Ph.D. Thesis

A Markov sow herd model for on-farm decision support

by Lluís Miquel Plà
Supervisor: Jesús Pomar

University of Lleida

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CONTENTS

DEDICATION

This thesis is dedicated to my daughter Alba, who died suddenly at the age of 6 month. It becomes difficult to express what Alba represents for my family and for me. Without doubts, she marked everybody who met her, even though her short staying among us. I would like to highlight the strong influence that she exerted and is exerting in the consolidation of ours believes in the goodness of life, in the greatest of man and the faith in God.

DEDICATÒRIA

Aquesta tesis està dedicada a la meva filla Alba, que va morir sobtadament quan tenia 6 mesos. És difícil expressar el que l'Alba representa per la meva família i per mi, però és indubtable que a tots els que la vam conèixer ens va marcar malgrat la seva curta estança entre nosaltres. Voldria destacar la influència que va exercir i exerceix en la fonamentació de les nostres creences en la bellesa de la vida, en la grandesa de l'home i la fe en Déu.

DEDICATORIA

Esta tesis está dedicada a mi hija Alba, que murió de forma súbita cuando tenía 6 meses. Resulta difícil expresar lo que Alba representa para mi familia y para mí, pero es indudable que nos ha marcado a todos los que la conocimos, a pesar de su corta estancia entre nosotros. Quisiera destacar la influencia que ejerció y que sigue ejerciendo en la consolidación de nuestras creencias en la belleza de la vida, en la grandeza del hombre y la fe en Dios.

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After I completed my Engineering studies, I left the university for one year. At the meantime, I decided to start a new degree in Mathematics to complete my educational background on statistics and operational research. At the time, I was not sure if I would come back for my PhD. It was Professor Joan Estany, today a good friend, who lead me to Dr. Pomar. He agreed to take me on as a PhD student once I entered as Assistant professor in the Department of Mathematics, at University of Lleida. I have had a very valuable, productive, and enjoyable experience at UdL-IRTA Center, more exactly in the "Área de producción animal" lead by Dr. Noguera who also encouraged me and facilitated both access to BD-porc system and initial financial support for the research. I also want to mention the colleagues in the Department of Mathematics that, directly or indirectly have encouraged me during this time. Specially Josep Conde who help me in the understanding, formulation and notation of the mathematical model and Carles Capdevila and Silvia Miquel for their personal support. During this time I also had the opportunity to meet other people who made interesting comments and gave me suggestions to improve the final results or supporting further research. That is the case of Dr. Kristensen and Dr. Ríos to whom I'm very grateful.

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Abstract

Spanish pig sector has gone through a deep change during recent times, that is due basically to the increase in competitiveness and the globalisation process of the economics. Furthermore, technological advances and the increasing degree of specialisation have maden possible the development and adoption of advanced tools for decision support. In this context, the objective of this Thesis has been to formulate and implement a dynamic estochastic model representing the productive behaviour of a sow herd, based on Markov decision processes. The model was aimed to be used in field condicions to analyse different management alternatives on reproduction and replacement, supporting farm managers in the decision-making process.

The semi-Markovià decision model and the derived Markov decision model (embedded Markov process) have demonstrated to be useful in the representation of management alternatives on reproduction and replacement. The availability of data from individual farms has allowed the validation of the model in real situations. The validation also has served to show how the model can not be applied indiscriminately on any farm. Previously, it has been required to assess the fit of the model in specific farm conditions. Also, it is shown that when the model has been used to calculate the population structure at equilibrium was not necessary a transition matrix being time step constant. Instead have been possible to consider transitions associated to biological states that are easier to estimate, more precise and provided computational time savings. Hence the model, as it was formulated, allowed a faster evaluation of management alternatives on reproduction and an efficient impleemntation of algorithms to optimise replacement policies.

The implementation of the semi-Markov model into a DSS (DSS: Decision Support Systems) has shown the potential on-farm use of the model. The development of the DSS makes easier the availability of complex models to less specialised users. The DSS allows the farm manager to evaluate on-farm different management alternatives on reproduction and optimise replacement decisions. Moreover, the integration of the DSS in a management information system (BDporc®[1]) makes easier the spreading of it over swine enterprises, as well the obtention of new variables like the number of services by mating, heat detections, pregnancy detection, facilities, etc, can help to increment model precision. The sophisticated dessign of the DSS interface has improved the interpretation of the model results, that not always is right direct. The addition of sensitivity analisis capability provided insight about the impact of changes in critical components of the model, that quite often result in a more interest than a single precise result. Finally, the sow herd model formulated in a flexible way, was able to be adapted to differents goals with minimum changes, thus it contribute to improve the knowledge about the effect of different management alternatives on overall economic efficiency of the system.

[1] http://www.bdporc.es

Resum

El sector porquí Espanyol ha sofert recentment profuns canvis degut bàsicament a l'augment de la competéncia i al proces de globalització econòmica. Ademés, els avenços tecnològicos i el grau creixent d'especialització en el sector afavoreixen el desenvolupament i l'adopció de eienes avançades per a la pressa de decisions. En aquest context, l'objectiu d'aquesta Tesi ha estat formular i implementar un model dinàmic estocàstic del comportament productiu d'un remat de truges, basat en processos de decisió Markovians i capaç d'esser utilitzat en condicions reals. La finalitat del model és representar alternatives de maneig reproductiu i de reposició en explotacins porcines per assistir als grangers, tècnics i gerents en la pressa de decisions en granja.

El model de decisió semi-Markovià i el Markovià que s'en deriva del primer han demostrat ser models útils per la representació de les estratègies productives de maneig reproductiu i de la reposició. La disponibilitat de dades de camp de granges individuals ha permés la validació del model en situacions reals. La validació ha servit també per mostrar com el model no pot ser aplicat indiscriminadament a qualsevol granja, previament s'ha d'assegurar l'ajust del model a les condicions concretes de cada explotació. També s'ha ficat de manifest que quan el model s'utilitza per a calcular l'estructura de la població a l'equilibri no és necessari que la matriu de transició representi el pas de temps constant, la qual cosa ha permés treballar amb transicions associades als estats biològics que són més fàcils d'estimar (embedded Markov process), ademés, proporcionen estalvis computacionals que permeten una avaluació més ràpida d'alternatives de maneig reproductiu i la implementació d'algorismes d'optimització pel problema de la reposició més eficientes.

La implementació del model de decisió semi-Markovià dins d'un sistema d'ajut a la pressa de decisions (DSS: Decision Support Systems) ha mostrat l'ús potencial del model en granja. El desenvolupament del DSS ha facilitat la disponibilitat d'un model complex com el presentat a potencials usuaris menys especialitzats. El DSS permet al granger avaluar a peu de granja diferentes alternatives productives, analitzar la sensibilitat dels paràmetres que consideri crítics i optimitzar la política de reposició. Ademés, la integració del DSS en un sistema de gestió informatitzat (BDporc®[2]) facilita la difusió del DSS en empreses de producció porcina i també l'obtenció de noves variables com el número de serveis por monta, la detecció de zels, la detecció de la gestació, instalacions, etc, que poden ajudar a incrementar la precisió dels resultats. El disseny sofisticat del interface del DSS ha millorat la interpretació dels resultats del model que no sempre és inmediata. La incorporació de l'análisi de sensibilitat permet estudiar i profunditzar en els components crítics del model que sovint resulta més important que l'obtenció d'un resultat precis. Finalment, el model de remat formulat de forma flexible, és capaç d'adaptar-se a diferents propósits amb canvis mínims, la qual cosa contribueix a una millor comprensió dels efectes de diferentes alternatives de maneig reproductiu sobre la millora de la eficiència econòmica de tot el sistema productiu.

[2] http://www.bdporc.es

Resumen

El sector porcino en España ha sufrido recientemente profundos cambios debido básicamente al aumento de la competencia y al proceso de globalización económica. Además, los avances tecnológicos y el creciente grado de especialización en el sector favorecen el desarrollo y la adopción de herramientas avanzadas para la toma de decisiones. En este contexto, el objetivo de esta Tesis es presentar la formulación e implementación de un modelo dinámico estocástico del comportamiento productivo de un rebaño de cerdas, basado en procesos de decisión Markovianos y capaz de ser usado en condiciones reales. La finalidad del modelo es representar alternativas de manejo reproductivo y de reposición en explotacines porcinas para asistir a granjeros, técnicos y gerentes en la toma de decisiones en granja.

El modelo de decisión semi-Markovianos y el Markoviano que deriva del primero han demostrado ser modelos útiles en la representación de las estrategias productivas de manejo reproductivo y de la reposición. La disponibilidad de datos de campo de granjas individuales ha permitido la validación del modelo en situaciones reales. La validación ha servido también para mostrar como el modelo no puede ser aplicado indiscriminadamente en cualquier granja, previamente se asegurar el ajuste del modelo a las condiciones concretas de cada explotación. También se ha puesto de manifiesto que cuando el modelo se utiliza para calcular la estructura de la población en equilibrio no es necesario que la matriz de transición represente de paso de tiempo constante, con lo cual ha sido posible trabajar con transiciones asociadas a los estados biológicas que son más fáciles estimar (embedded Markov process), además, proporcionan ahorros computacionales que permiten una evaluación más rápida de alternativas de manejo reproductivo y la implementación de algoritmos de optimización para el problema de la reposición más eficientes.

La implementación del modelo de decisión semi-Markoviano dentro de un sistema de ayuda a la toma de decisiones (DSS: Decision Support Systems) ha mostrado el uso potencial del modelo en granja. El desarrollo del DSS ha facilitado la disponibilidad de un modelo complejo como el presentado a potenciales usuarios menos especializados. El DSS permite al granjero evaluar a pie de granja diferntes alternativas productivas, analizar la sensibilidad de los parámetros que considere críticos y optimiza la política de reposición. Además, la integración del DSS en un sistema de gestión informatizado (BDporc®[3]) facilita la difusión del DSS en empresas de producción porcina y también la obtención de nuevas variables como el número de servicios por monta, la detección de celos, la detección de la gestación, instalaciones, etc, que pueden ayudar a incrementar la precisión de los resultados. El diseño sofisticados del interface del DSS ha mejorado la interpretación de los resultados que no siempre es inmediata. El análisis de sensibilidad incorporado permite estudiar y profundizar en los componentes críticos del modelo que a menudo resulta más importante que el disponer de un resultado preciso. Finalmente, el modelo de rebaño formulado de forma flexible, es capaz de adaptarse a distintos propósitos con cambios mínimos, lo que redunda en una mejor compresión de los efectos de diferentes alternativas de manejo reproductivo a fin de mejorar la eficiencia económica de todo el sistema productivo.

[3] http://www.bdporc.es

Chapter 1. GENERAL INTRODUCTION

1. *Introduction*

The decision-making process on sow farms is a complex process, due to the number of variables affecting sow production and the uncertainty of biological response of the sows. On the other hand, the environment in which farmers must operate is changing, sow production has seen many technological advances in recent times and economic performance is affected within the European Union (EU) by the CAP (Common Agricultural Policy) and by the globalisation process of economics. This situation can be expressed as an increase in the number of uncertainties for decision-makers. In this context, tools that may help sow farmers in dealing with uncertainties would be useful in supporting them in their decision-making processes. Methodologies involved are disciplines like mathematics and operational research allowing the formulation of simulation models more or less complex. Hence, mathematical models representing the production behaviour of a livestock herd are an interesting tool for livestock research and development (Sorensen, 1990). Much research work is being carried out to adapt different methodologies in a computerized framework and increase its solving power. However, the fact is that most farmers hardly ever use formalized decision support systems (DSS), do not employ computerized planning models, and do not take their financial accounting statements as a data base for economic analysis and planning (Ohlmer, Olson and Brehmer, 1998). In order to improve this situation, researchers should invest more efforts in livestock modeling. A good way to do this would be to validate and refine decision models for on-farm use with suitable data, to implement appropriate interfaces for the management and use of complex models and to cover adequately the needs of farm managers.

2. *Main Topics Involved*

Today, the implementation of tools to support decision-making processes should be based in three main topics: knowledge of the real system we want to give support to, mathematical foundation of the model we are going to implement and information technologies used in the computerised development. In this way it becomes essential to identify the nature of the process we want to represent, in order to be able to formulate good enough models and to provide good enough solutions.

2.1. Context of Spanish swine production

The pig sector has gone through a deep transformation in the last thirty years, especially since Spanish entry into the European Union (EU). Today it is a leading sector for Spanish agricultural economics with an increasing importance. It is the first sector in livestock production and pig meat is the first meat in Kg consumed per person and year. Spain has become the second European country in pig production with 3 million metric tones of pig meat and more than 23 million heads. This is still behind Germany with a production of near 4 million metric tones of pig meat and 27 million heads (FAOstat, 2000). However, it is expected that Spain will achieve leadership in the near future.

The way of achieving such a leading position involves all aspects of swine production, not only managerial techniques but also environmental economics. For example, farming enterprises have tended to be more capital intensive, the pig industry has increased in scale and in degree of specialisation required due to an increase of their competitiveness. Therefore, in order to reduce production costs a concentration process of firms in this sector has taken place. As result, the control over the production process is stronger and decisions are taken by fewer people in the Spanish pig sector. This situation makes the sector receptive to new management information systems dealing with real problems affecting sow farm production, and it has also made it easier to adopt new decision tools and promote the application of new modeling and optimisation techniques.

2.2. Mathematical formulations of livestock herd models

The type of model constructed depends on both its anticipated use and the nature of the system. We are concerned with livestock farm models where production process is closely related to the reproduction cycle. Those models are basically focused on reproduction and replacement. In this sense, existing livestock farm models could be a reference for new sow herd model developments because they share a similar logical structure. Published livestock herd models were reviewed by Jalving et al. (1992) and

Kristensen (1993). All models in both reviews described the production cycle during several reproductive cycles or parities. Previously, Glen (1981), Kennedy (1988) and Sorensen (1990, 1992) had made other interesting surveys emphasising different aspects of livestock modeling.

Jalving et al. (1992) focused her survey in dairy cows and sow models, and classified them in simulation and optimization models. What simulation models had in common was the random way they determined some parameters during simulation runs. Also, simulation models were able to follow changes in the herd over time. On the other hand, all optimization models were based on dynamic programming techniques. Optimization techniques have been applied to farm models as single components, only Kristensen (1992) considered a multi-component model to optimize the replacement in dairy cows. Kristensen (1993) directly reviews the historical development of dynamic programming techniques applied to the replacement problem. One common problem of dynamic programming applications is the problem of dimensionality but Kristensen (1996) demonstrates that, many times, that problem could be circumvented and some restraints could be added in a general MDP formulation.

Ben-Ari et Gal (1986) identified ciclicity, variability and repeatibility as main problems in livestock herd modeling. Although this was done only for dynamic programming problems, it remains true for livestock modeling in general. Since then, many advances have been seen in order to solve or circumvent such problems. The simplest approaches consisted of putting together different subsystems in a computerised simulation model. Major refinements were needed when a mathematical model of a livestock herd wanted to be formulated. Such methodological advances have not been followed by an increasing utilisation in field conditions of computerised tools including those mathematical approaches. Reasons for this could be the difficulty, common to operational research studies, in getting specific parameters (White, 1984) and suitable data for validation (Sorensen, 1990).

2.3. Information Technologies

Information technologies are changing modern society. These changes are possible thanks to the simultaneous development of computers, accompanied by a decrease in prices, an increase in computational capacity and an improvement of software implementations. All these combined factors have allowed the spread of personal computers and their use in all areas of human endevour including agriculture and farm management. Therefore, the amount of data and information that farmers, technicians and advisers can deal with has increased. In this sense, information technologies are being updated to take maximum advantage of data analysis in order to extract information that is useful for improving the effectiveness of managerial decision making, especially in complex tasks (Turban, 1991). This can be possible by the inclusion of more complex mathematical models in computerized management information systems. To date, different computerised-based systems have been developed to support decision processes in sow farms, for example expert systems (Huirne et al. 1990; Pomar et al., 1992; Enting et al., 2000) or decision support systems (Huirne, 1990; Jalvingh, 1992). The natural computerised framework for decision models are DSS, in which we are concerned.

Formally, a DSS is an information system that supports the process of making decision. DSS allow decision-maker to retrieve data and test alternative solutions. Most of them are initially designed for research purposes, and therefore, they can be troublesome to operate in field conditions. It is assumed that the user has a solid knowledge of the problem entity, but, however, what he really requires is a clear and well-reasoned advice about different management strategies (Kamp, 1999). Hence, the user interface for on-farm use tends to resemble an expert system which represents a good complement for DSS. Because of this, there is a trend to design such systems modularly, getting them able to be integrated into different information systems, and when possible providing access by Internet.

In general, none of previous referred models took into account farm data. This fact does not facilitate the inclusion of those models within a practical DSS for on-farm use. The development of new farm-oriented decision tools is not easy, decision models should be adapted to farm data availability and farmers' demands. Nowadays, the main challenge to researchers is the development of a modeling approach that makes a farmer's decision making easier when faced with a large number of uncertainties, and this, in such a way that it is computationally feasible and operational.

The development of this thesis should be branched as a main component in a new research line on decision support systems, which has been started at UdL-IRTA Center in Lleida. The UdL-IRTA Center hosts the Official Spanish databank and its own management information system called BDporc®[4]. It contains data of about 350.000 sows grouped in around 1100 sow farms. The BDporc (2000) has allowed the establishment of different standard references, as well as, to exploit the databank at a research level. Thus, the availability of the databank inspired the development of a practical decision model for on-farm use. Moreover, it allowed us to validate the model from specific farm parameters and detect problems derived from a lack of data.

3. Outline of the thesis

The purpose of this thesis is to formulate a mathematical model representing sow management production and reproduction. The formulation is made by using a dynamic stochastic approach, a semi-Markov decision model or its equivalent embedded Markov decision model. The approach presented assumes steady-state situation. The representation wants to reflect available information on sow farms during a stable enough period through specific farm parameters. With respect to model features we want to identify strong and weak points at formulation and practical levels in order to determine suitable environments for on-farm use and later refinements. The model will be included in a decision support system integrated in a more general pig information system (Bdporc, 2000). Hence, the differences between the model developed here and the models presented up to now are its formulation for specific farm data, its statistical validation, its integration on a general MIS for on-farm use and the explicitation of problems detected in such adaptation.

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[4] GTEP-IRTA before 1998 (Noguera et al., 1992)

collaboration between the Animal Production Area of the UdL-IRTA[5] Center the Department of Agricultural Engineering and the Department of Mathematics of the University of Lleida.

The thesis has been written in seven chapters. In this first Chapter we have presented a brief introduction to the Thesis. In Chapter 2 we present a review of different methodologies applied in sow herd management modeling. Chapter 3 introduces the general formulation of the model used and refined along the thesis, as well as, describes programming techniques that will be applied later on. In Chapter 4 we develop the model implementation and its validation with real farm data. In Chapter 5 we go on to discuss the suitability of the model to be included into a DSS. The inclusion into a DSS is completed with the optimization algorithm that provides optimal management rules for the replacement problem. This chapter emphasizes how the DSS could be applied, for example, it compares different management strategies concerning, basically, reproduction and replacement management decisions and the integration of the model in a more general pig information system. Finally, in Chapter 6 we document practical problems in the optimisation of sow herd management and suggest some areas for future work. We try to determine weak and strong points in its formulation, we review assumptions of the model and their practical consequences. A brief discussion about the suitability of the steady-state approach is also included. We finish with a general discussion about this approach, pointing to future developments for improving actual applications of the model and its formulation when needed.

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Chapter 2. REVIEW OF MATHEMATICAL MODELS OF SOW HERDS

Plà, L.M. [1,2]

[1] *University of Lleida, Department of Mathematics*

Pl. Victor Siurana 1 E-25003 LLEIDA

[2] *R+D Centre. UdL-IRTA, Area of Animal Production*

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Abstract

This paper is a survey of different livestock herd models described in the literature which make use of different mathematical methodologies. Different models are studied and grouped depending on its basic model formulations. We focus basically on the more common models classified in simulation, dynamic programming, linear programming and Markov decision models. In a first stage we recall modeling foundations of previous models, and after we review specifically sow herd models published until now. Special attention is paid to main variables, source of parameters, validation, output and intended use. We shall try to detect weak and strong points in model formulation for further developments and practical use on-farm.

1. Introduction

Herd management is the process by which certain goals of the farm manager, expressed as amount of product, are achieved by consuming a corresponding amount of production factors. In order to be able to combine these factors in an optimal way is necessary to know main interrelations among them and their influence on the final productivity of the system. The herd system can be understood as a set of different interrelated elements, breeding-animals, that acts as a whole face to exogenous solicitations. It is usual to make system simplifications in order to get practical herd models although conserving the essence of the real system. The challenge of the livestock modeller is to represent what is essential in the system in order to find relevant answers from a problematic situation that may initially seem chaotic (Huntley et al., 1990).

Mathematical models representing the production behaviour of a livestock herd are used for a long time in livestock research and development. Livestock herd models, in general, and sow herd models, in particular, are important tools to analyse different herd management strategies. Through herd models, researchers first, and farm managers after can understand better real farm behaviour. Nevertheless, research models get used to be quite complex initially because the system is so, but also are little effective for practical use. For example, programming models published in the 70's dealt with several hundred of states (Arendonk et al, 1978), and in the 90's the number increase until several millions of them (Houben et al., 1996). Rigid constraints restraining the scope of the model have been relaxed to represent fairly the system and efforts have been made in solving or circumventing problems related to complex models. Therefore, the ability to represent complex systems and solve huge problems has increased, but this ability is not corresponded with an augmentation of farmer's demand of such decision support tools (Kamp et al., 1999). Recent advances in computing and mathematics allow thinking in improvements on the effectiveness of herd models for on-farm use.

The objective of this paper is to review mathematical foundation of existing sow herd models. A sow herd model is defined as a model which try to represent the productive and reproductive behaviour of a group of breeding sows over time. Hence, the use of sow herd models is mainly focused on reproduction and replacement management. Because this framework is also common to other livestock species, other references are introduced to exemplify different mathematical models not used yet in swine production. It is in the aim of the review to detect strong and weak points making models more or less suitable for practical use. This work is intended as a first steep for future extensions in order to help the development of more practical tools for on sow farm decision support.

2. THE MODELLING OF SWINE PRODUCTION

2.1. A sow farm as a system.

In order to represent a sow farm it seems relevant to describe farm operations related to sows, because a sow farm is indeed a set of sows driven under the same management policy. Thus, the lifespan of a sow usually starts when it is purchased or reared as a gilt and introduced on farm after a recommended quarantine. Weight, age and observed heats are parameters to take into account when mating gilts at first time. Normally, when heats are detected means that gilts and sows are ready to be mated. At this stage, they are staying in the breeding facility (Fig. 1). For gilts there are different breeding techniques as mating them at second or third heat detected. After matings, gilts and sows are controlled in order to confirm the pregnancy. Once the pregnancy is positively confirmed they are moved to the pregnant facility. Otherwise, it is considered that conception has failed and they remain in the breeding facility for subsequent re-mating in next oestrus. Several days before farrowing (normally seven days), pregnant sows are moved to the farrowing facility. Farrowing facilities are usually organised in rooms to hold batches of sows, depending on intended batch size management and scheduled production. Sows remain there until weaning, after what they are moved to the breeding facility for re-starting a new productive cycle. Litters weaned, instead, get used to be left for few days more, before they are moved to the nursery facility. Farrowing rooms are cleaned, sterilised and closed for a drying period before to receive a new batch of sows. In case the farm holds rearing facilities, could be possible to rear gilts for replacing culled sows, but nowadays is more common to purchase gilts from outside. Depending on system production, litters exiting the nursery can be sold or reared in growth-finishing facilities to be sold at the end of the fattening process.

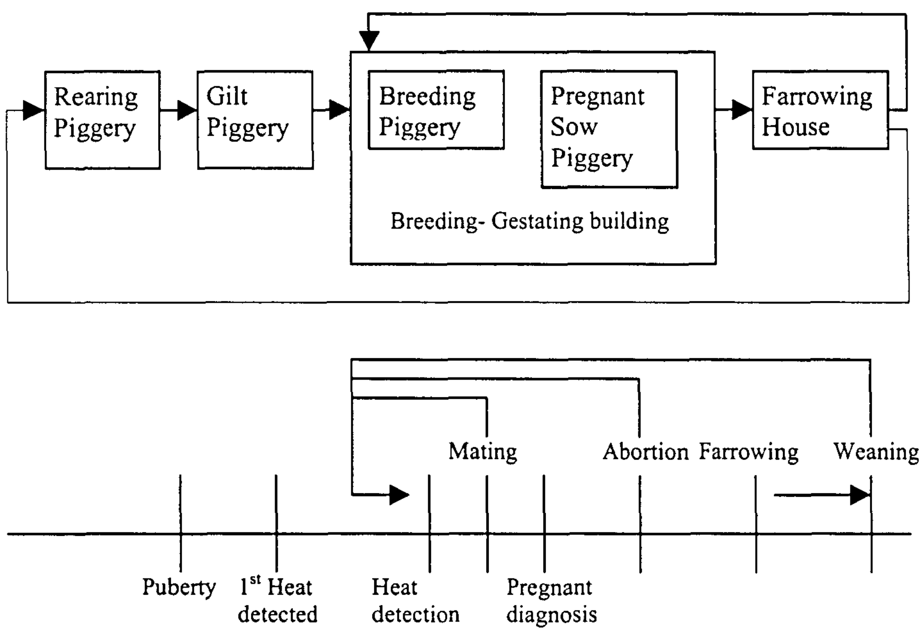

Figure 1. Physical and biological representation of a sow farm

As it is described, a sow herd is a complex system taking into account the number of factors that are involved in their productive behaviour. The production unit in the herd is a pig female and herd production is intimately related with reproduction processes.

That is because the final product, measured in number of piglets weaned/sold or in Kg of meat sold, depends basically on fertility and prolificacy of sows. Then, the sow herd system can be modelled using different approaches. One of them takes the sow as reference, and splits the sow's lifespan into different reproductive states which are bounded by events (e.g. gestation bounded by fertile mating and farrowing, lactation bounded by farrowing and weaning, etc). On the other hand, each physical facility can be organised in different buildings on farm and hosts sows remaining in a determinate state (e.g. gestation facility hosts pregnant sows, lactation facility or farrowing house hosts lactating sows, etc.). Therefore, another possible approach takes the availability and occupancy of such facilities as reference. These approaches are schematised in figure 1.

2.2. The modelling process

The modelling process can be described in different stages as it is shown in figure 2. First of all, the sow herd, as real system, is the entity to be represented. A natural description of what are observed can lead to formulate an empiric herd model more or less complex depending on the number of variables considered. Observations can come from experiments performed under a controlled situation or from statistics obtained from a record-keeping data bank or literature. The empiric herd model can be used to establish or study empiric relationships among variables, which can be used after a successful validation in prediction making about system evolution.

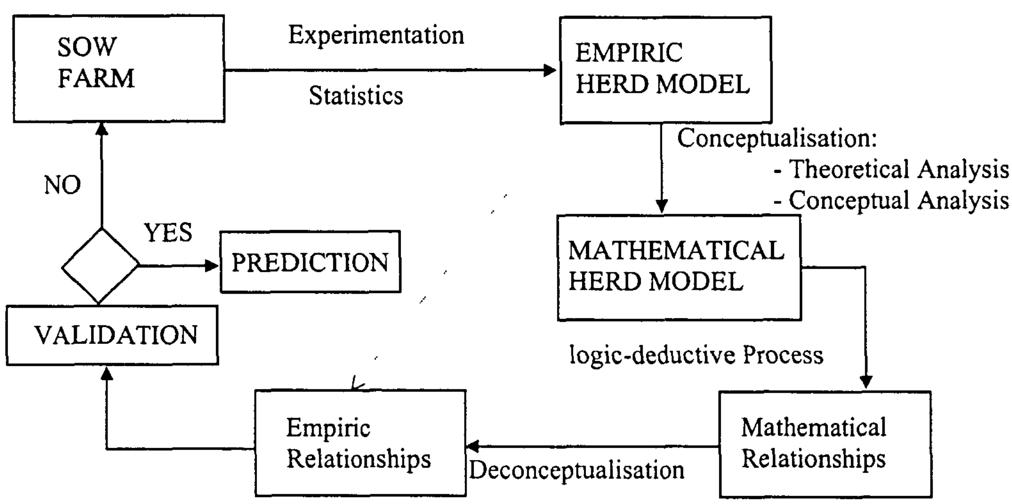

Fig. 2 The modelling process (adapted from Ríos, 1995)

Prediction can be addressed to solve practical problems, for on-farm decision support, or just to gain knowledge in hypothetical situations or system behaviour, for research or educational purpose. An effort to generalise the model to situations other than observed implies a more conceptual work on the empiric herd model. As result, a more detailed mathematical herd model can be formulated. The study of mathematical results facilitates the identification of mathematical relationships that have to be interpreted in empirical terms. Empiric relationships should be understood as the consequences of the model respect to the aim for what the system is modelled. The validation of the model has to assess the suitability of the model, i.e. the acceptance of the resemblance between the system and its alternative representation. In case of acceptation, predictions would be reliable, otherwise if the validation fails the process would have to start again and the model refined or reformulated.

2.3. Main mathematical methodologies applied in livestock herd modelling

The use of mathematical modelling in livestock production has a long pedigree, but swine production has received relatively little attention. In order to make a proposal of a sow herd model, it would be useful to get a general overview of different mathematical methodologies applied to represent livestock herd models, not only in swine systems.

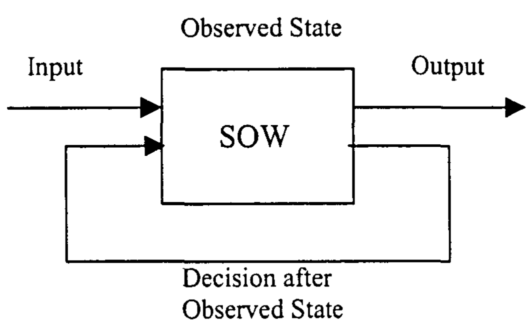

Figure 3.- General dynamic decision model representation

The class of decision models considered can be represented by a feedback graph (Figure 3) in which, after observing the state of the system, the farmer makes a decision in order to lead the farm to desirable productivity levels. In general uncertainty is always present in this kind of problems, but can be reduced to problems of decision under risk if we assume known, in some way, the probabilistic mechanism for uncertainties. A usual simplification used in herd modelling is to consider a herd composed by different units that more or less act in the same way. Therefore the farm becomes a complex system which is made up of an aggregation of small units following the same law of motion. The motion of the system can be represented and controlled through different elements described by decision variables.

All of the optimisation models are dynamic programming models or Markov decision models. We could represent all that models by following recursive expression:

$$V^R{}_n(s)=r^R(s)+V^R{}_{n-1}(s_{n-1})$$

where $V^R{}_n$ represents the economic value at time n, that the decision-maker, the farmer, obtains after following a decision policy, R, that determines at each n-time which decision to make. The economic value at time n, $V^R{}_n$, depends on present economic value gained at time n, r^R, plus accumulated economic value until time n-1. This expression is also known as Bellman's equation. Working this expression we can derive the more usual expression for dynamic programming and Markov decision models:

$$V^R{}_n(s)=r^R(s)+\sum_{j\in S} p^R(j\,|\,s)\,V^R{}_{n-1}(j)$$

where P is the matrix of transition probabilities representing transitions among states for each time step.

All these formulation determine input parameters needed, for example, functional V would depends on states, decisions, probabilities of transition, economic function and eventually by the time. The transition matrix would depends on the number of states in which the system is split on, and which transitions are allowed or which decision policies are considered.

In the operation of a sow farm unit several factors interact dynamically. Models for breeding and replacement policies should incorporate such interactions, but not all of them are possible to be implemented since many of the relationships are not fully understood and the limitations of particular techniques. To be of practical value for planning, a model of sow production must contain sufficient detail of the operation of the real system. On the other hand this type of approach may add complexity without providing the insight that can be obtained by analysing the results of a less sophisticated model. Therefore, many of the models are restricted to particular aspects of sow production assuming a stability of the reminding over a wide range of parameters. For instance, in intensive pig fattening operations, many of the models has concentrated on evaluating feeding and marketing policies, with mathematical programming models being widely used. Mathematical models have also incorporate some of the other activities associated with operations of this type, but the use of these models has been restricted by the nature of the simplifying assumptions. For example, Tess et al. (1983) included growth functions without taking into account feed composition. The milk production of an individual sow varies during the lactation period and depends on litter size and feeding policy. However, many sow production models have been restricted to feed consumption, and researchers have considered only average or overall milk production levels, when considered. Models should also consider the risks arising from uncertainty in, for example market prices, weather or disease outbreaks.

Approaches to such systems are made using simulation models, linear programming models and dynamic programming models including Markov decision models. Their development has followed different ways in last decades. All of them are conceived as decision models and methodologically can be classified in simulation and optimisation herd models. Simulation models are well suited to dealing with the variability and complex nature of livestock production, the first attempt was made by Tess et al. (1983) and Allen and Stewart (1983), who modelled and joined mathematically several subsystems with more or less simple links. However, simulation models may result difficult to establish some of the underlying relationships.

In order to deal with discrete events like conception, sex of offspring and death, a deterministic model has to use classes of animals as the simulation unit (Tess et al., 1983; Allen and Stewart, 1983; Jalvingh et al., 1992). Usually stochastic models have the female animal as a simulation unit (Singh et al., 1986; Pettigrew et al., 1987; Pomar et al., 1991). Discrete events are controlled by pseudorandom number generators and suitable probability distributions. Some stochastic models use pseudorandom number generators not only for discrete events but also for some continually distributed variables like live weight changes and milk production. Hence, the reproduction process takes a relevant paper and makes event driven simulation models to be advantageous face to continuous time models. Different mathematical models had been published before (Kennedy, 1988), but simplifications required for an adequate solving process by old computers prevent its practical use on farm. The evolution of computational power allows the formulation of more complex simulation models (e.g. Pettigrew et al., 1986; Singh, 1986; Pomar et al. 1991). Almost all simulation models are themselves described as a partially stochastic in order to express that not all the parameters are determined randomly. Neither, not always distributions for each parameter are known, therefore uncertainty is approached stating an "a priori" distribution and then bayesian methods are also possible.

Mathematicals models used in herd livestock representation are usually discrete models, and easily they are optimisation models intended to solve in the best way a well-defined problem. Thus we find Dynamic programming models (Huirne et al. 1990), Markov Decision Processes (Kristensen et al., 1988), linear programming (Jalving et al. 1992) and dynamic optimization based on control theory (Chavas et al., 1985). Chavas et al. (1986) presented a dynamic model which purpose is to emphasise dynamic aspects of pig production against static approaches. Through Markov models graph models like bayesian networks or influence diagrams have been introduced, but they are only used for modelling concrete aspects of livestock production and programming techniques based on them are not yet available (Kristensen, 1993). Also, realistic mathematical models previously unsolvable have taken advantage of computational power and have started to be solved (Huirne et al., 1990; Houben et al., 1990; Kristensen, 1996).

3. SOW HERD MODELS

3.1. Selected models.

In this section 12 sow herd models are reviewed, simulation is the methodology most often used to represent sow herds. The 12 models share their interest to model a sow herd, taken individual sow behaviour as reference, but not all of them are aimed for the same purpose. These models were reviewed to illustrate different mathematical approaches to sow farm problems needing a herd representation. Most of them were able to determine the effect of changes in reproduction or replacement, others considered the effect of changes in feeding and only one considers as well as genetic aspects. Different characteristics of the sow herd models analysed here are summarised in Table 1.

Authors	Year	Aspects	Model	Title
Allen and Stewart	1983	R	S	A simulation model for a swine breeding unit producing feeder pigs
Tess et al.	1983	R,F,E	S	Simulation Of Genetic Changes In Life Cycle Efficiency of Pork Production I. A Bioeconomic Model
Plà et al.	1998	R,RP,E	OP-S	A sow model for decision aid at farm level.
Dijkhuizen et al.	1986	RP,E	OP	Economic optimization of culling strategies in swine breeding herds, using the "PORKCHOP computer program"
Marsh	1986	R,E	S	Economic decision making on health and management in livestock herds: examining complex problems through computer simulation
Pettigrew et al.	1986	R,E	S	Integration of factors affecting sow efficiency: a modeling approach
Signh	1986	R,E	S	Simulation of swine herd population dynamics
de Roo	1987	R,G,F	S	A stochastic model to study breeding schemes in a small pig population.
Huirne et al.	1990	R,RP,E	OP	An Application of Stochastic Dynamic Programming To Support sow replacement decisions
Pomar et al.	1991	R,F	S	Computer simulation model of swine production systems: III. A dynamic herd simulation model including reproduction.
Jalving et al.	1992	R,RP,E	S	Dynamic probabilistic modelling of reproduction and replacement management in sow herds. General aspects and model description
Kristensen	1996	R,RP,E	OP	Multi-level hierarchic Markov processes as a framework for herd management support

R: reproductive, RP: replacement, E: economics, F: feeding, G: genetics
S: simulation, O: optimisation

Table 1. Sow herd models reviewed

One criterion to classify them is to know what is the aim for which they were building, thus we find that most of them were used for research purpose and the only objective was to represent farm dynamics in a suitable way, including available variables. Only models presented by Jalving et al. (1992) and Plà et al (1998) were aimed explicitly for use on field conditions and they introduce the possible use of specific farm data to run the model, but only Plà et al. (1998) did it with real farm data.

The problem entity for the herd model includes management factors like reproduction and replacement policies and for cattle and sheep often also grazing management. In this sense sow herd models could be considered simpler than models of ruminants, but the shorter productive cycle of sows and its sensibility to diseases increases the variability of expected outputs and makes the representation and control of the system difficult too. The usual time horizon of interest in order to evaluate the impact of different management strategies is therefore typically several years.

Optimization models are models that represents herd dynamics by transitions between different (reproductive) states, so they are discrete in time by nature. One difference among these models is the temporary pattern of such transitions that are represented weekly except Dijkhuizen et al. (1986) and Plà et al. (1998). Weekly transitions were chosen due to the usual scheduling of farm activities by weeks, and the election of a constant time transition matrix, but introduced some imprecision to force all (reproductive) states to be weekly-based. Dijkhuizen et al. (1986) considered transitions by parities, but they were not concerned with a fine representation of herd dynamics. Plà et al. (1998) considered the Markov decision process embedded in a semi-Markov decision model to solve original problem, hence they obtained savings in calculation and a more natural state representation. A different approach had been presented previously by Kristensen (1996) that exploited the structure of the transition matrix. He presented a hierarchic model based on the partition of the transition matrix in different sub-models. The advantage was a structuration of the problem besides an improvement in the handling of large models. All of the authors considered time-homogeneous transition probabilities, rewards and deterministic management policies. In this way they assure the ergodicity of the stochastic process represented and its convergence to a steady-state distribution although it is not mentioned. Therefore, the optimisation process is related with this steady-state distribution, and the common optimisation criterium is expected average reward per unit of time. All the authors solve the problem directly, and only Huirne et al. (1990) solve it approximately by successive iterations.

Simulation models represented sows in the herd according a pre-stated management policy. Sows were simulated sequentially, only Singh (1986) considered a synchronised simulation of the herd, thus he was able to represent a batch management. Only Allen and Stewart (1983) and Singh (1986) accounted for production facilities. On the other hand, Tess et al. (1983) and Pomar et al. (1991) accounted for growth process and nutrition requirements in more detail.

Within a range, different levels of nutrition have a limited effect on production in the current period. However, nutrition directly affects body condition at farrowing which influences the time after farrowing before the sow can again conceive. The practical application of feeding policies is limited because of the constantly changing nature of the input-output relationships. Since livestock feeding is a sequential decision process it can be modelled using DP.

Random numbers generators are used to create observations for individuals animals, such as production, survival and conception. As a result of using a random number generator, multiple runs are needed to obtain a reliable estimate of the average results of the herd. Moreover, the inclusion of random numbers in models for on-farm decision support is a possible source of confusion and reduction in the acceptability of the model and its results to the user, particularly when a number of replications is not large enough.

3.2. Input Parameters of the models.

Input parameters of the models depend on which kind of model we refer to, normally optimisation models have a more clear formulation than simulation models. To simulation models input parameters get used to be larger because the aim of such models is more general and flexible.

Optimisation models (Dijkhuizen et al., 1986; Huirne et al. 1990; Kristensen et al., 1996; Plà et al. 1998) are based on sow herd dynamics by means of a partition in states of the sow lifespan as it is represented in figure 1. The more general partition is proposed by Dijkhuizen et al., (1986) who considered parity-specific parameters (probability of survival, discount rate, marginal profit per parity, length, maximum number of parities allowed, deviation of typical parity-specific litter size). Parameters considered by remainding optimisation models (Huirne et al., 1990; Kristensen et al., 1996; and Plà et al., 1998) were similar. These parameters can be grouped in stage and state variables, economic inputs and transition probabilities. Main state variables accounted for gestation, lactation, interval weaning to first mating and interval between matings. Final number of states differed mainly due to different time pattern, only Plà et al. (1998) took directly into account spacific-state time interval (e.g. lactation length, gestation period, etc.). More states were added to represent better the variability of production and changes in production level. Most of the data used to study model behaviour is extracted from literature and less from real farms. Plà et al. (1998) presented specific-farm parameters calculated from farms stored in a management information system. Dijkhuizen et al. (1986), Huirne et al. (1990) and Kristensen et al. (1996) extracted parametres values from literature or considered standard values just to illustrate model operation. Authors considered mean parameters (e.g. gestation length, duration of lactation, oestrus interval, etc), without taking into account their specific variability.

Simulation models included random parameters characterised by a specific distribution and not a constant value. Biological production parameter are quite similar to all models and include conception rate, number of live pigs born/litter, mortality rates at different stages, length of gestation, weaning to first oestrus interval, oestrus cycle length and growth rate per state. The way these parameters are taken into account and valued depended on the model structure, design and objective. Marsh (1986) and Singh (1986) considered empirical distributions. For example, Singh (1986) considered empirical distributions of Hawaii's sow farms to generate values for litter size, mortality rates and weaning to 1st oestrus interval, but also random distributions for other parameters e.g. gestation and oestrus cycle length. In general, distributions used for random generation of input parameters were normal univariate when continuous variables are represented or real uniform in case to represent transitions between states. Although several authors used other distributions to represent weaning to oestrus interval (e.g. log-normal by Pettigrew et al. (1986) and exponential by de Roo (1986)). Allen and Stewart (1983) applied normal distribution to generate the age at puberty, weight at puberty, oestrus cycle, gestation period and litter size. Real uniform is only generated when individual behaviour is represented (De Roo, 1986; Marsh, 1986; Pettigrew et al., 1987; Pomar et al., 1991), if not the rate is directly applied to the herd (Tess et al., 1983; Allen and Stewart, 1983; Singh, 1986; Jalvingh et al., 1992; Plà et al, 1998). Uniform distribution is basically applied to represent conception success or not and culling reasons. As culling reasons infertility or reproduction problems and injuries were the most usual. For example, Allen and Stewart (1986) also considered culling based on parity limit and death while other authors were more explicative detailing infertility and more culling reasons (Singh, 1986; Pettigrew et al., 1987; Pomar et al., 1991).

Tess et al. (1983) and Pomar et al. (1991) based respective models upon growth process and feeding requirements, so they approached to the system under a nutritionist point of view. Tess et al. (1983) did it in a deterministic way whereas Pomar et al. (1991) built a stochastic model. Therefore, Pomar et al. (1991) accounted for interactions between nutrition and reproduction parameters in detail, but in general feeding requirements were large simplified in remainding models. As example, Allen and Stewart (1983) considered daily feed intake of pigs in nursery by age at weaning and chronological age like most of the authors, who just considered daily feed intake by stage (Singh, 1986; Jalvingh et al., 1992; Plà et al, 1998).

Pomar et al. (1991) took some parameters from previous simulation models (Tess et al., 1983; Allen and Stewart, 1983; Singh, 1986; Pettigrew et al. 1987) but they included other factors affecting sow behaviour and a more precise description of ovulation and growth processes by a set of equations. Otherwise, they did not represent the availability of facilities that was considered by several authors (Allen and Stewart, 1983; De Roo, 1986; Singh, 1986; Pettigrew et al. 1987). Allen and Stewart (1983) accounted for floor requirements and a limit was established for the farm while Pettigrew and al. (1987) fixed a maximum number of farrowings per week as limit. De Roo (1986) and Singh (1986) considered available places physically distributed among different buildings: breeding, gestation, farrowing, nursery and growing finishing. De Roo (1986) was the only who considered selection indices for sows and boars, beside other parity-dependent parameters.

Finally we can remark that not all of the simulation models included economic inputs, as optimisation models did (de Roo, 1986; Allen and Stewart, 1983; Pomar et al.,1991). Simulation models are more concerned in a description of a sow herd emphasising different aspects.

3.3. Outputs of the models

Outputs of the models are related with its purpose. In simulation models there were more outputs than in optimisation models. Optimisation models were aimed to find a maximum or a minimum, for example Kristensen (1996) found the optimal replacement policy for sows, like Huirne et al. (1993) and Dijkhuizen et al. (1986). After that, depending on the author an analysis of sensitivity or post-optimum is performed. Thus, Dijkhuizen at al. (1986) offered technical indexes and a sensitivity analysis of several variables while Huirne et al. (1993) just calculated some performance indices.

Respect to the simulation models, there are a wide variety of outputs depending largely on its construction. Then Marsh (1986) presented a lot of outputs classified in seven categories: Population, Performance indices, Reproductive performance, Monthly graphics, Cash flow analysis, Income statement and Livestock valuation. They were the same categories he used in a previous dairy model and inspired in commercial information systems. Singh (1986), Jalvingh et al. (1992) and Plà et al. (1998) presented different outputs related to herd dynamics. More specifically, Singh (1986) calculated statistics about herd dynamics. In addition to different prices and costs he computed annual incomes, costs ands rate of return for economic analysis. Similarly, Jalvingh et al. (1992) calculated technical and economic variables derived from the distribution of sows over states at equilibrium. The most important were the value of piglets and the slaughter value of culled sow, costs of replaced gilts and number of litters per sow per year and percent of reinseminations. Plà et al. (1998) calculated differently and individually for each farm analysed technical and economic variables, but also derived from the distribution of sows over states at equilibrium. They provided, as well, different graphics related to sow distribution over states. Tess et al. (1983) and Pomar et al. (1991) considered the animal growth in their models, therefore they showed plots of body weight of sows. Tess at al. (1983) appended growth curves, performance indices and some rates of biological efficiency while Pomar et al. (1991) appended statistics describing flow of animals between stages of life cycle in the herd, average sow age per day and simulated number of animals per day. Allen and Stewart (1983), Pettigrew et al. (1986) and de Roo (1987) were more concrete in calculating outputs. Thus, Allen and Stewart (1983) calculated the means of some production characters: litter size at birth, pigs born/sow/year, pigs weaned/sow/year, conception rate and Kg of pig sold per Kg of feed. Pettigrew et al. (1986) calculated sows days/pig, Pigs/sow/year, pigs/litter and litters/sow by year of simulation. Finally, de Roo (1987) calculated number of sow, farrowing index, number of inseminations, litters size at birth, litters size at weaning, statistics of culling reasons, breeding boars, imbreeding index and graphics of the effect of selection on fat, lean, growth (g/day) and feed intake.

3.4. Validation of the models

Not all of the reviewed models were validated. For example, optimisation models were not validated, they were presented as deterministic models dealing with well-defined problems. Optimisation models were mainly interested in showing mathematical methodologies to solve specific problems. For instance, Kristensen (1996) presented a new approach to sow herd modelling, hierarchical Markov decision models, based on a refinement of standard Markov decision processes in order to show its benefits. Validation in these papers was out of their purpose. Instead a formal validation, other authors as Dijkhuizen et al. (1986) and Huirne et al. (1990) determined the effect of changing conditions in some major parameters, just to gain insight into the model behaviour.

Alternatively, several validation methods were used in simulation models. Authors presenting simulation models agreed that it is difficult to achieve a fully validation because neither all parameters were known in practice nor suitable data for validation were available. An alternative used by several authors was to describe precisely the model without any other test to validate it (Singh, 1986; de Roo, 1987). In some cases, the common strategy was to perform a verification based on a detailed description of the model and a checking for the correct running of the model at several points in the life cycle including the final summation of inputs and outputs. For verification Allen and Stewart (1983) used two statistics, number of sows and gilts in the system and time in phase. For partial validation, Tess et al. (1983) and Pomar et al. (1991) evaluated different outputs as lactation weight pattern, final body composition, litter weight at birth and at weaning, feed/gain ratios and milk production, while Allen and Stewart (1983) compared pig weaning weights (at birth and at 18 Kg) with those referred in the literature. Marsh (1986), Jalvingh et al. (1992) and Plà et al. (1998) presented a model behaviour study based on sensitivity analysis, afterwards they compared general results with results obtained from management information systems. No statistical evaluation was presented in previous papers, only Marsh (1986) and Plà et al. (1998) did it. Marsh (1986) argued that his simulation model was based on the reproductive cycle of the sows and therefore the focus of the validation should be the reproductive events as predicted by the model. He used a non-parametric test, the Kolmogorov-Smirnov test, to test whether the observed and the simulated samples of farrowing to first oestrus interval derived from the same distribution. On the other hand, Plà et al. (1998) considered the sow herd distribution over states calculated by the model and the actual distribution observed, they used a non-parametric test, the Chi-square test, to test whether both distributions derived from the same.

Different runs of the model should be taken into account in order to get an estimate of model outputs and its variability. Hence, the number of runs and the simulated time could be considered also as outputs in simulation models. Only, Huirne et al. (1990) reported CPU time for the optimisation process.

3.5. Implementation and integration opportunities

Usually, researchers were who self-programmed their applications, at least in a first stage. Most of the models are intended for research or educational purpose and only few of them express their aim to be used on-farm (Dijkhuizen et al., 1986; Marsh et al., 1986; Jalving et al., 1993; Plà et al. 1998). These facts may explain why user interfaces are not well enough elaborated for farmers or advisers. Procedural languages were the most common programming languages used in software implementation, for example Marsh (1986) programmed his model in ANSI C, Huirne et al. (1993) and Jalvingh et al. (1992) used Pascal and Plà et al. (1998) Extend™, based on C, to do it. Instead, some simulation models were implemented using specialised programming languages for simulation like SLAM II (Allen and Stewart, 1983; Pomar et al. 1991) or GSSP (Singh, 1986). The rest of papers did not mention how the models were implemented.

The use on-farm of such models is strongly related to their integration in existing information systems as modules. For instance, PORKchop (Dijkhuizen et al., 1986) pointed to possible transfer of relevant data from PigCHAMP (Stein et al., 1983) and VAMPP (Buurman et al., 1986). PigORACLE (Marsh, 1986) was built as a module of PigCHAMP. TACTSys was a management information system for tactical decision support integrating different models (Jalvingh et al., 1993; Huirne et al., 1990). BD-Porc system (2000) is a management information system that contains the official databank of Spanish pig production and the model of Plà et al. (1998) can be included as a module. However the use of the computerised models as stand-alone applications is not completely successful to date (Kamp, 1999). The model interface is very important for a practical use and in acceptation by farmers or decision-makers.

3.6. Further applications and related works

Outputs of the models were not enough to have a scope of the advantages of using such models, therefore, several authors included brief examples of use. For instance, Allen and Stewart (1983) compared alternative management practices of 3 and 6 week lactations. Pettigrew et al. (1986) simulated several alternatives to compare them (decreased mortality, more uniform age at puberty, split weaning, increased litter size and increased prolificacy). Dijkhuizen et al. (1986), Huirne et al. (1990) and Jalvingh et al. (1992) did an analysis of sensibility for main productive variables in order to check their impact on model performances.

In general, most of the reviewed models were used in later works that provide more precise examples of potential applications. For example, to study the occupancy of facilities based on the model of Singh (1986) were published (Singh, 1986b). Similarly Lippus et al. (1996) and Plà et al. (2000) applied the model of Jalvingh et al. (1992) and Plà et al. (1998) respectively to study the same problem. Also it raised examples of applications in field conditions, for example Alsop et al. (1994) used the model of Jalvingh et al. (1992) with empirical data and Plà et al. (2001) built a decision support system for on-farm use. The model of Huirne et al. (1990) was also used in different works to evaluate replacement alternatives (Huirne et al., 1990; Jalvingh et al., 1993), as well as it happened with that of Dijkhuizen et al. (1986) used to analyse economic reasons in replacement (Dijkhuizen et al., 1990). Houben et al. (1990) modified the model of de Roo (1987) to calculate litters/sow/year, pigs weaned/sow/year, profit/sow and profit/herd. Later on, they applied their model to compare the outputs of different insemination and replacement policies in order to find the more suitable combination of them. Sometimes, reviewed models were included as a part of a bigger system described elsewhere (Tess et al., 1983a; Tess et al., 1983b; Tess et al., 1983c; Pomar et al., 1991a; Pomar et al., 1991b; Pomar et al., 1991c).

3.7. Risk management

To obtain statistically significant results from a stochastic model, as simulation models are, it is necessary to generate a large number of independent observations on the random variable of interest. Therefore, Singh (1986) run the model 10 consecutive years taking a sample per year and used the Student t distribution to test the average income and to obtain the 95% confidence interval of the yearly average income. Pettigrew et al. (1987) replicated each alternative three times and compared them by ANOVA in a completely random design.

Optimisation models ignore uncertainty associated to their results, although it is considered for most of the parameters of the model. Therefore, the result performed of optimisation models is not reliable and represent an orientation for potential users.

4. DISCUSSION AND OUTLOOK

Researchers have developed a large number of mathematical models of sow herd systems, but because of the complexities of the system models concentrate on particular aspects of farm operations (Glen, 1987). Reviewed sow herd models are focused on reproduction as main process determining herd production, as consequence sow reproductive behaviour is taken as reference and discrete reproductive events avoid the use of continuous models. As well, variability in sow performance during sow lifespan induces the formulation of dynamic models. Other aspects like replacement and growth are also considered depending on the aim of the models. Thus, different sow herd models have been reviewed focusing on their mathematical approach in order to settle strong and weak points. There are coincidences in the modelling approach for reviewed herd models at mathematical level, although Jalvingh (1992) argues that an enormous variation in structure is observed among the models.

Initially, many of such models have been developed as research tools and teaching aids and few of them are used directly by farmers. Several researchers have attempted to incorporate mathematical models in management information systems suitable for use by individual farmers, but without a widespread acceptance by farmers (Kamp, 1999). Debertin et al. (1981) has suggested that whenever models can run interactively or the results can be available quickly, the use of DSS including mathematical models can have a significant impact on farmer's decision making behaviour.

One advantage of herd models is that they can be used instead the real system, but in order to be a reliable tool they have to be validated. After validation, the use of the model can increase the knowledge about the system or it can serve to solve real system problems. Quite often difficulties appear in validation tasks and if they are not dealt properly the effectiveness of the model to support decision tasks is compromised. When operational validation of whole sow herd models is not possible, it is important to exploit other activities which can improve confidence in the model (Sorensen, 1990). Understanding and accepting the overall structure of the model is often a major source for building up user confidence.

In research models the interface need to be highly flexible because the model may be used to explore assumptions and hypotheses but this can became a disadvantage for on farm use (Sorensen, 1990). Applications based on models should be self-sufficient and prevent mistakes. Interfaces are not prepared for farmers. Therefore, the interface should be simple, easy-to-use, intuitive and under the user's point of view free of model mistakes or inconsistencies. Such requirements would therefore need a sophisticated user interface, even more for on-farm use. It is supposed that users have a level of knowledge about the model that many times is not corresponded. A solution to satisfy these requirements would be the appending of expert systems as various authors proposed (Turban, 1991; Huirne, 1990). In this architecture, the expert system would manage the model and display conveniently the output to the user, i.e. it provides suitable answers to the user within a structured framework and throughout an intuitive interface that avoid the direct interaction between the user and the model. This trend may change slightly in future in order to provide access to such applications through Internet.

The use of models can be promoted by adapting them to field conditions. In this way, a major weakness of current models would be that they do not consider other factors, such as feeding and marketing policy, that influence the economic efficiency of livestock production (Glen, 1987). Bearing in mind that the system itself is the main source of knowledge. Hence, management information systems are relevant for knowing in deep what had happened in a farm. Usual data record-keeping systems are centred in the collection of reproduction events. Usual events recorded are matings, farrowings, weanings and abortions. Besides these events, more informative data is added, like number of piglets born alive, dead, weaned, reared, sold, idetification of sows, type of mating, etc. The amount of data collected depends on facilities and advantages that farmers or decision-maker obtain instead of it. If the value of information is zero, it is not worthy to collect data providing such naught-information. Therefore, databanks containing economic and health/disease data can be increasingly developed once it has been seeing their utility. Data collection is a time demanding task and the short counterbalance for decision-maker can frustrate the recollection. These facts could explain why decision tasks are not already well-supported on-farm and why users are not confident with new information technologies supporting decision processes.

Unfortunately, a little lack of knowledge is always present, so uncertainty is also present. When input parameters are not known with complete certainty, failure of the simulated results to correspond to actual production data can be due to an invalid model, faulty input data or both.

Although previous optimisation models are of stochastic nature by including probabilities, their resolution is based on an equivalent deterministic model which is really solved. That is because the optimal criterion is the expected value of an economic function without considering variability and uncertainty. As it has been seen, there were usual both technical and economic outputs, however it is not so to take into account price fluctuations. Reason for that would be the uncontrollable nature of prices, which is difficult to model. Hence a usual procedure is to consider a deterministic behaviour of prices, even constant.

As it has seen, at present research models are good for learning, but not so for advising. Simple models are not explored enough to establish its level of utility before the formulation of complex ones. It is needed that decision models be reliable tools to assure future spreading.

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Chapter 3. SOW MODEL FOR DECISION AID AT FARM LEVEL

Plà, L.M. [1,2]; Conde, J. [1] and Pomar, J. [2,3]

1 University of Lleida, Department of Mathematics

Pl. Victor Siurana 1 E-25003 LLEIDA

2 R+D Centre. UdL-IRTA, Area of Animal Production

3 University of Lleida, Department of Agro-forestry Engineering

Avda. Rovira Roure 177. E-25198 LLEIDA

Paper by Lluís M. Plà; Josep Conde and J.Pomar in Giron, F.J. (Eds.), Applied Decision Analysis. Kluwer Academic Publishers, Boston, pp. 47-62.

Abstract

This paper presents a friendly implementation of a dynamic sow model. The aim of the model is to represent sow production through reproduction and replacement management at farm level. A first proposal is validated and later applied to optimize herd dynamics from real farm data. Optimization provides simple rules that farmers can apply to improve their profits. Realistic dynamic models include a large number of state and decision variables, therefore some simplifications are needed both in order to obtain a useful model and to included it in a decision support system (DSS) running on a PC.

Keywords: Decision aid, dynamic model, sow management.

1. Introduction

In general, a farmer is faced with the problem of influencing the behavior of a probabilistic system, like a farm, as it evolves through time. He does this by making decisions or choosing actions. The main question is to determine wich sequence of actions causes the system to perform optimally with respect to some predetermined performance criterion. Since the farm is not static, decisions must anticipate the opportunities and costs associated with futur system evolution, see Chavas et al. (1985).

Mathematical models representing production behaviour of a herd are a popular tool in livestock research. Several models that simulate various biological, physical and management factors influencing population dynamics have been developed in dairy production , and less in pig production, Huirne et al. (1993). Within the Information Technologies Group of Animal Production Area of the IRTA (Institut de Recerca Tecnico Agroalimentaries) in colaboration with the Mathematics Department of Lleida University decision sow models are being developed to apply modelling techniques that could be integrated in DSS in order to support farmer decision tasks. In this paper we present a friendly implementation of a dynamic sow model, its aim is to represent sow production through reproduction and replacement management at farm level, see Plà et al. (1996). The model is farm specific, its complexity is designed to be enough to solve real problems in field conditions avoiding unpractical complexities that sometimes some research models contain.

2. *Markov Decision Processes*

Markov decision processes and Markov chains are narrowly related, so that is easy to induce a new stochastic model from firsts (see, Puterman (1994)), in particular a Markov chain. We assume that S and A are dicretes.

We consider now the space Ω:

$\Omega = \{\omega \mid \omega = (i_1, a_1, i_2, a_2, \dots),\ i_n \in S,\ a_n \in A\} = \{S \times A\}^{\infty}$

An element of ω represents a sequence of states and actions. We refer to ω as a possible path. Consider also random variables X_n and Y_n:

$X_n : \Omega \to S$ and $Y_n : \Omega \to A$

which take values in S and A respectively, defined by:

$X_n(\omega) = i_n$ and $Y_n(\omega) = a_n$

for $n \in N$. X_n represents the system state and Y_n represents the action made at time n.

Given a policy $R = \{d_n\}_{n \in N}$ where d_n is a decision function, it will be stationary if $d_n = d$ for all $n \in N$; thus, the same decision function is considered at each period. We will indicate it as $R = (d)^{\infty}$. Function d in the deterministic case is like a random variable, we will be denoted by $d: S \to A$.

Now we can define a probability P^R on $(\Omega, \mathcal{B}(\Omega))$ by means of the following equations:

$$P^R\{X_1 = i\} = q_1(i) \qquad (1)$$

$$P^R\{Y_n = a \mid X_n = i\} = 1_{\{a\}}(d(i)) \qquad (2)$$

$$P^R\{X_{n+1} = j \mid X_n = i, Y_n = a\backslash\} = p_n(j \mid i, a) \qquad (3)$$

where (1) are the initial probabilities and (3) is the transition probability associated to policy R, defined by:

p_n: $(S\times A)\times \mathcal{B}(S) \rightarrow [0,1]$

such that

1.- $p_n(B|\cdot)$ is a random variable in S for each $B\in \mathcal{B}(S)$.

2.- $p_n(\cdot|i,a)$ is a probability on $(S\times A, \mathcal{B}(S\times A))$ for each $(i,a)\in S\times A$.

Thus, the probability of a path $\omega = (i_1,a_1,i_2,a_2,\ldots)$, is given by:

$$P^R(\omega) = q_1(i_1)1_{\{a1\}}(d(i_1))\cdot \prod_{n>1} p_n(i_{n+1}|i_n, a_n)\ 1_{\{an\}}(d(i_n))$$

Note that the policy determines P^R explicitly through (2) and implicitly trhough (3). If R is a markovian policy, then:

$P^R\{Y_n = a|\ X_1=i_1, Y_1=a_1,\ldots, X_n=i_n\} = P^R\{Y_n = a|\ X_n = i_n\}$

and

$P^R\{X_{n+1}=j|X_1=i_1, Y_1=a_1,\ldots, X_n=i_n, Y_n=a_n\} == P^R\{X_{n+1}=j|X_n=i_n, Y_n=a_n\}$

so that the induced stochastic process $\{X_n; n\in N\}$ is a discrete-time Markov chain.

Let us now consider a family of reward functions$(r_l)_{l\in N\times S}$, that is real functions, r_l:$S\times A\rightarrow \boldsymbol{R}$, $\mathcal{B}(S)\times \mathcal{B}(A)$-measurable and bounded. The reward function represents the costs or income obtained in a time×state, l, depending on the arrival state and action made.

Finally, a Markovian decision process is a process containing the following elements: $\{N, S, A, p_n(\cdot|s,a), r_{n,l}(\cdot,a)\}$

3. Model formulation

We have defined the elements needed in Markov decision process from a mathematical point of view. Now we are going to identify these elements within the real world. We assume the process time homogeneous, that is, $p_n(j|i,a)=p(j|i,a)$ and $r_{n,l}(j,a)=r_l(j,a)$. Each sow and its succesors are represented by a Markov decision process. The structure of the model is homomorphic with the sow's life. The model is formulated to account as accurately as possible for biological and economic inputs needed in sow production dynamics representation.

3.1. State and Action sets

We are concerned in sow production, so main traits that may influence the future behaviour of the animal would have to be represented in the model. It is obvious that this doesn't mean taking into account all of controlled traits to get a good model. However, realistic Markov decision models used to be very complex, and it is usual to encounter several computational problems derived from the course of dimensionality, see Kristensen (1993).

The sow's lifespan is modelled via a Markov chain, therefore an animal begins when it is purchased as a new replacement gilt coming on heat for the first time and finishes when it is sold to slaughter or it has had an involuntary indisposition. The replacement can be made immediately or a delay may be considered.

Sows in a farm will be found in one of the possible states $S=\{s_i| \ i=1,\ldots,N\}$ and states are ordered as much as possible depending on the sows's life. The set $S=B\cup E$, is finite ,where $E=\{e_{ijkl}|$ i: reproductive states, j: productive cycle, k: production level l: genetic merit} and B represents the set of some artificial states like involuntary indisposal, delay in replacement or mortality.

The sow model can simulate herd production indirectly by aggregating individual sow performance, then the farm is represented by a state vector $\{\Pi_n\}$ where each component represents the probability of sows remaining in their corresponding state. The state vector is made up of four variables: parity number, production level, number of unsuccessful breedings in the present parity and reproductive state.

The initial vector $\Pi_1=(\pi_{11}{}^R, \pi_{12}{}^R,\ldots, \pi_{1N}{}^R)$ can be determined from farm recorded data or set as a unit vector representing the farm's beginning, $\Pi_1=(1,0,\ldots,0)$. Optionally this vector can be modified by hand. In case π_{1i}^{R} were calculated, the procedure is to accumulate the number of sows at each considered state and its normalization provides the initial distribution of the model, thus

$$\tilde{\pi}_{1i}^{R} = \frac{n_i}{\sum_{i=1}^{N} n_i}$$

where n_i is the number of sows staying at i-state during the period chosen for estimation, normally one year, and $\sum_{i=1}^{N} n_i$ is the total number of sows in the same period.

For each state, a set of actions is given, the action set, $A=\{a_i \mid i=1,\dots, N'\}$, is finite and includes all possible controls that the farmer can carry out on the farm. Actions at the sow level all include replacement as one of the alternatives. For solving replacement problems two actions are normally considered $A=\{Keep, Replace\}$, however it is possible to add more actions. The state vector has its relative action vector whose components represent actions taken for each component of the state vector. Actions are given by decision function, so the action vector in a deterministic case equals the policy R applied.

3.2. Transitions and rewards

The model considers a sow in a process moving from one state to another, not all transitions being considered, only those for which there could be a logical biological justification (Figure 1). This assumption is determined in fact by field conditions, although some filters are introduced to prevent inconsistencies in farm data.

When the order of the chain happens to be greater than 1, it can be reduced to a first order chain by suitable definition of composite states, as a result the states number increases. These transformations are used for example to take into account production level, but it also may be useful if we want to consider transitions in a daily or weekly basis in order to have an accurate representation of sow age and to be able to take tactical decisions, but therefore the model becomes more complex, even when most elements of the transition matrix with not feasible transitions equal to zero (Huirne et al., 1993, Plà et al. 1996).

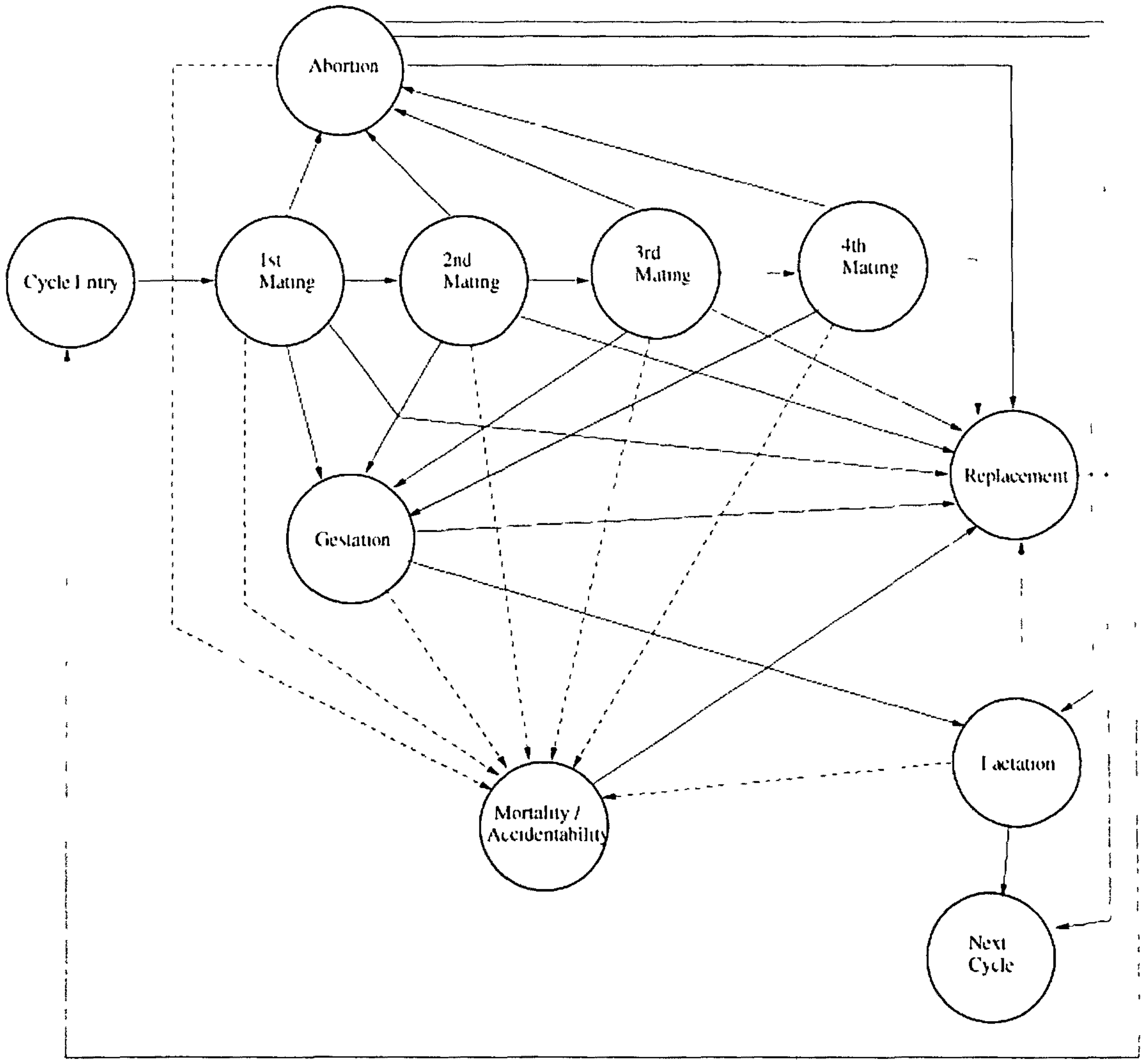

Figure 1. Graphic representation of reproductive sow cycle

The probability transitions were estimated from GTEP-IRTA data-bank or from literature when it was necessary, specially for default values. GTEP-IRTA system is a Pig Management Information System (Noguera et al., 1995) one of the most importants in Spain. Individual farm data is available and for each farm data recorded for each animal may be classified in reproductive animals inventory and input data: matings, parities and weanings and reproductive animal casualties. Then maximum likelihood estimates of probability transition are computed, see Billingsley (1961):

$$\tilde{p}(j|i,a) = \frac{n_{ij}}{n_i}$$

where n_{ij} is the number of sows moving from i state to j state, and $n_i = \sum_{k \in S} n_{ik}$ is the total number of sows passing through i state at the same period of time, usually one year. In case action Replace was taken, then $p(j|i,a)=1$ if j is the replacement state, and 0 otherwise. In general transition probabilities are associated to certain policies. A subset of the data-bank records could be considered to account for these alternative transitions and estimates would be obtained in the same way.

For the moment, the quality of the replacement sow is assumed to be unrelated to that of the particular sow being replaced, all sows having the same expected quality. The model does not take into account improvement in the genetic merit of sows to produce piglets, but it is possible to account for variation in prolificity and its repeatibility.

For any stationary policy $R=(d)^{\infty}$:

$$r^R(i)=\sum_{a \in A}\sum_{j \in S} r_t(j,a)p(j|i,a)1_{\{a\}}(d(i))$$

represents the expected risk at time n and in the state i. The expected value at decision epoch n may be calculated by

$$r(i,a) = \sum_{j \in S} r_t(j,a)p(j|i,a)$$

If R is a Markovian policy, we refer to bivariate stochastic process $\{(X_n, r(X_n,Y_n)); n \in N\}$ as a Markovian rewarded process.

The economic consequences of the decisions made are reflected in the reward function. If at a decision epoch the action a is chosen in state i and the system evolve to j, then an immediate reward $r_t(j,a)$ is obtained, this may occur with a probability $p(j|i,a)=p_{ij}^{\,a}$. In this way the model quantifies the gains or costs obtained in a swine herd.

4. Optimality Criteria for infinite-horizon

An infinite planning horizon implies that an optimal policy is stationary, see Puterman (1994). We assume from now on that we will have stationary rewards and probability transitions, bounded rewards ($|r(i,a)| \leq M < \infty$ $\forall a \in A$, $\forall i \in S$) with S and A finites. The discount factor, $0 < \lambda < 1$ express the time preference of the decision maker.

The Markov decision problem that we are concerned with can be optimised under different objective functions:

Expected total discounted reward. Let $v_\lambda^R(i)$ represent the expected total reward over the decision making infinite horizon, if policy R is used and the system is in state i at the first decision epoch. Then,

$$v_\lambda^R(i) = E_i^R \left\{ \sum_{n=1}^{\infty} \lambda^{n-1} r(X_n, Y_n) \right\}$$

which can be easily seen to be equal to:

$$v_\lambda^R(i) = E_i^R \sum_{n=1}^{\infty} \sum_{j \in S} \lambda^{n-1} r(j, a_j) P^R \{X_n = j, Y_n = a_j \mid X_1 = i\}$$

when

$P^R\{X_n=j, Y_n=a_j|X_1=i\} = P^R\{Y_n=a_j|X_n=j\} P^R\{X_n=j|X_1=i\}$.

This criterion maximizes the total discounted net revenues per animal. Such a criterion is relevant where a limiting housing capacity is the most limiting herd constraint. For induction purposes we will use the equivalent formulation:

$$v_\lambda^R(i) = r(i,a_i) + \sum_{j \in S} \lambda p(j \mid i, a_i) v_\lambda^R(j) \quad (4)$$

Average reward.

$$g^R(i) \equiv \lim_{n \to \infty} \frac{1}{n} E_i^R \left\{ \sum_{t=1}^{n} r(X_t, Y_t) \right\}$$

If all stages are of equal length, and also, when all states in S are recurrent $g^R = g^R(i) = g^R(j)$, then the function can be rewritten as:

$$g^R = \sum_{i \in S} \Pi_i^R r(i, a_i)$$

where Π_i^R is the limiting state distribution under policy R, and g^R is referred to as stationary reward. Average reward can be calculated by standard matrix methods, but enumeration does not provide an efficient procedure for computing optimal solutions.

For example, for $|S|=k$ and $|A|=2$ we will have 2^k possible stationary policies. However induction methods are more suitables. For its calculation we will use the equivalent formulation:

$$f^R(i)=r(i,a_i) - g^R + \sum_{j\in S} p(j \mid i,a_i) f^R(j) \qquad (5)$$

where f^R's are the so-called relative values of policy R and they represent for each starting state i, the expected total difference between the reward and the stationary reward, that is

$$f^R(i) \equiv \lim_{n\to\infty} \frac{1}{n} E_i^R \left\{ \sum_{t=1}^{n} \left(r(X_t, Y_t) - g^R \right) \right\}$$

Practical experience shows that the optimal policies under expected total discounted reward and average reward are almost identical.

Average reward per unit of output. If a herd restraint is imposed on the physical output we can use this criterion.

$$g^R(i) = \frac{g_r^R}{g_m^R(i)} = \frac{\sum_{i\in S} \Pi_i^R r(i,a_i)}{\sum_{i\in S} \Pi_i^R m(i,a_i)}$$

where $m^R(i)$ is a function defined like r but represents the output produced in the state i when policy R is applied. This function is also relevant when the criterion of maximization is the average reward over time in a model where the stage length varies. To manage such models we must transform it to an equivalent equal stage length model as Howard (1971) and Puterman (1994) discuss. In that case the physical output represents the stage length. For computing reasons we use the equivalent formulation:

$$f^R(i)=r^R(i) - g^R m^R(i) + \sum_{j\in S} p(j \mid i,a_i) f^R(j) \qquad (6)$$

When an average criterion is used, the set of simultaneous equations (5) or (6) determines the relative value of each state and the average reward per unit of time under a certain policy.

5. Model Implementation

The model has been constructed to be used on a personal computer. It runs with default values according to model formulation if no farm data is available. It allows the user to update these values automatically with estimations made from his own farm records or by hand; further, it is always possible to introduce single modifications. Therefore, the model can be farm specific. There are also a set of filters that can be applied to obtain more elaborate parameters, some for keeping out errors and some for testing different management alternatives.

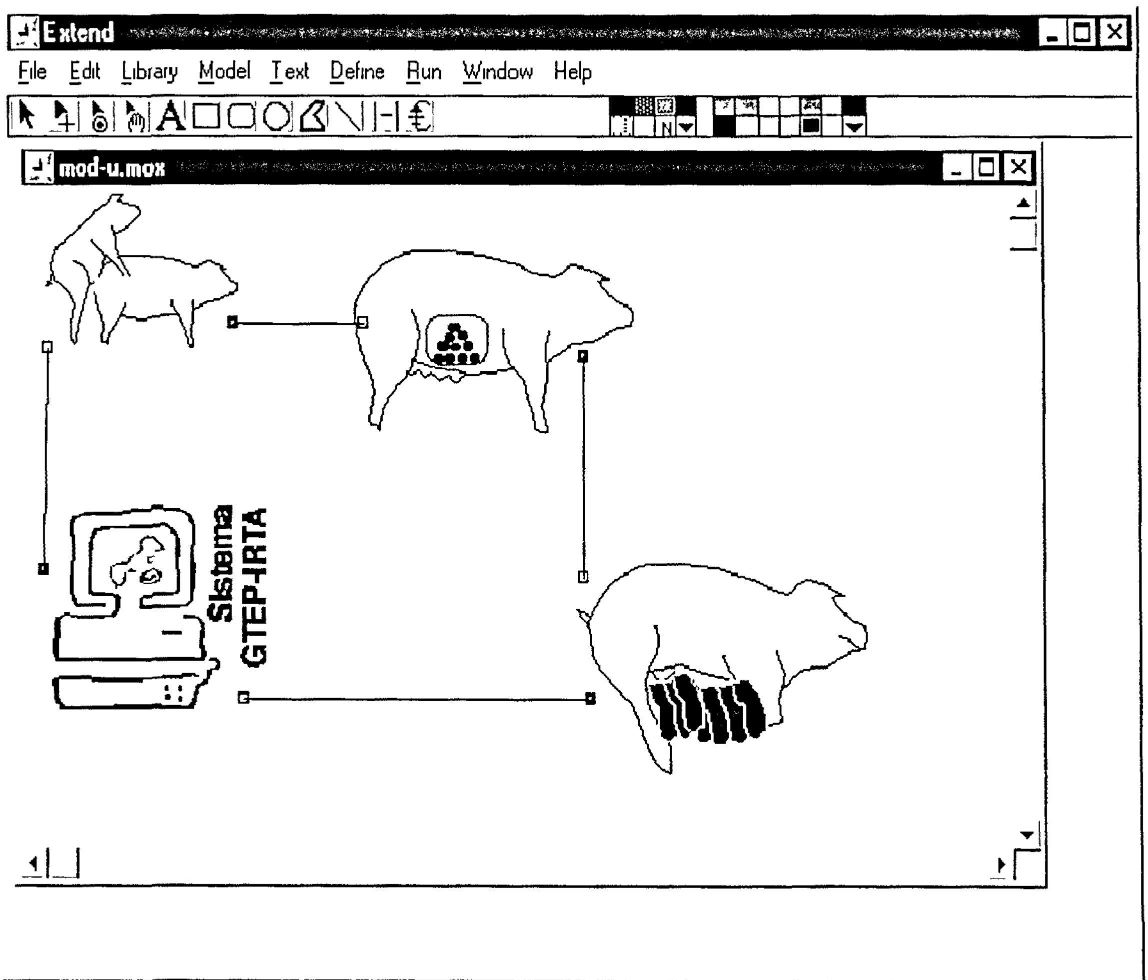

Figure 2. Model interface

The sow model prototype was developed with EXTEND™, an advanced simulation tool for decision support available on personal computers under different operative systems. The advantages of such implementation are very significant and are related to object oriented programming. The process has been split into objects that can be easily identified graphically with the real subprocess represented, see Figure 2. These objects are in fact a partition of the transition function and can be considered as submodels. This implementation helps us to structure the model better, modify it and develop it more extensively.

On the other hand, there are also many customizable graphic interfaces and other utility libraries to show off relationships between components in the model. Consequently, the model is an easy-to-use tool that can be used to simulate sow production dynamics in different ways, the most interesting is for reproducing its behaviour under some management alternatives. This implementation becomes more comprehensive for the user unlike some models published up to now.

6. A model application

In order to discuss the suitability of the prototype, a test was carried out with a basic model. The first step was a static analysis derived from the limit distribution at equilibrium. Comparing model outputs with the real ones. Several farms were chosen randomly among medium size farms from GTEP data bank. After that, we were able to carry out a systematic search for optimal management policies and the evaluation of the impact of alternative management policies at operational level that may be used to optimize the replacement policy in a farm. The main purpose is to create operational replacement guidelines under various conditions concerning reproductive performance and delay in replacement.

A limitation to some model variables were established. Hence, maximun lifespan allowed is 15 parities, maximum litter size is 20 piglets, and as soon as an animal is replaced a new gilt is introduced. Availability is not a constraint.

Also some economic parameters were fixed, see Table 1. Incomes in the model are calculated from the value of the piglets born alive and weaned (5,500 Ptas each), and the slaughter value of culled sows (25,000 Ptas each). The variable costs are the cost of replacement gilts (25,000 Ptas each) and feed costs. Feed costs are calculated depending on daily intake, reproduction state and feed type. The discount factor in the model is based in a real annual interest rate of 6%.

Feed (Kg)	Cost (Pts)	Consumption (day)	Concept	Ptas.
Open sows	21	2.0	Fix costs	98000 pspy[1]
1st repetition	21	2.0	Variable costs	5000 pspc[2]
2nd repetition	21	2.0	Replacement	25000[3]
3rd repetition	21	2.0	Slaughter	25000[3]
Gestation	21	2.5	Piglet	5500[4]
Lactation	22	4.0		
Abortion	21	2.0		
Piglets	40	0.2		

[1] pspy: per sow per year, [2] pspc: per sow per cycle, [3] Ptas. per sow, [4] Ptas. per piglet.

Table 1. Economic inputs used by the model

Cycle	LS[a]	LW[b]	IT1M[c]	G[d]	L[e]	A[f]	Oe[g]	Age[h]
1	10.67	9.69	0.05	114.4	31	64.1	26.8	238.7
2	10.52	9.66	6.56	114.4	31	64.1	26.8	
3	11.60	10.58	6.01	114.4	31	64.1	26.8	
4	11.28	10.07	5.46	114.4	31	64.1	26.8	
5	11.97	10.78	4.88	114.4	31	64.1	26.8	
6	11.68	10.66	4.98	114.4	31	64.1	26.8	
7	11.44	10.68	6.47	114.4	31	64.1	26.8	
8	11.32	10.57	4.92	114.4	31	64.1	26.8	
9	10.49	9.51	6.05	114.4	31	64.1	26.8	
10	10.00	9.50	5.21	114.4	31	64.1	26.8	
11	10.75	10.00	4.75	114.4	31	64.1	26.8	

[a] Litter size, [b] Litter weaned, [c] Intervat to 1st mating, [d] Gestation, [e] Lactation, [f] Abortion, [g] Oestrus, [h] Age of gilt at the beginning.

Table 2. Litters and length of states by cycle from **Farm1** during 1996

For technical results we need to establish the probability of litter size and litter weaned. Average litters of a sow can be used directly, but it is also possible to determine their expected values taking into account the parity number, and production level. The length of states is fixed in the basic model, although it can be taken randomly in a more general formulation, see Table 2.

The marginal probabilities of conception are dependent on the number of unsuccessful breedings and the parity number, see Table 3.

Cycle	CR(1)[a]	CR(2)[b]	CR(3)[c]	CR(4)[d]	AR[e]	CR[f]
1	90.8	100.0	0.0	0.0	0.61	11.24
2	94.0	88.9	100.0	0.0	0.66	9.43
3	91.0	100.0	0.0	0.0	1.39	10.07
4	95.3	100.0	0.0	0.0	1.57	11.54
5	96.7	100.0	0.0	0.0	0.0	13.01
6	92.9	71.4	50.0	100.0	1.01	11.88
7	97.5	100.0	0.0	0.0	0.00	19.05
8	98.1	100.0	0.0	0.0	0.00	26.79
9	97.3	100.0	0.0	0.0	5.41	53.85
10	100.0	0.0	0.0	0.0	14.29	43.75
11	100.0	0.0	0.0	0.0	0.00	50.00

[a,b,c,d] Conception rate for 1st, 2nd, 3rd and 4th mating,

[e] Abortion Rate [f] Casualty Rate

Table 3. Main rates obtained by cycle from **Farm1** during 1996

The stochastic formulation makes it possible, among other things, to account for other involuntary reasons of culling. The casualty rate is calculated for each farm and is divided between mortality/accidentability and involuntary disposal. The marginal probabilities of involuntary disposal are based on the parity number and reproductive state.

6.1. Validation

Model verification was performed by checking for both mathematical and logical consistency. The probability distribution over states at equilibrium is determined and compared with real distribution for validation, but we have also calculated two kinds of indexes: performance indexes and technical indexes. Performance indexes are based on limit distribution of the herd and they are designed specially to study model dynamics. One of the tests performed with the limit distribution was to compare it with real distribution through a chi-square test. Some results for herd distribution in 1996 are given in Table 4.

Farm	# sows	χ_n^2	*n*
1	140	48.88	77
2	184	158.30	105
3	266	82.15	70

Table 4. Chi-square test for limit distribution

Technical indexes are equivalents to those of the GTEP-IRTA system because they are used more and better understood by extension advisers. Sets of input values can be filtered and evaluated by comparing the results of the corresponding herd distribution at equilibrium. The output of the model can be applied in economic analysis, as well as for comparing management alternatives. Some results can be derived directly from the simulated distribution of sows over states; others need more calculations.

From a technical point of view the real technical indexes are very close to indexes calculated by the model at equilibrium, see Table 5. For productivity and mortality the results are not so close. In fact these results are in accordance with the model formulation and its assumptions. This is because there are some farms that although technical indexes are good for them, present more differences among sow distribution by cycle and through different states.

TECHNICAL	INDEXES	Theoretical	Real
	Average of Sows	140	140
	Average of Sires	6	6
	# Productive Sows	116	116
PRODUCTIVITY	Piglets/Present Sow/Year	23.6	26.3
	Piglets/Sow/Year	25.8	25.9
	# Piglets weaned	3690	3695
	# Litters weaned	335	336
	% Abortions	2.08	2.24
# Piglets/Litter	Litter size	11.29	12.29
	Alive	11.29	11.48
	Dead	0	0.81
	Weaned	10.24	10.59
	% Mortality	9.28	7.74
Reproductive Rates	# farrowing/ Sow/Year	2.45	2.45
	# farrowing/ Present Sow/Year	2.30	2.49
	Index of farrowing	89.0	88.4
	% Repetitions	11.2	11.6
	Interval weaning-oestrus (d)	5.53	5
	Interval weaning-fertile mating	6.72	6
	Lactation (d)	28	28
	Interval between farrows (d)	149	149
Age of Sow	1st Farrow (d)	340	341
	Farrowing Sows (month)	30	30
	Replaced	51	51
	Age of replaced Sow (month)	38	37
	# Litters/Sow replaced	7.0	6.1
	% Sows replaced	37.7	36.3

Table 5. Output of technical Indexes

6.2. Optimization

The problem can also be optimized finding the policy $R^{*}=(d)^{\infty}$ that prescribes an action for each system state and maximize the average expected profit. That is, when it is most profitable, in the mean, to replace a sow. All parameters in the model are estimated from GTEP-IRTA data bank, see Noguera et al. (1995) or from literature, but in fact it is designed to be farm specific.

Under infinite planning horizon the policy iteration method may be applied. Unlike the value iteration method, it always provides an optimal policy.

Algorithm:

Choose an stationary policy R.

Policy evaluation. For current rule R, compute the unique solution $\{g^R, f^R\}$ depending on optimality criteria. The problem is to solve a $|S| \times |S|$ system of linear equations.

Total discount revenues. For all $i \in S$ we must solve (4).

Average rewards per time. For all $i \in S$ we must solve (5) considering additional equation: $f(k) = 0$ where k is an arbitrary chosen state.

Average rewards per unit of physical output. For all $i \in S$ we must solve (6) considering additional equation: $f(k) = 0$ where k is an arbitrary chosen state.

Policy improvement. For each state i find the action $d = a \in A$ that maximizes the objective function, $v(i)$, and put $d(i)=a$.

$$\max_{a \in A} = r(i,a) - g^R + \sum_{j \in S} p(j \mid i, a_i) f^R(j)$$

So that we can build a new policy R', if R'=R stop, an optimal policy is found, otherwise go back to 2.

The policy iteration algorithm converges after a finite number of iterations, the proof (see Puterman, 1994 or Howard, 1971) is based in finitness of state and action set and the improvement of succesive policy iterations. It is a robust algorithm that converges very fast in specific problems. The number of iterations is practically independent of the number of states and normally varies between 3 and 15.

Cycle	Mating 1	Mating 2	Mating 3	Mating 4
1	K	R	R	R
2	K	K	R	R
3	K	K	K	K
4	K	K	K	K
5	K	K	K	K
6	K	K	K	R
7	K	K	K	R
8	K	K	K	R
9	R	R	R	R
10	R	R	R	R
11	R	R	R	R

K:Keep action, R:Replace action

Table 6. Optimal mating policy for **Farm1**

The algorithm was tested with Farm1, the parameters for the model were those calculated before. The output is showed in Table 6. We usually implement the evaluation step of the policy iteration by using Gaussian elimination to solve the linear system. In this case it was enough because there was only 180 states as maximum, but this formulation is equivalent to another with 2100 states on a weeekly time base. In case we have a large state space this method it may be computationally prohibitive.

A modified policy iteration algorithm cited by Puterman (1994) is available. This algorithm tries to get the best of the two most used algorithms in solving DP problems: policy iteration and value iteration. The key is to avoid the policy evaluation for each policy improvement step, therefore, a partial policy avaluation is carried out by using value iteration methods, which are less expensive in computational time. This algorithm seems to solve in a similar way problems that Kristensen (1993) formulate as hierarchic Markov process applied in farm management.

7. Conclusion

The model formulated may become very large, but we want to use a small version in order to get an idea of its benefits in solving real cases. Results obtained on the model are promising because the implementation is easy to use and is not always necessary to formulate very complex models for solving common problems. Very often farmers want to have simple rules to apply on their farms, and researchers are not able to provide them with comprehensible answers. Such models provide more insight into the technical and economic consequences of changes in performances, prices and management policies.

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Chapter 4. A MARKOV DECISION SOW MODEL REPRESENTING THE PRODUCTIVE LIFESPAN OF HERD SOWS

Plà, L.M. [1,2]; Pomar, J. [2,3] and Pomar, C. [4]

[1] *University of Lleida, Department of Mathematics*
Pl. Victor Siurana 1 E-25003 LLEIDA
[2] *R+D Centre. UdL-IRTA, Area of Animal Production*
[3] *University of Lleida, Department of Agro-forestry Engineering*
Avda. Rovira Roure 177. E-25198 LLEIDA
[4] *Dairy and Swine Research and Development Centre*
Agriculture and Agri-Food Canada, P.O. Box 90 - 2000 Route 108 east
Lennoxville, Quebec, Canada, J1M 1Z3

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Abstract

A Markov decision sow model has been developed to represent the productive and reproductive lifespan of herd sows. This model precisely describes the herd structure at equilibrium based on actual farm data. Model outputs are the herd structure at equilibrium, and technical and economic indexes. Validation has been performed by comparing observed and simulated outputs from specific farm data. A complementary validation using a statistical $\chi 2$ test based on Pearson's statistic is proposed to compare herd distributions at equilibrium. The model is intended to be used by farmers and runs on micro computers.

Keywords: Sow herds, semi-Markov Decision Model, Modelling, Validation.

1. Introduction

Today, decision-making in livestock production systems and in particular on pig farms is more difficult than in the past. Reasons for this include the intensification of production, the increase of competitiveness and the reduction of marginal profits. Rapid changes in market prices, production methods, biotechnology and communication systems contribute to increase the number of production strategies and the level of uncertainty. To maintain profitability, the farmer needs to estimate animal and economic responses to changing production conditions. In this way, decision-oriented models are increasingly needed and they are becoming essential in the process of decision-making.

Several mathematical models representing dairy cow and sow herd dynamics have been developed in the past focussing on reproduction and replacement decisions as reviewed by Jalving (1992a, 1993) and by Kristensen (1993). Due to conceptual similarity in reproduction and replacement problems between dairy and swine farms, dairy models have often been modified to represent the sow herd dynamics (Dijkhuizen et al., 1986a, Dijkhuizen et al., 1986b, Huirne et al 1988, Huirne, 1990, Jalving et al., 1992a, Jalving, 1993, Kristensen, 1993, Kristensen et al. 1996). However, sow models are scarce in the literature probably because of the difficulty of representing the productive cycle of the sow which is faster and much subject to variation than the dairy cow cycle. In all cases, different mathematical methodologies have been used to develop these population models. Markov decision models are the most suitable to simulate herd dynamics since they can take into account within herd variation and represent the reproductive cycle of individual sows. Furthermore, they can be formulated either as probabilistic models to study different management strategies (e.g. Jalving et al. 1992a) or extended to Markov decision programming models (e.g. Huirne et al. 1988, 1993) to optimize the reproduction cycle.

The first published dynamic programming optimisation model for helping the culling of individual sows was developed by Huirne et al. (1988). More recently, Jalving et al. (1992a) developed a swine herd model that evaluated the technical and economic consequences of tactical decisions concerning reproduction and replacement of sows. This work completed the previous one in order to estimate the consequences of management strategies, useful to find sub-optimum strategies. The model of Jalving et al. (1992a) was based on Markov chains designed to run on a weekly basis. However, this approach is not the most suitable when we are only interested in system behaviour in steady-state conditions. In general, all these models are validated subjectively by comparing simulated results to observed data. In few cases, models had been validated by comparing statistically simulated and observed data (Sorensen, 1990).

The objective of this project was to develop a semi-Markov decision sow model representing the productive lifespan of herd sows. A new statistical validation methodology is proposed to evaluate the quality of the fit between simulated and observed data. The proposed model is aimed to help decision-makers in evaluating the animal and economic consequences of reproduction and replacement strategies taken by the farmer. For simplicity, only the stationary approach (steady state) is presented in this first paper.

2. Model design and implementation

Management of sow herds can be understood as a multistage process where farmers choose management rules to improve net returns over time. In the model, sows are production units that evolve through different productive states following the same laws of motion. Sows are then characterized by their common productive behaviour and by the fact that they react equally to the same management strategies. Following these assumptions, the productive behaviour of the farm as a whole is the result of the aggregation of sows at the farm level. Thus, sows evolve through discrete intervals or stages (interval between farrowings, interval from weaning to first mating, interval between matings, interval from farrowing to weaning, interval from abortion to mating, interval from mating to departure from the farm, etc) where they can be in one of a finite number of states (gestation, lactation, etc.). It is at the end of any stage that the farmer can act to influence sow productivity in both current and future stages. The distribution of sows over states gives the herd structure. Total herd profit is then calculated by accumulating for each animal and for each state their economic performances. In this way, the impact of different management strategies can be analyzed and compared.

Productive states are basically defined from the reproductive cycle and lifespan of sows. Thus, sows can be empty, gestating or lactating and each reproduction cycle is represented. Management strategies are those related to reproduction and replacement decisions and they are maximal number of reproductive cycles and the maximal number of matings per cycle. Genetic and nutrition effects are not considered in this model. The model herein developed represents the productivity of a sow herd based on its population structure at equilibrium and some management strategies.

The model has been implemented in three main modules (Figure 1). The first module calculates farm specific input parameters from farm data. These parameters are the probabilities that a sow goes from one productive state to another, the average sojourn time of sows in each state, mortality rates, culling rates and average litter sizes at birth and at weaning. Other input parameters non extracted from farm data are production costs, incomes and farm management strategies. Observed herd structure and several technical and economic indexes are also calculated from farm data for further verification. Farm specific parameters are used by the second module that contains the model itself. The model calculates the herd population structure at equilibrium in terms of number of sows per state and rate of sows moving from one state to another. The last module calculates several technical and economic indexes from the simulated population, compares these indexes to those calculated by the first module from observed data and finally, it compares statistically the observed and calculated population structures.

The model was developed in EXTEND^{TM} (1997), an advanced tool for the development of models simulating decision-making processes. The model is available on PC under Windows or MacOS.

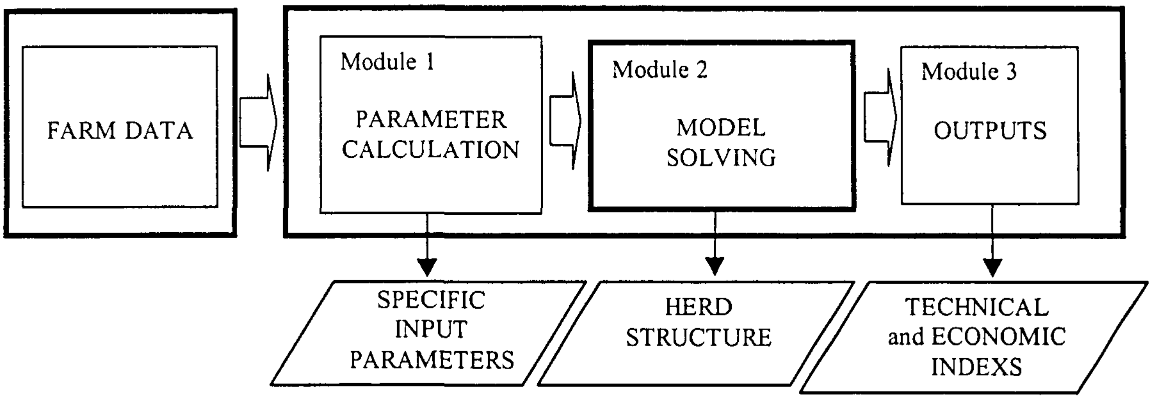


Figure 1. General design of the model

2.1. Model formulation

The model herein implemented has been developed based on the general Markov decision model presented by Plà et al. (1998). In this model, sows move from one state to another through transitions. Time interval between successive transitions is called a stage. Stages depend on states and they are variable. Actions can be taken by farmers at the end of any stage in order to control the farm. Therefore, the represented multistage process is semi-Markovian. However, it is modelled as a standard Markovian process because only the herd in steady state situation is analyzed (Howard, 1971; Puterman, 1994). The overall productive process has been represented into the model as a discrete and stationary Markov process homogenous in time.

Markovian decision models are characterized by the following elements: states S, actions A, transition probabilities and the reward function. The transitions of sows from one state $i \in S$ to another $j \in S$ are subjected to actions $a \in A$ that are chosen by farmers to modify the system behaviour. States and actions are considered finites. In an infinite planning horizon $\Omega = \{S \times A\}^{\infty}$ represents the set of all possible system paths (ω, i.e. all possible sequences of states and actions, $\omega = (i_1, a_1, i_2, a_2, \ldots, i_n, a_n, \ldots) \in \Omega$). The sequence of actions is the result of a policy or strategy, D.

For each policy D, $P^D = (p^D_{ij})$ is the transition matrix. In a Markovian based model, future states S are defined as being only conditioned by the present state and not by the manner to which the present state is reached, that is,

$$P^D\{X_{n+1} = i_{n+1} | X_1 = i_1, Y_1 = a_1, \ldots, X_n = i_n, Y_n = a_n\} = P^D\{X_{n+1} = i_{n+1} | X_n = i_n, Y_n = a_n\}, \quad (1)$$

where $X_n(\omega) = i_n$ and $Y_n(\omega) = a_n$ are random variables that take values in S and A, respectively.

For any system path $\omega \in \Omega$, each action has some immediate economic effect and influences productivity in both current and future stages. Thus, decision-makers will establish a management strategy D according to their production objectives. The reward function, r, represents the farmer preferences in a decision theory context and it can be used in the building of a performance criterion as follows:

$$B_n^D(i) = r_i^D + \sum_{j \in S} p_{ij}^D B_{n-1}^D(j) \quad (2)$$

where the reward function r_i^D is the expected net return from a sow in the i-state and taking a-action determined by policy D, and $B_n^D(i)$ is the total return expected after n transitions from the initial state i. Expression (2) can be reformulated depending on different considerations and assumptions (e.g. total discounted expected returns or average expected returns per unit of time). Once a performance criterion is established, it is possible to analyze and compare different policies.

The sow reproductive cycle is represented in Figure 2 which also illustrates sow's states and transitions. The productive life of a sow begins when entring the farm as a gilt. All gilts are purchased from outside the farm and they remain in a waiting state until first mating. Home-grown sows are not represented into the model since they are rare in commercial Spanish conditions. The productive life ends when sows are sold to the slaughterhouse or when they die.

In the model, sows can be culled depending on their reproductive condition and cycle or when the maximum number of matings or maximum number of cycles is achieved. Sold or dead sows can be replaced immediately or after some farm-specific delay. In this later case, sows stay in the fictive state "Purchasing new gilts" (Figure 2). However, average replacement time is rarely known or may change over time. In the same way, sows may wait in the "Interval No mating to Sold" or "Interval Mating to Sold" states (Figure 2). Model default values assumes that replacement and replaced sows have the same genetic potential and therefore, will have similar reproductive performances. The model simulates possible improvements on genetic merit in prolificacy throughout a rate of genetic improvement (Kristensen, 1988, Kristensen, 1993). Furthermore, the nutritional effects on reproduction are taken into account through reproductive parameters.

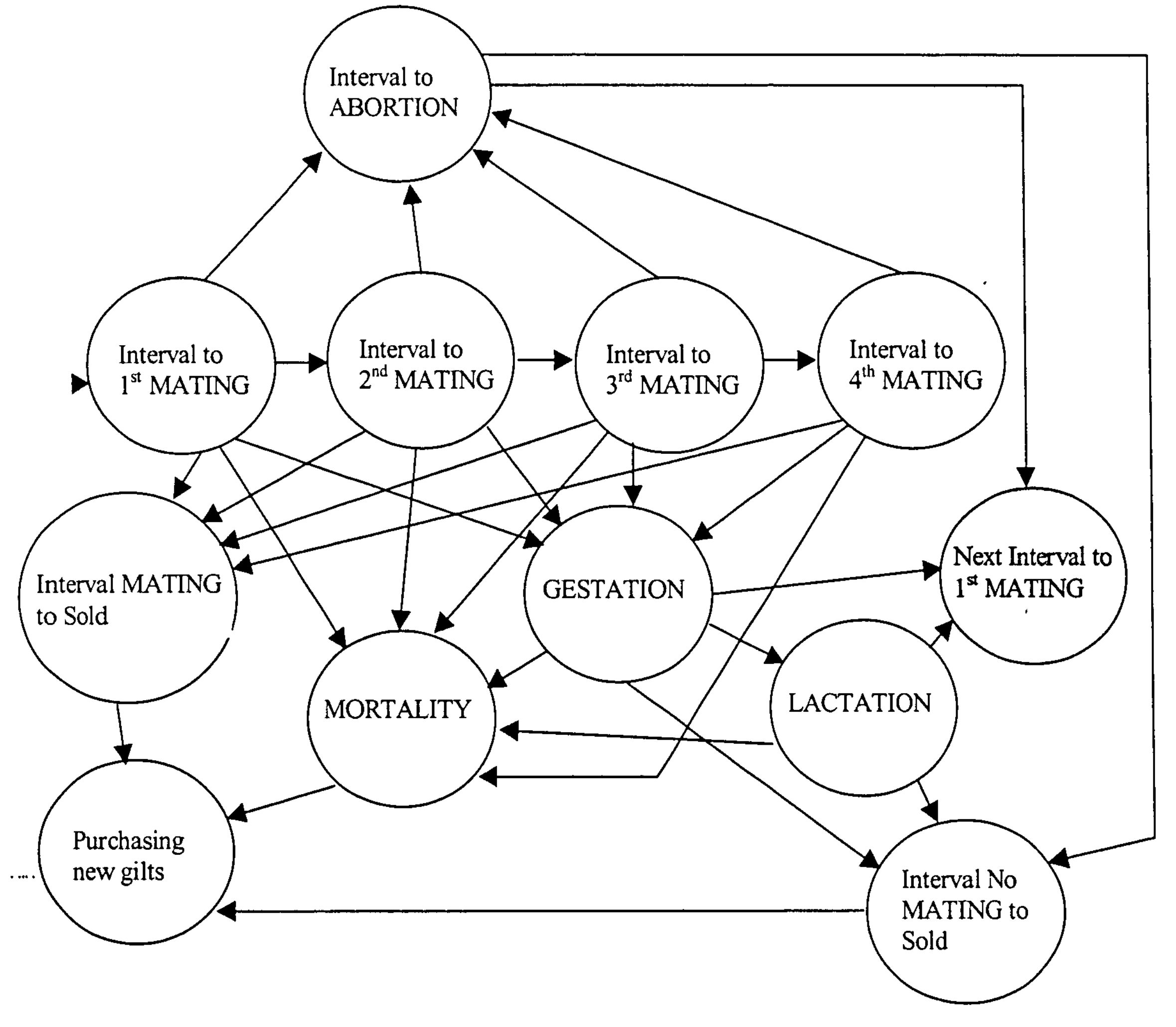


Figure 2. States and transitions of a sow cycle.

Farm data records rarely include heat or pregnancy detection information and therefore, the corresponding state of a sow is not known until a new mating or farrowing is recorded. Then, it is assumed that every sow that starts a stage in one state finishes the stage in the same state.

2.2. Estimation of population parameters

Under the assumptions made in this model, the dynamics of a herd under any given policy can be represented as a finite irreducible and aperiodic Markov chain. Transition probabilities are estimated from farm data during a fixed period of time. We assume that farm data represent farmer's management policy and therefore, no more transitions than observed are used in the estimation of probabilities. Maximum likelihood probabilities are calculated for each transition (Billingsley, 1961) as follows:

$$\hat{p}(j/i,a)=\frac{n_{ij}}{n_i} \qquad (3)$$

where n_{ij} is the number of reproductive sows passing from i to j state, when action a is taken and $n_i = \sum_{k\in S} n_{ik}$ is the total number of reproductive sows that have passed throughout state i during the considered period. In this study, represented actions are those related to sow's replacement (i.e. keep and replace). The replace-action means that one sow is moved from states "Interval mating to sold", "Interval no mating to sold" or "Mortality" to "Purchasing new gilt" with probabilities equal to one.

Average time between transitions τ_i^D, that is, the expected time in days that sows spend in any state is also calculated. These values are used to estimate average sow's age over states, the reward function, feed consumption and production costs. Every state represented in the model (Figure 2) has an average time interval associated to it. Main intervals are: gestation, lactation, between mating intervals, between last mating and replacement interval, etc.

2.3. Economic input parameters

Economic parameters are used in the model to define economic scenarios in order to calculate economic outputs and to compare economic results from different management strategies. This economic analysis is similar to the one used by other authors (Dijkhuizen et al, 1986b, Huirne et al., 1991, Jalving et al., 1992b, Kristensen, 1993).

Present net returns per state are calculated considering an annual discount rate of 6%. Total incomes result from both weaned piglets (33.06 euros/piglet) and slaughter sows (150.25 euros/sow). Piglet and sow average prices are reported from the main local auction market on a per animal basis. Different costs are considered in the model. Fixed costs are estimated at 588.99 euros per year for each sow in the farm. Variable costs are of 10.00 euros per cycle. Other variable costs include gilts purchasing (150.25 euros/gilt), artificial insemination (20.05 euros/mating) and feeding cost. Feeding cost is calculated based on an average daily feed intake and feed type cost depending on reproductive condition (Table 1). Default values assumes no delay between purchasing and replacement of sows.

Table 1. Average feed intake and feed costs.

State	Feed Cost (euros/Kg)	Feed intake (Kg/Day)
Open sows	0.13	2.0
Until 1st repetition	0.13	2.0
Until 2nd repetition	0.13	2.0
Until 3rd repetition	0.13	2.0
Gestation	0.13	2.5
Lactation	0.14	4.0
Until Abortion	0.13	2.0
Piglets	0.24	0.2

Parameters presented in Table 1 are used to define the reward value for each state, r_i^D, which represents the expected net return. Net return per state is defined from incomes and costs:

$$r_i^D = \beta_i \cdot (PCs_i + Lw_i \cdot PCw - VC_i - C_i \cdot PCf_i \cdot \tau_i - PCg) \qquad (4)$$

where

$\beta_i \cdot = e^{-q\,\tau_i}$ is the discount factor for state i, taking q as an annual interest rate;

PCs_i is the value per sow sold to the slaughterhouse when i is one of the sold states. Otherwise is 0;

$Lw_i \cdot$ is the number of piglets weaned per litter when state i is a lactation state. Otherwise is 0;

PCw is the value of weaned piglets;

VC_i are variable costs per sow depending on state i. They include veterinary, artificial insemination and other expenses;

$C_i \cdot$ is the daily feed intake per sow in state i (Kg);

PCf_i is the feed price (euros/Kg);

τ_i· is the time interval of state i;

PCg is the gilt value in the "purchasing gilt" state. Otherwise is 0.

2.4. Model Outputs

Model outputs are calculated from both farm data and simulated herd structure at equilibrium. These outputs are referred as observed or real and simulated or theoretical outputs, respectively. Model outputs are those related to the population distribution at steady-state conditions plus some technical and economical indexes (Figure 3).

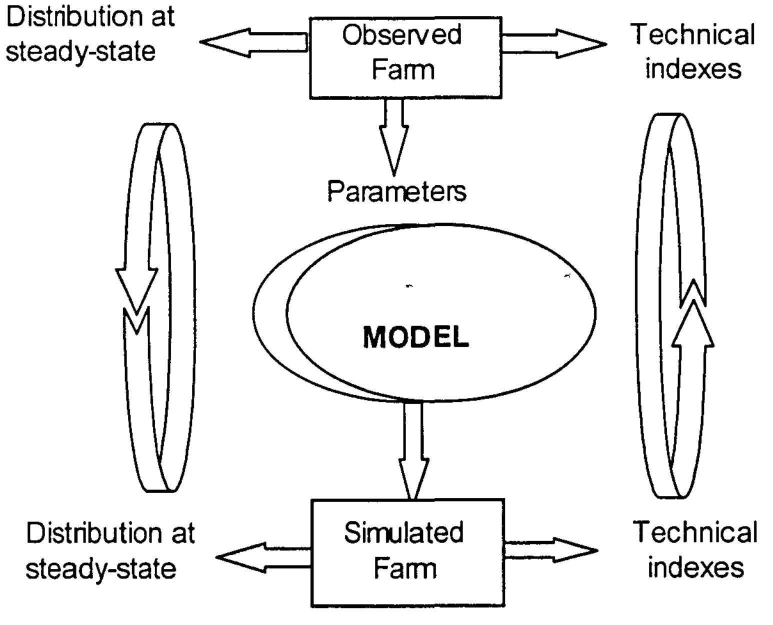


Figure 3. Model validation process

The observed structure of population, $\Pi_o=(\pi_{11}{}^D, \pi_{12}{}^D,..., \pi_{1|S|}{}^D)$, is estimated from farm data recorded during the period of study. Each component of the population structure $\pi_{oi}{}^D$ is estimated by accumulating present number of sows in each state and by normalizing the resultating vector. Observed technical indexes are then calculated following the specifications of the GTEP-IRTA® Spanish pig management information system (Noguera et al., 1992).

The expected herd distribution at equilibrium, $\Pi_s=(\pi_1{}^D, \pi_2{}^D,..., \pi_{|S|}{}^D)$, is calculated solving the following linear system of equations:

$$\pi_j^D = \sum_{k \in S} \pi_k^D \hat{p}_{kj}^D \qquad j \in S$$
$$\sum_{j \in S} \pi_j^D = 1 \qquad (5)$$

where $\hat{p}_{kj}^D$ represents the transition probability estimate for a sow to pass from k to j state as calculated in (3), D is the policy derived from farm data and $\{\pi_j^D, j \in S\}$ represents the distribution at equilibrium when policy D is followed. From equation (5) and the reward function we can obtain the expected average net return per sow on farm:

$$B^D = \sum_{j \in S} \pi_j^D r_j^D \qquad (6)$$

or the expected average net return per sow per day:

$$g^D = \sum_{j \in S} \frac{\pi_j^D r_j^D}{\pi_j^D \tau_j^D} \qquad (7)$$

We should note that (6) and (7) are just two of the alternative formulations that can be derived from (2). Economic indexes like variable costs, feed cost, replacement cost, incomes from piglets and sows sold to slaughterhouses and gross margin are calculated as follows:

$$\mathrm{FC} = \mathrm{k}\sum_{j \in S} \beta_j \pi_j^D \qquad \mathrm{VC} = \sum_{j \in S} \beta_j \pi_j^D \mathrm{VC}_j$$
$$\mathrm{FeC} = \sum_{j \in S} \beta_j \pi_j^D \mathrm{C}_j \mathrm{PCf}_j \tau_j \qquad \mathrm{RC} = \sum_{j \in S} \beta_j \pi_j^D \mathrm{PCg} \qquad (8)$$
$$\mathrm{WI} = \sum_{j \in S} \beta_j \pi_j^D \mathrm{Lw}_j \mathrm{PCw} \qquad \mathrm{SI} = \sum_{j \in S} \beta_j \pi_j^D \mathrm{PCs}_j$$
$$\mathrm{GM}=\mathrm{SI}+\mathrm{WI}-\mathrm{RC}-\mathrm{FeC}-\mathrm{VC} \qquad \mathrm{NP}=\mathrm{GM}-\mathrm{FC}$$

where FC are fixed costs, VC are variable costs, FeC are feed costs, RC are replacement costs, WI are incomes from weaned piglets, SI are incomes from slaughter sows, GM is gross margin and NP is net profit. Furthermore, expected technical indexes are also calculated following the specifications of the same pig management information system referred to before (IRTA, 1996) but based on steady-state conditions. The expected average net profit as well as all other indexes are given in yearly basis. Finally, observed and calculated population structures are compared statistically using Pearson's statistic (Billingsley, 1961).

3. *Validation of the model.*

3.1. The evaluation process

Model verification was first performed by checking for both mathematical and logical consistencies. Thus, virtual examples were used to ensure the appropriateness of model calculations. Particular emphasis was given to the distribution of the population at equilibrium.

To evaluate the behaviour of the model in real conditions and the suitability of the steady-state approach, a random sample of ten farms was chosen from GTEP-IRTA®. Of these farms, three were discarded because of the inconsistency of their data (high number of missing data, etc.). Specific input parameters were obtained as indicated in the previous section for the seven remaining farms (referred here as farms 1 to 7). All data used in this study were collected during year 1997. Also, observed herd structure and several technical and economical indexes were calculated. Finally, each farm was simulated and the results compared to those observed in the data base during the same period. Observed data and simulated results were compared graphically and numerically. Also, a non-parametric test (Pearson's statistic; χ^2 –test) was used to check the goodness of fit. The desegregated values of χ^2-components were also provided and used to detect model components requiring further development. The following sections describe in detail the validation process outlined in Figure 3.

3.2. Estimation of farm input parameters

All input parameters are farm specific and are calculated for each farm by the first module of the model (see Figure 1). For simplicity, only input parameters obtained for farm 4 are shown in this paper (see Tables 2, 3 and 4). Farm 4 was chosen because it was considered representative of the average farm in GTEP-IRTA® data-bank. Table 2 gives averages and deviations of time intervals for each state. These parameters are assumed to be independent of the reproduction cycle. Also, some of these parameters are the result of farmer's management strategy and are therefore specific to each farm. For some of these parameters differences between farms may be important, while within-farm variation is generally small. That is the case for the interval from last farrowing or weaning to market (Table 2) which shows important variation within farm and also between periods and farms (data not shown). We can also note that, in general, the interval from last mating to market is greater than the interval from last farrowing or weaning to market. Sometimes, unusual values can be observed for the interval between matings. These values may result from reproduction or management problems and they should be understood as part of the real management strategy. Two of the less variable parameters were gestation length, biologically determined, and age of gilts. Other parameters such as gestation with abortion length or the interval between matings have standard deviations that depend on the number of observations.

	Mean (days)	SD (days)
Gestation length	114.8	1.3
Lactation length	29.6	2.1
Gestation with abortion length	90.0	0.0
Interval from last mating to market	39.9	0.5
Interval from last farrowing or weaning to market	16.9	1.9
Interval between matings	23.7	1.9
Age of purchased gilts	243.2	1.0

Table 2. Mean and standard deviation of time intervals for farm 4

Table 3 shows litter size, litter weaned and the interval from weaning to first mating per cycle in farm 4. Information from this and prior tables are used to estimate reward function values. In general, values from Tables 2 and 3 can be used directly as mean values (deterministic simulation) or they can be determined randomly taking into account the standard deviation (stochastic simulation) in order to reflect the variability-risk features of the problem (White, 1988).

Cycle	LS[a]		LW[b]		IT1M[c]	
	Mean	SD	Mean	SD	Mean	SD
1	9.37	1.81	8.47	1.65	1.00	0.00
2	10.69	1.49	9.14	1.48	9.21	2.48
3	11.42	1.79	8.97	1.47	6.97	1.31
4	11.05	1.65	8.95	1.54	7.14	2.10
5	11.43	1.86	8.78	1.59	7.00	2.09
6	10.45	1.78	9.05	1.24	6.24	1.24
7	10.87	1.80	8.47	1.54	6.40	1.36
8	10.73	1.86	9.27	0.87	7.73	2.72
9	9.82	1.50	4.64	1.88	9.18	2.82
10	7.40	1.98	6.20	2.27	7.60	1.40
11	11.33	1.30	7.00	2.26	7.33	1.12

[a] LS: litter size at farrowing

[b] LW: litter size at weaning

[c] IT1M: Interval from weaning to 1st mating (days).

Table 3. Average and standard deviations of litter size, litter weaned and interval from weaning to first mating by cycle for farm 4

Marginal probabilities calculated from farm 4 data are shown in Table 4. Marginal probabilities for conception rates are affected by the number of unsuccessful matings and by the reproduction cycle. Conception rates are expressed in relation to all matings per cycle. Abortion marginal probabilities are also specific for each reproduction cycle. Culling marginal probabilities are specific for each reproduction cycle and state. Because farmers rarely indicate the reason of sow replacement, culling rates in Table 4 also include casualties. Only conception rates for first and second matings are given in Table 4 because it was unusual to find farms with more than two matings per cycle. A 100 % conception rate is given to parities 3 and up because farm data is inconsistent. Abortion rates are also low because they are rarely reported by farmers. Culling rates of 100% in the last cycle and state are given in order to end the sow lifespan.

	Marginal probabilities[a]					
Cycle	CR(1)	CR(2)	AR	CRO	CRG	CRL
1	68.42	100.0	0.0	15.56	2.22	4.44
2	82.76	100.0	0.0	6.45	0.00	0.00
3	100.0	0.0	0.0	3.03	6.06	0.00
4	100.0	0.0	0.0	8.33	4.17	0.00
5	100.0	0.0	0.0	0.00	4.17	0.00
6	100.0	0.0	4.76	4.55	4.55	4.55
7	100.0	0.0	0.0	16.67	0.00	0.00
8	100.0	0.0	0.0	8.33	0.00	0.00
9	100.0	0.0	0.0	0.00	26.67	20.00
10	100.0	0.0	0.0	0.00	33.33	33.33
11	100.0	0.0	0.0	20.0	20.0	100.0

[a] CR: conception rates at 1st and 2nd mating; AR: abortion rates; CRO: culling rates for open sows; CRG: culling rates for gestating sows; CRL: culling rates for lactation

Table 4. Marginal probabilities per cycle in farm 4

Some problems were sometimes found when calculating farm specific parameters by the first module. These problems were due to the lack of relevant data describing these parameters or to a low number of observations. To reduce the impact of such problems, in particular when estimating parameters in small farms, other estimation methods like empirical Bayes estimates instead of maximum likelihood estimates of transition probabilities could also be considered (Billiard et al., 1995).

3.3. Technical results and discussion

Results from each farm were obtained and evaluated. Farms were then classified according to differences between farm data and model predictions. For simplicity, only outputs from farms 1 and 4 are presented in this section. Farm 4 showed the best fit between observed and predicted population distribution parameters while farm 1 was the worst.

The main technical indexes (Table 5) obtained from simulated and observed farm data were in reasonable agreement in all farms. These indexes were selected from GTEP-IRTA® system and calculated in the same way. Observed and simulated productivity indexes are generally in agreement between simulated and observed data with the exception of Abortion. Abortions are scarce and therefore distributions are unstable.

	Farm 1		Farm 4	
INDEXES	Observed	Simulated	Observed	Simulated
Sows in farm (#)	184	184	96	96
Piglets/Sow/Year (#)	23.0	23.2	20.6	20.3
Litters weaned/Year (#)	418	420	197	194
Abortions (%)	1.50	1.14	0.54	1.92
Born alive (# piglets/litter)	10.42	10.70	10.42	10.55
Weaned (#piglets/litter)	9.33	10.47	8.58	8.67
Mortality (%)	10.40	11.46	17.66	17.88
Farrowing/ Sow/Year (#)	2.47	2.45	2.40	2.34
Index of farrowing (%)[a]	85.0	90.0	79.7	79.1
Repetitions (%)	15.1	10.0	20.3	20.9
Interval weaning-oestrus (d)	6	5	7	7
Interval weaning-Fertile mating (d)	6	7	8	12
Lactation length (d)	28	28	30	30
Interval between farrows (d)	148	149	152	156
Age at 1st Farrowing (months)	12	12	12	12
Age at Farrowing (months)	34	29	30	26
Age at replacement (months)	42	29	35	36
Litters/Sow replaced (#)	6.6	4.7	5.1	5.6
Sows replaced (%)	38.53	27.7	45.63	43.9

[a] Succesful number of farrowings over number of matings.

Table 5. Simulated and observed technical indexes for farms 1 and 4

The number of piglets per litter is also slightly different between simulated and observed indexes. This difference is explained by differences in herd structure because the number of piglets per litter per cycle is the same (see Table 3). Reproductive rates are rather similar, although the farrowing index (i.e. successful number of farrowings over number of matings) and Repetitions index are different in farm 1. Observed and predicted Lactation lengths are almost equal because this variable is not affected by herd structure. Other sow intervals are affected by herd structure and thus can show some discrepancies between simulated and observed values. Normally, the difference between simulated and observed Weaning to Fertile mating Interval should be the same as the difference in farrowing Intervals because gestation and lactation lengths are close. Age of sows at first farrowing is the same in all farms because farmers use similar management strategies. Age of sows at farrowing is an index that provides a pondered age of sows in the farm. In both farms, this simulated age is higher than observed. In fact, the model applies more uniformly casualty rates associated to each farm. The largest absolute differences in farm 1 between simulated and observed indexes are those related to sow's replacement (Rate of sows replaced and Age of replaced Sows). Consequently, differences in Litters per replaced sow are observed as well. Analysis of these indexes shows that farm 1 has more differences between observed data and simulated results than farm 4, and that farm 1 presents some important differences in indexes affected by the dynamics of the herd. Examples of these indexes are those reporting mating repetitions, abortions and replaced sows.

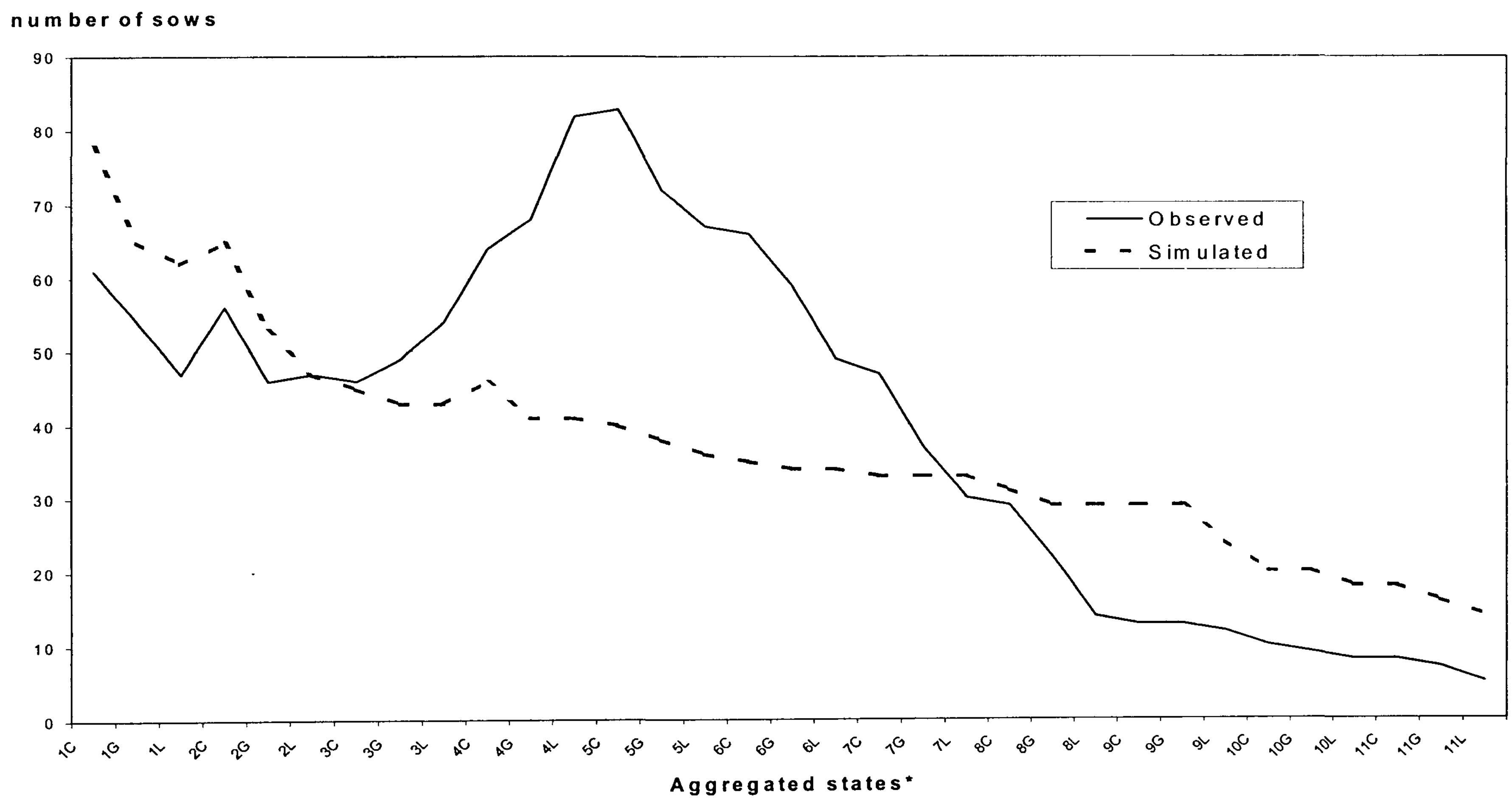
The number of sows per cycle and state for farms 1 and 4 are presented in Figures 4 and 5. The open state includes sows waiting for insemination while the gestating state also includes sows which abort. Graphically, simulated results from farm 4 seem to be closer to observed data than those from farm 1. In this former farm 1, the number of sows in cycles 4 to 7 diverges strongly between observed and simulated data. This disagreement seems to be the result of a rapid increase in the number of gilts introduced in the herd a few years before data collection. In fact, the model represents the population distribution in steady-state conditions while farm data is an instant picture of the herd population which may not be at equilibrium. However, farm 4 seems to be in steady-state conditions as shown by the agreement between observed and simulated data (Figure 5).

The χ^2-test was used to provide an objective measure of the goodness-of-fit between observed and simulated data. This test allows to compare the distribution ($H_0:\Pi_o=\Pi_s$) of the observed and simulated populations. The χ^2-test from all farms are presented in Table 6. Large values of χ^2 indicate that the null hypothesis is rather unlikely. The α-value gives a quantitative measure of the model goodness-of-fit. Low α-values indicate that discrepancies between observed and simulated data population are likely. Thus for farms 1, 2 and 6 the population distributions are different. Discrepancies between observed and simulated data may originate from different causes. First, the model may not represent precisely the farm reality. Incorrect data from farms may also be responsible for the disagreement between simulated and collected data. However, it is important to note that a good model with appropriate farm data may also give different population distributions. The degrees of freedom for the α–value calculation are related to productive states of the model and these states are specific to each farm.

Table 6. χ^2-test for limit distribution

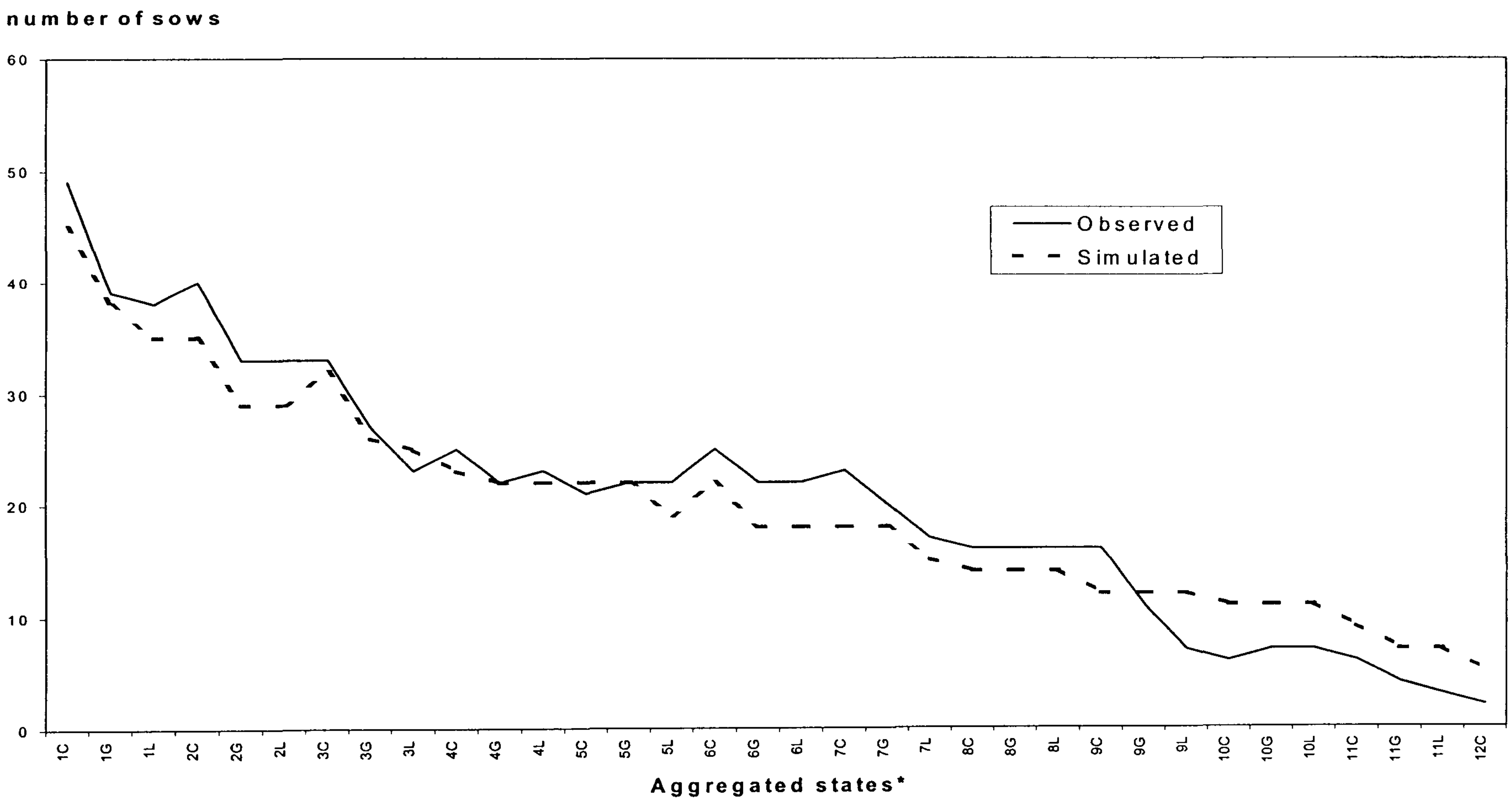
Farm	# Sows	χ^2_{n-1}	n	α
1	182.4	386.22	67	0.000
2	162.0	173.64	60	0.000
3	49.9	43.23	50	0.740
4	92.9	42.67	55	0.867
5	138.2	35.80	57	0.861
6	220.1	373.71	38	0.000
7	251.0	49.11	47	0.731

The study of the χ^2 components can be used to overview the influence of each state and cycle on the total χ^2 value (Table 7). For example, the open sow state at the 10^{th} cycle of farm 4 (see Table 7) represents 25% of the total χ^2 value (42.67 cf Table 6). At this same state but at the 2^{nd} cycle, χ^2 component-values are also important. It is possible to appreciate these discrepancies in Figure 5. Nonetheless, the analysis of the χ^2 components is much more powerful to appreciate differences that may exist between simulated and observed herd distribution data than the simple visual analysis.



*Aggregated states are C: representing open sows waiting to be mated, G: gestating sows waiting the farrowing or abortion and L: lactating sows.

Figure 4. Observed and simulated herd structure for farm 1 during year 1997.



*Aggregated states are C: representing open sows waiting to be mated, G: gestating sows waiting the farrowing or abortion and L: lactating sows.

Figure 5. Observed and simulated herd structure for farm 4 during year 1997.

Cycle	1st Mat	2nd Mat	Gestation	Lactation	Abortion	Mat-Sold	FWA-Sold
1	0.146	0	1.215	1.322	0	0.202	0.305
2	4.558	0.272	0.397	0.239	0.101	0.229	0.131
3	0.590	0.198	1.263	0.291	0	0	0.046
4	0.494	0	0.039	0.547	0	3.557	0.041
5	0.381	0	0.012	1.192	0	0.520	0
6	1.312	0.201	0.019	1.526	0	0.119	0.046
7	0.656	0.380	0.322	1.358	0	0.128	0.129
8	0.896	0	1.210	0.924	0	0.313	0.156
9	0.117	0	0.904	0.263	0	0.102	0.127
10	10.583	0	1.059	0.401	0	0	0.127
11	0.044	0	0.835	0	0	0	0.127

Mat-Sold: stage corresponding from last mating to sold

FWA-Sold:stage corresponding from farrowing, weaning or abortion to sold

Table 7. Components of χ^2 value corresponding to each state in farm 4.

3.4. Economic results and discussion

Economic results, including average expected profit, depend on both population structure derived from the model or from the farm and the economic scenario to be applied. Users can easily modify population and economic parameters defining the herd and the economic scenario to be simulated. An example of economic results produced by the model is shown in Table 8. Within farms, differences between simulated and observed farm results are generally in reasonable agreement with the exception of replacement costs and incomes from slaughtered sows. These differences between replacement costs and incomes from slaughtered sows are still important when compared between farms.

Farms with greater replacement costs have a greater slaughter value, and vice versa. The main income comes from weaned piglets and it is related to sow prolificacy, piglet mortality rate and reproduction efficiency. Variable and feed costs are quite close between farms. In farms 5 and 1, however, they are slightly higher, in farm 5 probably due to a shorter reproductive cycle and in farm 1 to a lower culling rate. Finally, farm 5 is the most and farms 4 and 6 are the less profitable farms with more than 90 euros per sow per year differences. These differences are explained by animal productivity (i.e. piglets weaned per sow per year) which is the main determinant of farm economical efficiency as shown in farms 5 and 4.

Replacement cost and slaughter income differences between observed and simulated results are determinant of the final net profit. Economic results show how the model in general can represent economical indexes which represent farm production based on weaned piglets, but it fails many often in indexes representing costs and incomes related with replacement.

Apart from comparing economic indexes within and among farms, it is also possible to explore other production alternatives from the animal or economical perspectives by performing sensitivity or post-equilibrium analyses. These analyses are essential for the farmer to gain insight related to the farm production process and to identify strategies that can improve the production efficiency and the expected net profit.

		Farm 1	Farm 2	Farm 3	Farm 4	Farm 5	Farm 6	Farm 7
#AS	O	182.4	162.0	49.9	92.9	138.2	220.1	251.0
VC	S	43.80	41.15	43.12	42.33	44.10	41.76	42.04
	O	43.59	40.68	42.25	41.81	43.19	42.57	42.74
FeC	S	116.55	113.87	113.16	115.38	115.98	111.40	112.83
	O	116.56	113.41	114.24	114.89	117.01	110.98	113.02
RC	S	55.52	83.86	71.14	70.14	60.62	160.62	157.42
	O	95.71	178.12	197.46	142.44	114.77	87.34	71.98
WI	S	783.97	705.64	789.19	683.42	807.91	696.68	647.55
	O	782.57	686.97	825.54	671.51	850.75	684.45	653.16
SI	S	33.31	50.32	42.68	42.08	36.37	96.37	94.45
	O	57.42	106.87	118.48	85.46	68.86	52.40	43.19
GM	S	601.42	517.07	604.46	497.65	623.58	479.27	429.72
	O	584.15	461.63	590.06	457.83	644.64	495.96	468.61
NP	S	597.37	512.51	589.67	489.70	618.24	475.92	426.78
	O	580.10	457.07	575.27	449.88	639.30	492.61	465.67

AS: Average of Sows; VC: Variable Costs of sows; FeC: Feed Cost; RC: Replacement Cost; WI: Piglets sold; SI: Slaughter sold; GM: Gross Margin; NP: Net Profit.

Table 8

Economic Results expressed in euros per sow per year (O: observed, S: simulated).

4. Concluding remarks

The proposed model is a semi-Markov decision model. It is shown that it can be used to represent specific swine farm situations. It has been formulated differently than other similar sow or cow Markov models already published (Dijkhuizen et al., 1986a, Huirne, 1990, Jalving, 1993, Kristensen et al., 1996). The main difference between the proposed and previous model formulations is in the definition of states and stages, which in our model represent natural states and stages of the lifespan of the sow. Also, the model approach herein presented can be understood as static because time is not driving model behaviour and is only concerned with steady state situations. This model structure was chosen to avoid splitting state variables into a greater number of states in order to improve the precision and speed of the model.

Former models estimate simulation parameters from previous published data assuming that these parameters can be applied to all farms. Model parameters in the proposed model are estimated from specific farm data to precisely describe the animal and management characteristics of each simulated farm. This precise caracterization of the farm allows to represent precisely each farm and to compare farm results in terms of actual data and simulation results. Technical indexes were also introduced to improve model applicability because they are more familiar to farmers and technicians. However, it is difficult to objectively evaluate the resemblance between simulated results and observed values. This situation was also true when comparing results in previous sections and in previous published reports. Because of this difficulty, we have completed the validation process by testing central hypothesis of the model with the χ^2 test. This statistical test provides a more objective measure of the model goodness of fit. The χ^2-components analysis helps to understand the model behaviour, especially when discrepancies between observed and simulated are not homogeneously distributed over states distributions.

Even with the statistical χ^2 test, it is not always easy to assert the suitability of the model for one specific farm. Exogenous variables like health problems, feeding changes, etc. may affect herd dynamics and make it difficult to explain differences that may appear between actual farm data and simulation results. In any case, this model is a powerful tool to study the components of the herd in steady-state situations, to answer questions of “What if...”, etc. Because of the stationary approach, our model can be considered equivalent to others with static orientation in terms of expectations.

Nevertheless, comparing results of herds at equilibrium is a good method of evaluating long term management alternatives as suggested by Jalving et al. (1992a, 1992b). Furthermore, this model can be combined with optimization techniques that can provide management strategies ensuring optimum farm performance. In fact, dynamic programming is one of such techniques with a recursive procedure to calculate optimal policies using equation (2).

In conclusion, the semi-Markov model herein presented precisely describes the herd structure at equilibrium based on actual farm data. In the future the model remains open for further improvements like by introducing optimization procedures, by making the model dynamic, etc.

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Chapter 5. A DECISION SUPPORT SYSTEM BASED ON A MARKOV DECISION SOW MODEL

Plà, L.M. [1,2*], Pomar, C. [4], Pomar, J. [2,3]

[1] *University of Lleida, Department of Mathematics*
Pl. Victor Siurana 1 E-25003 LLEIDA
[2] *R+D Centre. UdL-IRTA, Area of Animal Production*
[3] *University of Lleida, Department of Agro-forestry Engineering*
Avda. Rovira Roure 177. E-25198 LLEIDA

[4] *Dairy and Swine Research and Development Centre*
Agriculture and Agri-Food Canada, P.O. Box 90 - 2000 Route 108 east
Lennoxville, Quebec, Canada, J1M 1Z3

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Abstract

A sow herd decision support system (DSS) was developed based on a semi-Markov decision process and implemented using object-oriented programming. The DSS was designed to help farmers and/or advisers in the decision making process. A friendly user interface was implemented in order to facilitate the interaction with the decision model. The DSS provides several tools to test the model's suitability in relation to available farm data. The DSS inputs include technical and economic parameters that characterise specific farm production situations. Technical parameters are those defining sow production (e.g. abortion rates, sow mortality and fertility rates, prolificity, etc.) and management characteristics (e.g. lactation length, maximum number of matings allowed, maximum number of farrowings, etc.). Economic parameters are related to the economic environment (e.g. feed prices, value of weaned piglets, value of purchased gilts, value of slaughtered sows, etc.). The DSS outputs include calculated results as technical and economic indexes, herd structure and sensitivity analysis of some parameters assuming that the herd population is in steady-state. This approach has been preferred because it allows the evaluation of the impact of different management strategies on future sow performance throughout comparative static analysis.

Keywords: Decision Analysis, Markov Decision Process, DSS, sow herds.

1. Introduction

Management in the swine industry is a complex process where a large number of controlled and uncontrolled factors involved in production efficiency need to be considered. The uncertainty and the degree of specialisation in livestock production systems have increased dramatically over recent time. Also, new management tools dealing with more complex production environments are being implemented by larger enterprises in response to their competition. Therefore, information technology represented by computer-based systems is potentially a good way to pack complex knowledge, thus becoming available for 'less sophisticated' users (Kamp, 1999). Decision Support Systems (DSS) are suitable tools to facilitate the farmer's response to uncertainty and changing production conditions. Because of that, the interest on practical DSS applications is becoming greater and greater in modern, efficiently managed, swine production systems.

DSS are based on one or more mathematical models. A mathematical model is a simplified representation of a real system using mathematical formulations. In the past, several livestock models were developed for herd management (see Jalving, 1993 and Kristensen, 1993, for an overview). These models were developed mainly for research purposes and consequently, and therefore they are not well suited for working directly in field conditions. Thus, few of them have been incorporated into a DSS for on sow farm utilisation (Dijkhuizen et al., 1986; Huirne et al., 1988; Huirne, et al.1991; Jalving et al., 1992a; Jalving et al., 1992b; Huirne et al., 1993). One reason for this could be their complexity which is mainly inherited from the mathematical model. Its complexity makes it difficult to learn, even by technicians and advisors (Kamp, 1999). Therefore, a successful DSS requires a friendly interface, a suitable model and an adequate problem solving capability. To enhance the confidence in these DDS applications, they must be successful in solving problems and in giving useful information to farmers.

The development and implementation of a DSS application based on a semi-Markov decision sow model (Plà et al., 1998, Plà et al., 2001) is presented in this paper. The general purpose of the DSS is to provide a mathematical and computerised representation of sow-farm systems in order to simulate farm responses according to management strategies. Then, the DSS is intended for on-farm use and for educational purposes. The DSS provides estimations of farm expected net revenues according to the simulated management alternatives. Moreover, a module for checking the appropriateness of model parameters has been added to test their suitability for practical use.

2. General description of the system

The DSS has been developed to run on a personal computer with Windows 95, Windows 98 or Windows NT 4.0 operating systems. It has been implemented using Delphi™ v. 4.0 (1998) from Borland Inprise Inc., taking advantage of the object oriented programming and visual interfaces. The DSS includes several components, each of which interacts with a central Interbase® database. The software was designed to run alone or within a pig information system (Noguera et al., 1992; Noguera et al., 1995).

The basic functional structure and the data flow of the DSS are shown in Fig. 1. Data processing is performed by the semi-markov decision sow model which includes the mathematical equations representing sow herd dynamics. The DSS needs technical and economic input parameters to characterise a specific farm production situation. Technical parameters are those defining the animal's production potential (e.g. abortion rates, sow mortality and fertility rates, prolificity, etc.) and management characteristics (e.g. lactation length, maximum number of matings allowed, maximum number of farrowings, etc.). Economic parameters are related to the economic environment (e.g. feed prices, value of weaned piglets, value of purchased gilts, value of slaughtered sows, etc.). According to user's preferences, model outputs can be viewed on the screen, printed or stored for further studies (Figure 1).

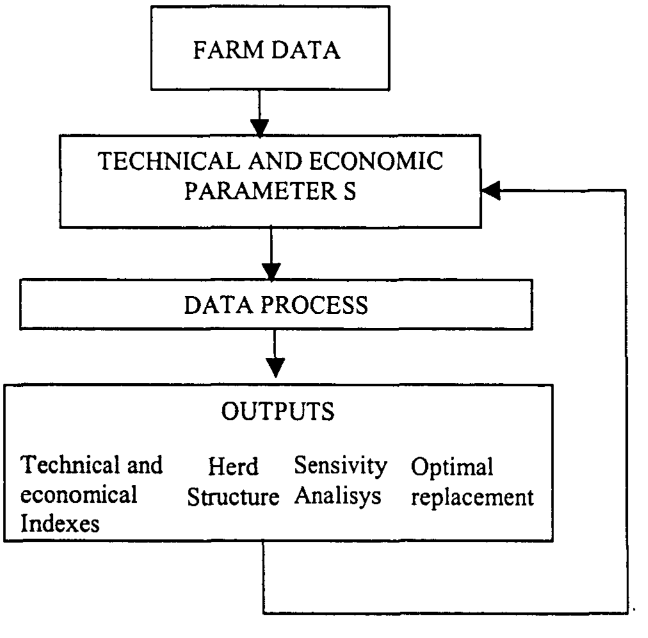

Figure 1. Information flow of the DSS

An important characteristic of the proposed DSS is that model inputs, technical and economic parameters, can be directly extracted from farm data to represent actual farm production conditions. Otherwise, the user can specify or modify these parameters. The DSS can also start the simulation with default parameters.

Basic outputs are technical and economic indexes, herd structure and sensitivity analysis of some parameters. These outputs can be presented using graphs, tables and reports. Optimal replacement strategy is also calculated from model outputs. All outputs and associated parameters are stored according to farm and date. Once stored, these outputs can be shown individually or grouped in order to be compared in different ways. For instance, model simulations based on subsequent periods allow the user to approximate the evolution of farm herd structure over a period of time. Furthermore, comparisons are possible not only among outputs but also between outputs and actual farm data. Finally, the sensitivity analysis permits the user to estimate the impact of changes in model parameters on farm structure, technical indexes and potential farm net revenue. In a similar way, the user can test the effect of parameter changes on farm performance ("What if...?" analysis). An optional test provides an objective measure of the goodness-of-fit between observed and simulated data.

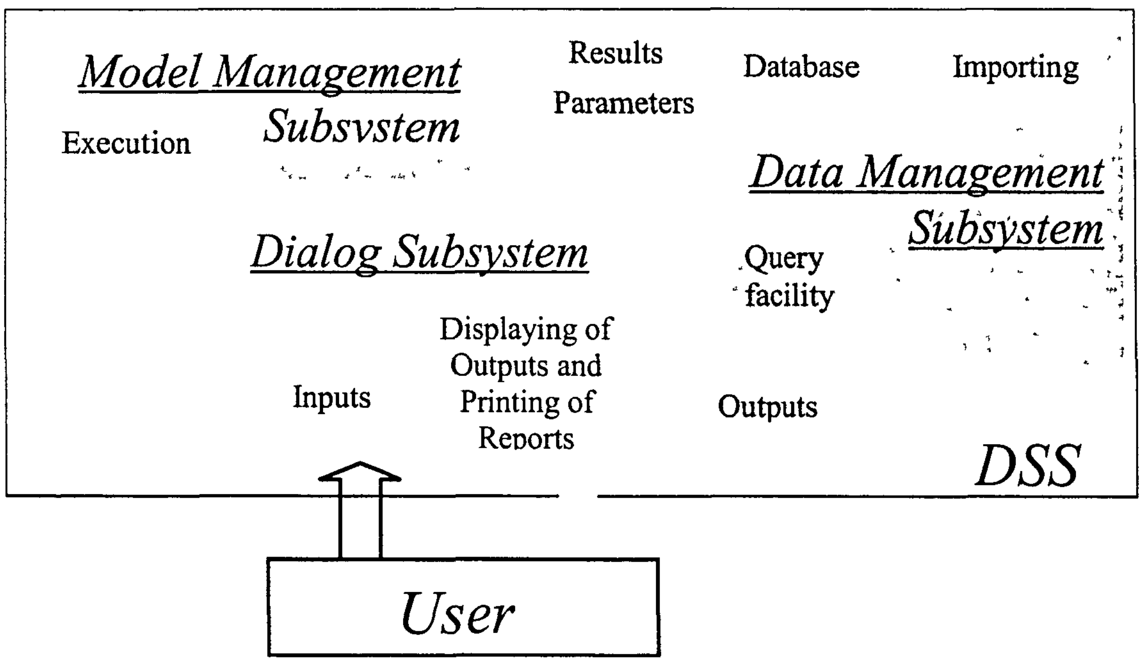

Figure 2 Architecture of the DSS

3. *DSS architecture and implementation.*

As proposed by Sprague (1980) and by Turban (1990) the DSS has been structured in three main subsystems: the data management, the model management and the dialog subsystems. These subsystems and their components are shown in figure 2. The model management subsystem reads the parameters, performs the calculations and gives the results. The data management subsystem contains the DSS-database and different facilities to manage data. Finally, the dialog subsystem contains the interface between the user and the application. In figure 2, arrows represent connections between the different subsystems. For example, the data management subsystem imports and processes farm data in order to calculate farm parameters which are used by the model. After that, model results are stored in the database and displayed when required by the user. Furthermore, the dialog subsystem can start the execution of the model, browse through the DSS-database, make queries, modify parameters and show actual or past stored outputs.

3.1. The model management subsystem

The model management subsystem contains the model used to simulate sow herd behaviour. The mathematical model included in the DSS was based on a general sow semi-Markov decision model (Plà et al. 1998). The model represents a sow farm as a controlled process observed at discrete points in time for values of state variables in which the sow lifespan is broken down into. At these points, actions can be taken in order to control the productive behaviour of sows determining in the next period the probability distribution of state variables and expected reward. State variables of economic importance are sow age, sow reproductive condition and litter size. Actions related to replacement strategies are “keep” and “replace”. As a Markovian process, state variables are defined assuming that future state values are conditioned only by the present state and not by the manner in which the present state was reached. The time interval between actions (called stage) depends on which state the sow is in. Actions to be taken in each possible state are specified by a decision policy and can be understood as the management strategy of the farmer. The transition probabilities and rewards (expenses or incomes) between states are assumed stationary. Thus, expected performance under a given management strategy is stable over time. Under these conditions, herd production for a given decision policy can be represented by a finite irreducible Markov chain.

Markovian decision analysis provides an appropriate framework to evaluate technical and economic consequences of different management strategies. By using common properties of Markov chains, expected annual net returns can be calculated for the sow herd when a stable management strategy is implemented. The expected actual net value of an infinite time horizon can also be determined by incorporating the discount rate into the transition matrix. Input model parameters are transition probabilities, productive parameters and also remaining parameters that serve to define the economic scenario. With these parameters, the model gives the herd structure at equilibrium, which is used to test the goodness of fit, between the real data and model outputs, to derive technical and economic indexes and to perform a sensitivity analysis. Also, this formulation facilitates the application of programming techniques for solving the replacement problem and finding the optimal replacement policy.

Incomes considered by the model come mainly from weaned litters, and replaced sows. Fixed costs for buildings and equipment are determined by per sow and per year cost. Variable costs per sow and cycle include veterinary, water and labour expenses, as well as, other variable costs as replacement gilts, feed and artificial insemination expenses. Feed cost is calculated based on daily feed intake, reproductive state and feed type. An annual discount rate of 6% is also considered to account for the time value of the costs and incomes. For simplicity, it is assumed that sows are replaced after culling without delay. More detailed information on the model structure and its inputs is described in Plà et al. (2001).

3.2. The data management subsystem

The data management subsystem was implemented under the InterBase® multi-user database management system from Borland Inprise Inc. The data management subsystem includes the following elements (see Fig. 2):

- Database management system: creates, accesses and updates the database. Typical functions of this system are, storing, retrieving and access control of data from the database.

- Importing facility: gives access to external farm data, when farm data is stored in a different database than the DSS database and not accessible by the Database management system.
- Query facility: allows the user to access to data when requested by the Dialog subsystem of the DSS.
- Database: contains the data needed by the DSS.

3.2.1 General data structure

Database structure is described in Figure 3 using an entity-relation diagram. Entities should not be understood as tables, since they can be related to one or more physical tables in the final implementation of the DSS database (DSS-DB). The DSS-DB stores all the parameters used by the semi-Markov decision model and its outputs.

The total number of the DSS-DB entities is five (Fig. 3), three for storing traits and specific parameters of the farm (Farm, Period and Parameter-cycle) and two for associated outputs (Population and Index). The Farm entity stores the data used to identify farms. In this entity the identification code is the primary key and the owner, address, postal code, city and e-mail are secondary keys. Through the primary key, all registered farm data can be accessed. The farm entity is related to the Period entity, the former indicating the period in which the farm has been analysed. Main attributes for Period entity are the farm identification code and the time interval. Time interval is defined by the first and the last period dates. Other Period entity attributes are maximum number of cycles, maximum number of matings, expected lengths of lactation, gestation, oestrus cycle, gestation with abortion, the time interval between the last event and culling, the time interval before first mating and a variable representing the goodness-of-fit of model outputs. Other parameters needed by the model subsystem are stored in the Parameter-cycle entity and they are specific to each reproductive cycle. These parameters consist of time interval from last reproductive cycle until first mating, litter size, litter weaned, conception rates for matings one to four, casualty and culling rates for open, gestating and lactating sows, abortion rate, average feed intake and price per Kg of feed.

The Herd-structure entity contains, for each period, the structure information of the herd, i.e., number of sows per cycle and state. The Index entity contains the technical and economic indexes. All of these fields are replicated three times to store observed, simulated, and optimal replacement policy information. The relationship between the Period and Index entities is not one to one because depending on farm data not all indexes can be calculated while a period can have different set of indexes.

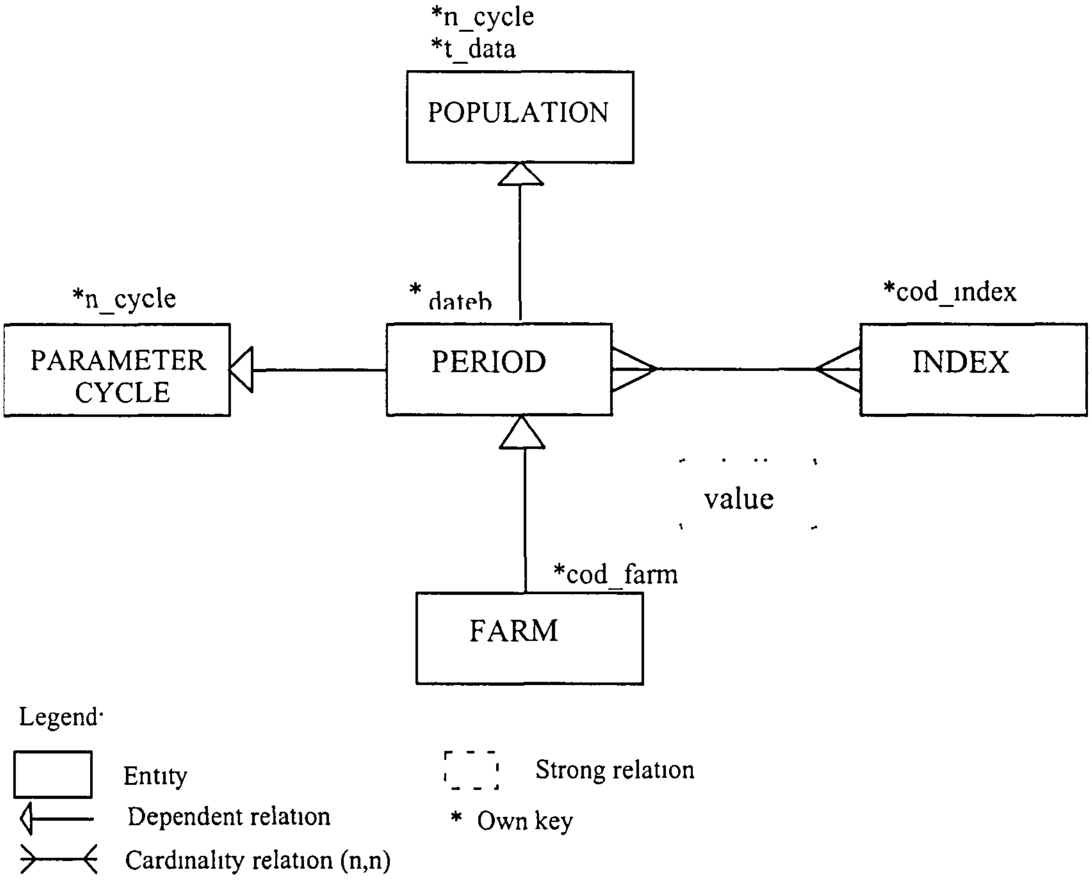

Figure 3. Database structure

3.2.2 Processes related with the Database

The main processes related with the database are those updating or querying the database. Briefly, in logical order:

* The first process only affects the Farm entity and concerns the importation of external farm data. Importation allows the DSS to create the DSS-DB for that farm. During this process, the imported farm data is filtered to avoid inconsistencies.

* The second process is activated when a new period to be analysed is selected. Period and Parameter-cycle entities are involved. All technical and economic parameters for that period are then calculated and stored. After that, the mathematical model of the DSS model subsystem is ready to be run.

* The third process stores the model outputs after the simulation. Fields in tables storing simulation and optimal information are updated. These tables belong to Herd-structure and Index entities.

* The forth process concerns the statistical test results. This test compares observed and simulated data as described by Pla et al. (2001). The chi-squared value, their components according to state and the significance level are stored in the Period entity.

* The fifth process is activated if requested by the user after modification of input model parameters. Tables are updated and later stored.

All these processes are activated through the corresponding options in the menu bar. The menu bar is managed by the Dialog subsystem described below.

3.3. The Dialog Subsystem

The Dialog Subsystem of the DSS is a graphic user interface (GUI) used to access all features of the DSS. The general appearance of the interface is similar to other Windows based applications. The menu bar of the application is composed of six menus: File, Decision analysis, Reports, Tools, Window and Help (Figure 4). Under the menu bar there is the button bar, there are three buttons by default: import a new farm data, display a farm data and exit. On the right side of the button bar there are three little boxes showing the period of time (initial and ending date) and the farm identifier activated at each moment. Most of the commands in the File, Window and Help menus act just like they do in other Windows applications. For instance, the File menu lets the user import, export, open, save, and print farm data. Menus that are specific to the DSS are Decision analysis, Reports and Tools. The Decision analysis menu contains options related to period management, that is, create, delete and open one or more periods of a farm to be analysed, as well as, chose the analysis to be performed (single or multiple simulation run, or analysis of sensitivity). The Reports menu let the user browse among different output displays. Outputs can be presented as reports (e.g. herd structure, chi square components), editable or fixed tables (e.g. parameters of the model, optimal strategy of replacement), and graphics (e.g. herd structure, litter size, culling rate). All of these displays can be printed. The Tools menu is only available to the system manager user to modify reference prices as feed prices, or standard references of technical and economic indexes from different databanks.

Figure 4. Main menu of the DSS

The DSS has different levels of authorised access in order to preserve database integrity. Standard users can handle different periods, run the model, view and print results stored previously, but they can not import data from new farms nor modify or update economic parameters and external references. The second is the system manager who has access to all features of the application.

4. *Program operation and application*

To operate the DSS, the user starts selecting the farm and choosing the period of time to retrieve farm specific parameters. Parameters can be adjusted if necessary to represent farm specific situations. As stated before the DSS can be used to simulate farm responses according to management strategies, to estimate farm net revenues, to study sensitivity analysis of farm specific parameters, to perform static comparative analysis of reproductive and replacement decisions, and finally to estimate optimal replacement strategies.

The static comparative analysis deals with technical and economic comparisons between the effects of different farm parameters and between replacement management alternatives. That is, the user can modify input parameters and obtain an approximation to their technical or economic impact. The DSS offers two strategies to perform this analysis, the simple analysis based on one by one comparisons and the sensitivity analysis. In the first, the user modifies the parameters defining a new management policy. Modifiable parameters are grouped in different categories and ordered in tables

(Figure 5). For each time interval, average length and standard deviation is displayed. By clicking on the Simulate again button the model is activated. Comparison between the new and old management policy can then be performed. Also indexes can be compared to standard references of the more common indexes taken from a management information system (BD-Porc®,2000).

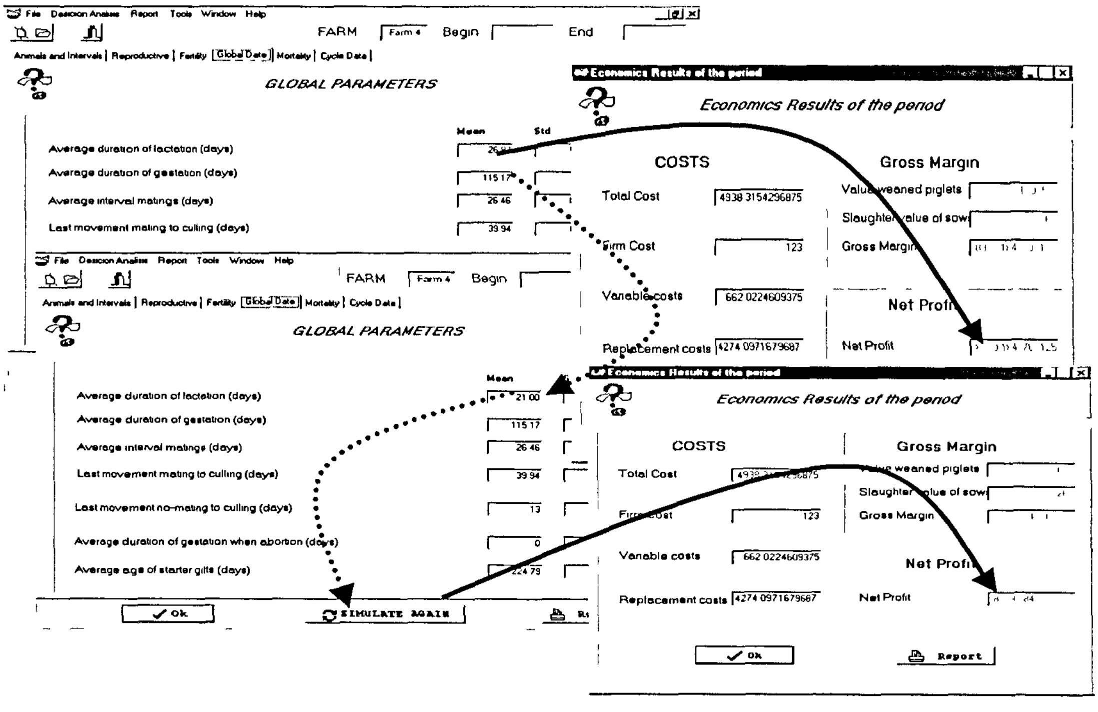

Figure 5. Exploring new alternatives by parameter edition and recalculation.

The sensitivity analysis option is accessible from the Decision analysis menu. Sensitivity analysis automatically applies variation to one by one farm specific parameters, performing multiple analysis. The aim of such analysis is to study the effect of expected parameter variation on farm results. In Table 1 results from a farm (Farm 4 discussed in Plà et al., 2001) are shown. Farm parameters were modified from –1 STD to +1 STD. Variation on economic farm performance are expressed in Euros per sow. Positive values indicate that increasing the value of the parameter will increase the farm of net revenue. Negative values indicate that increases in parameter values will decrease farm economic performance. The absolute value observed in this analysis represents the averaged increment of the net revenue of the farm between the two central STD. High parameter sensitivity can jeopardise the economic prosperity of the farm when farm parameter values gives wrong evaluations. However, we should improve these parameters to better farm economic performance. Results shown in table 1 indicate that litter weaned is the most sensitive parameter of those analysed. Therefore, taking this into consideration, farmers should be very careful to maintain or improve this parameter in order to improve the economic farm efficiency. However, the sensitivity of litter weaned on farm revenues decreases with the increase of the reproductive cycles. Although having less impact, the interval to first mating and culling rates also have to be controlled by farmers to maintain or improve farm efficiency.

Table 1.

Expected average economical farm performance variation for changes in parameter values from –STD to +STD (results in Euros per sow).

Cycle	LW[a]	IT1M[b]	CR(1)[c]	CR(2)[d]	CRO[e]	CRG[f]	CRL[g]
1	-108,1	21,2	2,5	0,4	-3,7	-6,4	-0,5
2	-92,1	19,5	1,0	0,2	-2,9	-5,6	0,5
3	-83,7	15,2	-1,2	0,0	-0,6	-4,6	-0,2
4	-72,3	14,4	-1,3	0,0	-1,1	-4,0	1,2
5	-70,0	12,3	0,7	0,0	-0,0	-2,9	2,7
6	-65,1	11,6	0,6	0,0	1,3	-1,8	2,1
7	-58,9	9,8	0,2	0,0	2,5	-1,0	1,9
8	-50,2	8,7	0,3	0,0	2,3	0,2	3,4
9	-45,8	8,6	0,4	0,0	1,8	1,5	2,6
10	-36,8	4,0	0,2	0,0	1,1	0,3	1,3
11	0,0	1,1	0,0	0,0	1,5		

[a] Litter weaned, [b] Interval to 1st mating, {[c],[d]}Conception rate for 1st and 2nd mating, {[e,f,g]}Culling rate for O:open G:Gestating L:Lactating sows.

Although the simulation of farm management alternatives, the estimation of farm net revenues and the sensitivity analysis could be performed under a static comparative analysis, sometimes they can become self-sufficient, for example in teaching or exploring management alternative feasibility.

Taking advantage of the Makovian framework, optimization procedures for replacement strategies can be used (Kristensen, 1996; Plà et al., 1998). One of these procedures is applied to the replacement problem obtaining a management guideline for maximal number of matings or maximal number of cycles (Figure 6). Optimisation is performed taking into account parameters selected by the user. Optimal policy is obtained using the improvement policy algorithm (Howard, 1970; Puterman, 1994), which is one of the standard dynamic programming algorithms.

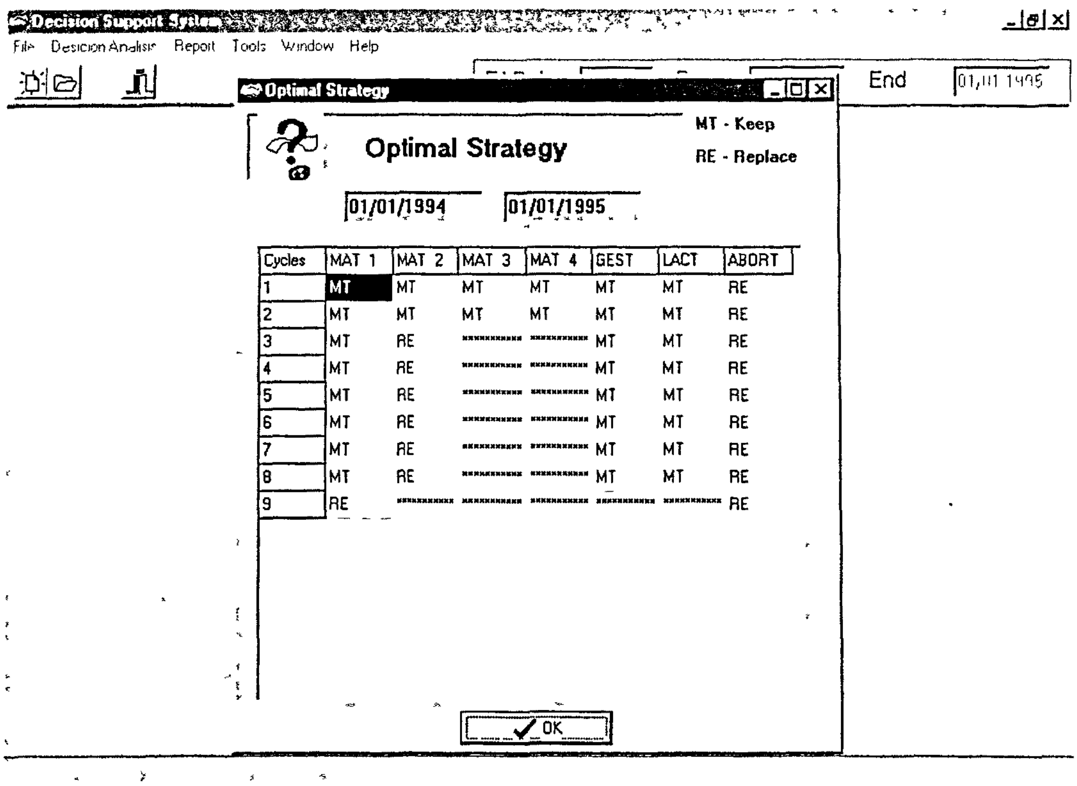

Figure 6. Optimal replacement strategy.

To evaluate the stationarity of the farm in terms of herd structure, the DSS allows the user to compare herd population structure graphically and statistically. Graphic comparison is a first approach that can be complemented with a statistical test based on Pearson's statistic used to check the goodness-of-fit (χ^2 –test). With these comparisons the correctness of the stationarity assumption and the robustness of the outputs provided by the DSS in field conditions can be evaluated. In addition, the time evolution of herd population structure can be studied by performing sequential graphic and statistical coparisons as shown in figure 7. This analysis can be performed automatically when the option Group of Periods is selected in the Decision Analysis menu.

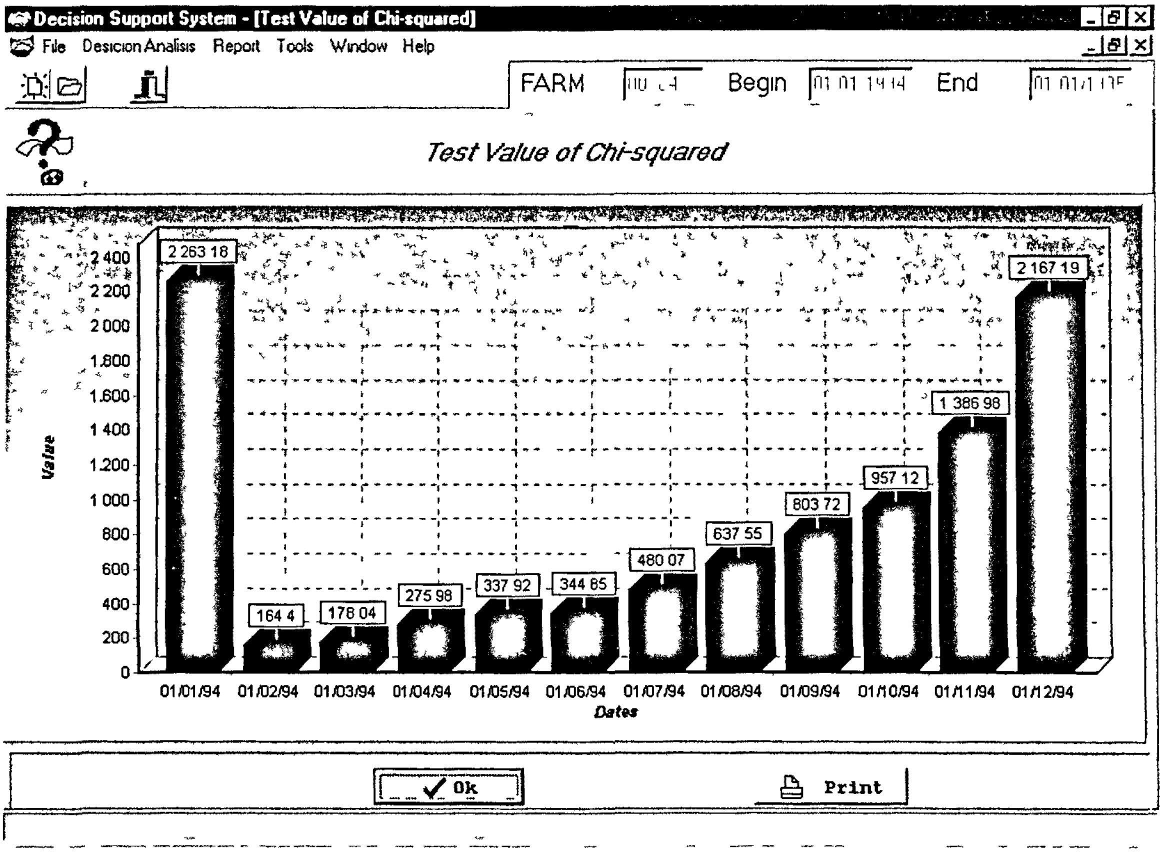

Figure 7. Chi value over time.

5. Discussion and conclusions

The DSS presented here is a system that can be used by farmers and farm advisors to analyse production factors associated with reproductive and replacement management strategies of sow farms. Decision support is based on a comparative stationary analysis performance from model outputs. The user can explore and evaluate different reproductive and replacement management alternatives and therefore can anticipate problems and make the most appropriate decisions, i.e. with a major positive impact. For instance, sensitivity analysis allows the identification of the most important parameters that can reduce or improve the farm economic efficiency. For example litter weaned seems to be the most important parameter affecting the economic efficiency. Reducing the number of unproductive days like those before first service seems also to be important.

Technical and economic indexes are calculated to help users to understand the results from the mathematical model. In this sense, the use of technical indexes such as number of piglets weaned per sow per year, interval between farrowing, number of farrowing per sow per year and even the distribution of sows per state makes the DSS outputs more familiar to farmers and technicians and makes it easier to understand. By comparing farm indexes with other reference indexes, farmers can built their own production goal. Afterwards, exploring different alternatives by static comparative analysis or a sensitivity analysis allows the user to choose management strategies that will improve production performance. The knowledge generated by these simulations can be incorporated into the decision process.

A practical output of the DSS derived from the mathematical formulation is the resolution of the replacement problem. Several decisions like keeping or replacing a sow can be optimised. For example, in figure 7 we observe that optimal sow life is of 9 reproductive cycles. Also, no more than one mating seems to be profitable on this farm. The resemblance between real and simulated farm behaviour is essential when using the DSS for on-farm decision making. Parameters obtained from herds may sometimes lead to disappointing system results, mainly because of the inadequacy of the extracted parameters. Therefore, it is essential to assess the suitability of the population parameters before being used for further simulations. The proposed Pearson's statistic measures the distance between real and simulated distributions, that is, the difference between the observed and the simulated herd structure. This statistic is better understood when complemented with the corresponding graph distribution of sows over states. Large values of the Pearson's statistic indicate that the DSS-outputs are rather unlikely to fit the real farm. The χ^2 distribution and tabulated α-values provide a quantitative measure of the model goodness-of-fit when the Pearson's statistic follows the same distribution. Low α-values indicates that discrepancies between observed and simulated population data are likely. Discrepancies between observed and simulated data may be due to different causes. Thus, the model may not precisely represent the farm reality, the model's adequacy in representing herd behaviour was demosnstrated elsewhere (Plà et al., 2001). Incorrect or missing farm data is the most frequent cause of discrepancies between simulated results and real data. Also, parameter values obtained from populations that have been modified and are far from steady-state conditions would not be appropriate to represent the farm. Hence, the use of a statistic test in conjunction with farm parameters enhances the applicability of the DSS on farm conditions.

Implementation of the DSS using a relational database in conjunction with object oriented programming has been proved to be useful when refinements of the database or the processes are required. Relational database facilitates and allows complex queries and database maintenance. For example, the insertion of new data or new fields on the database can be performed without disturbing the DSS operation. The friendly environment developed for this DSS includes the graphic interface and also the multi-language capability. On the other hand, the DSS can be fully integrated into a general pig management information system as an add-in module. Finally, the efforts invested in the design of the interface have dramatically improved the utilisation of the DSS by people having little experience of mathematical modelling or computers. Training time needed by users was estimated in three sessions of one hour each for beginners and half this time for users already trained in pig management information systems.

The DSS is a convenient tool for researchers and teachers for the study of sow farm management effects and optimal strategies. The static orientation of the DSS makes it inappropriate to study interactions and adjustments between variables because we are not concerned with dynamic adjustments. In that sense, the model is simpler to use and the convenience for farmers and advisors should be dealt carefully. However, new functions can easily be added to the DSS by coding new methods, programming new objects and enlarging the DSS-DB.

In conclusion we have shown how the DSS can be used to simulate farm responses according to a predefined management strategy and estimating expected farm net revenues. Besides the study of sensitivity analysis of farm specific parameters, the performance of static comparative analysis of reproductive and replacement decisions give efficient support to the decision-making task. Finally an estimate of the optimal replacement strategy is also available. Such advantages are highlighted when we put the DSS in a friendly environment and accompanied by tools to check the appropriateness of outputs. The proposed DSS is a suitable tool for practical use alone or integrated into a general pig management information systems.

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Chapter 6. PRACTICAL PROBLEMS ON SOW HERD MANAGEMENT OPTIMIZATION

Plà, L.M. [1,2] and Pomar, J.[2,3]

[1] *University of Lleida, Department of Mathematics*

Pl. Victor Siurana 1 E-25003 LLEIDA

[2] *R+D Centre. UdL-IRTA, Area of Animal Production*

[3] *University of Lleida, Department of Agro-forestry Engineering*

Avda. Rovira Roure 177. E-25198 LLEIDA

Submited to *Computers and Operations Research*

Abstract

A Markov Decision Process (MDP) is a stochastic dynamic model that it serves quite well to represent livestock systems. MDPs are able to deal with uncertainty inherent in such systems and it provides a framework where programming techniques can be applied. Mathematical foundations including assumptions are very important when we want to build Decision Support Systems (DSSs) based on MDPs. If model assumptions are not fulfilled the model output might induce an erroneous decision support. Implementation and verification of MDPs assumptions is not easy, several practical problems related with a sow MDP implementation are presented and discussed.

Keywords: MDP, DSS implmentation, Herd Management Optimization.

1. Introduction

During last years, rapid changes in pig farms environment emphasise the need for models and methods capable of dealing with the uncertainty inherent in livestock systems. That is because decision models are attaining an increasing interest. Such models try to represent the system, as well as to provide a framework where programming techniques can be applied. The more often framework used for those programming tasks are dynamic programming (DP) and more specifically Markov decision processes (MDPs).

MDPs were born in 1960s, and now there are for branches: dicrete time MDPs, continuous MDPs, SMDPs, continuous time Markov decision drift processes and shocket MDPs. The analysis of large scale systems has always been a challenging problem to modelers, realistic models are mostly too complex to be computationally feasible. But it remains true that some traditional problems in such techniques are reducing their importance due to high performance of modern computers. May be now is more important to develop useful models from large scale systems without sacrificing significant accuracy in order to build more confidence tools to decision support.

A MDP is a dynamic stochastic model that is used, in general, taking into account several assumptions, i.e. Markovian property, homogeneity of transitions, ergodicity, horizon time, reward function, among others. But what happens if some of these assumptions are not hold by the model? We are conscious that previous solving methods are procedures developed for deterministic optimization problems and what it implies? When assumptions are incorrect, the model and solving techniques, like value iteration and policy iteration algorithms, or linear programming could be inappropriate for its use. This problem is related with the lack of knowledge about implicit or explicit hypothesis that mathematical models included, it is very important that researchers (and model-users) know and accept model assumptions.

In a rigorous way, all this assumptions and hypothesis about the model formulation should be checked in a correct field implementation, this may constitute the model validation. Nevertheless, a whole validation is not easy to do because many times model formulation is made by taking into account theoretic based hypothesis, which are difficult to demonstrate. Furthermore, in several livestock models proposed until now, there is, apart from other practical problems, a lack of relevant data that prevent the achievement of the validation (Sørensen, 1990).

It is obvious that concurrence makes information essential in livestock economics, in this way decision support systems (DSS) become the natural information system where decision models are implemented in order to support farmers in decision making process. Although these applications are usually used for on farm decision support, few farmers are able to use them. In this work we present some problems in the development of a sow-DSS based on a MDP and integrated in a more general pig information system. The model is used to illustrate different problems mentioned above, and the impact on the results in case different assumptions fails. As far as possible, the examples will be taken from real farms.

2. Model Formulation

The dynamics of a MDP is described by a controlled Markov process. In this way, the model we want to formulate deal with a sow farm as a dynamic system evolving over time where the probabilistic law of herd motion can be controlled by taking decisions. Also, an economic effect happens as a consequence of decisions that are sequentially made when the herd evolves over time. An infinite horizon is assumed and the goal is to find an optimal policy, that is a control rule which optimizes some predetermined criteria. Standard optimality criteria for MDP have been the expected discount total reward or the limiting reward per unit over an infinite time horizon.

A MDP is defined as a set of $\{S, A, (p^t_{ij}(a)), r^t(i,a)\}$ where S denotes the set of all possible system states, A is the set of allowable actions, $(p^t_{ij}(a))$ is the transition probabilities matrix at time t and $r^t(i,a)$ is the reward that the decision maker receives at time t as a result of choosing action $a \in A$ in state $i \in S$. When positive, $r^t(i,a)$ may represent an income, and when negative a cost.

Decision rules are functions, d_t, which specify the action choice when the system occupies a state at time t. Then, a policy or strategy, $R=(d_1, d_2, \ldots)$, for controlling the system is a prescription for taking actions at each stage. In principle a control rule may be quite complicated if prescribed actions would be history dependent, but for simplicity we will just consider Markovian policies. We assume that the process is stationary, i.e. $(p^t_{ij}(d_t(i)) = (p^t_{ij})^R = (p_{ij})^R$, $r^t(i,a) = r(i,a) = r_i^R$ and $d_t = d$ for all $t \in N$. It happens that stationary policies are fundamental in the theory of infinite planning horizon.

If the limiting reward per unit is used as a criterion, we should know that if S and A are finites ($|S|$, $|A| < \infty$), r_i^R is bounded and the MDP is recurrent (also true for unichain MDPs), i.e. if the transition matrix, $(p_{ij})^R$, corresponding to every deterministic stationary policy consists of a single recurrent class, then there exists a stationary limiting reward per unit optimal policy. Hence, under a given stationary policy, R, the stochastic process representing the sow herd distribution, $\{X_t\}_{t \in N}$, is an irreducible aperiodic dicrete time Markov chain (the irreducibility means that there are no transient states). We denote by $q=(q_1,q_2,\ldots,q_{|S|})$ the stationary distribution, i.e. the normalized solution of equation $q=q(p_{ij})^R$. As results, the objective function is reduced to search for a policy, R, that maximizes $g^R = \sum_{i=1}^{|S|} q_i r_i^R$, where g^R is the limiting reward per unit and q is the limiting state distribution of the Markov process under the policy R.

Decisions are supposed to be made only at fixed stages. However, in many optimization problems the times between consecutive transitions or stages are not identical but random, they constitute the semi Markov decision processes (SMDP). Denoting by X_n the state of the system at the n^{th} stage, it follows that under some predefined policy, R, the stochastic process $\{X_n\}_{n \in N}$ is a discrete time Markov chain, the so called embedded MDP, with one step transition probabilities, $(p_{ij})^R$ and corresponding reward function, r_i^R.

For the SMDP the formulation of a solving algorithm is not straightforward. However, by a stage length calculation, t_i^R, we can find the optimal policy throughout the embedded MDP maximizing $g^R=\sum_{i=1}^{|S|} q_i r_i^R \,/\, \sum_{i=1}^{|S|} q_i t_i^R$. An other possibility is by a data transformation, converting the SMDP into a MDP such that for each policy the reward function are the same in both models. The data transformation allows to apply standard algorithms as well as to follow the system at equals step time stages.

The procedure is as follows. Choose a number t with $0<t<min\ \{\ t_i^R\ \}$. Consider now the MDP given by

$$S'=S$$

$$A'=A$$

$$r'^R_i=r_i^R/\,t_i^R \qquad i \in S \text{ and } a \in A$$

$$p'^R_{ij}=\begin{cases} \dfrac{t}{t_i^R}\,p_{ij}^R & j \neq i \\ \dfrac{t}{t_i^R}\,p_{ij}^R + \left[1-\dfrac{t}{t_i^R}\right] & j = i \end{cases}$$

This MDP has the same class of deterministic policies as the original SMDP, this transformation is in fact an aperiodicity transformation (Puterman, 1994). By direct application of the optimality criterion it follows that $g'^R=\sum_{i=1}^{|S'|} q'_i\, r'^R_i = \sum_{i=1}^{|S|} q_i r_i^R \,/\, \sum_{i=1}^{|S|} q_i t_i^R = g^R$. An example of a similar tranformation seems to be used by Jalving et al. (1993) where they fixed t=1 week instead of $min\ \{\ t_i^R\ \}$. There exist other examples in dairy cows where t=1 year or t=1 month (Jalving, 1993; Kristensen, 1993).

3. Practical problems

A DSS to support decision tasks in a sow herd management was implemented based on an embedding MDP according to the assumptions given in Sec. 2. The sow production cycle used to establish the pattern of the model is presented in Fig. 1, this cycle is very related with the life span of the sow and its reproductive ability.

The use of the DSS was intended to be farm specific and integrated in a more general pig Information System (Plà et al., 1998), so parameters were calculated from different farms separatly. It was shown by Jalving (1993) that stationary sow herd distribution could be useful in reproductive and replacement management, but no discussion was made about the suitability of the model in specific farms. During the model development and its validation with specific farm data several problems affecting model structure and assumptions were detected (Plà et al., 1999). In general, problems were related with parameter calculations or assumptions that failed and needed to be reformulated. The reformulation was not always possible, then the optimal solution could be inconsistent with the real problem. The DSS should be able to manage such exceptions in order to extract useful information for the decision maker. A brief discussion of some of those problems is presented.

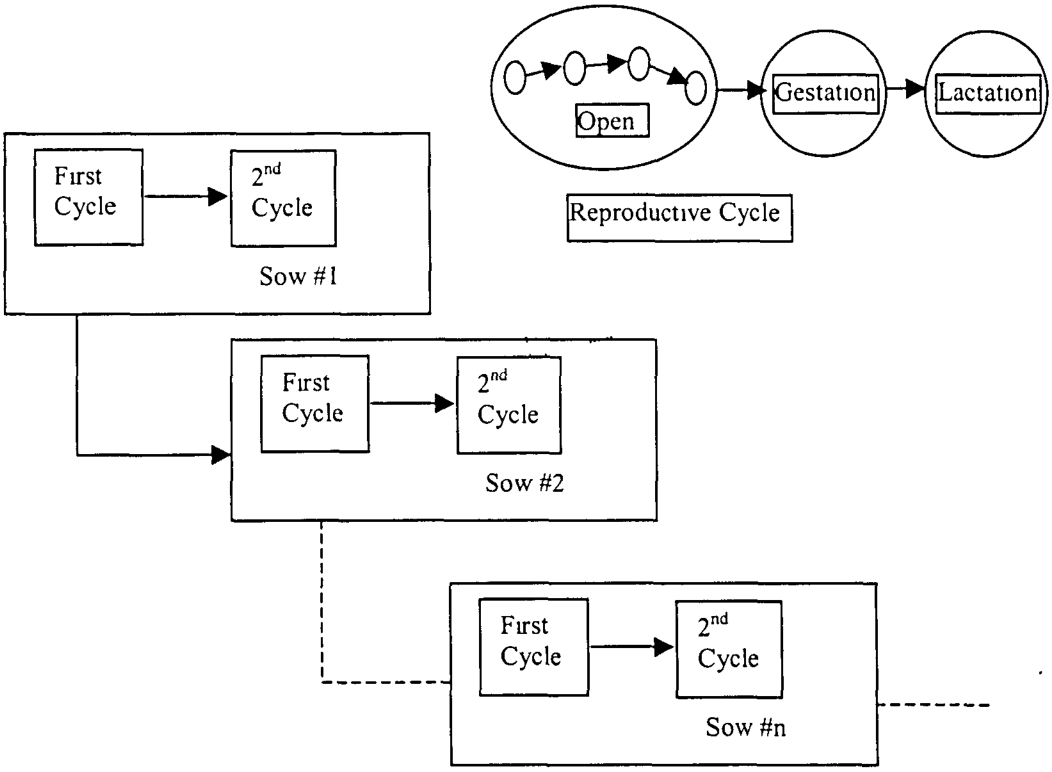

Fig. 1. Sow production cycle on farm.

3.1. Data registration

The first problem that we found in the development of a farm specific MDP is the quantity and quality of information available. This information could provide in some way the specificity wanted for the model.

The information for this study was extracted from GTEP-IRTA system data bank described by Noguera et al. (1992). The data registered in the data bank is used e.g. in the model parameters estimation. We can distinguish between technical and economical data. Technical registers of data include:

a) Reproductive animal inventory: identification number, sex, breed, animal origin, date of birth and date of entry into the farm.
b) Reproductive cycle: it includes mating dates, sow and boar identification, type of mating, parity number, number of born alive, number of stillborn,

mummified, fostered and discarded piglets, number of weaned and weaning date.

c) Reproductive animals casualties: animal identification, casualty date and cause.

On the other hand, economical data that are taken into account are the following:

a) Animal and feed inventory.
b) Animal purchase and sale: quantity, weigh, price, total amount and date.
c) Purchase of feed and other items: quantity, price, total amount and date.
d) Amortizations and financial expenses.
e) Energy, labour and other expenses (veterinary, insurance, taxes, etc...).

All these data are recorded in farms owned by different companies. Then, periodically data are transferred to the data bank host.

Model formulation is affected by data availability-variability. The main problem could be the definition of the state set, because it depends on registered data on farm. We can distinguish two main cases, first when a variable state is recorded but not always is observed (variability), and second a variable state that it is not recorded (availability). The first problem concerns the variability strictly observed within a specific farm and the variability represented by the model. Some extreme cases could be presented, for example, to consider or not a 4^{th} - 5^{th} mating, or how many reproductive cycles should be taken into account. Practical experience can help us when the matter is just to choose among different levels of a variable. Although in the second case the problem is related to the inexistence of observations and thus, several states are difficult to be considered if corresponding observations are not recorded. An example of that could be the hour time of the mating, the heat detection, the pregnancy detection, the number of services per mating, individual feed intake, etc. Even in those cases, solutions could be found depending on variables, these solutions would come from a knowledge generalisation like pregnancy detection rates or heat detection rates, that at least allow us to study their impact in herd economics.

The quality of data is another important topic. Data are filtered in order to avoid mistakes and inconsistencies, althought it was found that a 1% of data records were not wrong but suspicious, i.e. with some unusual value like a long lactation or a long interval between matings.

It should be notice that during 1998 GTEP-IRTA system became to a new management information system, named BD-Porc. Now, it is the official pig data bank of Spain supported by the Ministry of Agriculture. In order to improve the level of information more registrations of new farm variables are going to be added in BD-Porc .

3.2. Transition probabilities estimation.

Once the state space wasdefined and fixed, the estimation of transition probabilities could be done. We assumed for the model validation that data were representing the management policy used by farmers, then from Sec. 2 we could derive that farm records corresponded to a sample of an irreductible first-order Markov chain. Then statistical inference methods about Markov chains were available.

If it happens that the order of the chain is greater than 1, it can be reduced to a first-order chain by suitable definition of composite states. This is usually done when sow productivity wants to be represented e.g. Huirne (1990) and Jalving (1993) defined different states to represent the effect of previous litter size over actual litter size, and similarly Kristensen (1993) defined for dairy farms a state variable representing the cow production level.

The way to estimate different probabilities is accounting during a certain period of time, T, for different transitions, $n_{ij},(t)$ with $t \in T$. Then, maximum liklihood estimates of p_{ij} can be calculated as follows:

$$\hat{p}_{ij} = \frac{\sum_{t=1}^{t=T} n_{ij}(t)}{\sum_{k \in S} \sum_{t=1}^{t=T} n_{ik}(t)} \qquad \text{Eq. (1)}$$

From Eq. (1) it would be possible to test the stationarity of the process, i.e. H_0: $(p^t_{ij})^R = (p_{ij})^R$, and alternative hypothesis H_1: $(p^t_{ij})^R \neq (p_{ij})^R$. Although, due to the large number of transitions to test it was prefered to test directly for stationary distribution, even more when stationary distrubution is central for the optimal criterion. The test used was the χ^2 –test based on Pearson's statistic:

$$\chi^2 = \left(\sum_{i\in S}\sum_{k\in S}\sum_{t=1}^{t=T} n_{ik}(t)\right)\cdot\sum_{j\in S}\frac{\left(\dfrac{\sum_{j\in S}\sum_{t=1}^{t=T} n_{ij}(t)}{\sum_{i\in S}\sum_{k\in S}\sum_{t=1}^{t=T} n_{ik}(t)} - q_j\right)^2}{q_j} \qquad \text{Eq. (2)}$$

that has an assimptotic χ^2 distribution with $|S|-1$ degrees of freedom. In this way we have a measure of the goodness of fit, and also if the data do not fit the model we can study each component of Eq. (2) and find out which states are worse fitted. This strategy allowed improving the model. For example, after observing the problems posed by the number of sows waiting to be sold, a deeper study drove to consider different states when the sow to be sold had finished a lactation or a gestation or just it had been mated. In Table 1. we observe how the stage corresponding to an open sow waiting its first mating in the 11th cycle has accumulated the 25% of the total χ^2 value.

Cycle	1st Mat	2nd Mat	Gestation	Lactation	Abortion	Mto Sold	NMtoSold
1	0.146	0	1.215	1.322	0	0.202	0.305
2	4.558	0.272	0.397	0.239	0.101	0.229	0.131
3	0.590	0.198	1.263	0.291	0	0	0.046
4	0.494	0	0.039	0.547	0	3.557	0.041
5	0.381	0	0.012	1.192	0	0.520	0
6	1.312	0.201	0.019	1.526	0	0.119	0.046
7	0.656	0.380	0.322	1.358	0	0.128	0.129
8	0.896	0	1.210	0.924	0	0.313	0.156
9	0.117	0	0.904	0.263	0	0.102	0.127
10	10.583	0	1.059	0.401	0	0	0.127
11	0.044	0	0.835	0	0	0	0.127

Mto Sold: stage corresponding from last mating to sold

NmtoSold:stage corresponding from farrowing, weaning or abortion to sold

Table 1. Example of χ^2 components. Herd distribution of a farm is compared with model stationary distribution derived from the same farm.

The true model formulated is a SMDP, but we only will consider the embedded MDP with the same state and action space, allowed transitions and rewards per state. It is considered that after each transition the farmer reviews sow performance in order to decide if that sow is kept or it is replaced, in this way each state is associate to one decision and a stage is delimited by two consecutive transitions. Time between transitions is estimate and its expected value is applied to define the reward function.

The length stage estimates gave sometimes problems in some farms in some states. One of the more problematic was the interval between matings. This interval was quite large depending on farm. Probably, it was caused by reproductive problems or bad heat detection. No changes were introduced in such estimates in order to reflect actual farm management and because neither farm had enough observations to make other considerations, there were found few farms with more than one mating per cycle. If it was necessary, it would be possible to split this stage to consider regular oestrus and non-regular ones.

3.3. Reward function

The reward function, r_i^R, represent the economic performance of choosing the action prescribed by policy R, when the system is in the state i. Several costs, like feed cost were calculated based on time expectation between transitions. Other costs or incomes were depending just on the actual state, like slaughter value when replace action was taken, or purchasing value for gilts. Piglets sold were also depending on litter weaned per sow at the end of lactation. No fluctuation prices were considered.

Reward function should represent decision-maker utility. The use of expected utility theory, where it is thought relevant and practicable, would normally be the natural extension, but practical difficulties do arise, and decision making is often undertaken in a manner not always compatible with expected utility theory. The determination of utility functions in practise, and even the acceptance of utility ideas by decision-makers pose difficulties. Nevertheless, in many cases the comparison between different policies may give us an approximate size of the utility value.

If the reward function denotes the utility, then the optimality criterion is a linear additive function that represent the preferences of a risk neutral decision-maker. For changing these preferences, the decision-maker might take into account both the optimality criterion and its variability.

3.4. Deterministic Policies

When the reward function is used with an expected criteria, then this allow to find optimal policies that are deterministic. It is only true if assumptions concerning the decision maker are fulfilled. In case that optimality criteria change and the environment becomes uncertain, that could not be true.

Unfortunatly, we normally assume risky environments and stationary behaviour that afford us a good framework to develop practical models, but these situations do not happen very often. It is interesting to know the system behaviour in some controlled environments

4. Conclusion

In this work we have tried to show that MDPs could be used to formulate good models that mimics the dynamics in a farm. Its practical success is conditioned by the degree of agreement with the represented system. The model is only a simplified representation of the sow herd. Simplifications in the model are introduced throughout assumptions, so it is shown that not always they are fulfilled. When we work on farm specific models, the actual situation of the farm affects strongly the model suitability. And even it is very difficult to test about they appropriateness much more if we work on large models.

All these considerations should be taken into account when a DSS are wanted to be build in order to avoid mistakes. In the same way that calculators crash when an incorrect operation is introduced, DSS for on field use should crash, i.e. does not provide any output, when central assumptions fails.

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Chapter 7. GENERAL DISCUSSION

1. Introduction

Spain is achieving a leading position in pig production over recent times within the European Union, today is the second pig producer behind Germany (FAO, 2000). On the one hand it is a consequence of an increasing specialisation and concurrence among pig producers. On the other, particular organisation of Spanish pig sector that concentrates in few hands the decision-making and makes easier the adoption of new technologies. Then, the interest to develop advanced and efficient decision support tools in a computerised framework has increased. In this context the purpose of this Thesis has been to formulate and implement a mathematical model representing sow herd management strategies on production, reproduction and replacement. The formulation of the model has been made using a dynamic stochastic approach, a semi-Markov decision model or its equivalent embedded Markov decision model. Also, the model has been included in a decision support system (DSS) integrated as an extension of a more general pig information system (BDporc®, 2000).

The thesis was started in Chapter 2 with a literature review of different methodologies applied until now in sow herd management modelling and to identify weak and strong points in the formulation of mathematical sow herd models. In Chapter 3 the general formulation of the model used and refined along the thesis has been introduced, as well as, programming techniques that will be applied later on have been described. In Chapter 4, the development of the model, its implementation and validation based upon individual real farm data have been presented. To allow and facilitate the use on-farm of the model by farm managers and advisers a DSS have been developed (Chapter 5). Moreover, we discuss the suitability of the model to support specific on-farm decisions and basic features of software implementation. The DSS is completed with the optimisation algorithm that provides optimal management rules for the replacement problem. Additionally, in the last Chapter (Chapter 6) a critical analysis about practical problems in the use of mathematical programming to optimise sow herd management strategies have been presented. That includes a brief discussion about the suitability of the steady-state approach.

To conclude the Thesis, this general discussion will focus on the overall insights obtained by the investigation that have been presented and discussed at the end of each chapter. We will finish present chapter pointing to future developments for improving current applications of the model and its mathematical formulation. The chapter ends with a summary of the main conclusions from the study.

2. The sow herd model.

2.1. General formulation

Sow farm modelling is intimately related with the reproduction process, which is a discrete process by nature. The reproduction process, discrete in time, is marked by well-known events like farrowing, weaning or mating. Consecutive events mark different reproductive states. Each time interval between events, i.e. each state, has their own distribution like gestation, lactation or oestrus interval. As well, the probabilistic laws of motion guiding transitions between reproductive states are also relevant to herd performances. In this context, farmers, advisers, technicians or managers review periodically sow performance and take decisions stage after stage[6] to control performance evolution and raise productivity to desired levels. Therefore, the review presented in Chapter 2 showed how models were formulated to represent sow farm production taking into account the discreteness of the process. That is, the usual approach to model the sow farm system was by discrete models that described the swine production cycle through several reproductive cycles or parities. Reviewed sow models were classified in simulation and optimisation models. Simulation models were more flexible, and they had in common the inclusion of some parameters determined in a random way during simulation runs. Simulation models dealt with variability more extensively and were able to follow changes in the herd over time, but validation was also more difficult. Essentially, they consisted of putting together different subsystems represented by a set of equations in a computer program. From methodological point of view, simulation models were the simplest approach. On the other hand, optimisation models were based on dynamic programming techniques and required major mathematical refinement, but their formulation previous optimisation is not validated.

Usually, optimisation techniques had been applied to farm models as single components and less frequently as a multi-component model (Kristensen, 1988). All those models included the replacement optimisation as main problem. At present, the traditional main problem related to dynamic programming applications, i.e. the problem of dimensionality, can be overcome many times with a suitable mathematical formulation. Surprisingly, computational and methodological advances have not been followed by an increasing utilisation in field conditions of computerised tools including complex mathematical approaches. Instead, models have been refined and applied to represent different situations like facility scheduling. Reasons for that could be the difficulty, common to operational research studies, in getting specific parameters and suitable data for validation.

In order to avoid several of the problems detected previously, a semi-Markov model, more precisely its equivalent embedded Markov decision model, was formulated (Chapter 3). The model has been implemented, developed, validated, refined and discussed along the Thesis. The model is basically focused on the reproduction management and replacement problems. The steady-state was assumed. Several reasons justify the election of such an assumption, results of steady-state herds constitute a basis for a sound comparison of strategies (Jalving, 1993), are fast to be calculated making easy the performance of analyses of sensitivity, and also, optimisation algorithms are based on them.

2.2. Validation

Validation of the proposed model has been considered central in the development of the Thesis to asses the model suitability of later applications in field conditions. Initially, several outputs of the model had been selected to validate the model. For example, observed and simulated indices (technical or economic) seemed similar and hence justified the validation (Chapter 3). However, considering the principal role that herd dynamics played in the model, sow herd distribution at equilibrium has been considered as criterion to validate the model. Moreover, such a criterion allowed a statistical test to compare the resemblance of both observed and simulated herd distributions over states.

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[6] A stage is understood as the time interval from decision to decision

The availability of a pig management information system (BDporc® system that held the Official Spanish databank) allowed the validation of the model under different practical conditions with individual farm data Results of the validation showed that near of the 50% of farms randomly selected fitted reasonably the model (Chapter 4) while some of remaining farms showed similar indices. Therefore, we proposed the application of the statistical test based on herd distribution previous to the use of the model to better determine the resemblance between the model and the farm to be represented Hence the test becomes important to avoid erroneous advises to users.

2.3. Advantages

Main advantages of the Markov decision model presented can be summarised as follows: (1) The original semi-Markov model represents more precisely sow production behaviour and its embedded Markov decision process has allowed the computationally faster calculation of the equivalent steady-state distribution. (2) The population structure at equilibrium (steady-state distribution) equals that calculated by stochastic simulation and can be compared at different points of time with observed herd distribution. When the χ^2 test is applied, tabulated α-values provide a quantitative measure of the model goodness-of-fit. Low α-values indicates that discrepancies between observed and simulated population data are likely. (3) The steady-state distribution allows the evaluation and comparison of different management strategies, as well as the performance of sensitivity analysis of critical parameters. (4) The flexibility of the Markov herd model allows to optimise replacement policies by using programming algorithms. The algorithm implemented and used was the policy iteration algorithm (Howard, 1971). (5) The Markov herd model can easily be used to explore hypothetical situations for educational purposes.

2.4. Disadvantages or limitations

During the development of the research, not all of the problems found could be overcome. Several disadvantages raised from the herd model, the main problem appeared when discrepancies between observed and simulated data might be due to different causes others than the own herd dynamics, i.e. feeding changes, disease outbreak, etc. Such situations express the inaccessibility of the steady-state distribution and hence other models have to be applied. Also, incorrect or missing farm data is the most frequent cause of discrepancies between simulated results and real data (Chapter 6). Hence, the use of a statistic test in conjunction with improved farm parameters estimates enhances the applicability of the model. An other problem is the risk-neutral criterion adopted in the steady-state approach beside the null variation in the outputs. It may be possible to have two different alternatives, different valued, but without idea about the uncertainty associated to them. Therefore simulation procedures could overcome such a problem.

This sort of disadvantages related to hypothesis assumed by the model is common to most of reviewed models (Kamp, 1999). Therefore is central to judge whether the model mimics the real system sufficiently enough to fulfil the purpose of represent sow farm management strategies on reproduction and replacement.

3. *Potential use of the model and future perspectives*

3.1. The inclusion of the model in a DSS

Formally, a DSS is an information system that supports the process of decision-making. So, we designed and implemented a DSS to held and handle the sow herd model defined previously for decision support on farm. These sort of information systems has been useful to provide less specialised users access to a complex mathematical model. The design and the implementation of the DSS was made to help farm managers and/or advisers for on-farm decision support, then formal aspects of the user's interface were important (Chapter 5). The user-friendly interface allowed the interaction between the model and the user. DSS inputs included technical and economic parameters. The DSS outputs included calculated results as technical and economic indices, herd structure, statistical test of fit and sensitivity analysis of some parameters. The availability of farm data during the phase of design allowed the assessment of the model's suitability. Moreover, the validation on-farm of the model allowed us to answer more fairly users' demands.

Despite of the stress put on developing a practical DSS, it is difficult, if not impossible, to embrace all productive situations in sow farms. More important is to prevent users from giving them bad advises and built reliable tools. The DSS presented here is a system that can be used to analyse production factors associated with reproductive and replacement management strategies of sow farms. To gain insight in the impact of different alternatives can be more important that have a value. Therefore, decision support is mainly based on a comparative stationary analysis performance from model outputs (Chiang, 1987). The user can explore and evaluate different reproductive and replacement management alternatives and therefore can anticipate problems and make the most appropriate decisions, i.e. with a major positive impact. For instance, sensitivity analysis allows the identification of the most important parameters that can reduce or improve the farm economic efficiency. In agreement with previous models litter weaned seems to be the most important parameter affecting the economic efficiency. Reducing the number of unproductive days like those before first service seems also to be important.

Technical and economic indexes are calculated to help users to understand the results from the mathematical model. In this sense, the use of technical indexes such as number of piglets weaned per sow per year, interval between farrowing, number of farrowing per sow per year and even the distribution of sows per state makes the DSS outputs more familiar to farmers and technicians and makes it easier to understand. By comparing farm indexes with other reference indexes, farmers can built their own production goal. Afterwards, exploring different alternatives by static comparative analysis or a sensitivity analysis allows the user to choose management strategies that will improve production performance. The knowledge generated by these simulations can be incorporated into the decision process.

3.2. Future perspectives

Future perspectives of the research are related to the improvement of the model formulation in terms of number of states considered, i.e. through the increase of available data to better estimate model parameters, or appending general information extracted from elsewhere, or representing each sow individually in the model, all that with the improvement of the user interface capabilities.

The improvement of the model formulation also can be achieved varying model assumptions that limit a larger applicability of the model. For instance, a non-homogeneous Markov decision model would be interesting to be investigated. Also, the stationarity of the model can lead to study elapsed period of convergence when changes in management are introduced.
Parameter estimation, obviously can be improved if we have more data, but also general knowledge could be incorporated in order to substitute lacking data. Alternative estimates could be useful, as for example those based on Bayesian methods.

Improvements on the model would revert on the DSS. At present, the stationary orientation of the DSS makes it inappropriate to study interactions and adjustments between variables because we are not concerned with dynamic adjustments. In that sense, the convenience for farm managers and advisors should be dealt carefully.

4. Main Conclusions

1. Markov and semi-Markov decision models are useful models to represent sow herd management strategies on production, reproduction and replacement.
2. The availability of data from individual farms has allowed the validation of the model in practical situations. Additionally, the validation has shown that the model can not be applied indiscriminately on any farm, previously it is needed to verify hypothesis assumed by the model to assert its right running for specific situations.
3. The partition of reproductive sow lifespan in states, as a part of the model formulation, has served to better represent and in a natural way the productive reality of specific farms.
4. When the model is applied to calculate population structure at equilibrium, it is not necessary that the transition matrix is time step constant. It is possible to work with transitions associated to natural transitions between states, easier to be estimated (embedded Markov process), and providing as well computational savings.
5. Computational savings obtained working with the embedded Markov model allowed us to evaluate management alternatives faster on production and reproduction. Also,

it allowed us to implement optimisation algorithms to solve replacement problems in a more efficient way.

6. The implementation of the semi-Markov model into a DSS has shown the potential on-farm use of the model. The development of the DSS facilitates the availability of complex models to less specialised users. Moreover, the solving capability of the replacement problem is a guideline to farm managers to take into account.
7. The integration of the DSS into a MIS facilitates the adoption of DSS by farm managers. It makes also easier the adaptation to real situations of the DSS becoming a stimulation to register more variables for improving answers and precision of the DSS.
8. The lack of data is the most limiting factor to formulate a more practical model. The appending of new information, like number of services, facilities, heat detection, pregnancy detection, etc, can greatly improve the model precision.
9. The interpretation of the results is not immediate. The insight into critical components by means of sensitivity analysis is often more important than the prediction of precise values. But, it is necessary the comprehension and analysis of results. The design of more sophisticated interfaces can improve this disadvantage.
10. Additionally, the sow herd model, as it is formulated, is well suited for different purposes with minimum changes. This flexibility results in a better understanding of the consequential effects of swine management and replacement strategies to improve the economical efficiency of the overall production system.

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