (0.15)	1.37 (0.18)	1.65 (0.20)	1.45 (0.19)	1.88 (0.23)	1.95 (0.25)	1.76
corr(c,y)	0.89 (0.03)	0.94 (0,01)	0.90 (0.02)	0.89 (0.02)	0.79 (0.03)	0.79 (0.03)	0.77 (0.03)	0.85
corr(n,y)	0,98 (0.01)	0.97 (0.01)	0.98 (0.00)	0.99 (0.00)	0.99 (0.00)	0.99 (0.00)	0.98 (0.00)	0.76
corr(h,y)	-	0.97 (0.01)	0.98 (0.00)	0.99 (0.00)	0.99 (0.00)	0.99 (0.00)	0.99 (0.00)	-
corr(i,y)	0.99 (0.00)	1.00 (0.00)	0.99 (0.00)	0.99 (0.00)	0.99 (0.00)	0.99 (0.00)	0.99 (0.00)	0.92

Notes:[a] Source (except for [c]): Hansen (1985 table 1).
[b] The standard deviations and correlations are sample averages of statistics computed for each of 400 simulations; each simulation consists of 115 periods. Numbers in parentheses are sample standard deviations of these statistics. All statistics are computed after first logging and then detrending the simulated time series using the Hodrick-Prescott filter.
[c] Source: Juster and Stafford (1991).

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Chapter 3

Differential Responses of Labor Supply Across Productivity Groups

Joint with Serguei Maliar

3.1 Introduction

There is a substantial amount of evidence at the micro level documenting differential responses of labor supply across productivity groups. In particular, more productive individuals: (i) enjoy a higher employment rate, (ii) have a lower volatility of employment and (iii) spend less time working at home.

Concerning fact (i), in the U.S. the male unemployment rate for professional, technical and managerial workers is 1.8%, while the same rate for laborers is 10%; in the U.K., male unemployment rates for non-manual and for unskilled labor are 2.3% and 18.7% respectively (see Johnson and Layard (1986), tables 16.5, 16.7). Similar tendencies are observed for other data sets, see e.g. Moscarini (1995).

Many empirical studies provide evidence in support of fact (*ii*). For instance, Ríos-Rull (1993) illustrates this fact by using age as a proxy for individual productivity, Rosen (1968) by looking at a particular industry and Kydland (1984) by studying the educational levels of employees. Using data for different age-sex groups, Hansen (1993) constructs labor-input series where individual workers are weighted by relative hourly earnings. He finds that the resulting efficiency units series display smaller fluctuations than the physical hours series. One will expect this result if low skilled workers represent a larger fraction of the labor force during the expansions than during the recessions.

Fact (*iii*) is documented by Ríos-Rull (1993). He partitions the PSID data on U.S. households into five productivity groups and calculates the average hours worked at home by each group. The results indicate that the workers with the highest productivity level work almost three times less at home than workers from the lowest productivity group. He also provides some evidence on the difference in hours of home work by age-groups; given that old workers are less productive, more hours worked at home for this group also supports this fact.

This paper constructs a simple model with heterogeneous agents which explains the above facts. Most features of our setup are standard in the real business cycle (RBC) literature. In particular, we assume rational expectations, endogenous production, a competitive environment, full information, a stochastic technology and complete markets. The economy is populated by agents who have different abilities in producing the output good and it is assumed that the differences in productivity across agents are permanent. We assume that agents can work in the market only a fixed number of hours but are free to choose the amount of time devoted to working at home. The labor choice is modelled as in Hansen (1985) and Benhabib, Rogerson and

Wright (1991).

Besides the standard features, our model has an important new element. We assume that individuals can affect the probability with which they receive a job offer through (costly) variations in search intensity. This modification allows us to overcome the problem of nonuniqueness of equilibria in Hansen's (1985) model when applied to heterogeneous agents settings.[1] We introduce the cost in terms of time, i.e. we assume that the probability of being employed is determined by time spent on job search. The more individuals search, the higher probability of finding a job they have. In terms of Hansen's lotteries, this labor arrangement is equivalent to participating in employment lotteries whose probabilities of success depend on the intensity of search.

There is a large body of literature incorporating heterogeneity in productivity in the RBC models. For example, Cho and Rogerson (1988) and Prasad (1995) construct models with two-member families in which the family members differ in skills. Cho (1995) develops a version of neoclassical model with temporary heterogeneities in individual productivity, and Merz (1996) introduces idiosyncratic productivity shocks in a model with matching frictions. These papers demonstrate that incorporating the heterogeneity in productivity helps to improve on aggregate predictions of the existing models. Also,

[1] Nonuniqueness can occur in the heterogeneous agents model for the following reason. Consider Hansen's model with homogeneous agents at the steady state and assume, as an example, that the probability of employment is 1/2. If agents do not discount the future, an agent's two-period expected utility is $2\cdot[u^e/2 + u^u/2]$, where u^e,u^u are utilities in two states. Observe that there is another allocation that gives the same two-period utility: in the first period a half of the population work and the other half is on vacation, and in the second, the groups interchange, i.e. $[1\cdot u^e + 0\cdot u^u] + [0\cdot u^e + 1\cdot u^u]$. In principle, the two allocations can not be ranked. Hansen (1985) exploits the homogeneity of agents and picks up only the symmetric allocation. A similar argument can not be applied in the heterogeneous agents case.

they show that the RBC models can produce fluctuations of physical units of labor which are larger than those of efficiency units and, thus, can account for the empirical findings of Hansen (1993). This literature, however, does not provide a framework for studying differences in labor decisions across productivity groups and, consequently, does not make it possible to explain the stylized facts outlined above.

Relatively few papers consider models which provide a way to work both at the aggregate and at the individual levels. Kydland (1984) introduces two types of agents, skilled and unskilled, in a standard divisible labor setup. He finds that the volatility of working hours is lower for skilled workers than for unskilled. Ríos-Rull (1993) considers the two-period overlapping generations model with perfectly divisible labor and home production. In his setup, ex ante homogeneous agents acquire skills and thus make different labor choices. The model predicts that skilled individuals work more hours in the market and less at home than unskilled ones. However, his model fails to account for a lower volatility of market hours of skilled workers over the business cycle.

In general, solving RBC models with heterogeneous agents is a complicated task. To simplify the solution procedure, we exploit results from aggregation theory. To be precise, starting from the individual maximization problems, we derive relationships which describe the economy's aggregate behavior in terms of aggregate variables and known productivity parameters. The property of aggregation allows us to solve the model with several agents at low computational costs.

We calibrate the model with five heterogeneous consumers to match key aggregate features of the U.S. economy. The results from simulations show that the model is successful at replicating the stylized facts outlined in the beginning. Specifically, it predicts that high productive agents have a higher employment rate, experience lower fluctuations in employment and work less at home. Also, the model does reasonably well at reproducing cyclical behavior of macroeconomic aggregates in the U.S. economy.

The paper is organized as follows. Section 2 describes the economy and defines the equilibrium. Section 3 derives the equilibrium conditions. Section 4 discusses calibration and simulation procedures. Section 5 reports the results from simulations. Section 6 concludes.

3.2 The model

The economy consists of S types of infinitely lived heterogeneous consumers, an output producing firm and an insurance company. The share of a type $s \in S$ in the total population is $d\omega_s$, $\int_S d\omega_s = 1$. Within each type there is a continuum of identical consumers with names on the unit interval. Agents are heterogeneous across types with respect to their labor productivity. The distribution of productivity parameters, $\{e_s\}_{s \in S}$, is exogenously given and does not change with time; for convenience, we assume $\int_S e_s d\omega_s = 1$.

The representative firm runs a production technology with two inputs, capital, k_t, and efficiency labor, n_t, both of which it rents from households. The production is subject to a multiplicative technology shock, θ_t. The firm maximizes period-by-period profits

$$\max_{\{k_t, n_t\}} \pi_t^p = \theta_t f(k_t, n_t) - r_t k_t - w_t n_t, \tag{3.1}$$

where the production function f has constant returns to scale, is concave, continuously differentiable, strictly increasing with respect to both arguments and satisfies the Inada conditions; θ_t follows a first-order Markov process. The initial level of technology θ_0 is given.

A representative consumer of type $s \in S$ (further, consumer, agent, etc.) maximizes expected life-time utility discounted at the rate $\delta \in (0, 1)$ by choosing leisure and consumption of market and home-made goods. The agent owns the capital stock and rents it to the firm. The capital depreciates at the rate $d \in (0, 1]$. In the beginning of each period, the agent is jobless. To find a job, (s)he needs to search. Job opportunities come at random, depending on individual search time and simple luck. "Good" luck means that the agent gets a job and supplies a fixed number of hours, $\overline{n}$, in exchange for the efficiency wage. In the case of "bad" luck, (s)he does not work in the market. Independently on whether the agent works in the market or not, (s)he can work at home.

Markets are complete, i.e. the agent can insure himself against unemployment as well as against aggregate uncertainty. In the beginning of each period, (s)he buys unemployment insurance. In the same period, the insurance contract pays out one unit of consumption if the agent is unemployed and zero otherwise. A one-period-ahead contingent claim which allows the agent to insure against the aggregate productivity shock $\theta' \in \Theta$ pays one unit of consumption good in period $t+1$ if the shock $\theta_{t+1} = \theta'$ and nothing otherwise; here, Θ denotes the set of all possible realizations of the technology shock.

Therefore, the problem solved by the agent is the following

$$\max_{\{x_{ts}\}} E_0 \sum_{t=0}^{\infty} \delta^t \left\{ \varphi(\pi_{ts}) U\left(c_{ts}^{me}, c_{ts}^{he}, l_{ts}^{e}\right) + (1 - \varphi(\pi_{ts})) U\left(c_{ts}^{mu}, c_{ts}^{hu}, l_{ts}^{u}\right) \right\} \quad (3.2)$$

subject to

$$\begin{aligned} c_{ts}^{me} + k_{t+1s}^{e} + p_{ts}y_{ts} + \textstyle\int_{\Theta} q_t(\theta)\, m_{t+1s}^{e}(\theta)\, d\theta = \\ k_{ts}(1 - d + r_t) + \overline{n} e_s w_t + m_{ts}(\theta_t), \\ c_{ts}^{mu} + k_{t+1s}^{u} + p_{ts}y_{ts} + \textstyle\int_{\Theta} q_t(\theta)\, m_{t+1s}^{u}(\theta)\, d\theta = \\ k_{ts}(1 - d + r_t) + y_{ts} + m_{ts}(\theta_t), \end{aligned} \quad (3.3)$$

$$\begin{aligned} l_{ts}^{e} &= 1 - \overline{n} - h_{ts}^{e} - \pi_{ts}; \\ l_{ts}^{u} &= 1 - h_{ts}^{u} - \pi_{ts}, \end{aligned} \quad (3.4)$$

$$\begin{aligned} c_{ts}^{he} &= g\left(h_{ts}^{e}\right), \\ c_{ts}^{hu} &= g\left(h_{ts}^{u}\right), \end{aligned} \tag{3.5}$$

where $\{x_{ts}\} = \left\{\pi_{ts}, c_{ts}^{mj}, h_{ts}^{j}, k_{t+1s}^{j}, y_{ts}, \left\{m_{t+1s}^{j}(\theta)\right\}_{\theta\in\Theta}\right\}_{t\in T}^{j\in\{e,u\}}$. Here, the superscript $j \in \{e,u\}$ refers to employed and unemployed states; l_{ts}^{j}, c_{ts}^{mj} and c_{ts}^{hj} denote leisure and consumption of market and home-produced goods chosen by the agent in state j. The variables k_{ts}^{j}, y_{ts}, $\left\{m_{t+1s}^{j}(\theta)\right\}_{\theta\in\Theta}$ denote individual holdings of capital, unemployment insurance and contingent claims, respectively. The prices of capital and labor are r_t and w_t. The price of one unit of unemployment insurance is p_{ts} and the price of a contingent claim $\theta \in \Theta$ in period t is given by $q_t(\theta)$. The home technology is represented by the function $g\left(h_{ts}^{j}\right)$, where h_{ts}^{j} is the time spent by the individual on working at home. Variable π_{ts} denotes the time dedicated to searching for a job in period $t \in T$. This time determines the probability of employed and unemployed states, $\varphi(\pi_{ts})$ and $(1-\varphi(\pi_{ts}))$. The function φ satisfies $\varphi' > 0$, $\varphi'' < 0$, i.e. we assume that higher individual search efforts increase the probability of getting the job, but at a diminishing rate. The function U is concave, strictly increasing and twice continuously differentiable in all arguments. The expectations operator, E_0, takes into account that the technology is stochastic. Initial holdings of capital and contingent claims, k_0 and $m_0(\theta_0)$, are given.

The insurance company maximizes period-by-period expected profits with respect to insurance holdings of each type

$$\max_{\{y_{ts}\}_{s\in S}} \pi_t^I = \int_S y_{ts} p_{ts} d\omega_s - \int_S (1-\varphi(\pi_{ts}))\, y_{ts} d\omega_s. \tag{3.6}$$

In other words, we assume that agents' searching time can be perfectly and costlessly monitored by the company.[2] This insurance company is an exten-

[2] If search efforts were not observable, then moral hazard problem would arise. This

sion of Hansen's (1985) risk-sharing arrangement to the heterogeneous case.

Definition. A competitive equilibrium is a set of contingency plans for individual allocations, $\left\{\pi_{ts}, c_{ts}^{mj}, h_{ts}^{j}, k_{t+1s}^{j}, y_{ts}, \left\{m_{t+1s}^{j}(\theta)\right\}_{\theta\in\Theta}\right\}_{t\in T,\, s\in S}^{j\in\{e,u\}}$; the factors of production, $\{k_t, n_t\}_{t\in T}$; prices for the factors of production, $\{r_t, w_t\}_{t\in T}$; prices for unemployment insurance, $\{p_{ts}\}_{t\in T,\, s\in S}$ and prices for contingent claims, $\{q_t(\theta)\}_{t\in T,\, \theta\in\Theta}$ such that given the prices, all agents maximize their utilities (3.2) subject to (3.3) − (3.5), the firm maximizes its profit (3.1), the insurance company maximizes its profit (3.6) and all markets clear. We assume that in equilibrium all model variables are nonnegative and the probabilities satisfy $0 \leq \varphi(\pi_{ts}) \leq 1$ for all $s \in S$ and $t \in T$.

3.3 Analytic results

It is a well-established fact that in an economy without distortions and with complete markets, a competitive equilibrium allocation belongs to the set of Pareto optimal allocations (First Welfare Theorem) and any Pareto optimal allocation can be supported as a competitive equilibrium with transfers (Second Welfare Theorem). It is also known that each Pareto optimal allocation is a solution to the so-called planner's problem which is the problem of maximizing the weighted sum of individual utilities subject to economy's resource constraint. These results imply that the equilibrium in a heterogeneous model like ours can be computed by using the following iterative procedure: fix weights on individual utilities, solve the planner's problem and use the solution to check whether the assumed weights are consistent with the individual life-time constraints; iterate on the above steps until the fixed point weights are found. More details on this algorithmic procedure can be found, e.g., in Garcia-Mila, Marcet and Ventura (1995).

issue is difficult to model, however, in a dynamic economy like ours; we leave it for future research.

The above algorithm is costly in terms of computational time because each iteration requires finding a solution to a dynamic stochastic model. Moreover, the cost increases with the number of agents in the economy because having more agents implies more parameters (weights) to iterate on. Consequently, the application of this algorithm is in practice very limited.

The solution procedure simplifies substantially if the aggregate dynamics of a heterogeneous economy do not depend on utility weights. This case is known as perfect or Gorman aggregation, see, e.g., Mas-Colell et al. (1995). Under perfect aggregation one can, first, solve for aggregate quantities and then recover individual variables from the aggregate solution. This property makes it possible to extend the model to include any number of agents at no additional computational cost compared to the representative agent case. For the reasons discussed above, we restrict our attention only to a version of the model which is compatible with aggregation. Specifically, we assume:

- A1: all agents have identical momentary utilities of the form

$$U(c^m, c^h, l) = \frac{\left[\left(c^m + c^h\right)^{\gamma} l^{1-\gamma}\right]^{1-\sigma} - 1}{1-\sigma}; \tag{3.7}$$

- A2: the home production function is $g(h) = Ah$;

- A3: the function φ has the form

$$\varphi(\pi) = \beta_1 + \beta_2 \ln(1 + \beta_3 \pi), \qquad \beta_1, \beta_2, \beta_3 > 0. \tag{3.8}$$

where $\beta_1, \beta_2, \beta_3$ are some parameters.

Let us comment on these assumptions. According to A1, market and home consumption goods are perfect substitutes. This assumption is obviously restrictive, but it is consistent with U.S. data. Eichenbaum and Hansen

(1990) find that one cannot reject the hypothesis about perfect substitutability between market goods and services from consumer durables (which can be interpreted as home-made goods). Further, following Ríos-Rull (1993), in A2 we assume that the home technology is linear in labor (i.e. it does not require inputs of capital) and that all individuals are equally productive working at home. This captures two important features of home activities that most of home work is labor intensive and that agents with different market productivities have roughly the same skills in preparing meals, cleaning, child care, etc. Finally, according to A3, the search technology is given by the flexible functional form (3.8), where the parameter β_1 corresponds to the probability of becoming employed if search time is zero and the parameters β_2 and β_3 reflect how the initial probability increases due to search. Under (3.8), the inverse of the first derivative of the function φ is linear in π; this property helps us to achieve aggregation.

The individual optimality conditions are derived in the appendix. It is also shown that risk averse agents will choose to insure themselves fully against unemployment and that employed and unemployed agents of the same type will always hold the same amount of capital and contingent claims. Here, we summarize the individual optimality conditions which we obtain under A1-A3

$$\pi_{ts} = \beta_2 \overline{n} \left(A^{-1} e_s w_t - 1 \right) - \beta_3^{-1}, \tag{3.9}$$

$$c_{ts}^{mu} + A h_{ts}^{u} = c_{ts}^{me} + A h_{ts}^{e} \equiv c_{ts}, \tag{3.10}$$

$$1 - h_{ts}^{u} - \pi_{ts} = 1 - \overline{n} - h_{ts}^{e} - \pi_{ts} \equiv l_{ts}, \tag{3.11}$$

$$\gamma A \cdot l_{ts} = (1 - \gamma) \cdot c_{ts}, \tag{3.12}$$

$$c_{ts}^{-\sigma} = \delta E_t \left[\left(1 - d + r_{t+1}\right) c_{t+1s}^{-\sigma} \right], \tag{3.13}$$

$$c_{ts} = \frac{c_t \lambda_s^{1/\sigma}}{\int_S \lambda_s^{1/\sigma} d\omega_s}, \tag{3.14}$$

$$\varphi(\pi_{ts}) c_{ts}^{me} + \left(1 - \varphi\left(\pi_{ts}\right)\right) c_{ts}^{mu} + k_{t+1s} + \int_\Theta q_t\left(\theta\right) m_{t+1s}\left(\theta\right) d\theta = k_{ts}(1 - d + r_t) + \varphi(\pi_{ts}) \overline{n} e_s w_t + m_{ts}\left(\theta_t\right), \tag{3.15}$$

where $c_t \equiv \int_S c_{ts} d\omega_s$ is total (market plus home) aggregate consumption and λ_s is the weight on utility of individual s in the associated planner's problem.

Let us briefly discuss the individual optimality conditions and analyze some of the model's implications at the individual level.

Equation (3.9) is informative and helps in understanding several properties of the model. First, it shows how innovations to technology induce fluctuations in the labor market. In particular, a positive shock increases the return to working in market sector compared to that in home sector and, in response, agents choose to search more for a market job. This implies that the level of employment increases. Secondly, the condition demonstrates that the model can account for fact (i) discussed in the introduction. According to (3.9), workers with high productivity always devote more time to searching and, therefore, always have a higher employment rate than workers whose productivity is low. Finally, the condition indicates that the model's predictions are consistent with the empirical regularity (ii). Indeed, using (3.8) and (3.9), one can show that $\partial\left(\partial\varphi_{ts}/\partial w_t\right)/\partial e_s < 0$. This inequality implies that the level of employment of highly productive agents is less responsive to wage fluctuations than that of the agents whose productivity is low or, in other words, that the volatility of employment in our model decreases with the productivity level.

It is a well-known fact that Hansen's (1985) model has one undesirable property: if leisure is a normal good, the unemployed agent enjoys a higher level of utility than the employed does. It happens because, in equilibrium, employed and unemployed agents have the same consumption level, but unemployed have a higher level of leisure. Our model does not have this implication: according to (3.10) and (3.11), total consumption and leisure of employed and unemployed agents of the same type are equal, and, thus, both enjoy the same level of utility.

Equations (3.12) and (3.13) determine the marginal rate of substitution between current consumption and leisure and between current and expected future consumption respectively. Further, condition (3.14) states that total consumption of each individual is a constant share of total aggregate consumption. This result is a consequence of complete markets under which the ratio of marginal utilities of any two agents remains constant over time. Finally, due to perfect risk sharing, agents of the same type face the same budget constraint (3.15) in both employed and unemployed states.

Using individual optimality conditions, we can derive the following set of restrictions on the aggregate variables of the economy

$$c_t^{-\sigma} = \delta E_t \left[(1 - d + r_{t+1}) \, c_{t+1}^{-\sigma} \right], \tag{3.16}$$

$$\pi_t \equiv \int_S \pi_{ts} d\omega_s = \beta_2 \overline{n} \left(A^{-1} w_t - 1 \right) - \beta_3^{-1}, \tag{3.17}$$

$$\varphi_t \equiv \int_S \varphi\left(\pi_{ts}\right) d\omega_s = \beta_1 + \beta_2 \int_S \ln \left[\beta_2 \beta_3 \overline{n} \left(\frac{e_s w_t}{A} - 1 \right) \right] d\omega_s, \tag{3.18}$$

$$n_t \equiv \overline{n} \int_S \varphi\left(\pi_{ts}\right) e_s d\omega_s = \overline{n} \beta_1 + \overline{n} \beta_2 \int_S \ln \left[\beta_2 \beta_3 \overline{n} \left(\frac{e_s w_t}{A} - 1 \right) \right] e_s d\omega_s, \tag{3.19}$$

$$c_t/\gamma - A(1-\pi_t) + A\overline{n}\varphi_t + k_{t+1} = \theta_t f(k_t, n_t) + k_t(1-d), \qquad (3.20)$$

where $k_t \equiv \int_S k_{ts} d\omega_s$. Condition (3.16) results from (3.13) and (3.14). Equations (3.17) – (3.19) follow after substituting (3.9) into the definitions of the corresponding variables. The aggregate resource constraint (3.20) is obtained after integrating the individual budget constraint (3.15), substituting conditions (3.10) – (3.12) and using the fact that, in equilibrium, aggregate holdings of contingent claims are equal to zero.

Equations (3.16)–(3.20) and the prices, $r_t = \theta_t \partial f/\partial k_t$ and $w_t = \theta_t \partial f/\partial n_t$, determine uniquely the equilibrium aggregate quantities $\{c_t, n_t, k_{t+1}, \pi_t, \varphi_t\}$, provided that the equilibrium exists and is unique. Observe that none of these conditions depends on individual variables. Precisely because of this fact we can solve the model without iterating on the utility weights.

Once the equilibrium aggregate quantities $\{c_t, n_t, k_{t+1}, \pi_t, \varphi_t\}$ are known, individual variables are simple to recover. Individual search time and the probability of being employed can be found using (3.9) and (3.8) respectively. To compute the remaining variables, we need to use the individual life-time budget constraint (this condition is derived in the appendix)

$$E_0 \sum_{t=0}^{\infty} \delta^t \frac{U_1(c_{ts}, l_{ts})}{U_1(c_{0s}, l_{0s})} \left[\varphi(\pi_{ts}) c_{ts}^{me} + (1-\varphi(\pi_{ts})) c_{ts}^{mu} - \varphi(\pi_{ts}) \overline{n} e_s w_t\right] = k_0', \qquad (3.21)$$

where U_1 is marginal utility of consumption and $k_0' = k_0(1-d+r_0) + m_0(\theta_0)$. This condition restricts the expected discounted value of life-time difference between consumption and labor income to be equal to initial endowment. Substituting (3.10) – (3.12), (3.14) in (3.21) and rearranging the terms, we obtain

$$\frac{\lambda_s^{1/\sigma}}{\int_S \lambda_s^{1/\sigma} d\omega_s} = \frac{k_0' - E_0 \sum_{t=0}^{\infty} \delta^t \left(c_0/c_t\right)^{\sigma} \left[\varphi(\pi_{ts})\overline{n}\left(A - e_s w_t\right) - A(1-\pi_{ts})\right]}{E_0 \sum_{t=0}^{\infty} \delta^t \left(c_0/c_t\right)^{\sigma} \left(c_t/\gamma\right)}. \tag{3.22}$$

This equation makes it possible to compute the individual utility weights. Given the weights, we can recover individual total consumption from (3.14) and subsequently, restore market and home consumption using (3.10) – (3.12).

3.4 Calibration and simulation procedures

We now move on to calibrate the model in order to be able to carry out quantitative experiments. The calibration of many parameters is standard (see e.g. Cooley and Prescott's, 1995, account of the calibration procedure). Because of the heterogeneous agents setup, however, we need to calibrate some further parameters on individual characteristics. In particular, we are to choose the number of heterogeneous agents in the model and their productivity levels.

Castañeda, Díaz-Giménez and Ríos-Rull (1995) and Ríos-Rull (1993) divide the Panel Study of Income Dynamics (PSID) sample for 1969-1982 in five equally-sized groups according to the individual wages and computed the groups' averages of several individual variables including the wage, the level of employment, the standard deviation of employment and hours worked at home. We use the results of these studies for calibrating the model and also, for testing the validity of the model's predictions. Given that the data are computed for five groups, in a subsequent paper we consider a version of the model with five heterogeneous agents. We will use the wage as a proxy for productivity.

Table 1 reproduces the levels of productivity and employment by groups.

The data in *Table* 1 allow us to compute aggregate employment, $\varphi = \sum_{s=1}^{5} \varphi_s$, and aggregate labor input, $n = \sum_{s=1}^{5} \varphi_s e_s$. We will calibrate the model so that in the steady state it reproduces these two moments.

We assume that market output, y_t^m, is produced according to the Cobb-Douglas production function, $y_t^m = \theta_t k_t^\alpha n_t^{1-\alpha}$, and that the technology shock follows the law of motion $\ln \theta_t = \rho \ln \theta_{t-1} + \varepsilon_t$, where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$; the autocorrelation coefficient, ρ, and the standard deviation, σ_ε, are equal to 0.95 and 0.01 respectively. Aggregate output produced at home is $y_t^h = Ah_t$, where $h_t \equiv \int_S (\varphi(\pi_{ts}) h_{ts}^e + (1 - \varphi(\pi_{ts})) h_{ts}^u) \, d\omega_s$.

To make our results comparable to existing studies, we use standard parameters whenever it is possible. The values $(d, \delta, \alpha, h, \bar{n})$ are borrowed from Benhabib et al. (1991), where h denotes steady state level of average home hours. The ratio of net investment to output, i/y^m, is set to 0.25, the value which is used in the RBC models without home production, see e.g. Cooley and Prescott (1995). This is done because in our case home technology does not require capital. Given that in the steady state investment is used to cover depreciation of capital, $i = dk$, the assumed value i/y^m implies the capital to output ratio $k/y^m = 10$. The latter is roughly consistent with the estimate of capital to output ratio in the U.S. economy obtained by Christiano and Eichenbaum (1992) $(k/y^m = 10.62)$.

To calibrate average search time π, we use the following considerations. Barron and Gilley (1981) estimate the time spent by the typical unemployed individual on the job search as approximately eight and two-third hours per week. The results of Arellano and Meghir (1992), and Burgess and Low (1992) suggest that about one-third of employed agents participate in on-the-job search. Assuming that both employed and unemployed have the same intensity of search, these numbers imply that the average agent spends about 2.4% of his discretionary time (total time minus personal care) on job search.

Table 2 summarizes the parameters which are fixed for all simulations.

The remaining parameters to choose are $(\sigma, A, \gamma, \beta_1, \beta_2, \beta_3)$. Regarding the coefficient of risk aversion, σ, we consider two different values, namely, 1.0 and 5.0. Further, using the properties of the Cobb-Douglas production function and equations (3.10) – (3.12), one can derive the following relationships:

$$A = \frac{y^h/y^m}{h/n} \cdot \left(\frac{1/\delta - 1 + d}{\alpha} \right)^{\alpha/(\alpha-1)}, \quad \gamma = \frac{1 - i/y^m + y^h/y^m}{1 - i/y^m + y^h/y^m \cdot (1 - \pi - \varphi \cdot \bar{n})/h},$$

where y^h/y^m denotes the ratio of home output to market output. Given y^h/y^m, these formulas provide a basis for calibrating A and γ. To calibrate the ratio y^h/y^m, we use the results of existing studies. Eisner (1988) provides a summary of the literature measuring the magnitude of the home production and reports estimates of the ratio y^h/y^m in the interval of $(0.2, 0.5)$. Benhabib et al. (1991) argue that in a model without government taxation, the relative size of the home production may not be too high. Consequently, they use the ratio 0.26. Presumably, in our case, this ratio might be even lower since we assume that the home technology does not require capital. Based on this, we consider two alternative values, 0.15 and 0.20.

We are left to calibrate the parameters of the search technology (3.8). Evaluating equations (3.17), (3.18), (3.19) in the steady state and substituting the values (π, φ, n), we obtain the system of three equations with three unknowns, $(\beta_1, \beta_2, \beta_3)$. The solution to this system gives us the values of the search parameters.

Table 3 reports the parameters $(A, \gamma, \beta_1, \beta_2, \beta_3)$ computed under two values of the home output to market output ratio.

For all numerical experiments, we set the initial aggregate capital, k_0, equal to the steady state value and assume that the initial technology shock is $\theta_0 = 1$.

We solve for aggregate quantities $\{c_t, n_t, k_{t+1}, \pi_t, \varphi_t\}$ which satisfy equations (3.16) – (3.20) by using the parametrized expectations algorithm, see e.g. Den Haan and Marcet (1990). The length of simulations is 10000; the conditional expectation in (3.16) is parameterized by a second-order exponentiated polynomial. To find utility weights, we approximate the conditional expectations in (3.22) by the corresponding averages which are computed across 400 simulated data sets of the length 10000. The statistics in *Table* 4 and *Table* 5 are the averages of the corresponding variables. The averages are computed across 400 simulated data sets of the length 115. Numbers in parenthesis are standard deviations of the statistics. Before computing the second moments of aggregate variables, we log and detrend the series by using the Hodrick-Prescott filter under the standard penalty for variation in quarterly data, 1600.

3.5 Simulation results

This section analyses quantitative implications of the model. First, we focus on aggregate dynamics. After that we turn to the predictions at the individual level.

3.5.1 Aggregate predictions

In *Table* 4, we report the first and the second moments of aggregate variables generated by the representative agent (RA) version of the model and by the heterogeneous agent version of the model under three alternative sets of the parameters. For comparison, we also provide the predictions of the standard representative agent model with home production and the corresponding statistics for the U.S. economy.

At the aggregate level, most of the properties of the heterogeneous model are similar to what is found in standard RBC models with homogenous agents. Similar to the model of Benhabib et al. (1991), our model produces a negative cross-correlation of market output with hours worked at home. Further, the model predicts that efficiency hours are less volatile than employment, implying that the low productive workers represent a larger fraction of labor force during the expansion than during the recession. This implication is in agreement with the empirical finding of Hansen (1993). Comparing the cases $\sigma = 1.0$ and $\sigma = 5.0$ shows that an increase in the coefficient of risk aversion does not affect significantly the aggregate predictions of the model except for the correlation between capital and output which becomes too low.

The main shortcoming of the model is the small fluctuations of employment over the business cycle. The results imply that under the benchmark value of $y^h/y^m = 0.20$, the volatility of employment is 0.128 which is about 10 times less than the empirical counterpart. Comparison of the cases $y^h/y^m = 0.15$ and $y^h/y^m = 0.20$ shows that an increase in the ratio y^h/y^m improves on this statistic. In our model, however, this ratio cannot be too high. The reason is that a high ratio y^h/y^m also implies a high return to home hours. This can result in that individuals with low productivity have a return to working at home which exceeds the market wage and, therefore, they will choose not work in the market at all. This does not seem like an entirely convincing explanation for unemployment.

The implied low volatility of labor market variables is not particularly related to the present model but is instead a more generic property of heterogeneous agents models. For example, in the overlapping generations model with heterogeneous agents considered by Ríos-Rull (1993), the standard deviation of hours is 0.089 (see Ríos-Rull's table 7) which is even further away from the empirical estimates than our results. In the two-agents version of the standard neoclassical model studied by Garcia-Mila et al. (1995), this statistic is 0.001 (see their table 9). In fact the last paper argues that if the heterogeneous model is calibrated to match cross sectional observations, then such models have more difficulties in accounting for time series stylized facts than a similar representative agent setups.

Our results confirm this conjecture. As we see from the table, the representative agent version of our model ($e_s = 1$) can generate the standard deviation of employment equal to 0.813 which is reasonably close to the corresponding empirical value. The improvement relative to the heterogeneous agents case is due to a single difference in the calibration procedure, which is the choice of the parameter β_2. Specifically, in the representative agent case, $\varphi\overline{n} = n$ and, consequently, two of the restrictions (3.17) – (3.19) used for calibrating β's in the heterogeneous model become identical. Therefore, we set β_2 to an arbitrary value, namely, 0.80, and find β_1 and β_3 from the remaining two conditions; this gives us $(\beta_1, \beta_2, \beta_3) = (0.87, 0.80, 2.04)$. It turns out, however, that these values cannot be assumed for calibrating the heterogeneous model because they imply negative search time for low productivity groups. This simple exercise indicates that the set of the parameters which is consistent with cross sections can be very different from the one under which the model has the best chance to account for time series facts.

Another deficiency of the model is the degree to which employment and productivity (y^m/φ) are correlated with output. In the model these correlations are nearly perfect while in the data they are substantially lower. This failure is not surprising given that most of the existing RBC models dramatically exaggerate these statistics (for a discussion see, e.g., Christiano and Eichenbaum, 1991). Our results indicate that this problem cannot be resolved within our simple framework.

3.5.2 Individual predictions

In *Table* 5, we report the levels of employment, the standard deviations of employment, time worked at home, market consumption and search time for the five productivity groups predicted by the model under two alternative values of the home output to market output ratio. For comparison, we also provide the corresponding quantities in the U.S. economy. We report only the case $\sigma = 1.0$ since the case $\sigma = 5.0$ implies practically identical results.

The model can successfully account for a number of the moments of individual variables. It predicts that high productive individuals search more, and, as a result, have a higher employment rate. Furthermore high productive agents experience lower fluctuations in employment, work less at home and consume more. All of these predictions are in line with the empirical evidence. Notice also that the levels of employment are close to those in the data while the standard deviations of employment are somewhat lower than the empirical values. These results are very promising and imply that key features of the individual data can be accounted for by this rather simple heterogeneous agents model.

Regarding the time worked at home, Ríos-Rull's (1993) estimate of average annual home hours is equal to 461, which in terms of normalized to unity discretionary time corresponds to 0.075. Benhabib et al. (1991) argue that the average share of time worked at home is substantially higher, namely, 0.28. Given that we assume the latter value for calibrating the model, we normalize the data of Ríos-Rull (1993) respectively. As we see, the model can successfully account for the corrected distribution of home hours.

One problem is that the model generates unrealistically little cross-group variability in consumption and, consequently, in welfare levels (up to few percents). This shortcoming is due to the assumption that all individuals have the same initial wealth. Indeed, in our model market consumption increases with the agent's utility weight, which in turn, is an increasing function of the initial wealth. As in the micro data, the correlation between the level of productivity and wealth is positive,[3] therefore, the variability of consumption will rise once the heterogeneity in initial wealth is introduced. While we can computationally handle such heterogeneities very easily, we did not include them because of the lack of empirical evidence. Thus, it is somewhat unclear whether the model is consistent or inconsistent with the data along this dimension.

Finally, the model implies that time spent by agents on job search increases across productivity groups from several minutes to more than one hour per day. In fact, the implications of the model with respect to search are difficult to test because most of the existing data sets on the individual behavior do not provide the corresponding data. Several empirical papers construct measures of the intensity of search and use them for analyzing the relation between search, employment and productivity. Barron and Gilley (1981) find that the level of employment is positively related to search. Barron and Mellow (1979) analyze the determinants of search intensity for unemployed workers and find a strong positive effect of education; Arellano and Meghir (1991) report the same tendency for the employed. The relation between search and wages depends on the employment status: for unemployed, past wages have a positive effect on search intensity (Barron and Mellow, 1979); for employed, the effect of wages is negative (Arellano and Meghir, 1991).

In short, the model's predictions at the individual level are consistent with all empirical regularities except for a negative effect of wages on the search intensity of employed. Arellano and Meghir (1991) argue that the last tendency reflects the fact that search time has a higher opportunity cost for workers whose wages are high than for those whose wages are low. Provided that their inference is correct, the failure of the model results from

[3]See e.g. Garcia-Mila, Marcet and Ventura (1995).

the assumption of the one-period labor contracts. Indeed, in our model the agents may choose not to work in the market instead of searching because they get unemployed at the end of each period; all of them have the same opportunity cost of search, which is home work. Introducing a possibility of long-term labor contracts would presumably help to improve on the model's predictions along this dimension.

3.6 Concluding comments

We have analyzed a quantitative general equilibrium model with permanent heterogeneity in productivity with the aim of explaining differential responses of labor supply across productivity groups. The simulation results show that the model is successful in reproducing most of the key features of the data. In particular, at the individual level, which is our special matter of interest, it can account for the stylized facts which we outline in the introduction. At the aggregate level, it can generate the cyclical behavior of aggregate quantities which is reasonably close to that in the U.S. economy. Yet the heterogeneous version of our model does not produce better aggregate predictions than the associated representative agent setup. This result suggests that introducing heterogeneity is not a necessary condition for a model to be successful in explaining macroeconomic fluctuations in the real world economies. We also stress the computational ease with which our analysis was carried out. This computational aspect of the analysis was obtained due to the use of aggregation theory.

The model failed along some dimensions and this provides valuable insights into the main avenues for future research. Concerning aggregate predictions, it produces too little volatility of employment. As we have already pointed out, this shortcoming is partly due to heterogeneity. Specifically, it is more difficult to match both time series and cross sectional facts than only time series facts as in the representative agent case. To some extent, the lack of volatility of aggregate employment is due to the linear home technology. Presumably, introducing a more general production function for home goods would improve the model's performance. The main shortcoming at the individual level is that the model generates a positive correlation between productivity and search time for all workers while in the data this correlation is positive for the unemployed but negative for the employed. This deficiency is attributed to a simplified structure of the labor markets. A reasonable guess is that the inclusion of a possibility of long-term contracts will result in that highly productive employed workers have higher opportunity cost of search and, therefore, a lower intensity of search than workers with low productivity. Such extensions will be considered in future research.

3.7 Appendix

To derive the individual optimality conditions, we use the value function representation of the agent's problem

$$\begin{aligned} \max\nolimits_{\{x_{ts}\}} \; V_s\left(k_t, k_{ts}, \{m_{ts}(\theta)\}_{\theta\in\Theta}, \theta_t\right) = \\ \varphi(\pi_{ts})\left[U\left(c_{ts}^{me} + Ah_{ts}^{e}, l_{ts}^{e}\right) + \delta E_t V_s\left(k_{t+1}, k_{t+1s}^{e}, \left\{m_{t+1s}^{e}(\theta)\right\}_{\theta\in\Theta}, \theta_{t+1}\right)\right] + \\ (1-\varphi(\pi_{ts}))\left[U\left(c_{ts}^{mu} + Ah_{ts}^{u}, l_{ts}^{u}\right) + \delta E_t V_s\left(k_{t+1}, k_{t+1s}^{u}, \left\{m_{t+1s}^{u}(\theta)\right\}_{\theta\in\Theta}, \theta_{t+1}\right)\right] \end{aligned} \tag{3.23}$$

$$\text{s.t.} \qquad (3.3),\ (3.4),$$

where $\{x_{ts}\} = \left\{\pi_{ts}, c_{ts}^{mj}, h_{ts}^{j}, k_{t+1s}^{j}, y_{ts}, \left\{m_{t+1s}^{j}(\theta)\right\}_{\theta\in\Theta}\right\}_{t\in T}^{j\in\{e,u\}}$ and V_s is the value function of agent $s \in S$.

The first order conditions for unemployment insurance holdings, capital, hours worked at home, and holdings of contingent claims in employed and unemployed states respectively are

$$\varphi(\pi_{ts})p_{ts}U_1(c_{ts}^e, l_{ts}^e) = (1 - \varphi(\pi_{ts}))\,(1 - p_{ts})\,U_1(c_{ts}^u, l_{ts}^u); \tag{3.24}$$

$$U_1(c_{ts}^e, l_{ts}^e) = \delta E_t \frac{\partial V_s\left(k_{t+1}, k_{t+1s}^e, \{m_{t+1s}^e(\theta)\}_{\theta\in\Theta}, \theta_{t+1}\right)}{\partial k_{t+1s}^e},$$
$$U_1(c_{ts}^u, l_{ts}^u) = \delta E_t \frac{\partial V_s\left(k_{t+1}, k_{t+1s}^u, \{m_{t+1s}^u(\theta)\}_{\theta\in\Theta}, \theta_{t+1}\right)}{\partial k_{t+1s}^u}; \tag{3.25}$$

$$AU_1(c_{ts}^e, l_{ts}^e) + U_2(c_{ts}^e, l_{ts}^e) = 0,$$
$$AU_1(c_{ts}^u, l_{ts}^u) + U_2(c_{ts}^u, l_{ts}^u) = 0; \tag{3.26}$$

$$U_1(c_{ts}^e, l_{ts}^e)q_t\,(\theta_{t+1}) = \frac{\partial V_s\left(k_{t+1}, k_{t+1s}^e, \{m_{t+1s}^e(\theta)\}_{\theta\in\Theta}, \theta_{t+1}\right)}{\partial m_{t+1s}^e(\theta_{t+1})} P\,(\theta_{t+1}, \theta_t)\,,$$
$$U_1(c_{ts}^u, l_{ts}^u)q_t\,(\theta_{t+1}) = \frac{\partial V_s\left(k_{t+1}, k_{t+1s}^u, \{m_{t+1s}^u(\theta)\}_{\theta\in\Theta}, \theta_{t+1}\right)}{\partial m_{t+1s}^u(\theta_{t+1})} P\,(\theta_{t+1}, \theta_t)\,; \tag{3.27}$$

where $c_{ts}^j = c_{ts}^{mj} + Ah_{ts}^j$ is the agent's total consumption; U_i refers to the derivative of the utility function with respect to the i-th argument. Notice that given that the shock follows a first order Markov process, the probability distribution of θ_{t+1}, $P\,(\theta_{t+1}, \theta_t)$, depends only on the previous period shock, θ_t, and not on the whole history of the economy.

The equilibrium price of insurance is $p_{ts} = (1 - \varphi_{ts}\,(\pi_{ts}))$. This together with (3.25) gives the risk sharing condition

$$U_1(c_{ts}^e, l_{ts}^e) = U_1(c_{ts}^u, l_{ts}^u). \tag{3.28}$$

From the last equality and conditions (3.26) it follows that

$$U_2(c_{ts}^e, l_{ts}^e) = U_2(c_{ts}^u, l_{ts}^u). \tag{3.29}$$

Equations (3.25), (3.27) and (3.28) imply that the holdings of capital and contingent claims in both states are the same, i.e. $k^e_{t+1s} = k^u_{t+1s}$ and $m^e_{t+1s}(\theta_{t+1}) = m^u_{t+1s}(\theta_{t+1})$. Substituting these results into the state contingent constraints (3.3) gives the equilibrium holdings of unemployment insurance

$$y_{ts} = \overline{n}e_s w_t - c^{me}_{ts} + c^{mu}_{ts}. \tag{3.30}$$

Finding $\partial V_s / \partial k_{ts}$, updating it and combining the resulting condition with (3.25) and (3.28), we obtain the standard intertemporal condition

$$U_1(c^e_{ts}, l^e_{ts}) = \delta E_t \left[(1 - d + r_{t+1}) U_1 \left(c^e_{t+1s}, l^e_{t+1s} \right) \right]. \tag{3.31}$$

Similarly, finding $\partial V_s / \partial m_{ts}(\theta_t)$, updating it and using (3.27) and (3.28), we get

$$\delta U_1(c^e_{t+1s}, l^e_{t+1s}) \cdot P(\theta_{t+1}, \theta_t) = U_1(c^e_{ts}, l^e_{ts}) \cdot q_t(\theta_{t+1}). \tag{3.32}$$

This condition implies that the ratio of marginal utility of any two agents s, $s' \in S$ is constant over time and can be represented as

$$\frac{U_1(c^e_{ts}, l^e_{ts})}{U_1(c^e_{ts'}, l^e_{ts'})} = \frac{\lambda_{s'}}{\lambda_s}, \tag{3.33}$$

where λ_s is the agent-specific parameter.[4] Using condition (3.30) and the results that $k^e_{t+1s} = k^u_{t+1s}$ and $m^e_{t+1s}(\theta_{t+1}) = m^u_{t+1s}(\theta_{t+1})$, we can replace the state contingent constraints (3.3) by a single one

$$\varphi(\pi_{ts}) c^{me}_{ts} + (1 - \varphi(\pi_{ts})) c^{mu}_{ts} + k_{t+1s} + \textstyle\int_\Theta q_t(\theta) m_{t+1s}(\theta) d\theta = \\ k_{ts}(1 - d + r_t) + \varphi(\pi_{ts}) \overline{n} e_s w_t + m_{ts}(\theta_t). \tag{3.34}$$

[4] In fact, λ_s is agent's utility weight in the associated planner's problem.

Therefore, the agent faces the same constraint (3.34) independently on his employment status. Maximization of (3.23) subject to (3.4) and (3.34) with respect to π_{ts} gives

$$U(c^e_{ts}, l^e_{ts}) - U(c^u_{ts}, l^u_{ts}) + U_1(c^e_{ts}, l^e_{ts})\left(\overline{n}e_s w_t - c^{me}_{ts} + c^{mu}_{ts}\right) = \frac{U_2(c^e_{ts}, l^e_{ts})}{\varphi'(\pi_{ts})}. \quad (3.35)$$

Under the functions U and φ given in (3.7) and (3.8), conditions (3.35) ,(3.28), (3.29), (3.26), (3.31), (3.33) and (3.34) can be rewritten as (3.9) – (3.15) respectively.

Let us derive the individual expected life-time budget constraint. Multiplying (3.34) by $\delta U_1(c^e_{ts}, l^e_{ts})/U_1(c^e_{t-1s}, l^e_{t-1s})$, substituting (3.32) and taking conditional expectation, E_{t-1}, from both sides, we get

$$k_{ts} + E_{t-1}\left[\delta \frac{U_1(c^e_{ts}, l^e_{ts})}{U_1(c^e_{t-1s}, l^e_{t-1s})} m_{ts}(\theta_t)\right] =$$

$$E_{t-1}\left[\delta \frac{U_1(c^e_{ts}, l^e_{ts})}{U_1(c^e_{t-1s}, l^e_{t-1s})}\left(c^{me}_{ts}\varphi(\pi_{ts}) + (1-\varphi(\pi_{ts}))\, c^{mu}_{ts} - \varphi(\pi_{ts})\,\overline{n}e_s w_t\right)\right] +$$

$$E_{t-1}\left\{\delta \frac{U_1(c^e_{ts}, l^e_{ts})}{U_1(c^e_{t-1s}, l^e_{t-1s})}\left[k_{t+1s} + E_t\left(\delta \frac{U_1(c^e_{t+1s}, l^e_{t+1s})}{U_1(c^e_{ts}, l^e_{ts})} m_{t+1s}(\theta_{t+1})\right)\right]\right\}.$$

Starting from $t = 0$, we use this condition to substitute recursively for future variables. Applying the law of iterative expectations to the resulting equation, we obtain the expected life-time budget constraint (3.21).

Table 1. The distribution of productivity and employment in the U.S. economy

Variable	Group 1	2	3	4	5
Productivity, e_s	0.415	0.694	0.887	1.144	1.859
Employment, φ_s	0.846	0.905	0.920	0.924	0.925

Source: Castañeda, Diaz-Giménez and Ríos-Rull (1995, table 8); the average productivity is normalized to 1.

Table 2. The model's parameters

Parameter	d	δ	α	h	$\tilde{n}$	n	φ	i/y^m	π
Value	0.025	0.99	0.36	0.28	0.33	0.302	0.904	0.25	0.024

Table 3. The model's parameters

Case/Parameter	γ	A	β_1	β_2	β_3
$y^h/y^m=0.15$	0.808	0.599	0.863	0.0300	173.8
$y^h/y^m=0.20$	0.770	0.804	0.805	0.0387	758.6

Table 4. Selected statistics for U.S. and artificial economies

Statistic	RA model	Heterogeneous model			RA model (BRW) [a]	U.S. economy [a]
	$y^h/y^m=0.20$, $\sigma=1.0$	$y^h/y^m=0.15$, $\sigma=1.0$	$y^h/y^m=0.20$, $\sigma=1.0$	$y^h/y^m=0.20$, $\sigma=5.0$		
	First moments					
y^h/y^m	0.207	0.152	0.202	0.201	0.260 [c]	0.2-0.5 [c]
k/y^m	10.279	10.256	10.262	10.358	-	10.00 [c]
i/y^m	0.255	0.256	0.256	0.260	-	0.250 [c]
φ	0.906	0.904	0.904	0.904	-	0.904 [c]
h	0.284	0.283	0.282	0.282	0.280 [c]	0.280 [c]
π	0.025	0.024	0.024	0.024	-	0.024 [c]
	Percentage standard deviations					
σ_k	0.350	0.355	0.362	0.326	0.393	0.661
	(0.074)	(0.076)	(0.075)	(0.068)		
$\sigma_{y^m/\varphi}$	0.608	1.244	1.213	1.192	0.667	0.905
	(0.080)	(0.164)	(0.150)	(0.150)		
σ_i	4.092	4.155	4.292	3.742	4.668	4.907
	(0.533)	(0.539)	(0.530)	(0.482)		
σ_c^m	0.589	0.424	0.433	0.479	0.872	0.853
	(0.094)	(0.072)	(0.071)	(0.062)		
σ_h	2.445	0.576	0.574	0.644	1.197	-
	(0.408)	(0.090)	(0.086)	(0.086)		
σ_n	-	0.057	0.095	0.093	-	-
		(0.008)	(0.012)	(0.013)		
σ_φ	0.813	0.067	0.128	0.124	1.283 [b]	1.496 [b]
	(0.109)	(0.009)	(0.018)	(0.019)		
σ_π	38.598	2.091	1.994	1.951	-	-
	(22.808)	(0.278)	(0.247)	(0.246)		
σ_y^m	1.420	1.310	1.341	1.315	1.710	1.740
	(0.187)	(0.173)	(0.165)	(0.165)		
	Correlations with output					
$corr(k, y^m)$	0.118	0.098	0.109	0.014	0.090	0.280
	(0.071)	(0.065)	(0.066)	(0.067)		
$corr(y^m/\varphi, y^m)$	1.000	1.000	0.999	1.000	0.750	0.510
	(0.000)	(0.000)	(0.000)	(0.000)		
$corr(i, y^m)$	0.982	0.986	0.984	0.996	0.940	0.960
	(0.005)	(0.004)	(0.004)	(0.001)		
$corr(c^m, y)$	0.907	0.844	0.816	0.981	0.690	0.760
	(0.015)	(0.022)	(0.026)	(0.005)		
$corr(h, y^m)$	-0.991	-0.927	-0.928	-0.992	-0.760	-
	(0.005)	(0.013)	(0.014)	(0.002)		
$corr(n, y^m)$	-	0.999	0.999	0.999	-	-
		(0.000)	(0.000)	(0.002)		
$corr(\varphi, y^m)$	0.999	0.999	0.999	0.997	0.940 [b]	0.860 [b]
	(0.000)	(0.000)	(0.000)	(0.003)		
$corr(\pi, y^m)$	0.676	0.999	0.999	0.999	-	-
	(0.327)	(0.000)	(0.000)	(0.000)		

Notes: [a] Source: Benhabib, Rogerson and Wright (1991, table 1).
[b] These statistics are computed using physical hours worked.
[a] Source: see discussion in section 4.

Table 5. The distributions of individual variables in artificial and U.S. economies

Group	Individual variable				
	Employment	St.deviation of employment (%)	Time worked at home [a]	Market consumption [a]	Search time
	Model economy: $y^h/y^m=0.15$				
1	0.866	0.18	0.316	0.817	0.0006
2	0.896	0.14	0.295	0.829	0.0115
3	0.907	0.13	0.284	0.836	0.0191
4	0.917	0.12	0.270	0.844	0.0292
5	0.935	0.11	0.236	0.865	0.0572
	Model economy: $y^h/y^m=0.20$				
1	0.834	0.63	0.325	0.808	0.0015
2	0.895	0.22	0.295	0.833	0.0120
3	0.911	0.18	0.282	0.843	0.0193
4	0.927	0.16	0.267	0.855	0.0290
5	0.951	0.13	0.232	0.883	0.0559
	U.S. economy [b]				
1	0.846	2.28	0.394	-	-
2	0.905	2.21	0.351	-	-
3	0.920	1.92	0.282	-	-
4	0.924	1.74	0.213	-	-
5	0.925	1.37	0.160	-	-

Notes: [a] Time worked at home and market consumption are group's averages; the group's average of a variable x_{ts} is defined as $x^e_{ts}\varphi(\pi_{ts})+x^u_{ts}(1-\varphi(\pi_{ts}))$.

[b] Source (except for time worked at home): Castañeda, Díaz-Giménez and Ríos-Rull (1995, table 8).
Source for time worked at home: Ríos-Rull (1993, table 2); the average time is normalized to 0.28.

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Chapter 4

Inequality and Business Cycle

Joint with Serguei Maliar

4.1 Introduction

We examine the relation between distributive and aggregate dynamics in a heterogeneous agents version of the stochastic growth model by Kydland and Prescott (1982). Following Kydland (1984), we assume that agents differ in initial wealth and non-acquired skills; the skill differentials are permanent. The agents's preferences are of Cobb-Douglas type. The level of output in the economy depends on exogenous shocks. Markets are complete so that the agents can insure themselves against uncertainty by trading state contingent claims. This set of assumptions is sufficient for the existence of a representative consumer in the sense of Gorman (1953). Therefore, the first implication of the model is that the distributions and, consequently, inequalities have no effect on the time-series behavior of macroeconomic aggregates.

The converse to the above is not true, however. The model predicts that the economy's performance at the aggregate level plays a determinant role for the dynamic behavior of inequalities. Specifically, we show that the evolution of wealth and income distributions in our economy can be fully characterized in terms of aggregate time series and a set of time-invariant agent-specific parameters. Further, we show that the time-series properties of such inequality measures as the coefficients of variation of wealth and income can be expressed only in terms of macroeconomic aggregates and several parameters. The values of such parameters will depend on particular assumptions about the distribution of skills and the initial distribution of wealth, however, the relation between the coefficient of variation and aggregate variables will not. This fact makes it possible to test the relation between the behavior of inequalities and aggregate dynamics in the model by using only aggregate time-series data and aggregate measures of inequality; the distributions do not have to be specified explicitly.

Distributive implications of neoclassical models have been analyzed by Stiglitz (1964), Chatterjee (1994) and Caselli and Ventura (1996). Our work differs from these studies in two respects. First, we examine the cyclical properties of inequality measures, while the previous literature concentrates on transitional dynamics of distributions in deterministic setups. Second, we assume that the agents value both consumption and leisure, whereas the earlier papers neglect the issue of labor-leisure choice.

We test the implications of the model by using the data on the U.K. economy. To evaluate the empirical validity of the hypothesis that the inequalities do affect the behavior of the economy at the aggregate level, we perform a number of tests of the time-series exogeneity of main macroeconomic aggregates such as consumption, output, investment etc. with respect to two aggregate measures of inequality, the coefficient of variation and Gini index of personal distribution of income. We find no sufficient empirical evidence of Granger-causality. To investigate the role of aggregate time-series in determining of the behavior of inequality over the business cycle, we evaluate the explanatory power of the theoretical relation between the coefficient of variation of personal income and the macroeconomic aggregates. We find that aggregate fluctuations can account for almost 90% of the total variability in the coefficient of variation of income. Therefore, the predictions of the model are in agreement with the data.

The rest of the paper is organized as follows. Section 2 formulates the model and summarizes the aggregation results. Section 3 studies the distributive dynamics and derives the relation between the inequality measures and macroeconomic aggregates. Section 4 describes econometric methodology and discusses empirical results. Finally, section 5 concludes.

4.2 The model

We consider a heterogeneous agents version of the standard stochastic growth model. The economy is populated by a set of infinitely-lived agents S; the measure of agent s in the set S is ds with $\int_S ds = 1$. The agents differ in skills and initial endowments. The skills of an agent $s \in S$ reflect the amount of efficiency labor e^s corresponding to a unit of the agent's labor efforts. We assume that the skills remain constant over time and across states of nature. For convenience, we normalize the skills such that $\int_S e^s ds = 1$. The timing is discrete, $t \in T$, where $T = 0, 1, ..., \infty$.

Agent $s \in S$ maximizes the expected sum of momentary utilities, discounted at the rate $\delta \in (0, 1)$, by choosing a path for consumption, c_t^s, and leisure, l_t^s. The utility function $u(c_t^s, l_t^s)$ is Cobb-Douglas. In period t, the agent owns capital stock k_t^s and rents it to the firm at the rental price r_t. Also, he supplies to the firm n_t^s units of labor in exchange for labor income $n_t^s e^s w_t$, where w_t is the wage paid for one unit of efficiency labor. The total time endowment of the agent is normalized to one, $n_t^s + l_t^s = 1$. Capital depreciates at the rate $d \in (0, 1]$. When making the investment decision, the agent faces uncertainty about the future returns on capital. We assume that markets are complete: the agent can insure himself against uncertainty by trading state contingent claims, $\left\{m_{t+1}^{s}(\theta)\right\}_{\theta\in\Theta}$, where Θ denotes the set of all possible realizations of productivity shocks. The claim of type $\theta \in \Theta$ costs $p_t(\theta)$ in period t and pays one unit of consumption good in period $t+1$ if the state $\theta \in \Theta$ occurs and zero otherwise. Therefore, agent $s \in S$ solves:

$$\max_{\left\{c_t^s, n_t^s, k_{t+1}^s, m_{t+1}^s(\theta)\right\}_{\theta\in\Theta, t\in T}} E_0 \sum_{t=0}^{\infty} \delta^t \frac{\left[\left(c_t^s\right)^{\gamma}\left(1-n_t^s\right)^{1-\gamma}\right]^{1-\sigma}-1}{1-\sigma} \tag{4.1}$$

$$\text{s.t. } c_t^s + k_{t+1}^s + \int_{\Theta} p_t(\theta)\, m_{t+1}^s(\theta)\, d\theta = (1-d+r_t)\, k_t^s + n_t^s e^s w_t g^t + m_t^s(\theta_t), \tag{4.2}$$

where $0<\gamma<1$ and $\sigma>0$ and g denotes the rate of labor-augmenting technological progress. Initial holdings of capital and contingent claims, k_0^s and $m_0^s(\theta)$, are given.

The representative firm operates the technology which allows to produce output from capital, k_t, and labor, h_t. Taking the prices r_t and w_t as given, the firm chooses the inputs to maximize period-by-period profits

$$\max_{k_t,\, h_t} \pi_t = \theta_t f(k_t, h_t) - r_t k_t - w_t h_t. \tag{4.3}$$

The production function $f(\cdot)$ has constant returns to scale, is strictly concave, continuously differentiable, strictly increasing with respect to both arguments and satisfies the appropriate Inada conditions. The variable θ_t represents exogenous technology shocks; it follows first-order Markov process

$$\log\theta_t = \rho\log\theta_{t-1} + \varepsilon_t \tag{4.4}$$

where $\rho \in [0,1]$ and $\varepsilon_t \sim i.i.d.N\left(0, \sigma_\varepsilon^2\right)$ for $\forall t \in T$. The value θ_0 is given. Aggregate capital and labor in the economy are given by $k_t = \int_S k_t^s ds$ and $h_t = \int_S n_t^s e^s ds$.

We define wealth W_t^s of agent $s \in S$ in period $t \in T$ as the total value of his end-of-the period portfolio expressed in terms of the period output price

$$W_t^s = k_{t+1}^s + \int_\Theta p_t(\theta)\, m_{t+1}^s(\theta)\, d\theta.$$

Definition. A competitive equilibrium in the economy (4.1) – (4.4) is defined as a sequence of contingency plans for a consumers' allocation $\{c_t^s, n_t^s, W_t^s\}_{t\in T}^{s\in S}$, for an allocation of the firm $\{k_t, h_t\}_{t\in T}$ and for the prices $\{r_t, w_t, p_t(\theta)\}_{\theta\in\Theta, t\in T}$ such that given the prices, the sequence of plans for consumers' allocation solves the utility maximization problem (4.1), (4.2) of each agent $s \in S$, the sequence of plans for the firm's allocation leads to zero-profit solution of the firm's problem (4.3) for $\forall t \in T$ and all markets clear. Moreover, the equilibrium plans are to be such that $c_t^s \geq 0$, and $1 \geq n_t^s \geq 0$ for $\forall s \in S$, $t \in T$ and $w_t, r_t, k_t \geq 0$ for $\forall \theta \in \Theta$, $t \in T$. We assume that the equilibrium exists, is interior and unique.

The assumptions of complete markets and Cobb-Douglas preferences allow us to make use of aggregation theory. Maliar and Maliar (1999a) show that a representative consumer utility maximization problem, which corresponds to the above heterogeneous agents economy is

$$\max_{\{c_t,h_t,k_t\}_{t\in T}} \quad E_0 \sum_{t=0}^{\infty} \delta^t \frac{\left(c_t^\gamma (1-h_t)^{1-\gamma}\right)^{1-\sigma} - 1}{1-\sigma} \tag{4.5}$$

$$\text{s.t.} \qquad c_t + k_{t+1} = (1 - d + r_t)\, k_t + h_t w_t g^t. \tag{4.6}$$

where c_t is aggregate consumption in the economy, $c_t = \int_S c_t^s ds$.

Further, there exists a function $\{f^s\}^{s\in S}$ such that $f_s > 0$ for $\forall s \in S$ and $\int_S f^s ds = 1$ that relates aggregate and individual equilibrium quantities

$$c_t^s = c_t f^s, \qquad\qquad n_t^s = 1 - (1 - h_t)\frac{f^s}{e^s}. \tag{4.7}$$

The function $\{f^s\}^{s\in S}$ has the meaning similar to that of a set of welfare weights in the corresponding planner's problem.

Finally, in the appendix, we show that the equilibrium distribution of wealth is determined by the individual recursive budget constraints

$$W_t^s = E_t \sum_{\tau=t+1}^{\infty} \delta^{\tau} \frac{u_1(c_\tau, h_\tau)}{u_1(c_t, h_t)} \left(c_\tau^s - n_\tau^s e^s w_\tau g^\tau\right), \tag{4.8}$$

where $u_1(c_\tau, h_\tau)$ is the marginal utility of consumption of the representative agent in a period τ. If a function $\{f^s\}^{s\in S}$ is specified, representative consumer problem (4.5), (4.6) and equations (4.7), (4.8) are sufficient to determine the equilibrium. Specifically, one can solve for the aggregate equilibrium quantities by using the utility maximization problem of the representative consumer model and restore the individual equilibrium allocations afterwards.

4.3 Distributive dynamics

This section characterizes the dynamic behavior of wealth and income distributions in and constructs the inequality measures for the model formulated in section 2. Consider the share of agent's wealth in total wealth, z_t^s, defined as

$$z_t^s = \frac{W_t^s}{\int_S W_t^s ds} = \frac{k_{t+1}^s + \int_\Theta p_t(\theta)\, m_{t+1}^s(\theta)\, d\theta}{k_{t+1}}.$$

The fact that $\int_S W_t^s ds = k_{t+1}$ follows from the market clearing condition for the claims, $\int_S m_{t+1}^s(\theta)\, ds = 0$, for $\forall t \in T, \theta \in \Theta$. By construction, we have $\int_S z_t^s ds = 1$. The first order conditions (FOCs) of individual problem (4.5), (4.6) imply that recursive constraint (4.8) can be rewritten as follows

$$z_t^s k_{t+1} = E_t \sum_{\tau=t+1}^{\infty} \delta^{\tau} \frac{u_1(c_\tau, h_\tau)}{u_1(c_t, h_t)} \left(\frac{c_\tau^s}{\gamma} - e^s w_\tau g^\tau\right). \tag{4.9}$$

Using (4.7) to express individual consumption c_τ^s in terms of aggregate consumption c_τ and the function f^s, we obtain

$$z_t^s k_{t+1} = E_t \sum_{\tau=t+1}^{\infty} \delta^\tau \frac{u_1(c_\tau, h_\tau)}{u_1(c_t, h_t)} \left(\frac{c_\tau}{\gamma} \cdot (f^s - e^s) + \left(\frac{c_\tau}{\gamma} - w_\tau g^\tau \right) \cdot e^s \right).$$

Integration of the previous equation over the set of agents yields

$$k_{t+1} = E_t \sum_{\tau=t+1}^{\infty} \delta^\tau \frac{u_1(c_\tau, h_\tau)}{u_1(c_t, h_t)} \left(\frac{c_\tau}{\gamma} - w_\tau g^\tau \right).$$

Combining the two previous equations, we have

$$z_t^s = e^s + \frac{(f^s - e^s)}{k_{t+1}} \cdot E_t \sum_{\tau=t+1}^{\infty} \delta^\tau \frac{u_1(c_\tau, h_\tau)}{u_1(c_t, h_t)} \cdot \frac{c_\tau}{\gamma}.$$

As the above relation is to be satisfied for any period $t \in T$, we can write the same condition in period $t = 0$ and eliminate the unknown term $(f^s - e^s)$. In this manner, we obtain the equation relating the wealth distributions in period t with that in period 0

$$z_t^s = z_0^s \cdot \xi_t + e^s \cdot (1 - \xi_t), \tag{4.10}$$

where $\xi_0 = 1$ and ξ_t for $\forall t > 0$ is defined as

$$\xi_t = \frac{k_0 \cdot E_t \sum_{\tau=t+1}^{\infty} \delta^\tau \frac{u_1(c_\tau, h_\tau)}{u_1(c_t, h_t)} \cdot c_\tau}{k_{t+1} \cdot E_0 \sum_{\tau=1}^{\infty} \delta^\tau \frac{u_1(c_\tau, h_\tau)}{u_1(c_t, h_t)} \cdot c_\tau}. \tag{4.11}$$

Notice that, instead of expressing z_t^s in terms of initial capital endowment, k_0 (or z_{-1}^s), we do it in terms of k_1 (or z_0^s). This assumption does not affect our subsequent results because any period can be considered as initial when doing econometric estimation of the model; however, it simplifies the notations.

Next, we focus on income distribution. It turns out that the evolution of personal income in our model depends not only on the agents' choices of consumption, working efforts and wealth holdings $\{c_t^s, n_t^s, W_t^s\}_{t \in T}^{s \in S}$, but also on a particular composition of their portfolios W_t^s. As a result, the income distribution in the model is not uniquely defined. To see this, consider the variable y_t^s which is defined as the share of income of agent $s \in S$ in the total income

$$y_t^s = \frac{r_t k_t^s + m_t^s(\theta_t) + n_t^s e^s w_t}{\int_S (r_t k_t^s + m_t^s(\theta_t) + n_t^s e^s w_t)\, ds}. \tag{4.12}$$

This formula implies that the agent's period income cannot be expressed in terms of wealth W_t^s only; it also depends on how much capital k_t^s and how much claims $m_t^s(\theta_t)$ of type $\theta_t \in \Theta$ are owned by the agent. The equilibrium composition of asset portfolios is not uniquely defined, however. Indeed, the economy has Θ types of state contingent claims and capital stock. Since the number of assets is greater than the number of the economy's states, one of the assets is linearly dependent on the others.

The intuition behind this result is that in the model the agents do not care about the period levels of income; they only do about the total expected life-time income. Consequently, they are indifferent between any sequences of portfolios as long as these sequences will give an identical expected payoff over the life. To deal with this problem, we are to impose further restrictions on the composition of agents' portfolios. The assumption which we adopt here is that the agents hold no claims but only physical capital. Since composition of portfolio has no effect on the rest of equilibrium allocations, this restriction is consistent with the definition of the equilibrium.

In what follows, we assume that the production function is Cobb-Douglas, $f(k,h) = k^{\alpha} h^{1-\alpha}$, in which case the agent's income share in period $t > 0$ can be written as

$$y_t^s = \alpha \cdot z_{t-1}^s + (1-\alpha) \cdot n_t^s e^s / h_t.$$

Substituting for n_t^s from (4.7) and for z_{t-1}^s from (4.10) into the last expression yields

$$y_t^s = \vartheta_t \cdot z_0^s + (1 - \vartheta_t) \cdot e^s, \tag{4.13}$$

with the variable ϑ_t being defined as

$$\vartheta_t = \alpha \cdot \xi_{t-1} - (1 - \alpha) \cdot \frac{k_0 \left(1 - 1/h_t\right)}{E_0 \sum\limits_{\tau=1}^{\infty} \delta^\tau \left(c_\tau / c_0\right)^{-\sigma} c_\tau / \gamma}. \tag{4.14}$$

In our model, the individual wealth and income shares, z_t^s and y_t^s, can be represented as the weighted averages of individual's skills and initial capital. Consequently, the distributions of wealth and income will be driven by changes in weights ξ_t and ϑ_t. Therefore, the distributive predictions of our model are fully characterized by macroeconomic aggregates, the distribution of initial wealth and the distribution of skills.

There are several alternative aggregate measures which are used to represent the time-series behavior of the inequalities in the real economies, e.g., the Coefficient of Variation (CV), Gini and Theil indices, quintiles, etc. A straightforward way to calculate the corresponding measures in our model would be the following: fix some distributions of skills and initial wealth, solve for aggregate quantities, restore income and wealth shares z_t^s and y_t^s by using (4.10) and (4.13) and, finally, compute the inequality indices of interest. However, there is one inequality measure, which is the coefficient of variation, that can be constructed for our economy without specifying the distributions explicitly. The CV of wealth is defined as

$$CV_t^w = \frac{1}{S} \cdot \int_S \left(z_t^s - 1\right)^2 ds. \tag{4.15}$$

Substituting (4.10) into (4.15) and rearranging the terms, we get

$$CV_t^w = a_1 + a_2 \cdot \xi_t + a_3 \cdot \xi_t^2, \tag{4.16}$$

where the constants a_1, a_2 and a_3 are given by

$$a_1 = \frac{1}{S} \cdot \int_S (e^s)^2 \, ds - 1, \qquad a_2 = \frac{2}{S} \cdot \int_S \left(z_0^s e^s - (e^s)^2 \right) ds,$$

$$a_3 = \frac{1}{S} \cdot \int_S \left((z_0^s)^2 - 2 z_{-1}^s e^s + (e^s)^2 \right) ds.$$

Parallel result can also be obtained for the CV of income

$$CV_t^i = \frac{1}{S} \cdot \int_S (y_t^s - 1)^2 \, ds. \tag{4.17}$$

Indeed, after substituting (4.13) into (4.17), we get

$$CV_t^i = a_1 + a_2 \cdot \vartheta_t + a_3 \cdot \vartheta_t^2. \tag{4.18}$$

Other inequality indices such as Theil, Gini and quintiles, etc. cannot be constructed in a similar way, however. Before computing these indices, the consumers must be ordered according to the level of wealth (income). This requires knowing exactly the-period distributions, which is not possible without solving for the equilibrium explicitly.

4.4 Empirical analysis

In this section, we test the relation between aggregate and distributive dynamics in the model by using empirical data. We focus on the following two implications of the model.

(*i*) Inequalities (distributions) do not affect macroeconomic aggregates.

(*ii*) Aggregate dynamics do affect inequalities (distributions).

Implication (*i*) follows by the fact that the model admits a representative consumer. Implication (*ii*) generalizes the results of the previous section.

For empirical analysis, we use the time-series data on the U.K. economy. The CV and Gini index of income are reproduced from Atkinson and

Micklewright (1992). As a proxy for efficiency hours, h_t, we use the level of employment. The time series for employment and population come from the IMF data set; the series for private consumption (called "private final consumption expenditure") and investment (called "gross fixed capital formation") come from the OECD data base. Capital stock is reconstructed to match investment. All the time series, except for the CV and Gini, are from 1960 and onwards; the data on the CV and Gini are from 1964 to 1985.

To evaluate implication (i) statistically, we test the exogeneity of macroeconomic aggregates in the time-series sense with respect to inequality measures. Specifically, let us consider Vector Autoregressive Process (VAR) of two groups of variables, X_t and I_t, such that

$$\begin{bmatrix} X_t \\ I_t \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} + \begin{bmatrix} \Phi_1 & \Phi_2 \\ \Phi_3 & \Phi_4 \end{bmatrix} \begin{bmatrix} X_{t-p} \\ I_{t-p} \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix}.$$

where X_{t-p} and I_{t-p} are the lagged vectors, A_1, A_2 are the vectors of constants, Φ_i, $i = 1, ..., 4$ are the VAR coefficients, and $\epsilon_{it} \sim i.i.d.N(0, \Omega_i)$, $i = 1, 2$. The group of the variables X_t is called block-exogenous in the time-series sense with respect to the groups of variables I_t if $\Phi_2 = 0$. If X_t is block-exogenous, then we say that I_t does not Granger-cause X_t. There are several procedures in the literature which test the restrictions of type $\Phi_2 = 0$, see Hamilton (1998). In the paper, we will use the test by Granger (1969).

If both X_t and I_t include only one variable, i.e. $X_t = x_t$ and $I_t = i_t$, the VAR process for the variable x_t can be written as

$$x_t = a_1 + \phi_{11} x_{t-1} + ... + \phi_{1p} x_{t-p} + \phi_{21} i_t + ... + \phi_{2p} i_{t-p} + \epsilon_{1t} \qquad (4.19)$$

where a_1 is a constant term. Under specification (4.19), Granger-causality test is equivalent to evaluating the significance of the coefficients $\phi_{21}, ..., \phi_{2p}$.

In *Table* 1, we report the results of regression (4.19) of the main macroeconomic variables such as consumption, capital, investment, working efforts and output on a lag of the corresponding variables and the Gini index of personal distribution of income. In all the cases considered, the significance of the regression coefficients associated with the Gini index is outside of 5% significance level. Therefore, the result of test does not imply sufficient statistical evidence of Granger-causality. We find that increase in the number of lags can decrease the significance of the coefficients associated with the Gini index. We have also tried to test the time-series exogeneity of blocks of macro variables with respect to the Gini index. We do not find sufficient empirical evidence of the Granger-causality either.

Subsequently, we perform a similar analysis by using as a measure of inequalities the CV of income. The results are provided in *Table* 2. As we see, the regression coefficients, which correspond to the CV, are more significant. In particular, the hypothesis that inequality does not affect investment can be rejected at 2% level of significance. However, the evidence of the statistical relation between investment and inequalities might have an explanation which is different from the Granger-causality. Investments depend heavily on the expectations about the future prices; as formulas (4.11), (4.18) imply, inequalities also do so. In this case, the relation between investment and inequalities could be a consequence of the fact that both of these are driven by the expectations, see, e.g., Hamilton (1998).

To check the empirical validity of implication (*ii*), we are to investigate the explanatory power of the relations between inequality and aggregate dynamics predicted by the model. In particular, formulas (4.11), (4.14), (4.18) imply that the coefficient of variation of income is related to macroeconomic aggregates according to

$$CV_t^i = a_0 + a_1\xi_{t-1} + a_2\xi_{t-1}^2 + a_3\left(1 - 1/h_t\right) + \tag{4.20}$$

$$+a_4\left(1 - 1/h_t\right)\xi_{t-1} + a_5\left(1 - 1/h_t\right)^2 + \epsilon_t,$$

where ϵ_t is the error term.

The empirical testing of relation (4.20) is complicated by the fact that the variable ξ_t cannot be constructed from the data directly due to the fact that it depends on the unobservable conditional expectation of the future consumption. To construct a proxy for this variable, we use the following procedure. First, we formulate the orthogonality conditions of the problem (4.5), (4.6) and estimate the model's parameters including those for the process for technology shocks by using the Generalized Method of Moments (GMM) procedure. Next, we restore the Solow residuals, approximate the decision rules for consumption and capital by flexible functional forms of state variables $c_t = c_t\left(k_t, \theta_t, \overline{\beta^c}\right)$ and $k_t = k_t\left(k_t, \theta_t, \overline{\beta^k}\right)$ and use the empirical data to estimate the vectors of the parameters $\overline{\beta^c}$ and $\overline{\beta^k}$. Finally, for each given pair of state variables $\{k_t, \theta_t\}$ in the data sample, we generate a large number of sequences for consumption and approximate the expectations as the averages across simulated data sets. The results were subsequently used to restore the variable ξ_t. More details on estimation procedure are provided in the appendix.

To analyze the effect of individual labor choice on inequalities, we also consider the economy where the consumers do not value leisure and supply labor inelastically. The conditions for ξ_t and CV^i in such economy do not change, however, $\vartheta_t = \alpha \cdot \xi_{t-1}$ and $h_t \equiv 1$ for $\forall t \in T$. Therefore, if agents do not value leisure regression equation (4.20) is to be estimated under an additional restriction that the coefficients a_3, a_4, a_5 are equal to zero.

The properties of the error term in regression equation (4.20) are not known and, thus, the OLS estimator is not appropriate. We compute the weighted iterative least square as described by Huber (1973). The results of the regression are reported in *Table* 3. First, we do the regression of the CV of income under the assumption of the non-valuable leisure. As we see, the constructed variable ξ_t alone can account for more than 84% of the total variation in the *CV* of income. Introducing labor markets in the model, increases this number to about 89%. The last column of the table contains the result of the Wald test that the coefficients a_3, a_4, a_5 are equal to zero; we see that it can be rejected at 5% level of significance. This results suggest that labor markets play no role in determining inequalities.

To check the robustness of our results, we run similar regressions by using the Gini coefficient as a dependent variable. We find that this modification decreases the fit of the model from 84% to 69% for the economy with nonvaluable leisure, and from 89% to 75% for the economy with valuable leisure. The results of the Wald test that $a_3, a_4, a_5 = 0$ imply no evidence on the existence of a relation between the inequalities and labor choice. Though, considering Gini index as dependent variable decreases the explanatory power of equation (4.20), the model's fit to the data is still high enough.

4.5 Conclusion

This paper studies distributive implications of a general equilibrium stochastic growth model with heterogeneous agents. The model admits a representative consumer and, thus, implies that there is no effect of distributions on aggregate dynamics. On the other hand, the model predicts that aggregate behavior of the economy plays a determinant role in dynamics of distributions and inequalities. We test these two implications of the model by using the time-series data on the U.K. economy. We find that the model's predictions are in agreement with the data. Our main finding is that fluctuations of macroeconomic aggregates can account for almost 90% of the total variability in the coefficient of variation of personal income in the U.K data.

4.6 Appendix

The recursive budget constraint:

With an interior solution, the *FOC*s of agent's $s \in S$ utility maximization problem (4.1), (4.2) with respect to insurance holdings, capital holdings and the transversality conditions are as follows

$$u_1(c_t^s, n_t^s)p_t(\theta) = \delta u_1(c_{t+1}^s(\theta), n_{t+1}^s(\theta)) \Pr\{\theta_{t+1} = \theta \mid \theta_t = \theta'\}_{\theta,\theta' \in \Theta}, \quad (4.21)$$

$$u_1(c_t^s, n_t^s) = \delta E_t\left[u_1(c_{t+1}^s, n_{t+1}^s)(1 - d + r_{t+1})\right], \quad (4.22)$$

$$\lim_{t \to \infty} E_0\left[\delta^t u_1(c_t^s, n_t^s)\left(k_{t+1}^s + \int_\Theta p_t(\theta)\, m_{t+1}^s(\theta)\, d\theta\right)\right] = 0, \quad (4.23)$$

where u_1, u_2 are the first order partial derivatives of the utility u with respect to consumption and labor and $c_{t+1}^s(\theta)$, $n_{t+1}^s(\theta)$ are equilibrium consumption and working hours as functions of the realization of the aggregate shock.

Condition (4.21) implies

$$E_{t-1}\left[\delta \frac{u_1(c_t^s, n_t^s)}{u_1(c_{t-1}^s, n_{t-1}^s)} m_t^s(\theta_t)\right] = \int_\Theta m_t^s(\theta)\, p_{t-1}(\theta)\, d\theta. \quad (4.24)$$

Further, *FOC* (4.22) together with the fact that k_t^s is known at $t-1$ yields

$$E_{t-1}\left[\delta \frac{u_1(c_t^s, n_t^s)}{u_1(c_{t-1}^s, n_{t-1}^s)} k_t^s (1 - d + r_t)\right] = k_t^s. \quad (4.25)$$

Multiplying each term of (4.2) by $\frac{\delta u_1(c_t^s, n_t^s)}{u_1(c_{t-1}^s, n_{t-1}^s)}$, taking the expectation E_{t-1} on both sides and using (4.24), (4.25), one can show that for all $t > 0$ the following condition holds

$$k_t^s + \int_\Theta m_t^s(\theta) p_{t-1}(\theta) d\theta = E_{t-1}\left[\delta \frac{u_1(c_t^s, n_t^s)}{u_1(c_{t-1}^s, n_{t-1}^s)}(c_t^s - n_t^s e^s w_t)\right] +$$

$$E_{t-1}\left[\delta \frac{u_1(c_t^s, n_t^s)}{u_1(c_{t-1}^s, n_{t-1}^s)}\left(k_{t+1}^s + \int_\Theta m_{t+1}^s(\theta) p_t(\theta) d\theta\right)\right].$$

Applying forward recursion, using the law of iterative expectations and imposing transversality condition (4.23), we have

$$W_{t-1}^s = k_t^s + \int_\Theta m_t^s(\theta) p_{t-1}(\theta) d\theta = E_{t-1}\sum_{\tau=t}^{\infty} \delta^\tau \frac{u_1(c_\tau^s, n_\tau^s)}{u_1(c_{t-1}^s, n_{t-1}^s)}(c_\tau^s - n_\tau^s e^s w_\tau g^\tau).$$

Updating the latter, we get (4.8) used in the main text.

Estimation procedure:

Step 1. We estimate the parameters of the model by using L. Hansen's (1982) GMM procedure. Our estimation closely follows the steps outlined in the paper by Christiano and Eichenbaum (1993). The parameters δ and σ are not estimated: we set $\delta = 1.03^{-0.25}$, which implies the subjective discount rate of 3% per year, and assume $\sigma = 1$, which corresponds to the log − log utility, $u(c,h) = \ln(c) + \eta \ln(1-h)$. Therefore, the set Ψ of the parameters to be estimated is

$$\Psi = \left\{d, \delta, \eta, \alpha, g, \rho, \sigma_\varepsilon^2\right\}.$$

The orthogonality conditions that we use for estimation are the following

$$E\left\{d - \left[1 - (dk_t/k_t) - (k_{t+1}/k_t)\right]\right\} = 0,$$

$$E\left\{\delta^{-1} - \left[\alpha(y_{t+1}/k_{t+1}) + 1 - d\right] c_t/c_{t+1}\right\} = 0,$$

$$E\left\{\eta - (1-\alpha)\left(y_t/h_t\right)\left(1-h_t\right)/c_t\right\} = 0,$$

$$\ln\theta_t = \ln\left(y_t\right) - \alpha\ln\left(k_t\right) - (1-\alpha)\ln\left(h_t\right) - (1-\alpha)\,gt - q,$$

$$\ln\theta_t = \rho\ln\theta_{t-1} + \varepsilon_t,$$

$$E\left[\varepsilon_t^2 - \sigma_\varepsilon^2\right] = 0,$$

where dk_t is the gross investment and q is the unconditional level of technology corresponding to given measurement units. In addition, we include the restrictions that the variables $\{c_t, k_t, y_t\}$ grow at the same rate g. The estimates are reported in *Table* 4.

Step 2. After estimation is done, we restore the Solow residuals, $\{\theta_t\}_{t\in T}$, and approximate the decision function for consumption and the law of motion for capital by linear functions

$$c_t = \beta_1^c + \beta_2^c k_t + \beta_3^c \theta_t,$$

$$k_{t+1} = \beta_1^k + \beta_2^k k_t + \beta_3^k \theta_t,$$

Using the data and the Solow residuals, we estimate the parameters by using a non-linear iterative least squares estimator. The results of estimation procedure are given in *Table* 5.

Step 3. We approximate the conditional expectation as follows

$$E_t\sum_{\tau=1}^{\infty}\delta^{\tau}\frac{u_1\left(c_\tau, h_\tau\right)}{u_1\left(c_{t-1}, h_{t-1}\right)}c_\tau \simeq \frac{1}{N}\sum_{n=1}^{N}\sum_{\tau=1}^{T}\delta^{\,\tau}\frac{u_1\left(c_\tau, h_\tau\right)}{u_1\left(c_{t-1}, h_{t-1}\right)}c_\tau.$$

Subsequently, for each pair $\{k_t, \theta_t\}_{t\in T}$ in the sample, we use the obtained laws of motion for the Solow residuals, consumption and capital to run N simulations of the length T. We choose $N = 400$, $T = 10000$. Subsequently, we compute the averages and use the results to restore the variable ξ_t according to (4.11).

Table 1. Granger-causality test of time-series exogeneity (Gini index)

X_t	*Constant*	X_{t-1}	$Gini_t$	R^2	*Granger test*
c_t	2.215 (28.107)	0.866 (0.149)	51.679 (47.671)	0.658	0.293
k_t	-35.856 (99.360)	1.034 (0.144)	66.432 (139.103)	0.904	0.639
dk_t	-26.089 (15.839)	1.035 (0.121)	71.703 (36.854)	0.809	0.067
h_t	0.139 (0.092)	0.602 (0.226)	-0.046 (0.094)	0.409	0.633
y_t	-42.024 (39.512)	1.002 (0.129)	120.907 (71.949)	0.771	0.110

Table 2. Granger-causality test of time-series exogeneity (*CV* index)

X_t	*Constant*	X_{t-1}	CV_t	R^2	*Granger test*
c_t	11.658 (23.219)	0.845 (0.147)	16.433 (12.894)	0.666	0.219
k_t	-143.003 (77.338)	1.231 (0.139)	77.280 (36.840)	0.922	0.050
dk_t	-17.949 (9.236)	1.030 (0.108)	23.998 (8.980)	0.834	0.016
h_t	0.116 (0.083)	0.638 (0.230)	-0.005 (0.026)	0.403	0.851
y_t	-22.928 (28.723)	0.981 (0.122)	38.161 (18.560)	0.786	0.055

Table 3. The theoretical relation between macro variables and inequality indices

	Constant	ξ_{t-1}	ξ^2_{t-1}	$(1-1/h_t)$	$(1-1/h_t)\ \xi_{t-1}$	$(1-1/h_t)^2$	R^2	*Test*
$CV_t^{\ 1}$	-3.282 (7.007)	6.451 (14.226)	-2.245 (7.211)	-	-	-	0.844	-
$CV_t^{\ 2}$	-4.797 (6.531)	10.571 (13.229)	1.483 (6.715)	0.489 (2.139)	5.429 (3.471)	1.339 (1.185)	0.886	0.044
$Gini_t^{\ 1}$	0.317 (2.874)	-0.365 (5.853)	0.397 ((2.976)	-	-	-	0.688	-
$Gini_t^{\ 2}$	-0.299 (2.562)	2.674 (5.925)	1.223 (2.813)	0.798 (1.105)	2.132 (1.646)	0.661 (0.535)	0.749	0.329

Table 4. The model's parameters estimated by GMM

Parameter	η	α	g	d	ρ	σ_ε
Value	2.030 (0.011)	0.302 (0.002)	0.022 (0.001)	0.103 (0.000)	0.808 (0.022)	0.020 (0.000)

Table 5. The estimated parameters of the decision rules for consumption and capital

Parameter	β^c_1	β^c_2	β^c_3	β^k_1	β^k_2	β^k_3
Value	-19.866 (16.712)	0.103 (0.029)	130.560 (10.919)	-92.810 (15.282)	0.998 (0.026)	93.103 (10.018)

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