Information, Behavior, and the Design of Institutions

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Submitted to

Departament d'Economia i d'Història Econòmica

at

Universitat Autònoma de Barcelona

in partial fulfillment of the requirements for the degree of

Doctor in Economics

June 2012

Acknowledgments

There is a single author on the cover of this dissertation, but concluding it would not be possible without the help, support and contributions of many of my colleagues and friends. I take this opportunity to thank some of them.

First of all, I am deeply indebted to my advisor, Joan de Mart´. His suggestions and advice were always invaluable, pointing to the right direction and trusting me, more than I could ever ask for, to follow the right path. It was an unfortunate event, the loss of our dear Professor Toni Calvó that led to me being Joan's student. But at this point I feel extremely fortunate to have had this opportunity.

I want to thank Salvador Barbera, Matthias Messner and Larbi Alaoui for accepting to be memebrs of the dissertation committee. And especially Salvador, together with Enriqueta Aragones for writting reference letters for me and reading and commenting on my work. I have enjoyed my interaction with the program's faculty over the years and I have benefited from many helpful comments. In particular I want to thank Caterina Calsamiglia, Inés Macho, Carmen Beviá, Clara Pnsat´, Evi Pappa, Pedro Rey, Jordi Massó, Guillaume Haeringer and Xavier Vila.

The bureaucratic burden for PhD students is not high, but their willingness to deal with it is as low as it gets. I thank Angels, Merce, Conxa, and the rest of the department's staff for their help on these issues over the years.

During these years I was lucky enough to interact with faculty outside from other universities and countries. Out of these I am particularly grateful to David Austen-Smith, Wiola Dzuda, and the rest of the Kellogg MEDS faculty for their hospitality during my stay in Evanston. I also want to thank Antonio Cabrales for very useful conversations on my work and his help that made visit abroad feasible.

And now my fellow students... Thanks to Sabine for being the mess we all love her to be. Even if getting to know her better was the only thing I got out of my "extra" years in the program, it was totally worth it! I thank Pau, for being a colleague, a fellow band member, but most of all, an incredibly dear friend, and more so every year. Thanks to Orestis and Dimitris for being the much more than my "fellow greeks" and putting some necessary mediterranean color to our work. I thank Riste for showing me again and again, probably unconsciously, how there

is a way of facing life that I would like to follow. I am very happy to have shared many moments of hard work and many more of relaxation with Tommaso, Diego and my other classmates from these years.

Somebody recently asked me what fulfills me in life. That person's answer to the question was: his work. I enjoy my work very much and I am very proud of its results. But none of this would be possible without the love and support of many people I have met over the years. For this I want to thank them all. My answer to the question was: my friends.

Finally, for all their love, caring , understanding and affection, I thank my brother and my parents. I dedicate this work to them. I hope it makes more sense than the "atmominima"...

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Chapter 1

General Introduction

The object of study in this dissertation is the interaction among the design of institutions, the behavior of economic agents, and information. By institutions we mean the set of rules and procedures that govern the way individuals and other economic agents behave within the economy or other social structures. Two examples of institutions that are studied here are collective decision-making rules and markets for goods with limited supply (chapters 2 and 3).

Any given institution creates specific incentives for the participating agents that form their behavior. Because of strategic interdependencies among economic agents, it is not always clear how a particular institutional design may translate in to agents' behavior. The aggregation of agents' behavior sometimes leads to collective behavior that is not obvious ex ante (ch. 3). At a more elemental level, prediction of behavior requires a good model of how individuals think, form beliefs and act in strategic situations. This should not be assumed as given (ch.4).

When these interactions take place in an uncertain world, information becomes crucial. What information is available and it's value to agents depends on the particular rules of social interaction. Agents on the other hand act on the basis of available information and the shape it may have. But information has some special properties that make it different than other goods in the economy. The attempt to understand the role of these special properties in the problem of designing institutions is what connects the different parts of this dissertation.

1.1 The value of information in collective choice

We start in chapter 2 by looking at an old question: what is the value of information? We do so from the point of view of groups.

Although some individuals may derive utility from simply "consuming" information, it is generally accepted that its value lies in reducing uncertainty and allowing for better decision making. While it is clear that information can have a positive value to economic agents, it is not always clear how to compare the value derived from one information structure to that of another. In a seminal paper, Blackwell (1951) shows that only a partial order of information structures is possible. Subsequent literature achieves more complete orderings of information structures by focusing on particular families of decision problems. We follow a similar approach focusing on collective choice problems instead of individual decision problems.

We set up a fairly general model of collective decision making through voting. Attention is restricted to groups in which members share a common objective. We examine whether it is possible to compare information structures in terms of the expected aggregate value they offer the group. Our first result shows that such a comparison is possible in some cases, for any such like-minded group and any possible voting rule. Still, we show that the instances where such comparisons are possible are very limited. The set of information structures that can be compared is extended if one poses restrictions upon the profile of group members' preferences or the voting rule. We apply some of our results to a model in which information is endogenous.

1.2 Demand in markets with limited supply

In chapter 3 we turn our attention to a different institution: markets. Specific features of markets, that a priori seem unrelated to information, can have significant effects on how agents shape their beliefs about the available goods. Beliefs determine demand which may be affected in unexpected ways.

Some markets are characterized by limited supply. When this is the case, interdependencies are created among market participants' actions and outcomes: some agents can obtain access to these goods only if others choose not to. If furthermore actions are motivated by the available information, interesting strategic effects are observed. In particular, agents that manage to obtain goods in such markets may suffer a winner's curse. This induces some agents to ignore their private information and pass on opportunities that come in limited supply. They do so without actually observing others' actions. The simple knowledge that others may have priority over oneself allows for the necessary inferences. Agents' behavior in these environments leads to theoretical considerations that we explore in this chapter. We offer insights that are relevant to market design.

We look at a model in which agents can invest in a project with a limited number of available slots. Agents have incomplete information about the projects expected payoffs. Based on that, they must decide whether to invest in the risky project or take a safe outside option. Slots are assigned following an exogenous priority order. Low priority agents may face a winner's curse: if they choose to invest and obtain a slot in the project it must be that agents with higher priority choose not to do so. In equilibrium, only high priority agents choose to invest when their private information indicates they should. Low priority agents take the outside option independently of their private information. This feature of equilibrium is maintained when we look at variations of the model with priorities assigned by lottery or determined by a Bernoulli process. We perform relevant comparative statics and compare equilibrium outcomes of our simultaneous action model with the ones from a social learning model. Our analysis highlights unexplored links between market design features and the performance of such markets. In particular, agents' knowledge of the priority order affects both demand and efficiency. Furthermore, herding behavior occurs even in the absence of social learning.

1.3 An experimental study of strategic thinking

In this last chapter of the dissertation we take a step back. Instead of looking at a particular institution we switch the focus to individuals and the way they behave across different strategic situations.

The order and observability of actions in a game determine the informational inferences players can make. Intuition suggests that such inferences require a higher level of sophistication when they concern actions that are not directly observed, like in simultaneous action games, compared to sequential games where a player can observe others' actions before making decisions. This intuition contrasts with the assumption of full sophistication embodied in the Bayes-Nash equilibrium concepts. Informational cascades the winner's curse may depend on, respectively, the ability or inability to make such inferences.

We use a novel experimental design in which subjects play, both simultaneously and sequentially, a game in which either of these phenomena can occur. We find that, in accordance to our intuition, some subjects participate in informational cascades in the sequential game and suffer a winner's curse in the simultaneous game. "Level-k" thinking and "cursed equilibrium" are theories that have been proposed to explain why an individual may suffer from the winner's curse in common value auctions and other environments. Nevertheless, according to these theories the same individual could not participate in an informational cascade. Therefore, our results contradict the predictions of both classical and behavioral theories.

Chapter 2

Information and Collective Choice in Like-minded Groups

> *...information is itself a commodity, being both scarce and valuable, but it has properties that make it unique.*
>
> *Prof. K. Arrow, XXIII Barcelona GSE Lecture*

2.1 Introduction

Juries, FDA committees, hiring committees are all examples of groups of individuals that must make a decision: to convict or to acquit, to approve a drug or not, to hire a particular candidate. Furthermore, all members of these groups are *like-minded* because they share a common interest. If they had perfect information about the problem at hand they would all agree on what decision the group should take: acquit the innocent, reject a dangerous drug, hire an appropiate candidate for the job. Disagreement may arise because of imperfections in the information that the group members hold.

Groups can usually choose between a number of different sources of information that can reduce uncertainty about the outcome of their decision. Juries can hear different witnesses or admit specific evidence; an FDA committee can choose from an array of clinical trials for a drug; a hiring committee can ask for references from different sources, or have the candidate take specific tests. Given limited resources, these like-minded groups often face the choice of a specific source of information over another. Answering the question "what information source is better for the group?" is not straightforward. In this paper we show why not and under what conditions one can give a clear answer to that question.

It is well known that such an answer is not straightforward even for individual decision makers. They can not be hurt by any additional information, but the seminal work of Blackwell (1951) (4) shows that one cannot always rank two alternative sources of information. They can be ranked when one source A is equivalent to source B plus some noise. In that case source B is preferred because it is, unambiguously, a more precise statistic of the state of the world than A. But, in general, which one is better depends on the decision problem at hand. Subsequent literature has tried to provide partial answers by looking at specific families of decision problems. We further discuss these studies in the literature review section.

When one looks at the problem from the point of view of a group, the problem of aggregating preferences con icts with the comparison of informativeness provided by different sources. The focus of this paper is the understanding and resolution of these con icting interests from the group's point of view. Therefore, we set up a collective choice problem, general enough to encompass different economic situations. The group faces a binary choice problem. The collective choice problem is resolved through voting. All information is public. In such a model and with preferences being common knowledge there is no scope for strategic manipulation of votes. What makes the problem of ranking preferences over information harder for a group? The following example illustrates the relevant difficulties.

Example 1. *Consider a hiring committee comprised by two members: Anne and Bob. They face the choice of whether or not to hire a candidate for a job. The candidate may, or may not, be a good fit for the job. Any of the two possibilities is equally likely. The table in figure 2.1 gives Anne and Bob's valuations of any possible outcome of their decision.*

(Anne, Bob)		Candidates condition	
		Fit for job	Not fit
Committee's decision	Hire	$(1,1)$	$(0,0)$
	Do not hire	$(0,0)$	$\left(\frac{1}{4},\frac{2}{3}\right)$

Figure 2.1: Anne's and Bob's valuation of possible outcomes

Both members of the committee would agree on the best decision if they knew whether the candidate is appropiate for the job or not. Note also that while both agree on their valuation of making a wrong decision in either case, they also agree that the opportunity cost of not hiring a good candidate is higher than the opportunity cost of hiring an unfit candidate. That is, they both have a bias towards hiring. The bias is higher in Anne's case. Let us assume that unanimity is required in order to hire the candidate.

Before taking a decision they have the choice to either have somebody interview the candidate or have him take a test. Each of these procedures can give some additional information. To keep the example simple, suppose that the outcome of both procedures can be deduced to a binary noisy signal, in the form of a recommendation: 'hire' or 'don't hire'. This recommendation is public: the result of the interview or the test is common knowledge for both individuals. The tables in the following figures show the likelihood of each recommendation in each possible case, for each one of the two procedures.

Likelihood of recomendation		Candidates condition	
		Fit for job	Not fit
Recomendation	Hire	$\frac{1}{2}$	$\frac{1}{3}$
	Do not hire	$\frac{1}{2}$	$\frac{2}{3}$

Figure 2.2: Interview

Likelihood of recomendation		Candidates condition	
		Fit for job	Not fit
Recomendation	Hire	$\frac{3}{4}$	$\frac{1}{2}$
	Do not hire	$\frac{1}{4}$	$\frac{1}{2}$

Figure 2.3: Test

The question we examine is which of the two procedures gives a higher expected value to the committee. After receiving a recommendation, Anne and Bob update their beliefs about the candidate's fitness for the job. They use their *posterior to calculate their expected value from each of the two possible actions and vote for the action that gives them a higher expected value. It turns out that following an interview, both Anne and Bob would vote to hire the candidate. That is, a negative recommendation from an interview is not strong enough to overcome their bias towards hiring the candidate. On the other hand, if instead of an interview they use a test, Bob would vote according to the recommendation while Anne would still vote for hiring the candidate, regardless of the recommendation. Given that unanimity is required for a hire, the committee always hires the candidate after an interview, but only hires him if this is recommended after a test. We can thus calculate the expected social welfare for the committee after each of the two procedures:*

$$
\begin{aligned}
W(\textit{interview}) &= & \frac{1}{2}(1+1) + \frac{1}{2}\cdot 0 = & \quad 1 \\
W(\textit{test}) &= \frac{1}{2}\left[\frac{3}{4}(1+1) + \frac{1}{4}\cdot 0\right] + \frac{1}{2}\left[\frac{1}{2}\cdot 0 + \frac{1}{2}\left(\frac{1}{4}+\frac{2}{3}\right)\right] = & & \quad \frac{43}{48}
\end{aligned}
$$

Social welfare is higher when using the interview.

It is interesting to notice that the test is actually a more precise procedure. The posterior belief after a recommendation to hire is the same in both procedures. But a recommendation not to hire from the test gives a higher posterior belief that the candidate is not fit, compared to the same recommendation from an interview. This means that less precise public information has a higher social value.

In this example it is easy to see that this happens because the less precise recommendation does not affect the committee's decision either way: Anne's and Bob's bias is too strong and the recommendation is too weak. The strong recommendation from the interview may affect the vote of Bob but not the vote of Anne, since she is more biased. Still, given the decision rule here, this means that with the test, the final outcome may change, Anne does not want.

We do not examine how Anne and Bob decide whether to use an interview or a test. We just make the comparison between the expected value generated by each of these procedures.

In this example we perform the exercise of comparing two different sources of public information for a particular group. It is always possible to perform such a comparison for a given pair of information sources, a particular group and a particular decision rule. In the remainder of the paper we show when such a comparison is possible for different combinations of individuals valuations and decision rules. As in the example we focus on situations with a binary choice and two possible states of nature. Unlike the example, we do not limit the analysis to binary signals but allow for the public signal to have any general form. Groups are allowed to have any possible number of members (even infinite). We only restrict individual valuations to be such that under perfect information all individuals would agree on the optimal choice.

2.1.1 Results

We show first that it is possible to establish a partial order on information sources such that a particular source gives a higher value than another, for any possible profile of individual valuations and any possible voting rule. Alas, the cases where such an order is possible are limited. We show for instance that the only binary signal that dominates other binary signals in such a way is the perfectly revealing signal.

The partial order can be extended if one restricts the domain of possible individual valuations. In particular, if we restrict attention to groups that a priori receive the same expected value from both alternatives, it is possible to establish an extended partial order on information sources, for any possible voting rule. We give examples of parametric families of such sources that can be ordered according to some parameter. These include power distribution functions and exponential functions.

For any group, there may exist an optimal voting rule that maximizes social welfare. Such an optimal rule pinpoints a particular individual within the group as a decisive voter. If there exists an ordering on information sources according to the expected value they give to the decisive voter, then this order holds for the group if the optimal voting rule is applied.

Finally we use some of our results in an application of collective choice design. We look at a case where the group may obtain information before making a decision. This depends stochastically on how many of the group members want the group to receive this public information. We show how this demand for information depends on the voting rule used and we characterize the rule that maximizes the expected social value for the group. We then use the fact that some information structures can be ordered for any voting rule and perform comparative statics. It turns out that the more informative the available information structure, the more conservative is the optimal voting rule.

Throughout our analysis we assume that individual values from different outcomes represent the individuals' true valuations. This does not affect our analysis since we only perform comparisons of information structures without considering any mechanism for choosing a particular one among the available alternatives. For the cases covered in our first result, individuals would have no incentive to misrepresent their valuations: when information sources can be ordered in such a way, all group members agree with the group's ordering. When this is not possible, individuals could have incentives to misrepresent their valuations if this could affect the choice of information source. Such manipulation would not make any sense in order to affect the group's final choice: given binary choices and the type of preferences we consider, voting is a strategy-proof method. By misrepresenting their valuations the agents cannot obtain a better outcome.

2.1.2 Literature review

The seminal contribution to the literature on information structure comparisons is Blackwell (1951) (4). There, an information structure is more informative than another if it is preferred by any decision maker for any possible decision problem.This strong condition induces a partial order on information structures. A more comprehensive order is offered by Lehmann (1988) (15), who focuses on specific decision problems that are monotone, and on information structures that satisfy a monotone likelihood condition. Persico (2000) (19), Athey and Levin (2001) (1) and Jewitt (2007) (14) move in the same direction and extend Lehmann's ordering for more general classes of monotone problems. In a recent contribution, Cabrales, Gossner and Serrano (2012) (5) provide a complete order on information structures, based on a measure of entropy in the decision maker's beliefs, in a class of investment problems. Ganuza and Penalva (2010) (7) take a different approach than these papers. They provide an ordering that is not based on any class of decision problems. Instead, they order different information structures in terms of the variability of conditional expectations they generate. They use this order to study the incentives of an auctioneer to disclose information. Similarly to all these papers, the present one shares the aim of comparing different information structures. It differs in doing so from the point of view of a group of agents instead of a single decision maker.

A series of papers by Gersbach (1991 (9), 1992 (10), 1993 (11), 2000 (12)) study the value of public information for groups that face a collective choice problem. Through different examples it is shown that it is possible for public information to be harmful for even a majority of voters. There are important differences between our work and Gersbach's contribution. Gersbach's approach is valid for a more general family of collective choice problems, with multiple alternatives and no restriction on preferences, but it only considers comparisons between two extreme cases: perfect uncerainty or perfect information. Our institutional restrictions in the choice problem allow us to compare situations with different degrees of uncertainty

Messner and Polborn (2012) (17) and Strulovici (2010) (21) are two papers that look at the attitude towards experimentation of groups of individuals, that make decisions collectively. In both of them the setup is dynamic. In the first one, the group faces the option to take a decision immediately, or to wait to obtain more information. In the second one, the group decides through voting whether it wants to continue experimenting with a particular policy. Continuous experimentation allows voters to learn about the policy's effects on their welfare. Both papers focus on the choice of the collective decision making rule and on how it determines the degree to which group members learn. Similarly, one way to look at what we do in this paper is to think that the group is engaged in a one stage experimentation game in which experimentation can take different forms and we look at which type of experimentation offers the highest value to the group.

The study of collective choice by groups of individuals that share a common goal, in environments with incomplete information, goes back to Condorcet in the 18th century and was studied later as well by Marshak and Radner (1972) (16) in their theory of teams. More recently, Austen-Smith and Banks (1996) (2) showed that strategic considerations may not allow the correct aggregation of information in such groups. Feddersen and Pesendorfer (1997) (6) show that if the size of the group goes to infinity, information is correctly aggregated. Persico (2004) (20), Gerardi and Yariv (2008) (8) and Gershkov and Szentes (2009) (13) study how the incentives of group members to acquire information depend on the design of the decision mechanism. Bergemann and Välimaki (2005) (3) survey the literature on information acquisition in the context of committees and other mechanism design problems. In all of this line of the literature, any information that the agents have or may acquire is a priori private. This gives rise to particular strategic considerations on their part when making decisions on whether or not to acquire information, on how they communicate with others or on how to vote. All these are absent in our setting: attitudes towards information depend strictly on individuals' valuations and the design of the decision process, not on the possible existence of any private information.

The possible value of public information to a set of individuals is studied in Morris and Shin (2002) (18). In their setting, individuals are involved in a game where actions have strategic complementarities. They find that more public information is socially beneficial when agents have no private information. When this is not true, more public information may hurt society. In their paper decision making is decentralized. Public information can help agents make better decisions but may also serve as a coordination device. In our setting, decision making is centralized. Public information's value lies solely in its instrumental function in improving decision making by reducing uncertainty about the state of the world.

2.2 The model.

In this section we setup a model of collective choice. Any results we obtain concerning the possibility of ordering information sources refer to this model.

2.2.1 Like-minded groups.

Consider a set of agents I (possibly infinite) that have to choose, jointly, between two possible collective actions, $x \in X = \{0, 1\}$. These may represent, for example, two alternative policies, or two different candidates for a post. We may refer to these simply as the "low" and "high action". There is an unknown state of nature that can have two possible values, $\theta \in \{0, 1\}$. We may also refer to these as the "low" and "high" state respectively. Agents share a common prior regarding the state of nature. Let π represent the ex-ante probability agents assign to the state of nature being high: $\pi = Pr(\theta = 1)$.

Individual valuations depend both on the collective action x and the state of nature θ and are given by a function $u_i(x, \theta, t_i)$. Notice that we only assume that all individuals' utility functions depend on the same variables, namely x, θ and an idiosyncratic parameter $t \in T \subseteq \mathbb{R}$. This parameter represents the possible bias of an individual towards either one of the actions the group may take. In particular, let $\lambda_i(t) = \frac{u_i(1,1,t) - u_i(0,1,t)}{u_i(0,0,t) - u_i(1,0,t)}$ be an agent's *bias function*. That is the ratio of the opportunity cost from choosing the low action in the high state, over the opportunity cost from choosing the high action in the low state. We make the following assumption:

Assumption 1. *The bias function $\lambda_i(t)$increasing in t and* *for $t > t'$,*

$$\lambda_i(t) > \lambda_j(t')\,,\ \forall i, j \in I$$

What this says is that individuals of a higher type are more biased in favor of the high action. Furthermore, it states that it is possible to order individuals in terms of their bias simply by knowing their type t and without any further information about their valuations. We take advantage of this fact to lighten notation and we from now on refer to the bias of individual i simply as $\lambda(t_i)$, omitting the subscript for λ.

It is helpful for our analysis to assume that agents are distributed over the type space T following a distribution $\xi(t)$ which may be either continuous or discrete. All agents have equal mass. The total mass of agents is normalized to 1: $\int_T \xi(t)dt = 1$. In the case of a discrete distribution the integral should be substituted by a summation.

We shall further assume that all agents together form what we name a *like-minded group (LMG)*[1].

Definition 1. *A set of individuals I is a like-minded group if:*

$$u_i(\theta, \theta, t_i) > u(1 - \theta, \theta, t_i),$$

$$\forall\ \theta \in \{0, 1\},\ t_i \in T, i \in I$$

Assumption 2. *I is a like-minded group.*

As can be seen from the definition, all members of a LMG agree on what the best action is in each state of nature. This does not mean that members of a LMG always agree on what action the group should take. Given the uncertainty about the state of nature there may be disagreement resulting from individual biases in favor of the lower or higher action.

Tables 2.4 and 2.5 show two examples of 2-member groups. The numbers in the cells represent the value for each group member of given the action in a specific state. Group 1 is not a LMG. Agents disagree on the optimal choice in the high state. Group 2 is a LMG. Assuming the numbers represent the values of agents 1 and 2 respectively we can compute $\lambda_1(t_1) = 1 > \frac{1}{4} = \lambda_2(t_2)$, which means $t_1 > t_2$: agent 2 is biased towards the low action.

[1]The term 'committee' is often used in the literature to describe groups with such preferences. But besides preferences, the term also has connotations of relatively small groups and `committee members` are often assumed to possess private information which they are expected to aggregate. These characteristics are not present in our model and we therefore prefer this alternative term.

	$\theta = 0$	$\theta = 1$
$x = 0$	10, 10	10,8
$x = 1$	8, 8	8,10

Figure 2.4: Group 1 is not a LMG

	$\theta = 0$	$\theta = 1$
$x = 0$	1, 100	0, -10
$x = 1$	0, 20	1,10

Figure 2.5: Group 2 is a LMG

2.2.2 Information.

The group receives a public signal $s \in S \subseteq \mathbb{R}$ (common to all individuals) about the state of nature before taking a decision. In particular, the signal s is distributed according to a cumulative distribution function $F_\theta(s)$ on the set S of possible signals. From now on we refer to either the information structure (the pair $\{F_0(s), F_1(s)\}$) or the distribution (the unconditional distribution $F(s;\pi)$) interchangeably or simply by F. We use $f_\theta(s)$ to denote the probability density function for continuous signals and use the same notation for discrete signals, implying $f_\theta(s) = Pr(s|\theta)$ for such signals. We assume that the distribution satisfies the *monotone likelihood ratio property (MLRP)*.

Assumption 3. $\frac{f_1(s)}{f_0(s)} \geq \frac{f_1(s')}{f_0(s')} \Leftrightarrow s > s'$.

In other words, higher public signals imply that the high state of nature $\theta = 1$ is more likely.

For the moment we make no further assumptions on S. While we use integrals over subsets of S in the analysis that follows, and unless mentioned otherwise, results also hold in the case of a discrete signal space and proofs can be obtained by substituting integrals with summations.

2.2.3 Individual and Collective Choice.

Individual choice

Before setting up our model of collective choice it is useful to understand how individuals behave in such a setup. Or, in different words, by looking at "groups" of a single individual.

Let $\phi_i : S \to X$ be a decision rule for an individual agent i. Given a decision rule ϕ_i and an information structure F, an agent's ex ante expected value is:

$$\mathcal{U}_i(\phi_i, F, t_i) = \pi \int_{s \in S} u_i(\phi_i(s), 1, t_i) f_1(s) ds + (1 - \pi) \int_{s \in S} u_i(\phi_i(s), 0, t_i) f_0(s) ds$$

Now let $\hat{\phi}_i : S \times T \to X$ be the optimal decision rule i.e. the one that maximizes the individuals expected payoff given the public signal. It can be easily seen that the *MLRP* on the information structure implies that $\hat{\phi}_i(s, t_i)$ is a threshold function.

Lemma 1. *There either exists a threshold $\tilde{s}_i(t_i)$ such that:*

$$\hat{\phi}_i(s, t_i) = \begin{cases} 1, & s > \tilde{s}_i(t_i) \\ 0, & otherwise \end{cases}$$

or $\hat{\phi}_i(s, t_i)$ is constant.

Proof. All proofs of lemma's and propositions can be found in the appendix. □

In particular, the threshold $\tilde{s}(t)$ is defined as follows:

$$\tilde{s}(t) = \left\{ s \ : \ \frac{f_1(s)}{f_0(s)} \frac{\pi}{1-\pi} \lambda(t) \leq 1 \right\} \tag{2.1}$$

Furthermore, this threshold is decreasing in the agent's type.

Lemma 2.

$$\tilde{s}(t) \geq \tilde{s}(t') \quad for \quad t < t'.$$

At this point we make one last assumption on the set of possible signals that comes with no loss of generality, but makes sure that the threshold is always well defined:

Assumption 4. *There exists $\underline{s} \in S$, with $\underline{s} < s$, $\forall\, s \in S \setminus \{\underline{s}\}$, and $F(\underline{s}) = 0$. We further assume $\underline{s} = -\infty$ if and only if $(-\infty, s') \in S$ for some $s' \in \mathbb{R}$*

For the analysis that follows it may be useful to remark that given the way we define the threshold, $\tilde{s}(t_i)$ is the highest value of s such that i still prefers $x = 0$ over $x = 1$. This is particularly important in the case of discrete signals.

Collective choice

The group makes decisions using a voting rule in the following way: Action $x = 0$ is taken if a proportion of at least $q \in [0, 1]$ members of the group agree to take this action. Members of the group agree to take an action if it is the optimal decision they would take as individual decision makers[2]. Therefore it is given by the function $\hat{\phi}_i(s, t_i)$. Thus, the group's decision function is:

$$x(q, s) = \begin{cases} 1, & \int_T \hat{\phi}(s, t)\xi(t)dt < q \\ 0, & otherwise \end{cases}$$

Given the preferences, it is easily seen that after receiving the public signal the group divides into two ordered subgroups.

Lemma 3. *Given a public signal $\hat{s}$, there exists $\tilde{t}(\hat{s})$ such that:*

$$\hat{\phi}(\hat{s}, t) = \begin{cases} 1, & t > \tilde{t}(\hat{s}) \\ 0, & otherwise \end{cases}$$

Furthermore, $\tilde{t}(s)$ is decreasing in s.

Some individuals of a low type with a threshold $\tilde{s}$ above the received signal consider $x = 0$ optimal. The rest of the group, that is higher type individuals, consider $x = 1$ as optimal. The groups final decision will depend on whether the mass of the first of these subgroups is larger than the quota q. If it is, then according to the decision rule, the group takes action $x = 0$. If not, it takes action $x = 1$. If the voting rule is simple majority we know that the group's decision always coincides with the vote of the median voter. The following lemma generalizes this idea for all possible q-rules.

Lemma 4. *The decisive type is the policy type $t_d(q)$ that satisfies:*

$$\int_{(-\infty, t_d)} \xi(t)dt < q \ \ and \int_{(-\infty, t_d]} \xi(t)dt \geq q$$

The group's decision function can be written as follows:

$$x(q, s) = \begin{cases} 1, & t_d(q) > \tilde{t}(s) \\ 0, & otherwise \end{cases}$$

It is easy to see why this result holds: to know whether the group takes action 0 or 1 we just have to know whether an individual with the decisive type t_d prefers

[2]As was mentioned in the introduction, given preferences individual preferences, voting under any q-rule is strategy proof. This means that voting for one's preferred choice is a dominant strategy.

0 or 1. Given lemma 3, we know that if an individual of type t_d prefers $x = 1$, so will all individuals to his right (of a higher type) and following the decision rule, $x(q,s) = 1$. If the individual prefers $x = 0$, then so will all individuals to his left as well, and their mass is more or equal to q and therefore $x(q,s) = 0$. Note that the *decisive type* is defined in a way that depends on the distribution of types for a particular group and is always well defined.

Now set-up a model of collective choice for LMG's, in the following section we examine whether it is possible to order different distributions with respect to the value they provide to the group in this framework.

2.3 Comparing Information Structures

The value of information lies in the degree to which it allows an agent to take better decisions. Therefore, when comparing two information structures, say F and G, one is said to be more informative than the other when it allows the decision maker to expect a higher payoff from a decision under this information structure rather than the other. Making such a comparison for a specific decision problem is not complicated. But when the same comparison is made for a different information problem the previous ranking in terms of informativeness may not hold. A vast literature in economics and statistics has studied the properties of information structures that allow such comparisons and giving orderings that are valid for more or less general families of decision problems. In this section we attempt a similar exercise. Instead of looking at individual decision problems, we focus on a collective choice problem. When is an information structure better for the group than another one?

We attempt to answer this question in terms of the collective choice model setup in the previous section. Still, for this question to have a content one must define the value of information for a group. As in Gersbach (1991 (9); 1992 (10); 1993 (11); 2000 (12)), we choose to follow a utilitarian approach. We consider the sum of individual group members' valuations following the collective choice made under a particular information structure to represent the value of this information for the group.

Definition 2. *A group's value from an information structure F is:*

$$\mathcal{V}(\hat{\phi}_d, F, \pi, q) = \int_T \mathcal{U}_i(\hat{\phi}_d, F, \pi, t_i, q)\xi(t_i)dt$$

Having defined the value of information for groups we can now define informativeness for a group. Like in the literature concerning comparisons of information structures for individual decision makers, we consider a particular information structure to be more informative than another if it offers a higher expected value to the decision maker. In our case the decision maker is the group. Formally, we use the following definition:

Definition 3. *We say that F is* more informative for the group *than G and denote it as:*

$$F \geqslant_I G$$

if and only if:

$$\mathcal{V}(\hat{\phi}_d, F, \pi, q) \geq \mathcal{V}(\hat{\phi}_d, G, \pi, q)$$

2.3.1 Strong dominance

It follows from these definitions that how we rank two distributions depends not only on the their characteristics but also on the heterogeneity and distribution group members' preferences, and the decision rule used to make the collective choice. Appropriate restrictions on these objects could deliver an answer to our question but with a loss in generality. We first attempt to provide a more general answer. This is in terms of conditions on distributions that, if satisfied, give a ranking of distributions that does not depend on the group's characteristics and the decision rule used. The only essential assumption is that of the group being like-minded.

The following definition formalizes the type of relation among distributions we look for:

Definition 4. *We say that F* strongly dominates *G if $F \neq G$ and the conditions of Proposition 1 hold for F and G. We denote such a relationship as*

$$F \gg G$$

We obtain a partial order of distributions in terms of strict dominance. Our first main result gives the conditions that allow us to rank two distributions in such a way.

Proposition 1. *Let I be a like-minded group with $t_i \in T$. Let F, G be two information structures and $\hat{s}_H(k) = argmax\{\frac{h_1(s_H)}{h_0(s_H)} \leq k\}$ for $(H, h) \in \{(F, f), (G, g)\}$.*

Then,

$$F \gg G$$

if and only if, for all $k > 0$,

$$F_0(\hat{s}_F(k)) \geq G_0(\hat{s}_G(k)) \tag{2.2}$$

and

$$F_1(\hat{s}_F(k)) \leq G_1(\hat{s}_G(k)) \tag{2.3}$$

To understand the conditions in Proposition 1 notice that: $F_0(\tilde{s}_F(t_d(q))$ is the probability that the group takes the right decision ($x = 0$) when the state is $\theta = 0$, while $1 - F_1(\tilde{s}_F(t_d(q))$ is the probability of taking the right decision ($x = 1$) when the state is $\theta = 1$. Changing the voting rule changes the type of the decisive voter and therefore also $\tilde{s}(t_d(q))$. If the conditions in the proposition hold, then for any possible voting rule and any possible group or, intead, any possible value of $\tilde{s}(t_d(q))$, the probability of making the right decision in any of the two states of nature under F is higher than under G. In other words, using F reduces both type I and type II errors.

As we show next, these are very strong requirements for comparing distributions. In particular, if $F \geqslant_I G$ for any group, then it must also be true for single individuals. That is, any individual decision with preferences as the ones in our model agrees on the ranking of F and G. Recall that in the example of the introduction, although individuals would separately agree on one ranking, the ranking for the group was the opposite.

Furthermore, given that by using F instead of G, both types of errors are reduced, all members in a group are better-off when the group makes the decision under F. It would be therefore useless for any member of the group to miss-report his true valuation if that could affect the choice of information structure. Still, although everybody would agree on the best distribution for the collective choice, there does not have to be agreement with the group's final decision. For a given public signal from F there may still be some individuals supporting the high action and others supporting the low action.

We now give an example of two distributions F and G for which the conditions in Proposition 1 are satisfied and hence $F \gg G$.

Example 2. *Let F and G be two distributions over four different values: S_H =*

$\{s_H^1, s_H^2, s_H^3, s_H^4\}$, *for* $H \in \{F, G\}$. *The table in Figure 2.6 gives the complete description of the two distributions.*

	$\mathbf{F_0}$	f_0	$\mathbf{F_1}$	f_0	$\frac{f_1}{f_0}$		$\mathbf{G_0}$	g_0	$\mathbf{G_1}$	g_1	$\frac{g_1}{g_0}$
$\mathbf{s_F^1}$	**0.9**	0.9	**0**	0	0	$\mathbf{s_G^1}$	**0.3**	0.3	**0**	0	0
$\mathbf{s_F^2}$	**0.975**	0.075	**0.025**	0.025	$\frac{1}{3}$	$\mathbf{s_G^2}$	**0.8**	0.5	**0.15**	0.15	$\frac{3}{10}$
$\mathbf{s_F^3}$	**1**	0.025	**0.1**	0.075	3	$\mathbf{s_G^3}$	**1**	0.2	**0.5**	0.35	$\frac{7}{4}$
$\mathbf{s_F^4}$	**1**	0	**1**	0.9	∞	$\mathbf{s_G^4}$	**1**	0	**1**	0.5	∞

Figure 2.6: These distributions are such that $F \gg G$

To check whether the conditions of Proposition 1 are satisfied we calculate the pairs $\{\hat{s}_F(k), \hat{s}_G(k)\}$ *for different values of* k *and see whether the inequalities (2.2) and (2.3) hold.*

$$\{\hat{s}_F(k), \hat{s}_G(k)\} = \begin{cases} \{s_F^1, s_G^1\}\,, & k \in (0, \frac{1}{3}) \\ \{s_F^2, s_G^1\}\,, & k \in [\frac{1}{3}, \frac{3}{10}) \\ \{s_F^2, s_G^2\}\,, & k \in [\frac{3}{10}, \frac{7}{4}) \\ \{s_F^2, s_G^3\}\,, & k \in [\frac{7}{4}, 3) \\ \{s_F^3, s_G^3\}\,, & k \in [3, \infty) \end{cases}$$

The graph of the two distributions in figure 2.7 allows to easily check that the inequalities (2.2) and (2.3) hold for each one of these pairs.

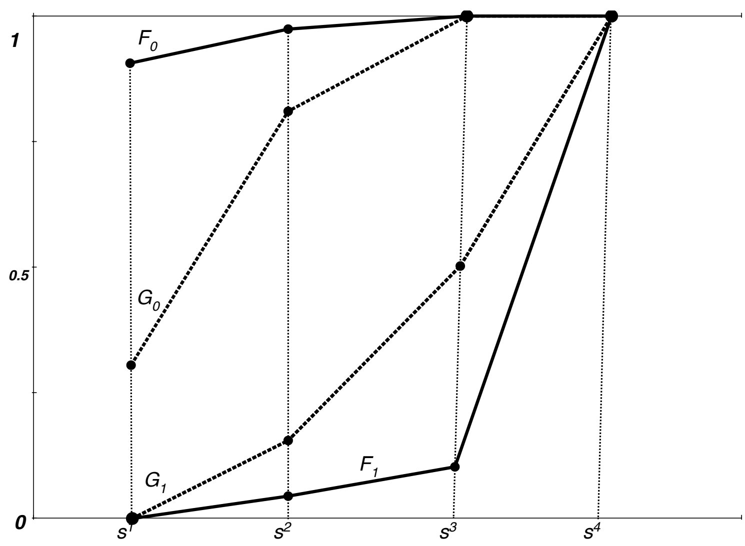

Figure 2.7: The cdf's for F and G.

It was mentioned already that the cases where a distribution strongly dominates another one are not common. Our next result re ects exactly that.

Proposition 2. *Let F, G be such that $F \gg G$.*

1. *If $G_1 = 0$ for some $s > -\infty$, then there exists $s_F \in S_F : Pr(\theta = 0|s_F) = 1$*
2. *If $G_0 = 1$ for some $s < \infty$, then there exists $s_F \in S_F : Pr(\theta = 1|s_F) = 1$*

To understand the statement in the proposition it is useful to note which possible forms of G are not included. If $S_G = \mathbb{R}$ and G is such that both $g_0(s_G)$ and $g_1(s_G)$ asymptotically tend to zero in both directions (as s increases or decreases), then Proposition 2 does not apply. If this is not true in either direction and $F \gg G$, then F must be such that the most extreme signal in the respective direction is fully revealing of the state.

As a consequence of Proposition 2, it turns out that the extend to which one can order distributions in terms of strict dominance is limited. Many situations where, intuitively, a certain information structure seems more informative than another are not included in this order.Furthermore, even some comparisons that are possible under the Blackwell criterion are excluded here. This becomes particularly striking when one considers comparisons among binary signals. The following corollary is an application of Proposition 2 to this case.

Corollary 1. *If $F \gg G$ and $S_H = \{s'_H, s''_H\}$, for $s'_H < s''_H$ and $H \in \{F, G\}$, then F must be fully revealing: $Pr(\theta = 0|s'_F) = Pr(\theta = 1|s''_F) = 1$.*

What is stated here is the following: a fully revealing signal is the only information structure that can strictly dominate a binary signal. This shows that the strict dominance relation is much stronger than any intuitive notion of informativeness one might have. Simply reducing the noise in a signal is not enough.

Why is this so? The best way to understand this is to return to example from the introduction. The signal from the interview is such that the group's choice is not affected by the signal's realization. The group always hires the candidate after an interview. This means that the group never makes the error of choosing not to hire when it should be hiring. Let's call this a type I error. It may make the error of hiring when it should not be doing it. This would be a type II error. A more precise signal, like the one from the test leads to a collective choice that depends on the realization of the signal. Therefore, after a test the group may commit a type I error with a positive probability. We know from Proposition 1 that under strict dominance both types of errors must be reduced. It turns out that it is always possible to construct such an example like the one in the Introduction with binary signals, unless the dominating signal is perfectly revealing. The same reasoning lies behind the more general result in the proposition.

One conclusion that we can draw at this point is that being able to make comparisons between distributions focusing only on their properties and without any restrictions on the groups preferences or decision rule is closer to an exception than to a rule. This gives rise to the question of whether one can extend this partial order by setting restrictions on either the group's preferences or the voting rule used. The next section deals with the first type of restrictions.

2.3.2 Restricted preferences

It is not hard to find pairs of distributions that satisfy inequalities (2.2) and (2.3) in Proposition 1 for a particular profile of group members' valuations. But these may no longer hold once we consider another group, containing more biased individuals. This happens, when the voting rule is such that one of these biased individuals becomes decisive. It is not hard to understand then, that if inequalities (2.2) and (2.3) hold for a particular group, they should also hold for any group where "extreme" biases are reduced. This idea is formalized in the following proposition.

Proposition 3. *Let F, G be two information structures and $\hat{s}_H(k) = argmax\{\frac{h_1(s_H)}{h_0(s_H)} \leq k\}$ for $(H, h) \in \{(F, f), (G, g)\}$. If:*

$$F_0(\hat{s}_F(k)) \geq G_0(\hat{s}_G(k)) \text{ and } F_1(\hat{s}_F(k)) \leq G_1(\hat{s}_G(k))$$

$$\forall\, k \in \left[\frac{1-\pi}{\pi}\frac{1}{\lambda(\bar{t})}\ ,\ \frac{1-\pi}{\pi}\frac{1}{\lambda(\underline{t})}\right]\ ,\ 0 < \underline{t} < \bar{t}$$

then

$$F \succcurlyeq_I G \quad \forall\, I\,,\ q\,,\text{ and } T \subseteq [\ \underline{t}\,,\ \bar{t}\ ]$$

Up to here, our results concern cases where one distribution is such that a decision can be made by the group in a way that the probability of making the correct decision is higher in any state of the world. We now turn to cases where this may not be true but a particular distribution still offers a higher expected value than another. To obtain such conditions we must make specific restrictions on the group's preferences. For these it is useful to define here a measure of the group's bias.

Definition 5. *The group's bias is:*

$$\Lambda(u, T) = \frac{\int_T [u_i(1, 1, t_i) - u_i(0, 1, t_i)]\xi(t_i)dt}{\int_T [u_i(0, 0, t_i) - u_i(1, 0, t_i)]\xi(t_i)dt}$$

Notice that the group bias is not the average of the group's members' biases. For instance, in the example of a LMG in Figure 2.5 the individual biases of the two group members are $\lambda(t_1) = 1$ and $\lambda(t_2) = \frac{1}{4}$. The average bias is then $\frac{5}{8}$. On the other hand, the group's bias, as defined here, is $\Lambda(u, T) = \frac{1+20}{1+80} = \frac{21}{81}$. That is, the magnitude of individuals' valuations matter.

It turns out that by restricting the group's bias to a particular value that depends on the prior belief about the state, one can find conditions that allow for a comparison between distributions that holds for any voting rule. These are given in the following result.

Proposition 4. *Let I be a like-minded group such that $\Lambda(u, T) = \frac{1-\pi}{\pi}$. Let F and G be two information structures and let $\hat{s}_H(k) = argmax\{\frac{h_1(s_H)}{h_0(s_H)} \leq k\}$ for $(H, h) \in \{(F, f), (G, g)\}$. Then*

$$F \succcurlyeq_I G\,,\ \forall\, q$$

if and only if,

$$F_0(\hat{s}_F(k)) - F_1(\hat{s}_F(k)) \geq G_0(\hat{s}_G(k)) - G_1(\hat{s}_G(k))$$

$$\forall\, k > 0$$

Example 3. *Here we give an example of a family of distributions that can be ordered in the way described in Proposition 4, assuming a uniform prior:* $\pi = \frac{1}{2}$. *Consider the family of power distribution distributions* $F(s;\alpha)$, *with* $F_0(s;\alpha) = 1-(1-s)^{\alpha}$ *and* $F_1(s;\alpha) = s^{\alpha}$, $s \in [0,1]$ *and* $\alpha \geq 1$. *This family satisfies the following properties:*

1. $F_0(s;\alpha) > F_0(s;\alpha')$ *and* $F_1(s;\alpha) < F_1(s;\alpha')$ *for* $\alpha > \alpha'$.

2. $\frac{f_1(\frac{1}{2};\alpha)}{f_0(\frac{1}{2};\alpha)} = 1$ *for all* $\alpha \geq 1$.

The following graph represents two distributions from this family with $\alpha > \alpha'$.

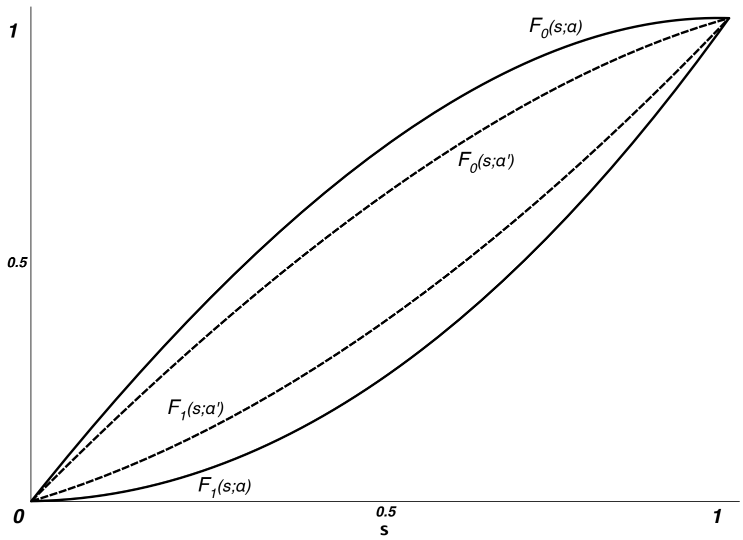

Figure 2.8: Two distributions from the power distribution family, with $\alpha > \alpha'$.

Let $\hat{s}(s) = \{s' : \frac{f_1(s;\alpha)}{f_0(s;\alpha)} = \frac{f_1(s';\alpha')}{f_0(s';\alpha')}\}$. *The following is true:*

- *From property 2 above we have that* $\hat{s}(\frac{1}{2}) = \frac{1}{2}$. *Then, one can observe from Figure 2.8 that*

$$F_0\left(\frac{1}{2};\alpha\right) - F_1\left(\frac{1}{2};\alpha\right) > F_0\left(\frac{1}{2};\alpha'\right) - F_1\left(\frac{1}{2};\alpha'\right)$$

- *For $s \in [0, \frac{1}{2})$ it must be $\hat{s}(s) < s$. Hence, it must be:*

$$F_0(s;\alpha) - F_1(s;\alpha) > F_0\left(\hat{s}(s);\alpha'\right) - F_1\left(\hat{s}(s);\alpha'\right)$$

- *For $s \in (\frac{1}{2}, 1]$ it must be $\hat{s}(s) > s$. Hence, again it must be:*

$$F_0(s;\alpha) - F_1(s;\alpha) > F_0\left(\hat{s}(s);\alpha'\right) - F_1\left(\hat{s}(s);\alpha'\right)$$

Summing up, the above means that the condition in Proposition 4 holds and thus, we can say that for a like-minded group with $\Lambda(u,T) = 1$ and a uniform prior, the family of power distributions can be ordered according to Proposition 4 in the following way: $F(s;\alpha) \geqslant F(s;\alpha')$ for all $\alpha > \alpha'$.

The previous example demonstrates that when restricting our attention to unbiased groups the scope for ordering distributions, in terms of informativeness for the group, increases. With the following example we demonstrate that, nevertheless, there are limits to the possibility of ordering distributions. The previous example may lead one to believe that state-wise stochastic dominance of the distributions[3] would be enough, but his is not true.

Example 4. *Consider two distributions F and G. The signal may take one of three possible values: $S = \{s^1, s^2, s^3\}$. The table in Figure 2.9 gives the complete description of the two distributions.*

	$\mathbf{F_0}$	f_0	$\mathbf{F_1}$	f_0	$\frac{f_1}{f_0}$	$\mathbf{G_0}$	g_0	$\mathbf{G_1}$	g_1	$\frac{g_1}{g_0}$
$\mathbf{s^1}$	**0.8**	0.8	**0.05**	0.05	$\frac{1}{16}$	**0.7**	0.7	**0.1**	0.1	$\frac{1}{7}$
$\mathbf{s^2}$	**0.95**	0.15	**0.4**	0.35	$\frac{7}{3}$	**0.85**	0.15	**0.5**	0.4	$\frac{8}{3}$
$\mathbf{s^3}$	**1**	0.5	**1**	0.6	12	**1**	0.15	**1**	0.5	$\frac{10}{3}$

Figure 2.9: These distributions can not be ordered, even when $\Lambda(u,T) = 1$

These two distributions can not be compared in the terms posited in Proposition 4. Suppose the group uses a voting rule such that $\lambda(t_d(q)) \in (\frac{3}{8}, \frac{3}{7})$. Then

[3] By this we mean the following: $F_0(s) > G_0(s)$ and $F_1(s) < G_1(s)$.

$\tilde{s}_F(t_d(q)) = s^2$ *and* $\tilde{s}_G(t_d(q)) = s^1$. *But in that case,* $F_0(s^2) - F_1(s^2) = 0.55$ *while* $G_0(s^1) - G_1(s^1) = 0.6$*, and the condition of Proposition 4 does not hold. The following graph depicts the cumulative distribution functions forboth F and G:*

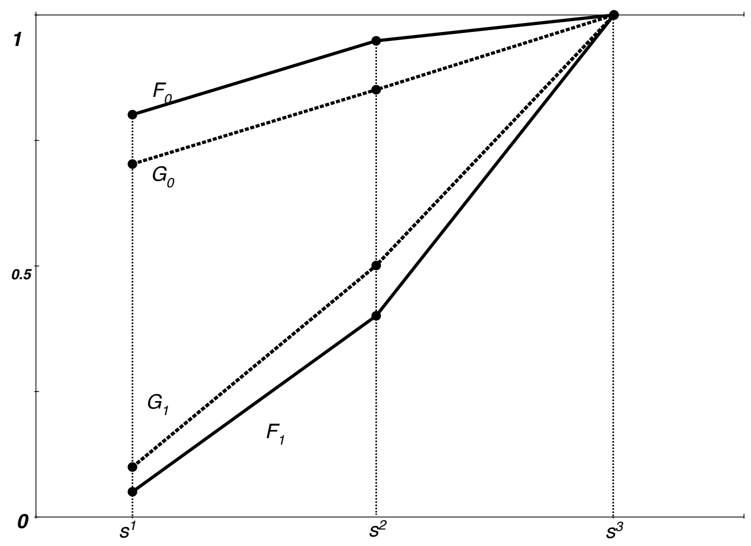

Figure 2.10: F and G can not be ordered for the group.

As can be seen from the graph, G_0 *stochastically dominates* F_0 *and* F_1 *stochastically dominates* G_1*. Yet, this is not enough to consider F more informative than G for any voting rule.*

Up to this point we have examined cases where an ordering of distributions is possible for any possible voting rule. In the remainder of this section we examine the possibility of ordering distributions when a particular voting rule is applied, and its role in the group's members demand for information.

2.3.3 The optimal voting rule

Our results show that it is possible to compare distributions without paying attention to the specific voting rule. Still, different voting rules affect the final outcome of the collective choice process. It makes sense then to ask whether there exists an "optimal voting rule", and if the answer is affirmative, what institutional elements determine it.

Our optimality criterion, in accordance with previous sections, is aggregate expected value maximization. As we show, it is possible for such an "optimal" rule to exist. Furthermore, it depends only on the profile of the group members' valuations.

Lemma 5. *The group's value is maximized for* $q^* = \{q : \lambda(t_d(q^*)) = \Lambda(u, T)\}$

According to Lemma 5 an optimal rule must be such that the bias of the decisive voter must coincide with the group's bias. In other words, the group's expected value is maximized if it makes decisions in the same way as an individual decision maker with the same bias would make them.

Whether such a member exists within the group is not guaranteed by any means. If group members form a continuous in the type space, then there should exist an individual with the required bias. On the other hand, when the set of agents is discrete there may not exist such a representative agent. The example of Anne and Bob in the introduction is such an instance. The group's bias is $\Lambda(u, T) = \frac{\frac{1}{4}+\frac{2}{3}}{1+1} = \frac{11}{24}$ and does not coincide with any of the two agents' biases because: $\lambda(t_{Anne}) = \frac{1}{4}$ and $\lambda(t_{Bob}) = \frac{2}{3}$. As a consequence, it seems easier to approach an optimal voting rule in large groups than in small ones.

2.3.4 The optimal voting rule and comparisons of information structures

Having defined the optimal voting rule is such way we apply it in the task of comparing distributions. This is done in the following result.

Proposition 5. *Let F and G be two information structures. Let* $q = q^*$. *If* $F \geqslant_d G$, *then* $F \geqslant_I G$.

According to this result, if the optimal voting rule exists and the decisive voter it defines is such that one distribution is more informative than the other for this individual, then the same is true for the group. The importance of this result lies in the fact that the use of the optimal rule not only maximizes the group's expected value. It also allows to use notions of informativeness established for individual decision problems to the group's problem. This means for instance that any set of distributions that can be ranked according to Blackwell's criterion for individuals, can be ranked in the same way for a group that uses an optimal voting rule. But Blackwell's criterion is not the only useful one. Our decision problem is such that when the group is comprised of a single individual, it falls within the class of monotone decision problems. Lehman (1988) gives criteria according to which distributions can be compared in the context of monotone decision problems. Under these same criteria it is possible to rank distributions that are not comparable according to Blackwell's notion of informativeness.

2.3.5 Group members' demand for information

We have looked at the comparison of information structures from two points of view: the group's and the individual's as a decision maker. Speaking in the terms of our introductory example, the first refers to the value of information to the group formed by Anne and Bob. The second refers to the value of information to either Anne or Bob in a case where one of them is deciding on his or her own whether or not to hire the candidate. The first is the main object of this paper. The second serves as a yardstick that allows us to measure the degree to which the notion of informativeness for the group departs from the notion of informativeness for individuals.

There is a third point of view though that may be useful when applying the concepts of this paper in economic models of collective choice: the value of information to individuals within the group. Again in terms of the example, this refers to the value of information to Anne when she must make a decision together with Bob. The next section considers an application where this value is relevant. In general it will may be relevant in cases where group members must take actions that in uence the choice of the information structure that is chosen to make the final decision. An example of this would be if group members are to vote on what source of information should be used.

Our next result characterizes this demand for information within the group. It should be no surprise that this demand depends on the voting rule that is used.

Let $\Gamma_q(F, G) = \{t : E_F[u(x(q, s_F), \theta, t)] \geq E_G[u(x(q, s_G), \theta, t)]\}$. That is, $\Gamma_q(F, G)$ is the set of the group's members that prefer distribution F over G or, in other words, all $i \in I$ such that $F \geqslant_i G$. The following lemma characterizes this set.

Lemma 6. *Let* $\{\underline{t}, \bar{t}\} = \{min\{T\}, max\{T\}\}$ *and* $\hat{t}(q) = \left\{t \;:\; \lambda(t) = \frac{1-\pi}{\pi} \frac{F_0(\tilde{s}_F(t_d(q)) - G_0(\tilde{s}_G(t_d(q))}{F_1(\tilde{s}_F(t_d(q)) - G_1(\tilde{s}_G(t_d(q))}\right\}$.

$$\Gamma_q(F,G) = \begin{cases} [\underline{t}, \bar{t}]\;, & F_0(\tilde{s}_F(t_d(q))) \geq G_0(\tilde{s}_G(t_d(q))) \;\; and \;\; F_1(\tilde{s}_F(t_d(q))) \leq G_1(\tilde{s}_G(t_d(q))) \\ [\underline{t}, \hat{t}(q)]\;, & F_0(\tilde{s}_F(t_d(q))) \geq G_0(\tilde{s}_G(t_d(q))) \;\; and \;\; F_1(\tilde{s}_F(t_d(q))) \geq G_1(\tilde{s}_G(t_d(q))) \\ [\hat{t}(q), \bar{t}]\;, & F_0(\tilde{s}_F(t_d(q))) \leq G_0(\tilde{s}_G(t_d(q))) \;\; and \;\; F_1(\tilde{s}_F(t_d(q))) \leq G_1(\tilde{s}_G(t_d(q))) \\ \emptyset\;, & otherwise \end{cases}$$

First, if F and G are such that given the voting rule there is a higher probability of taking the right decision in both states of the world under F, then all group members prefer the decision to be taken under F. Second, if the distributions are such that under F it is more likely to take the right decision in the low state but less likely in the high state, then there exists a type $\hat{t}$ that is indifferent between the two distributions, and all individuals to his left prefer F over G. This situation is reversed if under F it is more likely to take the right decision in the high state but less likely to do so in the low state. Finally, if a right decision in any state is more likely under G, then all agents prefer that distribution over F.

So, according to this result, group members are split: the ones of a lower type prefer one distribution while others of a higher type prefer the other one. Notice though that this division depends directly on the voting rule. Changing the voting rule not only moves the line of division. It can also lead to a switch of preferences for some group members.

This concludes our analysis of the possibility of comparing different distribution structures. In the following section we apply some of our results to a more structured collective decision problem. We do this to demonstrate the applicability of the tools we introduce in the study of collective decision making and the design of relevant institutions.

2.4 A model with endogenous information acquisition.

We now turn to an example of an application where we can make use of the ranking resulting from a comparison of information structures. In the model we consider, the group must make a decision. more information arrives if the group is willing to wait for it. Group members that want the group to obtain more information will try to prolong the waiting time. Others will press for an immediate decision. The more individuals press for either option, the more likely it is to happen. This is similar to the models of Gersbach (1992) (10) and Messner and Polborn (2012) (17) but with one big difference: there is no vote to decide whether or not to wait for more information. It is determined stochastically, depending on the proportion of group members that support this option.

As we know from our previous analysis, the demand for information in the group depends on the voting rule used. First we characterize the optimal voting rule for this setting. This differs from the optimal rule we described in the previous section where information was considered entirely exogenous. That is because now its effect on the demand for information, and hence on the likelihood of obtaining more information, must be taken in to account. The trade-off is the following. Positioning the voting rule away from q^* increases the demand for information within the group and therefore the likelihood to obtain more information before takingthe final decision. On the other hand, the final decision is not taken optimally any longer.

The optimal voting rule in this setting depends on the information structure that may provide the public signal. For some information structures that can be ranked according to Proposition 4, more informative distributions are associated with optimal rules for endogenous information that are further away from the optimal rule for exogenous information.

2.4.1 The model with endogenous information

To simplify the analysis, in this section we assume that I is a continuum distributed uniformly in the unit interval: $\xi(t) = 1$, $t \in [0, 1]$. Given this assumption we can economize on notation. In particular, note that now $t_d(q) = q$. This comes without any particular loss in generality, since we have not imposed any restrictions on $\lambda(t)$. We further assume that the group is unbiased, in terms of Proposition 4 : $\Lambda(u, T) = \frac{1-\pi}{\pi}$. This assumption is quite restrictive and not necessary in order to perform our analysis. Still, it lets us focus on the role of information when setting the optimal voting rule without having to bother about the group's bias or priors.

As mentioned before, the group faces two alternative scenarios: to have some additional information or none. Let the public signal be $\sigma \in \{\varnothing, s\}$, with $s \sim F(s)$. Again, in order to keep analysis tractable we focus on distributions with a continuous domain: $s \in [\underline{s}, \bar{s}]$. The following lemma characterizes the group's decision rule under no information:

Lemma 7. *The group's decision is given by*

$$x(q,\varnothing)=\begin{cases}1, & q>q^*\\ 0, & \textit{otherwise}\end{cases}$$

where

$$q^*=\left\{q\ :\ \lambda(q)=\frac{1-\pi}{\pi}\right\}$$

Let $\gamma(q)=\int_{t\in\Gamma_q}\xi(t)dt$ be the fraction of group members that prefer that the group receives public information before making a decision. According to Lemma 6, Γ_q takes the following form:

$$\Gamma_q(F,\varnothing)=\begin{cases}[o,\hat{t}(q)] & ,\ q>q^*\\ (\hat{t}(q),1] & ,\ q\leq q^*\end{cases}$$

where $\hat{t}(q)=\left\{t\ :\ \lambda(t)\frac{\pi}{1-\pi}\frac{F_1(\tilde{s}(q))}{F_0(\tilde{s}(q))}=1\right\}$. From this it follows that

$$\gamma(q)=\begin{cases}\hat{t}(q) & ,\ q>q^*\\ 1-\hat{t}(q) & ,\ q\leq q^*\end{cases}$$

It is important to note that given the definition of $\hat{t}(q)$ and the behavior of $\tilde{s}(q)$ with respect to q, the further away from q^* is q, the higher is the demand for information by group member's, captured by $\gamma(q)$.

Now we assume that whether or not the group receives a public signal depends on $\gamma(q)$. In particular we assume that there exists a function $\mu:[0,1]\rightarrow[0,1]$ such that $\mu\left(\gamma(q)\right)=Pr(\sigma=s)$. We assume μ is increasing in it's argument. One interpretation of this is that a contest ensues between group members: some exert effort to keep the group from deciding before information arrives, while others exert effort to accomplish the opposite. In this case, $\mu(\gamma)$ is the contest success function.

In such scenario, the aggregate utility of the group is given by:

$$W(q)=\mu\left(\gamma(q)\right)\mathcal{V}(\phi_d,F,\pi,q)+\left[1-\mu\left(\gamma(q)\right)\right]\mathcal{V}(\phi_d,\varnothing,\pi,q)$$

This is actually a piecewise function and depends on whether the voting rule q is higher or lower than q^*. We have: For $q\geq q^*$:

$$\begin{aligned}W(q) =& \pi \int_T u(1,1,t)dt + (1-\pi) \int_T u(1,0,t)dt \\ &+ \mu(\gamma(q))(1-\pi)F_0(\tilde{s}(q)) \int_T [u(0,0,t) - u(1,0,t)]dt \\ &- \mu(\gamma(q))\pi F_1(\tilde{s}(q)) \int_T [u(1,1,t) - u(0,1,t)]dt\end{aligned}$$

While for $q < q^*$:

$$\begin{aligned}W(q) =& \pi \int_T u(0,1,t)dt + (1-\pi) \int_T u(0,0,t)dt \\ &+ \mu(\gamma(q))\pi[1 - F_1(\tilde{s}(q))] \int_T [u(1,1,t) - u(0,1,t)]dt \\ &- \mu(\gamma(q))(1-\pi)[1 - F_0(\tilde{s}(q))] \int_T [u(0,0,t) - u(1,0,t)]dt\end{aligned}$$

To find the optimal voting rule one must solve the first order condition.[4] Again, given the piecewise nature of the function, we get two equations.
For $\mathbf{q} > \mathbf{q}^*$ we have the following[5]:

$$\frac{\partial \mu(\gamma(q))}{\partial \gamma}\frac{\partial \gamma(q)}{\partial q}F_1(\tilde{s}(q)) + \mu(\gamma(q))f_1(\tilde{s}(q))\frac{\partial \tilde{s}(q)}{\partial q} = \frac{\partial \mu(\gamma(q))}{\partial \gamma}\frac{\partial \gamma(q)}{\partial q}F_0(\tilde{s}(q)) + \mu(\gamma(q))f_0(\tilde{s}(q))\frac{\partial \tilde{s}(q)}{\partial q}$$

Notice that for $q \geq q^*$ we have $\gamma(q) = \hat{t}(q)$. That gives:

$$\mu(\gamma(q))f_0(\tilde{s}(q))\frac{\partial \tilde{s}(q)}{\partial q} + \frac{\partial \mu(\gamma(q))}{\partial \gamma}\frac{\partial \hat{t}(q)}{\partial q}F_0(\tilde{s}(q)) = \mu(\gamma(q))f_1(\tilde{s}(q))\frac{\partial \tilde{s}(q)}{\partial q} + \frac{\partial \mu(\gamma(q))}{\partial \gamma}\frac{\partial \hat{t}(q)}{\partial q}F_1(\tilde{s}(q))$$

[4]We do not check the second order condition. Uniqueness depends on particular functional forms but should in general not be an issue given the various monotonicity assumptions made so far. In any case, the first order condition characterizes the optimal rule, unless we have a corner solution which in this case would be unanimity. An interior solution is assumed in what follows.

[5]For $\mathbf{q} < \mathbf{q}^*$ we obtain a similar expression as the one we obtain here. The only difference is a negative sign in the RHS. Which of the two solutions represents the optimal voting rule depends on the specific functional forms for $\lambda(t)$ and μ. What interests us more here are the comparative statics that are of a similar nature in either case. We therefore focus analysis on this expression keeping in mind that the actual optimal voting rule may be characterized by the symmetric expression.

By implicit differentiation we get:

$$\frac{\partial \hat{t}(q)}{\partial q} = -\frac{\lambda(\hat{t}(q))}{\frac{\partial \lambda(\hat{t}(q))}{\partial t}} \frac{\partial \tilde{s}(q)}{\partial q} \frac{\left(\frac{F_1(\tilde{s}(q))}{F_0(\tilde{s}(q))}\right)'_s}{\frac{F_1(\tilde{s}(q))}{F_0(\tilde{s}(q))}}$$

Plugging this in to the last expression and simplifying we obtain the following:

$$\frac{f_0(\tilde{s}(q)) - f_1(\tilde{s}(q))}{F_0(\tilde{s}(q)) - F_1(\tilde{s}(q))} \frac{\frac{F_1(\tilde{s}(q))}{F_0(\tilde{s}(q))}}{\left(\frac{F_1(\tilde{s}(q))}{F_0(\tilde{s}(q))}\right)'_s} = \frac{\frac{\partial \mu(\gamma(q))}{\partial \gamma}}{\mu(\gamma(q))} \frac{\lambda(\hat{t}(q))}{\frac{\partial \lambda(\hat{t}(q))}{\partial \hat{t}}} \tag{2.4}$$

Let

$$\begin{aligned}\Delta(q) =& F_0(\tilde{s}(q)) - F_1(\tilde{s}(q))\\ =& F_0(\tilde{s}(q)) - [1 - F_1(\tilde{s}(q))] - 1\\ \propto& Pr(x = \theta|\sigma = s) - Pr(x = \theta|\sigma = \emptyset)\end{aligned}$$

That is, $\Delta(q)$ represents the improvement in the likelihood of making the correct decision after receiving a public signal, given q. Then 2.4 can be written as:

$$\frac{\Delta'_s(q)}{\Delta(q)} \frac{\frac{F_1(\tilde{s}(q))}{F_0(\tilde{s}(q))}}{\left(\frac{F_1(\tilde{s}(q))}{F_0(\tilde{s}(q))}\right)'_s} = \frac{\frac{\partial \mu(\gamma(q))}{\partial \gamma}}{\mu(\gamma(q))} \frac{\lambda(\hat{t}(q))}{\frac{\partial \lambda(\hat{t}(q))}{\partial \hat{t}}} \tag{2.5}$$

Expression (2.5) characterizes the optimal voting rule in this setup. It captures the trade-off we have described between optimal decision making and increased demand for information. The first is captured by the first term in the LHS which represents the proportional change of $\Delta(q)$. The other terms capture the change in demand for information. In particular, the RHS together with the second term of the LHS can be thought as the elasticity of the likelihood of receiving the public signal with respect to the the proportion of group members that want the public signal. This proportion depends on the individuals' biases, captured by the last term in the RHS and the form of the available information structure, captured by the second term in the LHS.

First note that for $q = q^*$ as defined in lemma 5, $\frac{\Delta'_s(q)}{\Delta(q)}$ must be zero,[6] since $\Lambda(u, T) = \frac{1-\pi}{\pi}$. Given that the terms in the RHS are strictly positive by definition,

[6] Rember the definition of $\tilde{s}$ given in lemma 2.1

q^* is not a solution to the equation in (2.5). Remember that $\tilde{s}$ is decreasing in t and therefore also in q. This implies, the solution in this case must be some $q^{**} > q^*$. This assures that $f_0(s) > f_1(s)$ which makes the LHS positive.

The optimal rule in this model with endogenous information is more conservative than the optimal rule when information is exogenous. To understand this, notice that q^* is such that the decisive voter must have a bias $\lambda(t_d) = \frac{1-\pi}{\pi}$ which is decreasing in π. This means that the decisive voter under q^* is biased towards the action that matches the state that is a priori. What we show here is that since $q^{**} > q^*$ it must be that

$$\lambda\left(t_d(q^{**})\right) > \lambda\left(t_d(q^*)\right)$$

Hence, the bias of the decisive voter according to q^{**} is closer to the state favored by the prior, compared to the decisive voter under q^*. The case of a uniform prior ($\pi = \frac{1}{2}$) offers a good illustration of this. In that case, $q^* = \frac{1}{2}$, which is simple majority. Then, the optimal rule in this model with endogenous information is some kind of super-majority.

2.4.2 Comparative statics with respect to information

How does the optimal voting rule here change for more informative public signals? It is at this point where Proposition 4 is useful. It allows us to determine whether a particular information structure is more informative than another for any voting rule. Thus, if distributions that satisfy the conditions in this proposition are considered, informativeness remains exogenous to the model and comparative statics with respect to information make sense.

We do this here using the family of power distributions defined in example 3. As we show, this family satisfies the conditions of Proposition 4. In particular, $F(s;\alpha)$ becomes more informative for higher values of α. The question is then, how does q^{**} change as α increases?

Remember that the family of distributions we consider is such that $F_0(s;\alpha) = 1-(1-s)^\alpha$ and $F_1(s;\alpha) = s^\alpha$. Plugging this in to the LHS of 2.5 gives:

$$\frac{s^\alpha}{1-(1-s)^\alpha}\frac{\frac{s^\alpha}{1-(1-s)^\alpha}}{\frac{s^{\alpha-1}\alpha}{1-(1-s)^\alpha}-\frac{s^\alpha(1-\alpha)^{\alpha-1}\alpha}{(1-(1-s)^\alpha)^2}}$$

This can be shown to be decreasing in α. It follows that q^{**} must be increasing in α. In other words, The better the possibly available information, the more conservative should the voting rule be.

When one obtains similar results performing this exercise with other families of distributions that can be ordered according to Proposition 4. Whether the is a more general relation between informativeness and the comparative statics on the voting rule in this model remains as a research question for the future.

2.5 Conclusions

We ask the question of whether an ordering of information structures is possible for a group of like-minded individuals. We answer by saying that only a partial, and very limited, order is possible in general. It can be extended if more restrictions are put on the group's profile of preferences.

What is important to understand is that the notion of "better information" as we understand it for individuals, even in its most restrictive form, as formalized by Blackwell, can not be applied to groups. Even if group members are like-minded, in that they agree on what should be done in a given state of the world. The reason is that uncertainty introduces disagreement. More precise information does not guarantee more agreement. This is why it may be better for the group to be "less informed".

There is an aspect of information we ignore throughout the analysis: it usually comes at some cost. In particular, more precise information is usually more costly. The omission of the costs from acquiring information in our analysis is intentional. It aims at emphasizing how the collective decision making process generates frictions that can reduce the value of information without even when there are no other costs to pay. Having established that, the question of how should a group proceed to acquire public information when it is costly, becomes particularly interesting. Public information has some of the characteristics of public goods. But as we show here it may also be a public "bad" for a subset of group members. What mechanism should be used to elicit individuals' valuations and choose among different sources of information and finance it's acquisition? What is the role of the voting rule in such a mechanism? These questions are left as future research.

2.6 APPENDIX: Proofs

Proof of Lemma 1. Agent i chooses $x = 1$ if:

$$\frac{f_1(s)\pi}{f(s;\pi)}u(1,1,t_i) + \frac{f_0(s)(1-\pi)}{f(s;\pi)}u(1,0,t_i) > \frac{f_1(s)\pi}{f(s;\pi)}u(0,1,t_i) + \frac{f_0(s)(1-\pi)}{f(s;\pi)}u(0,0,t_i)$$

$$\frac{f_1(s)}{f_0(s)}\frac{\pi}{1-\pi}\lambda(t_i) > 1 \tag{2.6}$$

Given the MLRP there must either be a threshold value $\tilde{s}_i(t_i)$ such that the inequality holds for $s > \tilde{s}_i(t_i)$ proving the first part of the lemma, either it always or never holds, and $\hat{\phi}_i(s, t_i)$ is constant. □

Proof of Lemma 2. This follows directly from inequality 2.6 in the proof of lemma 1 and the monotonicity assumption on $\lambda(t)$ □

Proof of Lemma 3. It follows from the monotonicity of $\hat{\phi}(s,t)$ with respect to t. □

Proof of Proposition 1. From the definition of $F \geqslant_I G$ we get the following inequality:

$$F_0\left(\tilde{s}_F(t_d)\right) - G_0\left(\tilde{s}_G(t_d)\right) \geq \frac{\pi}{1-\pi}\Lambda(u_i, \xi(t))\left[F_1\left(\tilde{s}_F(t_d)\right) - G_1\left(\tilde{s}_G(t_d)\right)\right] \tag{2.7}$$

where $\Lambda(u_i, \xi(t)) = \frac{\int_T [u_i(1,1,t_i) - u_i(0,1,t_i)]\xi(t_i)dt}{\int_T [u_i(0,0,t_i) - u_i(1,0,t_i)]\xi(t_i)dt}$ represents the group's bias.

Also, from the definition of $\tilde{s}(t)$ it follows that:

$$\frac{h_1(\tilde{s}(t))}{h_0(\tilde{s}(t))} \leq \frac{1-\pi}{\pi}\frac{1}{\lambda(t)}, \ \forall h \in \{f, g\}$$

The RHS of this inequality is always positive and decreasing in t. Then, for $\hat{t}(k) = \{t : \frac{1-\pi}{\pi}\frac{1}{\lambda(t)} = k\}$ it must be that $\tilde{s}(\hat{t}(k)) = \hat{s}(k)$. Thus, if the inequalities stated in the second part of the proposition hold for any k, they must also hold for any $\tilde{s}(\hat{t}(k))$. In such case, then the LHS in (2.7) is positive and the RHS is negative. This proves the sufficiency of these conditions for the informativeness relationship stated in the proposition.

We prove the "only if" part of the proposition by showing that if the inequalities do not hold, one can always two instances of groups and voting rules such that the informativeness ranking between F and G is different in each case. Consider a group $I = \{1, ..., n\}$ and $q = 0$. Suppose $t_1 < t_2 < ... < t_n$. Then we have

$t_d(q=0)=t_1$. Individuals' preferences are as follows:

$$
\begin{aligned}
u_i(1,1,t_i) &= t_i \\
u_i(0,0,t_i) &= 1 \\
u_i(1,0,t_i) &= u_i(0,1,t_i) = 0
\end{aligned}
$$

for all $i \in I$. If $F \succcurlyeq_I G$, then it must be that:

$$F_0\left(\tilde{s}_F(t_1)\right) - G_0\left(\tilde{s}_G(t_1)\right) \geq \frac{\pi}{1-\pi}\frac{\sum_I t_i}{n}\left[F_1\left(\tilde{s}_F(t_i)\right) - G_1\left(\tilde{s}_G(t_i)\right)\right] \tag{2.8}$$

Suppose $F_0\left(\tilde{s}_F(t_1)\right) \geq G_0\left(\tilde{s}_G(t_1)\right)$ and $F_1\left(\tilde{s}_F(t_i)\right) \geq G_1\left(\tilde{s}_G(t_i)\right)$. It is clear that inequality(2.8) may hold for $\sum_{i\neq 1} t_i$ low enough, but not for $\sum_{i\neq 1} t_i$ above a certain threshold. The same argument can be made for $F_0\left(\tilde{s}_F(t_1)\right) \leq G_0\left(\tilde{s}_G(t_1)\right)$ and $F_1\left(\tilde{s}_F(t_i)\right) \leq G_1\left(\tilde{s}_G(t_i)\right)$. This proves the necessity of the inequality conditions stated in the proposition in order for $\succcurlyeq_I$ to be valid for any I and any q. □

Proof of Proposition 2. We show the result for point 1. Point 2 follows from a symmetric argument. We show that given $F \gg G$ and $\frac{g_1(s)}{g_0(s)} > 0\,,\ \forall s \neq \underline{s}_G$, it must be $\frac{f_1(s)}{f_0(s)} = 0$ for some $s \neq \underline{s}_F$. Suppose not. Then $\inf\{\frac{f_1(s)}{f_0(s)}\} > 0$. There are two cases to consider:
<u>Case 1:</u> $\inf\{\frac{f_1(s)}{f_0(s)}\} > \inf\{\frac{g_1(s)}{g_0(s)}\}$.
Then there exists I such that for some q', $\frac{\pi}{1-\pi}\frac{1}{\lambda(t_d(q'))} \in \left(\inf\{\frac{g_1(s)}{g_0(s)}\}, \inf\{\frac{f_1(s)}{f_0(s)}\}\right)$. From the definition of $\tilde{s}(t)$, this means that:
$\tilde{s}_F(t_d(q')) = \underline{s}_F$ and $arg\inf\{\frac{g_1(s)}{g_0(s)}\} \leq \tilde{s}_G(t_d(q')) \geq \sigma$. This in turn means that $F_0(\tilde{s}_F(t_d(q'))) = F_1(\tilde{s}_F(t_d(q'))) = 0$ and $G_0(\tilde{s}_G(t_d(q'))) > 0$. This inequality holds because given our assumptions, $g_0(\tilde{s}_G(t_d(q')$ must be positive. If s_G is discrete in $[\,arg\inf\{\frac{g_1(s)}{g_0(s)}\}, \sigma]$ the inequality follows directly. If it is continuous in some interval then it must be true as well for the supremum of that interval. Notice then that $F_0(\tilde{s}_F(t_d(q'))) = 0 < G_0(\tilde{s}_G(t_d(q'))) > 0$ which violates $F \gg G$.
<u>Case 2:</u> $\inf\{\frac{f_1(s)}{f_0(s)}\} < \inf\{\frac{g_1(s)}{g_0(s)}\}$.
Then there exists I such that for some q', $\frac{\pi}{1-\pi}\frac{1}{\lambda(t_d(q'))} \in \left(\inf\{\frac{f_1(s)}{f_0(s)}\}, \inf\{\frac{g_1(s)}{g_0(s)}\}\right)$. From the definition of $\tilde{s}(t)$, this means that:
$\tilde{s}_G(t_d(q')) = \underline{s}_G$ and $arg\inf\{\frac{f_1(s)}{f_0(s)}\} \leq \tilde{s}_F(t_d(q')) \geq \sigma'$. This in turn means that $G_0(\tilde{s}_G(t_d(q'))) = G_1(\tilde{s}_G(t_d(q'))) = 0$ and $F_1(\tilde{s}_F(t_d(q'))) > 0$, for the same reason as with G_0 in case 1. Again this violates $F \gg G$. Thus for $F \gg G$ to be true when $\frac{g_1(s)}{g_0(s)} > 0\,,\ \forall s \neq \underline{s}_G$ it

must be $\frac{f_1(s)}{f_0(s)} = 0$ for some $s \neq \underline{s}_F$. This means that for that s $Pr(\theta = 0|s_F) = 1$. □

Proof of Proposition 3. Given the conditions in the proposition, inequality 2.7 must hold for any q, as long as $t_d \in [\underline{t}, \bar{t}]$. The restriction on T makes sure of that and thus the result holds. □

Proof of Proposition 4. The result follows directly by rearranging inequality (2.7). □

Proof of Lemma 5. Solving the FOC for the aggregate utility we obtain: $\frac{f_1(\tilde{s}(t_d(q^*)))}{f_0(\tilde{s}(t_d(q^*)))} = \frac{1-\pi}{\pi}\frac{1}{\Lambda(u_i,\xi(t))}$.
Combining this with inequality 2.6 in the proof of lemma 1 gives the definition of q^*. Monotonicity with respect to Λ follows from monotonicity of $t_d(q)$ and the MLRP. □

Proof of Proposition 5. From Lemma 5 we have $\lambda\left(t_d(q^*)\right) = \Lambda\left(u_i, \xi(t)\right)$. Plugging this into inequality (2.7) gives:
$F_0\left(\tilde{s}_F(t_d(q^*))\right) - G_0\left(\tilde{s}_G(t_d(q^*))\right) \geq \frac{\pi}{1-\pi}\lambda\left(t_d(q^*)\right)\left[F_1\left(\tilde{s}_F(t_d(q^*))\right) - G_1\left(\tilde{s}_G(t_d(q^*))\right)\right]$.
Since we assume $F \succcurlyeq_i G$, $\forall\, i \in I$, this inequality must hold, proving the point. □

Proof of Lemma 6. Note that:

$$E_F[u(x(q,s_F),\theta,t)] \geq E_G[u(x(q,s_G),\theta,t)]$$
$$F_0(\tilde{s}_F(t_d(q)) - G_0(\tilde{s}_G(t_d(q)) \geq \frac{\pi}{1-\pi}\lambda(t)[F_1(\tilde{s}_F(t_d(q)) - G_1(\tilde{s}_G(t_d(q))]$$

For $t = \hat{t}(q)$ this expression holds with equality, i.e. the individual of type $\hat{t}(q)$ is the one that is indifferent between the two distributions. The definition of $\Gamma_q(F,G)$ follows from the monotonicity of $\lambda(t)$ with respect to t. □

Proof of Lemma 7. Note that $t_d(q) = q$. We know that $x(q,\emptyset) = 1$ if

$$E[u_d(1,\theta,q)] > E[u_d(0,\theta,q)]$$
$$\pi u_d(1,1,q) + (1-\pi)u_d(1,0,q) > \pi u_d(0,1,q) + (1-\pi)u_d(0,0,q)$$
$$\lambda(q) > \frac{1-\pi}{\pi}$$

Remember that q^* is such that $\lambda(q^*) = \Lambda(u,T)$, and by assumption $\Lambda(u,T) = \frac{1-\pi}{\pi}$. This prooves the lemma □

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Chapter 3

Standing in Line: Demand for investment opportunities with exogenous priorities

A key factor in the organization of the economy is the set of beliefs that people have about each other. They change those beliefs by searching, by computing, by analyzing, and when looked at properly, this gives rise to some considerable anomalies when compared with the standard theories that I and many others have developed

Prof. K. Arrow, XXIII Barcelona GSE Lecture

3.1 Introduction

Markets for goods or investment opportunities are often characterized by limited supply: opportunities for micro-investments, initial private offerings (IPO's), offers in the housing market, and job offers in the labor market are some examples. When this is the case, interdependencies are created among market participants' actions and outcomes: some agents can obtain access to these opportunities only if others choose not to. If furthermore actions are motivated by the available information, interesting strategic effects are observed. The following "down-to-earth" example should make the nature of these effects clear to the reader.

You come back home after work in the evening and notice an add in the morning's paper offering 10 "Clean-your-house Robots" at a very low price to the first 10 persons to send a free sms to a specific number. At first glance this offer seems appealing. But before sending the sms you think again: given that the add was in the morning's paper and many hours have passed since it was published, the only chance of winning a robot for a low price is if less than 10 persons have sent the sms already. This would happen only if, unlike yourself, the vast majority of readers that saw the add during the day thought that this robot is probably useless. Sending the sms will either get you nothing or if you get something it will most likely be a big piece of junk taking away precious space in your house. A winner's curse!

The possibility of suffering such a winner's curse (WC) may induce some agents to ignore their private information and pass on opportunities that come in limited supply. They do so without actually observing others' actions. The simple knowledge that others may have priority over oneself allows for the necessary inferences. Agents' behavior in such environments leads to theoretical considerations that we explore in this paper. We offer insights that are relevant to market design.

Crowdfunding markets[1] present environments such as the one in our model. For example, profounder.com is one of many wesites that provide a platform for entrepreneurs to obtain funding from micro-investors. There is a limit to the total number of individuals that may finally invest.[2] The entrepreneur obviously wants to maximize the number of potential investors. But the existence of the limit can give rise to the WC reasoning of the "robot" example: a potential investor may argue that if, in spite of the limit in the number of investors, he becomes one of them then it is because others choose not to invest. If they do so because, according to their information, the project is not worth it, then maybe it is better for him not to invest. If more investors argue the same way, then demand for investing in the project may turn out to be low, contrary to the entrepreneur's desires.

We model this situation as a simultaneous choice game where the WC effect is internalized at the same time by all decision makers. In particular, our model considers a set of agents that face the opportunity to invest in a project. There is a limit to the total number of agents that may invest. If the number of agents

[1] According to the MacMillan Open Dictionary, crowdfunding is the use of the web or another online tool to get a group of people to finance a particular project. The American Jobs Act, from the current US administration, includes plans to work with the SEC to review securities regulations in ways that expand crowdfunding opportunities.

[2] The limit is imposed by US law. But one can imagine that even without legal limits, and given an average size of investment, there is a economic limit to the initial funding any entrepreneur can handle

that choose to invest exceeds the number of slots in the project, then these are assigned according to an exogenous priority order. Agents do not observe the actions of others. Thus when deciding to invest or not they don't know whether agents with a higher priority have invested or not and thus whether there is any available slot for them. Specifically, one can imagine the situation as one in which agents (investors) stand in a line and decide simultaneously whether or not to invest. The decision is taken without knowing what other agents choose to do. After decisions are made, the planner (entrepreneur) goes to the first agent in line and asks him for his decision. She assigns him a slot in the investment if he chose to invest and moves on to the next in line. The process continues until all agents have been asked or no more slots are available. Payoffs depend on whether or not an agent is assigned an investment slot. They further depend on an unknown state of nature which determines the returns of the investment. In a "good" state investing gives a high payoff, while in a "bad" state it is better not to invest. Each agent also has some private information concerning the state. This comes in the form of a binary noisy signal which points to a good or a bad state.

The main ingredients for our model are incomplete information, a common value and the limited supply of investment opportunities. The latter makes other investors' decisions relevant for everybody else, or, in particular, for those that follow in the line. Without limited supply, the problem becomes a sum of individual decision problems, independent of each other, since inferences about others' behavior are unnecessary. Only when supply is limited can one argue that being able to invest means that others with a higher priority have not done so. Incompleteness of information and the common value turn this argument to the WC argument described above. Awareness of the WC drives equilibrium behavior in our model: individuals in the front of the line decide according to their private information; the ones that stand further back, ignore their private information and simply do not invest. This is because for the ones standing in front, whether they get a slot does not depend on what others do. The ones in the back can only obtain a slot if the ones in front choose not to invest. This restriction of the market allows them to make equilibrium inferences about the private information of the agents that stand in front of them in the line. If several agents in front choose not to invest it must be that their private information points to the "bad" state. In that case it might be better not to invest, even if one's own private signal points to a good state. Thus individual behavior in equilibrium depends on one's position in the line.

Once we understand individual behavior in our model we can see how other factors may affect the equilibrium in such a market. Anything that can affect the strength of the WC can have an impact. For instance the number of available slots: the WC argument's strength is different in the case of only one available slot compared to the case of 50 slots. For an agent in position 51 obtaining the single available slot is almost certainly a consequence of the investment opportunity being bad. In the second case, even when the state is good, it is enough for a single agent of the 50 preceding in the line to get a wrong signal for a slot to be available. Another interesting issue is the knowledge an individual has about his position in the line. This may not always be perfect and it has an impact on the number of agents that choose to play informatively (follow their signal) or herd (ignore their signal).

Notice that we assume no complementarities among investors' actions. Whether others invest or not does not affect the quality of the investment. It may simply reveal their private information. Thus, we have that factors such as the knowledge about the priority order and the size of the supply of investment opportunities, both unrelated to the quality of the investment and the investor's payoff from it, become determinant for the demand for the investment slots.

In the base-line model, we assume that agents know the exogenous priority order, that is each agent knows exactly his position in line. After a detailed analysis of this case we consider an alternative scenario where priorities are determined by a lottery. The realization of this lottery takes place after investment decisions are made. This scenario represents the other extreme: agents have no knowledge of where they stand in line. We also consider an intermediate case with a Bernoulli arrival process that generates a random assignment. In each period an agent arrives with a given probability. Each agent is aware of this process but does not know how many other agents have arrived before him. Still,the date of arrival gives him some idea about the distribution of this number that allows him to build an expectation of the probability of getting a slot.

We fully characterize equilibrium behavior in our model. When the position in the line is known, agents in the front choose whether or not to invest according to their private information. The ones further back ignore their private information and choose not to invest. Equilibrium in the Bernoulli arrival process model shares the same features. When the priority is set by a lottery, all agents choose not to invest with positive probability even when their private information indicates they should do so. Increasing the available slots affects agents differently, depending on their position in line. When an agent's position in line is greater than the available slots, but close to that, the increase in the number of slots reduces the WC effect and makes investing more attractive. The contrary is true for agents further back in line. With high uncertainty about priority the final direction of the effect depends on the specific parameters.

3.1.1 Literature Review

Rock (1986) (29) studies the market for IPO's which is an example of a market with incomplete information and limited supply. He uses a "lemons market" type of model to explain the underpricing of initial public offerings (IPO's). In his model uninformed investors compete with informed ones. The first face a winner's curse since they know they can invest only if informed investors consider the offering price too high with respect to the expected market price. The issuer must therefore underprice in order to attract the uninformed investors. A significant body of empirical literature has followed, trying to verify this explanation of IPO underpricing (see Ljungvist, 2007 (22), for a survey). In our model we obtain the winner's curse is of a different nature. There is no asymmetry in information between agents. We show that it is the market design features that determine the strength of the curse. Our model does not share the aim of explaining IPO underpricing. Still, our results suggest that if such underpricing is due to a winner's curse effect, any empirical strategy trying to identify such effect must take into account the institutional settings of the market studied and possibly take advantage of any variation in these.

The paper by Thomas (2011) (32) shares with us the interest in studying markets with limited availability of different goods and incomplete information. However, there are several differences both in the approach and the kind of results obtained. Her paper examines the situation in which different agents acquire information about different alternatives through experimentation. The fact that some of these are limited in supply gives rise to strategic interactions when agents decide on the duration of experimentation. Still, agents here do not learn from one another, whether by observation or in equilibrium. The strategic incentives for choosing one alternative are of a preemptive nature. In our case it is equilibrium beliefs that push an agent to choose something contrary to his information. Furthermore, in Thomas' paper the priority over choices is endogenous. Agents decide when to stop experimenting and grabbing an option. We focus on exogenous priorities.

Given that the marketplace we study does not involve prices, the literature on matching markets is a natural place to look for parallelisms. One approach to matching markets looks at specific matching games. Perhaps the first attempt of such an approach has been the work of Becker (1973) (6). Within this literature and more recently, some papers have considered, as we do, environments with incomplete information and a common value. In particular, a paper that is closer to our work and that is the first to identify the type of winner's curse that in uences behavior in our model is the one by Lee (2009) (20). He looks at the decentralized college admissions market and finds a rationalization for the "early admissions" system on the basis of this curse. We focus on a centralized market and see how the use of a matching mechanism can create the curse. This induces herding on the part of some participants in order to avoid it. Another example is Chade (2006) (9). He looks at a decentralized marriage market and detects what he calls the acceptance curse. A participant can infer information by the event of being accepted by a partner at a given point in time. This acceptance may mean that one's value is higher than what one thought about oneself. This is different from the curse in our model where the information generating the curse comes from the equilibrium play of competing agents and concerns the value of the chosen alternative, and not one's own value.

Generalized matching markets with incomplete information were first studied by Roth (1989) (30), and the literature remains active (see for example Ehlers and Massó, 2007 (13); ,Pais and Pinter, 2008 (28)). Incompleteness in these examples concerns knowledge about others' preferences on the part of a participant in the matching market. This literature is interested in understanding the stability and strategy-proofness of matching mechanisms. Chakraborty et al. (2010) (10), follows this line of research and introduces the additional element of value interdependency among participants.

We study how in the presence of incomplete information and a common value agents can make inferences about others' information in equilibrium and the effect of such strategic considerations on the market's performance. In our model it is the assignment mechanism that is used to resolve the problem of limited supply that allows for such inferences. Milgrom and Weber (1982) (25), McAfee and McMillan (1987) (24) study similar effects that arise in auctions. Outside the realm of markets, Austen-Smith and Banks (1996) (4) and Feddersen and Pesendorfer (1997)(15) first studied the implications of such strategic considerations in voting and collective decision making.

The idea that rational individuals may take decisions ignoring their private information is not new. We have just mentioned the case of strategic voting, but probably the most prominent case is the one of social learning and informational cascades (Banerjee, 1992 (5);, Bikhchandani et al., 1992 (7)). This literature studies the case where individuals with imperfect information and a common value move sequentially and can observe the actions of some or all predecessors before making a decision. Gale and Kariv (2003) (16) and Acemoglu et al. (2008) (1), study the case where agents learn through their social network. An informational cascade starts when an individual ignores his private information because the information inferred by observing others' actions points to the other direction. Since his action conveys no new information, all individuals following him act in the same way. Herding behavior does not occur if actions of others were not observed. In Callander and Hörner (2009)(8), for instance, the exact actions of others' are not observed, but only the aggregate choices. Herding in these cases ceases to be an equilibrium feature. If actions are taken simultaneously agents should follow their information. But not if there is limited capacity. This is what happens in our paper and what builds a bridge with the informational cascade model. The general environment is the same but in our case actions are simultaneous and one of the two choices has limited capacity. For a general overview of the literature on social learning in markets the reader should look at the books by Chamley (2004) (11) and Vives (2010) (33).

Agents in our model are fully rational. It is not clear whether this is the right assumption in such a model, since different approaches find experimental and empirical evidence point to different directions. On the one hand, there is evidence that individuals are sophisticated enough to infer information from others' actions triggering informational cascades (Anderson and Holt, 1997 (3); Hung and Plott, 2001 (18); Alevy et al., 2007 (2); Goeree et al., 2007 (17)). On the other hand, evidence points to the opposite direction with respect to sophistication and its relation to the winner's curse. Both in the lab and the real world the majority of individuals fail to take the WC into account (Kagel and Levin, 1986(19); Lind and Plott, 1991 (21)). Our simple model provides a framework in which both situations can be tested. We use it in a related paper (Louis, 2011(23)) to test whether the same individuals are sophisticated enough to follow herds, but not so sophisticated as to avoid the winner's curse. This type of behavior is not predicted by the salient theories of play for games with incomplete information and a common value, such as "level-k reasoning" (Stahl and Wilson, 1995 (31); Nagel, 1995 (27); Crawford and Irriberi, 2007 (12) or "cursed equilibrium" (Eyster and Rabin, 2005 (14)).

3.2 The model

Agents. There are $n \geq 2$ agents that must choose whether or not to invest in an investment opportunity presented to them. Let $x_i \in X = \{I, O\}$ denote the choice of agent $i \in N = \{1, ..., n\}$. There are only $k < n$ available slots in the investment. This means, it is not possible for all agents to invest. Whether an agent is assigned to one of the available slots is determined by a mechanism $f : \{I, O\}^n \rightarrow \{I, O\}^n$. The assignment follows an exogenous priority order. An agents index denotes the agent's priority: agent i has priority over agent j if $i < j$. Let $f_i : \{I, O\} \times \{I, O\}^{N-1} \rightarrow \{I, O\}$ denote the outcome of the assignment for agent i given his and others' choices. The following holds:

$$f_i(x_i = I, x_{-i}) = \begin{cases} I & if \quad \left|\{x_j = I,\ j < i\}\right| < k \\ O & ,\ otherwise \end{cases}$$
$$f_i(x_i = O, x_{-i}) = O$$

Information. The state of nature is $\theta \in \Theta = \{G, B\}$. Agents have a uniform *common prior* about the state of nature. This means that the a priori probability of θ taking either value is $\frac{1}{2}$. [3] Before making a choice, each agent receives a noisy private signal $s_i \in S = \{g, b\}$ about the state of nature. Private signals are independent conditional on the state of nature. The following table indicates the probability of the signal taking either value conditional on the state θ.

		θ	
		G	B
s_i	g	q_G	$1 - q_B$
	b	$1 - q_G$	q_B

Payoffs: An agent's utility function has the following form:

$$u_i\left(f(x_i, x_{-i}), \theta\right) = \begin{cases} 1, & f(x_i, x_{-i}) = I \ \ and \ \ \theta = G \\ 0, & f(x_i, x_{-i}) = I \ \ and \ \ \theta = B \\ \gamma, & f(x_i, x_{-i}) = O \end{cases}$$

with $0 < \gamma < 1$. In other words, the payoff of an agent that obtains a slot in the investment is normalized to 1 if the state is "good" and 0 if the state is "bad".

[3] Considering non-uniform priors is also possible. Since it does not affect results in a particularly interesting way we choose not to do so, in favor of expositional clarity.

When an agent chooses not to invest or does not obtain a slot, he gets $\gamma \in (0, 1)$.

We can view this parameter as the value of a safe outside option. In the case of micro-investors it could be the return one gets by keeping the money in the bank. This being the same for agents that directly choose not to invest and for the ones not obtaining a slot implies that there is no cost from choosing to invest. This may not be true in some occasions. For instance, participating in an IPO may involve non-negligible transaction costs that are independent of whether or not one obtains shares of the company in the end. Adding cost for investing in our model is possible and mathematically tractable. Nevertheless, it will become clear further on that including such costs here would only reinforce our results about agents behavior in such a market. Hence, not including them makes both our results stronger and the exposition cleaner.

Coming back to the image of agents standing in a line, one can view the model we have described in the following way. An agent's index denotes his position in the line. Given the limited availability of investment slots, deciding to invest does not guarantee the agent a slot. The assignment mechanism works in a way that an agent that chooses to invest obtains a slot only if less than k agents standing in front of him, to invest. If an agent's position (index) is less than k than obtaining a slot only depends on his own decision.

In real markets, one's position in the line might depend on one's time of arrival when a the "first-come, first-served" method is used. Or it might depend on some priority assigned by the seller or planner. In the micro-investment example for instance, the entrepreneur might want to give priority to close friends and family over other investors. In a market for "public protection" housing there might be social criteria that determine the priority of potential buyers.

For the moment we assume that each agent knows exactly his position in the line. This assumption can be strong and we later relax it in different ways. Still is useful to start of this way for two reasons. On one hand it allows for a better understanding of the forces that determine equilibrium behavior. On the other hand it is an important building block in the calculation of equilibria in the other environments we explore later on.

As was mentioned in the introduction, agents decide whether or not to invest without observing what others choose to do. There is neither communication among agents nor the possibility for social learning by observing other actions. In some environments this makes sense. For instance in IPO's, investors must decide whether or not to participate in a simultaneous fashion. The stronger argument for this assumption will be clear once we present our results. As we shall see, the equilibrium behavior of agents in our model shares characteristics with the behavior of agents in models of social learning. In particular the fact that some agents ignore their private information and choose a particular action. By obtaining these results with agents acting simultaneously we show how this behavior can emerge in such an environment and what factors drive it.

We will further assume that the following condition holds:

Condition 1.

$$\frac{1-q_G}{q_B} < \frac{\gamma}{1-\gamma} < \frac{q_G}{1-q_B}. \tag{3.1}$$

This condition makes the problem interesting. It makes sure that when an agent has no further information than his own private signal, his best response depends on the signal's content. A signal $s_i = g$ indicates that investing is a "good" choice. A signal $s_i = b$ indicates it is better not to invest. As will become clear further on, were this not true, *all* agents would choose to invest (for low γ) or not to invest (for high γ) independently of their signal.

Up to now we have defined a set of agents that can take actions out of a particular set and have a particular type which is given by their private information. Their actions lead to payoffs that depend on the state of nature and on the actions of other agents. The environment is further defined by the available slots, the value of the outside option and the precision of the private information. All these define a bayesian game $\mathcal{G} = \langle N, \Theta, \{X, S, u_i, \}_{i \in N}, k, \gamma, q_G, q_B \rangle$. The relevant concept that we use to solve such a game is the one of *Bayesian Nash equilibrium*. In our specific context this equilibrium refers to a strategy for each agent that describes the action the agent takes depending on his private information: $x_i^* : S \to X$. The strategy must be such that and that it maximizes his expected payoff from the game given all other players' strategies: $E[u_i(x_i^*, x_{-i}^*)] \geq E[u_i(x_i^{'}, x_{-i}^*)], \forall i \in N$, and given his beliefs about others' private information.

3.3 Equilibrium behavior

As was mentioned, agents in our model can neither communicate or observe each others' actions. If there was no limit in the number of available slots, or simply $k \geq n$, then each agent would obtain a slot if he chose to invest. Our model would reduce to a sum of n individual decision problems in which each agent would choose according to his signal. Restricting the supply of slots forces agents to make strategic considerations when making their decision. In particular, agents standing at positions beyond k realize that they can obtain a slot only if less than k of the preceding agents choose not to invest.

We now use the simplest possible example to demonstrate how such strategic considerations affect agents' behavior in such a game.

Example 1. *In this example we consider only two agents: $i \in N = \{1, 2\}$. The capacity limit is the lowest possible: $k = 1$. Let us also assume that $q_G = 1$. Condition 3.1 then reduces to $q_B > 1 - \frac{1-\gamma}{\gamma}$ and we assume this holds. Notice that with this choice of parameters for the signal accuracy, if a player observes a signal $s_i = b$ he knows that the state of nature is $\theta = B$ with probability 1. This is because there is zero probability of obtaining such a signal when the state is $\theta = G$. Agent 1 stands in line in front of agent 2, or in other words, he has got priority over agent 2. This means that agent 2 can obtain the slot only if agent 1 chooses not to invest. For agent 1 the outcome depends only on his own choices.*

First consider agent 1. He is the first in line. Whether he obtains a slot depends only on his choice. Since Condition 3.1 holds, his decision depends on his private signal. If $s_1 = g$ he chooses to invest: $x_1(g)^* = I$. If $s_1 = b$, then he chooses not to invest: $x_1^*(b) = O$.

Agent 2 is second in line. He chooses simultaneously with agent 1. Therefore, even if he chooses to invest he does not know whether or not he will obtain the single slot. This depends on agent 1's choice. If agent 1 chooses not to invest then agent 2 can obtain the slot if he chooses to invest. If agent 1 chooses to invest, then there is no slot available for agent 2 and he gets the outside option. Still, he knows that agent 1's decision depends on the private signal s_1. He also knows that his own decision only matters when agent 1 chooses not to invest. He must therefore decide conditioning on this event. But agent 1 chooses not to invest only when he observes $s_1 = b$ and this is only possible when $\theta = B$. Thus agent 2 knows that his decision matters only when the state is "bad" and in that case he should not invest. Notice that this does not depend on s_2, the signal observed by agent 2. Therefore, agent 2 decides not to invest, independently of his private signal: $x_2^*(s_2) = O$.

The two agents in this example end up playing very distinct strategies in equilibrium. The first agent follows his signal, while the second agent ignores it and chooses not to invest. From now on we shall refer to the strategy of agent 1 as informative play and to to the strategy of agent 2 as herding.

Informative play: *The strategy in which an agent i chooses according to his signal:*

$$x_i(g) = I\,,\;\; x_i(b) = O$$

Herding: *The strategy in which an agent ignores his private signal and does not invest:*

$$x_i(g) = x_i(b) = O$$

First of all one should note that the reasoning that leads agent 2 to choose such a strategy is based entirely on the fact that the number of slots is limited. Were this not the case it would not be possible to make any inferences about agent 1's actions and information.

The second point to notice is that the behavior of both agents would be the same in equilibrium if there were more agents standing behind them in the line. What is more, it is easy to see that any agent standing behind agent 2 would also herd in equilibrium. This is because, since agent 2 is herding he does not affect any other agents. Thus the hypothetical agent 3 faces the exact same situation as agent 2 and also chooses to herd. The same would be true for any other agent standing in line after them.

This simple example demonstrates the main feature of equilibrium in such games. Agents standing in the first positions of the line play informatively. After some point in the line agents switch their equilibrium strategy to herding. The point where the switch takes place lies at a position grater than the number of available slots. The following proposition formalizes this result.

The result of the example is generalized in the following proposition.

Proposition 1. *Consider a game $\mathcal{G} = \langle N, \Theta, \{X_i, S_i, u_i,\}_{i \in N}, k, \gamma, q_G, q_B \rangle$ and assume Condition 3.1 holds. There is a unique Bayesian Nash equilibrium in this game. In equilibrium, all agents with index $i < \hat{m}(k, \gamma, q_G, q_B)$ play informatively. All others herd and choose $x_i = O$, independently of their signal. Furthermore, $\hat{m}(k, q_A, q_B) > k$.*

Proof. *All proofs can be found in the appendix.* □

What drives this result is the same as in the two-agent example. Agents with an index higher than k know that they can obtain a payoff higher than their outside option only if the state is "good" and less than k agents of the ones in front of them choose to invest. But given that agents in the front of the line play informatively, conditioning on the event that less than k agents choose to invest (which means that less than k agents received a signal $s_i = g$) reduces the probability of the state being good. There is an increased probability of obtaining a slot in the "bad" state. This is the winner's curse effect. This effect becomes stronger the further back one stands in the line. Therefore, eventually agents switch away from informative play as we move towards the back, in order to avoid the winner's curse.

One important feature of this result is that a significant number of agents never choose to invest. This means that with positive probability less than k agents invest and obtain a slot, even when the state of nature is "good". This ex-post inefficiency is reminiscent of the same inefficiency encountered in the social learning model. We study that further on when we make a comparison between the two different models: our own and a social learning model, where agents decide sequentially, with a limited availability of investment slots.

For now we must point out that the equilibrium is efficient. The number of agents playing informatively maximizes the sum expected payoffs. This is stated in the following proposition.

Proposition 2. *Given k, γ, q_G, q_B that satisfy condition 3.1, the unique equilibrium strategy profile of a game $\mathcal{G} = \langle N, \Theta, \{X_i, S_i, u_i,\}_{i \in N}, k, \gamma, q_G, q_B \rangle$ with known priorities, for any N, is ex ante efficient. Another pure strategy profile of the game is ex ante efficient if and only if the same number of agents play informatively as in the equilibrium profile.*

The reason why this holds is simple. The number of agents playing informatively is such that any agent that plays informatively has an expected payoff higher than what he obtains by herding, which is the outside option. If less agents play informatively, then they are forgoing to possibility of a higher expected payoff. If more agents play informatively, then some have an expected payoff smaller that their outside option. Both these cases result in a smaller sum of expected utilities and are therefore inefficient.

Another feature of the equilibrium to note is the sorting of agents and strategies. Low index agents play informatively while high index agents herd. This means that what to an external observer might seem as some sort of correlation between priorities and preferences or information is simply rational equilibrium behavior of agents with identical preferences.

3.3.1 Comparative statics.

To get a better grasp of how equilibrium behavior depends on the various parameters of the model we perform comparative statics. It is important to understand what exactly is "moving" when we change one of the parameters. For that one has to understand the mechanism that underlies proposition 1.

As long as Condition 3.1 holds, agents that receive a signal $s_i = b$ never choose to invest. The ones that receive $s_i = g$ calculate their expected payoff from investing, taking into account the fact that to obtain a slot it must be that less than k agents in front of them invest. They compare this to the payoff from the outside option γ. Whether an agent plays informatively or herds depends on this comparison. Thus any effect of a change in parameters on equilibrium behavior must come through the effect the change has on the expected payoff from investing after observing $s_i = g$. This is given by the following function in which we assume all agent in front of i play informatively:

$$\begin{aligned} E[u_i(I,g)] =& Pr(G|g)\underbrace{\left[\overbrace{Pr\left(\left|\{s_j=g,\ j\le k\}\right|<k\,|\,G\right)\cdot 1}^{payoff.\ when\ a\ slot\ is\ free}+\overbrace{Pr\left(\left|\{s_j=g,\ j\le k\}\right|\ge k\,|\,G\right)\gamma}^{payoff\ when\ no\ free\ slot}\right]}_{payoff\ when\ state\ is\ ``good"} \\ &+Pr(B|g)\underbrace{\left[\overbrace{Pr\left(\left|\{s_j=g,\ j\le k\}\right|<k\,|\,B\right)\cdot 0}^{payoff.\ when\ a\ slot\ is\ free:\ WC}+\overbrace{Pr\left(\left|\{s_j=g,\ j\le k\}\right|\ge k\,|\,B\right)\gamma}^{payoff\ when\ no\ free\ slot}\right]}_{payoff\ when\ state\ is\ ``bad"} \end{aligned} \tag{3.2}$$

The first term of the second bracket in the RHS represents the winner's curse. It is the payoff an agent receives when investing and obtaining a slot when the state is "bad". The number of agents that receive a particular signal given the state follows a binomial distribution. Hence the probability of less than k agents to have received a signal $s_j = g$ given the state, is given from the cumulative density function (cdf) of the binomial distribution with the appropriate parameters. Let $F_{(m,G)}(k)$ represent the cdf of $Bin(m, q_g)$ and $F_{(m,B)}(k)$ represent the cdf of $Bin(m, 1-q_B)$. Thus we have:

$$\begin{aligned} E[u_i(I,g)] = & \frac{q_G}{q_G + 1 - q_B}\left[F_{(i-1,G)}(k-1) + \left(1 - F_{(i-1,G)}(k-1)\right)\gamma\right] \\ & + \frac{1-q_B}{q_G + 1 - q_B}\left(1 - F_{(i-1,B)}(k-1)\right)\gamma \end{aligned} \tag{3.3}$$

The equilibrium behavior of a particular agent is determined by whether this expression is above or below γ, the value of the outside option. When it is above, the agent invests. When it is below he herds.

The value of the outside option.

The value of expression 3.3 is increasing in γ. Still, the sum of the factors of γ is lower than 1. This means that as we increase γ 3.3 also increases but at a slower rate. So let us consider the last agent in line that plays informatively for some low γ. This means that for him $E[u_i(I,g)] > \gamma$. Now suppose we increase the value of the outside option. While both sides of the inequality increase, the RHS does so faster, so eventually it will switch. This agent will change his strategy from informative play to herding.

Here the value of the outside option is given relative to the possible payoffs of the investment that are normalized. These values would normally depend on whoever tries to attract the investors. We do not model such an agent in any form here. Still, what we learn here is that an entrepreneur trying to attract investors, can do so by making the investment more attractive relative to the outside option. This is assuming she has no other information that can be inferred by her choices. This result is similar to the one obtained in Rock (1986) (29), where he concludes that the seller in an IPO might want to underprice in order to attract the uninformed investors. We get a similar conclusion, but here we do not assume any asymmetry in information among investors.

The number of available slots.

The number of available slots is a parameter which a market designer can control to a significant extent in many markets. For instance an entrepreneur might decide the maximum number of investors she wants to take on board her project, or a department may decide within limits on the number of available openings.

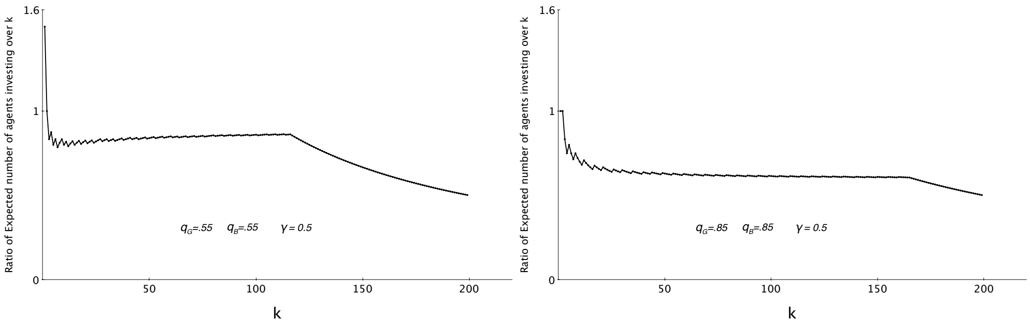
The effect on equilibrium from increasing the number of slots is clear cut: more agents play informatively.

Proposition 3. *Consider a game* $\mathcal{G} = \langle N, \Theta, \{X_i, S_i, u_i,\}_{i \in N}, k, \gamma, q_G, q_B \rangle$ *and assume Condition 3.1 holds. Then* $\hat{m}(k, q_A, q_B)$ *is increasing in* k*.*

To understand why this happens one must understand that it is the limited supply of slots that gives rise to the winner's curse. Obtaining a slot when the supply is limited happens only when "enough" preceding agents choose not to invest. When k is low, "enough" represents a large number of agents. When k is high, "enough" represents a small number of agents and thus a weaker winner's curse effect.

We must notice here that we obtain this clear-cut result for the case where priorities are known. For the cases we study further on with priority uncertainty this result may not hold, depending on the other parameters.

Although the number of agents choosing to invest increases with k it is interesting to see the rate of this increase. The following graphs in figure 3.1 show for two different levels of signal accuracy the ratio of the expected number of agents that choose to invest in equilibrium, over the number of available slots. When this ratio is above one, we expect excess demand. When the ratio is below 1 we expect excess supply. We observe that excess demand only occurs for low levels of k. The ratio drops off fast. This happens because of the effect of increasing k on the winner's curse. How it evolves further depends on the the other parameters. Here we see that for low levels of signal accuracy the ratio show a tendency to increase again, while for low levels of accuracy it continues decreasing. An explanation for that is that when accuracy is high the winner's curse effect remains persistent. More agents play informatively because more agents have an index below k but for agents with a higher index the effect is still there. When accuracy is low, the increase in k has a strong attenuating effect on the winner's curse. Therefore, not only agents with am index below k switch, but also a significant number of agents with a higher index. In both graphs there is a drop of the ratio in the end. This is



(a) low accuracy (b) high accuracy

Figure 3.1: Examples of how the demand for investment evolves with k

due to the fact that already all agents are playing informatively after that point. Hence, increasing k has no further effect on equilibrium.

The accuracy of information.

The accuracy of information in our model is represented by the parameters q_G and q_B. The higher these parameters are, the stronger is the signal the agents receive. As we explained, when an agent decides in equilibrium he also takes into account the signals of others that stand in front of him in the line. So suppose an agent receives a "good" signal. The higher the accuracy of the signal, the stronger the indication that the state is actually "good". But in equilibrium this agent may obtain a slot only if enough of the preceding agents choose not to invest. These agents must have received a "bad" signal. The higher the accuracy of the signals the stronger an indication it is that the state is actually "bad". Thus, the increase in accuracy has a positive effect through one's own signal but a negative effect through the signals of preceding agents.

Which effect dominates? This depends on where an agent stands in line. For equilibrium what matters is agent $\hat{m}$. If a change in the accuracy has a positive effect in his expected payoff, he (and maybe more agents) may switch from herding to informative play. If the effect is negative, then it is possible that some agents that played informatively, switch to herding. This would give a new $\hat{m}$ with a lower index.

Changes in q_G. An increase in q_G means that it is more likely to receive a signal $s_i = g$ when the state is "good". By bayesian logic it also means that having received a such a signal it is more likely that the state is "good" . Looking at expression 3.3 we can see how this creates the two opposite effects described. On one hand, the factor $\frac{q_G}{q_G+1-q_B}$ increases while the complementary factor $\frac{1-q_B}{q_G+1-q_B}$ decreases. This represents the positive effect from one's own private signal $s_i = g$ becoming stronger. At the same time though, the term in the brackets decreases, since $F_{(i-1,G)}(k-1)$ is decreasing in q_G. This represents the effect of the "bad" signals of preceding agents becoming stronger.

The graph in figure 3.2 shows an example of how changing q_G affects the shape of the function in expression 3.3. The points in the rectangle are the ones corresponding to the threshold agent $\hat{m}$.

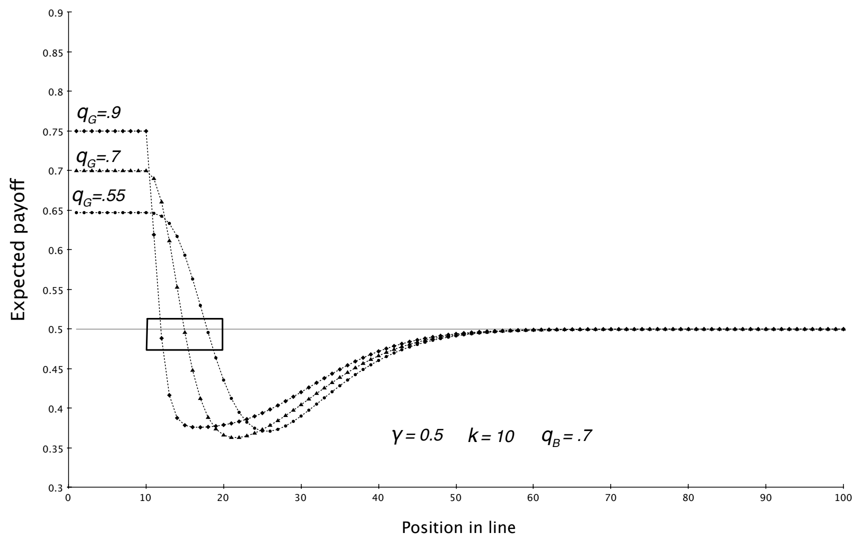


Figure 3.2: Comparative static with respect to q_G.

Changes in q_B. An increase in q_B means that it is more likely to receive a signal $s_i = b$ when the state is "bad". Again, by Bayesian logic it follows that having received a signal $s_i = g$ it is more likely that the state is "good". In expression 3.3 we can see the two opposite effects. The two fractions move in the same direction as before. Now it is in the term in the last parenthesis where we observe the opposite negative effect. This term decreases.

The graph in figure 3.3 shows an example of how changing q_B affects the shape of the function in expression 3.3. The points in the rectangle are the ones corresponding to the threshold agent $\hat{m}$. Note here that $\hat{m}$ moves to the opposite direction than when we were changing q_G.

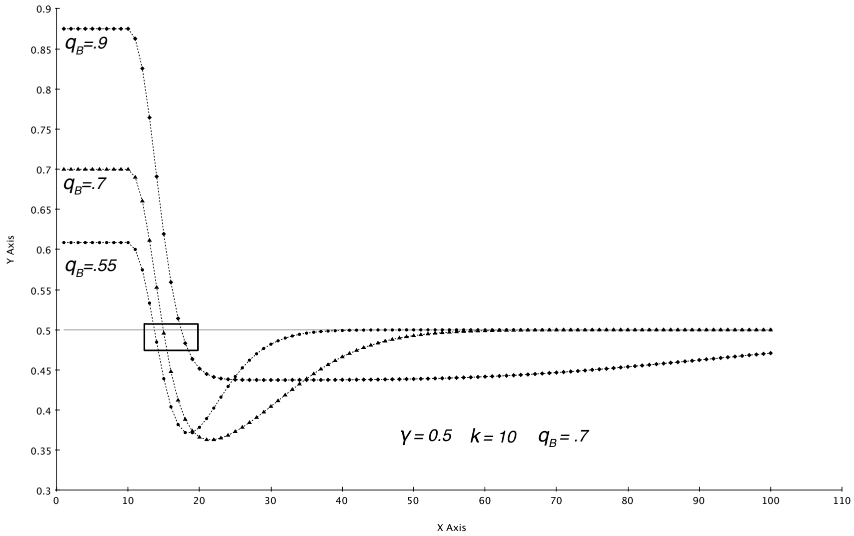


Figure 3.3: Comparative static with respect to q_B.

3.4 Simultaneous play vs. Social learning.

In our model agents do not learn from each other. There is no communication between them, nor is it possible to observe each others' actions. Yet, the behavior we observe in equilibrium resembles the one found in models with social learning in which agents take actions sequentially and can observe what others do (Banerjee, 1992 (5);, Bikhchandani et al., 1992 (7)). In this section we compare behavior in our model with the one in such a model. The social learning we consider follows the exact same setup as our model with one difference: agents take actions sequentially and observe the actions of the agents standing in front of them in the line. This is equivalent to adding a limited number of slots for one of the alternatives in the binary model in Bikhchandani et al. (1992). In the sequential model, the limited number of slots does not affect strategic behavior. Since agents observe the actions of others they can accurately their private information when making their own decision. If the slots are filled the game ends and remaining agents obtain their outside option. The interesting equilibrium feature in such a model is the possibility of an informational cascade emerging. After observing a particular sequence of actions an agent's beliefs about the state may be such that his private signal does not make a difference about the optimal action. In this case the agent herds (ignores his private information) and so do all agents after him in the line. Informational cascades can go either way with agents herding choosing to invest or not to do so. There is also the possibility of agents herding on the wrong decision.

The equilibrium outcome in the two models can be very similar. For instance, in the two gent model described in example 1, allowing the second agent to observe the action of the first agent, makes no difference in the outcome observed in equilibrium. The first agent may invest or not, depending on his signal, while the second agent always obtains his outside option. This happens because the equilibrium inferences made by the second agent in the simultaneous game mirror exactly the inferences he makes in the sequential game.

Such similarities persist in games with more players and different levels of k when the value of the outside option is low. Outcomes change when this value is high. We explain the intuition behind this phenomenon and use numerical simulations to demonstrate the result.

There are two types of mistakes agents can make: not investing when the state is "good" or investing when the state is "bad". The first type is costly when γ is low. That is when the outside option give a low payoff compared to that of a good investment. The second type is costly when γ is high.

In the simultaneous model, informational cascades serve as a mechanism to protect agents from these mistakes. By observing others, agents are able to make decisions based on more information than only their private signal. The "cost" of such a defense mechanism is that sometimes it produces "bad cascades", in which agents all herd on the wrong decision. Still, the probability of such a cascade is relatively low.

In the sequential model, there is again a low risk of committing the first mistake. In equilibrium a large number of agents plays informatively. For the ones that herd, choosing not to invest makes a difference only if the agents playing informatively leave free slots. But this rarely happens when the state is "good". Concerning the second type of mistake, investing in a "bad" state, the agents in the back of the line that herd are protected. Still, agents in the front of the line must rely solely on their private information and it is possible for them to make such a mistake. More so than agents in the sequential model that decide based not only on a single private signal.

So it turns out that what cause a difference in the outcome of the two models is the degree to which agents commit the mistake of investing in a "bad" project. When γ is low, such a mistake is not very costly and furthermore, "bad cascades" in that direction are not very likely in the sequential model. Therefore the outcomes of the models do not vary significantly. When γ is high, such a mistake becomes costly. "Good cascades" protect agents in the sequential model. In the simultaneous model agents commit this mistake more often.

From an efficiency point of view, when γ is high, the sequential game produces better outcomes. For a low γ outcomes do not differ much. In the simulations we perform, efficiency is slightly better in the simultaneous game for low a low k and slightly worst for higher k. Still, differences are of a very small magnitude.

From the point of view of demand, when γ is high there is a higher demand for investment in the simultaneous game, except for very low levels of k. For low γ again demand is higher in the sequential game, but only for very low levels of k is the difference significant.

The following graphs show the results of Monte-Carlo simulations performed in order to compare the outcomes of the two models. For these simulations we produce a vector of private signals. We calculate the equilibrium corresponding to this vector for each model for different levels of k. We repeat the process 10,000 times and take averages of our results. The parameters used in the simulations presented here are $n = 100$ and $q_G = q_B = q = 0.85$. We do the calculations for three different levels of the value of the outside option: $\gamma \in \{.4, .5, .6\}$. The first graph shows the difference in the total welfare (normalized to lie between zero and one) between the two models. Positive values indicate a higher welfare in the simultaneous model. The second graph shows the difference in demand for investment between the two models. Demand here is calculated as the fraction of slots filled in equilibrium. Positive values indicate a higher demand in the simultaneous model.

One can see in the graphs how the differences between the two models become pronounced when γ is high. The kind on the right side of both graphs is due to the fact that once k is high enough all agents play informatively in our model. Therefore increasing k further does not change the equilibrium behavior of agents. Still, it affects the normalized values of welfare and demand.

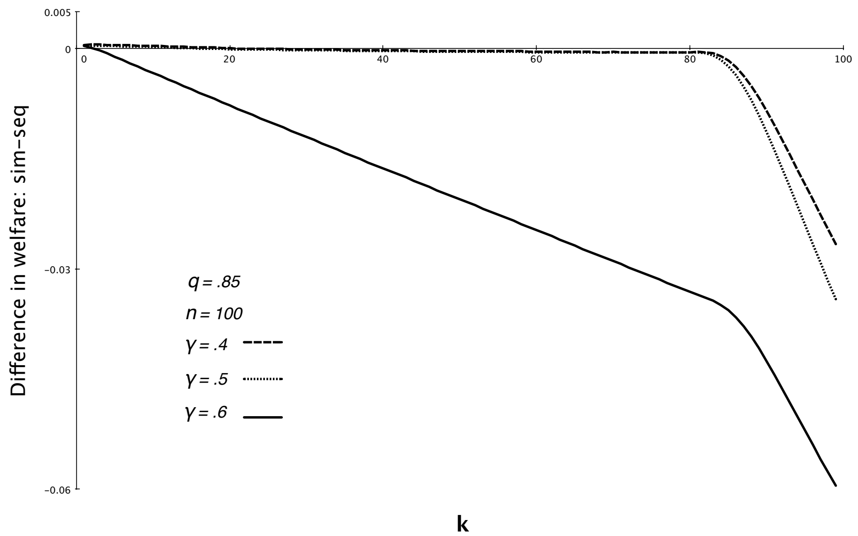


Figure 3.4: Difference in welfare in the two models.

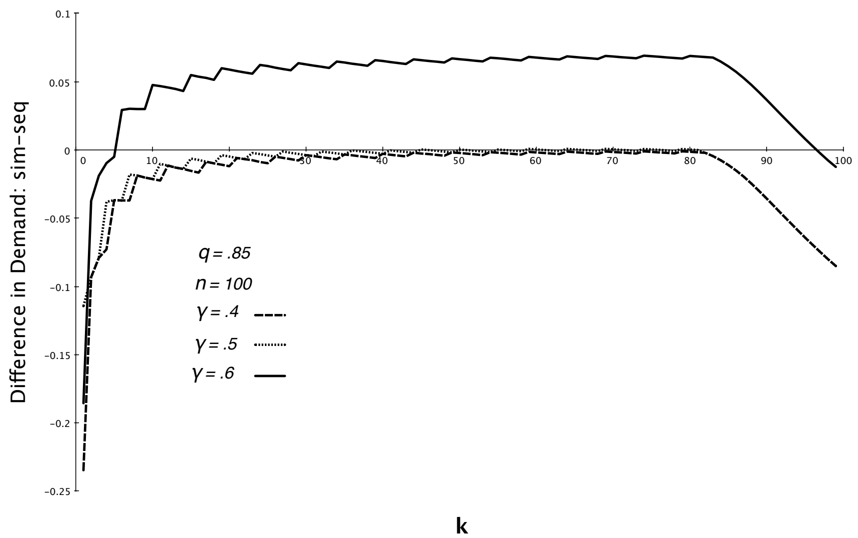


Figure 3.5: Difference in demand in the two models.

3.5 Priorities assigned by a lottery.

Up to this point we considered that each agent knew exactly his position in line. We now relax this assumption. In this section we consider the case where a lottery is used to determine the position in line of each agent. The lottery takes place after each agent makes his decision about whether or not to invest.

It makes sense to consider such a variation to our model for two reasons. First, it comes closer to some real life situations where such a mechanism is used, like some IPO's. In general, one could consider this as the other end of the spectrum of possibilities about what agents know about their priority. In reality, different cases might lie anywhere between the two extremes.

The second reason to consider this variation is a theoretical motivation. Notice that now all agents are ex-ante identical. Once they receive their private signal they are differentiated, but even at that point, all agents who observe the same private signal have exactly the same information and available choices. As we shall see, for some range of parameters there exists a symmetric equilibrium in which agents herd with a positive probability. This result highlights the fact that it is the institutional design of the market and not the heterogeneity of agents that give rise to the winner's curse effect. This is important for anybody looking at market data trying to identify such an effect. For instance in the "IPO underpricing" literature in finance the WC effect was described by Rock (1986) (29) but attributed to the existence of differentially informed agents. Empirical strategies trying to verify the theory relied on the existence of such heterogeneous groups. Our result suggests that the WC effect should be present even without differences in information between groups of agents.

From a technical point of view, the introduction of a lottery gives rise to multiple equilibria. Given that now agents are symmetric, we find it reasonable to focus on symmetric equilibria. It turns out there is a unique symmetric equilibrium in mixed strategies. We will denote the game with a lottery as $\mathcal{L}$. Let $L = \{1, ..., n\}$ denote the set of positions in line to which agents are assigned by the lottery.

Proposition 4. *Consider the game* $\mathcal{L} = \langle N, L, \Theta, \{X_i, S_i, u_i,\}_{i\in N}, k, \gamma, q_G, q_B \rangle$ *There exists a unique symmetric equilibrium in mixed strategies in the game with lottery determined priorities. For* k *sufficiently low and* γ *sufficiently high agents decide to herd with a positive probability.*

To understand where this result comes from one can think the following. If everybody else herds, then an agent knows that he can obtain a slot by choosing to invest. As long as he observes a "good" signal, this is a best response independently of the outcome of the lottery. Now as the probability of all other agents playing informatively increases, it becomes more and more likely to be placed in a position in the back of the line with a high probability of more than $\hat{m}$ agents in the positions in front playing informatively. In such a position the expected payoff is less than the outside option. If this probability is too high, then it is best for an agent to switch his strategy to herding. There is some level of this probability where an agent becomes indifferent between informative play and herding. It is easy to see that as the value of the outside option γ increases, this level becomes lower, since herding becomes more attractive. The opposite happens with the number of available slots k. This is because for a higher k there is a higher chance to be positioned through the lottery to one of the front spots in the line where one is immune to the winner's curse.

A natural question that rises is how the introduction of the lottery affect the characteristics of equilibrium. In particular, what effect does it have on herding behavior? While the symmetric equilibrium allowed us to highlight the existence of the winner's curse effect even with homogeneous agents, it does not lend it self for easy comparison to the equilibrium of the case where agents know their position in line. In the following proposition we have a comparison between pure strategy equilibria.

Proposition 5. *In the lottery game there exist pure strategy equilibria in which* $\tilde{m}(k, q_A, q_B, N) \leq N$ *agents play informatively. Furthermore, more agents play informatively in such an equilibrium than in the unique equilibrium of the game with known priorities:* $\tilde{m}(k, q_A, q_B \geq \hat{m}(k, q_A, q_B) - 1$.

In the game with no lottery any agent after $\hat{m}$ knows that his expected payoff from investing is less than his outside option and therefore herds. In the game with a lottery as long as there are at least $\hat{m}$ other agents playing informatively an agent can be unlucky and be assigned a position in the line after all these $\hat{m}$ or more agents and also get an expected payoff that is lower than his outside option. Still, this is only of the possible outcomes he faces. It is therefore not necessary that he prefers to switch to herding. Thus it is possible for such a profile with more than $\hat{m}$ agents playing informatively to be sustained as an equilibrium.

Combining this result with the one in proposition 2 it is easy to see that such an equilibrium is not efficient. Thus the uncertainty about priorities introduced with the lottery allows for inefficiencies to be introduced due to the fact that equilibria are possible in which the number of agents playing efficiently is higher than the efficient level.

3.6 A Bernoulli arrival process

In the Previous sections we have looked at two extreme cases concerning agents' knowledge of their position in line. In the first, they are perfectly informed about it while in the second they have no information whatsoever, since it is a lottery that determines it. Given the results obtained for these two cases, a natural question follows. What happens for "intermediate" cases of uncertainty about priority? By an "intermediate" case we mean one in which agents do not know their position with certainty, but still there is some heterogeneity among agents. Some know it is more likely for them to be in the front while others find it more likely to be in the back. How does such uncertainty and heterogeneity affect equilibrium behavior?

Modeling such a situation for a finite number of agents is not a trivial task. For agents' beliefs to be consistent it would require that the $n \times n$ matrix, representing each agents probability distribution for each position in the line, to be a doubly stochastic matrix[4].

We choose here a different approach. We use a Bernoulli arrival process to model an "intermediate" uncertainty case. In particular, we consider that time is divided in discrete intervals. In each time period t an agent arrives with positive probability p, the arrival rate. While the individual agent knows his time of arrival and the arrival rate at the time of making his decision he does not know the realized number of arrivals in the preceding periods. Coming back to the image of the line, one can think that the line exists inside a room. Agents arrive at the room's door and must make a decision before entering. They can not see how many agents have already entered the room before them. Once they make their decision and enter, they stand in line behind the one's that are there already, but cannot change their decision.

It is of course convenient to take time in this model at face value and consider it as a model of cases where "first come, first served" is used to allocate slots. This would of course fit the "robot example" in the introduction as well as many other cases of markets where such a rule is used. An alternative view would be think of time in the model as an metaphor for the information agents have about their

[4]Each row and each column add up to 1. This should be so because both the probabilities over positions for an agent must add up to 1 as the probabilities for any position to be filled by one of the agents

priority. The arrival time t could simply represent a private signal for the agent about his priority. The higher this signal, the more likely it is that the agent is actually in the back of the line. In applications this signal together with the arrival rate contains all the (noisy) information agents have about the total number of agents participating in the market and their individual priority.

Note that now the set of agents is not fixed. There is uncertainty about the total number of players in the game. This makes it a game with population uncertainty (Myerson, 1998 (26)). Still it is not a poisson game, which is the population uncertainty game usually studied, but which is not relevant in this context. To our knowledge it is the first instance of such a game in which population uncertainty is modeled by a Bernoulli process. Let us call it a Bernoulli game.

In such a game the set of agents is replaced by the set of types and a distribution over that set. An agent's type is determined by his time of arrival $t \in \mathbb{N} = \{1, ...\}$ and his private signal $s_t \in \{g, b\}$. The set of types is $T = \mathbb{N} \times \{g, b\}$. The distribution over the set of types is given by the arrival rate p and the accuracy of the signals q_G and q_B.

It turns out that this game has a unique equilibrium with similar characteristics to the equilibrium of the game with no uncertainty. The following proposition describes this result.

Proposition 6. *Consider the Bernoulli game* $\mathcal{B} = \langle T, \{p, q_G, q_B\}, \Theta, \{X_{(t,s)}, u_{(t,s)}, \}_{(t,s)\in T}, k, \gamma\rangle$ *and assume Condition 3.1 holds. There exists a unique Bayesian Nash equilibrium of this game. In equilibrium, all agents that arrive at* $t < \hat{t}(k, q_A, q_B, p)$ *choose according to their private signal. All others choose O, independently of their signal. Furthermore,* $\hat{t}(k, q_A, q_B, p) \in (\hat{m}(k, q_A, q_B), \infty)$.

This result has a similar avor to the one in proposition 1. There it was agents standing after a specific point in line that choose to herd. Here it is agents arriving after a specific point in time. The intuition that drives it is similar. Suppose everybody plays informatively. Agents arriving early see it as highly likely to be in the front of the line and therefore are happy playing informatively. Furthermore, their payoff does not depend on what others that arrive later do. The later an agent arrives, the more likely it is for him to be placed further back in the line. This depends on how many agents have arrived before him. Since all these agents will be playing informatively, we know from our previous results, that when the probability of being placed towards the back becomes high, eventually it is better to switch one's strategy to herding. The same is then true for all agents arriving after that point in time.

Concerning efficiency, we do not provide a formal result, but it is easy to see that this equilibrium is ex ante (before population uncertainty is resolved) efficient. This is for the same reasons as in the game with known priorities. Of course, once population uncertainty gets resolved but before the revelation of the state of nature, the equilibrium will generally not be efficient. It can only be efficient if the realized arrivals before time $\hat{t}$ equal exactly $\hat{m} - 1$. This will generally not be true.

Now that we have characterized behavior in this model of "intermediate" uncertainty about priorities we can look at one final issue. The relationship between this uncertainty and the behavior of agents. Looking at the equilibrium results for the two extreme cases (known priorities, lottery) one might think that there is a monotonic relationship between uncertainty and the incentives to herd. In particular it looks as if higher uncertainty about one's priority attenuates the winner's curse effect and makes informative play more attractive. In what follows we demonstrate by a counterexample that this is not always the case. The uncertainty about one's priority can have an effect on behavior, but the direction is not always the same. It depends on the whole parameter set.

In the following exercise we calculate the expected payoff of agents arriving at different time periods. For each agent we adjust the arrival rate in such a way that the expected number of earlier arrivals remains the same. For instance there can be two agents, one arriving at time t and the other at $t' > t$. Suppose the respective arrival rates are p and p' such that both agents the expected number of earlier arrivals is $p(t-1) = p'(t'-1) = \lambda$. Still, the variance of the distribution of earlier arrivals is different in each case. It must be $p(1-p)(t-1) < p'(1-p')(t'-1)$. One can therefore argue that in the second case the agent faces a higher uncertainty about his position in line. If the conjecture about the monotonic relationship between priority uncertainty and behavior was true, then we should expect that if the second agent plays informatively in the equilibrium of his game then so would the first agent in his respective game. And if the first herds in the equilibrium of his game, then so does the second in the equilibrium of the respective game. We use a numerical example to show this is not the case.

Example 2. *The graph in Figure 3.6 shows an example for a particular choice of parameters $k = 4$, $q_G = q_B = .733$, and $\gamma = .5$. Also, In this example we have fixed the expected number of earlier arrivals for each agent to $\lambda = 6.226$. From the properties of the Bernoulli distribution we have $\lambda = tp$. So given the time of arrival of an agent and in order to keep λ constant, we calculate a different* *arrival rate for each agent t: $p = \frac{\lambda}{t}$. This means that each agent we consider plays a different game. The horizontal axis shows the time of arrival of an agent. The vertical axis shows his expected payoff from deciding to invest after observing a signal $s = g$. That is: $E[u(I;(t,g)]$. An agent arriving at t plays informatively when this is higher than the value of the outside option, γ. The horizontal line in the graphs indicates the value γ. Thus, points above this line correspond to agents that play informatively in the equilibrium of their respective game. Notice that p is decreasing in t. The variance for each agent t is $tp(1-p) = \lambda(1-p)$ which is decreasing in p. This means that the later an agent arrives in this exercise, the higher the variance he faces.*

As can be seen in the graph, the monotonicity one might expect given our previous results is not there. Take an agent arriving at $t = 8$ and one arriving at $t = 14$. They both play in games where all parameters are the same except for the arrival rate. Still, this is such that they both expect the same number of agents to have arrived before them. The number of these previous arrivals is a random variable and has the same mean for both agents, but a different variance. This is larger for the agent arriving at $t = 14$. Assuming the agents both observe a private signal $s = g$, their expected payoff from choosing to invest is shown in the vertical axis. It is clear that fro the agent arriving at $t = 8$ it is best not to invest since he obtains a higher payoff from the outside option. The opposite is true for the agent arriving at $t = 14$.

The graph in Figure 3.7 can help explain this fact. The bell-shaped curves represent the distributions of previous arrivals that agents arriving at t and t' face. These have both the same mean λ. Thus the one for t' is a mean-preserving spread of the one for t. Once this uncertainty is resolved, agents find theme selves in a certain position m in the line. The quasi-U-shaped curve that spans horizontally represents the expected payoff from choosing to invest for an agent in position m that has observed a signal $s = g$. The straight horizontal line indicates the value of the outside option γ. The expected payoff from investing when observing $s = g$ for an agent arriving at t is calculated by taking the sum of the area below $E[u_m(I,g)]$ weighted by the corresponding binomial distribution. For an agent to decide whether or not to invest, this must be compared to the payoff from the outside option. The mean preserving spread for the agent that arrives later, at t', puts less weight close to the mean λ and more weight on the sides of the distribution. While this has a positive effect on the left side where the expected

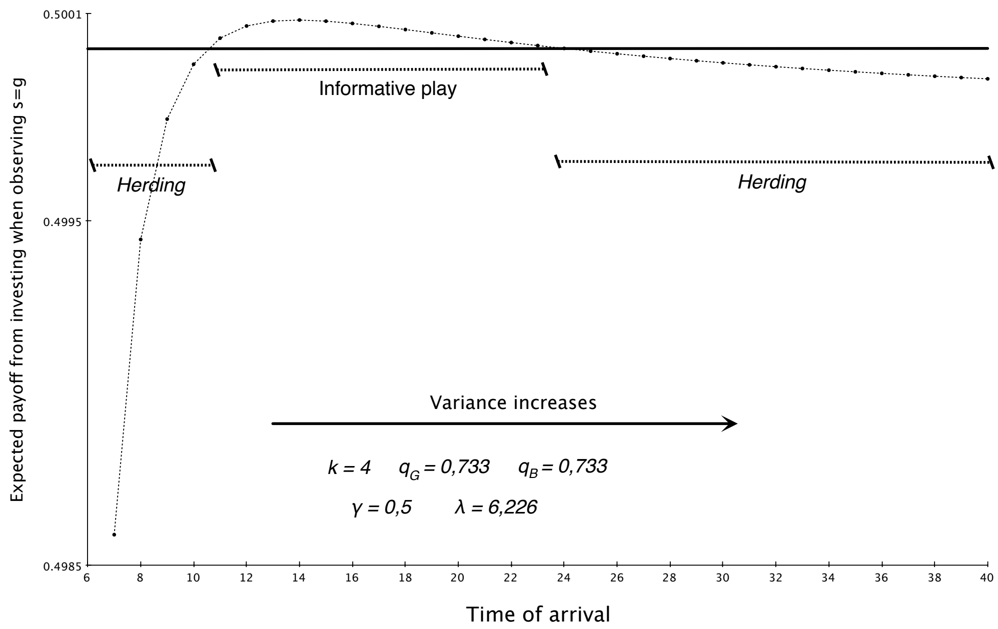


Figure 3.6: *An example of non-monotonicity in the relationship between uncertainty and behavior.*

payoff from investing is higher that γ it can have a negative effect on the right side where the expected payoff is less than γ. Which of the effects is stronger depends on the whole parameter set considered.

3.7 Conclusions

We presented a simple model of a market for *limited* investment opportunities. Incomplete information and a common value, combined with the limited offer of investment opportunities generate a winner's curse effect. Agents' equilibrium behavior depends on their priority, which is exogenous. We discuss how changes in the availability of investment slots, the accuracy of information and knowledge concerning the priority order can have an impact of the demand for the investment opportunities and the performance of the market in general.

In our model, agents face no budget constraint. Furthermore, the supply of investment opportunities is not connected to their payoff. This allows for a more tractable analysis. It is reasonable to think that in reality any change in the supply of investment slots should be connected to a change in the price of

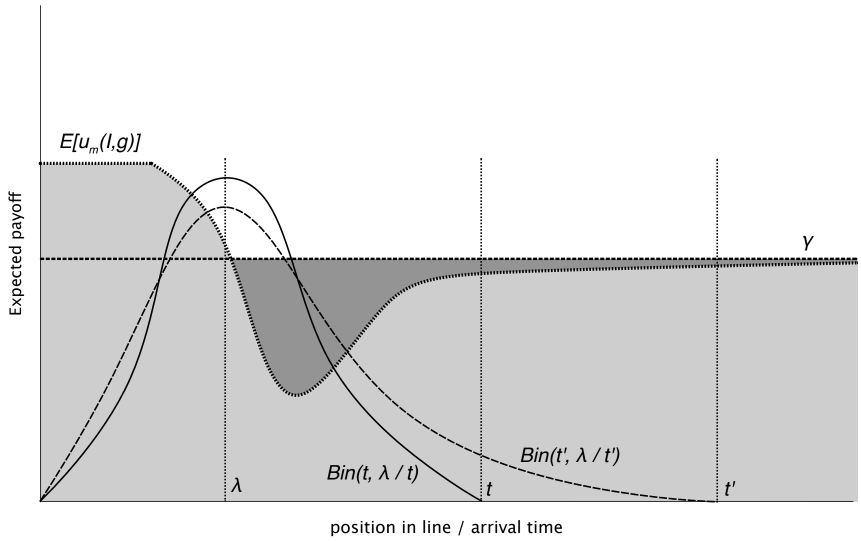


Figure 3.7: *What happens when the variance increases.*

investment and its attractiveness. For instance, an entrepreneur seeking up to 30,000 euros of capital for a new project can offer 15 slots for 2,000 euros each, or 20 slots of 1,500 euros each. Increasing the number of slots and maintaining the total capital constant makes each slot more affordable. On the other hand, the returns for each slot will also be lower. In our analysis abstract away from these issues. Nevertheless, our analysis suggests that such a change in the model would not affect the results concerning the existence of the winner's curse and its consequences.

In the literature on IPO underpricing[5], a winner's curse effect is identified as a theoretical possibility but is attributed to a problem of assymetry of information between perfectly informed and completely uninformed agents. Rock (1986) concludes that "...the institutional mechanism for delivering the shares to the public is irrelevant as far as the offer price discount is concerned.". In our paper, we show that institutions matter because a winner's curse can arise even when agents are symmetrically informed. It is the design of the market institutions that determines what the effects of the curse will be. An interesting extension to our model would be to allow for agents to decide whether or not to acquire information before deciding to invest. Given the winner's curse effect, even if the cost of information is low, some agents may decide not to acquire information

[5]See the survey by Ljungqvist (2004).

in equilibrium, giving rise to endogenous information asymmetry. Such a result would form a bridge between our model and the one by Rock. We are currently working on such an extension.

In this paper we find that social learning is not necessary for agents to make inferences about others' information and adapt their behavior accordingly. In environments with incomplete information and a common value, limited supply gives rise to herding behavior. Then, the particular mechanism used to assign priorities determines agents' demand. A natural next step is to think about implications for mechanism design in general. For example, our model can be viewed as a fixed price auction. How does it perform compared to a regular auction? How much information should participants have about others' actions or about their own priority? Building on the basis that we set here, we plan to further explore these issues both theoretically and experimentally.

We primarily focused our analysis on the buyers' side of the market. Even so our analysis shows how a seller can in uence demand by determining the relative payoffs and the available supply. These conclusions are based on the implicit assumption that the seller is uninformed about the state. If this is not the case, the situation becomes an interesting signaling game in which the seller can reveal information about the state through the choices of available supply and price. Another possible signaling vehicle that is worth exploring is a practice that is observed in some emerging crowdfunding platforms and other markets. The seller there sets a minimum demand threshold that must be covered to make the offer effective. In other words, if the minimum threshold is not reached no money changes hands. It can be interesting to study how a seller would optimally set such a threshold given its signaling content and the presence of the winner's curse. A model in which entrepreneurs use such a threshold to compete for investors is something we view as a potential route for future research.

Our results depend critically on the assumption of fully rational agents, sophisticated enough to be aware of the winner's curse and act accordingly. Whether actual individuals have this level of sophistication is a matter of debate. Experimental and empirical data on common value auctions are not conclusive. Nevertheless, besides contributing to this debate with another open question, our model also provides a useful tool: it represents a simple binary choice model in which the winner's curse appears. Therefore It can easily be used in experiments to test individuals' awareness of the curse or other related issues. Louis (2011) uses the two-agent version of the model from example 1 in such a way. Subjects play the game in the example both sequentially and simultaneously. The question is whether the same individual may be sophisticated enough to detect the course in the sequential game, but not in the simultaneous. It turns out that a significant portion of subjects fall in to this category, something that it not predicted neither by Bayesian-Nash equilibrium, nor by other alternative theories.

3.8 APPENDIX: Proofs

Proof. (**Proposition 1)** The fist k agents in the line face a simple decision problem. Whether they are assigned a slot or not does not depend on what others do. Therefore, given Condition 3.1 their dominant strategy is to play informatively. This means to follow their signal: $x_i = I$ when $s_i = g$ and $x_i = O$ when $s_i = b$. Any agent standing in position $m' > k$ gets a slot assigned only if less than k of the $m = m' - 1$ agents standing in front of him choose to invest. If this is not the case he obtains γ independently of his decision. He takes this into account when calculating his expected payoff from choosing whether or not to invest.

First let us consider agent $m' = k + 1$. and suppose he receives a private signal $s_{m'} = b$. All k agents standing on front of him play informatively and therefore their actions reveal their private signals. In other words, if for example k agents invest, it means that these k agents have received a private signal $s_i = g$. His expected payoff from choosing to invest is:

$$\begin{aligned} E[u_{k+1}(I)|b] =& Pr(G|b)\left[Pr\left(\left|\{s_j = g,\ j \le k\}\right| < k\,|\,G\right) + \left(1 - Pr\left(\left|\{s_j = g,\ j \le k\right| < k\,|\,G\right)\right)\gamma\right] \\ &+ Pr(B|b)\left[\left(1 - Pr\left(\left|\{s_j = a,\ j \le k\}\right| < k\,|\,B\right)\right)\gamma\right] \end{aligned}$$

His expected payoff from choosing not to invest is :

$$E[u_{k+1}(O)|b] = \gamma$$

Note that:

$$\begin{aligned} Pr\{G|b\} &= \frac{1 - q_G}{1 - q_G + q_B} \\ Pr\{B|b\} &= \frac{q_B}{1 - q_G + q_B} \\ Pr\left\{\#\{s_j = g,\ j \le k\} < k\,|\,G\right\} &= 1 - q_G^k \\ Pr\left\{\#\{s_j = a,\ j \le k\} < k\,|\,B\right\} &= (1 - q_B)^k \end{aligned}$$

We now show that when Condition 3.1 holds, the expected payoff from investing in this case is always lower than the one from not investing. Suppose not:

$$
\begin{aligned}
E[u_{k+1}(I)|b] &> E[u_{k+1}(O)|b] \\
\frac{1-q_G}{1-q_G+q_B}\left[1-q_G^k+q_G^k\gamma\right]+\frac{q_B}{1-q_G+q_B}(1-q_B)^k\gamma &> \gamma \\
\frac{1-q_G}{1-q_G+q_B}\left(1-q_G^k\right)+\frac{(1-q_G)q_G^k+q_B(1-q_B)^k}{1-q_G+q_B}\gamma &> \gamma \\
\frac{1-q_G}{q_B} &> \left(\frac{1-q_G}{q_B}+\frac{1-(1-q_B)^k}{1-q_G^k}\right)\gamma \\
\frac{1-q_G}{q_B} &> \frac{1-(1-q_B)^k}{1-q_G^k}\frac{\gamma}{1-\gamma} \\
(q_G > 1-q_B\,,\ \mathit{from\ Condition}\ 3.1) & \\
\frac{1-q_G}{q_B} &> \frac{1-q_G^k}{1-q_G^k}\frac{\gamma}{1-\gamma} \\
\frac{1-q_G}{q_B} &> \frac{\gamma}{1-\gamma}
\end{aligned}
$$

The last inequality contradicts Condition 3.1. This proves that for agent $m' = k+1$ it is a best response not to invest when observing $s_{m'} = b$. Note that this result does not depend on k. We can therefore extend it by saying that any agent $m' = m+1 > k$ that observes $s_{m'} = b$ and where all m preceding agents play informatively, best responds by not investing.

Now consider agent $m' = m+1 > k$ which receives signal $s_{m'} = g$ and suppose all m preceding agents play informatively. Let $F_{nX}(l)$ be the cumulative distribution of g signals for n players when the state of nature is X. Then l follows a binomial distribution and in particular F_{nG} is the cumulative distribution of $B(n, q_G)$, while $F_{n,B}$ is the one for $B(n, 1-q_B)$.

$$
\begin{aligned}
E[u_{m'}(I)|g] &> E[u_{m'}(O)|g] \\
Pr(G|g)\Big[Pr\left(\left|\{s_j = g,\ j \leq k\}\right| < k \,|\, G\right) & \\
+\left(1-Pr\left(\left|\{s_j = g,\ j \leq k\}\right| < k \,|\, G\right)\right)\gamma\Big] & \\
+Pr(B|g)\left[\left(1-Pr\left(\left|\{s_j = a,\ j \leq k\}\right| < k \,|\, B\right)\right)\gamma\right] &> \gamma \\
q_G\left[F_{mG}(k-1)+(1-F_{mG}(k-1))\,\gamma\right] & \\
+(1-q_B)\,(1-F_{mB}(k-1))\,\gamma &> \gamma(q_G+1-q_B) \\
q_G(1-\gamma)F_{mG}(k-1)-(1-q_B)\gamma F_{mB}(k-1) &> 0 \qquad (3.4)
\end{aligned}
$$

As long as 3.4 holds, player m' plays informatively. Next we show that the LHS of 3.4 is either increasing or quasi-convex with respect to m.

$$\begin{aligned}
E[u_{m'}(I)|g] &\gtrless E[u_{m'+1}(I)|g] \\
q_G(1-\gamma)F_{m,G}(k-1) \\
-(1-q_B)\gamma F_{m,B}(k-1) &\gtrless q_G(1-\gamma)F_{m+1,G}(k-1) \\
&\quad -(1-q_B)\gamma F_{m+1,B}(k-1) \\
q_G(1-\gamma)[F_{m,G}(k-1)-F_{m+1,G}(k-1)] &\gtrless (1-q_B)\gamma[F_{m,B}(k-1)-F_{m+1,B}(k-1)] \\
\frac{q_G(1-\gamma)}{(1-q_B)\gamma} &\gtrless \frac{F_{m,B}(k-1)-F_{m+1,B}(k-1)}{F_{m,G}(k-1)-F_{m+1,G}(k-1)}
\end{aligned} \tag{3.5}$$

Let $I_x(\alpha,\beta)$ denote the regularized incomplete beta function. Since F_{mG} and F_{mB} are binomial distributions we have:

$$\begin{aligned}
F_{mG}-F_{m+1G} &= I_{1-q_G}(m-k+1,k) - I_{1-q_G}(m-k+2,k) \\
&= I_{1-q_G}(m-k+1,k) - I_{1-q_G}(m-k+1,k) + \frac{q_G^k(1-q_G)^{m-k+1}}{(m-k+1)B(m-k+1,k)} \\
&= \frac{q_G^k(1-q_G)^{m-k+1}}{(m-k+1)B(m-k+1,k)}
\end{aligned} \tag{3.6}$$

Here $B(m-k+1,k)$ represents the beta function. Similarly we get:

$$\begin{aligned}
F_{mB}-F_{m+1B} &= I_{q_B}(m-k+1,k) - I_{q_B}(m-k+2,k) \\
&= \frac{(1-q_B)^k q_B^{m-k+1}}{(m-k+1)B(m-k+1,k)}
\end{aligned} \tag{3.7}$$

Thus from 3.5,3.6 and 3.7 we obtain:

$$\begin{aligned}
\frac{q_G(1-\gamma)}{(1-q_B)\gamma} &\gtrless \frac{(1-q_B)^k q_B^{m-k+1}}{q_G^k(1-q_G)^{m-k+1}} \\
\frac{1-\gamma}{\gamma} &\gtrless \left(\frac{1-q_B}{q_G}\right)^{k-1}\left(\frac{q_B}{1-q_G}\right)^{m-k+1}
\end{aligned}$$

When the RHS is smaller then $E[u_{m'}(I)|g] > E[u_{m'+1}(I)|g]$. It is easy to see that the RHS is increasing in m, since $q_B > 1-q_G$ (from Condition 3.1). It is easy to see that for $m = k$ which is the smallest possible value for m the RHS can be smaller than the LHS. We have:

$$\frac{1-\gamma}{\gamma} \gtrless \left(\frac{1-q_B}{q_G}\right)^{k-1} \frac{q_B}{1-q_G}$$
$$\frac{1-qG}{q_B} \gtrless \left(\frac{1-q_B}{q_G}\right)^{k-1} \frac{\gamma}{1-\gamma}$$

Which for sufficiently high k gives $LHS > RHS$. Still, as m grows the inequality must eventually switch and remain switched. This shows that $E[u_{m'}(I)|g]$ may be initially decreasing in m and then becomes increasing. This makes it either an increasing or a quasi-concave function of m.

We now show that it can not be that it is increasing and 3.4 holds. We do so by contradiction. Suppose it is. Then we have:

$$\frac{\gamma}{1-\gamma}\left(\frac{1-q_B}{q_G}\right)^{k-1}\left(\frac{q_B}{1-q_G}\right)^{m-k+1} > 1 \tag{3.8}$$

and from 3.4

$$\frac{1-\gamma}{\gamma}\frac{q_G}{1-q_B} > \frac{F_{mB}(k-1)}{F_{mG}(k-1)} \tag{3.9}$$

But then:

$$\begin{aligned}
\frac{\gamma}{1-\gamma}\left(\frac{1-q_B}{q_G}\right)^{k-1}\left(\frac{q_B}{1-q_G}\right)^{m-k+1} &< \frac{\gamma}{1-\gamma}\frac{1-q_B}{q_G}\left(\frac{q_B}{1-q_G}\right)^{m-k+1} && \{\text{since } q_G > 1-q_b\} \\
&< \frac{F_{mG}(k-1)}{F_{mB}(k-1)}\left(\frac{q_B}{1-q_G}\right)^{m-k+1} && \{\text{from 3.9}\} \\
&< \frac{\sum_{i=0}^{k-1}\binom{m}{i}q_G^i(1-q_G)^{m-i}}{\sum_{i=0}^{k-1}\binom{m}{i}(1-q_B)^i q_B^{m-i}}\left(\frac{q_B}{1-q_G}\right)^{m-k+1} \\
&< \frac{\sum_{i=0}^{k-1}\binom{m}{i}q_G^i(1-q_G)^{k-1-i}}{\sum_{i=0}^{k-1}\binom{m}{i}(1-q_B)^i q_B^{k-1-i}} \\
&= 1
\end{aligned}$$

Which contradicts 3.8! This shows that when $E[u_{m'}(I)|g]$ is increasing, 3.4 does not hold. Since we already showed that $E[u_{m'}(I)|g]$ becomes increasing in m as m grows, this shows that eventually as it does so an agent will not play informatively and so will all agents after him. Notice that the LHS of 3.4 goes to zero as m grows. This means the inequality never switches back. After one agent switches away from informative play, so do all agents after him.

□

Proof. (**Proposition 2)** Let $W(m)$ denote the sum of expected utilities from a pure strategy profile in which m agents play informatively. For the equilibrium profile we have:

$$W\left(\hat{m}(k,q_A,q_B)\right) = \sum_{m=1}^{\hat{m}(k,q_A,q_B)} E[u_m(x)] + (N - \hat{m}(k,q_A,q_B))\gamma$$

From the proof of proposition one we know that for an agent m playing informatively: $E[u_m(x)] > \gamma$ for $m < \hat{m}(k,q_A,q_B)$ and $E[u_m(x)] < \gamma$ for $m < -\hat{m}(k,q_A,q_B)$. It is therefore immediate to see that:

$$\begin{aligned}\sum_{m=1}^{m'} E[u_m(x)] + (N-m')\gamma &< \sum_{m=1}^{\hat{m}(k,q_A,q_B)} E[u_m(x)] + (N - \hat{m}(k,q_A,q_B))\gamma \\ &< \sum_{m=1}^{m''} E[u_m(x)] + (N-m'')\gamma\end{aligned}$$

Which summarizes to:

$$W(m') < W\left(\hat{m}(k,q_A,q_B)\right) < W(m'')$$

for $m' < \hat{m}(k,q_A,q_B) < m''$. Also note that this is independent of N, which proves the proposition. □

Proof. (**Proposition 3)**

$$\begin{aligned} E_{k+1}[u_m(A)|a] &> E_k[u_m(A)|a] \\ q_A F_{mA}(k+1) + (1-q_B)\left(1 - F_{mB}(k+1)\right) &> q_A F_{mA}(k) + (1-q_B)\left(1-F_{mB}(k)\right) \\ \frac{q_A}{1-q_B} &> \frac{F_{mB}(k+1) - F_{mB}(k)}{F_{mA}(k+1) - F_{mA}(k)} \\ &> \frac{f_{mB}(k+1)}{f_{mA}(k+1)} \\ &> \frac{\binom{m}{k+1}(1-q_B)^{k+1} q_b^{m-k-1}}{\binom{m}{k+1} q_A^{k+1}(1-q_A)^{m-k-1}} \\ 1 &> \left(\frac{1-q_B}{q_A}\right)^k \left(\frac{q_B}{1-q_A}\right)^{m-k} \end{aligned} \tag{3.10}$$

From the proof of proposition 1 we know that 3.10 must hold for $\hat{m}$. This player is the first (in order of priority) that herds. We show that increasing k increases $\hat{m}$'s expected payoff from playing informatively and therefore he eventually switches to that strategy. This makes $\hat{m}+1$ the first player to herd. □

Proof. (**Proposition 4)** We know from proposition 1 that given Condition 3.1 any agent that receives signal $s_i = b$ best replies by not investing. Let σ denote the probability with which an agent decides to play informatively. For an agent that receives signal $s_i = g$ the expected payoff from investing given that all other agents play strategy $\sigma \in [0,1]$ is:

$$E_\sigma[u(I)|g] = \frac{q_G}{q_G + 1 - q_B}\left[\frac{k}{N} + \frac{1}{N}\sum_{m=k+1}^{n-1}\sum_{i=0}^{k-1}\binom{m}{i}\sigma^i(1-\sigma)^{m-i}\right] + \frac{1}{N}\sum_{m=k+1}^{n-1}\sum_{i=k}^{m}\binom{m}{i}\sigma^i(1-\sigma)^{m-i}E[u_{m+1}(I)|g]$$

It is easy to sea that given the properties of the binomial distribution and the fact that as was shown in the proof of proposition 1 $E[u_{m+1}(I)|g] \leq \frac{q_G}{q_G+1-q_B}$, the above expression is decreasing in σ. The symmetric equilibrium strategy σ^* is the one that solves the following equation:

$$E_{\sigma^*}[u(I)|g] = \gamma \tag{3.11}$$

Note that for $\sigma = 0$ we have:

$$E_\sigma[u(I)|g] = \frac{q_G}{q_G + 1 - q_B} > \gamma$$

And for $\sigma = 1$ we obtain:

$$E_\sigma[u(I)|g] = \frac{q_G}{q_G + 1 - q_B}\frac{k}{N} + \frac{1}{N}\sum_{m=k+1}^{n-1} E[u_{m+1}(I)|g]$$

The RHS in the last expression can be less than γ if k is low enough or γ is high enough. When this is the case, 3.11 has a unique solution $\sigma^* \in (0,1)$. Otherwise, the unique symmetric equilibrium obtains for $\sigma^* = 1$. □

Proof. (**Proposition 5)** Consider a strategy profile in which $\tilde{m}$ agents play informatively and all others herd. It is easy to see that $\tilde{m} < k$ can not be an equilibrium profile. Suppose it were. Then an agent i that herds and observes $s_i = g$ is better of investing since he can obtain a slot and his payoff will be

$E[u_i(I)|g] = \frac{q_G}{q_G+1-q_B} > \gamma = E[u_i(O)|g]$. Thus it is not an equilibrium.

Consider $\tilde{m} \geq k$. Suppose i is in the set of agents that play informatively and receives signal $s_i = g$. His expected payoff from investing is:

$$E_{\tilde{m}}[u(I)|g] = \frac{q_G}{q_G + 1 - q_B}\frac{k}{\tilde{m}} + \frac{1}{\tilde{m}}\sum_{m=k+1}^{\tilde{m}-1} E[u_m(I)|g] \tag{3.12}$$

For $\tilde{m}$ to characterize an equilibrium profile it must be that:

$$\begin{aligned} E_{\tilde{m}}[u(I)|g] &\geq \gamma \\ E_{\tilde{m}+1}[u(I)|g] &< \gamma \end{aligned}$$

The first of these two conditions guarantees that no agent playing informatively has an incentive to deviate. The second does the same for the agents herding. We know that these conditions are necessary and sufficient from the properties of $E[u_m(I)|g]$ derived in the proof of proposition 1. It is immediate to see that $\tilde{m} < \hat{m}(k, q_A, q_B) - 1$ cannot be an equilibrium. By definition, $E[u_i(I)|g] > \gamma$, $\forall\ i < \hat{m}(k, q_A, q_B)$. Thus, such a profile would violate the second of the equilibrium conditions above. □

Proof. (**Proposition 6**) Let:

$$g_A(m,k) = \begin{cases} 1\ , & m \leq k \\ F_{mA}(k)\ , & m > k \end{cases}$$

and

$$g_B(m,k) = \begin{cases} 1\ , & m \leq k \\ F_{mB}(k)\ , & m > k \end{cases}$$

Then, given k:

$$
\begin{aligned}
E_t[u_{m+1}(A)|a] = & \\
&= \frac{q_A}{q_A + 1 - q_B}\left[\sum_{m=0}^{t}\binom{t}{m}p^m(1-p)^{t-m}\cdot g_A(m,k)\right] \\
&+ \frac{1-q_B}{q_A + 1 - q_B}\left[1-\sum_{m=0}^{t}\binom{t}{m}p^m(1-p)^{t-m}\cdot g_B(m,k)\right] \\
&= \frac{1}{q_A + 1 - q_B}\left[\sum_{m=0}^{t+1}\binom{t}{m}p^m(1-p)^{t-m}\left(q_A g_A - (1-q_B)g_B\right) + (1-q_B)\right] \\
&= \frac{1}{q_A + 1 - q_B}\left[\sum_{m=0}^{t+1}\binom{t}{m}p^m(1-p)^{t-m}E[u_{m+1}(A)|a] + (1-q_B)\right]
\end{aligned}
$$

Notice that for $m < \hat{m}(k, q_A, q_B)$, $E_t[u_{m+1}(A|a] > 0$, thus the above expression is positive. This means that all agents arriving at $t < \hat{m}(k, q_A, q_B)$ play informatively. This proves the minimum bound on $\hat{m}(k, q_A, q_B)$. From now on consider $t \geq \hat{m}(k, q_A, q_B)$.

Remember that $E[u_{m+1}(A)|a]$ is a quasi-concave function. Let $\tilde{m}$ be such that this function is decreasing for $m \leq \tilde{m}$ and increasing for $m > \tilde{m}$. From the proof of proposition 1 we have that $\tilde{m} > \hat{m}$. Then we have:

$$
\begin{aligned}
&(q_A + 1 - q_B)E_t[u_{m+1}(A)|a] = \\
&+\sum_{m=0}^{\tilde{m}}\binom{t}{m}p^m(1-p)^{t-m}E[u_{m+1}(B)|a] + \sum_{m=\tilde{m}}^{t}\binom{t}{m}p^m(1-p)^{t-m}E[u_{m+1}(B)|a] + (1-q_B)
\end{aligned}
$$

From stochastic dominance for the Bernoulli distribution we have that the first summation is decreasing in t. The second summation is always negative. The last term is constant with respect to t. Thus, as t increases, the expected payoff crosses zero at most once. $\hat{t}(k, q_A, q_B, p)$ is such that the expected payoff is negative for this value and positive for all smaller t. The agent arriving at $\hat{t}(k, q_A, q_B, p)$ ignores his private information and herds. All agents arriving after $\hat{t}(k, q_A, q_B, p)$ have the same expected payoff and herd as well. □

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Chapter 4

Seeing is Believing? An Experiment on Strategic Thinking.

The chain of information is based on inferences and information choices made by others. Can we really make a forecast if all forecasts depend on others' forecasts, and so on?

Prof. K. Arrow, XXIII Barcelona GSE Lecture

4.1 Introduction

According to a known anecdote, the comedian Groucho Marx (GM) once sent a message to the Friar's Club of Beverly Hills to which he belonged, saying: "Please accept my resignation. I don't want to belong to any club that will accept people as me as their member". [1] Is the behavior of the comedian in accordance with the prescriptions of classical game theory, embodied in the notions of the Bayesian Nash and Perfect Bayesian equilibria? Well, GM observed the club's action - accepting him as a member, inferred that such action conveyed the information that the club is not of his liking and therefore resigned. Learning from the actions of other agents is exactly what classical game theory prescribes. Still, were GM "perfectly bayesian" he would never have applied for the club in the first place! By applying he either gets rejected and stays out of the club, or accepted and becomes member of a club that reveals itself to be unacceptable to him, leading him to resign. Not applying keeps him out of the club saving him all the trouble from the start.

[1]This incident inspired M. Harris to name a theorem of Migrom and Stockey (1982) the "Groucho Marx theorem". That result is only tangentially related to this paper.

While one can only speculate why the comedian chose to apply to the club in the first place, let us assume he lacked the strategic sophistication needed for performing the above reasoning. In contrast to classic game theory, recent theories, such as "cursed equilibrium" or level-k reasoning try to explain why some agents may be less sophisticated than others. Nevertheless, no theory explains how GM can be sophisticated enough to resign from the club after being accepted, but not so in order to never apply in the first place. Can we observe such a behavior outside the realm of anecdotes? We offer experimental evidence that suggest we can. We develop and implement a novel experimental design that allows us to classify subjects according to their level of strategic sophistication and to identify individuals that behave like GM. A significant portion of our subjects fits this class. Based on some additional experimental evidence and the subjects' own interpretations of their behavior we conjecture upon the drivers of such behavior.

It must be made clear right from the start that what we are concerned with are players' initial responses to games. We do not consider the case where a game is played more times, allowing players to learn and develop better strategies.

To fix the ideas, let us analyze our example a bit further. We must note that reaching GM's conclusion as stated in the letter is not trivial in itself. One has to realize that others' actions are driven by their private information (in this case, the quality of the club) and infer this information by observing their actions. Let us call this *Information Inference from Observed Actions (IIOA)*. The reasoning prescribed by classical game theory and that should prevent GM from applying, adds an additional layer of sophistication. A player should form expectations about the outcomes of the game, given his actions, that are conditional on the possible actions of others *and* the information that drives them. Let us call this *Information Inference from Future Actions (IIFA)*.[2] IIOA is embedded in IIFA in the sense that an individual should be able to perform the latter only if she is able to perform the former. On the other hand, there is no obvious reason why ability to perform IIOA should imply an ability to perform IIFA. This subtle, although substantial point has escaped the attention of most analysis of strategic thinking.

A lack of strategic sophistication on part of some agents has recently been advanced as a possible explanation for the failure of the Bayesian Nash equilibrium notions to predict phenomena such as the winner's curse (WC) or the existence of trade in markets with adverse-selection problems. Eyster and Rabin (2005) (14) propose the notion of "Cursed equilibrium". A "cursed" player correctly predicts the distribution of other players' actions, but fails to recognize their informational

[2]Here use the term "future actions" for actions that have not been observed. This does not mean that these actions necessarily take place after one's decision. It means that the individual may only find out what these actions are after taking her own decision.

content. In our terms, they allow for players that are not able to perform IIOA, something that automatically excludes IIFA as well. The lack of sophistication of "cursed" players is directly connected to the existence of private information. In complete information environments such a player would not behave any differently than a "Nash type" player.

The theory of level-k reasoning on the other hand was advanced to explain behavior in complete information environments (Stahl and Wilson, 1995 (20); Nagel, 1995 (18)) and has recently been applied to games with private information(Crawford and Irriberi, 2007 (10)). It posits that players reason at different levels. Level-0 players pick an action randomly with a probability uniformly distributed over their action space. A level-k player believes all others to be of level $k-1$ and plays a best response to their strategy. In a game with private information, given the definition of level-0 players, a level-1 player's best response must depend on his private information alone. He fails to realize that other players may not be level-0 players and their actions may have and informational content.[3] Level-2 players consider everybody else to be a level-1 player and best reply to that. They recognize that others' actions reveal their private information. Although a level-2's behavior may still not coincide with a "Bayesian Nash" player's behavior, he is performing IIOA and IIFA (if necessary) in order to calculate his best response. Here, in contrast to "cursed equilibrium" theory, players are not considered unable to perform either IIOA or IIFA. Still, given the way players form their beliefs about others, level-1 players do not use neither of the two to calculate their best response.

While these theories allow for a lack of strategic sophistication that causes a failure to perform IIOA or IIFA, this is introduced in a way that does not capture the difference between the two. Actually, for most applications of such theories to date this failure is irrelevant. These theories have been applied to games with private information in order to explain empirical observations (field or lab data) that conventional game theory failed to predict. In each of these cases standard game theory expects players to either perform IIOA or the more complex IIFA. As long as a significant portion of players fails to perform either of the two, the alternative theories that allow for different levels of sophistication will generally perform better in terms of prediction. Whether these players fail only in IIFA or both is not an issue, since only one of the two is required in these applications. But if one needs a theory not only to fit existing data but to also make predictions of how different designs of markets or institutions affect the behavior of participants, this distinction becomes important. Consider an agent with the sophistication of GM: able to perform IIOA but not IIFA. Participating in a sealed bid common

[3] In the strict version of this theory, level-0 players exist only in the minds of level-1 players and one should not expect to observe any level-0 behavior.

value auction he is likely to be a victim of the WC. If on the other hand the design changes to that of an ascending price auction, the WC is less likely to be a problem for this agent. Similarly, his behavior as a juror should be different if the jury votes simultaneously or sequentially, observing what others vote.

We use an experimental design based on the model in Louis (2011) (17). It allows us to observe subjects' behavior in a game played both simultaneously and sequentially. We can therefore classify them based on that behavior in to different types. One of these matches the behavior of a GM type individual and it is distinct from the behavior predicted by any other theory.

4.1.1 Literature Review

The literature related to this paper can be divided in two broad and related themes. On the one hand there is research that looks at different games and tries to verify through experiments or empirical observations whether individuals' behavior is the one predicted by the appropriate game theoretic solution concepts. On the other hand, and more recently, scholars have developed and adapted new theories of strategic thinking that give predictions closer to the behavior observed in the lab and field. Our work moves to a new direction. Although this paper does not provide a new theory for strategic behavior, it points out some blank spots in existing theories. Furthermore, using an appropriate experimental design it demonstrates how because of these blank spots, existing theories fail to accurately describe individuals' strategic thinking in ways that can be relevant in economic applications.

It was a group of engineers that noticed for the first time a systematic and significant deviations of economic agents from the prescriptions of the classic game theoretic notions of Bayesian-Nash equilibrium. Capen, Clapp and Campbell (1971) (8) noticed how petroleum companies' returns from leases acquired through competitive bidding were lower than expected. Their explanation involved the failure of companies to take in to account the fact than winning the auction is an indication of overbidding, and adjust their bids accordingly. They termed this phenomenon a winner's curse. Thaler (1988) (21) provides a survey of the early empirical and experimental evidence supporting the existence of such curse. In our context, the winner's curse could be attributed to a failure to perform IIFA.

Moving from an environment of simultaneous actions to one of sequential actions, we find the literature on informational cascades. Banerjee (1992) (5) and Bikhchandani et at. (1992) (6) showed that in environments with a common value and social learning, agents can infer the information of others' by observing their behavior. This can give rise to rational herding behavior were a significant proportion of individuals end up taking the same action and ignoring their private information. Anderson and Holt (1997) (3) were the first to use a lab experiment to test for the existence of informational cascades. For more recent experimental studies in to the matter the reader can can look at Goeree et al. (2007) (15) and the references therein. Although the length and robustness of cascades is affected by the specific test environment, cascades do appear in the lab. Bikhchandani and Sharma (2001) (7) provide a survey of the related empirical literature while Drehmann et. al. (2005) (13) and Alevy et al. (2007) (1) use field experiments to detect the existence of cascades outside the lab. Putting informational cascades in to our context, it requires agents being able to perform IIOA.

The literature on voting touches on both IIOA and IIFA. Austen-Smith and Banks (1996) (4) were the first to notice that the ability of agents to perform IIFA may prevent a committee voting simultaneously to aggregate information efficiently. On the other hand, Dekel and Piccione (2000) (12) compare this situation with one in which agents vote sequentially, and assuming they fully rational (able to perform both IIOA and IIFA). Ottaviani and Sørensen (2001) (19) look at the possibility of informational cascades when voting is sequential. The idea of strategic voting as proposed by Austen-Smith and Banks has found support in experimental evidence in Guarnaschelli et al. (2000) (16) and Nageeb et al (2008) (2).

As was mentioned above, alternative theories that abandon perfect rationality in the strategic thinking of agents have been put forward, especially as a response to the failure of classic game theory to explain the persistence of the winner's curse in various applications. Eyster and Rabin (2005) (14) proposed the concept of "cursed equilibrium". Cursed agents fail to take in to account the informational content of others' actions. Translated to our context, a "cursed" individual is not able to perform IIOA and therefore, neither IIFA. Crawford and Iriberri (2007) (10) apply the "level-k" thinking model to environments with incomplete information. In such a model, level-1 individuals lack the sophistication to perform both IIOA and IIFA. Individuals of level-2 and higher can perform both. Neither "cursed equilibrium" nor "level-k" thinking allow for individuals of the GM type: able to perform IIOA but not IIFA.

Charness and Levin (2009) (9) also take a critical approach to these alternative theories, especially as far as the winner's curse is concerned. They use an experimental design that allows for the winner's curse to be detected, but precludes "cursed" behavior or "level-k" thinking as explanations. They find that simple bounded rationality, as opposed to unsophisticated strategic thinking, can go a long way in explaining the curse. Although our paper shares the critical nature of their approach, it uses a different strategy. We look at a situation where existing theories make predictions about individual behavior. We show that they fail to a significant extent.

Crawford, Costa-Gomes and Iriberri (2010) (11) offer a very complete survey of the literature in strategic thinking.

4.2 Experimental Design

The aim of our experiment is to verify the existence of individuals that seem able to perform IIOA but fail to perform IIFA. For brevity we shall call such an individual a GM type. To achieve that, we have subjects playing a game both simultaneously and sequentially. The game is such that a GM type individual is expected to play differently in each case and in a way not predicted by any of the relevant theories.

The game is based on an n-player matching market game with limited supply, found in Louis (2011) . Here we use the 2-player version. Players are offered a single object which they can accept or reject. Only one player can keep the object even if both accept it. Player 1 has priority. In the sequential version of the game, player 1 decides whether to accept or reject. If he accepts, he keeps the object and the game ends. If he rejects, player 2 is allowed to choose. If he accepts he keeps the object, and if he rejects then no player keeps the object. In the simultaneous version, both players must choose simultaneously. The object is assigned to any player that has accepted, respecting priority: if both accept player 1 keeps the object.

Payoffs depend on the state of nature which can be "good" or "bad" with a priori equal probability. If the state is "good", a player with the object gets a payoff of 1 and a player without the object gets zero payoff. If the state is "bad", a player with the object gets zero while a player without the object gets 1. Players have private information about the state of nature. This takes the form of a binary signal $s_i \in \{g, b\}$. If the state is "good", the probability of the signal being $s_i = g$ is $Pr(s_i = g|\text{"}good\text{"}) = 1$. If the state of nature is "bad", $Pr(s_i = b|\text{"}bad\text{"}) = q$. Given this information structure, if the signal is b, it perfectly reveals the state of nature to be "bad". The signal g means the state of nature is more likely to be "good".

In this game there is a possibility for player 2 to suffer from a type of winner's curse. The idea is the following: player 1 is not affected in any way by the actions of player 2. This means he faces a simple decision problem and therefore the most natural thing for him to do is to follow his private signal. This means to accept the object when observing $s_1 = g$ and rejecting it when $s_1 = b$. Notice that in this last case player 1 actualy knows that the state of nature is "bad". If player 1 plays like this, then player 2 can only keep the object when the state is "bad", and thus accepting it can only make him a victim of the winner's curse. Notice that whether the action of player 1 is observable or not (sequential or simultaneous play) is irrelevant for the existence of the WC. It is relevant as to what type of reasoning is required for player 2 to recognize the WC in order to avoid it. In the sequential game IIOA is enough. In the simultaneous game, the more sophisticated IIFA is required.

Before entering the details of how the design was implemented in the actual experiment, we brie y explain the different theoretical predictions about the players' behavior. It must be made clear from the start that our focus lies on player 2. It is this player that could use IIOA or IIFA and hence his ability to perform either affects his behavior. Player 1 is simply instrumental. Although part of the game, he faces a simple decision problem with no need or possibility to perform either reasoning process (IIOA or IIFA) and his behavior is not predicted to vary by any theory.

4.2.1 Theoretical predicitons

First let us fix notation. The subscript $i \in \{1,2\}$ denotes the player. Players makes a choice $x_i \in X = \{A, R\}$. The state of nature is $\theta \in \Theta = \{G, B\}$. Let $f_i : X^2 \to X$ be the assignment function. In particular,

$$f_1(x_1, x_2) = x_1$$

and

$$f_2(x_2, x_1) = \begin{cases} x_2, & for\ x_1 = R \\ R, & for\ x_1 = A \end{cases}$$

. For notational economy we use the set of choices to also denote the set of outcomes. The outcome A means the player keeps the object, while the outcome R means he does not keep it. Payoffs are given by

$$u_i\left(f(x_i, x_{-i}), \theta\right) = \begin{cases} 1, & f(x_i, x_{-i}) = A \ \&\ \theta = G \\ 1, & f(x_i, x_{-i}) = R \ \&\ \theta = B \\ 0, & otherwise \end{cases}$$

We say that a player *plays informatively* if his choice corresponds to his signal. That is if $x_i = A$ when $s_i = g$ and $x_i = R$ when $s_i = b$. On the other hand we say a player *herds* if he chooses to reject the object independently of his private signal. The case of always accepting does not come up in any situation so we have no name for it. Also, for the economy of the analysis we must note that if a player has private signal $s_i = b$, he knows for sure that the state of nature is "bad". Since we assume players to be individually rational, it is always optimal for them to reject after observing this private signal, independently of the other's choices. Thus the analysis below focuses on what players when observing private signal $s_i = g$.

Bayesian Nash equilibrium

Player 1
Player 1 faces a simple decision problem. The choice of player 2 does not affect his outcomes. If his private signal is $s_1 = b$ he knows the state of nature is $\theta = B$ and thus the optimal choice is to reject. If $s_1 = g$ then his expected payoff from accepting is:

$$\begin{aligned} E[u_1(x_1 = A, \theta)|s_1 = g] &= Pr(\theta = G|s_1 = g) \\ &= \frac{1}{1 + 1 - q} \\ &= \frac{1}{2 - q} \end{aligned}$$

His expected payoff from rejecting is:

$$\begin{aligned} E[u_1(x_1 = R, \theta)|s_1 = g] &= Pr(\theta = B|s_1 = a) \\ &= \frac{1 - q}{1 + 1 - q} \\ &= \frac{1 - q}{2 - q} \end{aligned}$$

It is easy to see that $E[u_1(x_1 = A, \theta)|s_1 = g] > E[u_1(x_1 = R, \theta)|s_1 = g]$ and therefore it is optimal to accept. Hence, player 1 plays informatively. Notice that this is true independently of whether the game is played simultaneously or sequentially. This is because player 1 knows he has priority over player 2.

Player 2 - Simultaneous play.
Now consider player 2. He calculates expected payoffs taking in to account player 1's strategy. Since player 1 plays informatively, player 2's expected payoff from accepting when $s_2 = a$ is:

$$\begin{aligned} E[u_2(f_2(A, x_1), \theta)|s_2 = g] = &\, Pr(\theta{=}G|s_2{=}g)\,[Pr(s_1{=}g|\theta{=}G) \cdot 0 + Pr(s_1{=}b|\theta{=}G) \cdot 1] \\ &+ Pr(\theta{=}B|s_2{=}g)\,[Pr(s_1{=}g|\theta{=}B) \cdot 1 + Pr(s_1{=}b|\theta{=}B) \cdot 0] \\ =&\, Pr(\theta{=}B|s_2{=}g) Pr(s_1{=}g|\theta{=}B) \\ =&\, \frac{(1 - q)^2}{2 - q} \end{aligned}$$

His expected payoff from choosing B when $s_2 = g$ is:

$$E[u_2(f_2(R,x_1),\theta)|s_2 = g] = Pr(\theta = B|s_2 = g) = \frac{1-q}{2-q}$$

We see that $E[u_2(f_2(A,x_1),\theta)|s_2 = g] < E[u_2(f_2(R,x_1),\theta)|s_2 = g]$ and thus player 2 rejects. Since he does the same when his signal is $s_2 = b$, we say player 2 herds. Notice that these calculations assume that player 2 is able to perform IIFA.

Player 2 - Sequential play.
Now player 2 observes player 1's choice before making his own. If player 1 accepts, then there is no choice to be made by player 2, since player 1 keeps the object, given his priority. If player 1 rejects, then player 2 must choose. But player 2 knows that player 1 rejects only after observing $s_1 = b$ which means that the state of nature must be "bad". Thus he also rejects independently of his signal. Notice here that this reasoning requires IIOA (but not IIFA).

Cursed equilibrium

A cursed equilibrium is the predicted outcome of a game played by cursed players. A fully cursed player can correctly predict the distribution of the others' actions but does not recognize the informational content of these[4]. According to the cursed equilibrium theory players' "cursedness' ' is measured by a parameter χ. With probability χ a players beliefs are the ones of a fully cursed player and with probability $1 - \chi$ they coincide with the ones of a rational player.

Player 1.
Here again player 1 faces a decision problem and his payoff is not affected by player 2's actions. Therefore it does not matter whether he is cursed or not. His optimal play is the same as in the case of bayesian-nash players for any degree of "cursedness" χ. Player 1 plays informatively.

Player 2 - Simultaneous play.
A fully cursed player 2 correctly predicts the distribution of player 1's actions.

[4]Such behavior gives rise to the winner's curse in auctions and other common-value environments, hence the term "cursed"

This means he believes that player 1 accepts with probability:

$$
\begin{aligned}
Pr(x_1 = A|s_2 = g) = Pr(s_1 = g|s_2 = g) &= \sum_{\theta\in\Theta} Pr(s_1 = g, \theta|s_2 = g) \\
&= \sum_{\theta\in\Theta} [Pr(s_1 = g|\theta) \cdot Pr(\theta|s_2 = g)] \\
&= \frac{1}{2-q} + (1-q)\frac{1-q}{2-q} \\
&= 1 - \frac{q(1-q)}{2-q}
\end{aligned}
$$

and rejects with the complementary probability:

$$
Pr(x_1 = R|s_2 = g) = \frac{q(1-q)}{2-q}
$$

Therefore, his expected payoff with these cursed beliefs from accepting after observing $s_2 = g$ is:

$$
\begin{aligned}
E_c[u_2(f_2(A, x_1), \theta)|s_2 = g] &= Pr(\theta{=}G|s_2{=}g) \left[Pr(x_1{=}A) \cdot 0 + Pr(x_1{=}R) \cdot \gamma\right] \\
&\quad + Pr(\theta{=}B|s_2{=}g) \left[Pr(x_1{=}A) \cdot 1 + Pr(x_1{=}R) \cdot 0\right] \\
&= Pr(\theta{=}G|s_2{=}g) \cdot Pr(x_1{=}R) \cdot \gamma + Pr(\theta{=}B|s_2{=}g) \cdot Pr(x_1{=}A) \\
&= \frac{1}{2-q}\frac{q(1-q)}{2-q} + \frac{1-q}{2-q}(1 - \frac{q(1-q)}{2-q}) \\
&= \frac{1-q}{2-q} + \frac{q^2(1-q)}{(2-q)^2}
\end{aligned}
$$

His (cursed) expected payoff from rejecting after observing $s_2 = g$ is:

$$
\begin{aligned}
E_c[u_2(f_2(R, x_1), \theta)|s_2 = g] &= Pr(\theta = B|s_2 = g) \\
&= \frac{1-q}{2-q}
\end{aligned}
$$

Since $E_c[u_2(f_2(A, x_1), \theta)|s_2 = g] > E_c[u_2(f_2(R, x_1), \theta)|s_2 = g]$, player 2 accepts. This means a fully cursed player 2 plays informatively.

Now consider the case where player 2 is not fully cursed, rather his degree of "cursedness" depends on the parameter χ. His expected payoff based on these partially cursed beliefs is a convex combination between the expected payoff calculated by a rational player and the one of a fully cursed player:

$$\begin{aligned}E_\chi[u_2(f_2(A,x_1),\theta)|s_2=g] = {}& \chi\cdot[\frac{1-q}{2-q}+\frac{q^2(1-q)}{(2-q)^2}]\\ &+(1-\chi)\cdot[\frac{(1-q)^2}{2-q}]\end{aligned}$$

and

$$\begin{aligned}E_\chi[u_2(f_2(R,x_1),\theta)|s_2=a] ={}& Pr(\theta=R|s_2=g)\\ ={}& \frac{1-q}{2-q}\end{aligned}$$

In order to play informatively the following condition must hold:

$$E_\chi[u_2(f_2(A,x_1),\theta)|s_2=g] > E_\chi[u_2(f_2(R,x_1),\theta)|s_2=g]$$
$$\Leftrightarrow$$

$$\begin{aligned}\chi\cdot[\frac{1-q}{2-q}+\frac{q^2(1-q)}{(2-q)^2}]+(1-\chi)\cdot[\frac{(1-q)^2}{2-q}] >{}& \frac{1-q}{2-q}\\ \frac{2-q}{2} <{}& \chi\end{aligned}$$

Thus there is a threshold level for χ. For values above the threshold player 2 plays informatively. For values of χ below the threshold he herds.

Player 2 - Sequential play.
Now player 2 observes the choice of player 1 so there is no question about whether he correctly predicts the distribution of player 1's actions. Still, a fully cursed player 2 does not realize that when player 1 rejects he does so because he observed $s_1=b$. When player 1 accepts the game stops, therefore we focus on the case where player 1 rejects. Player 2's expected payoff from accepting when observing $s_2=g$ is:

$$\begin{aligned}E_c[u_2(f_2(A,R),\theta)|s_2=g] ={}& Pr(\theta=G|s_1=g)\\ ={}& \frac{1}{2-q}\end{aligned}$$

The expected payoff from rejecting is:

$$\begin{aligned} E_c[u_2(f_2(R,R),\theta)|s_2 = g] =& Pr(\theta = B|s_2 = g) \\ =& \frac{1-q}{2-q} \end{aligned}$$

Again we see that a fully cursed player 2 has a higher expected payoff by accepting, so he does so.

Now we consider a partially cursed player 2. Given a value for χ his expected payoff from accepting is:

$$E_\chi[u_2(f_2(A,R),\theta)|s_2 = g] = \chi \cdot \frac{1}{2-q} + (1-\chi)\cdot \frac{(1-q)^2}{2-q}$$

and from rejecting:

$$\begin{aligned} E_\chi[u_2(f_2(R,R),\theta)|s_2 = g] =& Pr(\theta = B|s_2 = g) \\ =& \frac{1-q}{2-q} \end{aligned}$$

In order to play informatively the following condition must hold:

$$\begin{aligned} E_\chi[u_2(f_2(A,R),\theta)|s_2 = g] >& E_\chi[u_2(f_2(R,R),\theta)|s_2 = g] \\ & \Leftrightarrow \\ \chi \cdot \frac{1}{2-q} + (1-\chi)\cdot \frac{(1-q)^2}{2-q} >& \frac{1-q}{2-q} \\ \frac{1-q}{2-q} <& \ \chi \end{aligned}$$

Again there is a threshold as in the case of simultaneous play. For values above the threshold player 2 plays informatively. For values of χ below the threshold he herds.

It is important to notice that $\chi_{seq} = \frac{1-q}{2-q} < \frac{2-q}{2} = \chi_{sim}$. This means that for $\chi \in (\chi_{seq}, \chi_{sim}]$ a partially cursed player 2 herds in the simultaneous game but plays informatively in the sequential game.

Level-k reasoning

According to the theory of level-k reasoning, each player is of a type $k > 0$. A player of type k best responds to the strategy of players of type $k - 1$. For the theory to have a content one must define how level 0 players play. In this context it is reasonable to assume that level-0 players randomize uniformly over the two alternatives. Then, the best response of a level-1 player is to follow his own signal. This coincides with the optimal play of player 1 for both the simultaneous and the sequential game and both for rational and fully cursed players. For player 2 this coincides with the behavior of a fully cursed player 2. A level-2 player 2 should then best respond in the same way as a fully rational player and herd. This is true both for the simultaneous and the sequential game.

Notice that level-1 players here do not perform IIOA in the sequential game and similarly they do not perform IIFA in the simultaneous game. While the observed behavior is the same as in the case of cursed equilibrium, the reason such players play naively is not because they are assumed unable to recognize the informational content of other's actions and use it to calculate their best response. They dot not perform either of these processes because they wrongly believe that other's play in a very naive way and there is no informational content in their actions.

Prediction summary

If players follow the Bayes-Nash prototype, they are expected to herd independently of whether the game is played simultaneously or sequentially. Cursed equilibrium and level-k reasoning allow for players to be of a less sophisticated type. These are fully cursed and level-1 players respectively. Such less sophisticated players are expected to play informatively in both types of games. These theories also allow for more sophisticated players whose behavior is the same as that of Bayes-Nash type players. Cursed equilibrium leaves a window for a type of player that plays differently depending on whether the game is simultaneous or sequential. This would be a player that is "partially cursed". The theoretical basis for such a type of player is not very clear and in any case the predicted behavior is quite counterintuitive. Such a player is expected to play informatively in the sequential game and herd in the simultaneous version. In any case, none of the theories predicts a behavior such as the one described for the GM type, that is to play informatively in the simultaneous game and herd in the sequential one. The following table summarizes these results.

Game	Bayes Nash	Cursed equilibrium			Level-k reasoning		GM type
		Fully cursed or low χ	Medium χ	High χ	level-1	level-2	
Simultaneous	**Herd**	*Informative*	**Herd**	**Herd**	*Informative*	**Herd**	*Informative*
Sequential	**Herd**	*Informative*	*Informative*	**Herd**	*Informative*	**Herd**	**Herd**

4.2.2 Implementation

The experiment took place in the Pompeu Fabra University in Barcelona. Subjects were first and second year students from the Faculty of Economics and Business and the Faculty of Political and Social Sciences that participated in the experiment during the last 15 minutes of a lecture, using pen and paper. A preliminary experiment took place two weeks earlier with a small number of 3rd and 4th year students that acted as player 1 in the game. Subjects in the main experiment were randomly matched to one of the preliminary experiment's subjects and acted as player 2. Experiments with groups 1 to 5 took place in the Spring of 2011 with students of economics, while the remaining experiments were run in the Spring of 2012 with political science students.

Each subject received a document containing instructions and answer sheets for the experiment. Instructions were read out loud at the start and subjects had to complete a five question multiple choice comprehension test to show they understood the instructions. Not completing the test successfully, excluded them from any payment.

After that, each subject played two versions of the game. The different versions are described below. Subjects in the same group played the games in the same order. The information structure was reproduced by a method of urns. In the "good" state the urn only contained 10 white balls. In the "bad" state the urn contained 1 white ball and 9 black balls. This means that the parameter q in the experiment took the value $0,9$.

The strategy method was used to elicit subjects' strategies. They were asked to indicate whether to accept or reject conditional on observing a white or black ball. This was done both for practical reasons as well of "experimental economy". First it shortened the duration of the experiment, since no actual draws had to be made during the 15 minutes of the class[5]. Most importantly though, the strategy method allows us to collect a much greater number of useful observations. To understand why, one has to think that our interest is to see whether subjects play informatively or herd. Has the strategy method not been used, any observation coming from a

[5]in each group a subject volunteered to stay a few minutes after class and make a single draw which counted for all the subjects in the group

subject drawing a black ball (equivalent to $s_i = b$) would be practically useless.

After playing the game, subjects were asked to give an explanation for why they played the way they did in each game. They were also asked to report the grade with which they entered the university.

To determine payments one of the two games was chosen randomly and subjects were payed according to their outcome in that game. Subjects that kept the object in the "good" state or didn't do so in the "bad" state received a payment of 2 euros. These payments took place the next day in the premisses of the university.

Treatments

Each treatment consists in playing different versions of the game. While the game is always the same for player 1, we allow for 5 different versions to be played by player 2.

Version 1: Player 2 is asked to submit his strategy without observing the action of player 1. This is the straightorward simultaneous version of the game.

Version 2: Like in version 1, player 2 is asked to submit his strategy without observing the action of player 1. Still, his attention is called upon the fact that his actions only matter when player 1 rejects.

Version 2+: Player 2 is told he will have to make a choice after observing the action of player 1. Since there will be no need for a choice if player 1 accepts, player 2 is asked to submit a strategy for the case player 1 rejects.

Version 3: Player 2 observes the action of player 1. If player 1 accepts he does nothing. If player 1 rejects he submits his strategy.

Versions 1 and 3 represent the two versions of the game, simultaneous and sequential. 2 and 2+ represent a different framing of these two versions.

4.3 Experimental Results

The experiment was conducted with 5 different groups. Two versions were used in each group in different order. The following combinations were used: 13, 31, 12, 12+, 22+, 2+2. The table in figure 4.1 summarizes the composition of the different groups and their performance in the comprehension test.

Unfortunately, a large number of students that should normally attend the class of groups 4 and 5 showed up in the class of group 3. This led to an unbalanced

Group	1	2	3	4	5	6	7
Treatment	13	31	12	12+	22+	22+	2+2
# Subjects	74	72	69	20	20	54	34
Year	1st	2nd	2nd	2nd	2nd	2nd	1st
Correct Tests	91% (67)	83% (60)	80% (55)	75% (15)	40% (8)	70% (38)	68% (23)
Avg. grade	7.75	6.8	7.7	7.63	6.54	7.16	7.36

Figure 4.1: Descriptive statistics for the different groups

number of observations for these treatments. Furthermore, In group 5, more than half of the subjects failed the comprehension test. For these reasons we exclude results from this group from any further analysis.

The treatments used in groups 1 and 2 are the ones that give us some answers concerning the existence of GM types. We therefore focus our analysis on these groups. The results from groups 3 and 4 are relevant to understand the reasons behind a GM type's behavior. We refer to these results when discussing such explanations and the possibility for further experiments.

In each game a subject could submit one of four possible strategies: ("accept when drawing white", "accept when drawing black"), ("reject", "accept"), ("accept", "reject"), ("reject", "reject"). Notice that the first two are not individually rational. Drawing a black ball perfectly reveals the state to be "bad" and hence rejecting is optimal. Only 3 subjects submitted such strategy and of them only one had successfully completed the comprehension test. We choose to ignore these subjects for the rest of the analysis. The other two strategies correspond to informative play and herding.

Games with incomplete information are generally known to be complex for subjects. In our case, the fact that the game is played only twice and with no feedback about payoffs gives limited chances for subjects to get familiarized with the game. This is the reason for having subjects take the comprehension test. It is positive that the vast majority of subjects answered this test perfectly. We therefore choose to ignore from the rest of the analysis the subjects that have not answered correctly to one or more questions in the test. Including these subjects would not change any of our results.

The above exclusions leave us with 66 subjects in group 1 and 60 subjects in group 2. Note that while for version 1 (simultaneous) we have observations from all non-excluded subjects, for version 3 (sequential) we have observations only for subjects that were matched with players 1's that rejected in the preliminary session. This means that for version 3 we have 32 observations in group 1 and 24 observations in group 2.

First we look at subjects behavior in each version. The following graphs show the proportion of subjects that played informatively or chose to herd in each version for each group.

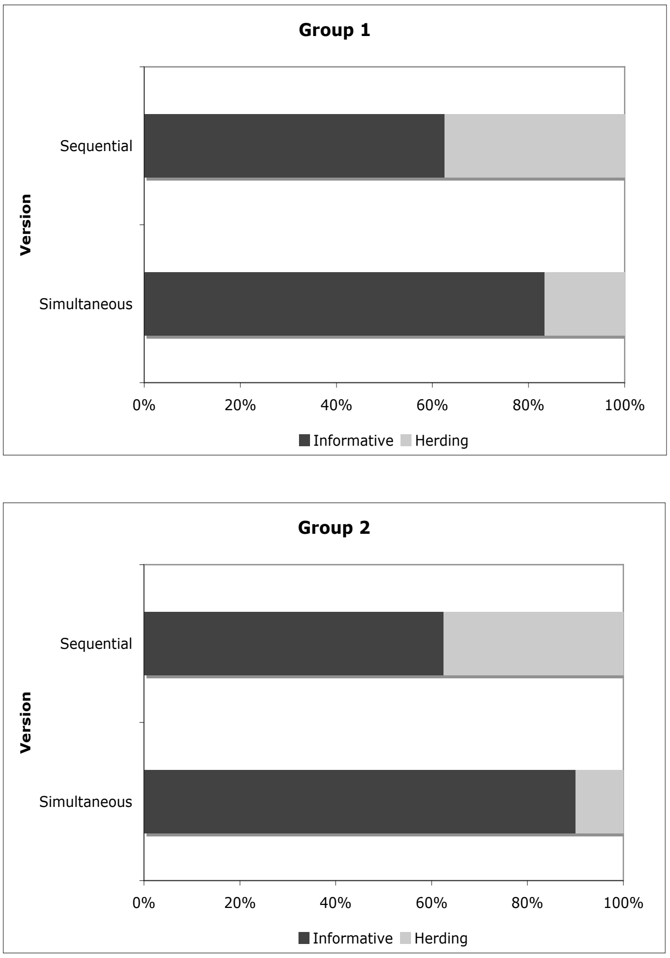


Figure 4.2: Proportion of play in groups 1 and 2

The reason for having subjects in different games play the same games in different order is to control for the possibility of "contamination". That is, playing one type of game first could have a an effect on the play of the second game. It turns out there are no statistically significant differences between the distribution of play in each game in the two groups (p-value of 0.6 and 1 for the simultaneous and sequential game respectively under the two-tailed Fisher's exact test). We shall therefore from now on group all observations together in a single sample.

We now compare play across versions. We have 126 observations for the simultaneous version and 55 for the sequential. The following graph shows the proportion of subjects that play informatively and the ones that herd in each version. 40% of subjects herd in the sequential version, while only 17% does so in the simultaneous one. This difference is statistically highly significant (p-value 0f 0.0025 under the one-tailed Fisher's exact test). Herding is the optimal play in both versions. Still recognizing this in the simultaneous game requires subjects to perform the more sophisticated IIFA. On the other hand, the simpler IIOA is needed to recognize herding as an optimal play in the sequential game. The higher proportion of subjects herding in the sequential game could re ect this difference in complexity among the two games.

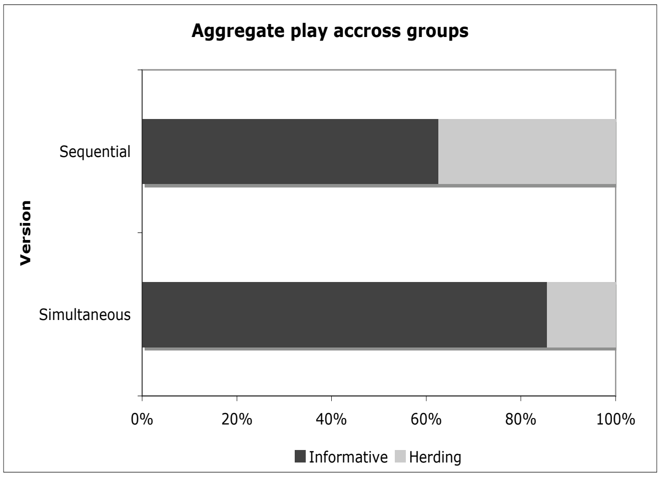


Figure 4.3: Aggregate play.

Still, this analysis simply identifies differences in aggregate behavior in each game. Our design allows us to do more than that and address the question of interest: do players behave as GM? To answer this question we look at every subjects behavior in both games. We then classify each subject according to this behavior. There are four classes of behavior. Behavior in three of these can be explained by existing theories. Behavior in the fourth class is the one of a GM type. More than one third of subjects are classified as GM types.

First we must note that for classification we can only use subjects that submited a strategy in both versions. This excludes subjects that were matched with a player 1 that accepted. Pooling subjects from both groups, this gives us 55 subjects to classify. There are four possible patterns of behavior: {"Informative", "Informative"}, {"Herd", "Herd"}, {"Informative", "Herd"}, {"Herd", "Informative"}, where the first element in the brackets denotes the behavior in the sequential version, and the second denotes behavior in the simultaneous version. The first pattern, playing informatively in both versions is the *naive* behavior. It corresponds to the behavior predicted for fully cursed types and level-1 types by the respective theories. The second pattern, herding in both versions is the *sophisticated* behavior. It is predicted as the behavior of Bayesian-Nash type players and level-2 players in the level-k reasoning model. The third pattern, informative in the simultaneous and herding in the sequential version, is predicted only by the cursed equilibrium theory for *partially cursed* players with an intermediate value of χ, the models parameter. The theoretical basis for such a behavior is not clear. The last pattern, herding in the sequential and informative play in the simultaneous version is the GM type's behavior.

The graph in Figure 4.4 shows the number of subjects that can be classified in each of the four classes. As one can see, the majority of subjects are classified as naive. 29 subjects (53%) enter this class. Only one subject is classified as sophisticated. 5 subjects are classified as partially cursed. Finally, 20 subjects (36%) are classified as GM types. This makes GM types the second largest class in our sample.

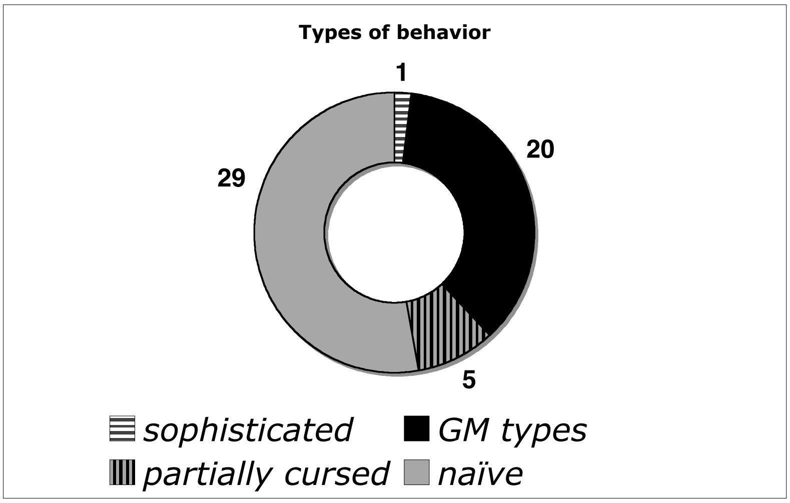


Figure 4.4: Types of behavior.

4.4 What drives behavior?

The results so far suggest that a behavior as the one we suggest could be empirically relevant. More than a third of the subjects in our sample behave in a way that can not be explained by any of the relevant theories of strategic thinking. While this finding indicates that the institutional setup, and in particular the observability of others' actions, is important in shaping agents beliefs and behavior, a question still remains: what drives such behavior?

Two general explanations seem plausible. One is related to the existence of a cognitive cost of reasoning that may differ between the different setups. The other explanation is related to the possibility that individuals use a different model of strategic thinking depending on whether or not they observe others' actions. The first seems more obvious and we present evidence that suggest that at least part of the observed behavior can be explained in this way. The second is more subtle and while we did not manage to produce conclusive evidence yet, we believe it plays a role in explaining the differences in behavior.

4.4.1 Cognitive cost of reasoning

The cognitive cost of reasoning (CCR) refers to the difficulty that an individual may face in making all necessary calculations that are required in order to compute one's best response in the game. In both versions of the game in the experiment, subjects' actions only matter once they are in the game node that follows a rejection by player 1. In the sequential game this is so because subjects know for a fact that they are at this node, while in the simultaneous game they have to reach this conclusion by calculating the possible outcomes from their actions in the other nodes of the game and realizing that in these other nodes, their actions do not matter. If for some players the CCR of these additional calculations is high, they may -consciously or not - choose not to perform them. The resulting behavior would be the one of a GM type.

If CCR drives behavior, then anything that can reduce this cost should push behavior more in line to sophisticated behavior. In group 3 versions 1 and 2 were used. Version 1 is the same as that of the simultaneous game used in groups 1 and 2. Version 2 is the version of the simultaneous game in which subjects are reminded that their actions only matter when player 1 rejects. There were 69 subjects in the group, out of which 55 answered the comprehension test correctly. In version 1, 44 played informatively while 11 chose to herd. In version 2, these numbers were 35 and 20 respectively. The increase in the number of players that herd in version 2 is statistically significant (p-value=0.004).

This finding suggests that for at least some of the subjects playing informatively, a behavior that is naive, the reason is a failure to realize that given the structure of the game their action only matters when player 1 rejects. This is the first step necessary in the IIFA reasoning process. Once they are reminded of this, as is done in version 2, they manage to perform the remaining steps of the reasoning by themselves. If this is so, then failure to perform IIFA is because of the complexity of the game and the associated higher CCR. It is not because of the inability to infer the information that drives other's actions. Note that it is this last part that is the basis for the notion of a "cursed equilibrium".

4.4.2 Different models of strategic thinking.

Another way of explaining the GM type behavior is that individuals may use different types (or models) of strategic thinking in different strategic situations. An example of different models of strategic thinking would be the different levels of reasoning in level-k thinking. But it could be something more varied and not necessarily a model already formalized by game theorists. What we assume is that the same individual may switch different models depending on the institutional setting. In particular, that individuals of the GM type use different models according to whether or not they observe the actions of other players. Note that this is different than a cognitive hierarchy model such as level-k thinking. Such models allow for different types of reasoning to be used by different individuals. What we suggest is that different types of reasoning may "co-exist" within the same individual.

In order to test this explanation we run treatments combining variation 2 with variation 2+ in both possible orders. Variation 2+ is similar to version 3 (the sequential game) but with an additional layer of the strategy method: subjects are told they will take their action after observing what player 1 does. Still, they are not told what this action is. Instead they are asked to submit their strategy for the case player 1 rejects, which is the case where they will have to take an action. Notice that these two versions of the game are equivalent. When subjects are submitting their strategy they do not know whether player 1 has accepted or rejected. Also, in both cases it is sugested to the subjects that their action only matters when player 1 rejects. Thus the CCR should be the same in both cases. The only difference is one of framing: in version 2 the game is framed as a simultaneous action game, while in version 2+ it is framed as a sequential game.

Results from these treatments (groups 6 and 7) are mixed. The following table summarizes the different types of behavior encountered in each treatment .[6]

[6]We avoid using the same terms as before to describe behavior of subjects that switch their strategy across the two versions of the game in these treatments. The reason is that unlike treatments 13 and 31, here subjects face the same game. Only in the case that our explanation of

	22+	2+2
Naive	42% (16)	48% (11)
Sophisticated	13% (5)	4% (1)
Other	3% (1)	4% (1)
Naive to Soph. (GM)	26% (10)	13% (3)
Soph. to Naive	11% (4)	26% (6)
Other switch	5% (2)	4% (1)

Figure 4.5: Types of behavior in treatments 22+ and 2+2.

'Naive' are the subjects that play informatively in both versions of the game. 'Sophisticated' are subjects that herd in both versions. 'Not rational' refers to subjects that always accept. 'Naive to Sophisticated' are subjects that play informatively in the the simultaneous framing and herd in the sequential framing (similar to GM types). 'Sophisticated to Naive do the opposite (similar to partially cursed). 'Not rational switch' always accept in one version and play informatively in the other.

As can be seen from these results, over 40% of subjects play differently in each version of the game. Given that both versions are equivalent, this could suggest that our explanation of different models of strategic thinking has some bite. The problem is that the direction of the switch seems to be related to the order in which the versions are played. This suggests that the switches may occur because of some "experimenter demand" effect.[7] To disentangle such an effect from our explanation of different models of strategic thinking we recur to subjects' self reported rationale for their decisions. A thorough analysis of these answers is still work in progress. Still, casual reading of the answers from subjects that switch their behavior from naive in version 2 to sophisticated in version 2+, suggest that our explanation of GM behavior can not be ruled out.

different models of strategic thinking is valid would it make sense to use the same terms. But this can not be established here and definitely not a priori

[7] See Zizzo (2010) (22). In treatments 13 and 31 there does not seem to be such an issue, since the majority of subjects that switch their behavior do so in the same direction irrespectively of the order of the game's versions.

4.5 Conclusions

Incomplete information characterizes a significant number of economic and social situations. It is therefore important to have a model of individuals' behavior in such situations. Bayesian Nash equilibrium concepts from classical game theory have provided a useful starting point. Nevertheless, situations like the appearance of the winner's curse in common value auctions demonstrate their possible limitations. Alternative theories have been advanced to explain these phenomena, including "cursed equilibrium" and "level-k" thinking. These theories offer interesting insights on the possible drivers of individual behavior in such environments. Still, the results in this paper show that they fail to capture an otherwise quite intuitive point that is crucial in games with incomplete information: observing others' actions is not the same as predicting they will happen. And therefore, inferring their informational content is "easier" in the first case than in the second.

Although we identify a type of behavior that is not predicted by any existing theory, we do not provide a complete theory to explain it. Still, we do offer two possible explanations each of which could explain part of the observed behavior. Differences in the cognitive cost of reasoning in different strategic situations is one of the possible drivers of this behavior and the experimental data seems to support it. The other explanation is the possibility of some individuals applying different models of strategic thinking in different institutional settings. We could not verify experimentally this explanation, but it still remains a possibility. The insights gained here can be the first step in developing a more complete theory of strategic thinking for situations of incomplete information with a common value.

The experimental designs used to study the winner's curse are commonly based on variations of auctions and the "company acquisition" game. Here we introduce a third alternative. We take advantage of the possibility to directly contrast behavior in a simultaneous action game with that in a sequential action game. But the fact that agents in this model only face a binary choice, allows to better control for complexity as a determinant factor. Given the continuing interest in studying the winner's curse we think that the design we introduce can serve as an additional alternative to be used in related experiments.

4.6 APPENDIX: Instructions

The next pages contain a sample of the original instructions used in the experiment. A translated version follows. During the experiment instructions were read out loud and subjects were asked not to turn pages unless instructed to do so.

Nombre:___________________________

DNI:______________________________

Vas a participar en un experimento económico sobre la toma de decisiones. Dependiendo de tus decisiones podrás ganar dinero.

Primero tienes que leer y entender bien las instrucciones. Tienes que demostrar que has entendido bien contestando a unas preguntas tipo test. Si contestas mal a las preguntas no puedes ganar dinero.

El experimento tiene 2 partes. En cada una te enfrentarás a una oferta y tendrás que tomar alguna decisión. Al final tienes que contestar unas preguntas adicionales.

Para determinar si ganas dinero, se escogerá aleatoriamente una de las dos ofertas y podrás ganar dinero en base de tus decisiones en esta oferta. Para saber si has ganado podrás consultar una lista con los ganadores que será colgada en el "Aula Global" de esta asignatura. Para ser pagado puedes pasar por el despacho 20.134 el Viernes 10/6 entre las 10.30-13.30 y 15.00-17.00. Es importante que lleves tu DNI y que los datos correspondan con los que has apuntado en esta hoja.

OFERTAS:

Tu y otro participante seleccionado al azar os encontráis con la siguiente oferta. Se os ofrece una bolsa que contiene 10 bolas. Podéis aceptar o no la bolsa. La bolsa puede ser "**buena**" (contiene solo bolas blancas) o "**mala**" (contiene algunas bolas negras). Hay un 50% de probabilidad que la bolsa sea buena o mala.

El otro participante tiene prioridad: dado que solo hay una bolsa si el la acepta, tu te quedas sin bolsa. Si el la rechaza y tu la aceptas te la quedas tu. Si los dos rechazáis la bolsa no la tendrá ninguno de los dos.

Los pagos dependen de si uno tiene la bolsa y si esta es "buena" o "mala". Si tienes la bolsa y es "buena", ganas 2 euros y si es "mala" no ganas nada. Si no tienes la bolsa y esta resulta ser "buena", no ganas nada. Si no la tienes pero resulta ser "mala", ganas 2 euros.

Si la bolsa es...:	**...buena**	**...mala**
si la tienes	***2***	*0*
si no la tienes	*0*	***2***

Antes de decidir, podrás sacar una bola de la bolsa y comprobar si es blanca o negra. Lo mismo hará el otro participante. Recuerda que si la bolsa es "buena", solo contiene bolas blancas. Por eso, si uno saca una bola negra, la bolsa tiene que ser "mala".

Preguntas de entendimiento:
Cada pregunta tiene una solo respuesta correcta. Marca tu respuesta poniendo en circulo la letra correspondiente.

1. *Si los dos aceptáis la bolsa:*
a. Te la quedas tu y ganas 2 euros si es “buena”.
b. Se la queda el otro y ganas 2 euros si es “buena”.
c. Te la quedas tu y ganas 2 euros si es “mala”.
d. Se la queda el otro y ganas 2 euros si es “mala”.

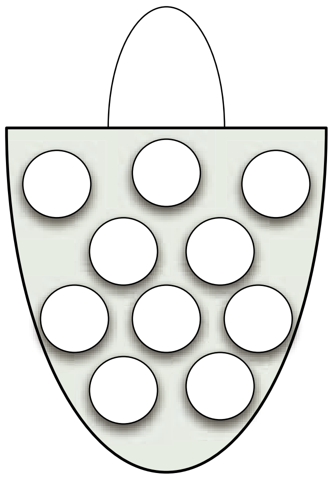
2. *Si tu aceptas y el otro participante rechaza:*
a. Te la quedas tu y ganas 2 euros si es “buena”.
b. Se la queda el otro y ganas 2 euros si es “buena”.
c. Te la quedas tu y ganas 2 euros si es “mala”.
d. Se la queda el otro y ganas 2 euros si es “mala”.

3. *Si rechazáis los dos:*
a. No se la queda nadie y ganáis 2 euros si es “mala”.
b. Te la quedas tu y ganas 2 euros si es “buena”.
c. No se la queda nadie y ganáis 2 euros si es “buena”
d. Se la queda el otro y ganas 2 euros si es “mala”.

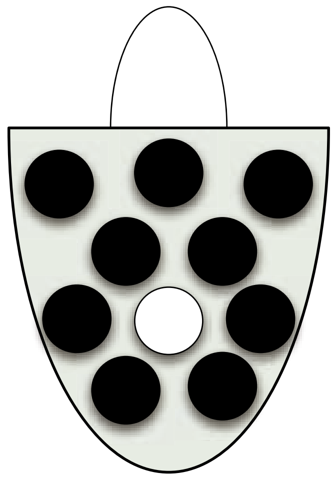
4. *Si uno saca una bola de la bolsa y esa es negra:*
a. Sabe que la bolsa tiene que ser “mala”.
b. Sabe que la bolsa tiene que ser “buena”.
c. No sabe con certidumbre si la bolsa es “buena” o “mala”.
d. Sabe que hay más probabilidad que la bolsa sea “mala”.

5. *Si uno saca una bola de la bolsa y esa es blanca:*
a. Sabe que la bolsa tiene que ser “mala”.
b. Sabe que la bolsa tiene que ser “buena”.
c. Sabe que hay más probabilidad que la bolsa sea “buena”.
d. Sabe que hay más probabilidad que la bolsa sea “mala”.

El siguiente dibujo muestra el posible contenido de la bolsa si es buena o si es mala.



Buena
Todas las bolas son blancas



Mala
9 bolas son negras

Hay la misma probabilidad que la bolsa sea buena o mala. Sacarás una bola de la bolsa para comprobar si e blanca o negra. Lo mismo hará el otro participante.

OFERTA 1

En esta oferta participas junto al participante **##** del otro grupo que ha sido escogido al azar. El tiene prioridad, pero tu tienes que tomar tu decisión sin observar la suya.
¿Que decisión tomarás? Aceptarás o rechazarás la bolsa?

(pon en circulo tu respuesta)

Si la bola que sacas tu es blanca: **Acepto** **Rechazo**

Si la bola que sacas tu es negra: **Acepto** **Rechazo**

OFERTA 2

En esta oferta participas junto al participante **##** del otro grupo que ha sido escogido al azar. El tiene prioridad, y observas que decide **RECHAZAR** la bolsa.

¿Que decisión tomarás? Aceptarás o rechazarás la bolsa?

(pon en circulo tu respuesta)

Si la bola que sacas tu es blanca: **Acepto** **Rechazo**

Si la bola que sacas tu es negra: **Acepto** **Rechazo**

PREGUNTAS

1. ¿Porque has tomado la decisión que has tomado en la oferta 1?

____

____

____

____

____

2. Si has cambiado de decisión en la oferta 2 ¿Porque lo hiciste?

____

____

____

____

____

3. ¿Cuál ha sido tu nota de selectividad?______________________

Name:___________________________

ID:_____________________________

You will participate on an experiment on decision making. Depending on your decisions, you may win cash.

First you must read well the instructions. You must prove that you understand them by answering a set of multiple choice questions. If you do not answer correctly you can not win any money.

The experiment has two parts. In each one you will face an offer and you will have to make a decision. In the end you must answer some aditional questions

In order to determine whether you have won cash, one of the two offers will be chosen randomly and you may win depending on your decision in that offer. To find out whether you have won you can check a li st with the winners that will be posted on the classe's intranet. In order to get payed you must come by office 20.134 on Friday 10th of June between 10.30-13.30 and 15.30-17.00. It is important to bring with you your ID containing the details that you have used on this sheet.

OFFERS:

You and another participant that has been selected randomly, face the following offer. You are offered a bag containing 10 balls. You may accept o reject the bag. The bag may be **"good"** contains only white balls) or **"bad"** (contains some black balls). There is a 50% chance that the bag is either "god" or "bad".

The other participant has priority: given that there is only one bag, if he accepts it you are left without the bag. If he rejects it and you accept, you get it. If you both reject the bag, neither one gets it in the end.

Payoffs depend on whether one has the bag and whether it is "good" or "bad". If you have the bag and it is "good", you win 2 euros and if it is "bad" you win nothing. If you do not have the bag and it turns out to be "good", you do not win anything. If you do not have it and it turns out to be "bad", you win 2 euros.

If the bag is...	...good	...bad
if you have it	***2***	*0*
if you do not have it	*0*	***2***

Before deciding, you will draw a ball from the bag and check whether it is black or white. The other participant will do the same. Remember that if the bag is "good", it only contains white balls. Therefore, if one draws a black ball, then the bag must be "bad".

Test of understanding:

There is a single correct answer to each question. Mark you answer by putting a circle around the corresponding letter.

1. *If you both accept the bag:*
a. You keep it and you win 2 euros if it is “good”.
b. The other one keeps it and you win 2 euros if it is “good”.
c. You keep it and you win 2 euros if it is “bad”.
d. The other one keeps it and you win 2 euros if it is “bad”.

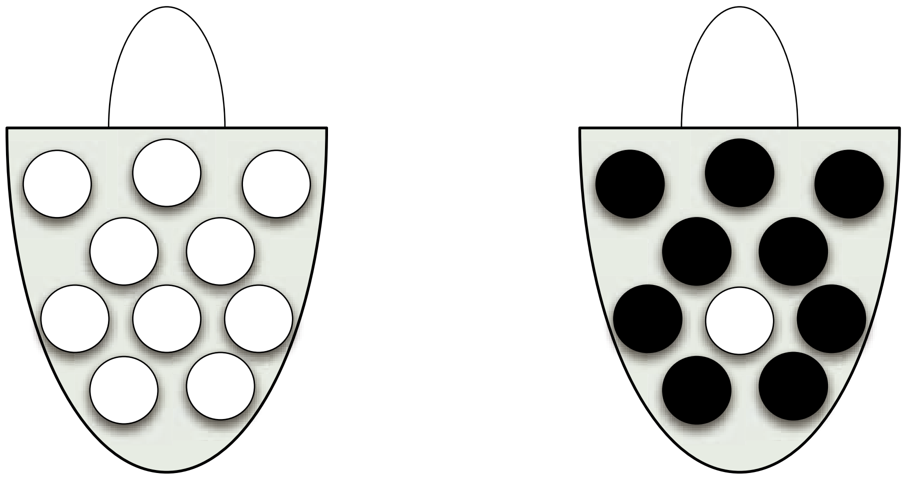
2. *If you accept and the other one rejects:*
a. You keep it and you win 2 euros if it is “good”.
b. The other one keeps it and you win 2 euros if it is “good”.
c. You keep it and you win 2 euros if it is “bad”.
d. The other one keeps it and you win 2 euros if it is “bad”.

3. *If you both reject:*
a. Nobody keeps it and you win 2 euros if it is “bad”.
b. You keep it and you win 2 euros if it is “good”.
c. Nobody keeps it and you win 2 euros if it is “good”.
d. The other one keeps it and you win 2 euros if it is “bad”.

4. *If one draws a ball and it is black:*
a. He knows that the bag is “bad".
b. He knows that the bag is “good".
c. He does not know for sure whether the bag is “good” or “bad”.
d. He knows it is more likely for the bag to be “bad”.

5. *Si uno saca una bola de la bolsa y esa es blanca:*
a. He knows that the bag is “bad".
b. He knows that the bag is “good".
c. He knows it is more likely for the bag to be “good”.
d. He knows it is more likely for the bag to be “bad”.

The following drawing depicts the possible content of the bag in case it is "good" and in case it is "bad".



Buena
Todas las bolas son blancas

Mala
9 bolas son negras

It is equally likely that the bag is "good" or "bad". You will draw a ball from the bag and ckeck whether it is black or white. The other participant will do the same.

OFFER 1

In this offer you participate together with participant no. ## from the other group which has been selected randomly. He/she has got priority, but you must take your decision without observing what he decides to do.
What will you do? Do you accept or reject the bag?

(put a circle around your answer)

If the ball I draw is white: **I accept** **I reject**

If the ball I draw is black: **I accept** **I reject**

OFFER 2

In this offer you participate together with participant no, ## from the other group that has been selected randomly. He/she has got priority and you observe that he/she decided to **REJECT.**
What will you do? Do you accept or reject the bag?

(put a circle around your answer)

If the ball I draw is white:	**I accept**	**I reject**
If the ball I draw is black:	**I accept**	**I reject**

QUESTIONS

1. Why did you take the decision above in offer 1?

____

____

____

____

____

2. If you changed your decision in offer 2, why did you do it?

____

____

____

____

____

3. What has been you university entry grade?_____________________

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