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Hawking radiation in NS5 and Little String Theory

Memòria presentada per
Oscar Lorente Espín
per optar al grau de
Doctor en Ciències

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A mis padres,
José y Anselma

En realidad no sabemos nada, pues la verdad yace en lo profundo.

Demócrito de Abdera

(...) ¿crees que los que están así han visto otra cosa de sí mismos o de sus compañeros sino las sombras proyectadas por el fuego sobre la parte de la caverna que está frente a ellos? (...)
–Examina, pues –dije–, qué pasaría si fueran liberados de sus cadenas y curados de su ignorancia, y si, conforme a la naturaleza, les ocurriera lo siguiente. Cuando uno de ellos fuera desatado y obligado a levantarse súbitamente y a volver el cuello y a andar y a mirar a la luz, y cuando, al hacer todo esto, sintiera dolor y, por quedarse deslumbrado, no fuera capaz de ver aquellos objetos cuyas sombras veía antes, ¿qué crees que contestaría si le dijera alguien que antes no veía más que sombras inanes y que es ahora cuando, hallándose más cerca de la realidad y vuelto de cara a objetos más reales, goza de una visión más verdadera, y si fuera mostrándole los objetos que pasan y obligándole a contestar a sus preguntas acerca de qué es cada uno de ellos? ¿No crees que estaría perplejo y que lo que antes había contemplado le parecería más verdadero que lo que entonces se le mostraba?.

Platón, *La República, Libro VII*

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This thesis contains:

- O. Lorente-Espin, P. Talavera, “A Silence black hole: Hawking radiation at the Hagedorn temperature,” JHEP **0804** (2008) 080. [arXiv:0710.3833 [hep-th]].
  Chapter 3, Section 3.3.
- O. Lorente-Espin, “Some considerations about NS5 and LST Hawking radiation,” Phys. Lett. **B703** (2011) 627-632. [arXiv:1107.0713 [hep-th]].
  Chapter 3, Section 3.4.
- O. Lorente-Espin, “Back-reaction as a quantum correction,”. [arXiv:1204.5756 [hep-th]]. Submitted to Physics Letters B.
  Chapter 5.

And also unpublished work.

Contents

Chapter 1

Introduction

Since ancient times some people have been interested in the world where they live and its environment. However, the unlimited human curiosity does not stop here, and goes beyond to the asymptotic limits of the universe. Questions about kinematics and dynamics of bodies, i.e. questions about motion, lead us to crucial responses dressed in consistent scientific theories. In this sense, gravitation has always been an special topic of study: from the physical philosophy of Aristotle to the free falling experiments of Galileo; from the Newton's law of universal gravitation to the Einstein's theory of general relativity. At the end of XVIII century, the mathematician physicist Pierre Simon Laplace and the cleric John Michell were influenced by the scientific ideas of Newton concerning the gravitation and light built by corpuscles. They were considering how gravitation would affect light, and if it would be possible that existed a star so massive and dense that light could not escape from its surface. Effectively, for a spherical star of fixed mass exists a minimum radius that acts as a frontier. For a values of radius lower than the minimum radius nothing can escape from the gravitational force at the star surface even the light. This star is named dark star. One century later, Einstein announced his theory of relativity changing our perception of the nature of space and time. A few years later, Schwarzschild found an intriguing solution to the Einstein's equations of general relativity. For a spherically symmetric body of fixed mass, neither with angular momentum nor electric charge, in vacuum, there exists a minimum radius known since then as *Schwarzschild radius* under which the body would collapse gravitationally to a space-time singularity. This object was called by John Wheeler, somewhat joking, black hole, nevertheless the astronomers have shown that such objects could exist in our universe. When an extremely massive compact object gravitationally collapses it could form a neutron star, however if it reaches the Chandrasehkar's limit nothing can stop the collapse and it will form a black hole. Another interesting scenario is the string theory framework, more concretely the AdS/CFT correspondence, where black holes are viewed as thermal states of a conformal field theory.

Nevertheless this thesis is basically founded in the semi-classical theory of black holes. It sheds some light over problems like the information loss or thermodynamical aspects of NS5 and LST black holes that although being constructed in string theory they will be studied using semi-classical methods. What we call semi-classical approach is: the background space-time is described by the Einstein's theory of general relativity, whereas the content of matter fields will be described by quantum field theory. Looking at the Einstein's equation of gravitation without the cosmological term

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu} \; , \tag{1.1}$$

on the left hand side it is seen the background geometry described by the general metric $g_{\mu\nu}$, the Ricci tensor $R_{\mu\nu}$ and the scalar curvature R; while on the right side one sees the energy matter content included in the energy-momentum tensor $T_{\mu\nu}$. A black hole is a classical solution of the equations of motion (1.1) in which there is a region of space-time that is causally disconnected from asymptotic infinity [1]. If we consider a spherically symmetric, non-rotating and uncharged distribution of matter collapsing under self-gravitation, when its radius is lower than the critical Schwarzschild radius the collapse cannot be stopped. The final result will be the matter ending up in an infinite density singular point, while the background metric will be the Schwarzschild metric,

$$ds^2 = -\left(1 - \frac{2GM}{rc^2}\right) dt^2 + \frac{1}{1 - \frac{2GM}{rc^2}} dr^2 + r^2 d\Omega_2^2 \; , \tag{1.2}$$

and M is the black hole mass. The event horizon radius is $r_0 = 2GM$. Hereafter we adopt the Planck units convention: $\hbar = c = G = k_B = 1$, except in some cases where we will restore some units for convenience. The Schwarzschild solution is the unique spherically symmetric solution of the vacuum Einstein's equations ($R_{\mu\nu} = 0$) [2]. The *singularity* theorems of Hawking and Penrose [3, 4] guarantee the existence of singularities once the collapse of a body, not necessarily spherical, reaches a certain point. Geodesic incompleteness, i.e. a geodesic that cannot be extended within the manifold but ends at a finite value of the affine parameter, lead us to the singularity hidden behind a trapped surface, something like a no return barrier. The *cosmic censorship conjecture* preserves us to observe naked singularities formed in a gravitational collapse from generic initially non-singular state in an asymptotically flat space-time obeying the dominant energy condition. Thus the singularity of a black hole will be hidden behind a null-like hypersurface causally disconnected from the out space of the black hole called the *event horizon*. All the relevant physics of black holes take place on the event horizon, consequently all the work developed in this thesis is concerned with the event horizon of the studied black holes. For a deep technical study about the above topics of classical black holes we refer the readers to [1] and [5]. Another important characteristic of black holes is the *no hair theorem* that states: four-dimensional stationary, asymptotically flat, black hole solutions coupled to electromagnetic fields are fully characterized by three parameters, i.e. mass, angular momentum and electric charge.

In the seventies, Bekenstein stated that black holes have entropy and this is proportional to the area of the event horizon [6],

$$S_{BH} = \frac{A}{4G} \; . \tag{1.3}$$

The second law of thermodynamics states that the entropy of the Universe never decreases. However one could imagine a quantity of gas around a black hole, which has a certain entropy, falling towards the black hole. An observer only would see the gas outside the black hole, then only accounts for the entropy of this gas that it is vanishing from the view of the external observer. In order to save the second law, Bekenstein associated an entropy to the black hole proportional to its surface area. Furthermore, one observes that the surface area of a black hole never decreases, that is the *area theorem*. Therefore if two black holes merge, the area of the final black hole will be equal or greater than the sum of the area of the two initial black holes. This behavior is reminiscent of the second law of thermodynamics applied to the black hole area. Eventually it is fulfilled the generalized second law, which states that the sum of the entropy of the black hole plus the matter surrounding never decreases,

$$\frac{d}{dt}(S_{BH} + S_{matter}) \geq 0 \; . \tag{1.4}$$

One can establish a direct relation between the laws of thermodynamics and the mechanics laws of black holes through the relations [7],

$$E \leftrightarrow M \quad , \quad S \leftrightarrow \frac{A}{4G} \quad , \quad T \leftrightarrow \frac{\kappa}{2\pi} \; , \tag{1.5}$$

where A is the event horizon area, κ is the surface gravity of the black hole and E, S, T are the usual thermodynamical variables respectively energy, entropy and temperature. The first law of black hole mechanics can be identified with the first thermodynamics law,

$$dM = \frac{\kappa}{8\pi} dA + \text{Work terms} \rightleftarrows dE = TdS - PdV \ . \tag{1.6}$$

However, after the work of Bekenstein, Hawking found that actually one can speak about the thermodynamics of black holes. In [8] Hawking found that black holes radiate a thermal spectrum of particles, since then called *Hawking radiation*, at a temperature $T_H = \frac{\hbar\kappa}{2\pi}$. However, this result shows that the temperature of a black hole is inversely proportional to its mass, having thus a negative specific heat. Therefore when a black hole radiates it loses its mass, it evaporates and eventually disappears, and this fact will lead us to the information loss problem.

Following an heuristic picture, Hawking radiation is produced by vacuum quantum fluctuations around the black hole where the gravitational field is strong. For black holes of large masses the curvature invariants are sufficiently small, hence one can work in a semi-classical regime where a theory of quantum gravity is not needed. Moreover, due to the *no-hair theorem* one can only know three charge parameters as mentioned above, thus all physics will be independent of the details of the initial configuration of matter that will forms the black hole. If the black hole completely evaporates away with a thermal spectrum, the final state of the radiation cannot have any information of the initial matter state. This violates the principle of information conservation, which it is fulfilled both in classical and quantum mechanics. In classical mechanics this principle is embodied in Liouville's theorem on the conservation of phase space volume. In quantum mechanics the principle of information conservation is expressed as the unitarity of the S-matrix. We have seen that it is not the case for the black hole evaporation, where the final state will not be related in a one-to-one to the initial state, thus violating the unitarity of the time evolution operator. Furthermore, the final state cannot be entangled if the black hole has completely evaporated. Initially the outgoing particle created by the quantum fluctuations was in a mixed state with the ingoing particle, and the outgoing radiation was entangled with the ingoing particles. Therefore, the final system, after the evaporation, will be described not by a pure quantum state but by a mixed state. There are a few alternatives in order to avoid this situation. One of them is the existence of a remnant of Planck size to which the outgoing radiation would be entangled. However the entanglement entropy is larger than the black hole entropy, which is really a huge number [1], thus the number of possible microstates of the remnant goes to infinity as the mass of the initial black hole increases. Another alternative, proposed by Hawking, is that black holes completely evaporate and the initial pure state evolves to a final mixed state in a theory of quantum gravity. In this case the description of the states is in terms of density matrices. Nevertheless this approach did not convince the quantum physicists community [9], due to the violation of quantum unitarity. It has been argued that the Hawking radiation carry out somehow the information of the collapsing matter, so that the black hole could completely evaporate and the process would not violate the unitarity.

Since then a lot of work has been done in order to solve the information paradox. A good candidate is string theory, more concretely the holographic conjecture and its AdS/CFT realization [10]. For example, counting microstates of the black hole in [11] the authors obtained a microscopic derivation of black hole entropy. For a good reviews on black holes in string theory see for example [12, 13, 14, 15]. Nevertheless, as we have mentioned above, in this thesis we will focus on semi-classical methods that enable us to obtain non-thermal spectra for the vast majority of black holes. This fact is due to taking into account the back-reaction of the metric and imposing energy conservation when the black hole radiates particles. Specifically, we have studied NS5 and Little String Theory (LST) black holes. We have calculated the Hawking radiation for both models, obtaining a non-thermal spectrum for NS5, whereas a purely thermal spectrum for LST.

[1]Taking into account the expression of the entropy using statistical mechanics $S = k \log\Omega$, where Ω is the number of microstates accessible to the macroscopic system, in this case a black hole of area A. For a black hole of solar mass one finds $10^{10^{78}}$ states.

Chapter 2

Semi-classical emission of Black Holes

Despite the absence of a complete theory of quantum gravity, one may hope to be able to say something concerning the influence of the gravitational field on quantum phenomena, for example the radiation emission carried out by black holes. One can study the quantum aspects of gravity in which the gravitational field is retained as a classical background, adopting the Einstein's general theory of relativity as a description of gravity, whereas matter fields are quantized in the usual way.

The Planck scale: Planck length $l_P = \sqrt{\frac{G\hbar}{c^3}} \approx 10^{-35}m$ and Planck time $t_P = \sqrt{\frac{G\hbar}{c^5}} \approx 10^{-44}s$; establishes the frontier at which a full theory of quantum gravity is necessary. Unlike the QED coupling constant $\frac{e^2}{\hbar c}$ the Planck length has dimensions, hence the effects become significant when the length and time scales of quantum processes of interest fall below the Planck scale. Thus the higher orders of perturbation theory become comparable with the lowest order. Nevertheless, when the distances and times involved are much larger than the Planck scale, the quantum effects of the gravitational field will be negligible, and a semi-classical theory appears to be valid. However, according to the equivalence principle all matter and energy, included the gravitational energy, couple equally strongly to gravity, thus the graviton is also subjected to an external gravitational field as could be a photon. Therefore quantum gravity will enter in a non-trivial way at all scales whenever interesting quantum field effects occur.

It may still be possible to work with a semi-classical approach. In the same way that in classical relativity one studies the propagation of gravitational waves in curved space-time, one can consider the graviton field as a linearized perturbation on the background space-time: $g_{\mu\nu} = g^{(0)}_{\mu\nu} + \hat{g}_{\mu\nu}$. The contribution of the dilaton to the left-hand side of Einstein's equations, can be casted in a form that might be included along with all the other quantum fields in the right-hand side of Einstein's equations, being part of the matter rather than the geometry.

On the other hand, the fact that the gravitational constant G has units of length square gives rise to a non-renormalizable theory of gravitation. hence the quantization of gravitational field has not been already accomplished. Nevertheless, one can truncate the expansion of the semi-classical theory (classical gravity plus quantum matter fields) at one-loop level for example. In this way, the finite number of divergences can be removed by renormalization of a finite number of physical quantities, thus the truncated theory could be considered renormalizable. Since important gravitational effects occur in quantum field modes for which the wavelength is comparable with some characteristic length scale of the background space-time, only in the vicinity of the microscopic black holes or in the early epochs of the Big Bang we can expect such gravitational effects. Otherwise, in the rest of the phenomenology one can study quantum field theory in curved space-time, i.e. in a semi-classical way. The fundamental Hawking's discovery of thermal emission by black holes [8] is a clear example of how gravity, quantum field theory and thermodynamics are closely interwoven. Henceforth all the work developed in this thesis gravitates in some way around such discovery.

We will see how curved space-time can create particles, henceforth we will not ever consider particle as a fundamental fixed concept, otherwise it might be considered as an observer-dependent object. For this study we have followed the notes in [16]. We consider space-time to be a C^∞ n-dimensional, globally hyperbolic, pseudo-Riemannian manifold [5]. We write the background metric $g_{\mu\nu}$ associated with the line element as

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \; ; \quad \mu, \; \nu = 0, 1, ..., (n-1) \; , \tag{2.1}$$

where x^μ, x^ν are the coordinates. We define the determinant as

$$g \equiv \mid \det g_{\mu\nu} \mid \; . \tag{2.2}$$

Now we want to consider the quantization of a field in the classical curved space-time defined by (2.1). The action is

$$S = \int \mathcal{L}(x) d^n x \ . \tag{2.3}$$

We will consider henceforth the quantization of a scalar field $\phi(x)$, thus the Lagrangian density will be

$$\mathcal{L}(x) = \frac{1}{2}\sqrt{-g}\left(g^{\mu\nu}(x)\partial_\mu\phi(x)\partial_\nu\phi(x) - [m^2 + \xi R(x)]\phi^2(x)\right) \ , \tag{2.4}$$

where m is the mass of the scalar field and $R(x)$ is the Ricci scalar curvature. Depending on the value of ξ we can consider two relevant cases: the conformally coupled case with $\xi = \frac{(n-2)}{4(n-1)}$ and the minimally coupled case with $\xi = 0$; we will work in the minimally coupled regime. Taking the variation of the action (2.3) with respect to the scalar field $\phi(x)$ equal to zero, one obtains the scalar field equation

$$\left(\Box + m^2\right)\phi(x) = 0 \ , \tag{2.5}$$

where $\Box$ is the D'Alembertian operator in curve space-time defined as $\Box \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu\phi = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right)$. See Appendix A for a discussion on notations and conventions.

The scalar product between two solutions is defined as

$$(\phi_1, \phi_2) = -i\int_\Sigma \left[\phi_1(x)\nabla_\mu\phi_2^*(x) - \phi_2^*(x)\nabla_\mu\phi_1(x)\right]\sqrt{-g}\ d\Sigma^\mu \tag{2.6}$$

with $\nabla_\mu \equiv \partial_\mu$ for a scalar field; $d\Sigma^\mu = n^\mu d\Sigma$ is the area element for a Cauchy surface Σ in the globally hyperbolic space-time, with n^μ a future directed unit vector orthogonal to the space-like hypersurface. The value of the scalar product is independent of the Σ, see [5]. There exists a complete set of mode solutions $u_i(x)$ of (2.5) which are orthonormal in (2.6),

$$(u_i, u_j) = \delta_{ij} \ , \ \ (u_i^*, u_j^*) = -\delta_{ij} \ , \ \ (u_i, u_j^*) = 0 \ . \tag{2.7}$$

Then the scalar field ϕ can be expanded in terms of this modes,

$$\phi(x) = \sum_i [a_i u_i(x) + a_i^\dagger u_i^*(x)] \ . \tag{2.8}$$

The theory is covariantly quantized invoking the commutations relations

$$[a_i, a_j^\dagger] = \delta_{ij} \ , \ \ [a_i, a_j] = [a_i^\dagger, a_j^\dagger] = 0 \ , \tag{2.9}$$

where we must identify a_i and $a_i^\dagger$ with the annihilation and creation operators respectively. The vacuum state corresponding to this modes should be constructed through

$$a_i|0\rangle = 0 \ , \quad \forall i \ , \tag{2.10}$$

where $|0\rangle$ is defined as the vacuum state. However, we bump into an ambiguity crucial for the creation of particles by curved spaces. In curved space-times the Poincaré group is no longer a symmetry group of the space-time, thus there will be no Killing vectors at all with which to define positive frequency modes. In Minkowski flat space-time there is a natural set of modes defined in the natural rectangular coordinate system in which the vacuum is invariant. These coordinates are associated with the Poincaré group, the action of which leaves the Minkowski line element unchanged. The modes, in Minkowski space-time, are eigenfunctions of the Killing vector $\partial/\partial t$ with eigenvalues $-i\omega$ for $\omega > 0$. In this way we have a well-definite positive frequency modes, whereas this is not the case for curved space-times. If we consider, for example, the formation of a black hole by the gravitational collapse of an amount of matter, e.g. a star; the metric is time dependent during the collapse. Therefore a mode solution that was purely positive frequency in the null past infinity of the black hole, will be partly negative when reaches the null future infinity. Near the event horizon of the black hole the mode is very blue shifted, there will be a mixing of frequencies that is independent of the details of the collapse in the limit of late times, it depends only on the surface gravity, κ, that measures the strength of the gravitational field on the horizon. Eventually, the mixing of positive and negative frequencies leads to particle creation. So that it does not exist a privileged system of coordinates in which the field ϕ can be decomposed into natural frequency modes. Therefore, one can consider a second complete orthonormal set of modes $\bar{u}_j(x)$ in which the field ϕ is expanded

$$\phi(x) = \sum_j [b_j \bar{u}_j(x) + b_j^\dagger \bar{u}_j^*(x)] \ , \tag{2.11}$$

b_j and $b_j^\dagger$ will be annihilation and creation operators respectively in the decomposition of the scalar field into the new modes, which also fulfill the quantization rules

$$[b_i, b_j^\dagger] = \delta_{ij} \ , \quad [b_i, b_j] = [b_i^\dagger, b_j^\dagger] = 0 \ . \tag{2.12}$$

Moreover this decomposition defines a new vacuum state $|\bar{0}\rangle$

$$b_j|\bar{0}\rangle = 0 \ , \quad \forall j \ , \tag{2.13}$$

and a new Fock space.

The new modes can be expanded in terms of the old ones

$$\bar{u}_j = \sum_i (\alpha_{ji} u_i + \beta_{ji} u_i^*) \ . \tag{2.14}$$

Conversely,

$$u_i = \sum_j (\alpha_{ji}^* \bar{u}_j - \beta_{ji} \bar{u}_j^*) \ . \tag{2.15}$$

These are the Bogoliubov transformations and the matrices α_{ij} and β_{ij} are the Bogoliubov coefficients that can be evaluated, taking into account the definition of scalar product, as

$$\alpha_{ij} = (\bar{u}_i, u_j) \ , \ \beta_{ij} = -(\bar{u}_i, u_j^*) \ . \tag{2.16}$$

Furthermore, one can expand the old annihilation-creation operators in terms of the new operators, equating (2.8) with (2.11) and making use of (2.14), (2.15) and (2.7),

$$\begin{aligned} a_i &= \sum_j (\alpha_{ji} b_j + \beta_{ji}^* b_j^\dagger) \ , \\ a_i^\dagger &= \sum_j (\beta_{ji} b_j + \alpha_{ji}^* b_j^\dagger) \ , \end{aligned} \tag{2.17}$$

conversely,

$$\begin{aligned} b_j &= \sum_i (\alpha_{ji}^* a_i - \beta_{ji}^* a_i^\dagger) \ , \\ b_j^\dagger &= \sum_i (\alpha_{ji} a_i^\dagger - \beta_{ji} a_i) \ . \end{aligned} \tag{2.18}$$

Two properties are accomplished by the Bogoliubov coefficients

$$\sum_k (\alpha_{ik} \alpha_{jk}^* - \beta_{ik} \beta_{jk}^*) = \delta_{ij} \ , \tag{2.19}$$

$$\sum_k (\alpha_{ik} \beta_{jk} - \beta_{ik} \alpha_{jk}) = 0 \ . \tag{2.20}$$

From (2.17) it follows that the two Fock spaces defined by the modes u_i and $\bar{u}_j$ are different as long as $\beta_{ji} \neq 0$, for example: $a_i|\bar{0}\rangle = \sum_j \beta_{ji}^* |\bar{1}_j\rangle \neq 0$. Therefore, the expectation value of the number operator,

$$N_i = a_i^\dagger a_i \ , \tag{2.21}$$

for the number of u_i mode particles in the vacuum state defined by the $\bar{u}_j$ modes, i.e. $|\bar{0}\rangle$, is

$$\begin{aligned} \langle\bar{0}|N_i|\bar{0}\rangle &= \langle\bar{0}|a_i^\dagger a_i|\bar{0}\rangle \\ &= \sum_j \langle\bar{0}|\beta_{ji}\beta_{ji}^* b_j b_j^\dagger|\bar{0}\rangle \\ &= \sum_j |\beta_{ji}|^2 \,, \end{aligned} \tag{2.22}$$

where we have taken into account the commutations relations (2.12). Thus the vacuum of the $\bar{u}_j$ modes contains $\sum_j |\beta_{ji}|^2$ particles [1]. Therefore, if any $\beta_{ji} \neq 0$, the $\bar{u}_i$ will contain a mixture of positive u_j and negative u_j^* frequency modes, and particles will be present.

As a conclusion, a curved space-time creates particle. In terms of field theory one can understand that the stress-energy tensor on the right of the Einstein's equations, which causes a strong gravitational field, is the source of the new created particles. But not only in curved space-time we can detect the creation of particles. In Minkowski flat space-time an accelerating observer in a vacuum state observes a thermal spectrum of particles, see [17]. The idea is that observers with different view about positive and negative frequency modes will disagree on the particle content of a given state.

2.1 Hawking radiation

Hawking found in [8] that a thermal flux of particles is emitted by the black holes when one combines quantum field theory with classical gravity. He described the background space-time geometry using the Einstein's general relativity, whereas he treated the content matter as a quantum field. During the gravitational collapse to a black hole the space-time is not stationary, thus we expect particle formation. The infinite time dilation at the event horizon involves that the Hawking radiation be independent of the detailed collapse. Next we will briefly develop the Hawking calculation following the notes in [18]. For a Schwarzschild space-time one solves the Klein-Gordon equation $\Box\phi = 0$ corresponding to a scalar massless field, which can

[1] We could also consider the continuum limit simply changing the sum over j by an integral onto energy: $\sum_j \to \int_0^\omega d\omega'$

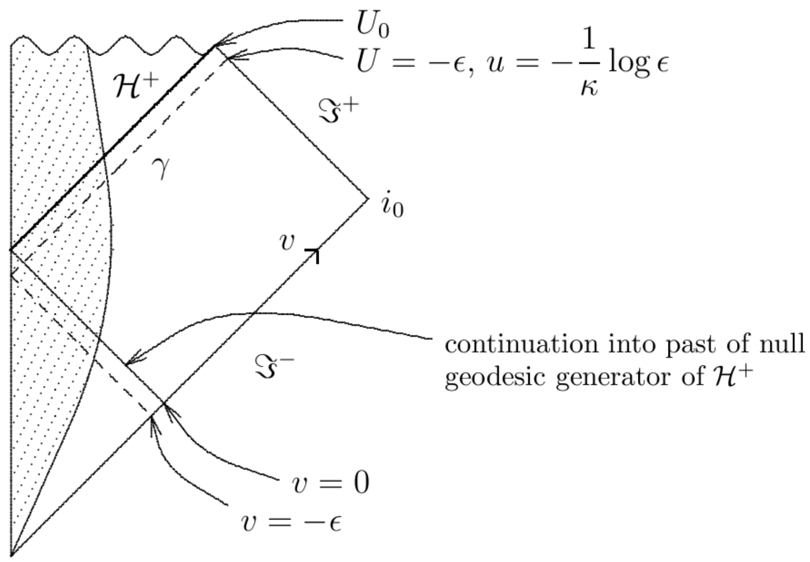

Figure 2.1: Penrose diagram of a spherical collapsing body. A null ray γ is traced back from the future null infinity $\mathcal{I}^+$.

be decomposed into an stationary term, a radial part and the angular part through the spherical harmonics

$$\phi = \left(Ae^{-i\omega t} + A^* e^{i\omega t}\right) R(r) Y_{lm}(\theta, \varphi) \ . \tag{2.23}$$

A positive frequency outgoing mode solution at the future null infinity $\mathcal{I}^+$ can be written as

$$\phi_\omega \sim e^{-i\omega u} \ . \tag{2.24}$$

Defining the null coordinates

$$v = t + r_* \quad , \quad u = t - r_* \ , \tag{2.25}$$

where r_* is the tortoise coordinate, v is the advanced time coordinate (or ingoing coordinate) and u is the retarded time coordinate (or outgoing coordinate), one can see that the outgoing mode (2.23) is defined by the outgoing coordinate parameter at a frequency ω. As Hawking proposed in [8], one can trace a null ray γ back in time from $\mathcal{I}^+$, which is the particle's world-line in optical approximation, see Figure 2.1. This approximation will be justified since near the event horizon at late times the mode ϕ is blue-shifted. The ray γ that reach $\mathcal{I}^+$ later is propagating more close to the horizon. Hence one can define the null geodesic generator γ_H of the event

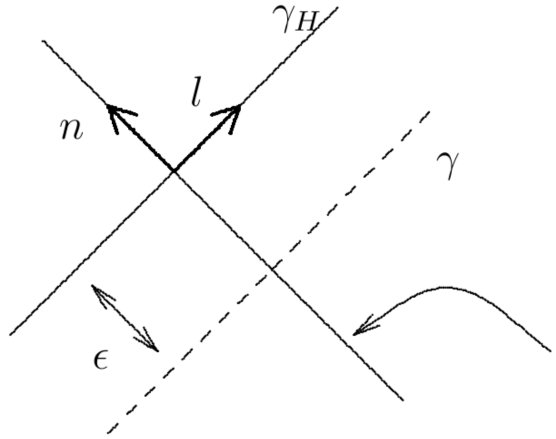

Figure 2.2: Parallel transport of the unitary vectors n and l through the future event horizon.

horizon $\mathcal{H}^+$ as the ray γ at $t \to \infty$. Any ray γ is specified giving its affine distance to γ_H along an ingoing null geodesic through $\mathcal{H}^+$, whose affine parameter will be the Kruskal coordinate, thereby $U = -\epsilon$. Then, taking into account the definition of the Kruskal coordinates

$$U = -e^{-\kappa u} \quad , \quad V = e^{\kappa v} \ , \tag{2.26}$$

near horizon one obtains

$$u = -\frac{\log \epsilon}{\kappa} \ , \tag{2.27}$$

and

$$\phi_\omega \sim \exp\left(\frac{i\omega}{\kappa} \log \epsilon\right) \ , \tag{2.28}$$

for the outgoing mode. Therefore it is possible to find the outgoing mode ϕ_ω at the past null infinity $\mathcal{I}^-$ by parallel transporting two defined unitary vectors n and l along the γ_H, see figures 2.2 and 2.3. The vector l is defined as the null vector tangent to the horizon, whereas n is defined as the future-directed null vector which is directed radially inward and normalized to $l \cdot n = -1$. The continuation of γ_H gets $\mathcal{I}^-$ at $v = 0$, whereas the continuation of the ray γ gets $\mathcal{I}^-$ along an outgoing null geodesic on $\mathcal{I}^-$ at the affine distance parameter $v = -\epsilon$. It is pointed out that

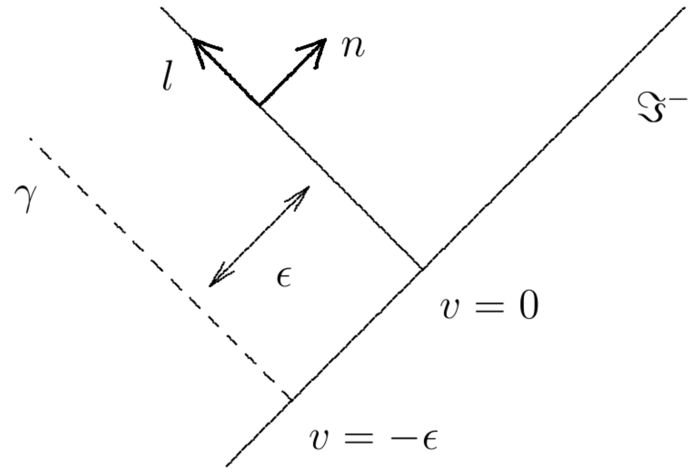

Figure 2.3: Parallel transport of the unitary vectors n and l through the ingoing null geodesic.

an ingoing null ray starting at $\mathcal{I}^-$ with $v > 0$ never reaches $\mathcal{I}^+$ since it crosses the event horizon $\mathcal{H}^+$. Thus the outgoing mode on $\mathcal{I}^-$ is

$$\phi_\omega(v) \sim \begin{cases} 0 & v > 0 \\ \exp\left(\frac{i\omega}{\kappa}\log(-v)\right) & v < 0\ . \end{cases} \tag{2.29}$$

By Fourier transforming

$$\bar{\phi}_\omega = \int_{-\infty}^{\infty} e^{i\omega' v}\phi_\omega(v)dv\ , \tag{2.30}$$

one obtains the following relation demonstrated in [18]

$$\bar{\phi}_\omega(-\omega') = -e^{-\frac{\pi\omega}{\kappa}}\bar{\phi}_\omega(\omega') \quad ,\ \omega' > 0\ . \tag{2.31}$$

Eventually, a positive definite frequency mode on $\mathcal{I}^+$ becomes a mixed positive and negative frequency mode on $\mathcal{I}^-$. Thus identifying the Bogoliubov coefficients as

$$\begin{aligned} \alpha_{\omega\omega'} &= \bar{\phi}_\omega(\omega') \\ \beta_{\omega\omega'} &= \bar{\phi}_\omega(-\omega')\ , \end{aligned} \tag{2.32}$$

it is accomplished the following relation between the Bogoliubov coefficients

$$\beta_{ij} = -e^{-\frac{\pi\omega}{\kappa}}\alpha_{ij}\ . \tag{2.33}$$

Now taking into account the relation (2.19)

$$\sum_k (\alpha_{ik}\alpha^*_{jk} - \beta_{ik}\beta^*_{jk}) = \left(e^{\frac{\pi(\omega_i+\omega_j)}{\kappa}} - 1 \right) \sum_k \beta_{ik}\beta^*_{jk} = \delta_{ij} \, , \tag{2.34}$$

and taking $i = j$

$$\sum_k |\beta_{ik}|^2 = \frac{1}{e^{\frac{2\pi\omega_i}{\kappa}} - 1} \, . \tag{2.35}$$

Actually the inverse process is needed, namely start with a positive frequency mode on the past null infinity $\mathcal{I}^-$ that propagates until it becomes a mixed positive and negative frequency mode on the future null infinity $\mathcal{I}^+$. The final result for the expectation value of the number of particles created and emitted to $\mathcal{I}^+$ is

$$\langle N \rangle_{\mathcal{I}^+} = \frac{1}{e^{\frac{2\pi\omega}{\kappa}} - 1} \, . \tag{2.36}$$

This result corresponds to a Planck distribution for black body radiation at the Hawking temperature

$$T_H = \frac{\hbar\kappa}{2\pi} \, . \tag{2.37}$$

So far we have considered that all the thermal radiation emitted by the black hole arrives to the future null infinity $\mathcal{I}^+$ without any change in the amplitude of the wave function. However, some emitted radiation will be partially scattered back to the event horizon. This fact is due to the gravitational potential barrier around the black hole, where some fraction of radiation will be reflected back to the hole, acting thus as a filter for the emitted radiation. Taking into account this effect we have to modify the orthonormal condition (2.19) by

$$\sum_k (\alpha_{ik}\alpha^*_{jk} - \beta_{ik}\beta^*_{jk}) = \Gamma \, , \tag{2.38}$$

where Γ_i is known as the greybody factor and it accounts for the deviation from pure black body spectrum, then the number of emitted particles will be

$$\langle N \rangle_{\mathcal{I}^+} = \frac{\Gamma}{e^{\frac{2\pi\omega}{\kappa}} - 1} \, . \tag{2.39}$$

Greybody factors have a relevant importance because successful microscopic account of black hole thermodynamics should be able to predict them. For example, it is shown in [19] that D-branes provide an account of black hole microstates which is successful to predict the greybody factors. There exists a vast literature on how to compute greybody factors in the context of the quantum field theory in curved space-time, e.g. [20, 21, 22, 23, 24, 25, 26].

2.2 Euclidean path integral and Hawking temperature

One can better understand the result that black holes radiate thermally appealing the Euclidean path integral formalism [27]. If one works with imaginary time coordinate setting

$$t = i\tau \ , \tag{2.40}$$

for a four-dimensional spherically symmetric black hole we obtain a positive definite metric known as Euclidean metric,

$$ds_E^2 = f(r)d\tau^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega_2^2 \ , \tag{2.41}$$

where $f(r)$ is a metric function defined as $f(r) \equiv \left(1 - \frac{r_0}{r}\right)$ being r_0 the radius of the event horizon. This metric still presents a coordinate singularity at $r = r_0$, so that one performs a change of coordinates going to the Rindler sector. For a general four-dimensional spherically symmetric background we define the proper length as

$$\rho = \int \sqrt{g_{rr}}\ dr \ . \tag{2.42}$$

Expanding around r_0 we write the metric function $f(r)$ near horizon as

$$f(r) = f(r_0)'(r - r_0) \ . \tag{2.43}$$

Then the new Rindler radial coordinate will be

$$\rho = \lim_{r \to r_0} \left[2 \sqrt{\frac{(r - r_0)}{f(r)'}} \right] \ . \tag{2.44}$$

Thus for the Schwarzschild black hole, i.e.: $g_{rr} = \frac{1}{f(r)}$ with $f(r) = \left(1 - \frac{2M}{r}\right)$ and $r_0 = 2M$ in Planck units; the Euclidean Schwarzschild metric is

$$ds_E^2 = \rho^2(\kappa d\tau)^2 + d\rho^2 + r^2 d\Omega_2^2 \ , \tag{2.45}$$

where κ is the surface gravity [2] and equals to

$$\kappa = \frac{1}{4M} \ . \tag{2.46}$$

[2] The surface gravity of a black hole is defined as the acceleration of a static particle near the event horizon measured by an asymptotic observer. It can be calculated with the formula $\kappa^2 = -\frac{1}{2}\left(\nabla_\mu \zeta_\nu\right)\left(\nabla^\mu \zeta^\nu\right)$ evaluated at the event horizon, where ζ_ν is a Killing vector. See [28] for a rigorous study.

The metric in the $\rho - \tau$ plane is just the plane polar coordinates if one identifies τ with period $8\pi M$. In general, the coordinate singularities on the horizon (conical singularities) of Euclidean black hole metrics can be removed by identifying

$$\tau \to \tau + \frac{2\pi}{\kappa} , \tag{2.47}$$

so that the imaginary time coordinate τ is periodic with period $\frac{2\pi}{\kappa}$. Therefore the Euclidean functional integral must be taken over fields that are periodic in τ with period $\frac{2\pi}{\kappa}$.

The Euclidean path integral is

$$Z = \int \mathcal{D}[\phi] \; e^{-S_E[\phi]} , \tag{2.48}$$

where S_E is the Euclidean action. Taking the integral over fields that are periodic in imaginary time with period $\hbar\beta$, one can write (2.48) as

$$Z = tr \; e^{-\beta H} , \tag{2.49}$$

which is the thermodynamic partition function corresponding to a quantum system with Hamiltonian H at the temperature given by $\beta = \frac{1}{k_B T}$, being k_B the Boltzmann constant.

In order to see this last result we consider the probability amplitude to go from an initial field configuration ϕ_1 on the space-like hypersurface at t_1 to a field configuration ϕ_2 on the hypersurface at t_2. This amplitude is determined by the matrix element $e^{iH(t_2-t_1)}$ [29]. Also we can calculate the amplitude as a path integral over all fields ϕ between t_1 and t_2 with ϕ_1 and ϕ_2 as fields on the initial and final hypersurface respectively. Thus

$$\langle \phi_2, t_2 | \phi_1, t_1 \rangle = \langle \phi_2 | e^{iH(t_2-t_1)} | \phi_1 \rangle = \int \mathcal{D}[\phi] \; e^{iS[\phi]} . \tag{2.50}$$

Then if one considers that the interval time is imaginary and equal to β,

$$t_2 - t_1 = i\beta . \tag{2.51}$$

Choosing as boundary conditions,

$$\phi_1 = \phi_2 \tag{2.52}$$

on the two hypersurfaces, and summing over all field configurations ϕ_n, one obtains on the left of (2.50) the partition function Z of a quantum system, i.e. the expectation value of $e^{-\beta H}$ summed over all states, at a temperature $k_B T = \beta^{-1}$. Furthermore, using the Euclidean action on the right of (2.50), we finally obtain

$$Z = \sum_n \langle \phi_n | e^{-\beta H} | \phi_n \rangle = \int \mathcal{D}[\phi] \; e^{-S_E[\phi]} \; . \tag{2.53}$$

Therefore the partition function for the field ϕ at temperature T is given by a path integral over all fields in Euclidean space-time, which is periodic in the imaginary time direction with period $\beta = (k_B T)^{-1}$. So that fields in Schwarzschild space-time in particular, and in curved background in general, will behave as if they were in a thermal state with temperature $T_H = \frac{\hbar \kappa}{2\pi k_B}$, or using Planck units,

$$T_H = \frac{\kappa}{2\pi} \tag{2.54}$$

where T_H is the Hawking temperature of a black hole at which the quantum field theory is in equilibrium. We point out that the equilibrium of a Schwarzschild black hole at Hawking temperature is unstable. From (2.46) and (2.54) we see that a black hole that emits radiation loses its mass hence its temperature increases, therefore the specific heat capacity of Schwarzschild black hole is negative.

2.3 Hawking radiation as tunneling

One way to solve semi-classically the information loss paradox is proposed in [30], where the authors obtains a non-thermal emission spectrum corresponding to a Schwarzschild black hole. The problem is addressed considering the emission of radiation by a black hole as a tunneling process. The key idea is that the energy of a particle changes its sign as it crosses the event horizon. The heuristic picture [31] shows a virtual pair of particle and antiparticle created just inside the horizon. Then the positive energy virtual particle can tunnels out, it materializes as a real particle and propagates to the infinity. These particles will be seen by an asymptotic observer as Hawking flux radiation. Conversely, the virtual pair could be created just outside the horizon, in that case the negative energy particle can tunnels inwards the black hole. In both cases the negative energy particle is absorbed by the black hole, thus the mass of the black hole decreases in the same amount of the positive energy released out through the emitted particle. The fact that black hols decreases its mass supports the quantum gravity idea that black holes can be regarded as highly excited states. Anyway the total energy of the system is conserved. The idea that black holes lose mass by absorbing negative energy is studied in [16].

In the WKB approximation the tunneling rate probability is related to the imaginary part of the action for the classically forbidden path,

$$\Gamma \sim e^{-2\mathrm{Im}S} \,. \tag{2.55}$$

The tunneling is between two separated classical turning points which are joined by a complex path. Nevertheless in this case it does not preexist a barrier, but it is just created by the outgoing particle itself. As the total energy must be conserved during the emission of radiation by the black hole, when particles are emitted the hole loses mass. Therefore, if the black hole loses mass it shrinks its event horizon to a new small radius, and the contraction will depends on the energy of the outgoing emitted particle [32].

We introduce the method of tunneling emission considering at first a line element of a four-dimensional spherically symmetric black hole.

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_2^2 \,, \tag{2.56}$$

where the metric function is

$$f(r) \equiv 1 - \frac{r_0}{r} \,, \tag{2.57}$$

being r_0 the event horizon radius. In order to avoid coordinate singularities at the event horizon we will write the metric in regular Painlevé coordinates [33], thus we obtain a smooth behavior through the horizon. Just to say that the Painlevé time coordinate is nothing more than the proper time of a radially free-falling observer [34]. Then if we shift the time coordinate to proper time coordinate

$$t \to t - g(r) \,, \tag{2.58}$$

where $g(r)$ is a function that depends only on the radial coordinate, we can write the new metric as

$$ds^2 = -f(r)dt^2 + 2f(r)g(r)'dtdr + \left(f(r)^{-1} - f(r)g(r)'^2\right)dr^2 + r^2 d\Omega_2^2 \,, \tag{2.59}$$

Also demanding that the constant time slices be flat,

$$f(r)^{-1} - f(r)g(r)'^2 = 1 \quad \Rightarrow \quad g(r)' = \frac{\sqrt{1-f(r)}}{f(r)} \,. \tag{2.60}$$

Eventually the metric, written in Painlevé coordinates, is

$$ds^2 = -f(r)dt^2 + 2\sqrt{1-f(r)}\,dtdr + dr^2 + r^2 d\Omega_2^2 \,. \tag{2.61}$$

Considering the Schwarzschild solution in Planck units with

$$f(r) \equiv 1 - \frac{2M}{r} \ , \tag{2.62}$$

being M the mass of the black hole, one obtains for the metric in Painelevé coordinates,

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + 2\sqrt{\frac{2M}{r}} \, dtdr + dr^2 + r^2 d\Omega_2^2 \ . \tag{2.63}$$

We can see that these coordinates are stationary and not singular through the horizon. Then we can define a vacuum state demanding that it annihilates the modes with negative frequency. Now consider a radial null geodesic

$$\dot{r} = \pm 1 - \sqrt{\frac{2M}{r}} \ , \tag{2.64}$$

with the plus (minus) sign corresponding to outgoing (ingoing) geodesics respectively. However, we have to modify the geodesic equation when self-gravitation is included, then we will not consider the emission of point particles but the propagation of shell particles. Self-gravitating shells in Hamiltonian gravity were studied in [35]. Keeping fixed the ADM mass [36] and allowing the black hole mass to vary, we see how the metric backreacts due to the emission of a shell particle, hence $M \to M - \omega$ in order to keep energy conservation.

The wavelength of the radiation is of the order of the size of the black hole. Nevertheless, when we trace back the outgoing wave, we point out that the wavelength is blue-shifted, justifying thus the use of the WKB approximation (2.55). In order to simplify, one could consider the propagation of an s-wave, neglecting then the angular part of the background metric (2.63). Thus using the Birkhoff's theorem one can decouple gravity from matter. Therefore, the imaginary part of the action for an s-wave outgoing positive energy particle will be

$$\text{Im}S = \text{Im} \int_{r_{in}}^{r_{out}} p_r dr = \text{Im} \int_{r_{in}}^{r_{out}} \int_0^{p_r} dp_r dr \ . \tag{2.65}$$

The particle crosses the horizon from r_{in} to r_{out} with $r_{in} > r_{out}$ due to the shrinking of the horizon when the particle is emitted and the metric backreacts. Making use of the Hamilton's equation $\dot{r} = \frac{dH}{dp_r}$ and writing the Hamiltonian as $H = M - \omega$, we obtain

$$\text{Im}S = \text{Im} \int_{M}^{M-\omega} \int_{r_{in}}^{r_{out}} \frac{dr}{\dot{r}} \, dH = \text{Im} \int_0^{\omega} (-d\omega) \int_{r_{in}}^{r_{out}} \frac{dr}{1 - \sqrt{\frac{2(M-\omega)}{r}}} \ . \tag{2.66}$$

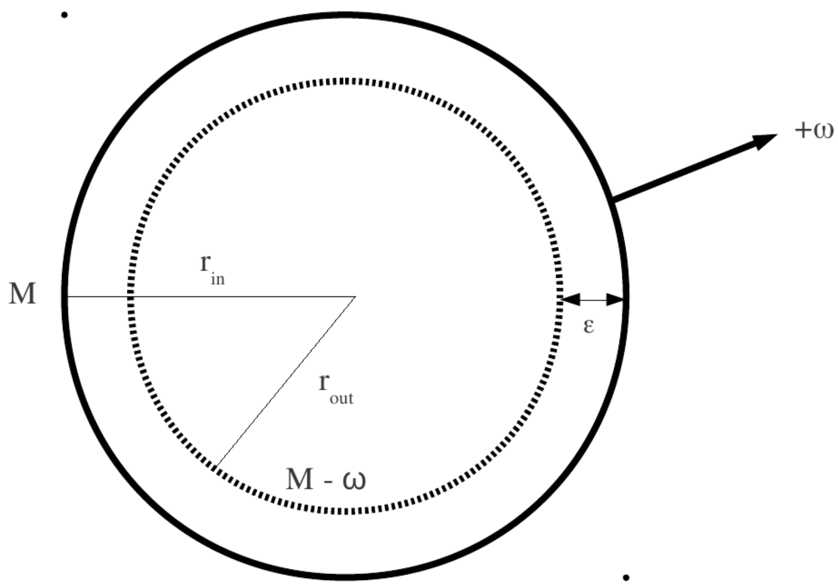

Figure 2.4: Diagram picture of the tunneling approach. A particle of energy $+\omega$ is emitted by a black hole of initial mass M and initial radius r_{in}. After the emission the event horizon shrinks, ϵ, and the black hole loses mass.

In the last integral there is a pole in the upper half plane of integration. In order to perform the integral we use the Feynman prescription to displace the pole from ω to $\omega - i\epsilon$ deforming the contour around the pole. We just can see how the particle tunnels along forbidden classical path between $r_{in} = 2M - \epsilon$ just inside the horizon and $r_{out} = 2(M - \omega) + \epsilon$ just outside. Hence the imaginary part of the action will be

$$\mathrm{Im}S = 4\pi \left(M\omega - \frac{\omega^2}{2} \right) . \tag{2.67}$$

Finally from (2.55) the emission rate of the tunneling process is

$$\Gamma \sim e^{-8\pi\left(M\omega - \frac{\omega^2}{2}\right)} . \tag{2.68}$$

We point out that we can write the above result in a statistical mechanics fashion in terms of the change of the entropy as

$$\Gamma \sim e^{\Delta S_{BH}} , \tag{2.69}$$

where ΔS_{BH} is the change of the Bekenstein-Hawking entropy according to the area law, being the entropy before the emission $S_i = 4\pi M^2$ and after the emission $S_f = 4\pi(M - \omega)^2$. It is very interesting to notice from the expression (2.68) that the emission is not purely thermal. Taking into account the backreaction of the metric and imposing energy conservation we obtain a non-thermal emission reflected in the presence of the ω^2-term. This fact leads us to think that some sort of correlations exist between the emitted particles, carrying out some degrees of freedom that enables us to recover the information lost in the black hole. Of course, if we neglect the quadratic energy term we obtain the Planck spectrum

$$\rho(\omega) = \frac{\Gamma_\omega}{(e^{\omega/T} - 1)} \frac{d\omega}{2\pi} , \tag{2.70}$$

at a Hawking temperature $T_H = \frac{1}{8\pi M}$, where Γ_ω is the greybody factor.

One can perform the same analysis in the Reissner-Nordstrom black hole obtaining similar conclusions. However, in order to simplify, we only consider the emission of uncharged particles, otherwise we must consider the electromagnetic interactions between the particles and the electromagnetic field of the black hole. The line element for the Reissner-Nordstrom charged black hole is

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 + \frac{1}{\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)} dr^2 + r^2 d\Omega_2^2 , \tag{2.71}$$

being Q the charge of the black hole. In Painlevé coordinates this metric is written as

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 + 2\sqrt{\frac{2M}{r} - \frac{Q^2}{r^2}}\, dtdr + dr^2 + r^2 d\Omega_2^2 . \tag{2.72}$$

A radial null geodesic for a outgoing uncharged particle is

$$\dot{r} = 1 - \sqrt{\frac{2M}{r} - \frac{Q^2}{r^2}} . \tag{2.73}$$

As in the Schwarzschild case we compute the imaginary part of the action for the emission of a shell of energy ω

$$\mathrm{Im} S = \int_M^{M-\omega} dH \int_{r_{in}}^{r_{out}} \frac{dr}{\dot{r}} = \int_0^\omega d(-\omega) \int_{r_{in}}^{r_{out}} \frac{dr}{1 - \sqrt{\frac{2M}{r} - \frac{Q^2}{r^2}}} . \tag{2.74}$$

In order to evaluate the radial integral we perform the following change of coordinates

$$u = \sqrt{2Mr - Q^2} \quad \Rightarrow \quad du = \frac{M}{u} dr , \tag{2.75}$$

thus the radial integral in terms of the u coordinate is

$$\int \frac{u(u^2 + Q^2)}{M\left(u(u - 2M) + Q^2\right)} du . \tag{2.76}$$

The integral has a pole at $u = M \pm \sqrt{M^2 - Q^2}$, where plus/minus sign corresponds to the outer/inner horizon position, effectively if we apply the Cauchy's theorem we obtain a residue value of $\frac{\left(M+\sqrt{M^2-Q^2}\right)^2}{\sqrt{M^2-Q^2}}$. Now if we take into account the self-gravitation [37], then replacing M by $M - \omega$ and integrating the energy, the imaginary part of the action becomes

$$\begin{aligned} \mathrm{Im}S &= -2\pi \int_0^\omega \frac{\left((M-\omega) + \sqrt{(M-\omega)^2 - Q^2}\right)^2}{\sqrt{(M-\omega)^2 - Q^2}} \, d(-\omega) \qquad (2.77) \\ &= 2\pi \left[M \left(M + \sqrt{M^2 - Q^2} \right) - (M-\omega) \left(M - \omega + \sqrt{(M-\omega)^2 - Q^2} \right) \right] . \end{aligned}$$

Eventually we can also see quadratic energy terms, thus the emission rate (2.55) is non-thermal,

$$\Gamma \sim e^{-4\pi \left[M\left(M+\sqrt{M^2-Q^2}\right) - (M-\omega)^2 - (M-\omega)\sqrt{(M-\omega)^2-Q^2} \right]} . \qquad (2.78)$$

2.4 The complex path method

Another semi-classical method in order to calculate the particle production near the event horizon of black holes was proposed in [38]. The complex path method has the advantage that avoids the Kruskal extension of the space-time thus one can work with the usual spherical coordinates, and hence it is not needed to compute the Bogoliubov coefficients. We will show the method in the simple case of four-dimensional spherically symmetric background (2.56) and (2.57). We only consider the $r-t$ sector relevant for the emission process, so that the effective two-dimensional metric is

$$ds^2_{eff} = -f(r)dt^2 + \frac{1}{f(r)} \, dr^2 . \qquad (2.79)$$

Now we consider the propagation of a massless scalar field in this two-dimensional background, then the Klein-Gordon equation of motion

$$g^{\mu\nu} \nabla_\mu \nabla_\nu \phi = 0 , \qquad (2.80)$$

can be written as

$$\left[\frac{1}{f(r)} \partial_t^2 - \partial_r \left(f(r) \partial_r \right) \right] \phi = 0 . \qquad (2.81)$$

Then if we take the WKB ansatz solution

$$\phi \sim e^{\frac{i}{\hbar} S(t,r)} , \qquad (2.82)$$

where we have written $\hbar$ explicitly, and substituting in (2.81) we get

$$\frac{1}{f(r)}\left(\frac{\partial S(t,r)}{\partial t}\right)^2 - f(r)\left(\frac{\partial S(t,r)}{\partial r}\right)^2 + \\ + \frac{\hbar}{i}\left(\frac{1}{f(r)}\frac{\partial^2 S(t,r)}{\partial t^2} - f(r)\frac{\partial^2 S(t,r)}{\partial r^2} - \frac{df(r)}{dr}\frac{\partial S(t,r)}{\partial r}\right) = 0 \; . \tag{2.83}$$

Now taking the expansion of the action in terms of $\left(\frac{\hbar}{i}\right)$

$$S(t,r) = S_0(t,r) + \sum_{n=1}^{\infty}\left(\frac{\hbar}{i}\right)^n S_n(t,r) \; , \tag{2.84}$$

substituting into the equation of motion (2.83), and neglecting terms of the order $\left(\frac{\hbar}{i}\right)$ and higher; we obtain at leading order in the action a Hamilton-Jacobi equation,

$$\frac{\partial S_0(t,r)}{\partial t} = \pm f(r)\,\frac{\partial S_0(t,r)}{\partial r} \; , \tag{2.85}$$

whose solution is

$$S_0(r_2,t_2;r_1,t_1) = -\omega(t_2-t_1) \pm \omega\int_{r_1}^{r_2}\frac{1}{f(r)}\,dr \; . \tag{2.86}$$

The plus/minus sign corresponds to the ingoing/outgoing particle respectively whereas ω is the energy of the absorbed or emitted particle. Henceforth, we will neglect the time part which accounts for the stationary phase of the solution and does not affect the final result. In order to evaluate the integral of the radial part of the solution (2.86) we must take into account that the position of the event horizon r_0 stay between the turning points r_1 and r_2, thus when we integrate from $r_1 < r_0$ to $r_2 > r_0$ we bump into a singularity at r_0, which makes the integral divergent since $f(r_0) = 0$. Therefore we might carry out an integration over the complex plane, specifying what complex contour we will use in order to perform the integration around the pole r_0. Following the prescription used in [38], we use as a contour of integration the infinitesimal semi-circle above r_0 for outgoing particles on the left of the horizon ($r < r_0$) and ingoing particles on the right of the horizon ($r > r_0$), so that displacing the pole $r = r_0 - i\epsilon$. Whereas, for incoming particles on the left and outgoing particles on the right of the horizon the contour will be a semi-circle below r_0, being just the reversed time situation of the previous case, thus displacing the pole $r = r_0 + i\epsilon$.

If we consider an outgoing particle at $r_1 < r_0$, the contour of integration lies on the upper half-complex plane, see Figure 2.5, and the radial integral in (2.86) can

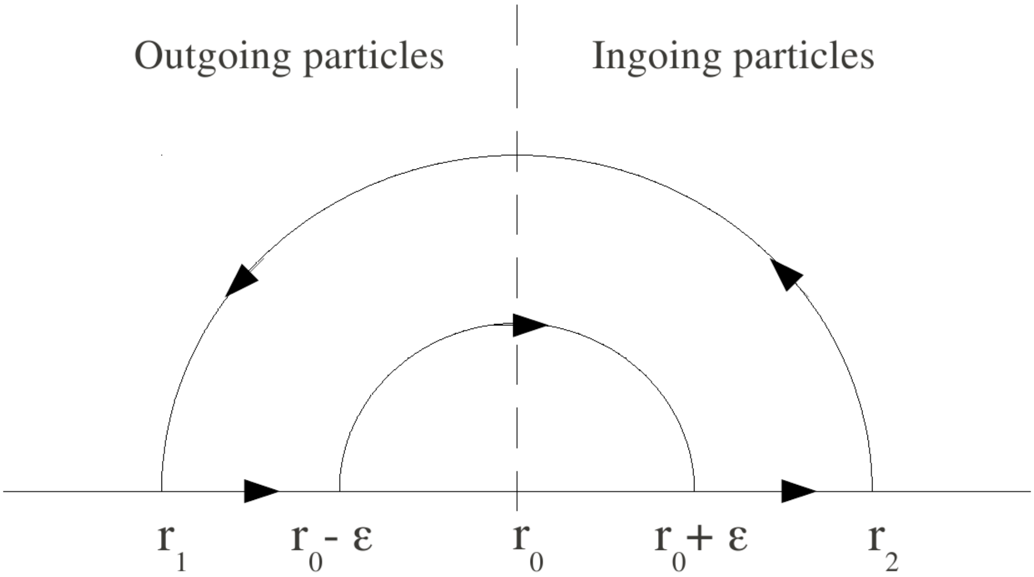

Figure 2.5: Emission action integral (2.86). Contour of integration in the complex plane corresponding to an outgoing particle on the left of the event horizon r_0, between the turning points r_1 and r_2.

be written as

$$S_0^e = -\omega \lim_{\epsilon\to 0} \int_{r_0-\epsilon}^{r_0+\epsilon} \frac{dr}{f(r)} + (\text{Real}) \, , \tag{2.87}$$

where the contribution to the integral in the range $(r_1, r_0-\epsilon)$ and $(r_0+\epsilon, r_2)$ is real. Then, taking into account that the residue of the function $f(r)$ is $i\pi r_0$, the result of the complex integration is

$$S_0^e = i\pi\omega r_0 + (\text{Real}) \, . \tag{2.88}$$

We can show that it is the correct result if we perform the change of variables into complex plane: $r = r_0 + \rho e^{i\theta} \;\Rightarrow\; dr = i\rho e^{i\theta} d\theta$, then the integral becomes

$$\int_{r_0-\epsilon}^{r_0+\epsilon} \frac{dr}{f(r)} = \int_{r_0-\epsilon}^{r_0+\epsilon} \frac{r}{r-r_0}\, dr = \int_{\pi}^{0} \frac{r_0+\rho e^{i\theta}}{r_0+\rho e^{i\theta}-r_0}\, i\rho e^{i\theta} d\theta \, . \tag{2.89}$$

Now considering that we have infinitesimally displaced the pole, we take the limit

$$\lim_{\rho\to 0} = \int_{\pi}^{0} (r_0+\rho e^{i\theta}) i d\theta = -i\pi r_0 \, . \tag{2.90}$$

The same result had been obtained if we had considered the propagation of an ingoing particle. In this case we might choose the contour of integration lying in the

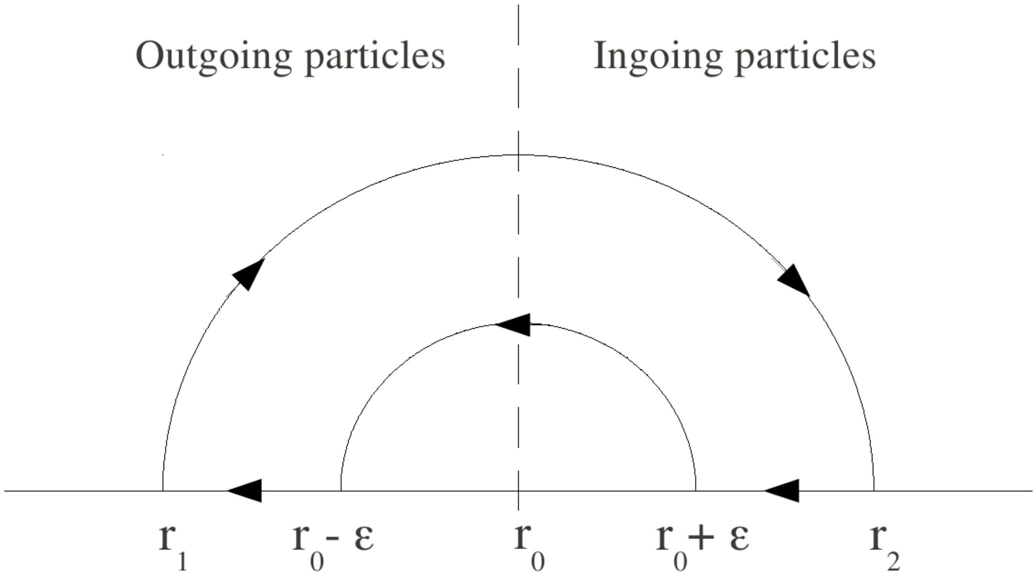

Figure 2.6: Absorption action integral (2.86). Contour of integration in the complex plane corresponding to an ingoing particle on the right of the event horizon r_0, between the turning points r_1 and r_2.

lower half-complex plane. The result (2.88) corresponds to the *emission* action at leading order for a massless scalar particle.

Next, we perform the same analysis for an ingoing particle at $r_2 > r_0$ that crosses the horizon being absorbed by the black hole. Choosing the contour in the upper half-complex plane, see Figure 2.6, we get

$$S_0^a = -\omega \lim_{\epsilon \to 0} \int_{r_0+\epsilon}^{r_0-\epsilon} \frac{dr}{f(r)} + (\text{Real}) \; . \tag{2.91}$$

One obtains the same result considering an outgoing particle with a contour of integration lying in the lower half-complex plane. Then the *absorption* action at leading order for a massless scalar particle will be

$$S_0^a = -i\pi\omega r_0 + (\text{Real}) \; . \tag{2.92}$$

We are going to use the saddle point approximation that enables us to compute the semi-classical propagator in flat space-time for a particle propagating from (t_1, r_1) to (t_2, r_2), [39],

$$K(r_2, t_2; r_1, t_1) = N \exp\left[\frac{i}{\hbar} S_0(r_2, t_2; r_1, t_1)\right] \; . \tag{2.93}$$

Therefore taking into account the definition of probability,

$$P = |K(r_2, t_2; r_1, t_1)|^2 \ , \tag{2.94}$$

we will have for the emission probability,

$$P_e \sim \exp\left[-\frac{2\pi\omega r_0}{\hbar}\right] \ , \tag{2.95}$$

and for the absorption probability,

$$P_a \sim \exp\left[\frac{2\pi\omega r_0}{\hbar}\right] \ . \tag{2.96}$$

Thus, the relation between the emission and absorption probability it is just

$$P_e = \exp\left[-\frac{4\pi\omega r_0}{\hbar}\right] P_a \ , \tag{2.97}$$

that when compared with the thermodynamical result

$$P_e = e^{-\beta\omega} P_a \ , \tag{2.98}$$

allows to identify the temperature as

$$\beta^{-1} = T = \frac{\hbar}{4\pi r_0} \ . \tag{2.99}$$

For the Schwarzschild case where $r_0 = 2M$ we obtain $T = \frac{\hbar}{8\pi M}$, which it is just the correct Hawking temperature.

Thus the complex path method reviews the study of particle production in curved space-times and permits to obtain the correct Hawking temperature. In the next chapter we will see how this method can be implemented taking into account the back-reaction of the metric.

Chapter 3

Hawking radiation in Little String Theory

3.1 LST, thermodynamics overview

Little String Theory (LST) is a non-gravitational six-dimensional and non-local field theory believed to be dual to a string theory background. LST is defined as the decoupled theory on a stack of N NS5-branes. For some good reviews see [40, 41, 42, 43, 44, 45, 46, 47]. In the limit of a vanishing asymptotic value for the string coupling $g_s \to 0$, keeping the string length l_s fixed while the energy above extremality is fixed, i.e. $\frac{E}{m_s}$ = fixed, the processes in which the modes that live on the branes are emitted into the bulk as closed strings are suppressed. Thus the theory becomes free in the bulk, but strongly interacting on the brane. In this limit, the theory reduces to Little String Theory or more precisely to (2,0) LST for type IIA NS5-branes and to (1,1) LST for type IIB NS5-branes [47].

We shall consider the non-extremal case, from where we shall deduce the thermodynamics properties of the black hole. Even if the Hawking's area theorem applies in Einstein frame, where the weak energy condition is satisfied [48], we have cross-checked that all our claims concerning the semi-classical emission can also be obtained from the string frame. For a discussion see Chapter 6.

We take the ten-dimensional action corresponding to a scalar field ϕ propagating in the NS5 background,

$$S = \frac{1}{2k_{10}^2} \int \sqrt{-g} \left(R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{12} e^{-\Phi} H_{(3)}^2 \right) d^{10}x \ , \tag{3.1}$$

where k is a constant, R is the Ricci curvature scalar, Φ the dilaton field and $H_{(3)}$ a strength field. Taking the variation of the action with respect to the strength field, scalar field and metric respectively, we obtain the equations of motion for the three-form $H_{(3)}$,

$$\partial_\mu (\sqrt{-g} e^{-\Phi} H^{\mu\nu\rho}) = 0 \ , \tag{3.2}$$

the scalar Klein-Gordon equation,

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \phi) + \frac{1}{12} e^{-\Phi} H^2 = 0 \ , \tag{3.3}$$

and the Einstein's equations

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R &= \frac{1}{2} \left(\partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} \partial_\beta \phi g^{\beta\alpha} \partial_\alpha \phi \right) + \\ &+ \frac{1}{12} e^{-\Phi} \left(3 H_{\mu ab} H_{\nu cd} g^{ac} g^{bd} - \frac{1}{2} H^2 g_{\mu\nu} \right) \ . \end{aligned} \tag{3.4}$$

The throat geometry corresponding to N coincident non-extremal NS5-branes in the string frame [49] is

$$ds^2 = -f(r) dt^2 + \frac{A(r)}{f(r)} dr^2 + A(r) r^2 d\Omega_3^2 + \sum_{j=1}^{5} dx_j^2 \ , \tag{3.5}$$

where dx_j^2 corresponds to flat spatial directions along the 5-branes, $d\Omega_3^2$ corresponds to 3-sphere of the transverse geometry,

$$d\Omega_3^2 = d\theta^2 + \sin^2\theta \, d\varphi^2 + \sin^2\theta \, \sin^2\varphi \, d\psi^2 \ , \tag{3.6}$$

and the dilaton field is defined as

$$e^{2\Phi} = g_s^2 A(r) \ . \tag{3.7}$$

The metric functions are defined as

$$f(r) = 1 - \frac{r_0^2}{r^2} \quad , \quad A(r) = \chi + \frac{N}{m_s^2 r^2} \ , \tag{3.8}$$

the location of the event horizon corresponds to $r = r_0$. The black hole mass is related with r_0 through the relation $M \sim r_0^2$, see Appendix C and Chapter 6 equation (6.11) for a exact relation in string frame and Einstein frame respectively. We define the parameter χ which takes the values 1 for NS5 model and 0 for LST, these are the unique values for which exist a supergravity solution. In addition to the previous fields there is an $NS-NS$ $H_{(3)}$ form along the S^3, $H_{(3)}=2N\epsilon_3$. According to the holographic principle the high spectrum of this dual string theory should be approximated by certain black hole in the background (3.5). The geometry transverse to the 5-branes is a long tube which opens up into the asymptotic flat space with the horizon at the other end. In the limit $r\to r_0$ appears the semi-infinite throat parametrized by (t,r) coordinates, in this region the dilaton grows linearly pointing out that gravity becomes strongly coupled far down the throat. The string propagation in this geometry should correspond to an exact conformal field theory [50]. The boundary of the near horizon geometry is $R^{5,1}\times R\times S^3$. The geometry (3.5) is regular as long as $r_0\neq 0$.

We are going to reduce the metric (3.5) to the $r-t$ sector, relevant for the forthcoming sections. At first step we take the scalar field action

$$S=\frac{1}{2k_{10}^2}\int_M d^{10}x\sqrt{-g}\left(R-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-\frac{1}{12}e^{-\Phi}H_{(3)}^2\right)\quad . \tag{3.9}$$

Performing a change to tortoise coordinate, see (3.5): $dr_*=\frac{\sqrt{A(r)}}{f(r)}dr$, we expand the ten-dimensional action as

$$\begin{aligned}S=&\frac{1}{2k_{10}^2}\int dt dr_* d\theta d\varphi d\psi\prod_{j=1}^{5}dx_j\; r^3A(r)^2\sin^2\theta\,\sin\varphi\,(g_se^{-\Phi})^{5/2}\left[\frac{f(r)}{\sqrt{A(r)}}\times\right.\\ &\times\left(R-\frac{e^{-\Phi}}{12}H_{(3)}^2\right)+\left(\frac{1}{2\sqrt{A(r)}}(\partial_t^2-\partial_{r_*}^2)-\frac{f(r)}{2r^2A^{3/2}}\left(\partial_\theta^2+\frac{1}{\sin^2\theta}\partial_\varphi^2+\right.\right.\\ &\left.\left.\left.+\frac{1}{\sin^2\theta\,\sin^2\varphi}\partial_\psi^2\right)-\frac{f(r)}{2\sqrt{A(r)}}\sum_{j=2}^{6}\partial_j^2\right)\phi(t,r)S(\Omega_3)\,e^{i\sum k_jx_j}\right]\,,\end{aligned}\tag{3.10}$$

where we have decomposed the scalar field into $r-t$, 3-angular and 5-brane parts. Our following approximations are based on three main steps:

1. We only consider the propagation mode of an s-wave.
2. We only take into account a subset of states of the Hilbert space such that the eigenstates of momentum parallel to the NS5-brane vanish.
3. We take the near horizon limit, $r\to r_0$.

Eventually we come back to the original r radial coordinate, obtaining for the action

$$S = \frac{\mathrm{Vol}(\mathbf{S}^3)\mathrm{Vol}(\mathbf{R}^5)}{2k_{10}^2}\int dtdrA(r)^2e^{-2\Phi}\left(-\frac{1}{f(r)}\partial_t^2+\frac{f(r)}{A(r)}\partial_r^2\right)\phi(t,r)\ , \qquad (3.11)$$

where $\mathrm{Vol}(\mathbf{R}^5)$ stands for the volume of the NS5-branes and $\mathrm{Vol}(\mathbf{S}^3)$ is the volume of the 3-sphere. From (3.11) we find out that the scalar field can be seen as $(1+1)$-dimensional scalar field $\phi(t,r)$ propagating in the background

$$ds^2_{eff} = -f(r)dt^2 + \frac{A(r)}{f(r)}dr^2\ , \qquad (3.12)$$

together with an effective dilaton field

$$e^{2\Phi} = g_s^2 A(r)\ . \qquad (3.13)$$

Henceforth we are going to work with this two-dimensional effective metric.

Concerning the black hole thermodynamics we will construct the thermal states of the black hole following the same analysis of Chapter 2, Section 2.2. Working in imaginary time coordinate $t = i\tau$, we will write the positive Euclidean metric in Rindler coordinates. The radial Rindler coordinate is

$$\rho = \lim_{r\to r_0}\left[2\sqrt{\frac{A(r)(r-r_0)}{f(r)'}}\right]\ . \qquad (3.14)$$

Thus the Euclidean metric in Rindler coordinates is

$$ds_E^2 = \rho^2(\kappa d\tau)^2 + d\rho^2 + A(r)r^2d\Omega_3^2 + \sum_{j=1}^{5}dx_j^2\ , \qquad (3.15)$$

where we have defined κ as

$$\kappa = \frac{f(r_0)'}{2\sqrt{A(r_0)}}\ , \qquad (3.16)$$

which it is precisely the surface gravity of the NS5 and LST black holes. In the footnote of Section 2.2 we had given a simple explicit formula in order to calculate the surface gravity [28],

$$\kappa^2 = -\frac{1}{2}\left(\nabla^\mu\zeta^\nu\right)\left(\nabla_\mu\zeta_\nu\right) \qquad (3.17)$$

evaluated at the event horizon, where ζ_ν is a Killing vector. For the NS5 and LST stationary black holes we choose the Killing vector

$$\zeta^\nu = (-\partial_t, \zeta^i) \quad \text{with} \quad \zeta^i = 0 \ , \ i = 1, ..., 9 \ ; \tag{3.18}$$

and its covariant form

$$\zeta_\nu = g_{\nu\lambda}\zeta^\lambda = g_{tt}\,(-\partial_t) \ . \tag{3.19}$$

Calculating

$$\nabla^\mu \zeta^\nu = g^{\mu\lambda}\nabla_\lambda \zeta^\nu = g^{rr}\nabla_r \zeta^t \ , \tag{3.20}$$

and

$$\nabla_\mu \zeta_\nu = \nabla_r \zeta_t \ , \tag{3.21}$$

it is obtained the expression

$$\kappa = \frac{1}{2}\sqrt{-g^{tt}\cdot g^{rr}} \ \left(\partial_r g_{tt}\right) \ . \tag{3.22}$$

Evaluating this expression at the event horizon r_0, it is obtained the surface gravity. Concretely for NS5 and LST we obtain

$$\kappa = \frac{f(r_0)'}{2\sqrt{A(r_0)}} = \frac{1}{\sqrt{\chi r_0^2 + \frac{N}{m_s^2}}} \ . \tag{3.23}$$

Then identifying the period of the Euclidean time with $\tau \to \tau + \frac{2\pi}{\kappa}$ we avoid the conical singularity in (3.15), thus the imaginary time is periodic with period $\beta = \frac{2\pi}{\kappa}$. As it was demonstrated in Section 2.2 we can identify β^{-1} with the Hawking temperature T_H, thereby calculating the value of the surface gravity (3.16) we obtain the Hawking temperature for the NS5 and LST black holes,

$$T_H = \frac{\hbar}{2\pi\sqrt{\chi r_0^2 + \frac{N}{m_s^2}}} \ . \tag{3.24}$$

Notice that this value for LST ($\chi = 0$) is independent of the black hole radius, that is *fixed* even if many particles impinge on the black hole. This results holds at all orders in α' (inverse string tension) corrections, but receives modifications from higher genus [51, 52].

We would like to address the question whether an observer in a moving frame observes a temperature above the Hagedorn temperature. We know that in the near horizon limit of NS5, i.e. LST, the system reaches the maximum temperature, namely the Hagedorn temperature. One could think that a boosted observer may observe a temperature higher than the Hagedorn one, for this reason we want to verify the validity of this statement. We have evaluated the simplest case, namely a scalar particle-like observer who moves on an NS5-brane with constant velocity at a fixed distance r from the horizon of the LST black hole. We consider the orbit for which $x_1 = vt$. Relating the time coordinate t with the proper time τ (this is not the imaginary time) through $d\tau^2 = -g_{\mu\nu}dx^\mu dx^\nu$, one obtains

$$\frac{d\tau}{dt} = \sqrt{f(r) - v^2}\,. \tag{3.25}$$

The velocity is bounded by the local velocity of light thus we have to impose the constraint $v^2 \leq f(r)$. This relation brings us to a new coordinate of the horizon position seen by the moving particle, $r = \frac{r_0}{\sqrt{1-v^2}}$. Furthermore the Killing vector relevant for the process is $\zeta = -\partial_t + v\partial_{x_1}$. Therefore evaluating the surface gravity using this new coordinate r, we obtain the local temperature for the moving scalar particle

$$\bar{T} = \frac{\hbar(1-v^2)}{2\pi\sqrt{\chi\frac{r_0^2}{(1-v^2)} + \frac{N}{m_s^2}}}\,, \tag{3.26}$$

where we have used natural units, $c = 1$ and $v < 1$. We notice two important features. First of all, we see that in the $v \to 0$ limit we recover the result (3.24). Secondly, comparing the temperature for the particle-like observer (3.26) with the temperature defined by (3.24) for an asymptotic static observer, we see that the former is lower than the later. We conclude that the Hawking temperature of LST is a maximum bound and corresponds to the Hagedorn temperature. Unfortunately, we are not able to perform the same analysis for an accelerating particle-like observer. The main problem is that the path which the particle follows is not generated by a Killing vector field, this fact prevent us from using the surface gravity method in order to calculate the temperature.

Next, we calculate the entropy using the area law through the Bekenstein-Hawking entropy relation

$$S_{BH} = \frac{A_H}{4G^{(10)}\hbar}\,, \tag{3.27}$$

where A_H is the area of the event horizon and $G^{(10)}$ is the ten-dimensional gravitational constant. Working in string frame the area of the event horizon is

$$A_H = \int \sqrt{-g^{(8)}}\, d\theta\, d\varphi\, d\psi\, d^j x = \mathrm{Vol}(\mathbf{R}^5)\, 2\pi^2 \left(\chi r_0^2 + \frac{N}{m_s^2}\right)^{3/2}\,, \tag{3.28}$$

the factor $2\pi^2$ accounts for the volume of the 3-sphere, see Appendix B, and $-g^{(8)}$ is the determinant of the induced metric on the event horizon

$$d\tilde{s}^2 = A(r)\, r^2 d\Omega_3^2 + \sum_{j=1}^{5} dx_j^2 \,. \tag{3.29}$$

Then the Bekenstein-Hawking entropy is

$$S_{BH} = \frac{A_H}{4G^{(10)}\hbar} = \frac{\mathrm{Vol}(\mathbf{R}^5)\, \pi^2 \left(\chi r_0^2 + \frac{N}{m_s^2}\right)^{3/2}}{2G^{(10)}\hbar} \,. \tag{3.30}$$

We have seen that the LST temperature is independent of the black hole radius and therefore of the black hole mass. We could identify this temperature with the Hagedorn temperature. It has been argued, [53], that the energy, entropy and temperature of a CFT at high temperatures can be identify with the mass, entropy and Hawking temperature of the dual black hole. The Euclidean action for a LST black hole solution gives a vanishing contribution to the Helmholtz free energy: $\log Z = -\mathcal{I} = 0$, with Z being the string partition function. In that precise case the entropy and energy density are directly proportional to each other and the Bekenstein-Hawking entropy relation is fulfilled. Otherwise, one can compute the Komar energy E for the LST background, see Appendix C, either in Einstein frame [51, 54] or in string frame [46, 49] which satisfies the usual thermodynamic relation $S = \beta E$. This relation implies that the free energy of the system $\mathcal{F} = E - TS$ vanishes. This behavior suggests that at leading order the Hagedorn density of states at very high energies grows as $\rho(E) = e^{S(E)} \sim e^{\beta E}$ [45, 55]. At first sight one could think that a phase transition is present when the system evolves from NS5 to the near horizon limit of NS5, i.e. LST, but we have checked that it is not the case. Plotting the entropy (3.30) versus the temperature (3.24) we do not detect any critical point (Davies point) [56] that would signal a phase transition. Even working in thermodynamic geometry [57], writing the LST metric as a Ruppeiner metric $ds^2 = -3\sqrt{\frac{\pi G}{\hbar^2 M}}\, dS^2$, we do not detect any divergence in the scalar curvature that would signal a possible phase transition. However calculating the specific heat as $C = T\frac{\partial S}{\partial T}$, we have found that it has a negative value: $-3S$, showing that the theory is unstable. In the work [45], the authors show that loop/string corrections to the Hagedorn density of states of LST were of the form $\rho(E) \sim E^{\alpha} e^{\beta E}(1+O(\frac{1}{E}))$, where α is a correction factor. The temperature-energy relation thus becomes $\beta = \frac{\partial \log\rho}{\partial E} = \beta_0 + \frac{\alpha}{E} + O(\frac{1}{E^2})$, where $\beta_0 = T_H^{-1}$. The authors found that since α is negative the high energy thermodynamics corresponding to near-extremal 5-branes is unstable, the temperature is above the Hagedorn temperature and the specific heat is negative. This instability would be associated to the presence of a negative mode (tachyon) in string theory. The high temperature phase of the theory yields the condensation of this mode. The authors are lead again to the conclusion that the Hagedorn temperature is reached at a finite energy, being associated with a phase transition.

3.2 Semi-classical emission in NS5

We are going to calculate the Bogoliubov coefficients, partially following [58], corresponding to the NS5 black hole. We want to stress that it is not possible to make this computation for the LST black hole because it is not asymptotically flat. The creation of particles in the vacuum is due to a mixing of positive and negative frequency modes near the event horizon where the gravitational field is strong. Somehow we are observing the evolution of an initial positive frequency state in one definite vacuum to a final negative frequency state in another vacuum. Working in Heisenberg picture and evaluating the number operator between initial vacuum state, one observer in the final vacuum state will detect a number of particles created in the process. We are interested in the propagation of a massless scalar field ϕ in a geometry which is asymptotically flat. We decompose the scalar field into positive frequency ingoing modes $\{f_i\}$ in the past null infinity hypersurface $\mathcal{I}^-$

$$\phi = \sum_i (f_i a_i + f_i^* a_i^\dagger) \ . \tag{3.31}$$

The set $\{f_i\}$ form a complete orthonormal basis with well inner defined product (2.6). The operators a_i and $a_i^\dagger$ can be interpreted as annihilation and creation operators respectively that fulfill the usual commutation relation $[a_i, a_j^\dagger] = \delta_{ij}$ and $[a_i, a_j] = [a_i^\dagger, a_j^\dagger] = 0$. Thus one obtains a well defined vacuum state on $\mathcal{I}^-$

$$a_i |0_-\rangle = 0 \ . \tag{3.32}$$

At some point a black hole is formed and an event horizon appears. On the future horizon $\mathcal{H}^+$ there is not Cauchy data coming from the future null infinity hypersurface $\mathcal{I}^+$, in the same way on $\mathcal{I}^+$ we do not have Cauchy data coming from $\mathcal{H}^+$. Therefore the scalar field can also be decomposed as

$$\phi = \sum_i (p_i b_i + p_i^* b_i^\dagger + q_i c_i + q_i^* c_i^\dagger) \ , \tag{3.33}$$

where $\{p_i\}$ are positive frequency outgoing modes defined on $\mathcal{I}^+$ with its corresponding creation and annihilation operators b_i and $b_i^\dagger$. The modes $\{q_i\}$ are absorbed by the future event horizon $\mathcal{H}^+$ thus cannot escape to $\mathcal{I}^+$. These modes are not-well positive frequency defined modes because on $\mathcal{H}^+$ we cannot define positive (or negative) frequencies, having thus a mixing of positive and negative frequency modes. Nevertheless the choice of $\{q_i\}$ does not affect the calculation at the asymptotic limit since they are zero at $\mathcal{I}^+$. Also we can define a vacuum state on $\mathcal{I}^+$

$$b_i|0_+\rangle = 0\,. \tag{3.34}$$

The modes $\{p_i\}$ on $\mathcal{I}^+$ can be decomposed in terms of the incoming modes $\{f_i\}$

$$p_i = \sum_j (\alpha_{ij} f_j + \beta_{ij} f_j^*)\ . \tag{3.35}$$

In the same way we can relate the corresponding operators

$$\begin{aligned} b_i &= \sum_j (\alpha_{ij}^* a_j - \beta_{ij}^* a_j^\dagger)\ , \\ b_i^\dagger &= \sum_i (\alpha_{ij} a_j^\dagger - \beta_{ij} a_j)\ . \end{aligned} \tag{3.36}$$

These relations are known as Bogoliubov transformations and relate different modes expressed in different basis. We can see that operating with annihilation operator b_i in the vacuum state defined on $\mathcal{I}^-$ the result will be different from zero, if the coefficients β_{ij}^* are non-zero. Thus one has a mixing between positive and negative frequency modes. It can be calculated the number of particles created, i.e. the number of particles measured by an observer in the future null infinity in the vacuum defined on $\mathcal{I}^-$,

$$\langle 0_-|N_i|0_-\rangle = \langle 0_-|b_i^\dagger b_i|0_-\rangle = \sum_j |\beta_{ij}|^2\ . \tag{3.37}$$

In order to calculate the scalar field modes we must solve the Klein-Gordon equation for a massless particle $\Box\phi = 0$. In the background (3.5) this equation is written as

$$\left[-A(r)\frac{\partial^2}{\partial t^2} + f(r)A(r)\frac{\partial^2}{\partial y^2} + \frac{f(r)}{r^3}\frac{\partial}{\partial r}\left(r^3 f(r)\frac{\partial}{\partial r}\right) + \frac{f(r)}{r^2}L^2\right]\phi = 0\ , \tag{3.38}$$

where we have defined the angular momentum operator as

$$L^2 \equiv \frac{1}{\sin^2\theta}\frac{\partial}{\partial\theta}\sin^2\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta\sin\varphi}\frac{\partial}{\partial\varphi}\sin\varphi\frac{\partial}{\partial\varphi} + \frac{1}{\sin^2\theta\sin^2\varphi}\frac{\partial^2}{\partial\psi^2}\ . \tag{3.39}$$

We are looking for a solution of type

$$\phi = \left(Ae^{-i\omega t} + A^* e^{i\omega t}\right) R(r) \left(Be^{ikx} + B^* e^{-ikx}\right) Y_{l,m,m'}(\theta, \varphi, \psi) \ , \tag{3.40}$$

where we have decomposed the solution into stationary part, a pure radial term, the propagation through the flat-space brane directions x and the angular part in which the 3-dimensional scalar spherical harmonics satisfy

$$L^2 Y_{l,m,m'}(\theta, \varphi, \psi) = -l(l+2) Y_{l,m,m'}(\theta, \varphi, \psi) \ . \tag{3.41}$$

Thus the equation of motion can be written as

$$\left[A(r)\omega^2 - f(r)A(r)k^2 + \frac{f(r)}{r^3}\frac{\partial}{\partial r}\left(r^3 f(r)\frac{\partial}{\partial r}\right) - \frac{f(r)}{r^2} l(l+2)\right] R(r) = 0 \ . \tag{3.42}$$

Performing a change to tortoise coordinate

$$dr_* = \frac{\sqrt{A(r)}}{f(r)} dr \ , \tag{3.43}$$

and the standard functional change $R(r) = \frac{R(r_*)}{r}$, [39], we obtain a Schrodinger-type equation

$$\left[\frac{\partial^2}{\partial r_*^2} + \left(\omega^2 - f(r)k^2 - \frac{f(r)}{A(r)r^2} L^2\right)\right] R(r_*) = 0 \ . \tag{3.44}$$

Considering the propagation of an s-mode in the asymptotic limit we find the solution

$$R(r) = \frac{1}{r}\left(C_1 e^{-i\sqrt{\omega^2 - k^2}\, r_*} + C_2 e^{i\sqrt{\omega^2 - k^2}\, r_*}\right) \tag{3.45}$$

where C_1 and C_2 are constants. Eventually the scalar field takes the form

$$\begin{aligned} \phi \;=\; & \left(Ae^{-i\omega t} + A^* e^{i\omega t}\right) \times \left(Be^{ikx} + B^* e^{-ikx}\right) \times \\ & \times \frac{1}{r}\left(C_1 e^{-i\sqrt{\omega^2 - k^2}\, r_*} + C_2 e^{i\sqrt{\omega^2 - k^2}\, r_*}\right) Y_{l,m,m'}(\theta, \varphi, \psi) \ . \end{aligned} \tag{3.46}$$

If we only consider a subset of states of the Hilbert space such that the eigenstates of momentum parallel to the NS5-brane vanish, i.e. $k = 0$, the scalar field solution will be

$$\begin{aligned} \phi \;=\; & \frac{1}{r}\left(AC_1 e^{-i\omega(t+r_*)} + AC_2 e^{-i\omega(t-r_*)} + A^* C_1 e^{i\omega(t-r_*)} + A^* C_2 e^{i\omega(t+r_*)}\right) \times \\ & \times Y_{l,m,m'}(\theta, \varphi, \psi) \ . \end{aligned} \tag{3.47}$$

We introduce the advanced and retarded null coordinates

$$v = t + r_* \quad , \quad u = t - r_* \ , \tag{3.48}$$

and we use them as a canonical affine parameters in order to define the positive frequency modes. Substituting (3.48) in (3.47), we obtain for the incoming modes defined at $\mathcal{I}^-$,

$$f_{\omega' lmm'} \sim \frac{1}{\sqrt{2\pi\omega'}} \frac{F_{\omega'}(r)}{r} e^{i\omega' v}\, Y_{l,m,m'}(\theta, \varphi, \psi) \ , \tag{3.49}$$

and for the outgoing modes defined at $\mathcal{I}^+$,

$$p_{\omega lmm'} \sim \frac{1}{\sqrt{2\pi\omega}} \frac{P_{\omega}(r)}{r} e^{i\omega u}\, Y_{l,m,m'}(\theta, \varphi, \psi) \ . \tag{3.50}$$

$F_{\omega'}(r)$ and $P_\omega(r)$ are integration variables that contain tiny effects depending on r since this modes are calculated at the asymptotic. The normalization constant $\frac{1}{\sqrt{2\pi\omega}}$ is frequently used in the Klein-Gordon equation.

Next, we transform the discrete expressions (3.35), (3.36) and (3.37) to the continuous limit integrating the energy ω and considering the same value for the l, m, m' numbers, thereby we obtain

$$p_\omega = \int_0^\infty (\alpha_{\omega\omega'} f_{\omega'} + \beta_{\omega\omega'} f^*_{\omega'}) d\omega' \ , \tag{3.51}$$

$$b_\omega = \int_0^\infty (\alpha_{\omega\omega'} a_{\omega'} - \beta^*_{\omega\omega'} a^\dagger_{\omega'}) d\omega' \ , \tag{3.52}$$

and

$$N_\omega = \int_0^\infty |\beta_{\omega\omega'}|^2 d\omega' \ . \tag{3.53}$$

We calculate the Bogoliubov coefficients $\alpha_{\omega\omega'}$ and $\beta_{\omega\omega'}$ by Fourier transforming (3.51). Then substituting (3.49) into (3.51) and multiplying both sides by $\int_{-\infty}^{\infty} e^{-i\omega'' v} dv$, we obtain

$$\int_{-\infty}^{\infty} p_\omega e^{-i\omega'' v} dv = \frac{1}{\sqrt{2\pi\omega'}} \frac{F_{\omega'}(r)}{r} \int_0^\infty \left(\alpha_{\omega\omega'} 2\pi\delta(\omega' - \omega'') + \beta^*_{\omega\omega'} 2\pi\delta(-\omega' - \omega'')\right) d\omega' \ , \tag{3.54}$$

where the second term of the integral must be zero due to the properties of the delta distribution. The Bogoliubov coefficients can be written as

$$\alpha_{\omega\omega'} = \frac{r\sqrt{\omega'}}{F_{\omega'}(r)\sqrt{2\pi}} \int_{-\infty}^{\infty} p_\omega e^{-i\omega' v} dv \ , \tag{3.55}$$

$$\beta_{\omega\omega'} = \frac{r\sqrt{\omega'}}{F_{\omega'}(r)\sqrt{2\pi}} \int_{-\infty}^{\infty} p_\omega e^{i\omega' v} dv \ . \tag{3.56}$$

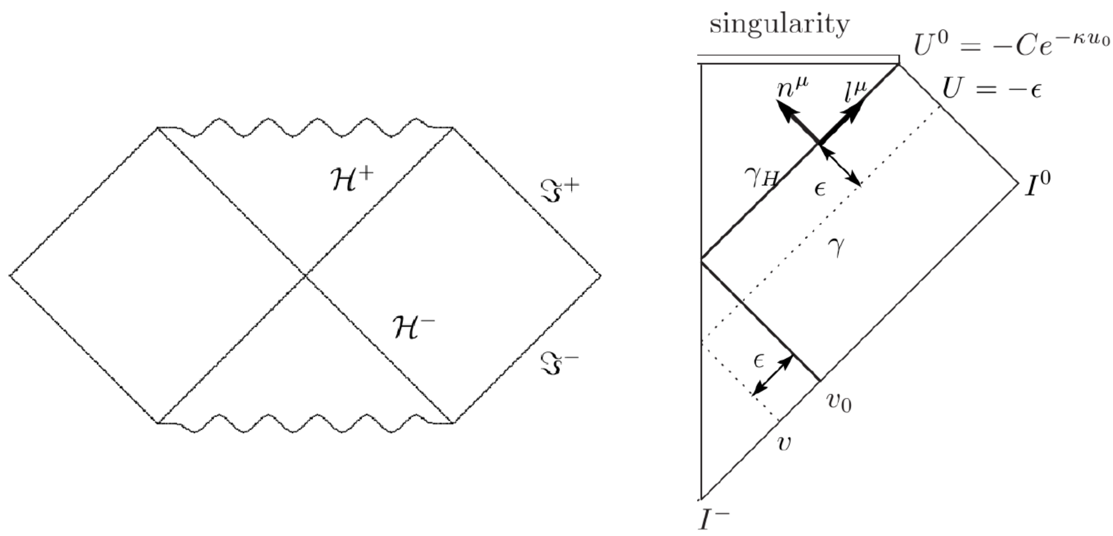

Figure 3.1: On the left: a Penrose diagram which shows the future and past null infinity and the future and past event horizon. On the right: a null ray is traced back in time from the future null infinity using the parallel transporting of two unitary vectors n and l. It is obtained a relation between the null coordinates v and u in order to express the outgoing modes p_ω in terms of the advanced null coordinate v.

In order to evaluate the integral (3.55) we must express the outgoing modes p_ω in terms of the advanced null coordinate v. It is considered a light ray traced backward from $\mathcal{I}^+$ as proposed in [8] and it is taken the Kruskal coordinate as the affine parameter on the past event horizon $\mathcal{H}^-$. We consider a mode p_ω propagating backward from the future null infinity hypersurface $\mathcal{I}^+$ and with zero Cauchy data on $\mathcal{H}^+$. Some part of this mode solution will be scattered over the black hole potential and will eventually emerge at $\mathcal{I}^-$ with the same frequency ω. On the other hand, some part of p_w will be partially scattered and reflected, eventually emerging at $\mathcal{I}^-$. This second part will produce the creation of some new particles measured by the asymptotic observer at $\mathcal{I}^+$. The modes will be extremely blue-shifted at $\mathcal{H}^+$ because the outgoing null coordinate tends to infinity, therefore we are able to use the optical theorem, which states that only the reflected part of the wave will be significant. Now consider a point x on the horizon, a null tangent vector l^μ on the horizon at x, and a future directed null vector n^μ at x which is normal to the horizon and directed radially inwards. This two vectors are normalized: $l^\mu \cdot n_\mu = -1$, see figure 3.1. Then we consider a null geodesic γ that starts at a point u_0, goes along $\mathcal{H}^+$, is reflected at the center, and emerges towards $\mathcal{I}^-$ along a path defined by v_0.

This is the last time on $\mathcal{I}^-$ that a ray will reach $\mathcal{I}^+$, for later times $v > v_0$, the ray will be absorbed by the black hole and never will reach $\mathcal{I}^+$. Any vector $-\epsilon n^\mu$, with ϵ small and positive, will connect the point x on the future event horizon to a nearby null surface of constant u, and therefore with a p_ω surface of constant phase. If we parallel transport the vectors l^μ and n^μ along the null geodesic γ which defines $\mathcal{H}^+$ until it intersects $\mathcal{H}^-$, the vector $-\epsilon n^\mu$ will always connect the event horizon with the same surface of constant phase of p_ω. The null geodesic will be reflected in the center at $r = 0$, will propagate towards $\mathcal{I}^-$, and the vector $-\epsilon n^\mu$ would then lie along $\mathcal{H}^-$. Then on $\mathcal{H}^-$ we use as a canonical affine parameter the Kruskal coordinate

$$U = -Ce^{-\kappa u} \ , \tag{3.57}$$

where κ is the surface gravity defined on the horizon, and C is a constant. The Kruskal coordinate U takes the values 0 on the future horizon H^+ and $-\epsilon$ on the null geodesic near the horizon. Thus from (3.57) it is obtained the relation

$$u = -\frac{1}{\kappa}(\log \epsilon - \log C) \ . \tag{3.58}$$

Otherwise the phase of the mode p_ω is related with the ingoing null affine parameter v when it propagates along the null geodesic γ towards $\mathcal{I}^-$. Remembering that for $v > v_0$ any ray will reach $\mathcal{I}^+$, for a null geodesic near the event horizon it is fulfilled $\epsilon = v_0 - v$. Furthermore the vector n^μ on $\mathcal{I}^-$ will be parallel to a Killing vector η^ν which is tangent to the null geodesic generator of $\mathcal{I}^-$,

$$n^\mu = D\eta^\nu \ , \tag{3.59}$$

where D is a constant. Finally we obtain the desired relation between the null affine parameters u and v,

$$u = -\frac{1}{\kappa}\left(\log(v_0 - v) - \log C - \log D\right) \ . \tag{3.60}$$

Substituting this relation in (3.50), we find the outgoing mode expressed in terms of the ingoing null coordinate v,

$$p_\omega(v) = \begin{cases} 0 & v > v_0 \\ \frac{P_\omega(r_0)}{r\sqrt{2\pi\omega}} \exp\left[-\frac{i\omega}{\kappa} \log\left(\frac{v_0 - v}{CD}\right)\right] & v < v_0 \ . \end{cases} \tag{3.61}$$

$P_\omega(r_0)$ is the radial function P_ω evaluated at the past event horizon. Therefore inserting (3.61) into (3.55) in a region near the horizon we obtain,

$$\alpha_{\omega\omega'} = \frac{\bar{P}_\omega(r_0)}{2\pi}\sqrt{\frac{\omega'}{\omega}}\,(CD)^{\frac{i\omega}{\kappa}} \int_{-\infty}^{v_0} (v_0 - v)^{-\frac{i\omega}{\kappa}} e^{-i\omega' v} dv \ . \tag{3.62}$$

We have collected the functions $F_{\omega'}(r_0)$ and $P_\omega(r_0)$ in a one single expression, $\bar{P}_\omega(r_0)$. Then performing the change of variable $x = v_0 - v$ and using the Gamma function

$$\int_0^\infty x^{\epsilon-1} e^{-tx} dx = \Gamma(\epsilon) t^{-\epsilon} \ , \tag{3.63}$$

we can evaluate the integral in (3.62). The Bogoliubov coefficient thus is

$$\alpha_{\omega\omega'} = \frac{\bar{P}_\omega(r_0)}{2\pi} \sqrt{\frac{\omega'}{\omega}} \, (CD)^{\frac{i\omega}{\kappa}} \, e^{-i\omega' v_0} \, \Gamma\left(1 - \frac{i\omega}{\kappa}\right) \, (-i\omega')^{-1+\frac{i\omega}{\kappa}} \ , \tag{3.64}$$

which is related with the coefficient $\beta_{\omega\omega'}$ by

$$\beta_{\omega\omega'} = -i\alpha_{\omega(-\omega')} \ , \tag{3.65}$$

where we have used the definitions (2.16). In order to calculate $\beta_{\omega\omega'}$ from (3.65) we analytically continue $\alpha_{\omega(-\omega')}$ anticlockwise around the singular point $\omega' = 0$. Thus we find

$$|\alpha_{\omega\omega'}| = e^{\frac{\pi\omega}{\kappa}} |\beta_{\omega\omega'}| \ . \tag{3.66}$$

In [18] it is demonstrated the lemma (3.66). Using the continuum orthonormal condition (2.19) between the Bogoliubov coefficients,

$$\int_0^\infty (|\alpha_{\omega\omega'}|^2 - |\beta_{\omega\omega'}|^2) d\omega' = \Gamma_\omega \ , \tag{3.67}$$

where we have considered the effect of the greybody factor Γ_ω that deviates the spectrum from a pure Planckian spectrum. Eventually, we obtain the average number of particles created in the mode p_ω on $\mathcal{I}^+$,

$$< N_\omega >= \frac{\Gamma_\omega}{e^{\frac{2\pi\omega}{\kappa}} - 1} \ . \tag{3.68}$$

From this expression we identify the Hawking temperature at which the NS5 black hole radiates away its energy

$$T_H = \frac{\hbar\kappa}{2\pi} = \frac{\hbar}{2\pi\sqrt{r_0^2 + \frac{N}{m_s^2}}} \ . \tag{3.69}$$

3.3 Hawking radiation via tunneling

In this section we basically reproduce the work [59], where we calculated the Hawking emission of NS5 and LST black holes. We analyze the thermal transition between

NS5-branes and LST. It is shown that once the near horizon limit is taken, i.e. LST, the emission is thermal even if back-reaction is taken into account. We remark that this fact is due to the LST mass-independent temperature. However, it is not the case for NS5, which shows a non-thermal emission and thus the possibility of recovering information through the correlations between the emitted particles.

We motivate this study since a central issue in the black hole information puzzle is the problem of low-energy scattering for ordinary quanta by an extremal black hole with a subsequent absorption and Hawking reemission. From a semi-classical point of view the final radiation turns to be that of an exact black body [60, 61]. It has been argued, but not demonstrated, that departures from thermal emission could explain black hole evaporation without lost of information and hence reconcile quantum mechanics with general relativity. In most of the approaches in the literature the role of the black hole is similar to that of a soliton in field theory, being gravity treated as a non-perturbative field to be added to the game once the spectrum and quantization rules to the particle-like objects have been put down by quantum mechanics rules. Although this view suffices in a semi-classical picture it can be inappropriate when one probes Planck scales.

One successful approach that overcomes partially this problem, incorporates the self-gravitation interaction in the radiation process [35]. The underlying idea in this model is extremely simple: the full hole-particle system is reduced to an effective one-dimensional system and for that purpose all the degrees of freedom are truncated to two dimensional. In particular the model for emission/absorption is still only suitable for regions of low-curvature and exclusively tackles the s-wave part of the short-wavelength radiation. This fact allows to employ the WKB approximation that makes any calculation almost straightforward. *All* the studies pursued within the mentioned approach reveal so far that Hawking radiation is not purely thermal. These results, although encouraging to explain the Hawking effect, are distressing and it is not clear which is the ultimate reason that allows *all* the black holes to have a non-thermal emission independently of their nature. Our aim is to present some features of the semi-classical geometry and Hawking radiation in a family of black holes with strict thermal emission even if back-reaction effects are taken into account.

We shall begin by outlining the most salient features of a simpler related model, LST, that is at the main core of the study. Many of the points that will arise here are implicitly or explicitly given in other works. Next we present the emission probability via tunneling in this model, and explain some details of the formalism. As a next step we elucidate a plausible "dynamics" that rides a NS5 setup towards its Hagedorn temperature and study the spectrum of the emission. As we shall see as temperature is increased in this process the spectrum, initially non-thermal, goes to a thermal one.

It should be stressed that thermal emission is not something peculiar of this metric space, but most probably a feature of a full family of spaces [62]. We also worked out a model which ultraviolet completion reduces to the previous one. In that sense one does not expect to obtain the very similar result as before for the decay width, because the emission/absorption process is produced near the horizon and must be insensible to the behavior of the radial asymptotic in the metric. As we shall see this does not turn to be the case.

3.3.1 Tunneling approach in LST

Following [30] we consider the emission of an s-wave massless scalar particle in the radial direction of (3.5). This will allow to use Birkhoff's theorem and decouple gravity from matter. In order to find the Hawking emission we bring the line element (3.5) to a smooth form near the horizon using a Painlevé-like transformation $t \to \hat{t} + g(r)$, which is nothing more than the proper time along the radial geodesic worldline [63]. This form will be more suitable to study across-horizon physics, for instance the tunneling of massless shells. In doing so, we consider a transformation with the property that at a constant time slice matches the geometry of LST space without a black hole immersion

$$ds^2 = \sum_{j=1}^{5} dx_j^2 + N \left(\frac{dr^2}{r^2} + d\Omega_3^2 \right) . \tag{3.70}$$

This is accomplished by choosing

$$f(r) = -\sqrt{N} \operatorname{arctanh} \left(\frac{r}{r_0} \right) , \tag{3.71}$$

which allows to rewrite (3.5) as

$$ds^2 = -f(r) d\hat{t}^2 + \sum_{j=1}^{5} dx_j^2 - 2\sqrt{N} \, \frac{r_0}{r^2} dr d\hat{t} + \frac{N}{r^2} \left(dr^2 + r^2 d\Omega_3^2 \right) . \tag{3.72}$$

The function (3.71) is time independent and as a consequence (3.72) remains stationary as was already the case for (3.5).

To describe the black hole emission we rely on the notion of virtual pair creation near the horizon [8]. Loosely speaking, if the pair is created inside the horizon the positive energy particle tunnels out while the antiparticle is absorbed by the black hole which horizon recesses. Alternatively the pair can be created just outside the horizon, in that case is the antiparticle which tunnels through the horizon, shrinking once more the size of the black hole while the particle escapes. In any of the cases the quantum state of the outside particle is not a pure state, and it is possible to compute the entanglement entropy between the particles that fall into the hole with those that escape to infinity.

This intuitive picture contains some drawbacks, the main one is the lack of understanding the origin of the source for the potential barrier to be tunneled across. The approach devised in [30, 32] overcome this by noticing that when a virtual pair of particles is created is the self-gravitating field of the emitted particle the source for the potential barrier to be tunneled across the horizon. In addition one has to take into account the energy conservation in the process: the ADM mass remains fixed while the black hole mass decreases when the quanta is emitted. This back-reaction deforms the initial metric and is implemented in (3.5) by shifting the black hole mass appearing in the warping factors, $M \sim r_0^2$. To be concrete, once the shell is emitted the correct warp factor would be proportional to $M - \omega$, with ω been the energy released in the emission. This would correspond to a new, smaller value for the radius r_1.

For an observer located at the radial infinity of (3.5), an object approaching r_0 is infinitely blue-shifted. This allows to apply a semi-classical treatment to the particle emission problem and with an extend to use the classical action, in the smooth coordinates (3.72), to describe the wave function $\Psi(r) \sim e^{iS_{\text{class}}}$. Keeping this in mind we evaluate the rate emission for massless particles in the sequel.

The metric (3.5) is stationary and the lagrangean density derived from it fulfills the simple relation $\mathcal{H} = -2\mathcal{L}$ with the hamiltonian density. For dynamics, one consideres only the radial coordinate the expression $\mathcal{L} = -\dot{r}p_r$ holds and the classical action reads as

$$S = \int_{r_{\text{in}}}^{r_{\text{out}}} p_r dr = \int_{r_{\text{in}}}^{r_{\text{out}}} \int_{M}^{M-\omega} \frac{dH}{\dot{r}} dr = -\int_0^{\omega} d\omega \int_{r_{in}}^{r_{out}} \frac{dr}{\dot{r}}\,, \tag{3.73}$$

being ω the maximum energy released by the shell. To obtain (3.73) we have applied Hamilton's equation, defined $\dot{r} := dr/d\hat{t}$ and pulls out factors that do not contribute to the imaginary part of the action. Inherently the expression (3.73) is obtained in the semi-classical regime, i.e. the emitted shell must be a *probe*, $\omega \ll$ M. This also is justified because for large black hole masses, much larger than the Planck mass, the only relevant field configurations taken into account by the WKB approximation are short wavelength solutions in a relative low curvature region. This, in addition, overcomes the ill-defined extremal limit [64].

For the geometry (3.72) the radial light-like geodesics are orthogonal to the surfaces of constant time on which r measures the radial proper distance and is given by

$$\dot{r} = \frac{1}{\sqrt{N}}(r \pm r_0)\,, \tag{3.74}$$

where the plus (minus) sign corresponds to the geodesic rays going towards (away from) the observer. Its general solution is $r = r_0\left(e^{\hat{t}/\sqrt{N}} \pm 1\right)$. Any radial light-like emission reach the future null infinity at $\hat{t} \to \infty$. While a light-like emission leaving the observer at $\hat{t} = 0$ reach the horizon at $\hat{t} = \ln 2/\sqrt{N}$. therefore, as one increases the number of NS5-branes the traveling time gets reduced.

Using the Feynman prescription $+i\epsilon$ to displace the pole, the imaginary part of (3.73) reads $\mathrm{ImS} = \pi\sqrt{N}\omega$. One does not fail to notice that: *i)* this result is independent of the black hole radius and *ii)* that no infinities arise in this calculation, so is mathematically well defined without any need for regularization. The previous relation, together with (3.24), leads to the rate emission

$$\Gamma \sim |\Psi(r)|^2 \sim e^{-\beta_0 \omega}\,, \tag{3.75}$$

where $\beta_0 = \frac{2\pi\sqrt{N}}{m_s} = T_H^{-1}$. The exponent contains the difference between the actions of the higher and lower black hole mass evaluated at the same and unique temperature for the system. The emission (3.75) follows a black body distribution and hence the LST black hole radiation is purely *thermal*.

The consequences of (3.75) are: *i)* all the corresponding states in the dual CFT must be a priori equally weighted,and *ii)* one can convince oneself that cluster decomposition applies and as a result the quantum state of Hawking radiation does not depend on the initial state of the collapsing body. In addition, this fact implies that the probability of emission of a shell of energy $\omega_1 + \omega_2$ is equal to the probability of emitting independently two shells with the same total amount of energy.

As the radiation comes always as a pure state, the Hilbert space can be factorized into two disjoint parts, $\mathcal{H} = \mathcal{H}_{\text{in}} \oplus \mathcal{H}_{\text{out}}$, which correspond to states located at the inner and outer sides of the event horizon, respectively. It will follow from the superposition principle that the state inside the horizon must be a unique state carrying no information at all. Summing up, this can be expressed in a somewhat muted fashion as: the black hole at the Hagedorn temperature does not interact with its environment and hence we can represent a state of the entire space as $|\psi(t)\rangle = |\psi_{\text{in}}(t)\rangle \otimes |\psi_{\text{out}}(t)\rangle$.

3.3.2 Locking information at the Hagedorn temperature

That the result (3.75) gives the correct behaviour for the LST system is intuitively clear in the semi-classical approach from the very beginning since in this type of black holes the temperature is not related with the black hole mass. It is precisely this fact which encodes the ultimate reason for the non-thermal behaviour in the model of [30]. To make this point more clear, instead of using the field content of LST we retain the full asymptotic, ten-dimensional CHS background [65]

$$ds^2 = -f(r)dt^2 + \sum_{j=1}^{5} dx_j^2 + \frac{A(r)}{f(r)}\, dr^2 + A(r) r^2 d\Omega_3^2\,, \tag{3.76}$$

and dilaton $e^{2\phi} = \chi + \frac{N}{m_s^2 r^2}$ with $\chi \equiv 1$ in (3.8). One then sees that the temperature depends on the black hole mass [66]. In this case the Hawking temperature can be determined by the surface gravity method at the event horizon and is given by

$$\beta_{\text{CHS}} = \beta_0 \sqrt{1 + \chi r_0^2/N}\,, \tag{3.77}$$

notice that it provides an infra-red cutoff for the radial coordinate. We have used χ as an eventual continuous variable that parameterizes the geometry (3.76). By no means, one should not understand that all the intermediate values correspond to supergravity solutions. Its utility is twofold: first the near horizon limit is recovered setting $\chi = 0$, and second it will also control the temperature; for instance $\chi \to 0$ increases the temperature to the Hagedorn one. The basic tenant is that (3.77) relates the temperature with the size of the black hole, thus as the black hole emits, not only the radius shrinks but also the temperature increases. This fact relates the emission with the thermodynamic properties of the black hole and contrary to the previous situation we expect that the radiation provides information on the black hole state.

As previously, the geometry at the horizon can be brought to a smooth form with a Painlevé-like change of coordinates

$$\begin{aligned} t &\rightarrow \hat{t} - r\sqrt{A(r)-\chi f(r)}\,\mathrm{arctanh}\left(\frac{r}{r_0}\sqrt{1-\chi\frac{f(r)}{A(r)}}\right) + \\ &+ r_0\sqrt{A(r)}\log\left[2r\left(\sqrt{\chi}+\sqrt{A(r)}\right)\right] . \end{aligned} \tag{3.78}$$

After using (3.78) the metric field (3.76) is reduced to

$$ds^2 = -f(r)d\hat{t}^2 + \sum_{j=1}^{5} dx_j^2 - 2\sqrt{A(r)}\,\frac{r_0}{r}d\hat{t}dr + A(r)\left(dr^2 \; + r^2 d\Omega_3^2\right) . \tag{3.79}$$

A calculation similar to (3.73) leads to the probability for a CHS black hole of mass M to emit a shell of energy ω

$$\Gamma \sim \exp\left(-2\pi\sqrt{N+M\chi}\;\omega + \frac{\chi\,\omega^2}{4\sqrt{N+M\chi}} + \ldots\right) , \tag{3.80}$$

where the ellipsis stand for terms proportional to higher powers of ω. Now for $\chi \to 1$ (3.80) is clearly non-thermal while for $\chi \to 0$ we recover once more the thermal emission (3.75). In view of this fact it seems wholly tenable that as the temperature is increased, $\beta_{\rm CHS} \to \beta_0$, the system evolves from non-thermal to thermal, and as a consequence an asymptotic observer could conjecture that the black hole internal degrees of freedom are reduced during the evaporation process and eventually one remains with a single state. The very same conclusions can be traced back from a stringy point of view if one consider the strings as the fundamental degrees of freedom of the black hole. In a flimsy language: as one approaches the Hagedorn temperature strings condense leaving a residual single one, a unique state that contains no information at all [67]. To substantiate this point we have computed, in the spirit of [68], some properties of a classical string located at the stretched horizon, i.e. a time-like curve slightly outside the global event horizon, that is of relevance in describing the evaporation process. We expect that for sufficiently large black hole masses both the proper distance between the stretched and the event horizon, $\sim \int_{\rm e.h.}^{\rm s.h.} dr\sqrt{g_{rr}}$, and the local Unruh temperature,

$$T_{\rm loc}(r) = \frac{1}{\beta\sqrt{f(r)}} , \tag{3.81}$$

are ballpark of the Planck order (up to a numerical factor of order 1). This implies that the stretched horizon must be almost coincident with the event horizon, $r_P \approx$

$r_0 + \delta$ for some positive and *infinitesimal* constant δ. Using (3.81) at the Planck radius and the Planck temperature, $T_P \sim G^{-1/2}$, we obtain

$$\delta \approx \frac{G\sqrt{GM}}{\beta_0^2 + 4GM\chi}\,, \tag{3.82}$$

where we have momentally reinstated the Newton constant G in the proper space-time dimension. For the CHS model $\delta \sim \sqrt{G/M}$, thus for large black hole masses one can consider that the stretched horizon is almost on top of the event horizon. As we increase the temperature the distance δ also increases up to reaching $\delta \sim G\sqrt{GM}/\beta_0^2$ at the Hagedorn temperature. At this point the stretched horizon is displaced towards the distant observer and swallows up all of space, provided we ensure the validity of the supergravity approximation

$$M \sim r_0^2 \gg N \gg 1\,. \tag{3.83}$$

In the CHS model all the thermodynamic quantities on the stretched horizon can be identify as those of the event horizon, with additional subleading terms suppressed by the black hole mass. This is in contrast with the outcome at the Hagedorn temperature where subleading contributions are no longer suppressed.

Let us continue examining the classical behavior of the stretched horizon and visualize the "number of states". For that purpose we calculate, in the two-dimensional flat Minkowski space at the Planck temperature T_P, the mass of a ring shaped string located between the boundary and the event horizon. It reads

$$m = \int_{\sqrt{GM}}^{\sqrt{GM}+\delta} 2\pi r \rho_P\, dr \approx \begin{cases} \frac{1}{GM}\,, & \text{if } \chi = 1; \\ \frac{M}{\beta_0^2} + \mathcal{O}\left(\frac{GM}{\beta_0^4}\right)\,, & \text{if } \chi = 0 \end{cases} \tag{3.84}$$

where we have used the behavior $\rho_P \sim G^{-2}$. Notice that (3.84) matches the result below (3.80): For the background (3.76) the string mass can be considered residual and in accordance the black hole mass remains to be almost $\sim GM$. Furthermore, the whole mass is localized inside the event horizon. As we increase the temperature the mass of the string forming a ring of radius r_P is of the order of the black hole mass and hence there must be only a residual mass in the interior of the event horizon. With the expectation of a small distortion with respect to the flat Minkowski space the approach of (3.84) is fully justified in this latter case. One can regard this phenomenon as a progressive melting of the strings as they encounter Hagedorn temperature conditions [69]. The energy of the strings states is so large when the

Hagedorn temperature is approached, that the strings on the horizon will tend to join forming a single one [70]. Thus the system evolves to a single state and consequently the entropy is reduced. This picture matches the view where black hole states at the Hagedorn temperature are in one to one correspondence with single string states.

3.3.3 Hawking emission via tunneling: Wrapped fivebranes

The metric (3.5) is the ultraviolet completion of a large family group of regular non-abelian monopole solutions in $\mathcal{N}=4$ gauged supergravity, interpreted as 5-branes wrapped on a shrinking S^2 [62]. In the following we shall deal with a thermal deformation of one of such metrics dual to $\mathcal{N}=1$ SQCD with a superpotential coupled to adjoint matter [71]. Analyzing the emission problem with the method outlined in Section 3.3.1 leads to the same result obtained in (3.24), i.e. a constant outward flux of particles independent of the black hole characteristics. The metric field in Einstein frame is given by

$$ds^2 = e^{\frac{\phi_0}{2}} r \left[-K(r)dx_1^2 + \sum_{j=2}^{4} dx_j^2 + N\alpha' \left(\frac{4}{r^2 K(r)} dr^2 + \frac{1}{\xi} d\Omega_2^2 + \frac{1}{4-\xi} d\tilde{\Omega}_2^2 \right) + \right.$$

$$\left. + \frac{N\alpha'}{4} \left(d\psi + \cos\theta d\varphi + \cos\tilde{\theta} d\tilde{\varphi} \right)^2 \right] , \quad K(r) = 1 - \left(\frac{r_0}{r} \right)^4 . \tag{3.85}$$

In addition we have a dilaton field which is linear $\phi = \phi_0 + r$ and a RR 3-form field.

First of all we truncate the theory to two dimensions: the radial and temporal one. To cast (3.85) in Painlevé coordinates we chose the function $f(r)$ in (3.71) as $f(r) = \sqrt{N} \log K(r)$. Then the truncated theory equivalent to (3.85) is rewritten as

$$ds^2 = e^{\frac{\phi_0}{2}} r \left(-K(r)dx_1^2 + 4N\alpha' \frac{dr^2}{r^2 K(r)} - 4\sqrt{N\alpha'} \frac{r_0^2}{r^3} dx_1 dt \right) . \tag{3.86}$$

To calculate the semi-classical emission one needs the radial null geodesics of the back-reacted metric. The mass scales as $M \sim r_0^4$, then the emission of a shell with energy ω translates in a shift in the radius, so $M - \omega \sim r_1^4$. This leads, after the emission, to the geodesic

$$\dot{r} = \frac{1}{2\sqrt{N\alpha'}} \, r \left(\frac{r_1^2}{r^2} \pm 1 \right) . \tag{3.87}$$

Its solutions are $r^2 = r_1^2 \left(e^{\pm x_1/\sqrt{N\alpha'}} \mp 1 \right)$, and one finds for timings the very same pattern as in the LST case.

Inserting the outgoing solution of (3.87) in (3.117) one obtains $\mathrm{ImS} = \pi\sqrt{N}\omega$, from where follows once more the behavior (3.75). Thus, most probably, all metrics which asymptotic completion is LST will emit thermally.

As in the LST case, one can check that using the mass density $m = r_0^4 e^{2\phi_0} N^{5/2}$ and entropy density $s = r_0^4 e^{2\phi_0} N^2$ [54] the emission entropy in (3.75) turns to be directly related with Hawking-Bekenstein entropy, $e^{-\beta_0\omega} = e^{\Delta S_{\rm BH}}$.

3.4 Complex path and anomalies in LST

In this section we reproduce the work [72] where we have studied the Hawking radiation of NS5 and LST using two semi-classical methods: the complex path and the gravitational anomaly. As in the previous section NS5 exhibits non-thermal behavior that contrasts with the thermal behavior of LST. We remark that energy conservation is the key factor leading to a non-thermal profile for NS5. In contrast, LST keeps a thermal profile even considering energy conservation because the temperature in this model does not depend on energy.

Since the pioneering proposal of Hawking that black holes can radiate [8], much work has been done in order to obtain a complete theory of quantum gravity. When Hawking announced his amazing results, a new powerful paradox emerged. The information loss paradox with the apparent violation of unitarity principle has consequences on well-established quantum mechanics. A recent effort in order to solve this paradox has been done studying different semi-classical approaches such as the tunneling method, studied in the preceding section, proposed by Parikh and Wilczek [30, 32], the complex path analysis [38, 73, 74] or the cancellation of gravitational anomalies [75, 76, 77].

In order to develop our study we have reduced the ten-dimensional metric of LST to two-dimensional one, see (3.10−3.12). Momentally we make a comment on the validity of this truncation of the metric (3.5) to two dimensions. The interesting points concern: *i)* the fate of dimensional and field content reduction on the S^3 modes is consistent [78]. *ii)* Furthermore, both the R^5 and S^3 wrap factors are independent of the (t, r) coordinates. As a consequence the equation of motions of these modes can be taken static and r independent, i.e. the emission in the $t - r$ plane does not alter the dynamics in the transverse coordinates to it. Hence all the physics will be analyzed within the propagation of massless particles in the $r - t$ sector of the metric. We have verified that the NS5 model shows a non-thermal emission whereas LST shows a thermal emission. This last conclusion matches with the Hagedorn properties of LST, namely the temperature of LST corresponds to the Hagedorn temperature.

Complex path method and anomalies yields the same results as the tunneling method, analyzed in [59], for the temperature and the emission rate. It is worth to mentioning that in the classical computation of the Bogoliubov coefficients all the results for emission rates shows thermal profiles due to the lack of energy conservation. This fact had driven Hawking to state that all the information that falls into the black hole is lost for ever, establishing in this way the information loss paradox. Nevertheless, one hopes to overcome this weird conclusion using semi-classical methods.

3.4.1 Complex path method

The complex path method has been developed in [38], in order to calculate particle production in Schwarzschild-like space-time and it was extended for different coordinate representations of the Schwarzschild space-time [73, 74]. Nevertheless complex path analysis had already been discussed by Landau and Lifshitz [79], where it was used to describe tunneling processes in non-relativistic semi-classical quantum mechanics.

We will follow the reference [38] in which the authors avoid to work in the Kruskal representation. They use the standard coordinates in the $r - t$ sector. However the method presents a disadvantage because one finds a coordinate singularity at the horizon. Nevertheless using the techniques of complex integration one bypasses the singularity. We also want to mention that the method of complex path leads to the same results with those in [27]. In both methods, for the Schwarzschild space-time and also as we will see in this section for NS5 and LST space-time, it has been found that the relation between emission and absorption probabilities is of the form

$$P_e = e^{-\beta\omega} P_a \ , \tag{3.88}$$

where ω is the energy of the emitted particles. We are tempting to compare this relation with the standard thermal Boltzmann distribution for blackbody radiation where β^{-1} is identified with the Hawking temperature. We have verified that this is the case, if we compare our results with the temperature calculated using the definition of surface gravity for example. It is noteworthy to say that this method allows one to derive temperatures for black holes comparing probabilities of emission and absorption but it is not able to calculate the spectrum of thermal radiation. In that sense the tunneling method is so far incomplete. To remove this shortcoming the authors in [80] presented a new mechanism.

In order to apply the complex path method to NS5 and LST we have constructed the semi-classical action obtained from Hamilton-Jacobi equations. Then we have computed the semi-classical propagator $K(r_2, t_2; r_1, t_1)$. Eventually we have calculated the emission and absorption probabilities.

We consider the equation of motion of a massless scalar particle $\Box\phi = 0$ in the background (3.12),

$$-A(r)\frac{\partial^2}{\partial t^2}\phi(t,r) + \frac{f(r)}{r^3}\frac{\partial}{\partial r}\left[r^3 f(r)\frac{\partial}{\partial r}\phi(t,r)\right] = 0 \ . \tag{3.89}$$

Using the standard ansatz solution

$$\phi(t,r) \sim e^{\frac{i}{\hbar}S(t,r)} \ , \tag{3.90}$$

and substituting in (3.89) we get an expression in terms of the action $S(t,r)$,

$$\begin{aligned}
&-A(r)\left(\frac{\partial S}{\partial t}\right)^2 + f(r)^2\left(\frac{\partial S}{\partial r}\right)^2 + \\
&+\frac{\hbar}{i}\left[-A(r)\frac{\partial^2 S}{\partial t^2} + f(r)^2\frac{\partial^2 S}{\partial r^2} + \frac{f(r)}{r^3}\frac{d(r^3 f(r))}{dr}\frac{\partial S}{\partial r}\right] = 0 \ ,
\end{aligned} \tag{3.91}$$

where we have collected the terms with $\hbar$ dependence. The following step is to write the action as an expansion in a power series of $(\frac{\hbar}{i})$,

$$S(t,r) = S_0(t,r) + \left(\frac{\hbar}{i}\right)S_1(t,r) + \left(\frac{\hbar}{i}\right)^2 S_2(t,r) + \ldots \ . \tag{3.92}$$

Substituting the above expansion in (3.91) and neglecting terms of order $(\frac{\hbar}{i})$ and higher, we obtain a non-linear first order partial differential equation which corresponds to the Hamilton-Jacobi equation of motion to the leading order in the action S,

$$-A(r)\left(\frac{\partial S_0(t,r)}{\partial t}\right)^2 + f(r)^2\left(\frac{\partial S_0(t,r)}{\partial r}\right)^2 = 0\ . \tag{3.93}$$

We are interested in the evaluation of the semi-classical propagator which inform us about the amplitude for a particle going from r_1 at time t_1 to r_2 at time t_2. In the saddle point approximation we get

$$K(r_2,t_2;r_1,t_1) = N\exp\left[\frac{i}{\hbar}S_0(r_2,t_2;r_1,t_1)\right]\ , \tag{3.94}$$

where N is a normalization constant. Applying separation of variables in (3.93) we get

$$S_0(r_2,t_2;r_1,t_1) = -\omega(t_2-t_1) \pm \omega\int_{r_1}^{r_2}\frac{\sqrt{A(r)}}{f(r)}dr\ , \tag{3.95}$$

the plus/minus sign corresponds to ingoing/outgoing particles respectively and ω is the energy of the emitted or absorbed particle.

The integral (3.95) is not well behaved if the horizon r_0 is within the region of integration. This turns to be the case since we are interested in the emission of particles through the event horizon, so the region of integration runs from inside the horizon to outside.

First we consider the propagation of an outgoing particle in the inner region $r_1 < r_0$. Applying the usual complex analysis tools, we deform the contour of integration around the pole r_0 in the upper complex half-plane. Obtaining for the radial part of (3.95)

$$S_0^e = \frac{i\pi\omega}{2}r_0\sqrt{A(r_0)}\ . \tag{3.96}$$

We will call it emission action because we simply consider the emission of an outgoing particle propagating from inside the horizon to the outside.

In the same way one proceeds with analogous analysis to evaluate the action at lowest order for absorbed particles. In that case we are considering the propagation of an ingoing particle in the outer region, $r_0 < r_2$. Deforming the contour of integration in the upper complex half-plane, eventually we obtain the same result as the emission process up to a change of sign. Now we are obtaining the absorption action for a particle that propagates from the region outside of the horizon to the inside

$$S_0^a = -\frac{i\pi\omega}{2} r_0 \sqrt{A(r_0)} \,. \tag{3.97}$$

We are interested in the expressions (3.96) and (3.97) in order to evaluate the probabilities of the emission and absorption processes. Thereby using the definition of the probability: $P = |K(r_2, t_2; r_1, t_1)|^2$, and substituting the expression for the corresponding actions, we finally obtain for the emission and absorption probabilities

$$P_e \sim \exp\left[-\frac{\pi}{\hbar}\omega r_0 \sqrt{A(r_0)}\right] \ , \ P_a \sim \exp\left[\frac{\pi}{\hbar}\omega r_0 \sqrt{A(r_0)}\right] \ , \tag{3.98}$$

where we have omitted the normalization constants. Eventually we are interested in writing the relation between emission and absorption probabilities,

$$P_e = \exp\left[-\frac{2\pi}{\hbar}\omega r_0 \sqrt{A(r_0)}\right] P_a \,. \tag{3.99}$$

At first sight we observe that the absorption process dominates over the emission, it is easier for the system to absorb than to radiate particles. Also we note some misleading behavior in the expression for the absorption probability (3.98), because we could think that one might get a probability absorption greater than 1. However we only have considered the spatial contribution of the action in order to calculate the probabilities of emission and absorption processes. Instead of this we must also have considered the time contribution as proposed in the work [81].

Comparing (3.99) with the same relation in a thermal bath of particles (3.88), we can identify the temperature of our system (taking $\hbar = 1$ and $m_s = 1$) as

$$T = \frac{1}{2\pi r_0 \sqrt{A(r_0)}} = \frac{1}{2\pi \sqrt{\chi r_0^2 + N}} \ , \tag{3.100}$$

that coincides with the value of temperature obtained in (3.24).

So far we have studied NS5/LST systems without backreaction. The next step is to consider the backreaction of the metric due to the emission process. Our starting point in the evaluation of the backreaction is the expression of the action for the emission process (3.96). In our NS5/LST model we have the following relation between the event horizon and the mass of the black hole: $r_0^2 \sim M$, where M is the mass of the black hole and the factors omitted here are not relevant to our study.

When the metric backreacts in the emission process the energy conservation implies that $r_0^2 \to r_0^2 - \omega$. The shrink of the event horizon rides the tunneling emission between turning points defined just inside and just outside of the event horizon. Once the emission has been carried out we perform the previous change in (3.96),

$$S_0^e = \frac{i\pi}{2}\omega\sqrt{\chi(r_0^2 - \omega) + N} \,. \tag{3.101}$$

Expanding in low energies we get

$$S_0^e = \frac{i\pi}{2}\left(\omega\sqrt{\chi r_0^2 + N} - \frac{\chi\omega^2}{2\sqrt{\chi r_0^2 + N}} + O(\omega)^3\right) \,. \tag{3.102}$$

Calculating the emission probability for both models we obtain

$$P_e \sim \begin{cases} \exp\left[-\frac{\pi}{\hbar}\left(\omega\sqrt{r_0^2 + N} - \frac{\omega^2}{2\sqrt{r_0^2+N}} + ...\right)\right] & \text{if } \ \chi = 1 \ \ (\text{NS5}); \\ \exp\left[-\frac{\pi}{\hbar}\omega\sqrt{N}\right] & \text{if } \ \chi = 0 \ \ (\text{LST}). \end{cases} \tag{3.103}$$

We see higher-order correction terms corresponding to the NS5 emission probability, which indicate that the emission is not purely thermal. On the other hand the emission probability expression corresponding to the LST model is exact, which indicates that the emission is purely thermal.

In this work, we have concluded that the results obtained from the tunneling formalism in [30] are nothing more than an extension of the Hamilton-Jacobi formalism taking into account the energy conservation, which induces the backreaction of the event horizon. In our particular case we are facing with an anomalous model, in the sense that it does not fulfill the previous expectations about non-thermal emission. The LST model emits thermal radiation irrespective whether the energy conservation holds, or not.

In order to analyze the deviations from the thermal behavior of the NS5 model it would be relevant to perform the computation of the greybody factors. So that we must solve the radial part of the equation of motion (3.89). As far as we know this equation cannot be solved analytically, therefore a numerical analysis is needed in order to show up the non-thermal character of the NS5 model. Even so, we can elucidate that the non-thermal behavior of the NS5 model comes from the throat region. In this region the dilaton grows linearly pointing out that gravity becomes strongly coupled far down the throat, and states with large quantum numbers exist. On the other hand, the near horizon limit of the NS5, i.e. LST, decouples the mode interactions between the bulk and the brane. The spectrum reduces to less excited states leading to a thermal behavior with the Hagedorn temperature, see [82] for a complete discussion.

3.4.2 Anomalies

In this section we will present another successful semi-classical method for the computation of the Hawking radiation from an evaporating black hole. The method is based on the cancellation of gravitational anomalies in a two-dimensional chiral theory taken as effective theory near the event horizon. This method was first proposed in [75]. Gravitational anomalies are anomalies in general covariance, i.e. general coordinate transformations (diffeomorphism), and they manifest the non-conservation of the energy-momentum tensor.

The authors in [75, 76] managed the treatment of gauge and covariant anomalies deriving an effective two-dimensional theory close to the horizon. They built an effective action performing a partial wave decomposition in tortoise coordinate and dropping potential terms which vanish exponentially fast near the horizon. Thus physics near the horizon can be described by an infinite collection of (1+1) fields with the metric reduced to the $r-t$ sector. In the aforesaid works the authors derived the Hawking radiation flux by anomaly cancellation, splitting the space-time into the near horizon region where the anomaly holds and the outside region where the conservation law is preserved. They carried out the calculation using the consistent chiral anomaly form of the energy-momentum tensor, see [83, 84],

$$\nabla_\mu T^\mu_\nu = \frac{1}{96\pi\sqrt{-g}}\epsilon^{\beta\delta}\partial_\delta\partial_\alpha\Gamma^\alpha_{\nu\beta} \tag{3.104}$$

and the covariant boundary condition at the horizon. It is the cancellation of this anomaly which led to the appearance of the Hawking radiation flux.

On the other hand it is known that there are two types of anomalies. Covariant anomalies, which transform covariantly under gauge or general coordinate transformations but they do not satisfy the Wess-Zumino consistency condition, And consistent anomalies, which satisfy the consistency condition but they do not transform covariantly under gauge or general coordinate transformations. In our study we adopt the procedure carried out in [77] where the author uses a more coherent frame, working with covariant forms both for the expression of the chiral anomaly and for the boundary condition. Unlike the previous works it is not necessary to split the space-time into two different regions: near the horizon and far from the horizon.

First of all we consider the physics near the horizon of the NS5 and LST models described by an infinite collection of (1+1) scalar field particles propagating in the background (3.12). It is not necessary to work with the full metric because only the $r-t$ sector is relevant to the emission processes, obtaining in this way the same results for the full theory as for the effective two-dimensional theory. In this frame we can consider that only the outgoing modes are present. The ingoing modes are lost into the black hole and they do not affect at the classical level. Nevertheless the total effective action must be covariant. Thereby the quantum contribution of these irrelevant ingoing modes will supply the extra term, a Wess-Zumino term, in order to cancel the gravitational anomaly providing the Hawking flux [76]. The loss of the ingoing modes behind the horizon of the black hole makes the effective theory chiral, obtaining consequently a gravitational anomaly [83, 84]. Following [77], we adopt the expression for the covariant form of the gravitational anomaly

$$\nabla_\mu T^{\mu\nu} = \frac{1}{96\pi\sqrt{-g}}\epsilon^{\nu\mu}\nabla_\mu R \,, \tag{3.105}$$

where R is the Ricci scalar and $\epsilon^{\nu\mu}$ is the Levi-Civitá tensor that in our case takes the values $\epsilon^{tr} = -\epsilon^{rt} = 1$ and zero for other contributions. The covariant boundary condition at the event horizon is

$$T^r_t(r = r_0) = 0 \,. \tag{3.106}$$

Noticing that we are working with a static metric, we evaluate the equation (3.105) for the effective two-dimensional theory in the $r-t$ sector. Eventually we get

$$\partial_r(\sqrt{-g}T^r_t) = \frac{1}{96\pi}g_{tt}\partial_r R \,. \tag{3.107}$$

The Ricci scalar for NS5 and LST models is

$$R = \frac{f'A'}{2A^2} - \frac{f''}{A} , \tag{3.108}$$

where the prime denotes derivative with respect to the coordinate r. Defining the new function

$$N_t^r \equiv \frac{1}{96\pi}\left(-\frac{ff'A'}{2A^2} - \frac{f'^2}{2A} + \frac{ff''}{A}\right) , \tag{3.109}$$

we can write (3.107) as

$$\partial_r(\sqrt{-g}T_t^r) = \partial_r N_t^r . \tag{3.110}$$

Then integrating the equation (3.110) we obtain

$$\sqrt{-g}T_t^r = b_0 + (N_t^r(r) - N_t^r(r_0)) , \tag{3.111}$$

where b_0 is an integration constant that can be evaluated implementing the covariant boundary condition (3.106). Doing so it yields the value $b_0 = 0$. Hence (3.111) becomes

$$T_t^r = \frac{1}{\sqrt{-g}}(N_t^r(r) - N_t^r(r_0)) . \tag{3.112}$$

The Hawking radiation flux is measured at infinity where the covariant gravitational anomaly vanishes. Therefore we compute the energy flux by taking the asymptotic limit of (3.112)

$$T_t^r(r \to \infty) = -\frac{1}{\sqrt{-g}}N_t^r(r_0) . \tag{3.113}$$

Evaluating (3.109) at the event horizon, r_0, and considering the value of the surface gravity $\kappa = \frac{1}{\sqrt{N+\chi r_0^2}}$, we finally obtain for the energy flux at infinity

$$T_t^r(r \to \infty) = \frac{1}{\sqrt{-g}}\frac{\kappa^2}{48\pi} , \tag{3.114}$$

which it is of course the Hawking radiation flux for a black hole.

3.5 Validity of the Semi-classical approaches

The previous analyses are based on a semi-classical approaches, and even if top of them one can implement some extra quantum corrections, the approaches are not free of assumptions and possible criticisms. For instance an observable effect of string theory is the very last steps in the black hole evaporation. In the usual picture the final evaporation process takes place at planckian temperatures and thus the last radiated particles would carry energy of order of the Planck scale. One wonders if at these energies the approach of the above section is still reliable. If it does, energy conservation imposes a constrain in the minimum size of the remnant, because the energy of the emitted particles can not exceed the remainder mass.

Common lore assigns to the previous optical approximation treatment a validity meanwhile the wavelength of the bulk probe is much smaller than the local curvature of space-time

$$\frac{1}{\text{momentum scale}} \ll \text{local curvature length scale}\,. \tag{3.115}$$

In terms of local coordinates, the curvature length scale, Δr, can be written as a function of the scalar curvature as $\Delta r = 1/\left(g_{rr}\sqrt{\mathcal{R}}\right)$. This function is bounded from below with a single minimum located at $r \approx r_0/2$, and then (3.115) leads to $p \gg 2/r_0$. As the black hole emits and shrinks, the momenta of the space-like geodesics probe must increase to fulfill the inequality (3.115). At some point the mass of the emitted probe would be larger than the remaining mass in the black hole and the semi-classical approach will break down.

Considering the behavior of the radial momenta $p_r \sim p_0 g_{rr}\dot{r} = \omega\frac{\sqrt{N+\chi r^2}}{r-r_0}$ as a function of the emitted particle energy, we can see that inequality (3.115) leads to

$$\omega \gg \frac{\sqrt{2N(3Nr^2+r_0^2)(2N+5\chi r^2)}}{r(r-r_0)(N+\chi r^2)}\,. \tag{3.116}$$

Notice that a particle near the horizon needs a large amount of energy in order to escape up to the boundary.

3.6 Further thermodynamic relations

One should keep in mind that any observable quantity is computed at the boundary and receives contributions from both supergravity solutions, (3.5) with $\chi = 0$ and $\chi = 1$. Usually in a given thermodynamic regime one solution dominates over the other and most of the bulk of the physical quantity can be computed by considering only one of them. We shall see in the sequel that this is not the case for these models.

The basic thermodynamic quantity at play is the Helmholtz free energy, that can be casted in terms of the action via the relation $\mathcal{F} = \mathcal{I}/\beta$. The action consists of two terms

$$\mathcal{I} = \mathcal{I}_{\text{grav}} + \mathcal{I}_{\text{surf}} \,. \tag{3.117}$$

The first term on the right hand side of equation (3.117) is

$$\mathcal{I}_{\text{grav}} = \frac{1}{2\kappa_{10}^2} \int_{\mathcal{M}} d^{10}x\sqrt{g} \left(R - \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{12} e^{-\phi} H_{(3)}^2 \right) , \tag{3.118}$$

being $\mathcal{M}$ a ten-dimensional space-time. While the second term of the equation (3.117) is the surface contribution

$$\mathcal{I}_{\text{surf}} = \frac{1}{\kappa_{10}^2} \oint_{\Sigma} K d\Sigma \,, \tag{3.119}$$

with Σ the boundary that encloses the ten-volume $\mathcal{M}$ in (3.118), K is the extrinsic curvature, $K_{\mu\nu} = n^\sigma \partial_\sigma g_{\mu\nu}$ and $n^\sigma \partial_\sigma$ is the outward directed unit normal vector.

If one calculates directly the action (3.117) for the solution (3.76) the result turns to be divergent. To regularize the solution we use an ultraviolet cutoff Λ that eventually will tend to infinity. Furthermore, we perform a fiducial renormalization, subtracting a reference background. It seems natural to choose the later as the corresponding extremal solution. The calculation is lengthly but straightforward: the on-shell Euclidean actions of the extremal and non-extremal solutions are given by

$$\mathcal{I}_{\text{e}} = \frac{\text{Vol}(R^5)\text{Vol}(S^3)}{2\kappa_{10}^2} \int_0^{\beta'} dt \left[\frac{3}{2}\Lambda^2 \left(\frac{3N + 4\Lambda^2\chi}{N + \Lambda^2\chi} \right) - \int_0^{\Lambda} dr \frac{N^2 r}{(N + r^2\chi)^2} \right] , \tag{3.120}$$

and

$$\begin{aligned}\mathcal{I}_{\rm ne} = \frac{\mathrm{Vol}(R^5)\mathrm{Vol}(S^3)}{2\kappa_{10}^2}\int_0^{\beta_{\rm CHS}(\Lambda)} dt \left[\frac{N(9r^2-5r_0^2)+4\chi r^2(3r^2-2r_0^2)}{2(N+r^2\chi)}\right.\\ \left. - \int_{r_0}^{\Lambda} dr \frac{Nr(N-\chi r_0^2)}{(N+r^2\chi)^2}\right] ,\end{aligned} \tag{3.121}$$

respectively. At the boundary, $\Lambda \to \infty$, the temperature of both solutions must be the same. For this purpose the temporal period in the extremal case is adjusted to be $\beta' = \beta_{\rm CHS}(\Lambda)\sqrt{F(\Lambda)}$.

For fixed, but otherwise arbitrary N and r_0, we find the renormalized action

$$\begin{aligned}\mathcal{I} = \lim_{\Lambda\to\infty}[\mathcal{I}_{\rm ne}-\mathcal{I}_{\rm e}] = \lim_{\Lambda\to\infty}\frac{1}{4\kappa_{10}^2}\frac{\mathrm{Vol}(R^5)(2\pi)^3}{(N+\kappa\Lambda^2)^{3/2}}\Big(-2\Lambda(2N+3\kappa\Lambda^2)(N+\kappa r_0^2)\sqrt{\Lambda^2-r_0^2}\\ + N^2(4\Lambda^2-2r_0^2)+2\kappa^2\Lambda^2(3\Lambda^2-2r_0^2)r_0^2+N\kappa(6\Lambda^4+\Lambda^2 r_0^2-3r_0^4)\Big) \to 0\end{aligned} \tag{3.122}$$

implying that the free energy of the system vanishes. This means that none of the actions dominate over the other, and to obtain an observable one has to add the contributions from both actions. In order to avoid a divergent behavior of the renormalized action, we point out that it is crucial to carry out the subtraction of the reference background before to take the limit in which the cutoff Λ tends to infinity.

It is also instructive to compute in an independent way some of the thermodynamic contributions to the Helmholtz free energy, $\mathcal{F} = E - TS = 0$. For instance the entropy goes as

$$S = \frac{\mathrm{Area}}{4G_{10}} = \frac{1}{2G_{10}}\mathrm{Vol}(R^5)\pi^2 r_0^2\sqrt{N+\chi r_0^2} = \frac{1}{4G_{10}}\mathrm{Vol}(R^5)\pi r_0^2\beta_{\rm CHS} , \tag{3.123}$$

and turns to be χ dependent, but the combination entering in the Helmholtz free energy it is not

$$T_{\rm CHS}S = \frac{1}{4G_{10}}\mathrm{Vol}(R^5)\pi r_0^2 = T_{\rm LST}S . \tag{3.124}$$

Notice that (3.123) matches the behavior described by (3.84): as $\chi \to 0$ the black hole degrees of freedom, i.e. strings, joint together up to forming a single state. As a consequence the entropy decreases.

We end this section by noticing that the exponent in (3.75) is just the variation of the Bekenstein-Hawking entropy. In this precise case the mass and the entropy density are proportional, hence it follows that $e^{-\beta\omega} = e^{\Delta S_{\rm BH}}$. This matches the statistical picture in which large fluctuations are suppressed and supports the idea that in this background the Bekenstein-Hawking area-entropy relation, $S_{BH} = A/4$, can be obtained by counting the degeneracy states [11].

3.7 Discussion and remarks

We have reviewed briefly some aspects about LST thermodynamics. We have exposed the thermal emission of LST due to the non-energy dependence of the Hagedorn temperature. In addition, we have evaluated the temperature experienced by a scalar particle-like observer, thereby we have verified that the Hagedorn temperature of LST is a maximum bound. Furthermore we have studied the Hawking radiation of the NS5 and LST black hole models using two semi-classical emission methods: the complex path method and the cancellation of the gravitational anomaly. It shall be stressed that using both methods we have recovered our previous results derived in [59] where we worked using the tunneling formalism. The complex path method [38, 74] shows how to evaluate the emission rate in the framework of the Hamilton-Jacobi formalism. We have proved that imposing the energy conservation, in order to take into account the backreaction of the metric during the emission process, we reproduce exactly the same results with those derived in the tunneling formalism [30, 32]. We would like to point out the advantage of the complex path method over the tunneling method. First of all, we avoid heuristic explanations about the tunneling mechanism in the process of the emission. Secondly, we work with the well-known Hamilton-Jacobi equations plus the imposition of the energy conservation. Finally, it is not necessary to change the standard coordinates of the metric into Painlevé coordinates. We conclude that the tunneling method is nothing more than the complex path method plus energy conservation.

We have verified that another successful method to evaluate Hawking radiation in NS5 and LST models is that of the cancellation of the gravitational anomaly [75]. However, it is argued that the method fails for non-asymptotically flat space-times like de Sitter space-time and Rindler space-time. In [85] it is proposed a new method based on the chiral nature of field theories in the near horizon region of the black hole, but does not depend on the existence of a chiral anomaly. It defines a new effective energy-momentum tensor (either in consistent or in covariant form same results are obtained) $\mathcal{T}_\nu^\mu = T_\nu^\mu \mp N_\nu^\mu$ that is conserved $\partial_\mu \mathcal{T}_\nu^\mu = 0$. The physics in the whole part of the manifold external to the horizon can be described without the need to decompose the background in two pieces, i.e. near horizon and asymptotic. Thus the new method works even for non-asymptotically flat space-times as de Sitter and Rindler space-times.

Summarizing, we have shown that all the above methods lead to a non-thermal emission for the NS5 black hole and to a thermal one for the LST black hole, see (3.103). The latter can be interpreted as the thermal limit of the former. The entire process of black hole evaporation, except for the final period when the black hole is of Planckian size, can be summarized according to the following patterns: Starting from the NS5 system at a given temperature we checked, in a semi-classical approximation, that the black hole emission is non-thermal (3.80). The black hole contains many degrees of freedom coupled with its environment. At this point the system is thermodynamically irreversible, and the entropy of the surrounding increases as the black hole emits. As the emission takes place the black hole temperature increases while, both the mass and the emission rate, decrease and the latter becomes pure thermal at the Hagedorn temperature (3.75). The interference term vanishes at this point and the black hole system is thermodynamically reversible and consists of a single state. This single state radiates, while the black hole temperature remains completely independent of its mass. Thus, as the LST black hole evaporates, its energy flux is exactly constant.

Once this point is reached, one could think that we deal with a stable remnant with zero entropy. That this is not the case can be inferred from the stringy corrections to the entropy as a function of the energy. This gives a thermodynamically unstable system [45] which in turn implies that the probability of emission diverges. In order to have a rough idea of the latter effect we use the area law relation but incorporating its first *quantum* corrections

$$S_c = \frac{\text{Area}}{4} + \alpha \log\left(\frac{\text{Area}}{4}\right) + \frac{\gamma}{\text{Area}} + \dots . \tag{3.125}$$

Taking into account the relations of the mass and energy densities, the black hole emission (3.75) is replaced at leading order by

$$\Gamma \sim \left(\frac{\text{Area}_1}{\text{Area}_0}\right)^\alpha e^{\Delta S_{\text{BH}}} = \left(1 - \frac{\omega}{M}\right)^\alpha e^{-\beta_0 \omega} . \tag{3.126}$$

The above expression together with the fact that the value of α is negative –the system is unstable– shows that the trend in (3.126) is that as the system evolves in time the emission increases, i.e. without further considerations at play the system would fully evaporate without leaving any relic behind it. This fact is clearly driven by the sign of α, which is negative. For a more complete discussion see [86]. The above picture relies in a truncation of (3.125) and as one approaches Planck scales one must consider that subleading contributions in (3.125) are enhanced and they wash out any solid conclusion.

We also have found that for theories which their ultraviolet completion is LST, the radiation is also that of a blackbody at a fixed temperature (3.24).

Finally, the cluster decomposition principle is a crucial physical requirement which states that very distant experiments produce uncorrelated results, thus establishing the local behavior of the field theory. Cluster decomposition principle states that if multi-particle processes are performed in N very distant laboratories, then the S-matrix element for the overall process factorizes. This factorization ensures a factorization of the corresponding transition probabilities, corresponding to uncorrelated experimental results. In the line of the works [87, 88] where the authors linked the existence of correlations among tunneled particles and the entropy conservation of the full system (black hole plus Hawking radiation), we have calculated the successive emission probabilities for two particles of energies ω_1 and ω_2 using (3.103) for each model, respectively. We have found that the NS5 model does not satisfy cluster decomposition

$$\ln \mid \Gamma(\omega_1+\omega_2) \mid - \ln \mid \Gamma(\omega_1)\Gamma(\omega_2) \mid = \frac{\omega_1\omega_2}{2\sqrt{N+r_0^2}} \,. \tag{3.127}$$

On the other hand we have found that the LST model satisfies cluster decomposition as we expected

$$\ln \mid \Gamma(\omega_1+\omega_2) \mid - \ln \mid \Gamma(\omega_1)\Gamma(\omega_2) \mid = 0 \,, \tag{3.128}$$

where $\Gamma(\omega_1)$ and $\Gamma(\omega_2)$ are the emission probabilities corresponding to a particle of energy ω_1 and ω_2, respectively and $\Gamma(\omega_1+\omega_2)$ is the emission probability of a particle with energy $\omega_1+\omega_2$. We have found that $\Gamma(\omega_1 \mid \omega_2) = \Gamma(\omega_1+\omega_2)$ is fulfilled at low energies, namely, the emission probability of a particle ω_2 conditioned by the previous emission of a particle ω_1 is the same as the emission of a single particle of energy $\omega_1+\omega_2$. With these results at hand we can conclude that NS5 black hole there are correlations among emitted particles. This fact is intimately related with the non-thermal emission rate (3.103). Regarding (3.127) one hopes that the successive Hawking emissions could preserve unitarity avoiding in such a way the information loss paradox. However it is not the case for the LST black hole where the thermal emission rate (3.103) leads us to cluster decomposition. Therefore the successive emissions of particles are independent among them, and thus the information of the initial states remains hidden.

3.8 Spectrum

So far we have calculated the temperature and the emission rate of NS5 and LST black holes. Nevertheless, in order to obtain a complete study we must evaluate the emission spectrum even taking into account the back-reaction of the metric. In [80], the authors calculated the Hawking radiation spectrum corresponding to a spherically symmetric static black hole. It is our aim to perform a similar analysis corresponding to NS5 and LST black holes. Our starting point will be the two-dimensional action (3.11) at leading order (3.95). This action can be written as

$$S_0(r,t) = \omega(t \pm r_*) \ , \tag{3.129}$$

where r_* is the tortoise coordinate defined as

$$dr_* = \frac{\sqrt{A(r)}}{f(r)} dr \ . \tag{3.130}$$

Then if we consider the outgoing/ingoing null coordinates

$$u = t - r_* \quad , \quad v = t + r_* \ , \tag{3.131}$$

we can define the right/left modes inside and outside of the black hole in the following way,

$$\begin{aligned} \phi^R_{in} &= e^{-\frac{i}{\hbar}\omega u_{in}} \quad , \quad \phi^L_{in} = e^{-\frac{i}{\hbar}\omega v_{in}}, \\ \phi^R_{out} &= e^{-\frac{i}{\hbar}\omega u_{out}} \quad , \quad \phi^L_{out} = e^{-\frac{i}{\hbar}\omega v_{out}} \ . \end{aligned} \tag{3.132}$$

The Kruskal coordinates corresponding to the inside and outside of the NS5 and LST black holes are defined, see [1], as

$$\begin{aligned} T_{in} &= e^{\kappa r^*_{in}} \cosh(\kappa t_{in}) \quad , \quad X_{in} = e^{\kappa r^*_{in}} \sinh(\kappa t_{in}), \\ T_{out} &= e^{\kappa r^*_{out}} \sinh(\kappa t_{out}) \quad , \quad X_{out} = e^{\kappa r^*_{out}} \cosh(\kappa t_{out}) \ , \end{aligned} \tag{3.133}$$

where κ is the surface gravity corresponding to each model. The two sets of Kruskal coordinates are then connected: $T_{in} \to T_{out}$ and $X_{in} \to X_{out}$, by the following transformation relation between the coordinates t and r

$$t_{in} \to t_{out} - i\frac{\pi}{2\kappa} \quad , \quad r^*_{in} \to r^*_{out} + i\frac{\pi}{2\kappa} \; . \tag{3.134}$$

Moreover, the null coordinates are also transformed as

$$u_{in} \to u_{out} - i\frac{\pi}{\kappa} \quad , \quad v_{in} \to v_{out} \; . \tag{3.135}$$

Eventually we have obtained a transformation relation between the left/right modes inside and outside the black hole,

$$\phi^R_{in} \to \phi^R_{out}\, e^{-\frac{\pi\omega}{\hbar\kappa}} \quad , \quad \phi^L_{in} \to \phi^L_{out} \; . \tag{3.136}$$

This last relation is precisely the relation that one obtains between the Bogoliubov coefficients in the standard study of the emission of Hawking radiation (2.31) and (3.66).

3.8.1 Blackbody spectrum

We are going to construct the density matrix operator corresponding to an outside observer for an n number of bosons and fermions. In this way we will be able to compute the average number of particles (bosons or fermions) emitted by the black hole. We will see that the emitted spectrum corresponds to the blackbody spectrum without taking into account the back-reaction of the metric. For a more detailed calculations see Appendix E.

We construct the physical state associated to a system of n number of non-interacting virtual pair of particles created inside the black hole,

$$|\psi\rangle = N \sum_n |n^L_{in}\rangle \otimes |n^R_{in}\rangle \; . \tag{3.137}$$

Applying the relations between modes (3.136), we get the physical state measured by the observer outside of the black hole which is necessary to evaluate the density matrix operator,

$$|\psi\rangle = N \sum_n e^{-\frac{\pi\omega n}{\hbar\kappa}} |n^L_{out}\rangle \otimes |n^R_{out}\rangle \; . \tag{3.138}$$

The normalization constant N is determined using the orthonormalization condition $\langle\psi_m|\psi_n\rangle = \delta_{mn}$,

$$N = \left(\sum_n e^{-\frac{2\pi\omega n}{\hbar\kappa}}\right)^{-\frac{1}{2}} . \tag{3.139}$$

For bosons ($n = 0, 1, 2, ...$), and fermions ($n = 0, 1$), the normalization constant is respectively

$$N_b = \left(1 - e^{-\frac{2\pi\omega}{\hbar\kappa}}\right)^{\frac{1}{2}} \quad , \quad N_f = \left(1 + e^{-\frac{2\pi\omega}{\hbar\kappa}}\right)^{-\frac{1}{2}} . \tag{3.140}$$

Henceforth we will only perform the calculations for the boson state system, and one can proceed in analogous way for the fermion system.

We write the density matrix operator for the boson system as

$$\rho_b = |\psi_b\rangle\langle\psi_b| = \left(1 - e^{-\frac{2\pi\omega}{\hbar\kappa}}\right) \sum_{n,m} e^{-\frac{\pi\omega(n+m)}{\hbar\kappa}} |n^L_{out}\rangle \otimes |n^R_{out}\rangle \; \langle m^L_{out}| \otimes \langle m^R_{out}| \; , \tag{3.141}$$

where $|n_{out}\rangle$ and $|m_{out}\rangle$ are orthonormalized outside eigenstates. Then tracing over the left modes we get the matrix density operator in terms of the right eigenmodes,

$$\rho^R_b = \left(1 - e^{-\frac{2\pi\omega}{\hbar\kappa}}\right) \sum_n e^{-\frac{2\pi\omega n}{\hbar\kappa}} |n^R_{out}\rangle\langle n^R_{out}| \; . \tag{3.142}$$

Now we compute the average number of particles detected at asymptotic infinity using the relation $\langle n\rangle = Tr(n\rho^R)$, where the trace is taken over $| \, n^R_{out}\rangle$ eigenstates. We have obtained for bosons and fermions respectively

$$\langle n_b\rangle = \frac{1}{e^{\frac{2\pi\omega}{\hbar\kappa}} - 1} \quad , \quad \langle n_f\rangle = \frac{1}{e^{\frac{2\pi\omega}{\hbar\kappa}} + 1} . \tag{3.143}$$

Both distributions correspond to a blackbody spectrum with Hawking temperature defined as $T_H = \frac{\hbar\kappa}{2\pi}$. We can identify the Bose-Einstein distribution for bosons and the Fermi-Dirac distribution for fermions.

3.8.2 Hawking radiation flux

Integrating over all energy range the expression for the average number of particles (3.143), we will give the flux of bosons and fermions, respectively, seen by an asymptotic observer,

$$F_\infty = \frac{1}{2\pi}\int_0^\infty \langle n_b\rangle \; \omega \; d\omega = \frac{\hbar^2\kappa^2}{48\pi} = \frac{\pi}{12}T_H^2 \; ,$$
$$F_\infty = \frac{1}{2\pi}\int_0^\infty \langle n_f\rangle \; \omega \; d\omega = \frac{\hbar^2\kappa^2}{96\pi} = \frac{\pi}{24}T_H^2 \; . \tag{3.144}$$

3.8.3 Back-reaction spectrum

During the emission of the Hawking radiation we have not considered the back-reaction of the metric. In this section our aim is to evaluate it. In [72] we had showed that the back-reaction effect was introduced imposing the energy conservation in the framework of the Hamilton-Jacobi formalism. We need to know how the modes are affected by the back-reaction of the metric. Looking at the transformation expression between the modes (3.136), we write the new transformation relation between the modes taking into account the back-reaction as

$$\tilde{\phi}^R_{in} \to \tilde{\phi}^R_{out}\, e^{-\frac{\pi\omega}{\hbar\tilde{\kappa}}} \quad , \quad \tilde{\phi}^L_{in} \to \tilde{\phi}^L_{out} \ , \tag{3.145}$$

where $\tilde{\phi}^{R,L}_{in,out}$ are the back-reacted modes and

$$\tilde{\kappa} = \frac{1}{\sqrt{\chi(r_0^2 - \omega) + \frac{N}{m_s^2}}} \ , \tag{3.146}$$

is the back-reacted surface gravity for the NS5 and LST models. In order to calculate the emission probability corresponding to a mode that tunnels through the event horizon and emerges to the outside of the black hole, we use the above relation between the right (outgoing) modes (3.145), obtaining

$$P_e = |\tilde{\phi}^R_{in}|^2 \to |\tilde{\phi}^R_{out}\, e^{-\frac{\pi\omega}{\hbar\tilde{\kappa}}}|^2 = e^{-\frac{2\pi\omega}{\hbar\tilde{\kappa}}} \ . \tag{3.147}$$

Therefore if we make an expansion at low energies we obtain for the emission probability observed by an asymptotic observer

$$P_e = \exp\left[-\frac{\omega}{T_H}\left(1 - \frac{\chi\omega}{2\left(\chi r_0^2 + \frac{N}{m_s^2}\right)} - \frac{\chi^2\omega^2}{8\left(\chi r_0^2 + \frac{N}{m_s^2}\right)^2} + ...\right)\right] \ , \tag{3.148}$$

where T_H is the Hawking temperature (3.24). This result coincides with [59, 72] but now we have avoided the problem of the temporal term showed in [81]. Moreover, we have evaluated the absorption probability corresponding to an incoming mode; in this case the left (incoming) mode that propagates toward the center of the black hole does not change, see (3.145), whereby the absorption probability will be

$$P_a = |\tilde{\phi}^L_{in}|^2 \to |\tilde{\phi}^L_{out}|^2 = 1 \ . \tag{3.149}$$

Thus using the principle of detailed balance, $P_e = e^{-\beta\omega} P_a$, we see from (3.147) and (3.149) that $\tilde{\beta} = \frac{2\pi}{\hbar\tilde{\kappa}}$. Then we can conclude that there exists an effective temperature for the back-reacted NS5 and LST black holes whose value is

$$\tilde{T} = \tilde{\beta}^{-1} = \frac{\hbar}{2\pi\sqrt{\chi(r_0^2 - \omega) + \frac{N}{m_s^2}}} \, . \tag{3.150}$$

Of course the effective temperature (taking into account higher order terms in ω) that appears in the emission rate (3.148) is nothing more than the expansion at low energies of the effective temperature (3.150). Eventually we can write the emission probability (3.148) in a more compact form as

$$P_e = e^{-\omega/\tilde{T}} \, , \tag{3.151}$$

which resembles the thermal emission corresponding to a perfect blackbody at temperature $\tilde{T}$. In the spirit of the work [89], we could think that when we do not consider the back-reaction of the metric the emission is purely thermal, $P_e \sim e^{-\omega/T_H}$, with a spectrum corresponding to a blackbody with temperature (3.24). On the other hand, when we are taking into account the back-reaction, i.e. energy conservation, the emission is not strictly thermal. Nevertheless, we can define an effective temperature (3.150) and consider that the black hole is emitting as a black body with this effective temperature, (3.151). We also see that the deviation from pure thermal behavior of the spectrum is

$$\frac{\tilde{T}}{T_H} = \left(\sqrt{1 - \frac{\chi\omega}{\chi r_0^2 + \frac{N}{m_s^2}}} \right)^{-1} \, . \tag{3.152}$$

Furthermore the results for the number of emitted particles (3.143) and fluxes (3.144) are subjected to the back-reaction effect through the factor $e^{\frac{2\pi\omega}{\hbar\tilde{\kappa}}}$, in which the back-reacted surface gravity, $\tilde{\kappa}$, appears. We may also expand at low energies the equations (3.143) and (3.144), obtaining higher order energy terms and thus the new spectrum deviates from the pure blackbody radiation spectrum.

3.9 Greybody factor

In this section we will compute the decay rate of an excited black hole into neutral scalars,

$$\Gamma = \sigma_{\text{abs}} \, \rho \left(\frac{\omega}{T} \right) \frac{d^4 k}{(2\pi)^2} \, , \tag{3.153}$$

given in terms of the thermal factor

$$\rho\left(\frac{\omega}{T}\right) = \frac{1}{e^{\omega/T} - 1} , \qquad (3.154)$$

and the classical absorption cross section, which corresponds to the greybody factor. In order to calculate the greybody factor of LST black hole we have basically followed the works [21, 23, 24]. We start considering the Klein-Gordon equation $\Box\phi = 0$ describing the propagation of a massless s-wave scalar particle minimally coupled to the fixed background (3.5). We obtain the absorption cross section as the ratio of the flux into the black hole at the future horizon to the incoming flux from the infinity,

$$\sigma_{\rm abs} = \frac{\mathcal{F}_{\rm abs}}{\mathcal{F}_{\rm in}} . \qquad (3.155)$$

Since we are interested in the $r-t$ sector of the metric we must solve the equation (3.89) in the background (3.12). In terms of the new variable $z = f(r)$, the equation (3.89) becomes

$$z\frac{\partial}{\partial z}\left(z\frac{\partial}{\partial z}\phi(z)\right) + \frac{\alpha}{(1-z)^2}\,\phi(z) = 0 , \qquad (3.156)$$

with

$$\alpha \equiv \frac{\omega^2 N}{4m_s^2} , \qquad (3.157)$$

hereafter for simplicity we take $m_s = 1$. There exist two possible approximations: *i)* the low-energy regime, $\omega\sqrt{N} \ll 1$ and *ii)* the dilute gas region, $r_0 \ll N$, for which the system resembles the $D1 - D5$ system in the limit $r_0, r_1, r_n \ll r_5$. Performing a function substitution of the form $\phi(z) \equiv z^\alpha(1-z)^\beta F(z)$, the equation (3.156) can be reduced to an hypergeometric equation, see e.g. [24]. Provided we choose

$$\alpha_\pm = \pm\, i\,\frac{\omega\sqrt{N}}{2} \quad , \quad \beta_\pm = \frac{1}{2}\left(1 \pm \sqrt{1-\omega^2 N}\right) , \qquad (3.158)$$

the solution of (3.156) becomes

$$\begin{aligned}\phi(z) \;=\; & C_1 z^{\alpha_+}(1-z)^{\beta_\pm}F(\alpha_+ + \beta_\pm, \alpha_+ + \beta_\pm; 1 + 2\alpha_+; z) + \\ & C_2 z^{\alpha_-}(1-z)^{\beta_\pm}F(\alpha_- + \beta_\pm, \alpha_- + \beta_\pm; 1 + 2\alpha_-; z) ,\end{aligned} \qquad (3.159)$$

where C_1 and C_2 are constants. The boundary conditions are

- At the event horizon ($z \to 0$): *purely ingoing waves.*
- At spatial infinity ($z \to 1$): *purely outgoing waves.*

Taking into account the first boundary condition we pick the first term on the right hand side of (3.159) as solution. Furthermore, both roots of β give the same result, thus henceforth we drop the subindex.

Expanding the solution for large r (or equivalently $z \to 1$) and neglecting the divergent solution, we obtain an asymptotic solution in the inner region

$$\phi_a(r) = C_a \frac{\Gamma(1 - i\omega\sqrt{N})\,\Gamma(-\sqrt{1-\omega^2 N})}{\Gamma\left(\frac{1-i\omega\sqrt{N}-\sqrt{1-\omega^2 N}}{2}\right)^2} \left(\frac{r_0}{r}\right)^{1+\sqrt{1-\omega^2 N}} , \tag{3.160}$$

where C_a is a constant. Evaluating the asymptotic solution in the outer region directly from equation (3.156), and using the Frobenius method it is found, in terms of the r variable,

$$\phi_a(\infty) = \sqrt{\frac{\pi}{2}} \left(A_1\, r^{-1-\sqrt{1-\omega^2 N}} + A_2\, r^{-1+\sqrt{1-\omega^2 N}}\right) , \tag{3.161}$$

where A_1 and A_2 are constants. Then if we match both solutions we find a relation between the constants,

$$C_a = \sqrt{\frac{\pi}{2}}\, r_0^{-1-\sqrt{1-\omega^2 N}} \frac{\Gamma\left(\frac{1-i\omega\sqrt{N}-\sqrt{1-\omega^2 N}}{2}\right)^2}{\Gamma(1 - i\omega\sqrt{N})\,\Gamma(-\sqrt{1-\omega^2 N})} A_1 \quad , \quad A_2 = 0 \, . \tag{3.162}$$

Imposing the second constraint one neglects the divergent modes at asymptotic infinity.

In order to obtain the behavior near the event horizon we expand the ingoing mode solution (3.159) around r_0 (or equivalently $z \to 0$),

$$\phi_h(r) = C_h \left(1 - \frac{r_0^2}{r^2}\right)^{-i\,\frac{\omega\sqrt{N}}{2}} , \tag{3.163}$$

where C_h is a constant. Then we match the solutions (3.160) and (3.163) at the matching point r_m, which fulfills $r_0 \ll r_m \ll r_5 = \sqrt{N}$, see [21], we thus obtain a relation between constants,

$$C_a = \left(\frac{r_0}{r}\right)^{-1-\sqrt{1-\omega^2 N}} \left(1 - \frac{r_0^2}{r^2}\right)^{-i\,\frac{\omega\sqrt{N}}{2}} \frac{\Gamma\left(\frac{1-i\omega\sqrt{N}-\sqrt{1-\omega^2 N}}{2}\right)^2}{\Gamma(1 - i\omega\sqrt{N})\,\Gamma(-\sqrt{1-\omega^2 N})} C_h \, . \tag{3.164}$$

Eventually comparing (3.162) with (3.164), we obtain the desired relation between the constant in the asymptotic solution and the constant of the near horizon solution,

$$A_1 = \sqrt{\frac{2}{\pi}} \left(1 - \frac{r_0^2}{r^2}\right)^{-i\,\frac{\omega\sqrt{N}}{2}} \left(\frac{1}{r}\right)^{-1-\sqrt{1-\omega^2 N}} C_h \, . \tag{3.165}$$

Finally, we compute the fluxes per unit solid angle corresponding to the LST background,

$$\mathcal{F} = \frac{1}{2i}\left(r^3 f(r)\phi^*(r)\frac{d\phi(r)}{dr} - c.c.\right) , \tag{3.166}$$

where *c.c.* stands for the complex conjugate of the first term. Therefore calculating the incoming flux from infinity and the absorbed flux at horizon, substituting in (3.166) the corresponding mode solution (3.160) and (3.163) respectively, we have found that both fluxes are identical. Then the greybody factor for LST black hole is

$$\sigma_{abs} \equiv \Gamma_\omega = 1 . \tag{3.167}$$

Thus the emission rate of LST will be

$$\Gamma = \frac{1}{e^{\omega/T_H} - 1}\frac{d^4k}{(2\pi)^2} , \tag{3.168}$$

where T_H is the Hawking temperature of LST, (3.24). This result matches the conclusion derived from the preceding sections: LST exhibits a purely thermal spectrum even taking into account corrections like the back-reaction of the metric.

3.10 Quasinormal modes

When a black hole is perturbed by absorbing a particle field or by emitting radiation, it starts to oscillates. The mode decay and the corresponding frequencies will be complex, being this damped modes the *quasinormal modes* (QNMs). The damping time is the inverse of the imaginary frequency and depends only on the black hole parameters, being independent of the initial perturbation. The black hole QNMs are just the solutions of the perturbation equations with the boundary conditions: only ingoing modes at event horizon and only outgoing modes at spatial infinity. Such boundary conditions single out a discrete set of complex frequencies $\{\omega_n\}$ that are usually labeled by an integer n, namely the overtone number. For a complete review of quasinormal modes see [90].

We find a useful application of QNMs in AdS/CFT correspondence, where a black hole can be viewed as a thermal state of a field theory. The decay of a scalar field is related with the decay of a perturbation of this thermal state. Then the QNMs are useful in order to obtain a timescale for the approach to thermal equilibrium [91].

It has been argued, [90, 91], that one can compute the QNMs as the poles of the Green function. In [92] the authors calculated the two-point function corresponding to the propagation of a massless minimally coupled complex scalar in a LST background. For the Euclidean version of the action (3.9) corresponding to a massless minimally coupled scalar particle, it is imposed the following boundary conditions:

- The solution of the equation of motion implied by the Euclidean action must be regular at the horizon, or at the tip of the cigar in the Euclidean case.
- Imposing a cut-off Λ at some distance along the linear dilaton tube, the solution must be constant at this surface when the cut-off goes to infinity.

Thus it is found a boundary term on the action evaluated in a solution that satisfy the above boundary conditions. Then using the solution (3.159) at the asymptotic limit, it is possible to evaluate the boundary term of the Euclidean action. Differentiating with respect to the sources the authors in [92] found the two-point function in momentum space,

$$\langle \mathcal{O}(p)\mathcal{O}^*(-p')\rangle \sim \frac{\Gamma(-\sqrt{1+\omega^2 N})}{\Gamma(\sqrt{1+\omega^2 N})}\,\frac{\Gamma^2\left(\frac{1+\sqrt{\omega^2 N}+\sqrt{1+\omega^2 N}}{2}\right)}{\Gamma^2\left(\frac{1+\sqrt{\omega^2 N}-\sqrt{1+\omega^2 N}}{2}\right)}\,\delta(p-p')\ , \qquad (3.169)$$

where we have neglected modes that are propagating on the NS5-brane, i.e. $k=0$. Therefore the quasinormal can be obtained as

$$\omega_n \sim i\, 2\pi T_H\left(n+\frac{1}{2}\right)\ , \qquad (3.170)$$

with Hawking temperature T_H defined in (3.24). This result matches the result for Schwarzschild black hole obtained in [89] (and references therein).

Chapter 4

Emission of fermions in LST

In this chapter we will study the tunneling of fermions through the event horizon of the LST background (3.5). We write the covariant Dirac equation in a general background [93] for a spinor field Ψ,

$$[\gamma^a e_a^\mu(\partial_\mu + \Gamma_\mu) + m]\,\Psi = 0\ , \tag{4.1}$$

where m is the bare mass of the particle; e_a^μ are the vielbein defined by the relation $g_{\mu\nu} = \eta_{ab}\, e_\mu^a e_\nu^b$ with $\eta_{ab} = \mathrm{diag}(-1, 1, 1, 1, ...)$, the latin indexes run for local inertial flat coordinates $(0, 1, 2, ...)$ whereas the greek indexes run for general coordinates $(t, r, \theta, ...)$. In LST the vielbein take the form

$$e_\mu^a = \mathrm{diag}\left(\sqrt{f(r)}, \sqrt{\frac{A(r)}{f(r)}}, r\sqrt{A(r)}, r\sqrt{A(r)}\sin\theta, r\sqrt{A(r)}\sin\theta\sin\varphi, 1, ..., 1\right)\ . \tag{4.2}$$

The spin connection is defined as

$$\Gamma_\mu = \frac{1}{8}[\gamma^c, \gamma^b] e_c^\nu \nabla_\mu e_{b\nu}\ , \tag{4.3}$$

where $\nabla_\mu e_{b\nu} = \partial_\mu e_{b\nu} - \Gamma^\lambda_{\mu\nu} e_{b\lambda}$ is the covariant derivative of $e_{b\nu}$ and $[\gamma^c, \gamma^b]$ the commutator of the gamma matrices. We choose for the gamma matrices [94],

$$\gamma^0 = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}\quad , \quad \gamma^k = \begin{pmatrix} 0 & -i\sigma^k \\ i\sigma^k & 0 \end{pmatrix}\quad , \quad k = 1, 2, 3\ ; \tag{4.4}$$

where the matrices σ^i are the Pauli matrices,

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\quad , \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\quad , \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\ . \tag{4.5}$$

The gamma matrices satisfies the Clifford algebra

$$\begin{aligned}
&[\gamma^a, \gamma^b] = -[\gamma^b, \gamma^a] \quad \text{if} \;\; \mathrm{a} \neq \mathrm{b}, \\
&[\gamma^a, \gamma^b] = 0 \quad \text{if} \;\; \mathrm{a} = \mathrm{b} \;, \\
&\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \;, \\
&\{\gamma^a, \gamma^b\} = 2\eta^{ab} \;.
\end{aligned} \tag{4.6}$$

Thus taking into account all the aforesaid definitions and properties we write the full ten-dimensional Dirac equation for the spinor field in the LST background,

$$\begin{aligned}
&\left[\gamma^0 \frac{1}{\sqrt{f(r)}}\,\partial_t + \gamma^1 \left(\frac{3A(r)'\sqrt{f(r)}}{4A(r)^{\frac{3}{2}}} + \frac{6\sqrt{f(r)}}{4r\sqrt{A(r)}} + \frac{f(r)'}{4\sqrt{A(r)f(r)}} + \sqrt{\frac{f(r)}{A(r)}}\,\partial_r\right) + \right. \\
&+ \gamma^2 \frac{1}{r\sqrt{A(r)}}(\cot\theta + \partial_\theta) + \gamma^3 \frac{1}{r\sqrt{A(r)}} \sin\theta\,\partial_\varphi + \gamma^4 \frac{1}{r\sqrt{A(r)}} \sin\theta \sin\varphi\,\partial_\psi + \\
&\left. + \sum_{j=5}^{9} \gamma^j \partial_{x_j} + m\right] \Psi = 0 \;.
\end{aligned} \tag{4.7}$$

4.1 Emission probability

Henceforth in order to study the probability emission of fermions we will be interested in the $r-t$ sector of the LST metric, see (3.12). Working with this effective two-dimensional metric, the Dirac equation corresponding to a spinor field is simplified to

$$\left[\gamma^0 \frac{1}{\sqrt{f(r)}}\,\partial_t + \gamma^1 \left(\frac{f(r)'}{4\sqrt{A(r)f(r)}} + \sqrt{\frac{f(r)}{A(r)}}\partial_r\right) + m\right] \Psi = 0 \;. \tag{4.8}$$

Taking into account the appropriate choice of the gamma matrices (4.4), we use for the spin-up and spin-down Dirac fields, respectively, the following WKB ansatz [95],

$$\Psi_\uparrow = \begin{pmatrix} \mathcal{A}(t,r)\xi_\uparrow \\ 0 \\ 0 \\ \mathcal{D}(t,r)\xi_\uparrow \end{pmatrix} \exp\left[\frac{i}{\hbar} S_\uparrow(t,r)\right] \;, \tag{4.9}$$

$$\Psi_{\downarrow} = \begin{pmatrix} 0 \\ \mathcal{B}(t,r)\xi_{\downarrow} \\ \mathcal{C}(t,r)\xi_{\downarrow} \\ 0 \end{pmatrix} \exp\left[\frac{i}{\hbar}S_{\downarrow}(t,r)\right] , \tag{4.10}$$

where S is the classical action, whereas $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ and $\mathcal{D}$ are arbitrary functions of the coordinates. Measuring the spin in the z direction the eigenvector of σ^3 for the spin-up and spin-down fields respectively are $\xi_{\uparrow} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\xi_{\downarrow} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. We will only solve the spin-up case and the spin-down case is solved analogously. Thus, we substitute the spinor-up field (4.9) and the gamma matrices γ^0 and γ^1 into the Dirac equation (4.8). Next, we apply the WKB approximation neglecting the $\hbar$ dependent terms, and after some algebra we eventually obtain the following set of equations for the spin-up case,

$$\begin{aligned} &\left(-\frac{1}{\sqrt{f(r)}}\,\partial_t S_{\uparrow}(t,r) + m\right)\mathcal{A}(t,r) - \sqrt{\frac{f(r)}{A(r)}}\,\partial_r S_{\uparrow}(t,r)\;\mathcal{D}(t,r) = 0 \;, \\ &\sqrt{\frac{f(r)}{A(r)}}\,\partial_r S_{\uparrow}(t,r)\;\mathcal{A}(t,r) + \left(\frac{1}{\sqrt{f(r)}}\,\partial_t S_{\uparrow}(t,r) + m\right)\mathcal{D}(t,r) = 0 \;. \end{aligned} \tag{4.11}$$

For the spin-down case we would obtain

$$\begin{aligned} &\left(-\frac{1}{\sqrt{f(r)}}\,\partial_t S_{\downarrow}(t,r) + m\right)\mathcal{B}(t,r) - \sqrt{\frac{f(r)}{A(r)}}\,\partial_r S_{\downarrow}(t,r)\;\mathcal{C}(t,r) = 0 \;, \\ &\sqrt{\frac{f(r)}{A(r)}}\,\partial_r S_{\downarrow}(t,r)\;\mathcal{B}(t,r) + \left(\frac{1}{\sqrt{f(r)}}\,\partial_t S_{\downarrow}(t,r) + m\right)\mathcal{C}(t,r) = 0 \;. \end{aligned} \tag{4.12}$$

In order to obtain non-vanishing values of the functions $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ and $\mathcal{D}$, (4.11) must fulfill the following condition,

$$\begin{vmatrix} -\frac{1}{\sqrt{f(r)}}\,\partial_t S_{\uparrow}(t,r) + m & -\sqrt{\frac{f(r)}{A(r)}}\,\partial_r S_{\uparrow}(t,r) \\ \sqrt{\frac{f(r)}{A(r)}}\,\partial_r S_{\uparrow}(t,r) & \frac{1}{\sqrt{f(r)}}\,\partial_t S_{\uparrow}(t,r) + m \end{vmatrix} = 0 \;. \tag{4.13}$$

Writing the action as an expansion in a power series of $(\frac{\hbar}{i})$,

$$S_{\uparrow}(t,r) = S_{0\uparrow}(t,r) + \left(\frac{\hbar}{i}\right) S_{1\uparrow}(t,r) + \left(\frac{\hbar}{i}\right)^2 S_{2\uparrow}(t,r) + ... \tag{4.14}$$

and making use of the WKB approximation (we neglect terms of order $(\frac{\hbar}{i})$ and higher); we finally obtain a non-linear first order partial differential equation which corresponds to the Hamilton-Jacobi equation of motion to the leading order in the action $S_{\uparrow}$,

$$-A(r)\left(\frac{\partial S_{0\uparrow}(t,r)}{\partial t}\right)^2 + f(r)^2\left(\frac{\partial S_{0\uparrow}(t,r)}{\partial r}\right)^2 + A(r)f(r)m^2 = 0\ . \qquad (4.15)$$

We point out that this equation for a massless particle is the same as the equation (3.93) in Chapter 3, Section 3.4.1. Therefore, for the emission of fermions in LST black holes we will obtain the same results for the emission probability (3.99) and temperature (3.100) that in the scalar case, even taking into account the back-reaction. We can conclude that the tunneling emission through the event horizon of the LST does not depend on the particle spin.

We would like to consider briefly the case for massive particles. If we solve (4.15) by taking into account the mass term, we obtain the action at leading order,

$$S_{0\uparrow}(t,r) = \omega t \pm \int_{r_{in}}^{r_{out}} \frac{\sqrt{A(r)}}{f(r)}\ \sqrt{\omega^2 - f(r)m^2}\ dr\ . \qquad (4.16)$$

The plus/minus sign corresponds to ingoing/outgoing particles respectively, r_{in} and r_{out} correspond to a position inside and outside of the black hole respectively and ω is the energy of the emitted or absorbed massive particle. We will obtain again the same results for the emission probability and temperature, even taking into account the back-reaction of the metric.

4.2 Fermion modes and greybody factor

In this section we are interested in finding the fermion modes of the Dirac equation (4.8) corresponding to the two-dimensional LST background (3.12). Now, in order to study the Dirac equation, we choose the following basis for the spinor field

$$\Psi(t,r) = \begin{pmatrix} \Psi_+(t,r) \\ \Psi_-(t,r) \end{pmatrix}\ , \qquad (4.17)$$

considering that each term is a two-component spin-up and spin-down spinor, $\Psi_+ = \begin{pmatrix} \Psi_+^{\uparrow} \\ \Psi_+^{\downarrow} \end{pmatrix}$ and $\Psi_- = \begin{pmatrix} \Psi_-^{\downarrow} \\ \Psi_-^{\uparrow} \end{pmatrix}$. Using this basis for the spinor field and the γ^0 and γ^1 matrices defined in (4.4), we obtain a two set of equivalent equations corresponding to the spin-up and spin-down fermion case. We will study the spin-up fermion case, equivalently we could also study the spin-down case. Therefore the Dirac equation becomes

$$\begin{aligned}
&\left(\frac{-i}{\sqrt{f(r)}}\,\partial_t + m\right)\Psi_+(t,r) - i\left[\sqrt{\frac{f(r)}{A(r)}}\,\partial_r + \frac{f'(r)}{4\sqrt{A(r)f(r)}}\right]\Psi_-(t,r) = 0\ ,\\
&\left(\frac{i}{\sqrt{f(r)}}\,\partial_t + m\right)\Psi_-(t,r) + i\left[\sqrt{\frac{f(r)}{A(r)}}\,\partial_r + \frac{f'(r)}{4\sqrt{A(r)f(r)}}\right]\Psi_+(t,r) = 0\ .
\end{aligned} \tag{4.18}$$

Next, we consider the following ansatz for the spinor field

$$\Psi_+(t,r) = \phi_+(r)\,e^{-i\omega t}\ ,\quad \Psi_-(t,r) = i\phi_-(r)\,e^{-i\omega t}\ . \tag{4.19}$$

Substituting this expressions into (4.18) and after doing algebra we obtain the following set of equations

$$\begin{aligned}
&\partial_r\phi_-(r) + \frac{f(r)'}{4f(r)}\,\phi_-(r) + \left(m\sqrt{\frac{A(r)}{f(r)}} - \omega\,\frac{\sqrt{A(r)}}{f(r)}\right)\phi_+(r) = 0\ ,\\
&\partial_r\phi_+(r) + \frac{f(r)'}{4f(r)}\,\phi_+(r) + \left(m\sqrt{\frac{A(r)}{f(r)}} + \omega\,\frac{\sqrt{A(r)}}{f(r)}\right)\phi_-(r) = 0\ .
\end{aligned} \tag{4.20}$$

We can solve this set of coupled equations. If we define

$$\eta_\pm(r) \equiv m\sqrt{\frac{A(r)}{f(r)}} \pm \omega\,\frac{\sqrt{A(r)}}{f(r)}\ , \tag{4.21}$$

we will obtain

$$\begin{aligned}
&\eta_+^{-1}(r)\,\phi_+(r)'' + \left(\partial_r\eta_+^{-1}(r) + \eta_+^{-1}(r)\,\frac{f(r)'}{2f(r)}\right)\,\phi_+(r)' + \\
&+\left(\partial_r\eta_+^{-1}(r)\,\frac{f(r)'}{4f(r)} + \eta_+^{-1}(r)\,\partial_r\left(\frac{f(r)'}{4f(r)}\right) + \eta_+^{-1}(r)\left(\frac{f(r)'}{4f(r)}\right)^2 - \eta_-(r)\right)\phi_+(r) = 0\ .
\end{aligned} \tag{4.22}$$

In order to simplify the resolution of the above equations, we consider the propagation of a massless fermion through the LST background (3.12). Substituting the values of $f(r)$ and $A(r)$ given in (3.8), into (4.21) and (4.22), eventually we obtain the propagation equation for a massless fermion mode,

$$4r^2(r^2-r_0^2)^2\,\phi_+(r)'' + 4r(r^2-r_0^2)(r^2+2r_0^2)\,\phi_+(r)' + (4\omega^2Nr^4 - 4r_0^2r^2 + 5r_0^4)\,\phi_+(r) = 0\ . \tag{4.23}$$

This equation admits the following solution

$$\phi_+(r) = \sqrt{r}\,(r^2-r_0^2)^{-1/4}\left(C_1\,(r^2-r_0^2)^{-\frac{i}{2}\omega\sqrt{N}} + \frac{C_2}{2i\omega\sqrt{N}}\,(r^2-r_0^2)^{\frac{i}{2}\omega\sqrt{N}}\right)\ , \tag{4.24}$$

where C_1 and C_2 are arbitrary constants.

The gravitational potential barrier around the black hole acts as a filter for the emitted radiation, therefore the spectrum detected at the asymptotic infinity is not a pure Planckian spectrum. The greybody factor accounts for this deviation from the purely blackbody spectrum, see (2.39). However LST exhibits a different behavior; the non-dependence of its temperature on the black hole mass leads to the fact that the emission is purely thermal, even taking into account back-reaction effects. Therefore, one expects that the spectrum shall be purely Planckian and the greybody factor takes the value 1, as we verified in Section 3.9. for massless scalar particles. But now, we are going to verify this assumption computing explicitly the greybody factor corresponding to the emission of a massless fermion in a two-dimensional effective background (3.12). We will follow the method of matching the solutions at asymptotic infinity of the black hole and near the horizon at a matching point r_m, see references [21, 22, 23, 24]. Basically we must calculate the flux

$$\mathcal{F} = \frac{1}{2i} \left(\phi_+^*(r) r^3 f(r) \partial_r \phi_+(r) - c.c. \right) \tag{4.25}$$

near the horizon of the black hole and at the asymptotic infinity. The ratio of the two fluxes is the absorption cross section, i.e. the greybody factor of the black hole. The mode solution at the near horizon limit is obtained imposing the propagation of ingoing modes as a boundary condition. Then if we expand the solution (4.24) near the horizon, we obtain

$$\phi_h(r) = C_h \, (r - r_0)^{-\frac{1}{4} - \frac{i}{2}\omega\sqrt{N}} \, , \tag{4.26}$$

where we have collected all the terms that are independent of the radial coordinate in the constant C_h. The flux (4.25) calculated at the near horizon limit is

$$\mathcal{F}_h = \frac{|C_h|^2}{2} \, \omega\sqrt{N} \, \frac{r \, (r + r_0)}{\sqrt{r - r_0}} \, . \tag{4.27}$$

Next we calculate the mode solution at the asymptotic limit. We must take into account that in this limit the metric function $f(r)$ fulfills the relation

$$\lim_{r \to \infty} f(r) = 1 \, . \tag{4.28}$$

Then, we solve equation (4.22) for the massless case using (4.28), and we obtain for the modes solution at the asymptotic limit

$$\phi_\infty(r) = C_\infty \, r^{i\omega\sqrt{N}} \, . \tag{4.29}$$

Now, the flux (4.25) computed in the asymptotic limit is

$$\mathcal{F}_\infty = \frac{|C_\infty|^2}{2}\,\omega\sqrt{N}\,r^2\ . \tag{4.30}$$

In order to find a relation between the constants C_h and C_∞ we match both solutions at the matching point r_m, which fulfills $r_0 << r_m$. Hence imposing the matching condition: $\phi_h(r_m) = \phi_\infty(r_m)$, we find the following relation between the constants

$$|C_\infty|^2 = \frac{|C_h|^2}{\sqrt{r_m - r_0}}\ . \tag{4.31}$$

Finally, if we calculate the greybody factor as the ratio of the ingoing flux through the horizon, $\mathcal{F}_h$, to the outgoing flux at the asymptotic limit, $\mathcal{F}_\infty$, we obtain

$$\Gamma_\omega \equiv \frac{|\mathcal{F}_h|}{|\mathcal{F}_\infty|} = 1\ . \tag{4.32}$$

This result indicates that for LST we will obtain a pure Planckian spectrum,

$$\rho(\omega) = \frac{1}{(e^{\omega/T}-1)}\,\frac{d\omega}{2\pi}\ , \tag{4.33}$$

in accordance with the result of Hawking, see (2.36). Effectively, one would expect this result since we have demonstrated how LST exhibits a purely thermal behavior, even taking into account the back-reaction of the metric.

Chapter 5

Back-reaction and quantum corrections

In a recent work we have shown how the back-reaction can be treated as a quantum correction, [96]. The novel semi-classical approach which will be presented here consists of the introduction of adequate quantum corrections into the $r - t$ sector of the black hole metric. Thus, we will obtain corrected values for the temperature, entropy and emission rate, which at leading order coincide with the results derived in the tunneling approach. Comparing this approach some semi-classical methods as: the tunneling method, the complex path analysis or the cancellation of gravitational anomalies; we conclude that we obtain similar results for the emission rate and Bekenstein-Hawking entropy, however we alsonotice the appearance of new terms. We also apply this technique to the Little String Theory. Interestingly, we find similar results for the entropy with those using string one-loop calculations, e.g. we have found the classical Bekenstein-Hawking entropy plus a logarithmic correction term.

We have seen in the previous chapters that during the radiation emission of black holes we enforce energy conservation, thus the metric back-reacts and the event horizon shrinks. When the black hole radiates the total ADM mass [36] is conserved, whereas the mass of the black hole decreases by the same amount of the energy that has been released by emission. According to the heuristic picture most commonly considered [31], the quantum vacuum fluctuations generate a pair of virtual particles; one member of the pair, for example the anti-particle, falls into the black hole while the other member of the pair, i.e. the particle, escapes towards the asymptotic infinity. The net effect would be as if the black hole had emitted a particle at the expenses of slowly decreasing its mass. Accordingly, we must consider the quantum nature of the emission process; thereby, we have been led to introduce quantum perturbations into the original static metric of the black hole in order to evaluate the back-reaction.

In this work we have considered a general metric with some sort of perturbations of quantum character. Eventually, we want to show that the back-reaction of the metric, imposing energy conservation, can be viewed as a quantum perturbation. Furthermore, we have analyzed the same sort of perturbations in LST.

5.1 Quantum correction on the metric

Consider a general metric in conformal-string frame with spherical symmetry defined in a d-dimensional space-time,

$$ds^2 = -f(r)dt^2 + \frac{g(r)}{f(r)}dr^2 + h(r)r^2 d\Omega_{d-2}^2 \ . \tag{5.1}$$

The event horizon is found at the radial coordinate position r_0 and $d\Omega_{d-2}^2$ defines the $(d-2)$-sphere. Since the radiation emission depends only on the $r-t$ sector of the metric, we are going to slightly modify those terms of the metric, furthermore we want that these changes on the metric accounts for quantum effects. In [97] the authors introduced quantum corrections considering all the terms in the expansion of a single particle action. Motivated by this work, we introduce the following perturbations on the radial and time part of the metric (5.1),

$$\delta g_{tt} = -f(r)\sum_i \frac{\xi_i \hbar^i}{\xi_i \hbar^i + r_0^{(d-2)i}} \quad , \quad \delta g_{rr} = \frac{g(r)}{f(r)}\sum_i \xi_i \frac{\hbar^i}{r_0^{(d-2)i}} \ , \tag{5.2}$$

thus the slightly perturbed metric, $\hat{g}_{\mu\nu} = g_{\mu\nu}^{(0)} + \delta g_{\mu\nu}$, can be written as

$$\begin{aligned} \hat{ds}^2 &= -f(r)\left(1 + \sum_i \xi_i \frac{\hbar^i}{r_0^{(d-2)i}}\right)^{-1} dt^2 + \frac{g(r)}{f(r)}\left(1 + \sum_i \xi_i \frac{\hbar^i}{r_0^{(d-2)i}}\right) dr^2 + \\ &\quad + h(r)r^2 d\Omega_{d-2}^2 \ , \end{aligned} \tag{5.3}$$

where ξ_i are positive dimensionless parameters. This choice of the perturbations has been motivated by dimensional analysis. The reduced Planck length ($\tilde{l}_P = \frac{l_P}{2\pi}$) in a

d-dimensional space-time is defined as $\tilde{l}_P^{d-2} = \frac{\hbar G^{(d)}}{c^3}$, where $G^{(d)}$ is the d-dimensional Newton's constant. In natural units ($G = c = 1$) we obtain the following dimensional relation $[\tilde{l}_P^{d-2}] = [\hbar]$. Since for the black hole metric (5.1) we have only one parameter with length dimensions, i.e. the event horizon r_0; we conclude that r_0^{d-2} must be proportional to $\hbar$.

The perturbed metric expression (5.3) deserves a few comments. Firstly, we should verify whether it is a solution of the Einstein equations. In fact, we notice that this is the case since the perturbations are independent of any of the coordinates. Secondly, we point out the modification of the particles velocity in the region near the event horizon. Causal propagation is limited to time-like and null particle trajectories with respect to the background (5.3), therefore in the case of null coordinates we find that the maximum velocity of photons has been shifted to a new value,

$$\hat{c} = c\left(1 + \sum_i \xi_i \frac{\hbar^i}{r_0^{(d-2)i}}\right)^{-1} . \tag{5.4}$$

In any case, we do not obtain superluminal propagation velocities. Eventually, we verify that the null energy condition is not affected by the inclusion of quantum perturbations, thus $T_{\mu\nu}e^\mu e^\nu \geq 0$, or equivalently $R_{\mu\nu}e^\mu e^\nu \geq 0$ for any null vector e^μ, is fulfilled near the event horizon.

Next, we are interested in studying how the Hawking temperature of the black hole is modified by the above perturbations. As usual, if we introduce the euclidean time, $\tau = it$, we get the corresponding Euclidean positive definite metric. Furthermore, taking into account the definition of the proper length, $d\rho^2 = g_{rr}dr^2$, together with the expansion of the metric function near the event horizon, $f(r) = f'(r_0)(r - r_0)$, we can define a new radial coordinate as

$$\rho = 2\sqrt{\frac{g(r)(r-r_0)}{f'(r)}}\Big|_{r\to r_0}\left(1 + \sum_i \xi_i \frac{\hbar^i}{r_0^{(d-2)i}}\right)^{1/2} . \tag{5.5}$$

We write the metric in Rindler coordinates,

$$\hat{ds}_E^2 = \rho^2\left(\frac{f'(r)}{2\sqrt{g(r)}}\Big|_{r\to r_0}\left(1 + \sum_i \xi_i \frac{\hbar^i}{r_0^{(d-2)i}}\right)^{-1} d\tau\right)^2 + d\rho^2 + h(r)r^2 d\Omega_{d-2}^2 , \tag{5.6}$$

where we point out the presence of the modified surface gravity due to the correction terms,

$$\hat{\kappa} = \frac{f'(r)}{2\sqrt{g(r)}}\Big|_{r\to r_0}\left(1+\sum_i \xi_i \frac{\hbar^i}{r_0^{(d-2)i}}\right)^{-1} . \tag{5.7}$$

We can remove the apparent conical singularity at the event horizon in (5.6) by identifying the imaginary (Euclidean) time coordinate with the period $\beta = \frac{2\pi}{\hat{\kappa}}$. We find that the effective temperature corresponding to the perturbed black hole is

$$\hat{T} = \frac{\hbar\hat{\kappa}}{2\pi} . \tag{5.8}$$

In this equation it is easily seen that the new temperature is just the standard Hawking temperature

$$T_H = \frac{\hbar}{4\pi}\frac{f'(r)}{\sqrt{g(r)}}\Big|_{r\to r_0} , \tag{5.9}$$

corrected by quantum perturbations.

5.2 Back-reaction viewed as a quantum correction

We would like to analyze how the metric is affected by the back-reaction, and consequently if we can consider such back-reaction of the metric as a quantum effect. Motivated by the idea that the emitted particles are quantum fields whose energy, ω in natural units ($\hbar = 1$), is also quantized; our aim is to show if we can treat the back-reaction of the metric as a quantum perturbation.

In order to interpret properly the quantum perturbation of the back-reacted metric, it is useful to show the relation between the mass and the event horizon of the black hole. For that purpose we have calculated the Komar integral, see Appendix C, associated with the time-like Killing vector K^ν. For the background (5.1) we have found the following relation,

$$M = \frac{\mathrm{Vol}(\mathbf{S}^{d-2})}{8(d-3)\pi G^{(d)}}\,\frac{f(r)'}{\sqrt{g(r)}}\,\left(r\sqrt{h(r)}\right)^{d-2}\Big|_{r\to r_0} , \tag{5.10}$$

where $\mathrm{Vol}(\mathbf{S}^{d-2})$ stands for the volume of the $(d-2)-$sphere and all quantities are evaluated at the event horizon. Moreover, we also impose the following three conditions on the space-time metric:

1. Spherical symmetry.
2. The background is asymptotically flat.
3. The metric function $f(r)$ is expressed as $f(r) = 1 - \left(\frac{r_0}{r}\right)^{d-3}$, depending on the mass through the event horizon r_0. For future convenience we write the metric functions $g(r)$ and $h(r)$ as $\left(1 + \frac{r_{i,j}^2}{r^2}\right)$, depending on the charges r_i and r_j, respectively, which are different from the mass charge. With this choice for the metric functions we see from the relation (5.10) that $M \propto r_0^{d-3}$.

Taking into account the above three conditions, and expanding in the energy of the emitted particle ω, we eventually write (5.1) as

$$\tilde{ds}^2 = -\tilde{f}(r)dt^2 + \frac{g(r)}{\tilde{f}(r)}dr^2 + h(r)r^2 d\Omega_{d-2}^2 , \tag{5.11}$$

where we have defined the new metric function $f(r)$ as

$$\tilde{f}(r) = f(r) + \frac{1}{r^{d-3}} \sum_i \frac{\omega^i}{r_0^{(d-3)(i-1)}} . \tag{5.12}$$

We motivate this expression for the expansion in the energy ω of the particle based on dimensional analysis, since we have just seen that r_0^{d-3} has energy-mass dimension. Working as in the above section we find the effective temperature, which is

$$\tilde{T} = \frac{\hbar}{4\pi} \frac{\tilde{f}'(r)}{\sqrt{g(r)}}\Big|_{r\to r_0} , \tag{5.13}$$

and taking the derivative of (5.12) at the event horizon we eventually obtain

$$\tilde{T} = T_H - \frac{\hbar(d-3)}{4\pi r_0^{d-2}\sqrt{g(r_0)}} \sum_i \frac{\omega^i}{r_0^{(d-3)(i-1)}} . \tag{5.14}$$

Since the heat capacity is negative, we can verify that this expression for the temperature works properly increasing its value when the black hole emits a particle of energy ω. To see this, we can rewrite equation (5.14) using the definition of the Hawking temperature (5.9) and imposing the above third condition, hence we get for the effective temperature

$$\tilde{T} = \frac{\hbar(d-3)}{4\pi\sqrt{g(r_0)}} \left(\frac{1}{r_0} - \frac{1}{r_0^{d-2}} \sum_i \frac{\omega^i}{r_0^{(d-3)(i-1)}} \right) . \tag{5.15}$$

Since the event horizon shrinks proportionally to $\omega^{1/(d-3)}$, we see from this last expression that at low energies the temperature increases with respect to the standard Hawking temperature (5.9).

Finally, if we compare the two expressions for the temperatures (5.8) and (5.14), we obtain definite values for the dimensionless parameters, ξ_i, in terms of the released energy, ω,

$$\xi_i = \left(\frac{r_0^{d-2}}{\hbar}\right)^i \frac{\omega^i}{r_0^{(d-3)i} - \omega^i} \ . \tag{5.16}$$

Therefore, looking at the metric (5.3) and its corresponding temperature (5.8), we conclude that back-reaction can be treated as a quantum perturbation leading us to the following expressions for the perturbed metric and effective temperature respectively,

$$\begin{aligned} \hat{ds}^2 &= -f(r)\left(1+\sum_i \frac{\omega^i}{r_0^{(d-3)i}-\omega^i}\right)^{-1} dt^2 + \frac{g(r)}{f(r)}\left(1+\sum_i \frac{\omega^i}{r_0^{(d-3)i}-\omega^i}\right) dr^2 + \\ &\quad + h(r) r^2 d\Omega_{d-2}^2 \ , \end{aligned} \tag{5.17}$$

$$\hat{T} = T_H \left(1+\sum_i \frac{\omega^i}{r_0^{(d-3)i}-\omega^i}\right)^{-1} \ . \tag{5.18}$$

We are going to specify all the aforesaid expressions in a simple four-dimensional, static and spherically symmetric background. Therefore we consider a Schwarzschild black hole which is asymptotically flat, the metric functions are defined as: $f(r) = 1 - \frac{2M}{r}$, $g(r) = h(r) = 1$ and the event horizon is at $r_0 = 2M$ in natural units. From (5.17) we write the perturbed back-reacted metric as,

$$\begin{aligned} \hat{ds}^2 &= -\left(1-\frac{r_0}{r}\right)\left(1+\sum_i \frac{\omega^i}{r_0^i-\omega^i}\right)^{-1} dt^2 + \frac{1}{\left(1-\frac{r_0}{r}\right)}\left(1+\sum_i \frac{\omega^i}{r_0^i-\omega^i}\right) dr^2 + \\ &\quad + r^2 d\Omega_2^2 \ . \end{aligned} \tag{5.19}$$

The Hawking temperature corresponding to a Schwarzschild black hole is $T_H = \frac{1}{8\pi M}$. When the black hole emits a single particle with energy ω, the new effective temperature at first order in energy expansion is

$$\hat{T} = \frac{1}{8\pi(M-\omega)}\left(1+\frac{\omega}{2M-\omega}\right)^{-1} \ . \tag{5.20}$$

Likewise at semi-classical level we calculate the Bekenstein-Hawking entropy using the area law, $S_{BH} = \frac{A}{4}$, in the presence of back-reaction effects, and obtain

$$\hat{S}_{BH} = 4\pi(M-\omega)^2 \ . \tag{5.21}$$

Furthermore we also calculate the emission rate through the relation $\Gamma \propto e^{-\omega/T}$, [27]. Therefore using the effective temperature (5.20) we obtain

$$\Gamma \propto e^{-8\pi\omega(M-\omega)\left(1+\frac{\omega}{2M-\omega}\right)} \ . \tag{5.22}$$

At low energies we notice that the emission rate can be written semi-classically as

$$\Gamma \propto e^{\Delta \hat{S}_{BH}} \ , \tag{5.23}$$

being the initial entropy $S_{BH}^{(0)} = 4\pi M^2$, and the final entropy is given by (5.21). We point out that this result coincides with the result in [30], where the emission rate also matches the statistical mechanics picture. We see that deviation from thermal behavior when a black hole emits particles is due to the energy conservation, moreover the temperature increases while the entropy of the black hole decreases properly during the emission process. Thus summarizing, we have seen that all the semi-classical results concerning the emission of particles are recovered when we consider the back-reaction as a quantum correction.

On the other hand, instead of calculating the entropy using the area law, we can evaluate the entropy through the first law of thermodynamics: $dM = TdS$, in the presence of back-reaction effects. Thus using (5.20) for the temperature we obtain

$$\hat{S} = 4\pi M^2 - 4\pi M\omega - 2\pi\omega^2 \log(2M-\omega) + \pi\omega^2 + \mathcal{O}(\omega^3) \ . \tag{5.24}$$

The first term is just the semi-classical area law, $S_{BH} = \frac{A}{4}$. Nevertheless, we interestingly point out the presence of a logarithmic correction term, which is considered as a one-loop correction term over the classical thermodynamics.

5.3 An example in string theory

5.3.1 Quantum corrections at action level

Previous to the study of the quantum corrections on the metric related with the tunneling emission of scalar massless particles, we would like to consider in this section the study of quantum corrections at the action level [97, 98] of LST. Our starting point will be (3.91), and the expansion in $\hbar$ of the action which we write as

$$S(t,r) = S_0(t,r) + \hbar S_1(t,r) + \hbar^2 S_2(t,r) + ... = S_0(t,r) + \sum_i \hbar^i S_i(t,r) \ . \tag{5.25}$$

Substituting the above expansion in the equation (3.91) we obtain a set of equations at each order of $\hbar$,

$$\begin{aligned}
\hbar^0 &: \; -A(r)\partial_t S_0^2 + f(r)^2 \partial_r S_0^2 = 0 \, , \\
\hbar^1 &: \; -2A(r)\partial_t S_0 \, \partial_t S_1 + 2f(r)^2 \partial_r S_0 \, \partial_r S_1 + iA(r)\partial_t^2 S_0 - if(r)^2 \partial_r^2 S_0 \\
&\quad -i \, \frac{f(r)}{r^3} \frac{d(r^3 f(r))}{dr} \, \partial_r S_0 = 0 \, , \\
\hbar^2 &: \; -A(r)\partial_t S_1^2 - 2A(r)\partial_t S_0 \, \partial_t S_2 + f(r)^2 \partial_r S_1^2 + 2f(r)^2 \partial_r S_0 \, \partial_r S_2 + \\
&\quad +iA(r)\partial_t^2 S_1 - if(r)^2 \partial_r^2 S_1 - i \, \frac{f(r)}{r^3} \frac{d(r^3 f(r))}{dr} \, \partial_r S_1 = 0 \, , \\
&\dots \, .
\end{aligned} \tag{5.26}$$

Interestingly each equation can be simplified using the preceding one, thus after a little bit of algebra we arrive at the following set of equations at each order in $\hbar$:

$$\begin{aligned}
\hbar^0 &: \; \partial_t S_0(t,r) = \frac{f(r)}{\sqrt{A(r)}} \, \partial_r S_0(t,r) \, , \\
\hbar^1 &: \; \partial_t S_1(t,r) = \frac{f(r)}{\sqrt{A(r)}} \, \partial_r S_1(t,r) \, , \\
\hbar^2 &: \; \partial_t S_2(t,r) = \frac{f(r)}{\sqrt{A(r)}} \, \partial_r S_2(t,r) \, , \\
&\cdots \, ,
\end{aligned} \tag{5.27}$$

where the metric functions were defined in (3.8). As the functional form of the above set of equations is identical, the solutions will not be independent, therefore each action term solution of the expansion (5.25) will be proportional to the leading order term S_0.

In order to write the quantum corrections terms proportional to $\hbar$ we will follow the dimensional arguments of [97], hence the LST corrected action will be of the form

$$S(t,r) = S_0(t,r) \left(1 + \sum_i \xi_i \frac{\hbar^i}{r_0^{3i}} \right) \, , \tag{5.28}$$

where r_0 is the event horizon. Then solving the equation (5.27) at leading order we will obtain the solution (3.95), thus the quantum corrected solution will be

$$S(t,r) = \left(\omega t \pm \omega \int_{r_{in}}^{r_{out}} \frac{\sqrt{A(r)}}{f(r)} \, dr \right) \left(1 + \sum_i \xi_i \frac{\hbar^i}{r_0^{3i}} \right) \, , \tag{5.29}$$

where plus/minus sign corresponds to ingoing/outgoing mode solutions. Taking into account the WKB approximation, $\phi \sim e^{-\frac{i}{\hbar}S(t,r)}$,

$$\phi_{in/out} = \exp\left[-\frac{i}{\hbar}\left(\omega t \pm \omega \int_{r_{in}}^{r_{out}} \frac{\sqrt{A(r)}}{f(r)}\, dr\right)\left(1 + \sum_i \xi_i \frac{\hbar^i}{r_0^{3i}}\right)\right] . \tag{5.30}$$

Next we will calculate the emission probability; instead of using the procedure carried out in Section 3.4.1, or in [72], we will use the method that follows. We must take into account that when a particle crosses the event horizon the nature of the $(r - t)$ coordinates changes. The time coordinate acquires an imaginary part, hence there will be a time contribution to the probability of the ingoing and outgoing particles. Taking into account this temporal contribution we will obtain the correct expression for the emission probability, see discussion around (3.99). Then we write the absorption and emission probability for an incoming and outgoing particle, respectively, as

$$P_{a/e} = |\phi_{in/out}|^2 = \exp\left[\frac{2}{\hbar}\left(\omega\, \mathrm{Im}\, t \pm \omega \int_{r_{in}}^{r_{out}} \frac{\sqrt{A(r)}}{f(r)}\, dr\right)\left(1 + \sum_i \xi_i \frac{\hbar^i}{r_0^{3i}}\right)\right] . \tag{5.31}$$

In the classical limit everything is absorbed without any reflection, thus $P_a = 1$, and this fact implies

$$\mathrm{Im}\, t = -\mathrm{Im} \int_{r_{in}}^{r_{out}} \frac{\sqrt{A(r)}}{f(r)}\, dr . \tag{5.32}$$

Otherwise when a particle crosses the horizon the time coordinate experiences the transformation

$$t \to t - i\pi \int_{r_{in}}^{r_{out}} \frac{\sqrt{A(r)}}{f(r)}\, dr , \tag{5.33}$$

whose imaginary part is precisely (5.32). Therefore evaluating the integral at the pole $r = r_0$ and substituting the value of $A(r)$, see (3.8), we eventually obtain

$$P_e = \exp\left[-\frac{2\pi}{\hbar}\sqrt{\chi r_0^2 + \frac{N}{m_s^2}}\left(1 + \sum_i \xi_i \frac{\hbar^i}{r_0^{3i}}\right)\omega\right] . \tag{5.34}$$

Finally, if we use the Boltzmann relation between the emission and absorption probabilities, $P_e = e^{-\omega/T} P_a$; we will obtain, comparing with (5.34) and remembering that $P_a = 1$, the quantum corrected temperature of the NS5 and LST black holes,

$$T_c = \frac{\hbar}{2\pi\sqrt{\chi r_0^2 + \frac{N}{m_s^2}}}\left(1 + \sum_i \xi_i \frac{\hbar^i}{r_0^{3i}}\right)^{-1} , \tag{5.35}$$

which is nothing more that the standard Hawking temperature of NS5 and LST (3.24) corrected by a sort of quantum perturbations.

5.3.2 Quantum corrections on the metric

In this section we are going to illustrate the preceding techniques in LST background, moreover we will elucidate some thermodynamical aspects. Following the above techniques we introduce the perturbations at first order modifying the function metric $f(r)$ given in (3.8),

$$f(r) \rightarrow f(r)\left(1+\frac{\omega}{r_0^2-\omega}\right)^{-1} . \tag{5.36}$$

Now the effective temperature will be

$$\hat{T} = T_H\left(1+\frac{\omega}{r_0^2-\omega}\right)^{-1} , \tag{5.37}$$

and also from (5.10) we have computed the ADM mass corresponding to the NS5 and LST black hole,

$$M = \frac{\mathrm{Vol}(\mathbf{R}^5)\pi}{4G^{(10)}}\left(\chi r_0^2+\frac{N}{m_s^2}\right) , \tag{5.38}$$

where $\mathrm{Vol}(\mathbf{R}^5)$ stands for the volume of the NS5-branes. In [54] and previously in [45] it was found that the Helmholtz free energy vanishes, $\mathcal{F} = E - TS = 0$. Therefore the entropy coincides with the semi-classical area law entropy,

$$S_{BH} = \frac{A}{4G^{(10)}\hbar} = \frac{\mathrm{Vol}(\mathbf{R}^5)\pi^2}{2G^{(10)}\hbar}\left(\chi r_0^2+\frac{N}{m_s^2}\right)^{3/2} . \tag{5.39}$$

Then, for an emission process taking into account back-reaction we shall consider the effective temperature (5.37). Moreover, considering the relation between the mass and the event horizon (5.38), we calculate the emission rate $\Gamma \propto e^{\omega/\hat{T}}$ at low energies,

$$\Gamma \propto e^{-\frac{\omega}{T_H}\left(1+\frac{\omega}{2M}+\mathcal{O}(\omega^2)\right)} , \tag{5.40}$$

which is in accordance with the statistical mechanics result, $\Gamma \propto e^{\Delta \hat{S}_{BH}}$, being $\hat{S}_{BH} \propto (M-\omega)^{3/2}$ the Bekenstein-Hawking entropy after the emission. This last result coincides entirely with the result in [72] for the NS5 black hole, however it is not the case for LST which disagrees in the factor $e^{\sim\frac{\omega}{2M}}$. In that work we calculated the emission rate modifying naively the mass factor that appears in the temperature, i.e. $M \rightarrow M - \omega$, without taking into account any temperature correction factor. Meanwhile in this work we have considered the effective temperature (5.37), obtaining in this way an interesting deviation from the pure thermal behavior found in [59, 72]. Therefore this result signals some sort of correction over the classical thermodynamics.

5.3.3 Discussion

The thermodynamics of the near horizon limit of NS5 presents a Hagedorn behavior, where the statistical mechanics of any string theory breakdown. At very high energy density, one can see from (3.24) that the Hagedorn temperature of LST is independent of the mass, thus at leading order the thermodynamics will be completely degenerate with a constant temperature. Furthermore, the entropy will be proportional to the energy, $E = T_H S$, hence the free energy is expected to vanish. In [51] the authors implemented string one-loop corrections at the near horizon limit of the NS5-brane thermodynamics to explain the Hagedorn behavior of LST. These corrections expand the phase space and introduce small deviations from the constant Hagedorn temperature. On the other hand, in the high energy regime LST become weakly coupled, thus being able to perform a perturbative holographic analysis, see [45]. In this work it is shown that LST has a Hagedorn density of states that grows exponentially: $\rho = e^{S(E)} \sim E^{\alpha} e^{E/T_H}$, then the authors computed the genus one correction to both the temperature and the density of states. As in [51] they found an entropy-energy relation with logarithmic corrections.

In our preceding study we have introduced, at semi-classical level, some sort of quantum energy corrections into the temperature for back-reaction processes. Now, our aim is to calculate the corrected entropy for the LST black hole. If we calculate the ADM mass-energy in Einstein frame: $ds_E^2 = \sqrt{g_s e^{-\Phi}} ds^2$, see Chapter 6, we find the relation between the mass and the event horizon. Hence we can write the corrected Hawking temperature corresponding to LST as

$$\hat{T}_H = \frac{\hbar}{2\pi\sqrt{\frac{N}{m_s^2}}} \left(1 + \frac{\omega}{\frac{4G^{(10)}M}{\mathrm{Vol}(\mathbf{R}^5)\pi} - \omega} \right)^{-1} . \tag{5.41}$$

We note that this temperature decreases when the LST black hole emits radiation, thereby the specific heat will be positive. Moreover, we verify that the Hagedorn temperature is the maximum temperature reached by the system and cannot be crossed. Now, integrating $dM/\hat{T}_H$ we obtain the corrected entropy,

$$\hat{S}(M) \approx \frac{M}{T_H} + \frac{\mathrm{Vol}(\mathbf{R}^5)\pi^2\omega\sqrt{N}}{2G^{(10)}\hbar m_s} \log(M) + \mathcal{O}\left(\frac{1}{M}\right) . \tag{5.42}$$

Thus, we have obtained the classical Bekenstein-Hawking term plus logarithmic corrections to the entropy of LST. As in [45] and [51], we have found that the logarithmic term is $\mathrm{Vol}(\mathbf{R}^5)$-dependent.

Finally, we would like to make a last comment on the thermodynamics of LST. Looking at the temperature (5.41), we are tempted to modify the plus sign of the correction factor by a minus sign. The purpose of this change of sign is to fit the usual behavior of classical black holes, e.g. Schwarzschild-like black holes increase their temperature when they emit radiation, and have a negative specific heat. In this case, we point out that, in accordance with [45], the logarithmic correction term of the entropy (5.42) will be negative, the temperature (5.41) will be above the Hagedorn temperature and the specific heat will be negative, therefore the thermodynamics will be unstable. Thus, if we perform a semi-classical analysis introducing quantum corrections on the metric, we are able to obtain similar results as working with string loop corrections.

Chapter 6

Einstein and conformal frame

In the previous chapters we have calculated some physical magnitudes as the temperature or the entropy of NS5 and LST black holes working mainly in string (or conformal) frame. However, in some cases, we have specified that we were working in Einstein frame. Actually we understand that physics must be frame independent, thus we guessed that some sort of scale factor should relate the two frames. In this section we have obtained the precise scale factor that relates the two frames.

A *conformal transformation* is a local change of scale (or geometry but not a change of coordinates) that leaves light cones *invariant*. Such transformations are defined as

$$\tilde{g}_{\mu\nu} = \Omega^2(x) g_{\mu\nu} \ , \tag{6.1}$$

where $\Omega(x)$ is a space-time non-vanishing function. We then say that the physical quantities are expressed in the conformal frame.

So far we have worked in string frame (3.5). On the other hand there exists a conformal transformation that relates the string action with the standard Einstein-Hilbert action [10]. The low energy action for ten-dimensional type IIB string theory can be written as

$$I = \frac{1}{16\pi G^{(10)}} \int_M d^{10}x \sqrt{-g} \left[e^{-2\Phi} (R + 4(\nabla\phi)^2) - \frac{1}{12} H_3^2 \right] \ . \tag{6.2}$$

Whereas the action in Einstein frame takes the standard Einstein-Hilbert form,

$$I_E = \frac{1}{16\pi G^{(10)}} \int_M d^{10}x \sqrt{-g} \left(R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{12} e^{-\Phi} H_{(3)}^2 \right) \ , \tag{6.3}$$

where Φ is the dilaton scalar field, H_3 is the $NS-NS$ form along the S^3 and ϕ is the scalar field. Thus the metric in string frame can be written in Einstein frame using

$$ds_E^2 = \sqrt{g_s e^{-\Phi}}\, ds^2 \ , \tag{6.4}$$

where g_s is the string coupling. Both metrics are related by a Weyl rescaling given by the dilaton. In the string frame the scalar field play the role of a spin-0 component of gravity, whereas in the Einstein frame the scalar field plays the role of a source matter field.

The question arises: 'Which frame is the physically relevant frame?' With the aim to answer this question we are going to calculate the entropy of the NS5 and LST black holes in Einstein frame, and compare it with the expression obtained in string frame, Chapter 3, Section 3.1. We start writing the metric (3.5) in Einstein frame by using the relation (6.4) and also using the definition of the dilaton (3.7),

$$ds_E^2 = -\frac{f(r)}{A^{1/4}(r)}dt^2 + \frac{A^{3/4}(r)}{f(r)}dr^2 + A^{3/4}(r)r^2 d\Omega_3^2 + \sum_{j=1}^{5}\frac{dx_j^2}{A^{1/4}(r)} \ . \tag{6.5}$$

If we then calculate the temperature we will notice that it is frame independent. Next, we are going to calculate the area of the event horizon corresponding to NS5 and LST black holes defined in the induced Einstein metric

$$d\hat{s}_E^2 = A^{3/4}(r)r^2 d\Omega_3^2 + \sum_{j=1}^{5}\frac{dx_j^2}{A^{1/4}(r)} \ , \tag{6.6}$$

for which the determinant is

$$\sqrt{-\hat{g}_E} = r^3\sqrt{A(r)}\,\sin^2(\theta)\sin(\varphi) \ . \tag{6.7}$$

The area of the event horizon is

$$\mathrm{A_H} = \int \sqrt{-\hat{g}_E}\, d\theta\, d\varphi\, d\psi \prod_{j=1}^{5} d^j x = V_5\, 2\pi^2 r_0^3\sqrt{A(r_0)} = \frac{\mathrm{Vol}(\mathbf{R}^5)\, 2\pi^2 r_0^2\,\sqrt{\chi m_s^2 r_0^2 + N}}{m_s} \ . \tag{6.8}$$

Comparing with (3.28) we can see that the value of the area of the event horizon is frame dependent. Then the Bekenstein-Hawking entropy is

$$S_{BH} = \frac{\mathrm{A_H}}{4G^{(10)}\hbar} = \frac{\mathrm{Vol}(\mathbf{R}^5)\,\pi^2 r_0^2\,\sqrt{\chi m_s^2 r_0^2 + N}}{2G^{(10)}\hbar m_s} \ , \tag{6.9}$$

and differs from (3.30) calculated in the string frame.

In order to calculate the total energy of the NS5 and LST black holes we use the Komar integral (C.1), see Appendix C. We choose as normal vectors in Einstein frame

$$e_t = -\sqrt{\frac{f(r)}{A^{1/4}(r)}} \quad , \quad e_r = \sqrt{\frac{A^{3/4}(r)}{f(r)}} \ . \tag{6.10}$$

The total energy (mass) is given by

$$E = \frac{\mathrm{Vol}(\mathbf{R}^5)\ \pi r_0^2}{4G^{(10)}} \ , \tag{6.11}$$

which differs from (C.5), therefore the Komar integral is also frame-dependent. Thus comparing the expressions of the entropy (6.9) and the energy (6.11) in Einstein frame with the entropy (3.30) and the energy (C.5) in string frame, we see that they are related by the metric function $A(r_0)$ at the event horizon,

$$\begin{aligned} S_{BH}^{(E)} &= A(r_0)^{-1} S_{BH} \ , \\ E^{(E)} &= A(r_0)^{-1} E \ . \end{aligned} \tag{6.12}$$

Therefore $A(r_0)$ acts as a scale factor for physical extensive quantities like entropy, whereas the temperature is an intensive quantity and its value does not change under conformal scalings. We can conclude that physical laws are invariant under conformal scalings. Only the values of extensive quantities change by a fixed scale factor.

Chapter 7

Summary, conclusions and outlook

After a brief outline in Chapter 1 about the properties of black holes, where we have introduced the information loss paradox, we have reviewed in the Chapter 2 how curved space-time, e.g. black hole backgrounds, creates particles. Hawking demonstrated that black holes with temperature T_H emit thermal radiation, and calculated its flux without taking into account the back-reaction of the metric. Afterwards we have presented two semi-classical methods, i.e. the tunneling approach and complex path method, that somewhat solve the information loss paradox stated by the work of Hawking. In Chapter 3, we have applied both semi-classical methods plus the covariant anomaly method in NS5 and Little String Theory (LST) black holes. We have calculated some thermodynamical quantities as the temperature and the entropy; furthermore, after reducing the ten-dimensional theory to a two-dimensional effective theory, we have calculated the emission rate and the corresponding fluxes taking into account the back-reaction of the metric. In Chapter 4, we have calculated the emission probability of fermions by NS5 and LST black holes obtaining identical results as for scalar particles. In Chapter 5, we have presented a novel method in order to introduce quantum perturbations directly in the black hole metric, that accounts for back-reaction effects. This method has been applied to a general stationary spherically symmetric metric, recovering similar results with the results of the semi-classical methods presented in the previous chapters. Moreover, when we have applied this method in LST's black hole we have obtained similar results with those derived in string one-loop theory. Finally in Chapter 6, we have calculated and compared some thermodynamical quantities as the entropy, using both Einstein frame and conformal frame.

In previous sections we have already been discussing some conclusions, now we will outline the most general salient features for the NS5 and LST black holes. In general we have seen:

- The Hawking radiation as tunneling approach solves partially the information loss paradox, since the emission rates obtained for a general class of black holes, e.g. Schwarzschild, Reissner-Nordstrom, stringy black holes as NS5, etc., are non-thermal. The lack of thermal behavior in the emission spectrum lead us to establish some correlations between the emitted particles, recovering at least all the information stored in the initial configurations that originate the black hole. This approach takes into account the back-reaction of the metric when a scalar massless shell particle is emitted by the black hole, imposing *energy conservation*. The emission rate is in accordance with the statistical mechanics results.

- The complex path semi-classical method allow us to calculate the emission probability of a black hole. Imposing energy conservation in order to implement the back-reaction of the metric we obtain again a non-thermal spectrum. This method, compared with the tunneling approach, has the advantage that avoids any heuristic interpretation of the emission mechanism. Moreover it is not needed to go to Painlevé coordinates, since one can works directly with spherical Schwarzschild coordinates.

- The factor that cancels the gravitational anomaly in a two-dimensional effective black hole metric is exactly the Hawking radiation flux of the black hole. We have calculated explicitly the spectrum in (3.144) and we have verified that this result matches the anomaly result (3.114) for massless scalar particles.

- The general clue for obtaining non-thermal spectra, in the vast majority of the black holes studied in the literature, is the imposition of *energy conservation* when one takes into account the *back-reaction* of the metric.

- We can define an effective temperature T_{eff}, which consists basically of the standard Hawking temperature of the black hole corrected by a factor depending on the energy of the particle emitted by the black hole when the back-reaction is taking into account. Then the emission rate will be the standard Boltzmann factor at a temperature T_{eff}, (3.151).

- Introducing some sort of quantum corrections in the $r-t$ sector of the metric we are able to analyze the back-reaction as a quantum perturbation. This corrections are built on dimensional grounds and satisfy the Einstein's equations, since are independent of any system of coordinates.

Regarding the concrete case of NS5 and LST we would like to remark the following aspects:

- The NS5 black hole shows the expected non-thermal spectrum, thus all the above conclusions are verified in this case. On the other hand, LST keeps its *thermal behavior* even taking into account the back-reaction of the metric, hence the greybody factor for LST is 1. NS5 does not accomplish cluster decomposition, therefore it would be possible to recover the information of the initial configurations that formed the black hole. This information could be encoded in the correlations between the emitted particles and would be released out when the black hole evaporates. Nevertheless LST satisfies cluster decomposition, and the emitted particles are not correlated. In fact, we point out the notorious property that the LST Hawking temperature is independent of its mass, thus this temperature is constant even if the black hole is emitting. Therefore the information remains hidden behind the event horizon of LST until it evaporates.

- LST is the thermal limit of NS5. When we explore the region near the event horizon of an evaporating NS5 black hole the temperature increases until it reaches the maximum temperature, i.e. the Hagedorn temperature, becoming then a single pure thermal state. This single state, i.e. LST, radiates a constant flux of energy at a constant temperature.

- The emission spectrum of fermions and scalar particles is the same either for NS5 or LST black holes. In general, even for other black holes, working with an effective theory in the $r-t$ sector of the metric the emission will be independent of the spin degrees of freedom.

- Introducing quantum corrections that accounts for the back-reaction in the metric of LST, and then studying the thermodynamics, we have obtained the same kind of logarithmic entropy corrections that are also found working in one loop string theory. The thermodynamics of LST presents a Hagedorn behavior, its specific heat is negative, hence the thermodynamics is unstable. Therefore, we can conclude that LST consists of a single unstable state.

We would like to finish with a brief outlook. One decade ago a new theory proposed a higher dimensional mechanism for solving the hierarchy problem [99]. In that framework the Planck scale can be reduced considerably [100, 101] until it reaches the TeV scale. There also exists some four-dimensional models which are able to reduce the Planck mass to the TeV scale [102, 103]. Then considering the results of the present thesis, where we have presented different semi-classical methods which lead us to non-thermal spectra results, it should be interesting to focus our attention to the study of the emission of gravitons at the LHC (Large Hadron Collider). Furthermore, it should be very interesting the study of quantum production [104] of small quantum black holes in scattering processes.

Appendix A

Calculus tools and notation conventions

We use two kinds of tensor indices: greek index $(\mu, \nu, \rho, ...)$ in general curved space-times and latin index $(a, b, c, ...)$ in flat space-time. We also use the Einstein notation of summing over repeated indices.

We choose the metric signature $(- + ...+)$. The expression η_{ab} represents the components of the Minkowski metric and $g_{\mu\nu}$ the general components of a curved space-time metric. We define the vielbeins as

$$e^{\mu}_{a} e^{\nu}_{b}\ g_{\mu\nu} = \eta_{ab}\ , \quad e^{a}_{\mu} e^{b}_{\nu}\ \eta_{ab} = g_{\mu\nu}\ . \tag{A.1}$$

The expressions $\frac{\partial}{\partial x^{\mu}}$ or ∂_{μ} or $,\mu$ represent a partial derivative. Whereas ∇_{μ} or $;\mu$ represents a covariant derivative. A prime over a function means partial derivative with respect to the radial coordinate: $f' \equiv \frac{\partial f}{\partial r}$, whereas a point over a function means derivative with respect to time coordinate: $\dot{f} \equiv \frac{\partial f}{\partial t}$. And finally $\mathcal{D}$ is the covariant Lorentz derivative. These derivatives are defined over the tensors and spinors ψ as

$$\begin{aligned} \nabla_{\mu} k^{\nu} &= \partial_{\mu} k^{\nu} + \Gamma^{\nu}_{\mu\rho} k^{\rho}\ , \\ \nabla_{\mu} \psi &= \partial_{\mu} \psi - \frac{1}{4} \omega^{ab}_{\mu} \Gamma_{ab} \psi\ , \\ \mathcal{D}_{\mu} k^{a} &= \partial_{\mu} k^{a} + \omega^{a}_{\mu b} k^{b}\ , \end{aligned} \tag{A.2}$$

where Γ_{ab} is the antisymmetric product of two gamma matrices. The connections are related by

$$\omega^{b}_{\mu a} = \Gamma^{b}_{\mu a} + e^{\nu}_{a} \partial_{\mu} e^{b}_{\nu}\ , \tag{A.3}$$

where the affine connection $\Gamma^{\rho}_{\mu\nu}$ are the Christoffel symbols

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} \left(\partial_{\mu} g_{\nu\sigma} + \partial_{\nu} g_{\mu\sigma} - \partial_{\sigma} g_{\mu\nu} \right) \ . \tag{A.4}$$

We use for the anticommutator the relation

$$[A, B] = AB - BA \ , \tag{A.5}$$

and for the commutator

$$\{A, B\} = AB + BA \ . \tag{A.6}$$

For the majority of the cases presented in this work we use Planck units

$$\hbar = c = G = 1 \ . \tag{A.7}$$

However in some cases we write the units explicitly for convenience.

Appendix B

N-sphere area

The n-sphere S^n of radius unit is defined as the hypersurface of radius $r = 1$ in the $(n+1)$-dimensional Euclidean space-time, where r is the radial coordinate in $(n+1)$-dimensional spherical coordinates $(r, \varphi, \theta_1, ..., \theta_{n-1})$. The induced metric on S^n in spherical coordinates is

$$d\tilde{s}^2 = r^2 d\Omega_n^2 \ , \tag{B.1}$$

where the volume form in S^n is

$$d\Omega^n \equiv d\varphi \prod_{i=1}^{n-1} \sin^i\theta_i d\theta_i \ . \tag{B.2}$$

The volume of the n-sphere S^n is the volume integral over all the sphere

$$\omega_n = \int_{S^n} d\Omega^n = \frac{2\pi^{\frac{n+1}{2}}}{\Gamma\left(\frac{n+1}{2}\right)} \ . \tag{B.3}$$

Using the properties of the gamma function

$$\begin{aligned} &\Gamma(x+1) = x\,\Gamma(x) \ , \\ &\Gamma(0) = 1 \ , \\ &\Gamma(1/2) = \sqrt{\pi} \ , \end{aligned} \tag{B.4}$$

for example we obtain

$$\begin{aligned} &\omega_1 = 2\pi \ , \\ &\omega_2 = 4\pi \ , \\ &\omega_3 = 2\pi^2 \ , \\ &... \end{aligned} \tag{B.5}$$

Appendix C

Komar integral and ADM energy

In this appendix our aim is to show the equivalence between the square of the event horizon and the mass of the NS5/LST black hole. We will calculate the Komar integral associated with the time-like Killing vector K^ν, for definition see e.g. [105]. For the ten-dimensional NS5 and LST the Komar integral is defined as

$$E = \frac{1}{8\pi G^{(10)}} \int_{\partial\Sigma} d^{(8)}x \sqrt{-g^{(8)}}\, e_\mu e_\nu \nabla^\mu K^\nu \ , \tag{C.1}$$

where the integral is carried out in the boundary $\partial\Sigma$ of a space-like hypersurface Σ. We will work in string frame (3.5). We define two normal vectors as

$$e_t = -\sqrt{f(r)} \quad , \quad e_r = \sqrt{\frac{A(r)}{f(r)}} \tag{C.2}$$

with the other components to be equal to zero. They fulfill the normalization condition $e_\mu e^\mu = -1$ and $e_\nu e^\nu = +1$. We therefore have

$$e_\mu e_\nu \nabla^\mu K^\nu = e_t e_r \nabla^t K^r \ . \tag{C.3}$$

The Killing vector is $K^\nu = (1, 0, 0, 0, ...)$, then

$$\nabla^t K^r = g^{tt}\nabla_t K^r = g^{tt}(\partial_t K^r + \Gamma^r_{tt} K^t) = \frac{1}{2} g^{tt} g^{rr} \partial_r(-g_{tt}) \ . \tag{C.4}$$

Substituting in (C.1) and integrating we obtain the total energy, or equivalently the ADM mass[1] corresponding to the NS5 and LST black holes,

$$M = \frac{\mathrm{Vol}(\mathbf{R}^5)\pi \left(\chi r_0^2 + \frac{N}{m_s^2}\right)}{4G^{(10)}} \ . \tag{C.5}$$

[1]For the NS5/LST backgrounds the components of the metric are time-independent at infinity, thus the Komar energy is equivalent to the ADM mass.

Appendix D

Gamma matrices

In this appendix we have followed the excellent notes in [15]. The 11-dimensional gamma matrices satisfy

$$\{\Gamma^a, \Gamma^b\} = 2\eta^{ab} , \tag{D.1}$$

with the relation

$$-i\Gamma_{11} \equiv i\Gamma^0 ... \Gamma^9 = \Gamma^{10} . \tag{D.2}$$

They are purely imaginary, thus

$$\Gamma^{a*} = -\Gamma^a \tag{D.3}$$

and anti-hermitian, except Γ^0,

$$\begin{aligned} &\Gamma^{0\dagger} = \Gamma^0 , \\ &\Gamma^{i\dagger} = -\Gamma^i , \quad i = 1, ..., 10 . \end{aligned} \tag{D.4}$$

Furthermore they are symmetric except Γ^0 which is antisymmetric,

$$\begin{aligned} &\Gamma^{0T} = -\Gamma^0 , \\ &\Gamma^{iT} = \Gamma^i , \quad i = 1, ..., 10 . \end{aligned} \tag{D.5}$$

More properties are

$$\epsilon \Gamma^{a_1 ... a_n} \psi = (-1)^{n + \frac{n}{2}} \psi \left(\Gamma^{a_1 ... a_n}\right)^T , \tag{D.6}$$

which is symmetric for $n = 0, 3, 4, 7, 8$ and antisymmetric for $n = 1, 2, 5, 6, 9, 10$,

$$\left(\epsilon \Gamma^{a_1 ... a_n} \psi\right)^\dagger = (-1)^{\frac{n}{2}} \psi \Gamma^{a_1 ... a_n} \epsilon , \tag{D.7}$$

and finally

$$\Gamma^{a_1\dots a_n} = i\,\frac{(-1)^{1+\frac{n}{2}}}{(11-n)!}\,\epsilon^{a_1\dots a_n b_1\dots b_{11-n}}\Gamma_{b_1\dots b_{11-n}}\ . \tag{D.8}$$

The ten-dimensional spinors and their definitions are the same in eleven dimensions. Moreover one can define the Weyl spinors as

$$\Gamma_{11} = -\Gamma^0\dots\Gamma^9 = i\Gamma^{10}|_{11-\mathrm{dim}}\ , \tag{D.9}$$

which is hermitian $(\Gamma_{11})^2 = 1$. Weyl positive chiral spinors and negative chiral spinors are defined as

$$\Gamma_{11}\psi^{(\pm)} = \pm\psi^{(\pm)}\ . \tag{D.10}$$

In ten dimensions, one can define Majorana-Weyl spinors. A useful representation is that in which the gamma matrices are imaginary and Γ_{11} can be expressed as

$$\Gamma_{11} = \mathbb{I}_{16\times16}\otimes\sigma^3 = \gamma^0 = \begin{pmatrix} \mathbb{I}_{16\times16} & 0 \\ 0 & -\mathbb{I}_{16\times16} \end{pmatrix}\ . \tag{D.11}$$

In the Majorana-Weyl representation every Majorana spinor can be expressed as a direct sum of one positive chiral spinor and one negative chiral spinor of 16 components,

$$\psi = \begin{pmatrix} \psi^{(+)} \\ \psi^{(-)} \end{pmatrix}\ . \tag{D.12}$$

Finally it is fulfilled the identity

$$\Gamma_{11}\Gamma^{a_1\dots a_n} = \frac{(-1)^{1+\frac{(10-n)}{2}}}{(10-n)!}\,\epsilon^{a_1\dots a_n b_1\dots b_{10-n}}\Gamma_{b_1\dots b_{10-n}}\ . \tag{D.13}$$

In four dimensions we are able to use the Majorana or Weyl representations. In a purely imaginary representation (Majorana), the chiral matrix is

$$\gamma_5 = -i\gamma^0\gamma^1\gamma^2\gamma^3 = \frac{i}{4!}\,\epsilon_{abcd}\gamma^{abcd}\ , \tag{D.14}$$

hermitian, imaginary and antisymmetric. The matrix $i\gamma^0$ is real and antisymmetric. The Majorana condition is that the spinors be real $\psi = \psi^*$. Finally we have the identity

$$\gamma^{a_1\dots a_n} = i\frac{(-1)^{\frac{n}{2}}}{(4-n)!}\,\epsilon^{a_1\dots a_n b_1\dots b_{4-n}}\gamma_{b_1\dots b_{4-n}}\gamma_5\ . \tag{D.15}$$

Appendix E

Average number of emitted bosons

In this appendix we have explicitly calculated some expressions of the Section 3.8.1. concerning to the blackbody spectrum of a black hole. We start with a state that describes a system of n virtual pair of particles inside the black hole,

$$|\psi\rangle = N\sum_n |n^L_{in}\rangle \otimes |n^R_{in}\rangle \ . \tag{E.1}$$

We want to write this physical state in terms of the out eigenstates. The reason is that outside of the black hole we can carry out observations. Thus taking into account the relation (3.136) between the modes inside and outside of the black hole, we obtain

$$|\psi\rangle = N\sum_n e^{-\frac{\pi\omega n}{\hbar\kappa}} |n^L_{out}\rangle \otimes |n^R_{out}\rangle \ . \tag{E.2}$$

In order to calculate the normalization constant N we make use of the orthonormalization condition between the orthonormalized states

$$\langle\psi_m|\psi_n\rangle = \delta_{mn} \ . \tag{E.3}$$

Thus considering two states $|\psi_n\rangle$ and $\langle\psi_m|$ we construct

$$\begin{aligned}\langle\psi_m|\psi_n\rangle &= \left(N\sum_m e^{-\frac{\pi\omega m}{\hbar\kappa}} \langle m^L_{out}|\otimes\langle m^R_{out}|\right)\cdot\left(N\sum_n e^{-\frac{\pi\omega n}{\hbar\kappa}} |n^L_{out}\rangle \otimes |n^R_{out}\rangle\right)\\ &= N^2\sum_{m,n} e^{-\frac{\pi\omega(m+n)}{\hbar\kappa}} \langle m^L_{out}|n^L_{out}\rangle \otimes \langle m^R_{out}|n^R_{out}\rangle \ . \end{aligned}\tag{E.4}$$

Then taking into account (E.3) we obtain

$$1 = N^2\sum_n e^{-\frac{2\pi\omega n}{\hbar\kappa}} \ . \tag{E.5}$$

The normalization constant corresponding to bosons ($n = 0, 1, 2, ...$) and fermions ($n = 0, 1$) is respectively

$$N_b = \left(1 - e^{-\frac{2\pi\omega}{\hbar\kappa}}\right)^{\frac{1}{2}} \quad , \quad N_f = \left(1 + e^{-\frac{2\pi\omega}{\hbar\kappa}}\right)^{-\frac{1}{2}} \ . \tag{E.6}$$

Eventually a state associated to a system of bosons inside the black hole can be written as

$$|\psi_b\rangle = \left(1 - e^{-\frac{2\pi\omega}{\hbar\kappa}}\right)^{\frac{1}{2}} \sum_n e^{-\frac{\pi\omega n}{\hbar\kappa}} |n^L_{out}\rangle \otimes |n^R_{out}\rangle \ . \tag{E.7}$$

The density matrix for a boson system is

$$\begin{aligned}
\rho_b &= |\psi_n\rangle\langle\psi_m| \\
&= \left(N_b \sum_n e^{-\frac{\pi\omega n}{\hbar\kappa}} |n^L_{out}\rangle \otimes |n^R_{out}\rangle\right) \cdot \left(N_b \sum_m e^{-\frac{\pi\omega m}{\hbar\kappa}} \langle m^L_{out}| \otimes \langle m^R_{out}|\right) \\
&= \left(1 - e^{-\frac{2\pi\omega}{\hbar\kappa}}\right) \sum_{n,m} e^{-\frac{\pi\omega(n+m)}{\hbar\kappa}} \left(|n^L_{out}\rangle \otimes |n^R_{out}\rangle\right) \cdot \left(\langle m^L_{out}| \otimes \langle m^R_{out}|\right) \\
&= \left(1 - e^{-\frac{2\pi\omega}{\hbar\kappa}}\right) \sum_{n,m} e^{-\frac{\pi\omega(n+m)}{\hbar\kappa}} \left(|n^L_{out}\rangle\langle m^L_{out}|\right) \otimes \left(|n^R_{out}\rangle\langle m^R_{out}|\right) \ . \qquad \text{(E.8)}
\end{aligned}$$

Tracing over the left modes

$$\langle m^L_{out}| \left(|n^L_{out}\rangle\langle m^L_{out}|\right) |n^L_{out}\rangle \ , \tag{E.9}$$

and taking into account the orthonormalization condition (E.3) we obtain

$$\rho^R_b = \left(1 - e^{-\frac{2\pi\omega}{\hbar\kappa}}\right) \sum_n e^{-\frac{2\pi\omega n}{\hbar\kappa}} |n^R_{out}\rangle\langle n^R_{out}| \ . \tag{E.10}$$

This expression corresponds to the density matrix for bosons in terms of the right outgoing modes. This modes will be detected at asymptotic infinity as the Hawking radiation. Finally, we calculate the average number of bosons detected at asymptotic infinity using the equation

$$\begin{aligned}
\langle n_b\rangle &= Tr(n \cdot \rho^R_b) \\
&= \left(1 - e^{-\frac{2\pi\omega}{\hbar\kappa}}\right) \sum_n n \cdot e^{-\frac{2\pi\omega n}{\hbar\kappa}} |n^R_{out}\rangle\langle n^R_{out}| \ . \qquad \text{(E.11)}
\end{aligned}$$

Tracing over the right outgoing modes

$$\langle m^R_{out}| \left(|n^R_{out}\rangle\langle n^R_{out}|\right) |n^R_{out}\rangle \ , \tag{E.12}$$

and taking into account the orthonormalization condition (E.3), we obtain the average number of emitted bosons

$$\langle n_b\rangle = \left(1 - e^{-\frac{2\pi\omega}{\hbar\kappa}}\right) \sum_n n \cdot e^{-\frac{2\pi\omega n}{\hbar\kappa}} = \frac{1}{e^{\frac{2\pi\omega}{\hbar\kappa}} - 1} \ . \tag{E.13}$$

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